Q1)

Ans) Given, $g_1 = f_1 + h_2 * f_2$ — (1)

$g_2 = f_2 + h_1 * f_1$ — (2)

Taking fourier transform of both sides.

$G_1(u,v) = F_1(u,v) + H_2(u,v) \cdot F_2(u,v)$ — (3)

$G_2(u,v) = H_1(u,v) \cdot F_1(u,v) + F_2(u,v)$ — (4)

Let, $G_1(u,v) = G_1$, $F_1(u,v) = F_1$, $F_2(u,v) = F_2$,

$H_1(u,v) = H_1$, $H_2(u,v) = H_2$

Equation (3) & (4) now become,

$G_1 = F_1 + H_2 \cdot F_2$ — (5)

$G_2 = H_1 F_1 + F_2$ — (6)

Substituting value of F_1 from eq. (5) into (6) we get,

$G_2 = H_1(G_1 - H_2 F_2) + F_2$

$\Rightarrow \hat{F}_2 = \dfrac{G_2 - H_1 G_1}{(1 - H_1 H_2)} = F_2$ — (7)

Taking inverse fourier transform

$F^{-1}(F_2) = F^{-1}\left(\dfrac{G_2 - H_1 G_1}{(1 - H_1 H_2)}\right)$ — (8)

$f_2(x,y) = F^{-1}\left(\dfrac{G_2 - H_1 G_1}{(1 - H_1 H_2)}\right)$ — (9)

Similarly,

$f_1(x,y) = F^{-1}\left(\dfrac{G_1 - H_2 G_2}{(1 - H_2 H_1)}\right)$ — (10)

The problem over here is that h_1 & h_2 are blur kernels, hence H_1 & H_2 are low pass filters. So at low frequency, both tend to one. Which in turn implies that $1 - H_1 H_2 \simeq 0$, so $\hat{F}_1$ & $\hat{F}_2$ blow up. Hence, we cannot use this formula for low frequency components.