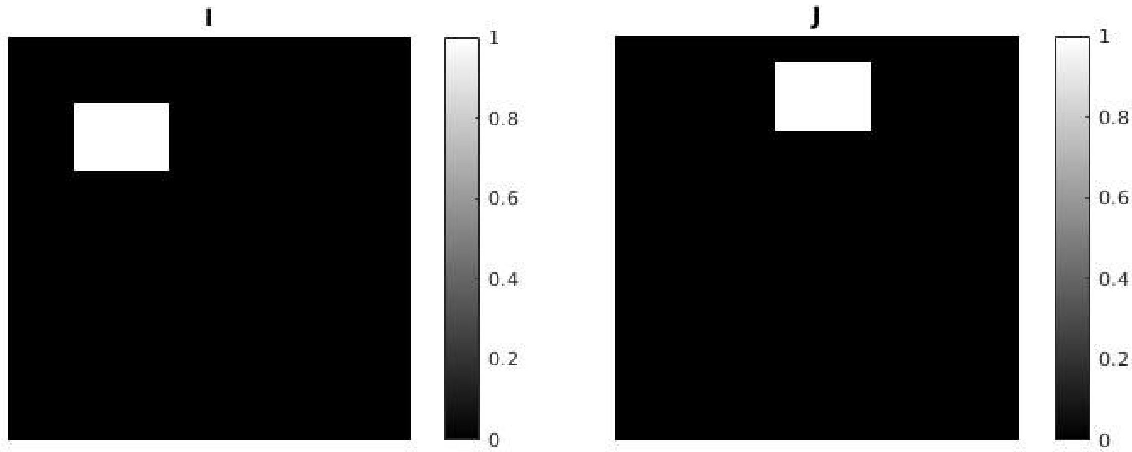
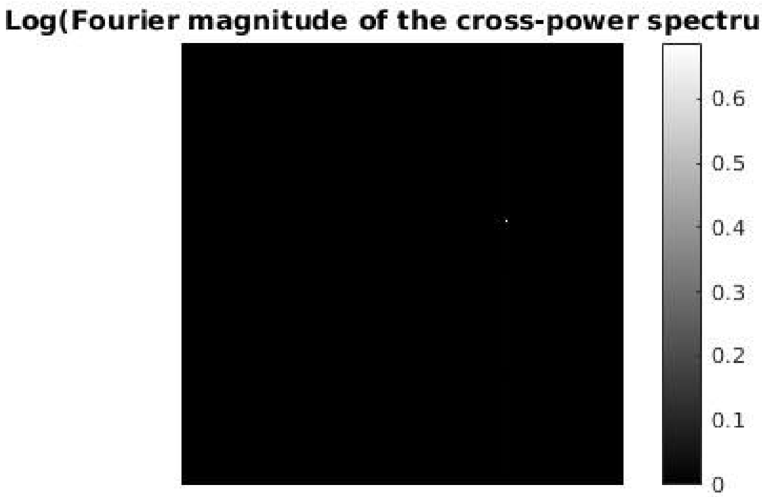
Q6)

