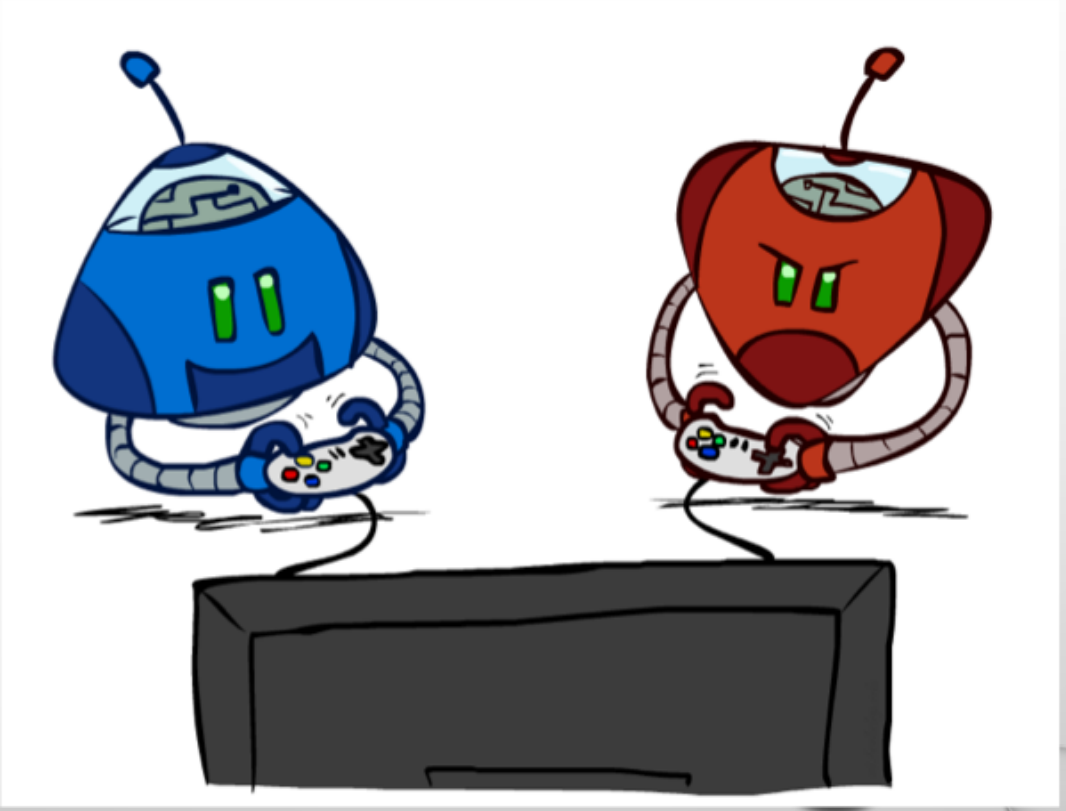
# Artificial Intelligence
# CE-417, Group 2
# Computer Eng. Department
# Sharif University of Technology

Fall 2020

By Mohammad Hossein Rohban, Ph.D.

Courtesy: Most slides are adopted from CSE-573 (Washington U.), original slides for the textbook, and CS-188 (UC. Berkeley).

# Adversarial Search Methods

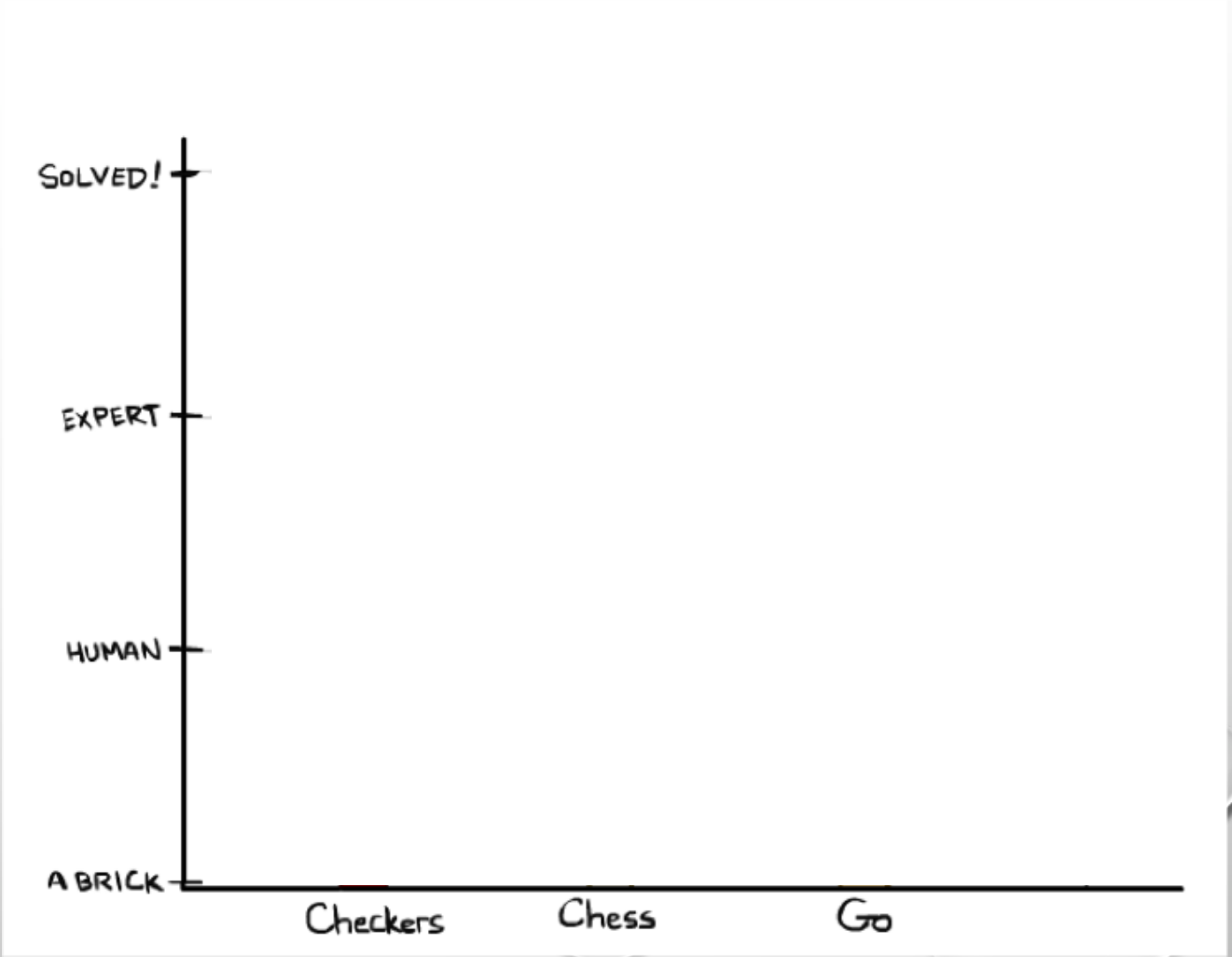


# Game Playing State-of-the-Art

- **Checkers:** 1950: first computer player. 1994: first computer champion: chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: checkers solved!

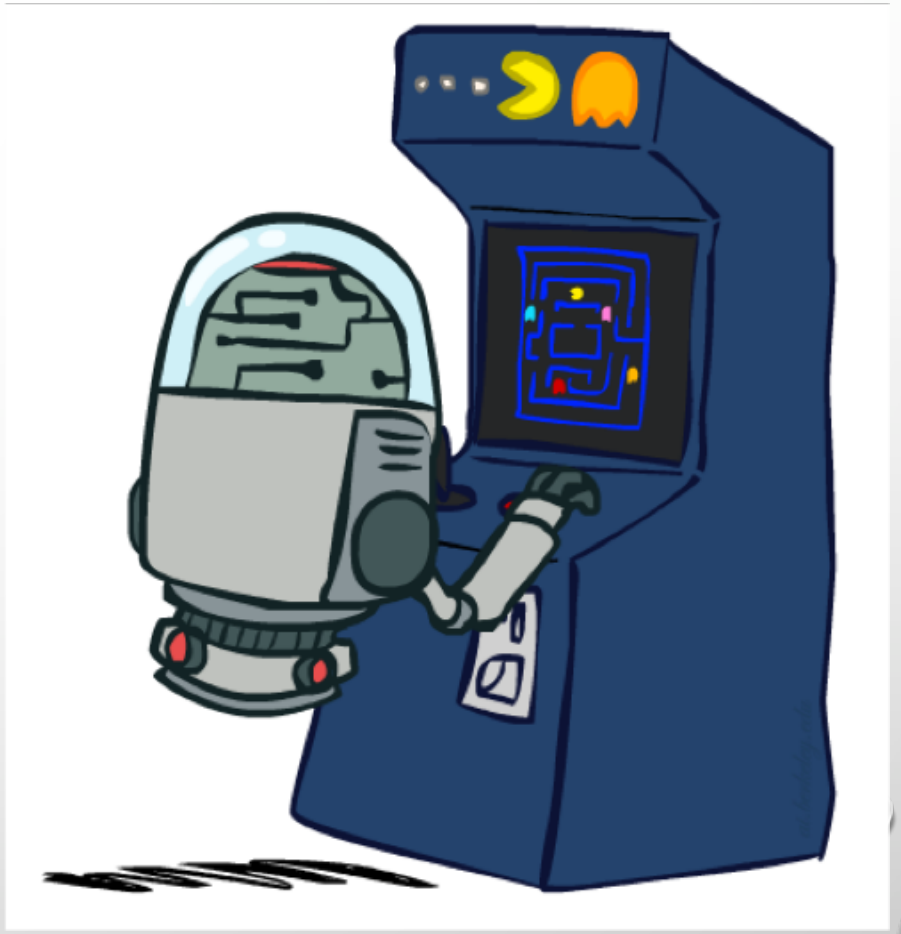
- **Chess:** 1997: deep blue defeats human champion Gary Kasparov in a six-game match. Deep blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go:** human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use monte carlo (randomized) expansion methods.



# Deterministic games

- Many possible formalizations, one is:
  - States: S (start at $s_0$)
  - Players: P={1, … , N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition function: $S \times A \rightarrow S$
  - Terminal test: $S \rightarrow \{T, F\}$
  - Terminal utilities: $S \times P \rightarrow R$

