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To prove C(x, y, p) where x, y : A and $p : x =_A y$ it suffices to prove $C(x, x, refl_x)$ for all x : A

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Suppose C(x,y,p) is a (dependent) type for each x,y:A and $p:x=_Ay$, and suppose there is a function $c:\prod_{x:A}C(x,x,\operatorname{refl}_x)$, then there is a function $f:\prod_{x,y:A}\prod_{p:x=_Ay}C(x,y,p)$.



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- If x : A then there is a special element $refl_x : Id_A(x,x)$ ("unit element")
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Universal property / Recursion principle for $\mathbb N$

Given any type A, and $a_0:A$, and a map $f:A\to A$, we get a unique map $rec_{(a_0,f)}:\mathbb{N}\to A$ such that



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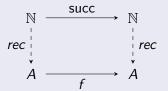
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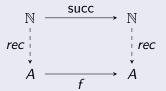
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• We say $\mathbb N$ is "constructed freely" from $0:\mathbb N$ and succ : $\mathbb N\to\mathbb N$

- A new way to construct new types in HoTT is to construct "free types on some generators", called higher inductive types
- Whereas "normal" inductive definitions (eg. \mathbb{N}) use elements and functions, we allow *paths*, i.e. elements of $Id_A(x, y)$.

Example: Interval

The interval I is constructed freely from 0_I : I, 1_I : I and the path seg : $Id_I(0_I,1_I)$

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Higher Inductive definition of the Circle - \mathbb{S}^1

 \mathbb{S}^1 is constructed freely from

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base: \mathbb{S}^1 (element)
loop: Id_{\mathbb{S}^1}(base, base) (path!)
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 \mathbb{S}^1 is the "free ∞ -groupoid" on these generators

- There is already a lot of structure implied by these generators for example
 - $\operatorname{refl}_{\mathsf{base}}: \operatorname{Id}_{\mathbb{S}^1}(\mathsf{base},\mathsf{base})$
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As for \mathbb{N} , since we defined \mathbb{S}^1 freely we get a universal mapping property, for mapping out of \mathbb{S}^1 .

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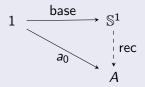
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We can "rank" our types into a hierarchy according to which level is homotopically trivial.

Propositional truncation

Let A be a type. Then the *propositional truncation* of A, written $||A||_{-1}$, is A "reduced to a logical proposition"

The type $||A||_{-1}$ is freely generated by the function $|a|:A\to ||A||_{-1}$ and the paths $\prod_{a,b:A}a=_Ab$.

Writing the usual axiom of choice:

$$\left(\left.\prod_{(X:F)}\left\|\left|\sum_{(x:A(X))}P(x,X)\right|\right|_{-1}\right)\rightarrow \left.\left\|\sum_{(g:\prod_{(X:F)}A(X))}\prod_{(X:F)}P(g(X),X)\right\|_{-1}\right.$$



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The hierarchy of types is divided into certain "h-level" accordingly

- At the −1-level we have types that are either contractible ("have only one point") or empty.
- At the 0-level we have "sets"
- At the 1-level we have ordinary groupoids ...

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- The *loop space* of a pointed type $\langle A, a \rangle$, denoted $\Omega(A, a)$ is the pointed type $\langle Id_A(a, a), refl_a \rangle$
- But $\Omega(A, a)$ is not a group! We need it to be a set first
- Thus $\pi_0(\Omega(A,a))$ is a set, and it can be shown to be a group

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Thank you for listening!

• More info: HomotopyTypeTheory.org