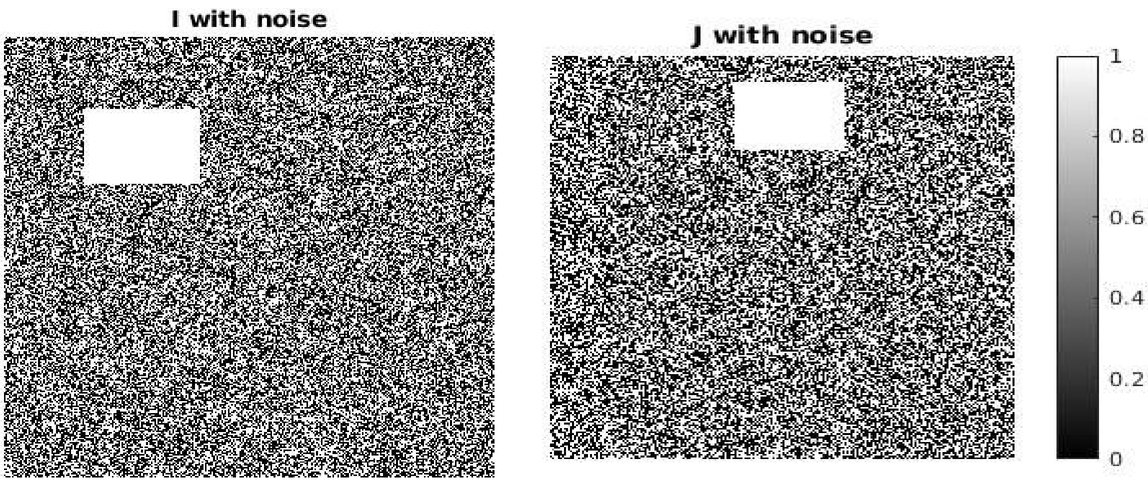



Cross spectrum:-

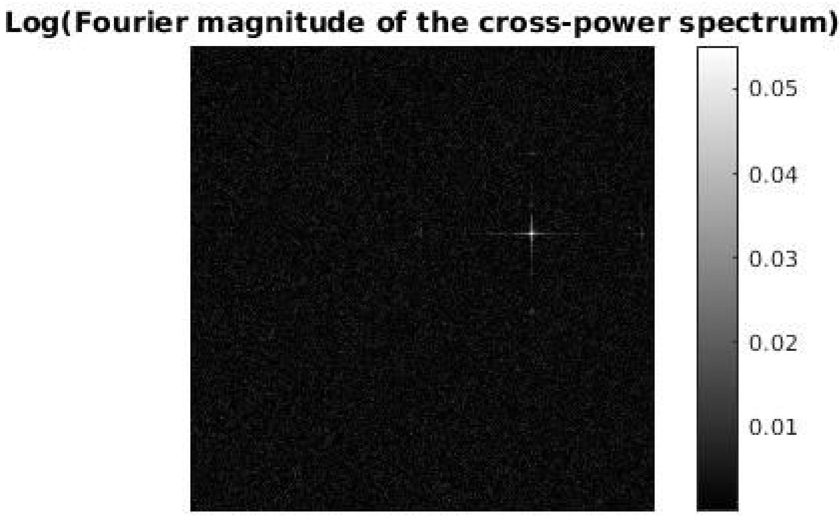



The maximum value is at [121,221]
Due to wraparound effect the maximum is observed at [121,221].
The translation is = [121-150, 221-150] = [-29,71] which is approximately the same as the translation applied.

With gaussian noise:-




Cross spectrum:-

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If the images were of size (N*N) the time required is of the order $O(N^2\log(N))$ .
For pixel wise comparison the order is $O(N^4)$. The time required is much more than what is taken by cross power spectrum.

For rotation angle:-

$f_2$ is rotated version of $f_1(x,y)$ with rotation $\theta_0$.

$$f_2(x,y) = f_1(x\cos\theta_0 + y\sin\theta_0, -x\sin\theta_0 + y\cos\theta_0)$$

Taking Fourier transform,

$$F_2(\xi,\eta) = F_1(\xi\cos\theta_0 + \eta\sin\theta_0, -\xi\sin\theta_0 + \eta\cos\theta_0)$$

Let $M_1$ & $M_2$ be magnitude of $F_1$ & $F_2$

$$M_2(\xi,\eta) = M_1(\xi\cos\theta_0 + \eta\sin\theta_0, -\xi\sin\theta_0 + \eta\cos\theta_0)$$

Using polar coordinates,

$$M_1(\rho,\theta) = M_2(\rho, \theta-\theta_0)$$

Now, again taking fourier transform, we get

$$F(M_1(\rho,\theta)) = e^{-j2\pi(u\theta_0)} F(M_2(\rho,\theta))$$

Using cross power spectrum,

Let $F(M_1) = F$ and $F(M_2) = F'$

$$e^{j2\pi\theta} = \frac{F \cdot F'^*}{|F \cdot F'|}$$

Taking inverse fourier transform, we get an impulse at $(0, \theta_0)$