Dan’s Thesis <3

Great work on the thesis Dr Nielsen. No Danglish to be found. Maths theses are sooo hard to read but I did my best for you baby. I thought you couldn’t spell ‘if’ at first, who knew iff was a word.

Abstract

‘Building on work of Wilson,’

Building on the work of Wilson

‘When the games are finite then we show that this results in a characterisation of the completely ineffable cardinals’

Consider removing then

Introduction

‘One consequence of this is that the property that j(κ) > θ always holds when κ is a θ-strong cardinal with j : V → M being the associated elementary embedding, do not always hold in the virtual world.’

Change do to does, (the property) does not always hold in the virtual world

Another consequence is Corollary 2.55, that “virtual Kunen inconsistencies”, being the existence of virtually rank-into-rank cardinals, happen exactly when we can separate the virtually prestrong cardinals from the virtually strongs.

Add a comma after 2.55

We introduce these and show that in Theorem 2.43 that the virtual Vopˇenka principle implies that On is Mahlo exactly when there are no virtually Berkeley cardinals, improving on a result by Victoria Gitman and Joel Hamkins.

I think this would read better if you put a comma after 2.43 and deleted the second ‘that’ (We introduce these and show that in Theorem 2.43, the virtual Vopˇenka…)

Chapter 1

We then spend some time on large cardinal theory, as it plays a prominent role in understanding how the virtual large cardinals in Chapter 2 compares to the other large cardinals.

Assuming cardinals are plural, should be compare instead of compares

In the interest of brewity

I think you mean brevity

When we’re dealing with elementary embeddings between set-sized structures, we will usually be interested in structures of the following form.

Consider using a colon instead of a full stop

One property of the strong cardinals that we will get back to in the next subsection and which will be important in Chapter 2 is the following.

As above

The core model5 K of a universe is the roughly speaking the subuniverse that strikes a balance between retaining the complexity of the universe while being as simple as possible.

Delete this ‘the’

‘We choose to define the complexity of a universe by its large cardinal structure.’

It didn’t copy over but you wrote complexity with the curly x, was this deliberate?

The famous example of this is the current situation with the superstrong- and strongly compact cardinals, that we simply do not know which one is stronger7

Is this meant to be hyphenated?

The intuitive idea is that every set in the universe can be “approximated” by a set in the subuniverse, and arose from a seminal theorem of Jensen, see [Schindler, 2014, Theorem 11.56], stating that 0 ] exists if and only if strong covering fails for L, defined as follows.

Again, you could use a colon here. At least you are consistent with not using colons.

‘and is still work in progress’

I don’t think this is incorrect, but we almost always say a work in progress

‘The takeaway here is that whenever we’re working with an initial segment of K then that segment will be build using the recursive steps (i) and (ii) above, carefully including extenders from V.’

Build to built

We touched on the weak covering property above, but for completeness we state most of the properties usually employed when working with K, here.

Where? I assume you mean in the thesis but not clear. Can probably just delete the word here.

Then the following are equivalent. / A key folklore lemma which we will frequently need when dealing with elementary embeddings existing in generic extensions is the following.

Colon : )

That’s it!

Chapter 2

As mentioned above, the above proof in fact shows something stronger:

Do you need above twice in this sentence? Maybe ‘As mentioned, the above proof in fact…’ or ‘As mentioned above, the proof in fact…’

Definition 2.8. Let θ be a regular uncountable cardinal and and α an ordinal.

So many ands

This embedding must also be the unique such embedding in all generic extensions,

‘The unique such embedding’ doesn’t seem to read right. Do you mean the embedding must be unique? Or maybe ‘the unique embedding’ as a thing.

Note that the proof of the above lemma shows that we, without loss of generality, may assume that the generic homomorphism in (i) exists in V , which we record here.

Some commas would make this read better, also there’s a space after the V that shouldn’t be there

so that we in V we have homomorphisms

Either ‘so that in V we have homomorphisms’ or ‘so that we in V have homomorphisms’

and we will see in Theorem 2.43 shows that uniformity of this filter is equivalent to there being no virtually Berkeley cardinals  the following lemma is the first implication:

Either ‘and we will see in Theorem 2.43 that uniformity of this filter…’ or ‘and we will see Theorem 2.43 shows that the uniformity of this filter…’

which is the critical point of proper class many πθ,

which is the critical point of proper class of many πθ?

Most of the cardinals turn out to downwards absolute to most inner models, including L.

Most of the cardinals turn out to be downwards absolute to most inner models, including L.

Having proving many positive results about the relations between the virtual large cardinals in the previous sections,

Having proven many positive results…

Starting from much stronger hypothesis

‘Starting from a much stronger hypothesis’ or ‘starting from much stronger hypotheses’

Chapter 3

i.e. that we are simply throwing in the sequences into our models

Delete in (i.e. that we are simply throwing the sequences into our models)

they simply follow the same strategy as in wfGθ γ (κ) but plugs in unions at limit stages.

