Q5)

We have to maximise $f^T Cf$ given f is a unit vector.

To solve, J = $\sum_{i=1}^{N} \|x_i - xbar - (e(x_i - xbar))e\|_2$ is minimum.

J = $\sum_{i=1}^{N} -(e^t(x_i - xbar))^2 + \sum_{i=1}^{N} \|x_i - xbar\|_2 = -e^t Se + \sum_{i=1}^{N} \|x_i - xbar\|_2$ S = (N-1)C

Minimizing J wrt e is similar to maximizing $e^t Se$ wrt e as other term does not have e.

Also $e^t e = 1$. So new equation is,

Z = $e^t Se - \lambda(e^t e - 1)$

Derivative wrt to e and setting to 0 gives,

Se = λ e

This implies that e is an eigen vector of S and $e^t Se$ gives the eigen value of S.

For maximizing $e^t Ce$, eigen-vector corresponding to highest eigen value is selected.

Similarly for maximizing $f^t Cf$, eigen-vector corresponding to second highest eigen value is selected as e is not equal to f.

Since f and e are the eigen-vector of C.

Ce = λ_1 e ….(1)

Cf = λ_2 f ….(2)

Multiply eq 1 by f^t and eq2 by e^t.

$f^t Ce = f^t \lambda_1 e$

$e^t Cf = e^t \lambda_2 f$

Since c is a symmetric matrix, $f^t Ce = (f^t Ce)^t = e^t Cf$.

So, $f^t \lambda_1 e = \lambda_2 e^t f$.

Since $\lambda_1 \neq \lambda_2$, $e^t f$ = 0.

Hence, f and e are orthogonal.