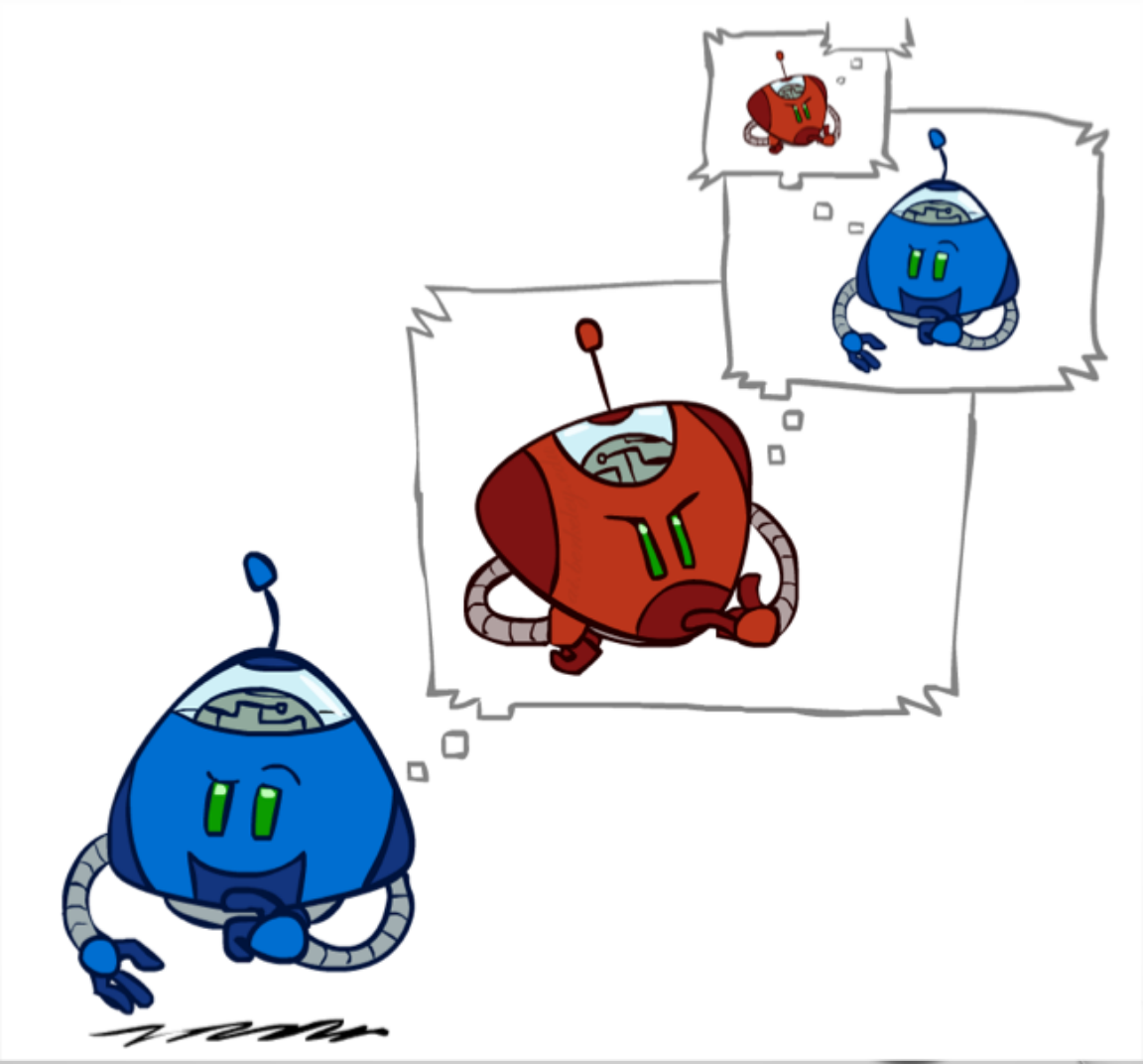


- Solution for a player is a policy: $S \rightarrow A$

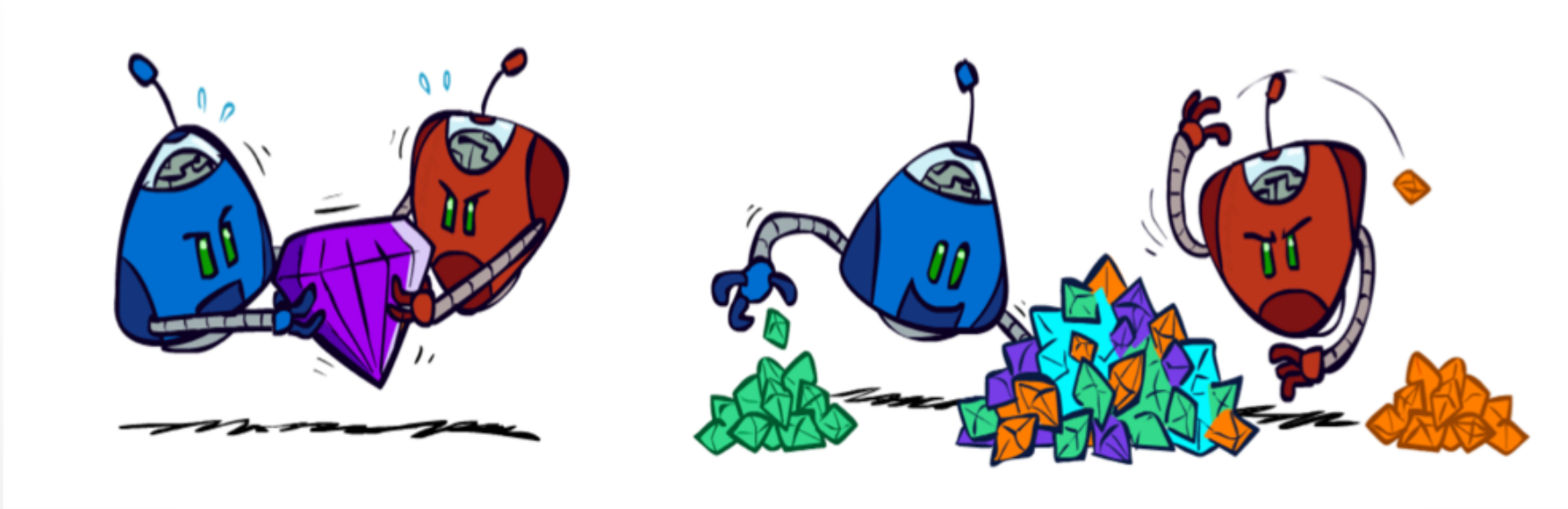
# Games vs. search problems

- "Unpredictable" opponent ⇒ solution is a **strategy** specifying a move for every possible state/opponent reply
- Time limits ⇒ unlikely to find goal, must approximate

# Adversarial Search

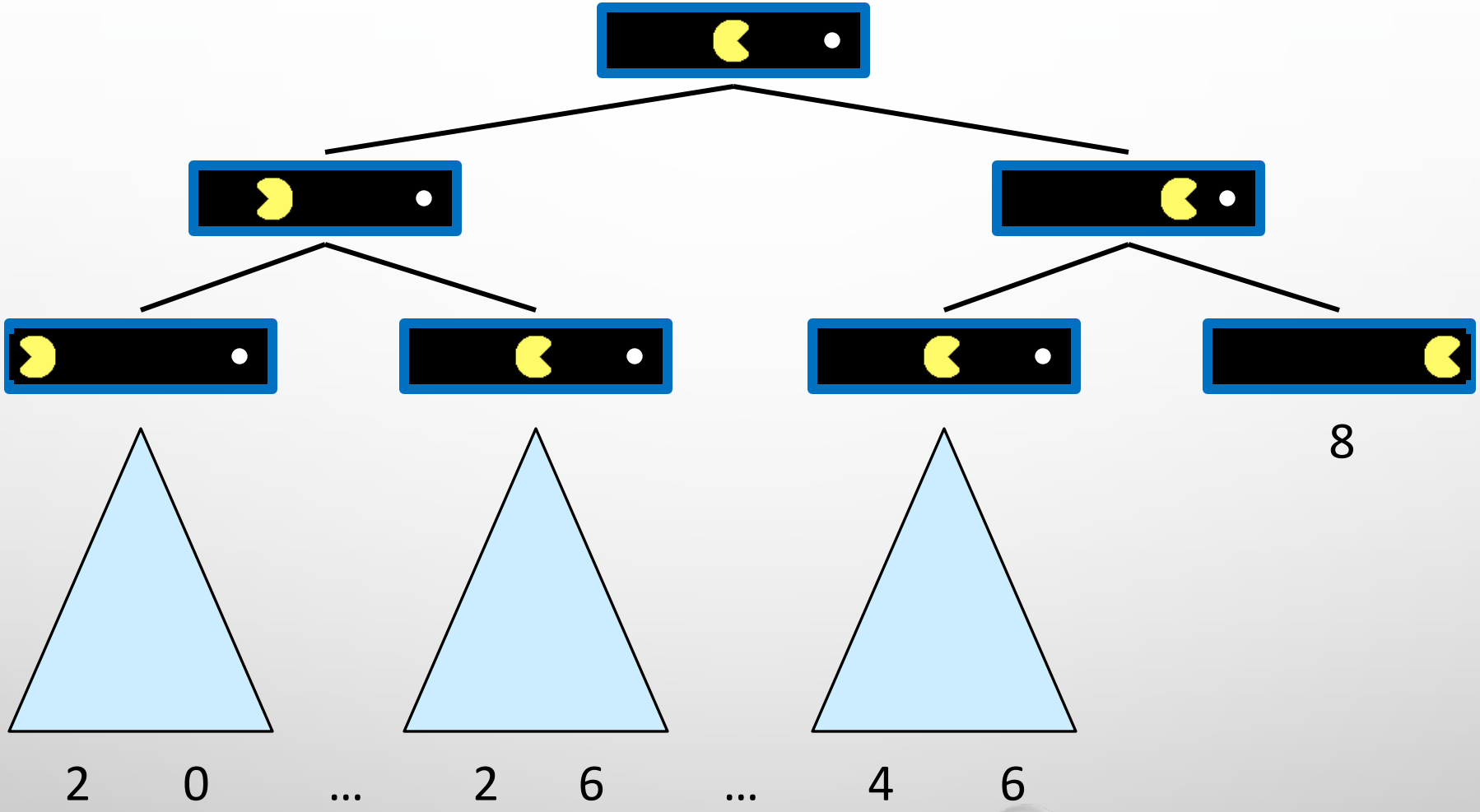


# Zero-Sum Games



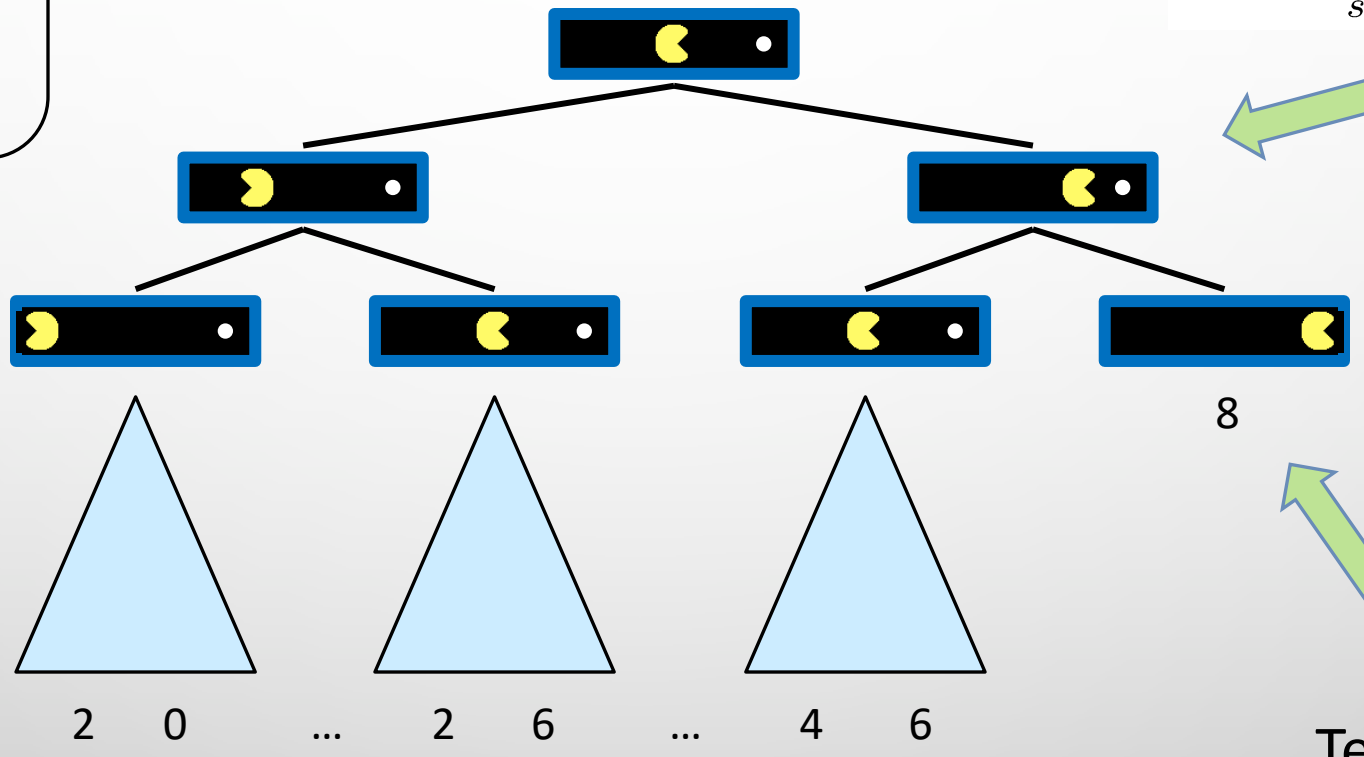
- **Zero-Sum Games**
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- **General Games**
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, & more are possible
  - More later on non-zero-sum games

# Single-Agent Trees



# Value of a State

Value of a state:
The best achievable outcome (utility) from that state



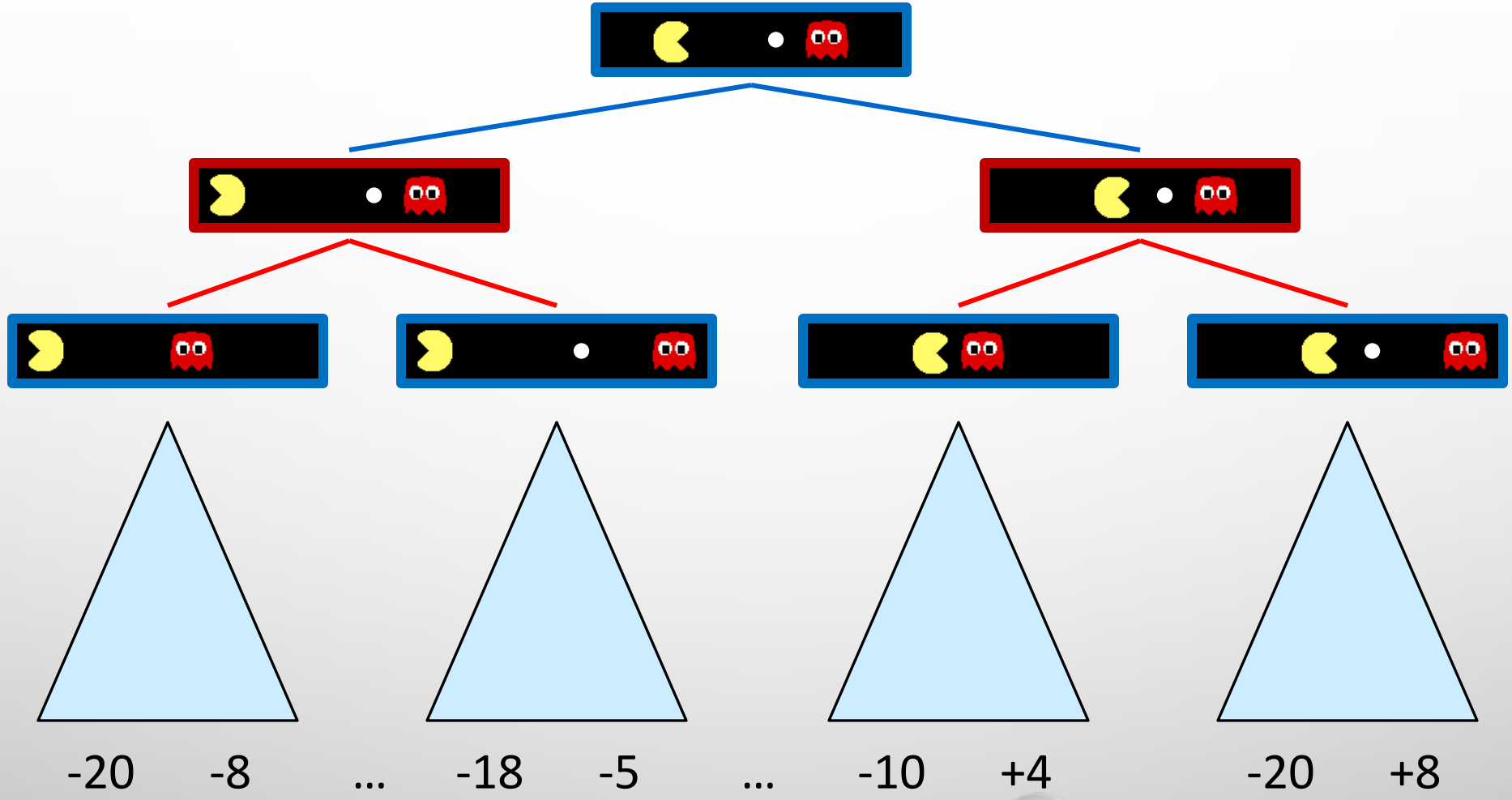

Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Terminal States:

