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Q/svm05.stex
% q06 ho07 ho08 h08f h09 h09f h11 ho11f ho12 ho12f ho13s ho13f ho14 ho14s ho14f ho15 ho16 ho17 ho18
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An SVM is trained with the following data:

i	1	2	3
x_i	$(-1,-1)$	$(1,1)$	$(0,2)$
y_i	-1	1	1

Let $\alpha_1, \alpha_2, \alpha_3$ be the Lagrangian multipliers associated with this data. (α_i is associated with (x_i, y_i).)

A

Using the polynomial kernel of degree 2, what (dual) optimization problem needs to be solved in terms of the α_i in order to determine their values?

Reminder: the polynomial kernel of degree 2 is:

$$K(x_i, x_j) = (x_i' x_j + 1)^2$$

Answer

$$\text{The Gram matrix for the linear kernel:} \quad G = \begin{pmatrix} 2 & -2 & -2 \\ -2 & 2 & 2 \\ -2 & 2 & 4 \end{pmatrix}$$

$$\text{The Gram matrix for the specified kernel:} \quad G = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 9 \\ 1 & 9 & 25 \end{pmatrix}$$

$$\begin{aligned} \text{Maximize:} \quad & \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2}\left(9\alpha_1^2 - 2\alpha_1\alpha_2 - 2\alpha_1\alpha_3 + 9\alpha_2^2 + 18\alpha_2\alpha_3 + 25\alpha_3^2\right) \\ \text{subject to:} \quad & \alpha_1 \geq 0, \ \alpha_2 \geq 0, \ \alpha_3 \geq 0, \quad -\alpha_1 + \alpha_2 + \alpha_3 = 0 \end{aligned}$$

B

The solution to the optimization problem is:

$$\alpha_1 = 1/8, \quad \alpha_2 = 1/8, \quad \alpha_3 = 0$$

a. What are the indexes of the support vectors? Circle them below.

Answer: 1 2

b. This SVM classifies the example x according to the sign of $w'\phi(x) + b$, where the transformation ϕ is implicitly defined by the kernel. Compute the value of the constant b. (This can be done without explicit computation of ϕ or w.)

Answer: Using the first support vector:

$$b = -1 - (1 * (-1) * 9 + 1 * 1 * 1)/8 = -1 - (-9 + 8/8)/8 = 0$$

c. What computation needs to be carried out to determine the classification of the point $x = (-1, 0)$ by this SVM?

Answer: $K(x_j, x) = (4, 0)$.

$$-\frac{1}{8}(4) + \frac{1}{8}(0) < 0$$

Therefore the classification of x is -1.

d. What computation needs to be carried out to determine the classification of the point $x = (1, 0)$ by this SVM?

Answer: $K(x_j, x) = (0, 4)$.

$$-\frac{1}{8}(0) + \frac{1}{8}(4) > 0$$

Therefore the classification of x is $+1$.