Question 2)

a) For computation of imaging matrix the following method is used:-

Ax = b where A = imaging matrix,

x = original image matrix in column form

b = radon transform obtained using radon function

We define x = USV' where U and V are orthogonal matrices.

For orthogonal matrices, $UU^T = I$.

$$Axx^T = bx^T$$

$$A = (bx^T) * (xx^T)^{-1} = (bVSU^T) * (USV^T VSU^T)^{-1}$$

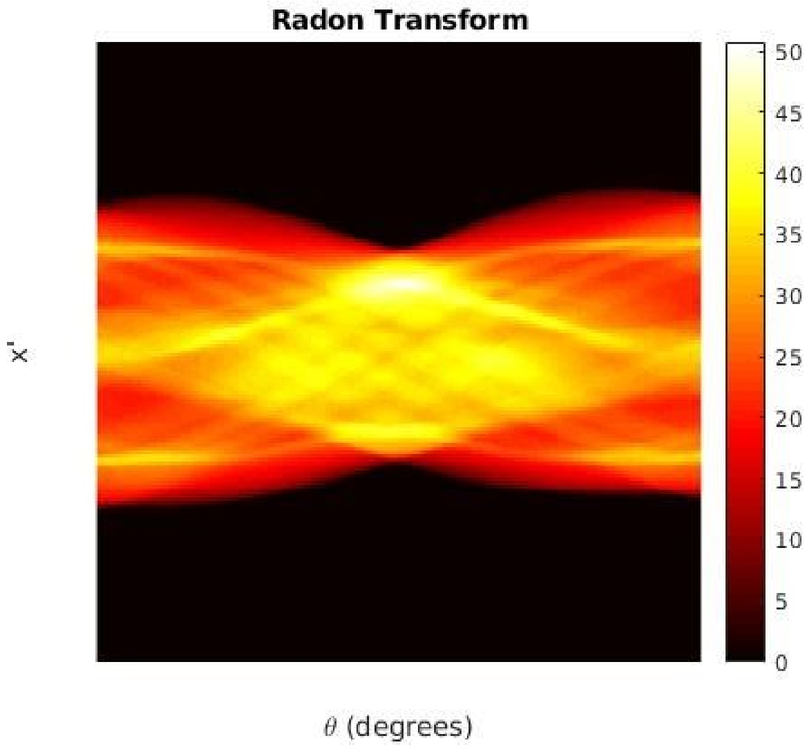
$$A = (bVSU^T) * (US^2U^T)^{-1} = (bVSU^T) * ((U^T)^{-1}S^{-2}U^{-1})$$

$$A = (bVSU^T) * (US^{-2}U^T) = b(VS\,U^T US^{-2}U^T)$$

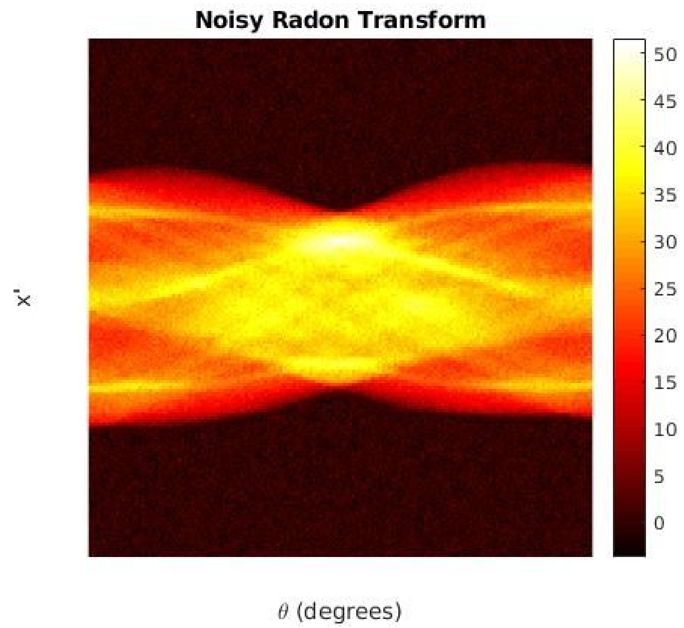
$$A = b(VS^{-1}U^T)$$

This technique speeds up the calculation of A matrix.