$$V(s) = \text{known}$$

# Adversarial Game Trees



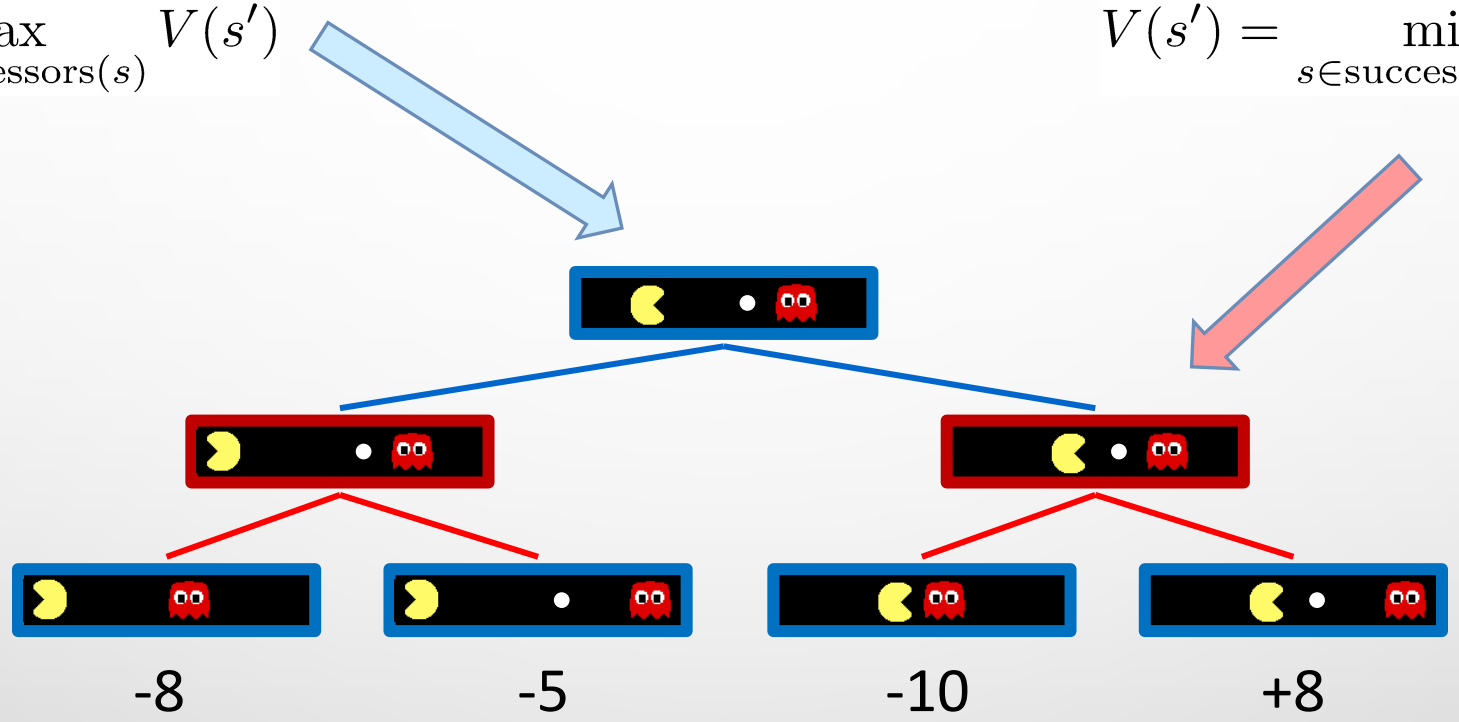
# Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



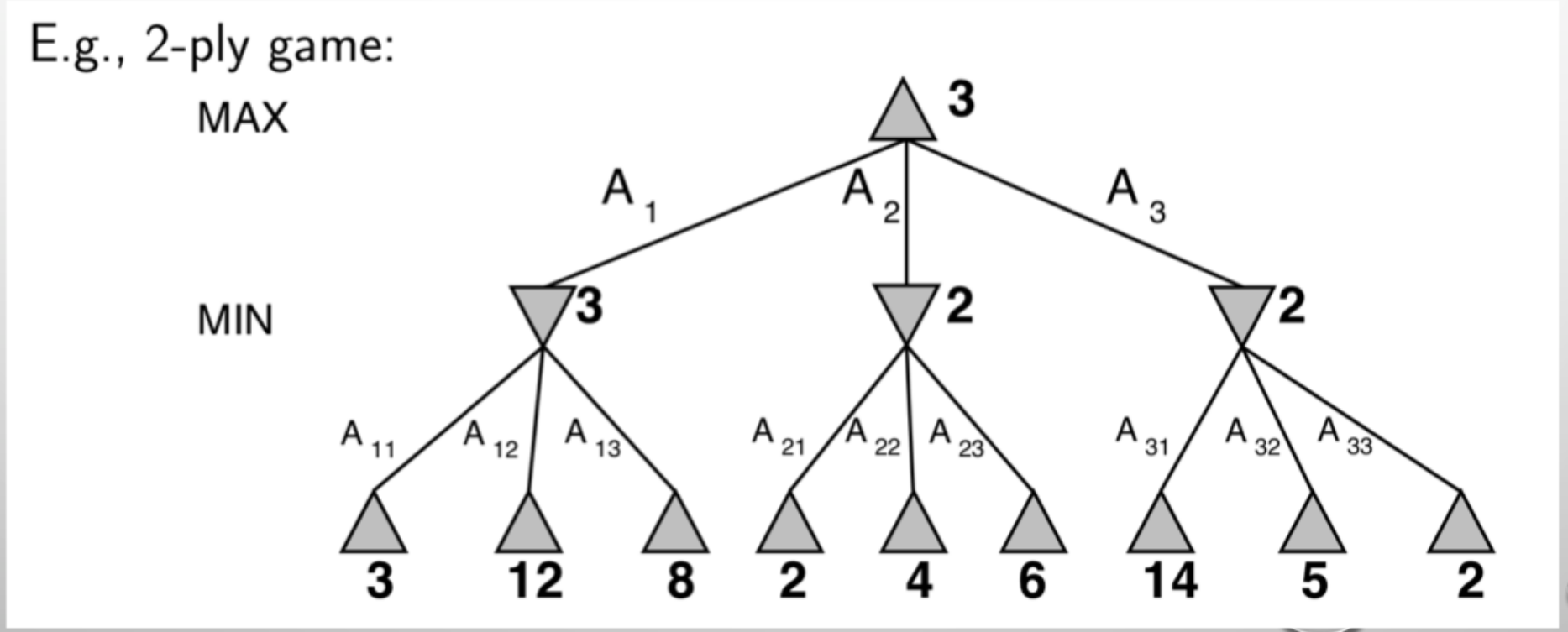

Terminal States:

$$V(s) = \text{known}$$

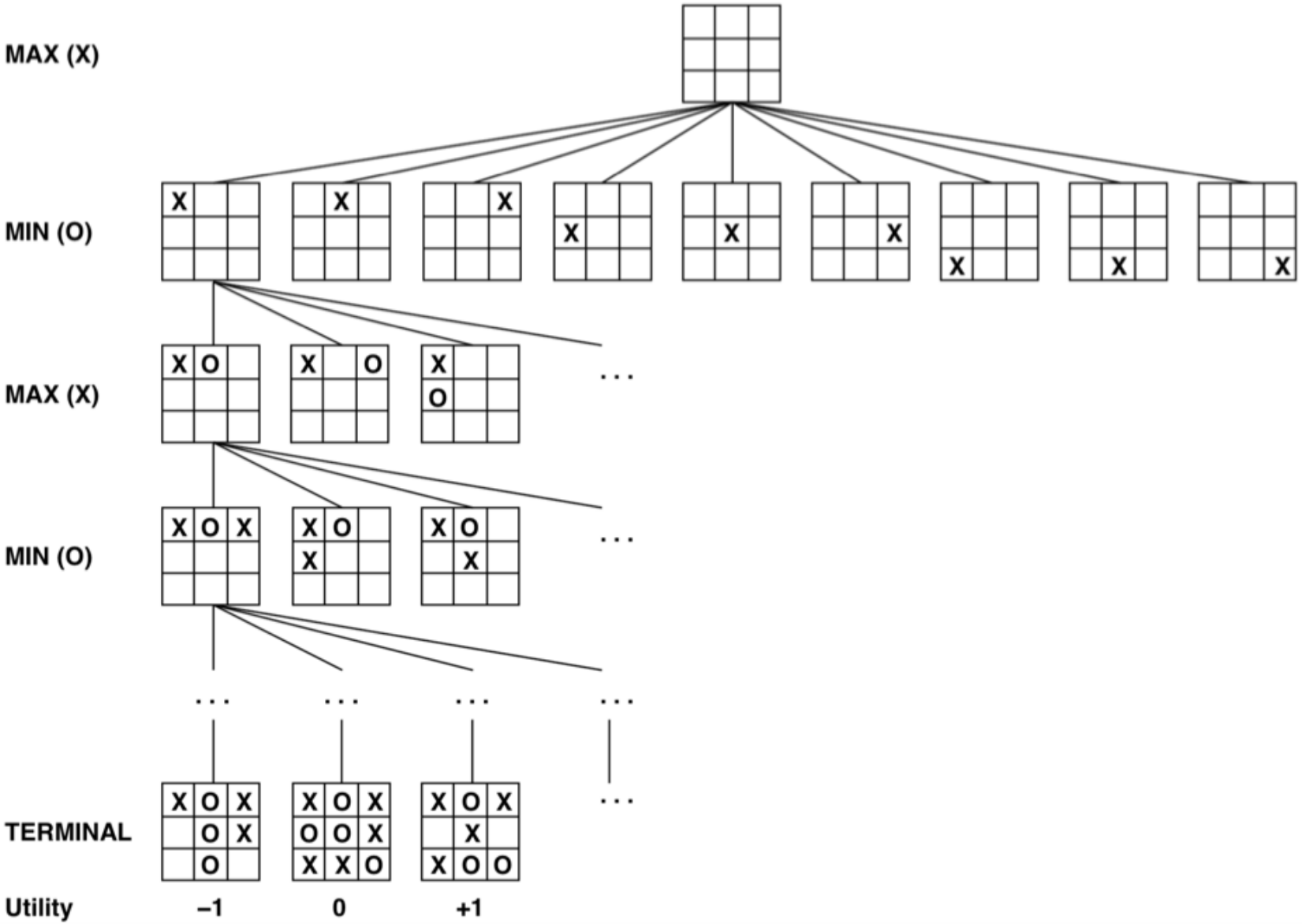
# Minimax Strategy

- Perfect play for deterministic, perfect-information games
- Idea: choose move to position with highest minimax value

