Q2)

Ans) (a) $g(x,y) = (h * f)(x,y)$ — ①

Taking fourier transform of both sides

$G(u,v) = H(u,v) \cdot F(u,v)$ — ②

Now, $\hat{F}(u,v) = \dfrac{G(u,v)}{H(u)} = F(u,v)$ — ③

So, $f(x,y) = \mathcal{F}^{-1}(\hat{F}(u,v))$

The fundamental difficulties we will face will be that as $h(x)$ is a convolution kernel to represent gradient, hence $H(u)$ is a high pass filter.

So at low frequency $H(u)$ tends to 0 which causes $\hat{F}(u,v)$ to blowup.

If noise is considered, $H(u)$ blows up amplifying noise in $f(x,y)$

(b) For 2-d gradient,

$g_x(x,y) = (h_x * f)(x,y)$ — ①

$g_y(x,y) = (h_y * f)(x,y)$ — ②

Taking fourier transform of eq ① & ②,

$G_x(u,v) = H_x(u,v)\, F(u,v)$ — ③

$G_y(u,v) = H_y(u,v)\, F(u,v)$ — ④

Using eq ③ & ④

$\hat{F}(u,v) = \dfrac{G_x(u,v)}{H_x(u,v)} = F(u,v)$ — ⑤

$\mathcal{F}(f(x,y)) = \hat{F}(u,v) = \dfrac{G_y(u,v)}{H_y(u,v)} = F(u,v)$ — ⑥

$f(x,y) = \mathcal{F}^{-1}(\hat{F}(u,v))$

Similar to the previous part, at low frequency $H_x(u,v) = H_y(u,v) \to 0$ and $\hat{F}(u,v)$ blows up.

To estimate good values of $\hat{F}(u,v)$ we can use eq ⑤ & ⑥ for appropriate boundary conditions.

$h_x(x,y)$ is gradient in x direction while $h_y(x,y)$ is gradient in y direction. $H_x(u,v)$ is a high pass filter in u while $H_y(u,v)$ is high pass filter in v.

(i) When ~~u = 0~~ u is small

Use equation (6) to estimate $f(x,y)$.

(ii) When ~~v = 0~~ v is small

Use eq. (5).

(iii) When u & v ~~= 0~~ are small.

We cannot estimate $f(x,y)$ accurately as $\hat{F}(x,y)$ blows up in both eq.