Question 3)

a)

Let us denote image histogram by, $h\,(\,I\,)$

Given the mass of the histogram $h\,(\,I\,)$ is 1.
Thus , $\int h\,(\,i\,) = 1$

Thus we can say, $pdf(\,I\,) = h\,(\,I\,)$

Let us split this histogram into 2 histograms, $h_1\,(\,I\,)$ and $h_2\,(\,I\,)$
With intensity intervals as **[0,a]** and **(a,1]** respectively.

Now, let us use the following notations
$Sub^{1,2} : w_0 = \int h_1\,(\,I\,)$ here $I \in$ **[0,a]**
$Sub^{2,2} : w_1 = \int h_2\,(\,I\,)$ here $I \in$ **(a,1]**

We can now calculate the mean value by,
$Sub^{1,2} : \mu_0 = \int (\,h_1\,(\,I\,) * I\,) / w_0$ here $I \in$ **[0,a]**
$Sub^{2,2} : \mu_1 = \int (\,h_2\,(\,I\,) * I\,) / w_1$ here $I \in$ **(a,1]**

Therefore, the mean of the whole image is $\mu = (\,\mu_0 * w_0\,) + (\,\mu_1 * w_1\,)$

b)

When the mean and the median of the histogram are same and equal to **a,** then we can say that,
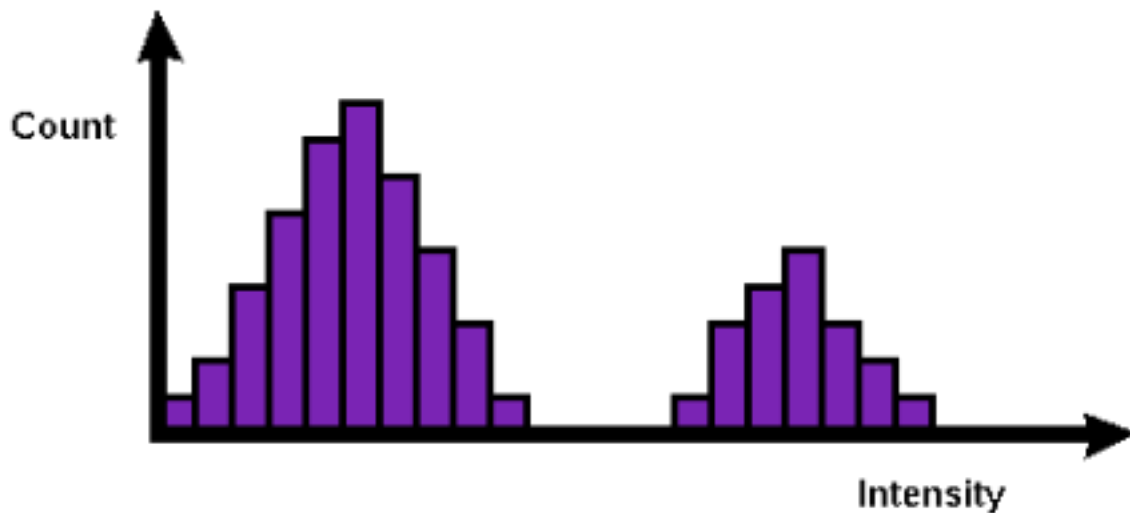$w_0 = \int h_1\,(\,I\,) = w_1 = \int h_2\,(\,I\,)$

But, $w_0 + w_1 = 1$ (since the mass of the orignal histogram = 1)
Thus we have $w_0 = w_1 = 0.5$

Therefore, the mean of the whole image is,
$\mu = (\,\mu_0 + \mu_1\,) / 2 = a$

c)

h

When the distribution of intensity values is as shown above, we can say that simple histogram equalization will not give good results. Insted we will have to use the technique explained in this question to get good contrast enhancement.

We can use **"a"** value as the median of intensity values. Then perform histogram equalization on **[0,a] and (a,1]** individually. By doing this we can individually perform enhancement for each of the 2 groups of intensities, whereas if we do in all together for **[0,1]** we will no longer treat these 2 groups differently and the contrast already present in the image due to these 2 groups will perish.

d)

As we can see above, simple histogram equalization doesn't enhance the background of the square properly. The close range of colors in the background cannot be distinguished easily even after contrast enhancement using histogram equalization, but when we use the modified histogram equalization, we can see a bit better enhancement in the background of this image. This is because the background and foreground undergo equalization individually. Hence contrast of the overall image is not stretched, contrast of individual region is stretched.