  = best achievable payoff against best play



# Game Tree (2-player, deterministic, turns)



# Minimax Implementation

Need ***Base case*** for recursion

```
def max-value(state):
    if leaf?(state), return U(state)
    initialize v = -∞
    for each c in children(state)
        v = max(v, min-value(c))
    return v
```

```
def min-value(state):
    if leaf?(state), return U(state)
    initialize v = +∞
    for each c in children(state)
        v = min(v, max-value(c))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

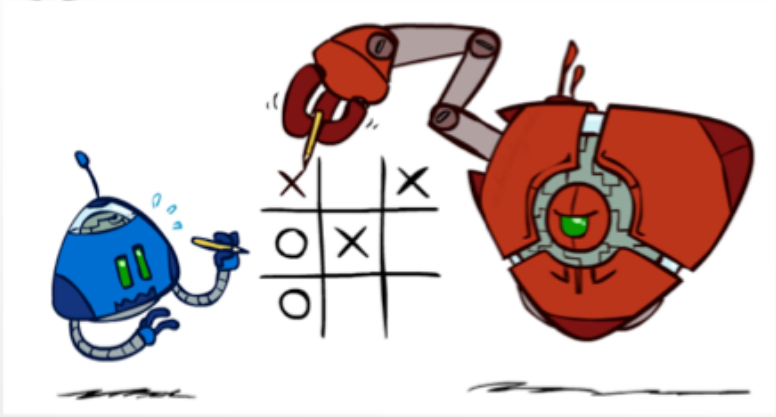
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

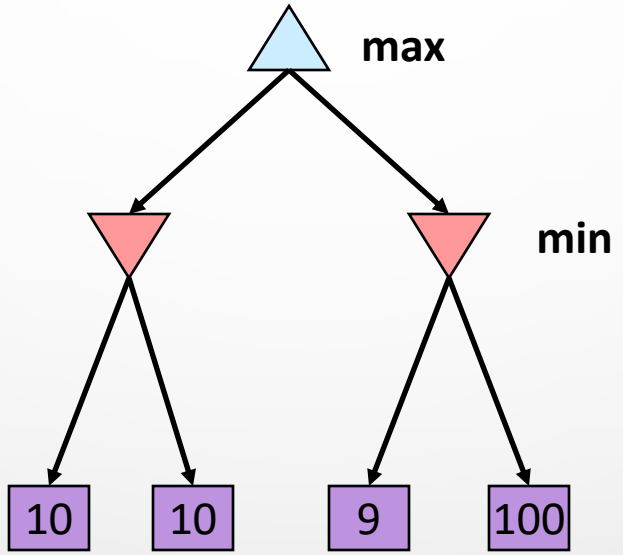
Slide adapted from Dan Klein & Pieter Abbeel - ai.berkeley.edu

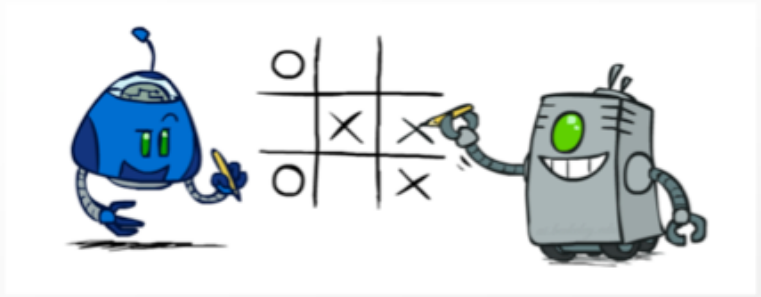
# Properties of minimax

- **Complete**:
  - Yes, if tree is finite (chess has specific rules for this)
- **Optimal**:
  - Yes, against an optimal opponent. Otherwise?
- **Time complexity**:
  - $O(b^m)$
- **Space complexity**:
  - O(bm) (depth-first exploration)
- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games ⇒ exact solution completely infeasible
- But do we need to explore every path?

# Properties of minimax (cont.)





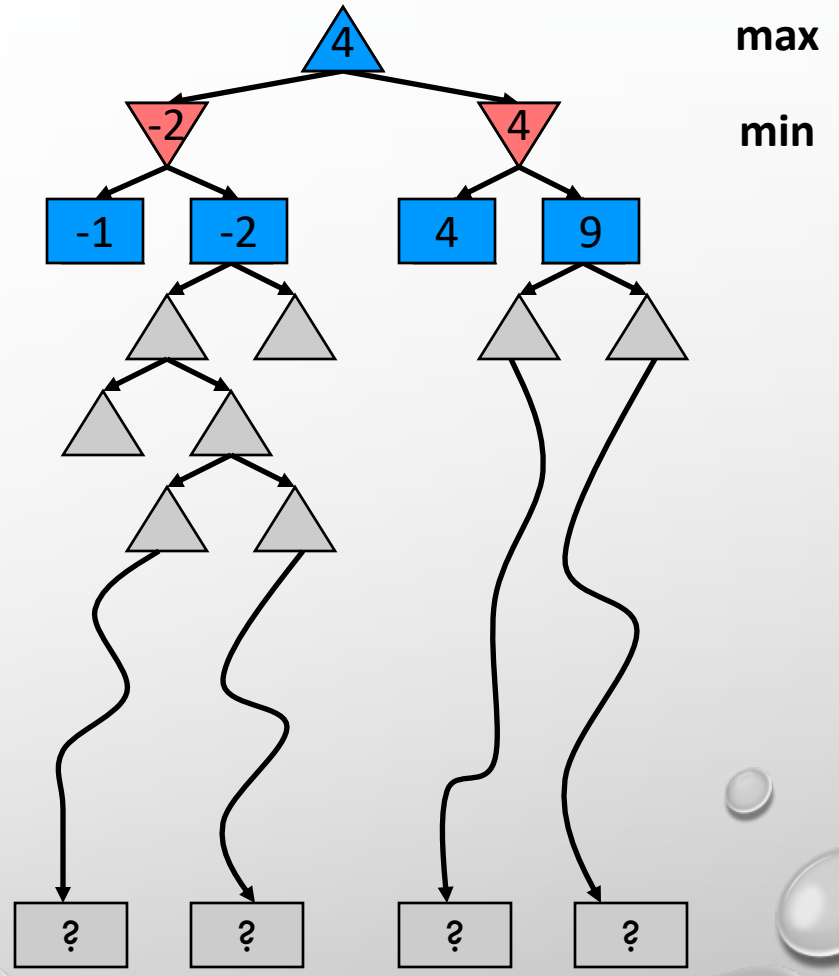



Optimal against a perfect player.  Otherwise?

# Resource Limits

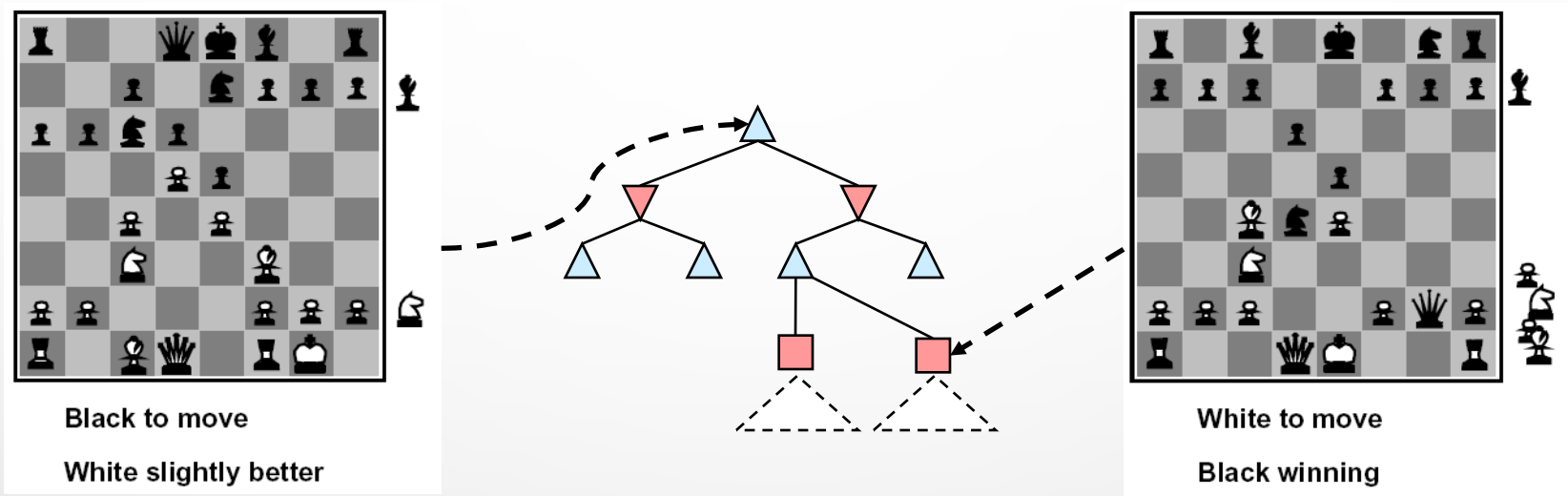
# Resource Limits

- Problem: in realistic games, cannot search to leaves!
- Solution: depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - $\alpha$-$\beta$ Reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More plies makes a big difference
- Use iterative deepening for an anytime algorithm



# Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



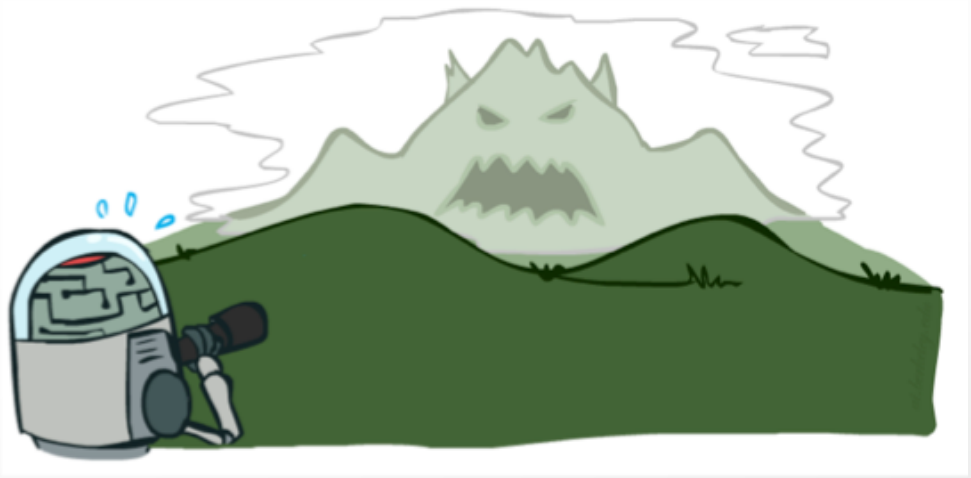

- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

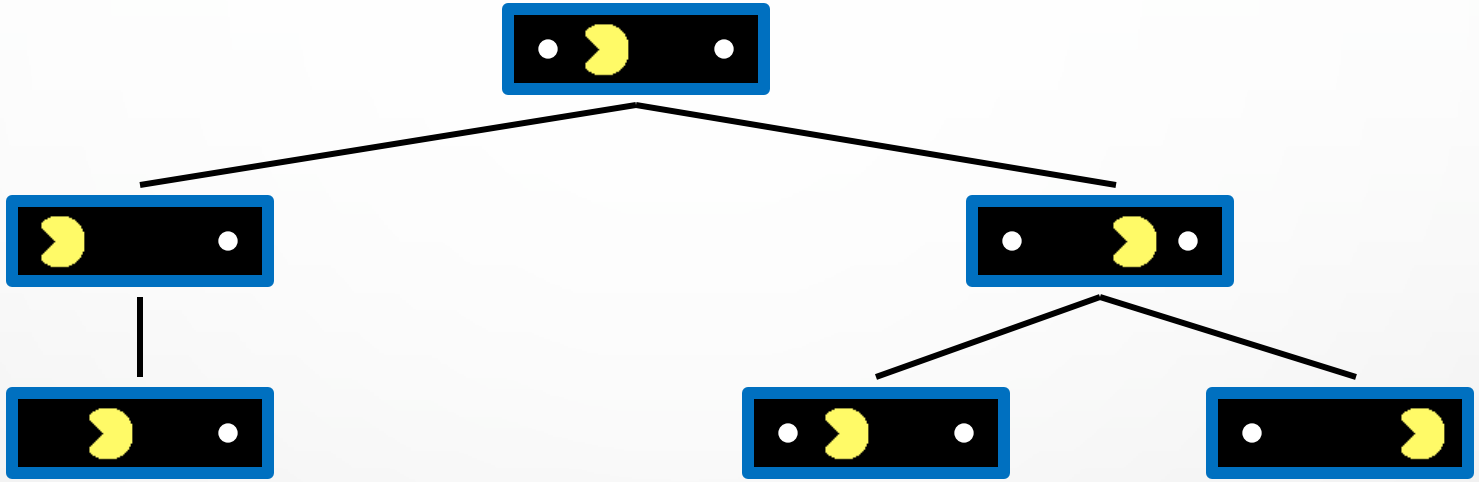
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

# Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

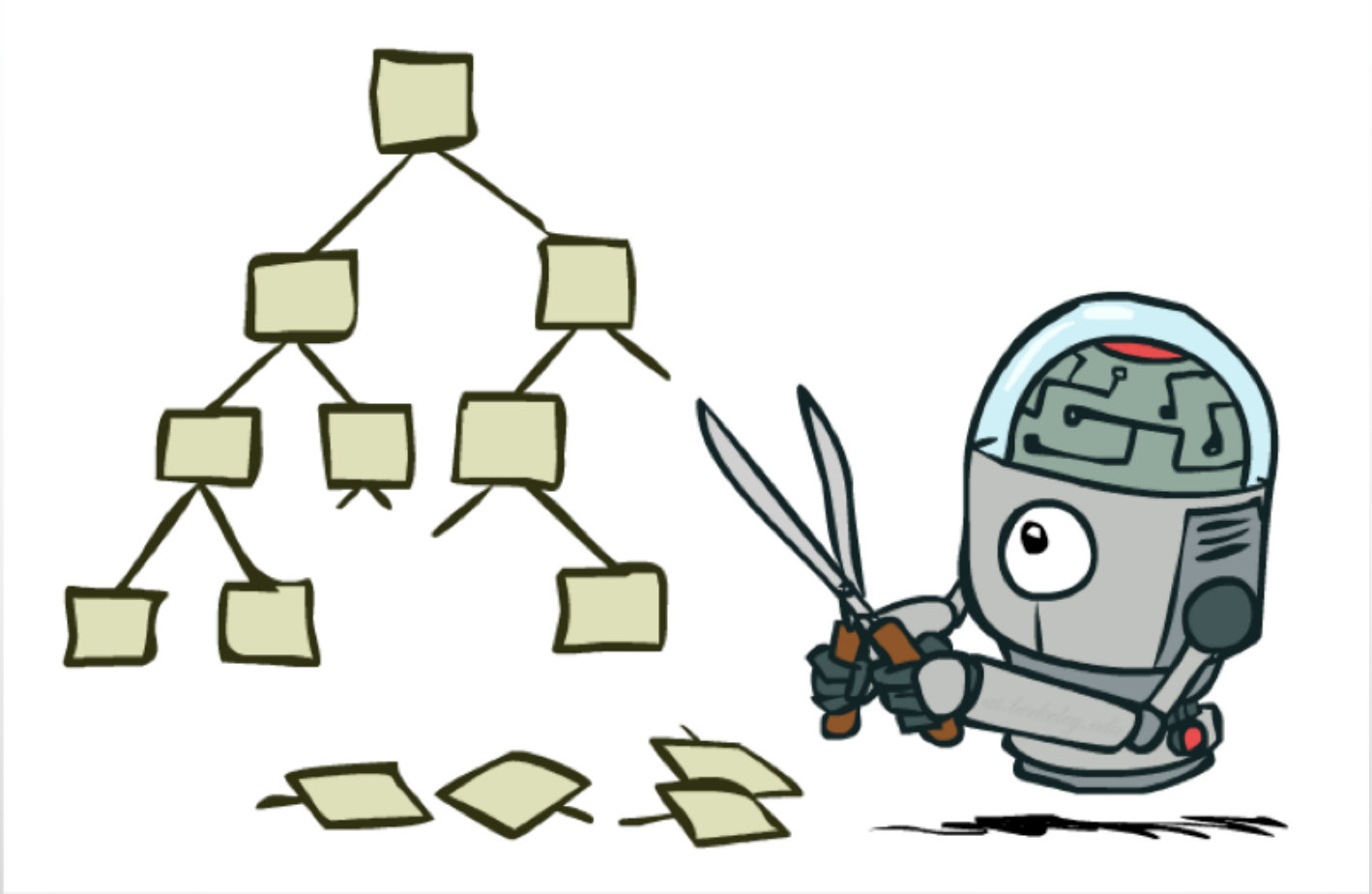


# Why Pacman Starves? (evaluation function matters)



- A danger of re-planning agents! (assume that evaluation function is 10 * number of eaten dots).
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

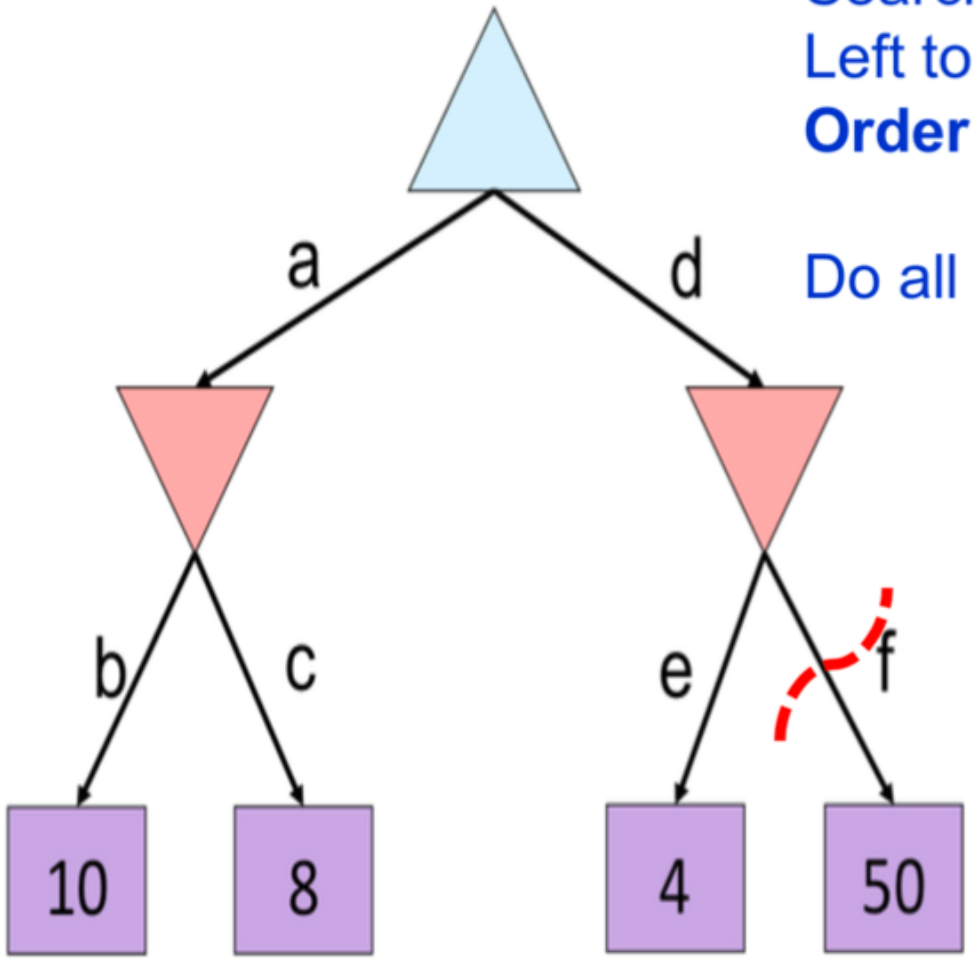
# Minimax pruning



# Minimax pruning

Max:

Min:



