

Mouse reflection is equiconsistent with weakly compacts

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ABSTRACT. We show that every uncountable regular cardinal κ satisfying mouse reflection is weakly compact in L .

DEFINITION 0.1. Let κ be a cardinal. Then **mouse reflection** holds at κ , written $\text{MR}(\kappa)$, if every mouse operator F which is total on H_κ is also total on H_{κ^+} . \dashv

THEOREM 0.2 (N.). *Let κ be an uncountable regular cardinal satisfying $\text{MR}(\kappa)$. Then κ is weakly compact in L .*

PROOF. We show that κ has the tree property in L , which by a result of Jensen is equivalent to being weakly compact in L . Let therefore $T \in L$ be a tree of height κ where every level has cardinality $< \kappa$. Define a mouse operator F_T as

$$F_T(x) := L_\gamma, \text{ where } \gamma \text{ is least such that } L_\gamma \models \text{``}T \restriction |x| \text{ has a branch''}.$$

Note that this is indeed a mouse operator as L_γ is a sound and (trivially) countably iterable premouse. Since T has height κ we see that F_T is total on H_κ , so by $\text{MR}(\kappa)$ it's also total on H_{κ^+} . Since regularity of κ implies that $|T| = \kappa$, $F_T(T)$ exists, so that T has a branch in L . \blacksquare