

Home Assignment 1 – Burst Image Deblurring

Burst mode is a modality available in virtually all modern digital cameras. It is a powerful and fun "hidden" feature in the phone native camera apps that allows to continuously capture multiple photos. It is especially useful for capturing the perfect moment when your subjects are moving. It is also a good idea to use it when your hand or camera is moving, in order to give you the best chance of getting a sharp shot with good composition. In addition, this is the main method for catching pictures through videos and performing detection like car detection. Many other applications could be implemented using this mode, such as taking space pictures and etc. Unfortunately, videos and bursts captured with hand-held cameras often suffer from a significant amount of blur, mainly caused by the inevitable natural tremor of the photographer's hand.

In this assignment we are going to simulate a very basic burst mode and a camera motion blur due to the hand movement. We will do so by synthesizing an image burst (sequence) from a single source image.

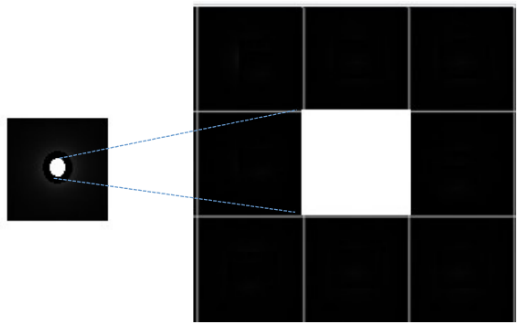
The settings in the assignment will be simplified. We will assume camera movement and hand tremor are all limited to a single plane, which is parallel to the image plane, and that all objects are at the same depth relative to the camera. Our goal is to utilize the blurred image burst to reproduce a single unblurred image, that should be as close as possible to the original one.

Thus we assume the following image setting: a static planar scene parallel to the image sensor (i.e., all objects are at the same depth relative to the camera) being captured in burst mode. More precisely, we assume a rapid sequence of N frames is captured at a frame-rate of $f = 1msec^{-1}$, while the camera moves due to the natural shaking of the hand.

The camera motion is assumed to be only parallel to the image plane (i.e., no off-plane rotation or translation. You can imagine that all the pictures in the burst are taken while the camera is moving on the same flat plane, with the lenses parallel to the image).

The motion trajectory (path) of the camera shake movement is given as the location of the sensor center $o(t)$ at time t in image plane units.

The image sensor has monochrome pixels arranged in an orthogonal grid with a $1\,\mu m$ step in each direction. Each pixel can be assumed to have an ideal box response of size $1\,\mu m \times 1\,\mu m$ (that is, it integrates light in the said box centered around the pixel location).



So if the grid size is 3×3 for example, one pixel will look on the grid as $1\mu m \times 1\mu m$ rectangle, like shown in the figure above.

The camera optics can be assumed to be an ideal anti-aliasing filter, such that the imaged scenes are band-limited according to the sampling rate of the pixels.

The camera has a global shutter that raises instantly to 1 when the exposure starts and drops immediately to 0 when the exposure finishes. The pixel integrates light in this interval. The same integration time $T = 1\, msec$ can be assumed for all frames in the burst. **Imaging noise can be considered negligible.**

1 Theoretical questions

1. Let $f(\mathbf{x})$ be the image that would be formed on the image plane if the camera was absolutely still. The actually formed digital image of the k-th frame can be described as

$$g_k[\mathbf{n}] = (f * h_k)(\mathbf{n}),$$

where $h_k(\mathbf{x})$ is the effective point spread function corresponding to the k-th frame. Express h_k in terms of the camera motion.

2. Express the Fourier transform of h_k.

3. Show that the action of the camera can be expressed fully in the digital domain as

$$G_k[\boldsymbol{\omega}] = F[\boldsymbol{\omega}] \cdot P_k[\boldsymbol{\omega}] \,\mathrm{sinc}\,(\boldsymbol{\omega})\,.$$

Write an expression for the frequency response of the discrete kernel P_k.

4. Write an upper bound on $|P_k[\boldsymbol{\omega}]|$.

5. Assume that on the interval $\tau \in [k, k+1)$, the camera moves with constant velocity v in the horizontal direction, $\mathbf{o}(\tau) = (\tau - k - 0.5)\, v\, \mathbf{e}_1$, where $\mathbf{e}_1 = (1, 0, 0)^{\mathrm{T}}$. Express $P_k[\boldsymbol{\omega}]$ in this case. How does $|P_k[\boldsymbol{\omega}]|$ depend on v?

6. Generalize the previous result to the case where on the interval $\tau \in [k, k+1)$, $\mathbf{o}(\tau) = \mathbf{q} + \tau\, \mathbf{v}$.

7. Suppose that N frames have been acquired such that in the k-th frame the camera was moving with some *unknown* but constant velocity $\mathbf{v}_k$. Given the frames $g_1, \ldots, g_N$ that have been pre-aligned, suggest a way to estimate f.

Notes:

- Please justify nontrivial mathematical transitions in your answers, and especially point out whenever one the imaging assumptions listed above is being used.
- A variety of approaches are possible for tackling question 7. We encourage you to invest time thinking about the problem, and look up related papers online. Please keep your answer concise and use the previous guiding questions and the material learned in class to establish a suitable method.

2 Practical questions

To do this exercise, please download the following materials from here:

- The source image: DIPSourceHW1.jpg
- The file with 100 trajectories (100_motion_paths.mat). Each trajectory k ($k = 0, \ldots, 99$) corresponds to the exposure time of the k^{th} frame and is sampled at 256 points $(X_k(i), Y_k(i) \mid i = 0 \ldots 256)$. Note that all the trajectories are aligned in the sense that every trajectory has a zero mean.

Part I: Forward problem synthesis

- Plot the trajectories resulted by the handshake.
- Generate and display the discrete point spread functions corresponding to each trajectory.
- Generate the blurred frame for each trajectory.

Part II: Solving the inverse problem

Now assume that you do not have the source image, and do not know the PSFs. You only have the blurred images burst that were generated in the previous assignment. Now your mission is to restore the source image - to get a new image that will be as close as possible to the original one.

Your measurement for the quality of your restoration will be the PSNR (peak signal to noise ratio) with respect to the source image (that is assumed known only for PSNR evaluation!).

Use the results of the theoretical part to produce an estimate of the source image given the first k frames in the burst. Plot the PSNR as a function of the number of frames.

3 Deliverables

- **For Part I:** Write the answers for questions 1-7 in a digital file.
- **For Part II:**
  - The code
  - 100 PSF matrices
  - 100 blurred images
  - 100 deblurred images
  - 100 PSNR values

Submit all the above mentioned deliverables by **7th December, 11:55PM**.

Submission will soon be available here.

Thanks and enjoy!
Alex and Omer