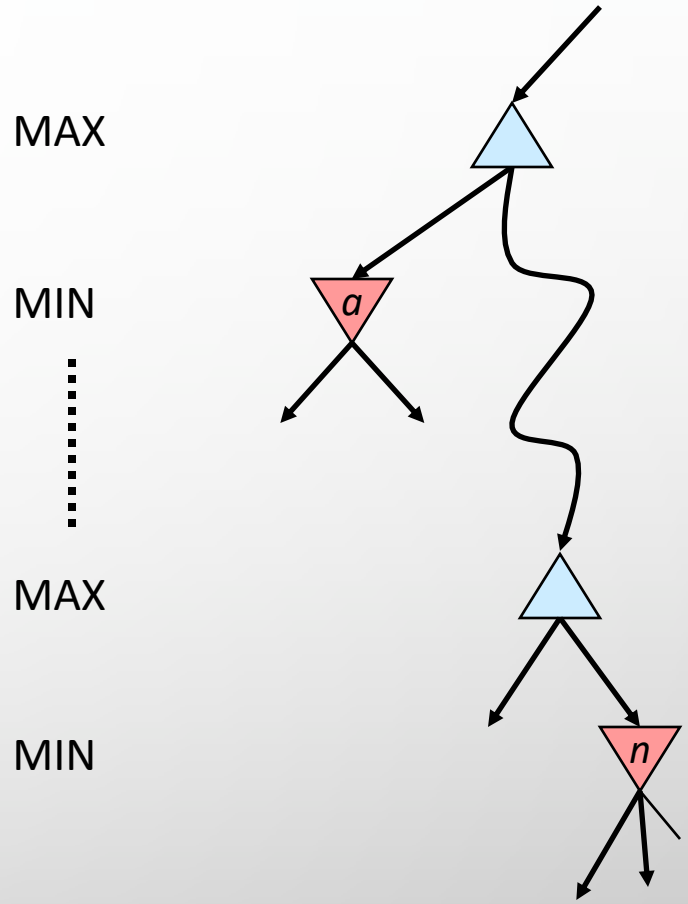

Search depth-first
Left to right
**Order is important**

Do all nodes matter?

Slide adapted from Dan Klein & Pieter Abbeel - ai.berkeley.edu

# Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *N*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If *n* becomes worse than *a*, MAX will avoid it, so we can stop considering *n*'s other children (it's already bad enough that it won't be played)
- MAX version is symmetric



# Alpha-Beta Implementation

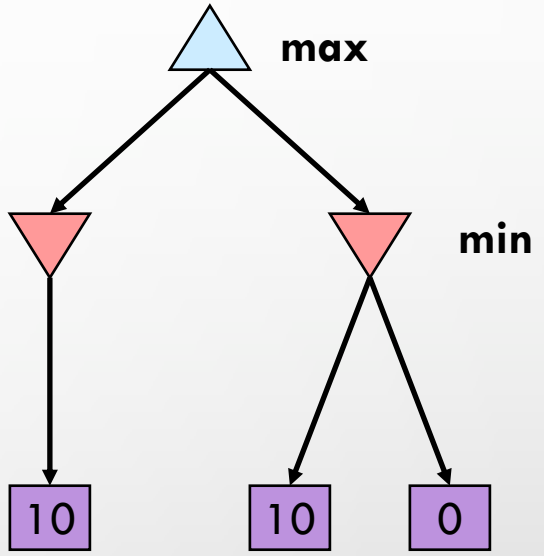
$\alpha$: MAX'S BEST OPTION ON PATH TO ROOT
$\beta$: MIN'S BEST OPTION ON PATH TO ROOT

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state, α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

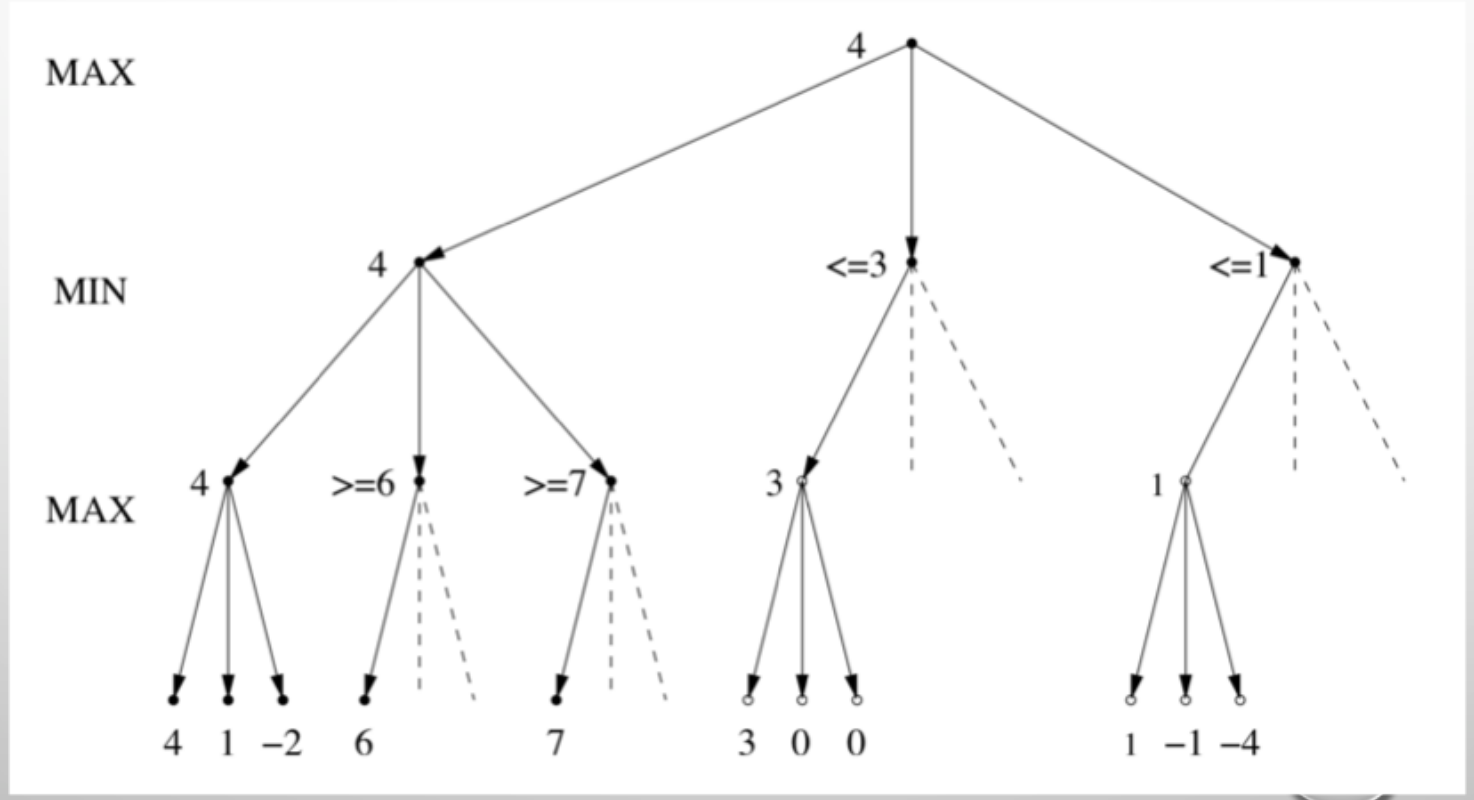
# Alpha-Beta Pruning Properties

- This pruning has no effect on minimax value computed **for the root**!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection
- Good child ordering improves effectiveness of pruning
- With "perfect ordering":
  - Time complexity drops to $O((2b)^{m/2})$ or better $O\left(\left(\sqrt{b}+0.5\right)^{m+1}\right)$
  - Doubles solvable depth!
  - Full search of, e.g. Chess, is still hopeless…
- With random ordering:
  - The total number of nodes examined will be roughly $O(b^{3m/4})$ for moderate b.
- This is a simple example of meta-reasoning (computing about what to compute)

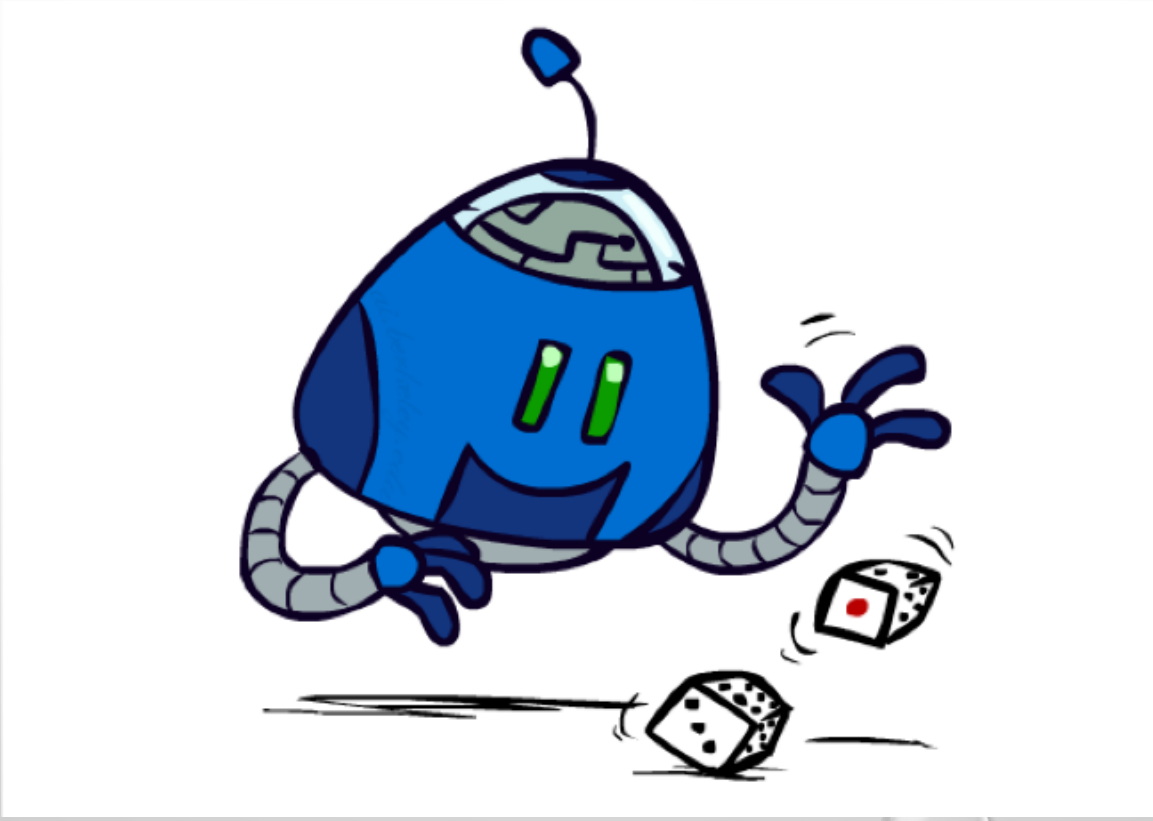


# Best Case Analysis of the Alpha-Beta pruning

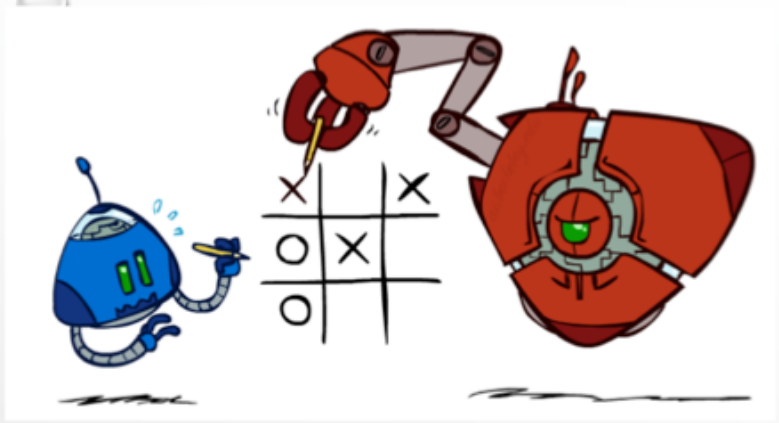
- In the best case, the minimax values are explored in descending order for MAX and in ascending order for MIN (why?)
- Can you write down a set of recursive equations to find the time complexity?

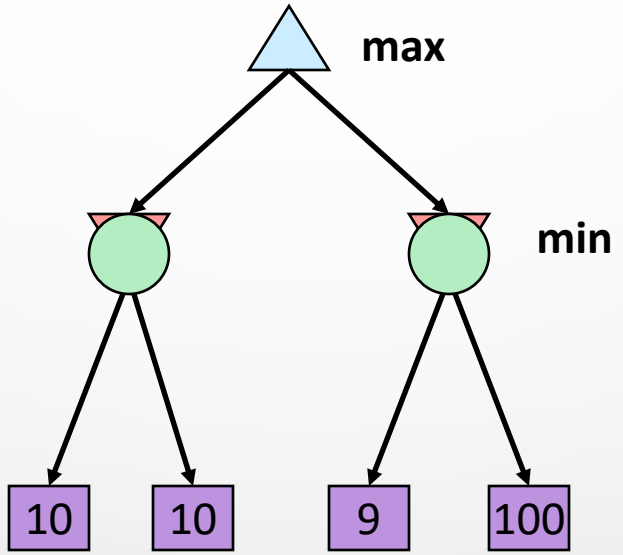


# Uncertain Outcomes



# Worst-Case vs. Average Case

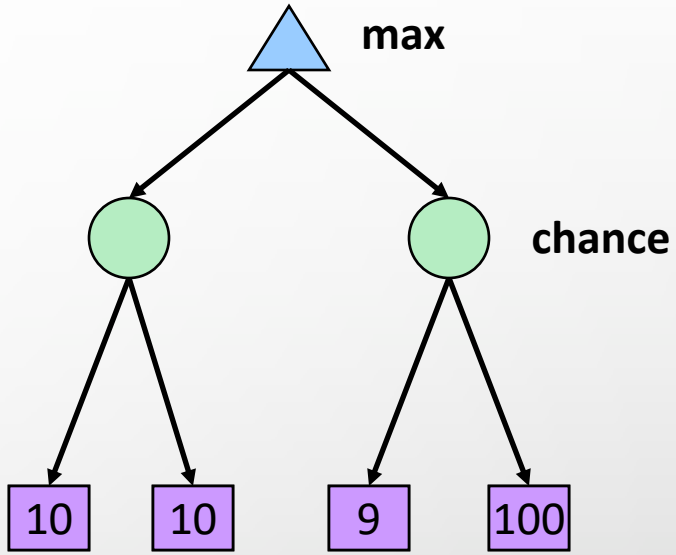





Idea: Uncertain outcomes controlled by chance, not an adversary!

# Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - i.e. Take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes



# Expectimax Pseudocode

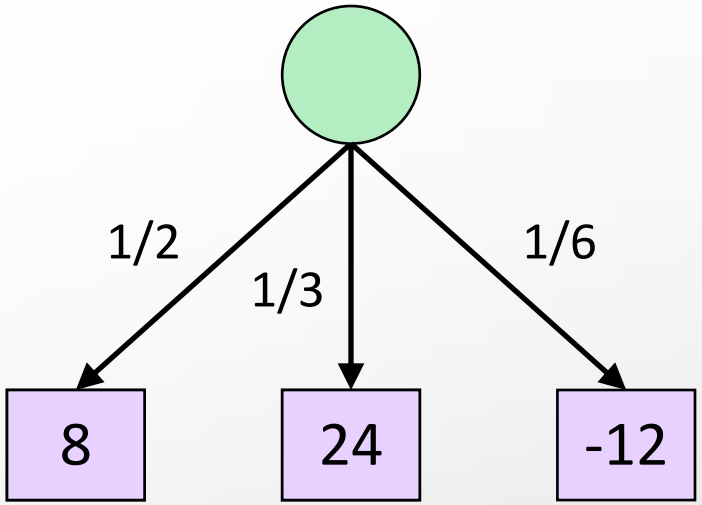
```
Def value(state):
    If the state is a terminal state: return the state's utility
    If the next agent is MAX: return max-value(state)
    If the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

