Virtual large cardinals

EUROPEAN SET THEORY CONFERENCE, VIENNA

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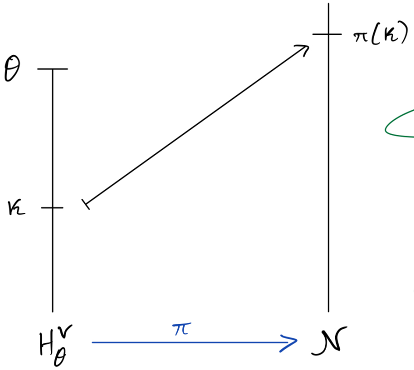
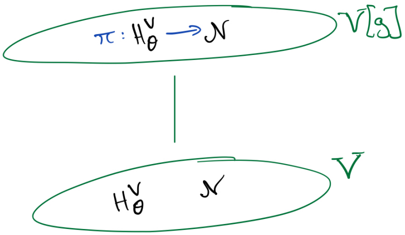
What are they?

Rough definition

A large cardinal κ defined via *set-sized* elementary embeddings is **virtual** if the elementary embeddings exist in a generic extension.

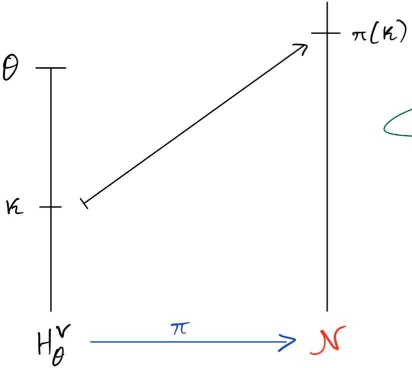


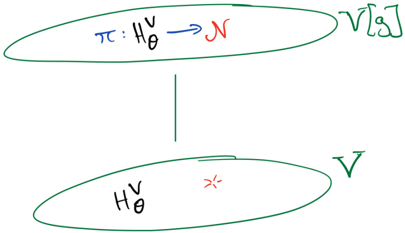


What are they?

Rough definition

A large cardinal κ defined via *set-sized* elementary embeddings is **generic** if the elementary embeddings and the target model exist in a generic extension.





Why should we care?

Theorem (Schindler '00)

A virtually strong cardinal is equiconsistent with $\mathrm{Th}(L(\mathbb{R}))$ being unchangeable by proper forcing.

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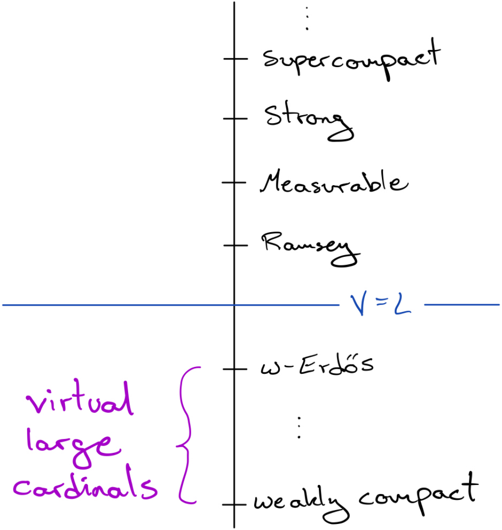
Theorem (Wilson '19)

A virtually Vopěnka cardinal is equiconsistent with $\Theta = \omega_2$ and Σ^1_2 being the class of all ω_1-Suslin sets.

Where are they?



How do they behave?

Theorem (Gitman)

Virtually strongs are equivalent to virtually supercompacts.

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Theorem (N.)

Virtually measurables are equiconsistent with virtually strongs.

How do they behave?

-picture of level-by-level virtuals-