

Example of AdaBoosting

The training data:

index:	0	1	2	3	4	5	6	7	8	9
x value:	0	1	2	3	4	5	6	7	8	9
y value:	1	1	1	-1	-1	-1	1	1	1	-1

The weak learner produces hypotheses of the form: $x < v$, or $x > v$. The threshold v is determined to minimize the probability of error over the entire data. (No sampling.)

Running the algorithm.

We start with the following probabilities:

p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

$t = 1$. The best threshold is between 2 and 3.

$$h_1(x) = I(x < 2.5)$$

$$\epsilon_1 = 0.3$$

$$\alpha_1 = 0.423649$$

$$q_i = 1.52753 \text{ for wrong, } 0.654654 \text{ for right}$$

Updating the probabilities:

index:	0	1	2	3	4	5	6	7	8	9
correct:	y	y	y	y	y	y	n	n	n	y
old p_i	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
pre-normalized p_i	.06547	.06547	.06547	.06547	.06547	.06547	.15	.15	.15	.06547
$Z_1 = 0.916515$										
new p_i	.07143	.07143	.07143	.07143	.07143	.07143	.16667	.16667	.16667	.07143

$$f_1(x) = 0.423649 I(x < 2.5), \quad 3 \text{ mistakes}$$

$t = 2$. Now a threshold between 2 and 3 gives error of 0.5, the threshold between 5 and 6 gives 0.28, the threshold between 8 and 9 gives 0.214, best.

$$h_2(x) = I(x < 8.5)$$

$$\epsilon_2 = 0.214$$

$$\alpha_2 = 0.6496$$

Updating the probabilities:

index:	0	1	2	3	4	5	6	7	8	9
correct:	y	y	y	n	n	n	y	y	y	y
pre-normalized p_i	.037	.037	.037	.137	.137	.137	.087	.087	.087	.037
$Z_2 = 0.82$										
new p_i	.045	.045	.045	.167	.167	.167	.106	.106	.106	.045

$$f_2(x) = 0.423649 \, I(x < 2.5) + 0.6496 \, I(x < 8.5), \quad 3 \text{ mistakes}$$

$t = 3$. The best threshold is between 5 and 6.

$$\begin{aligned} h_3(x) &= I(x > 5.5) \\ \epsilon_3 &= 0.1818 \\ \alpha_3 &= 0.7520 \end{aligned}$$

Updating the probabilities:

index:	0	1	2	3	4	5	6	7	8	9
correct:	n	n	n	y	y	y	y	y	y	n
pre-normalized p_i	.0964	.0964	.0964	.078	.078	.078	.05	.05	.05	.0964
$Z_3 = 0.77139$										
new p_i	.125	.125	.125	.102	.102	.102	.064	.064	.064	.125

$$f_3(x) = 0.423649 \, I(x < 2.5) + 0.6496 \, I(x < 8.5) + 0.752 \, I(x > 5.5), \quad 0 \text{ mistakes}$$