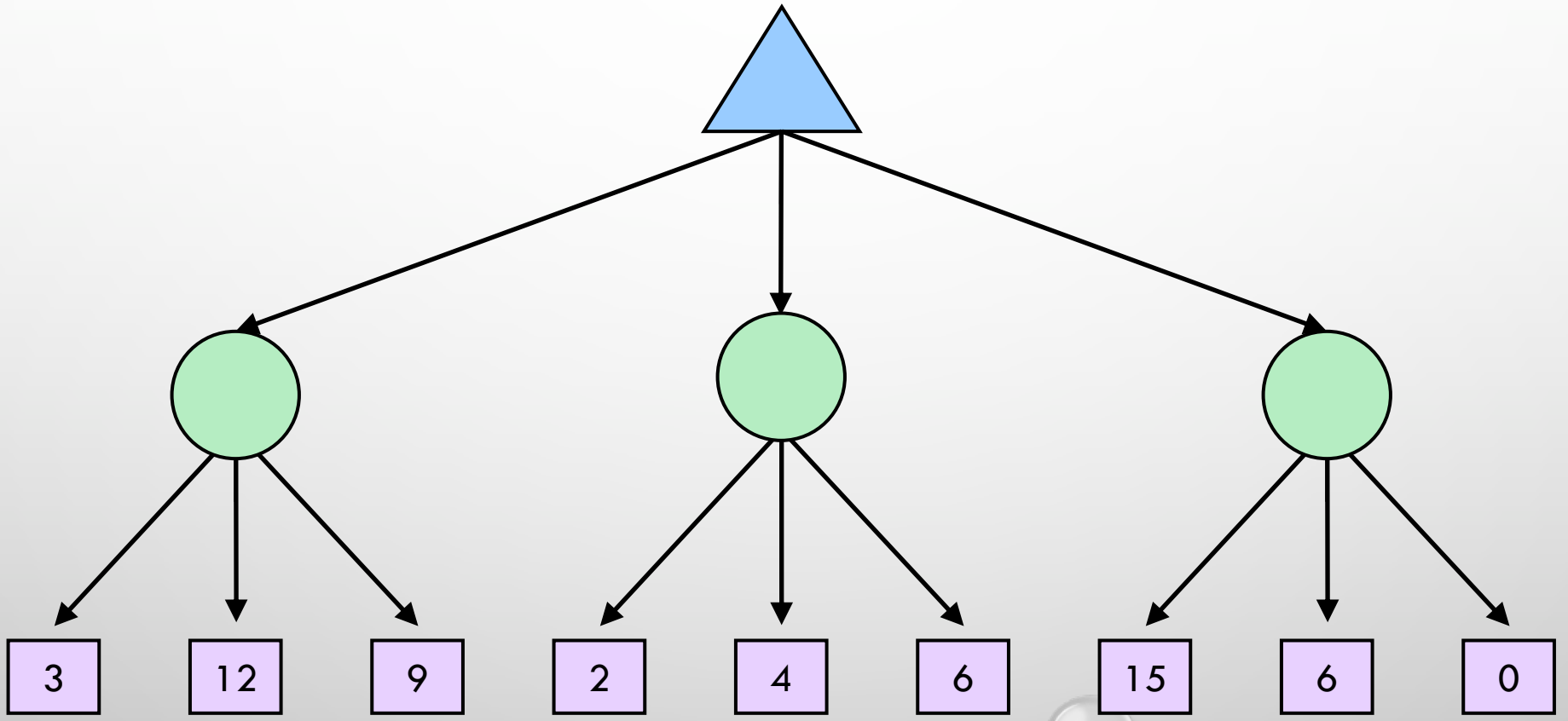
# Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

