

Virtual large cardinals

EUROPEAN SET THEORY CONFERENCE, VIENNA

Dan Saattrup Nielsen
University of Bristol

July 2019



**ALUMNI
AND FRIENDS**

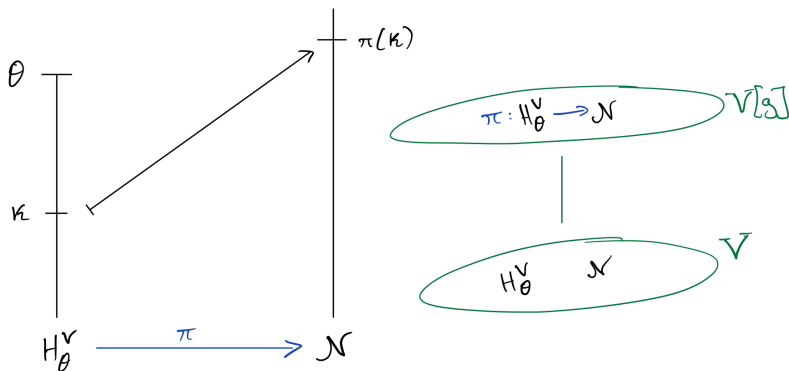


Engineering and Physical Sciences
Research Council

What are they?

Rough definition

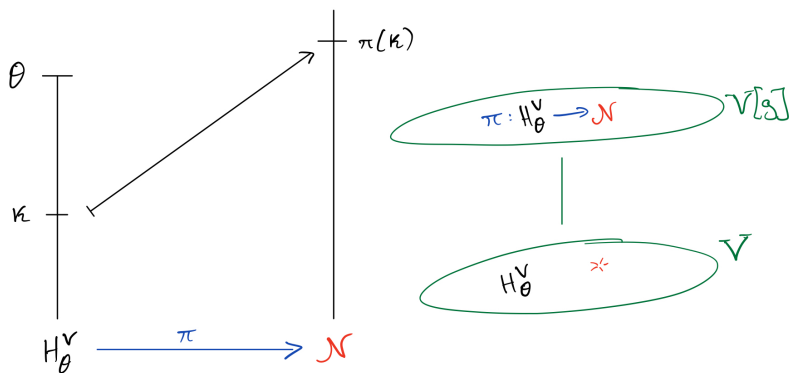
A large cardinal κ defined via *set-sized* elementary embeddings is **virtual** if the elementary embeddings exist in a generic extension.



What are they?

Rough definition

A large cardinal κ defined via *set-sized* elementary embeddings is **generic** if the elementary embeddings **and the target model** exist in a generic extension.



Why should we care?

Theorem (Schindler '00)

A virtually strong cardinal is equiconsistent with $\text{Th}(L(\mathbb{R}))$ being unchangeable by proper forcing.

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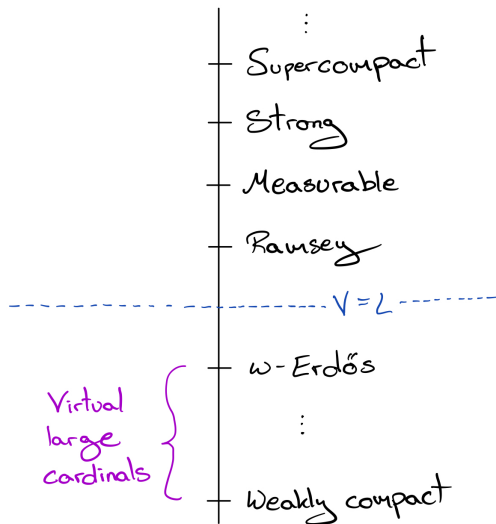
Theorem (Schindler-Wilson '18)

A virtually Shelah cardinal is equiconsistent with every universally Baire set of reals having the perfect set property.

Theorem (Wilson '19)

A virtually Vopěnka cardinal is equiconsistent with $\Theta = \omega_2$ and Σ_2^1 being the class of all ω_1 -Suslin sets.

Where are they?



How do they behave?

Theorem (Gitman)

Virtually strongs are equivalent to virtually supercompacts.

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Theorem (N.)

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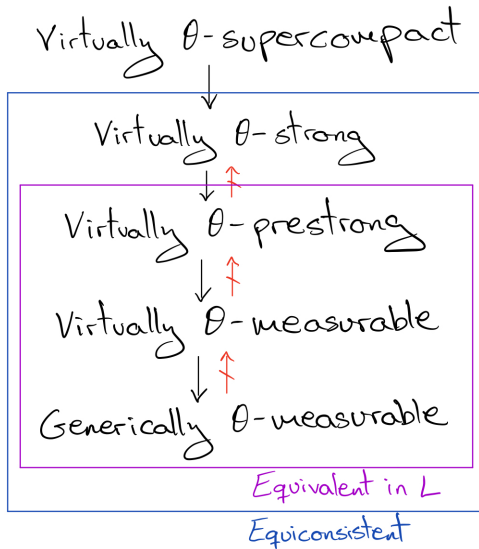
Theorem (N.)

Virtually measurables are equiconsistent with virtually strongs.

Theorem (Schindler-N.)

There exists a game representation for the generically measurables.

How do they behave level by level?



Future work

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- 2 How do virtually Woodin cardinals behave? Vopěnka cardinals?
- 3 Virtualising small embedding cardinals like Ramsey cardinals and below?
- 4 Game characterisations of other generic large cardinals?
- 5 Indestructibility properties?