1 Further questions

1.1 Virtually strongs & supercompacts

Question 1.1. Are virtually θ-strong cardinals, virtually θ-supercompacts and virtually θ-supercompacts ala Magidor all equivalent, for any uncountable regular cardinal θ?

1.2 Behaviour in core models

Question 1.2. What happens in larger core models? It seems that in both $L[\mu]$ and K below $0^\P$ we get that generically θ-measurables are equivalent to virtually θ-measurables, but the measurable in $L[\mu]$ is virtually measurable and not virtually κ^{++}-strong. What happens to winning strategies in $\mathcal{G}^\theta_\omega(\kappa)$ then?

1.3 Separation results

Question 1.3. Assume $V = L[\mu]$. Is every virtually θ-measurable cardinal also virtually θ-prestrong?

Question 1.4. Can we find a virtually ∞-measurable which isn't measurable?

1.4 Berkeleys

Question 1.7 in [**?**] asks whether the existence of a non-Σ_2-reflecting *weakly remarkable* cardinal always implies the existence of an ω-Erdős cardinal. Here a weakly remarkable cardinal is a rewording of a virtually prestrong cardinal, and Lemmata 2.5 and 2.8 in the same paper also shows that being

ω-Erdős is equivalent to being virtually club berkeley and that the least such is also the least virtually berkeley.[1]

Furthermore, they also showed that a non-Σ_2-reflecting virtually prestrong cardinal is equivalent to a virtually prestrong cardinal which isn't virtually strong. We can therefore reformulate their question to the following equivalent question.

Question 1.5 (Wilson)**.** If there exists a virtually prestrong cardinal which is not virtually strong, is there then a virtually berkeley cardinal?

[**?**] showed that their question has a positive answer in L, which in particular shows that they are equiconsistent. Applying our Theorem **??** we can ask the following related question, where a positive answer to that question would imply a positive answer to Wilson's question.

Question 1.6. If there exists a cardinal κ which is virtually (θ, ω)-superstrong for arbitrarily large cardinals $\theta > \kappa$, is there then a virtually berkeley cardinal?

Our results above at least gives a partially positive result:

Corollary 1.7 (N.)**.** *If there exists a virtually A-prestrong cardinal for every class A and there are no virtually strong cardinals, then there exists a virtually berkeley cardinal.*

PROOF. The assumption implies by definition that On is virtually prewoodin but not virtually woodin, so Theorem **??** supplies us with the desired. ■

The assumption that there is a virtually A-prestrong cardinal for every class A in the above corollary may seem a bit strong, but Theorem **??** shows that this is necessary, which might lead one to think that the question could have a negative answer.

[1] Note that this also shows that virtually club berkeley cardinals and virtually berkeley cardinals are equiconsistent, which is an open question in the non-virtual context.

1.5 Games

Question 1.8. If κ is generically θ-power-measurable, does player II then have a winning strategy in $\mathcal{G}_\omega^\theta(\kappa)$?

1.6 Ideals

Question 1.9. Is "ω-distributive (κ, κ)-distributive" ideal-absolute? Does it correspond to generically power-measurables?