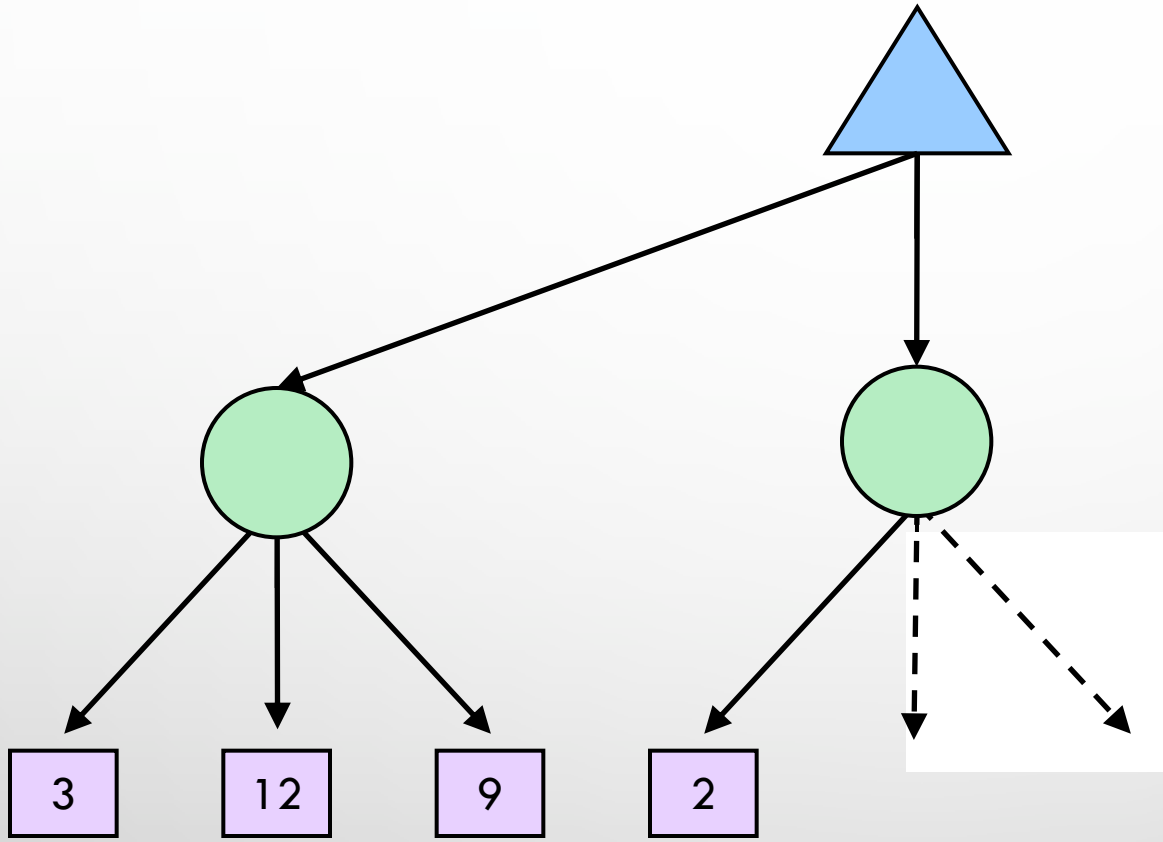


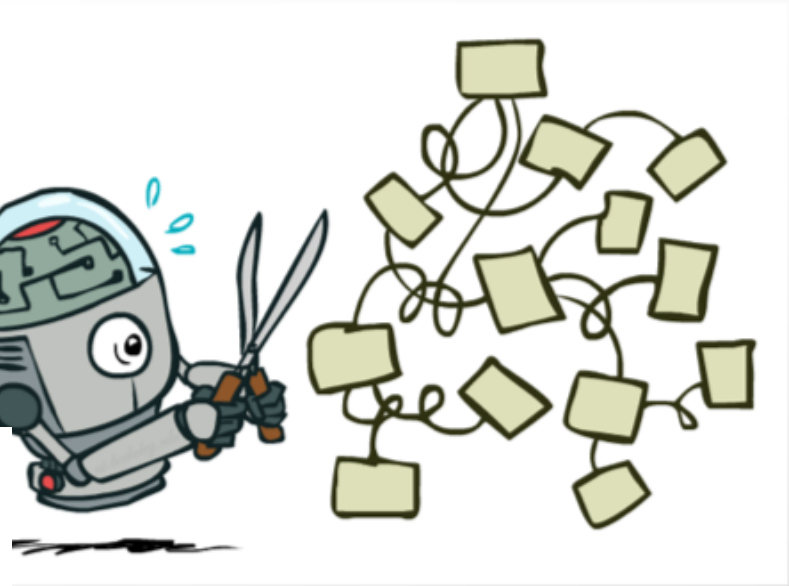

$$v = (1/2)\,(8) + (1/3)\,(24) + (1/6)\,(-12) = 10$$

# Expectimax Example

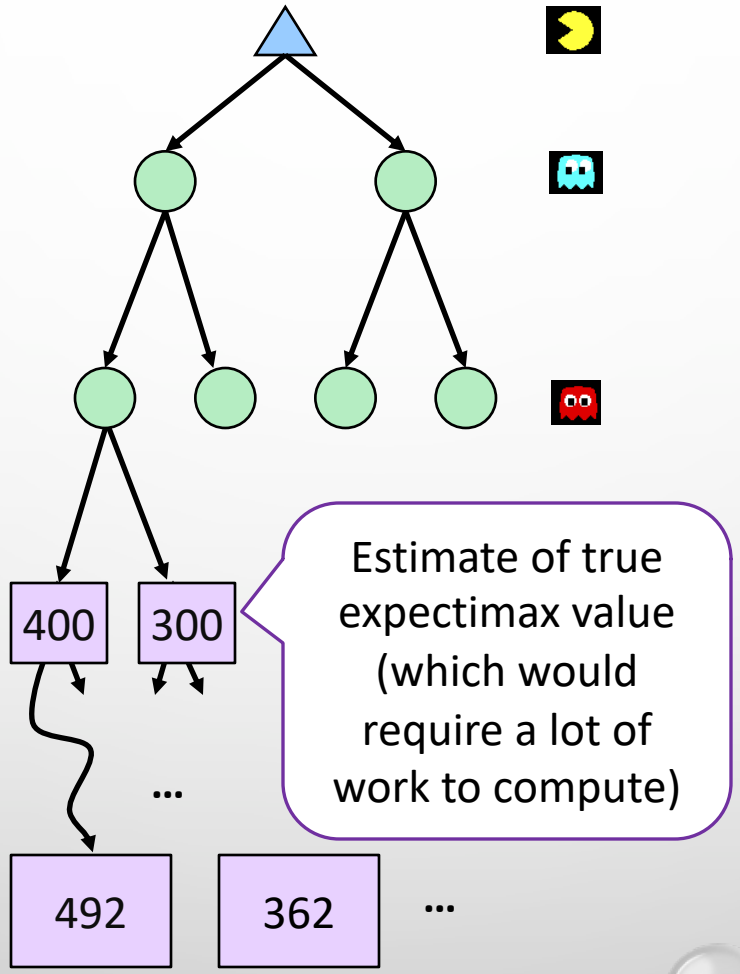


# Expectimax Pruning?

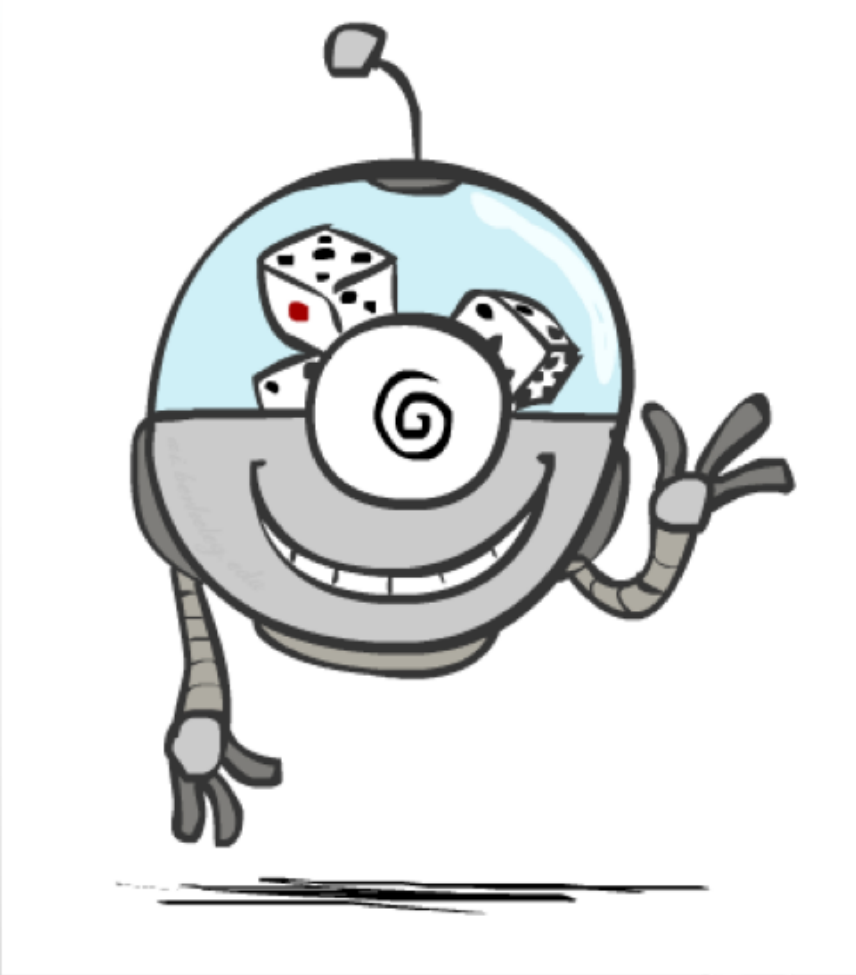


# Depth-Limited Expectimax



# Probabilities



# Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

0.25

- Example: traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: p(T=none) = 0.25, p(T=light) = 0.50, p(T=heavy) = 0.25

0.50

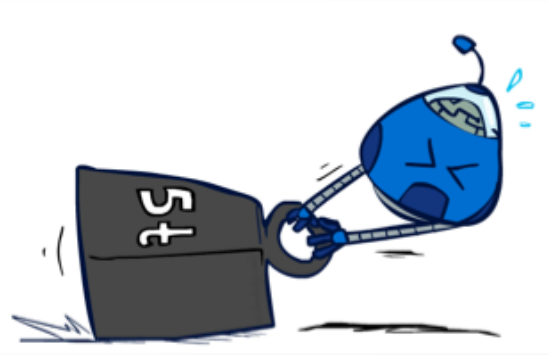
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

0.25

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, p(T=heavy | hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

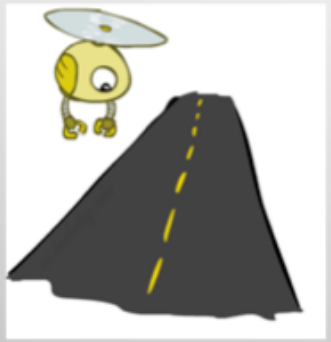
# Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes



- Example: how long to get to the airport?

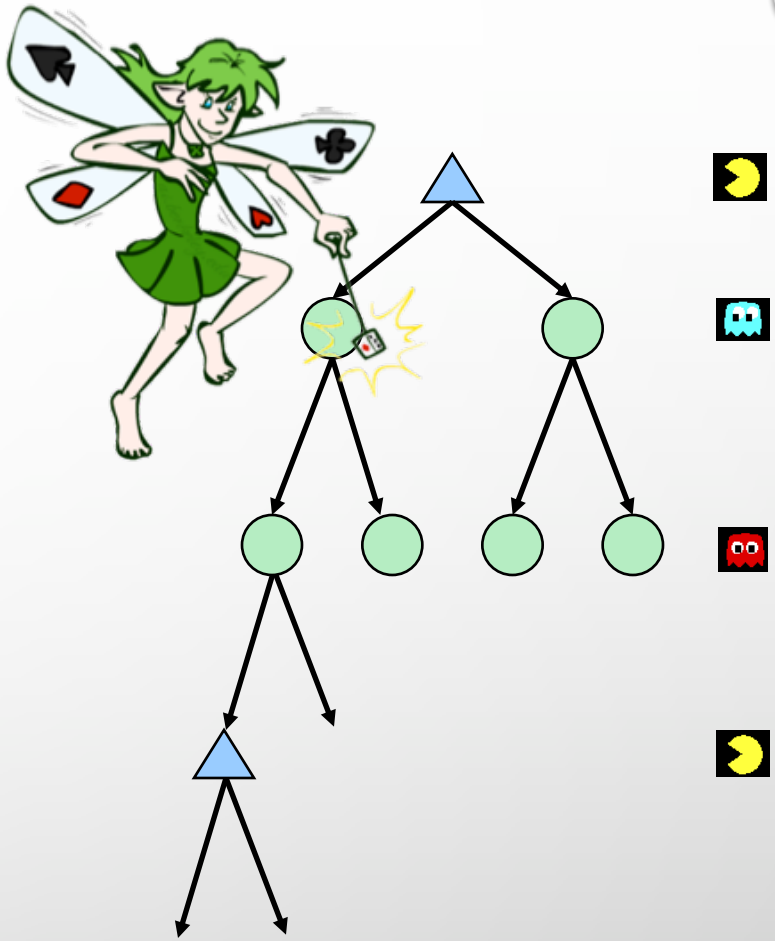
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min | | |
| | x | + | x | + | x | | |
| Probability: | 0.25 | | 0.50 | | 0.25 | | 35 min |



# What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!

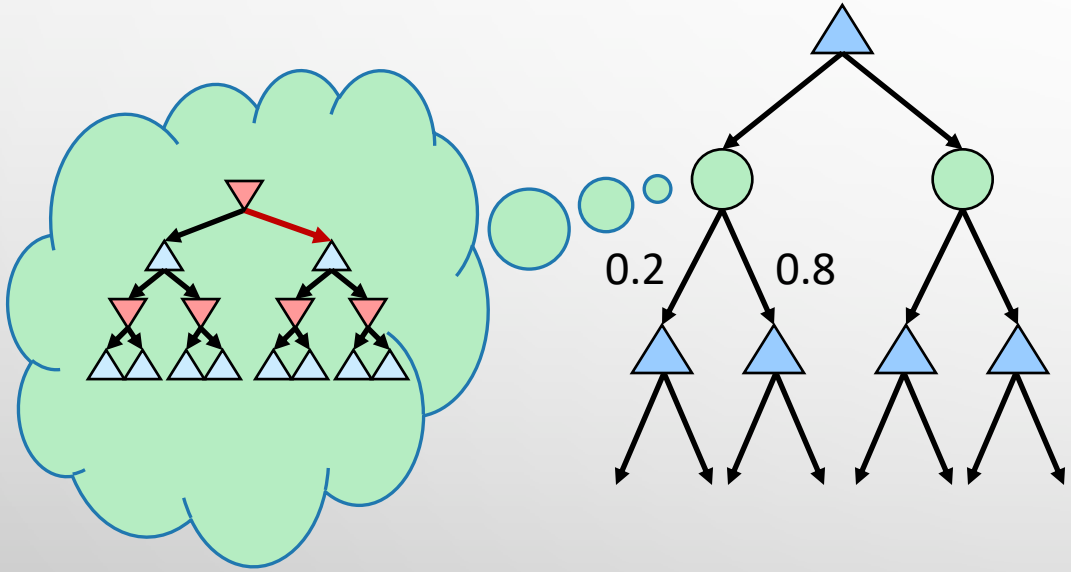
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



*Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!*

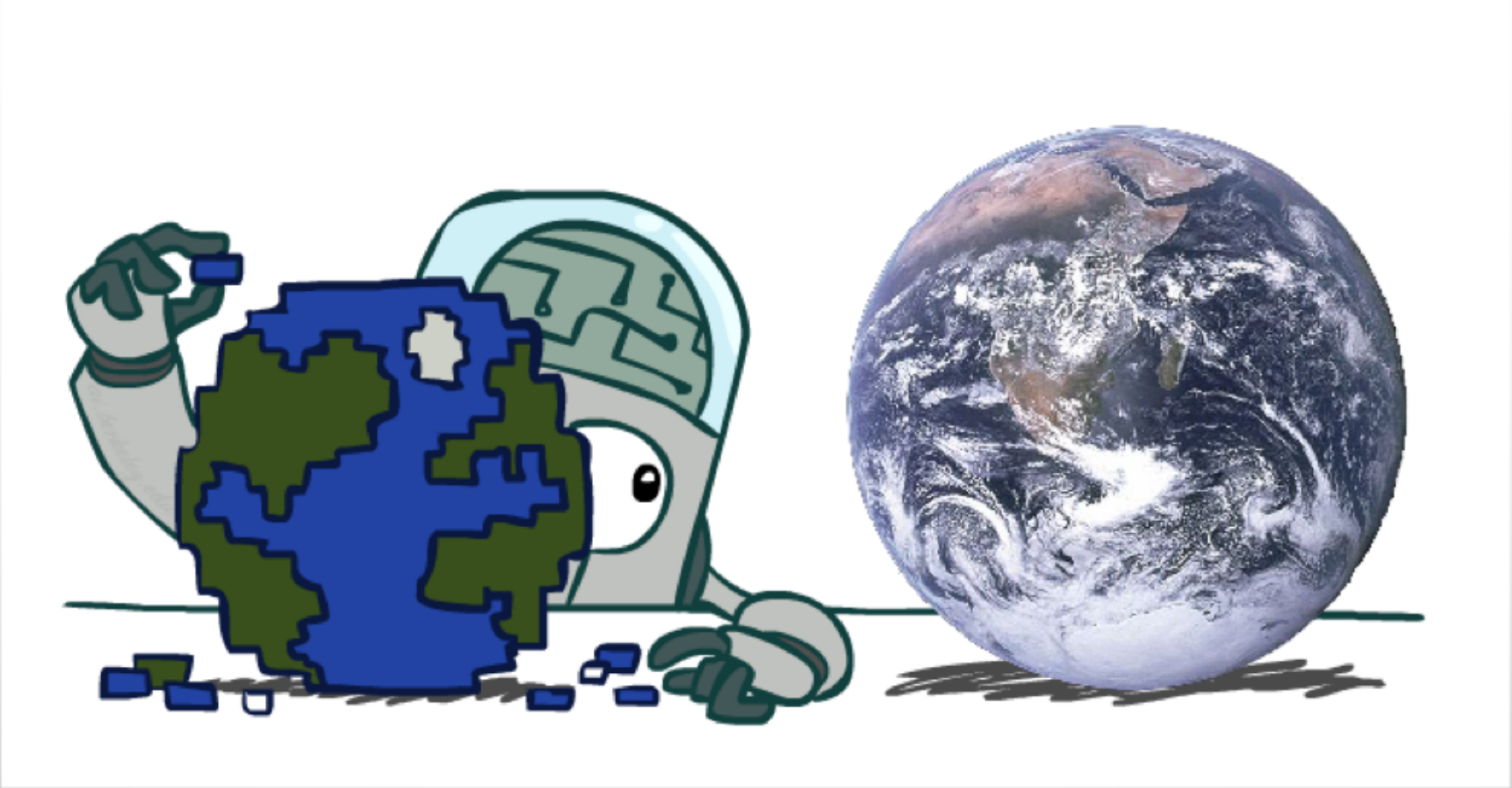
# Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: what tree search should you use?




- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

# Modeling Assumptions



# The Dangers of Optimism and Pessimism

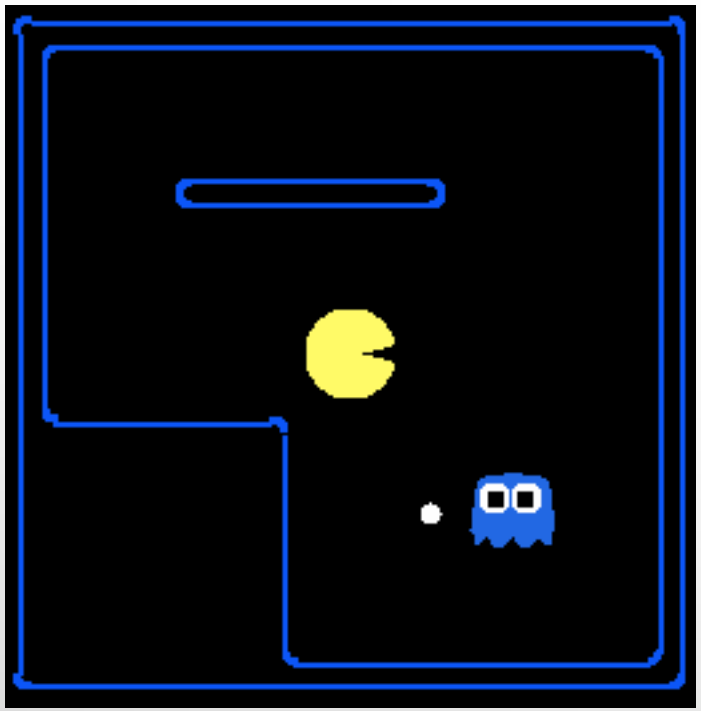
## Dangerous Optimism

Assuming chance when the world is adversarial


## Dangerous Pessimism

Assuming the worst case when it's not likely

# Assumptions vs. Reality

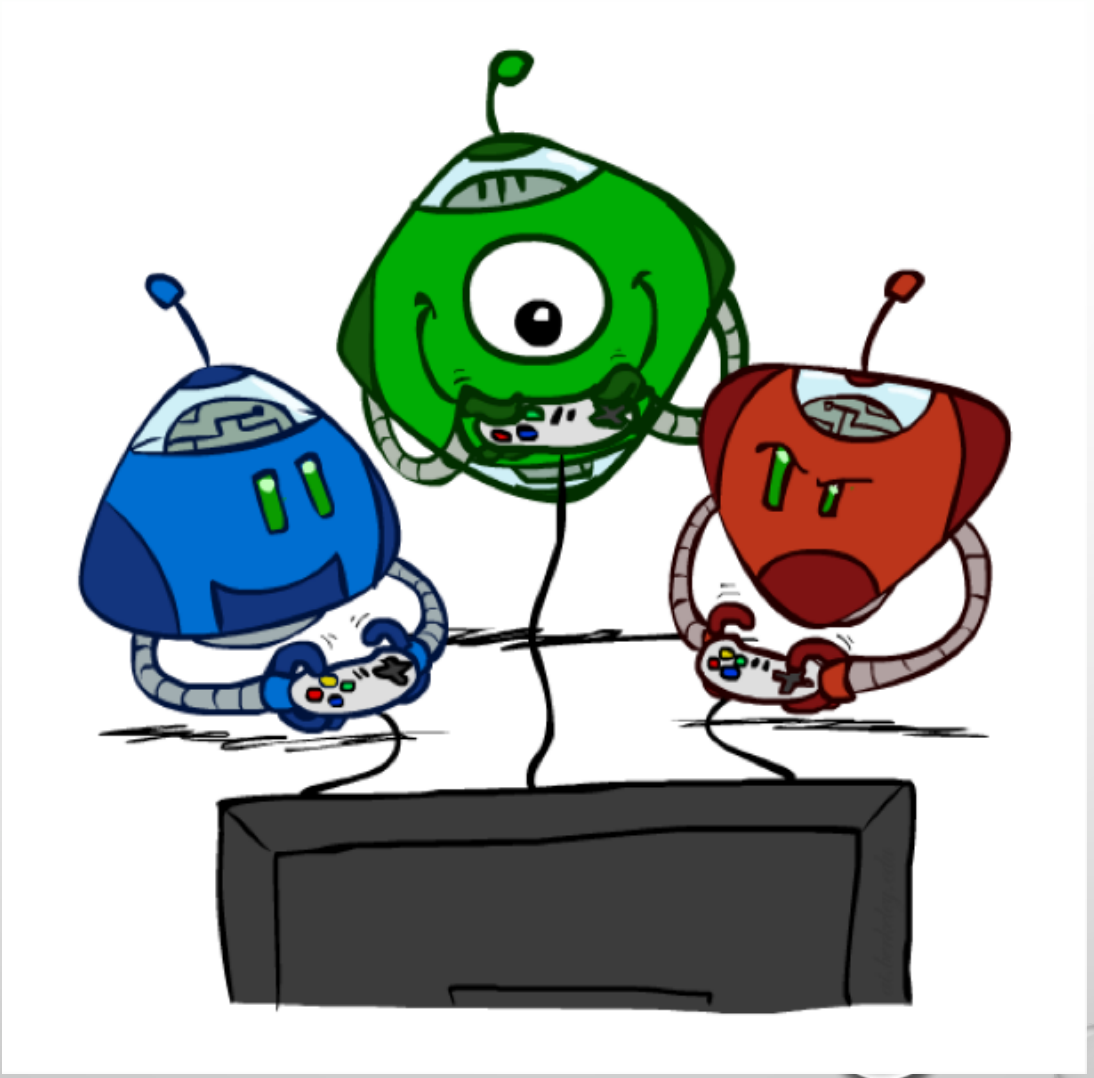


| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | Won 5/5<br>Avg. Score: 483 | Won 5/5<br>Avg. Score: 493 |
| Expectimax Pacman | Won 1/5<br>Avg. Score: -303 | Won 5/5<br>Avg. Score: 503 |

Results from playing 5 games

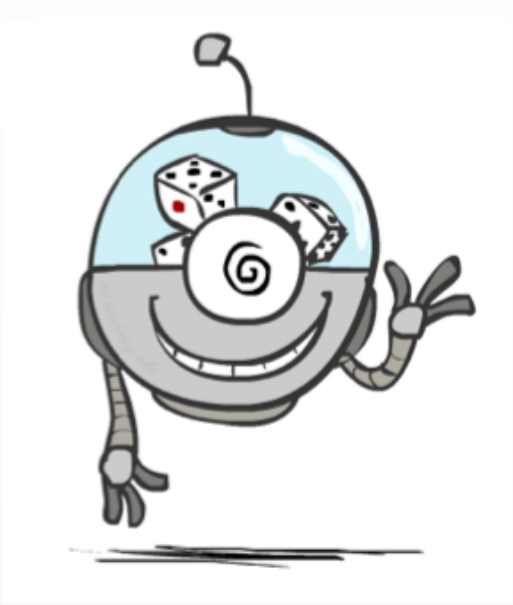
Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

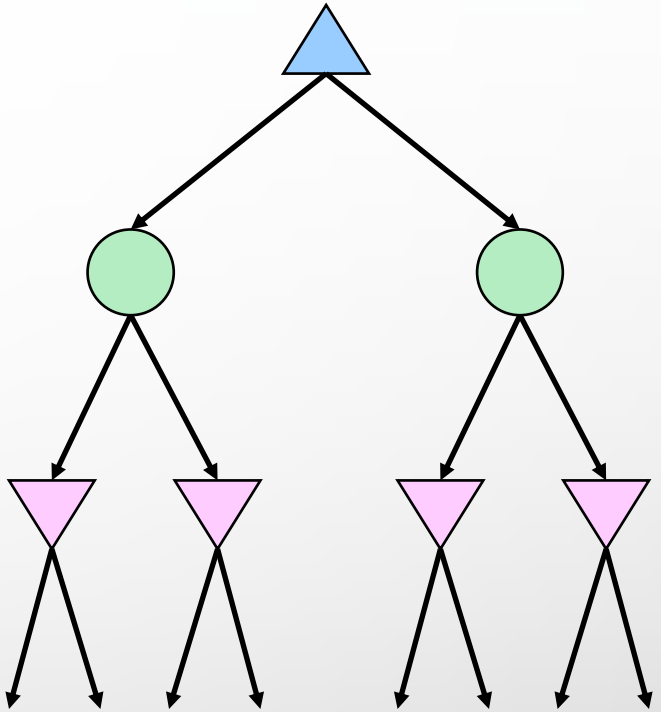
# Other Game Types

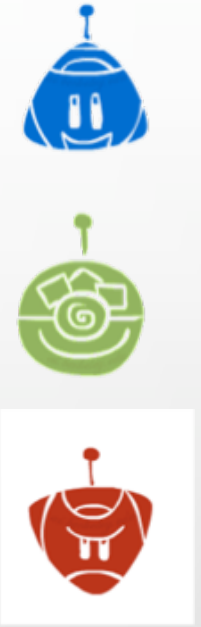


# Mixed Layer Types

- e.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children







# Example: Backgammon

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x $(21 \times 20)^3 = 1.2 \times 10^9$

- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…

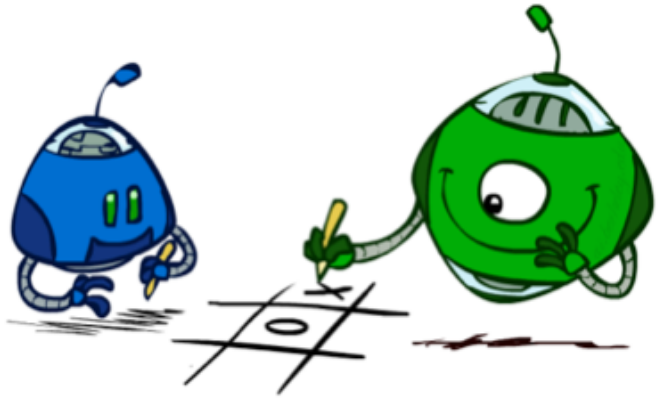
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

- 1st AI world champion in any game!



Image: Wikipedia

# Multi-Agent Utilities



- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and Competition dynamically…