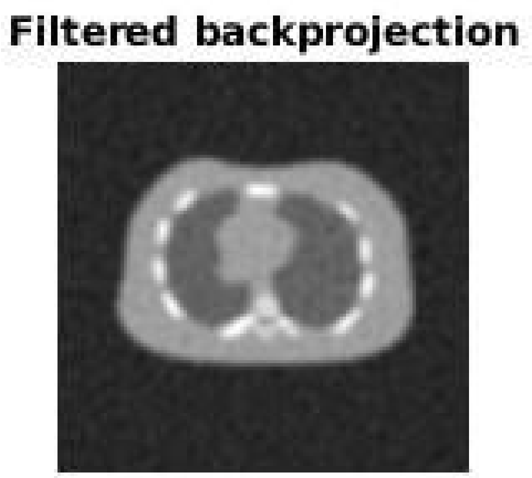
Radon transform of the image is:-



b) The sinogram data for the noisy image.



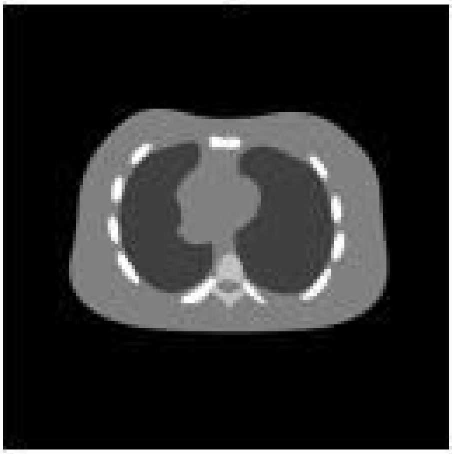
c) Shepp's filter is used for filtered backprojection of the noisy sinogram image.



RRMSE of the given image with the original image is 0.1789.

d) Using the tikhonov regularisation with gradient descent,
grad = A'*(A*x - sinogram)+lambda*x
x = x - grad*n
Final image:-

Tikhonov regularisation

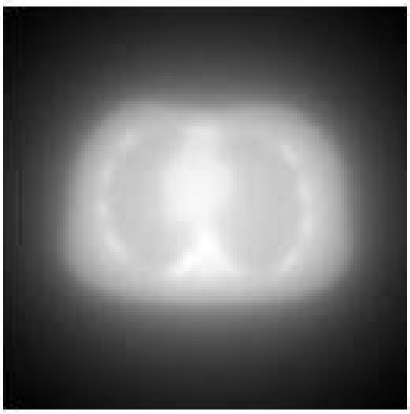


For lambda = 2 RRMSE = 2.1479e-04
lambda = 2*1.2 RRMSE = 2.5419e-04
lambda = 2*0.8 RRMSE = 2.2087e-04

For learning rate n = 0.01 RRMSE = 2.1479e-04
n = 0.01*1.2 RRMSE = 2.2006e-04
n = 0.01*0.8 RRMSE = 2.4883e-04

e) Using quadratic prior:-
Denoised Image obtained is:-

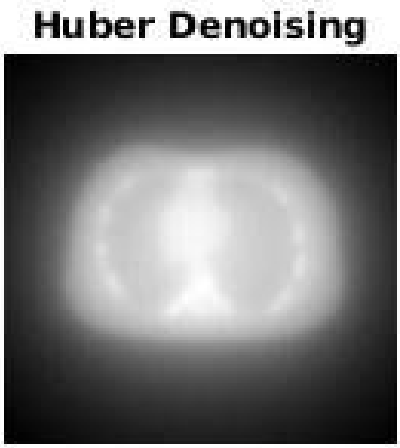
Quadratic denoising



For alpha = 0.003 RRMSE = 1.5532
alpha = 0.003*1.2 RRMSE = 1.5533
alpha = 0.003*0.8 RRMSE = 1.5532

Using huber prior:-

Denoised Image obtained is:-



For alpha = 0.0001 RRMSE = 1.5532
alpha = 0.0001*1.2 RRMSE = 1.5534
alpha = 0.0001*0.8 RRMSE = 1.5532

For gamma = 0.001 RRMSE = 1.5532
gamma = 0.001*1.2 RRMSE = 1.5532
gamma = 0.001*0.8 RRMSE = 1.5533

Using discontinuity adaptive prior:-

Denoised Image obtained is:-

For alpha = 0.0002 RRMSE = 1.5532
alpha = 0.0002*1.2 RRMSE = 1.5532
alpha = 0.0002*0.8 RRMSE = 1.5532

For gamma = 0.00225	RRMSE = 1.5532
gamma = 0.00225*1.2	RRMSE = 1.5533
gamma = 0.00225*0.8	RRMSE = 1.5533