Plug in unions or plugs in union? Seems like too many plurals, maybe it’s maths talk.

But now the rules of wfGθ γ (κ) ensures that µη ∈ M, so since

‘The rules (of ###) ensure that’ or ’the rule (of ###) ensures that’

Define a strategy for player II in C θ ω (κ) as follows:

The only time you ever use a colon after ‘as follows’. Remove to keep consistency. Actually you use a lot of colons in this chapter. As a chapter, it’s consistent!

Up to 3.1 (p72)

, in the sense that every <κ-sequence (in V) of measure one sets has nonempty intersection.

Sets is plural and intersection is singular, reads a bit strange to me

Genuine- and normal n-Ramseys are downwards absolute to L, for every n < ω.

Should this be hyphenated? If so, then shouldn’t normal also be hyphenated?

Every (n+1)-Ramsey is normal n-Ramsey in L, for every n < ω.

Every (n+1)-Ramsey is a normal n-Ramsey / the plural vs singular thing making my brain hurt

This case is special because, as we cannot ensure that the final measure in G θ γ (κ) is countably complete and so the existence of winning strategies might depend on θ, in contrast with the uncountable cofinality case.

The comma after because seems like an odd place to put a comma.

Maybe rearrange this to ‘This case is special because it is in contrast with the uncountable cofinality case, as we cannot ensure that the final measure in G θ γ (κ) is countably complete and so the existence of winning strategies might depend on θ.’

Or

‘This case is special; we cannot ensure that the final measure in G θ γ (κ) is countably complete and so the existence of winning strategies might depend on θ, and therefore it is in contrast with the uncountable cofinality case.’

Strategic ω-Ramseys are downwards absolute to L, and the existence of a strategic ω-Ramsey cardinal is equiconsistent with the existence of a virtually measurable cardinal. Further, in L the two notions are equivalent

Did you mean to spell existence with a curly x?

and our Propositions 3.12 and 3.40 shows that

Propositions is plural, shows should be show

pidgeon hole principle

Archaic spelling of pigeon / pigeonhole can be one word

In particular, the existence of a strategic (ω+1)-Ramsey cardinal is equiconsistent with the existence of a measurable cardinal.

I’m starting to think that the curly x is deliberate, maths language. But just checking.

In this subsection we define the strongly- and super Ramsey cardinals which were introduced in [Gitman, 2011]

This is correct if ‘strongly-Ramsey’ is hyphenated and ‘super Ramsey’ isn’t, otherwise you need to add or remove hyphens

If furthermore M ≺ Hκ+ then we say that κ is super Ramsey

‘Furthermore, if M < Hk+…’ reads better

Now, a first connection between the α-Ramseys and the strongly- and super Ramseys is the result in [Holy and Schlicht, 2018]

Same as above re. hyphens

If now κ instead was either genuine- or normal α-Ramsey for any limit ordinal

And

Then every genuine- and every normal α-Ramsey cardinal is downwards absolute to K

You can remove the hyphen

Theorem 3.54 (Sharpe-Welch). Let κ be an uncountable cardinal, α > κ, and fix A ⊆ α and a ⊆ κ. Let M := hJα[A], ∈, Ai and m := hJκ[a], ∈, ai. Assume the following:

You can also write fix with a curly x? What!

(iv) There exists a countably complete weakly amenable M-measure µ on κ. Then there exists a good set of indiscernibles I ∈ µ for m.

You’ll have to explain why sometimes you write fix and exists with a normal x and sometimes with a curly one. I’m going to stop pointing it out now, I’m sure it’s deliberate.

The next couple of results concerns the connection between the strategic α-Ramseys and the α-very Ramseys. We start with the following:

Results is plural, should be concern rather than concerns

we change player I’s strategy and let player I play H1 := HullHκ+ (Jκ[A0] ∪ {M0,M1, κ, A0, A1,hH(k) 0 , µ (k) 0 | k ≤ n0i}) ≺ Hκ+ and otherwise continues following some strategy,

Should be continue (let player 1 play X and otherwise continue following some strategy)

Do you mean some strategy or the same strategy?

Chapter 4

|Y ∩ Yα| < κ,

I think there should be a fill stop after k rather than a comma, as the next line is the start of a new sentence with a capital letter.

1. There exists a forcing poset P such that, in V P , there is a weakly amenable V -normal V -measure on κ;
2. There exists a (κ, κ)-distributive forcing poset P such that, in V P , there is a V -normal V -measure on κ;

Did you mean to write exists with a curly x?

I love the poem at the end, I’m happy you put some Danish in the thesis <3

Chapter 5

(I skipped ahead to a short chapter) = Looks good, didn’t spot any errors!