Practice 3

Theory of languages and machines

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1.1

a) L = $\{a^i b^j a^k \mid k > i + j\}$

b) L = {a i b j | j = i ∨ j = 2i}

$a^n b^{2n} = a^{n-1} a b^{2n}$

$|xy| = n <= n$, $|y| = 1 <= 1$

$a^{n-1} b^{2n}$, 2n != n-1 or 2n != 2(n-1) =>

$a^{n-1} b^{2n}$ not regular

c) L = {ω ∈ {a, b} ∗ | na(ω) < 2nb(ω)}

$a^n b^n = a^{n-1} a b^n$

$|xy| = n <= n$, $|y| = 1 <= 1$

$a^{2n} b^n$, not 2n < n =>

$a^{2n} b^n$ not regular

d) L = {ωωω | ω ∈ {a, b} ∗}

a2^n

|xy| <= n,

the pumping part, which has to be non-zero in length
cannot be of length greater than n

e) L = $\{a^{2^n} \mid n \geq 0\}$

$a^{2^n} = a a a^{2^n - 2}$

$|xy| = 2 <= n$, $|y| = 1 <= 1$

a^{2^n+1} not regular

f) L = {ω ∈ {α, γ} ∗ | ω = ω1 γ ω2 γ ... γ ωk, for k ≥ 0, each ωi ∈ α ∗ , and ωi≠ ωj for i≠ j}

1.2

Suppose F is regular.

Let L = {x | x begins with 1 a}.

L is regular, the intersection of two regular languages is regular, so the language L' = F ∩ L is regular.

p = pumping length of L'.

$L' = \{ab^nc^n \mid n \geq 0\}$, so ab^pc^p in L'.

By pumping lemma, ab^pc^p written as xyz $|y| > 0$, $|xy| \leq p$, and for each $i \geq 0$, $xy^iz \in L'$.

However

(i) if y includes a, the string xyyz has at least 2 a's
(ii) else, if y includes both b and c, the string xyyz has at least 2 substrings bc
(iii) else, y includes only b or only c, and so the number of b and the number of c in xyyz will not be equal.

so xyyz can't be in L', so F is not regular.

1.3

a)

This restatement does not change any of the original pumping lemma's criteria (|z2| = n implies |uv| <= n) and the proof is similar, but it does allow us to switch our focus of attention anywhere along a long string. The new pumping lemma statement allowed us to shift our attention to the 1s in the middle of the string, resulting in a simple proof.

b)

z1 = an , z2 = bn , z3 = cn ; z = z1 z2 z3 = anbncn is in L. Since z2 consists only of b, so does v; therefore the string z1uv2wz3 is anbn+|v|cn is not in L=>

L is not regular

1.4

to prove that a language L is not regular using closure properties to combine L with regular languages by operations that preserve regularity in order to obtain a language known to be not regular.

a) $L = \{a^nb^mc^{n+m} \mid n \geq m \geq 0\}$

$L' = \{a*b*\}$ is regular

$L'' = L' \cap L = \{a^nb^n \mid n \geq 0\}$

L'' is not regular => L is not regular

b) $L = \{\omega1\omega2 \in \{a, b\}* \mid |\omega1| = |\omega2| \text{ and } \omega1 \neq \omega2\}$

L' = { } is regular

L'' = L' ∩ L = { }

L'' is not regular => L is not regular

c) $L = \{a^nb^{2^k} \mid n, k \geq 1\}$

$L' = \{b*\}$ is regular

$L'' = L' \cap L = \{b^{2^k} | k \geq 1\}$

L'' is not regular => L is not regular

2

2.1

a) $L = \{\omega \in \{a, b\}* \mid (n_a(\omega) - n_b(\omega)) \bmod 3 = 2\}$

$n_a(\omega) \bmod 3 = 2$, $n_b(\omega) \bmod 3 = 0$

$n_a(\omega) \bmod 3 = 1$, $n_b(\omega) \bmod 3 = 2$

$n_a(\omega) \bmod 3 = 0$, $n_b(\omega) \bmod 3 = 1$

((aaa)+b)*∪((aa)(bbb)*)* ∪(a(bb))*

b) $L = \{\omega \in \{a, b\}* \mid (2n_a(\omega) + 3n_b(\omega)) \bmod 2 = 0\}$

$2n_a(\omega) \bmod 2 = 0$ ->

$(2n_a(\omega) + 3n_b(\omega)) \bmod 2 = 3n_b(\omega) \bmod 2$ ->

$3n_b(\omega) \bmod 2 = n_b(\omega) \bmod 2$ ->

Language that accept even number of b ->

(ba*ba*)*

c) $L = \{a^n b^m, n \geq 3, m \leq 4\}$

aaa+ ((bbbb) ∪ (bbb) ∪ (bb) ∪ (b) ∪ (ε))

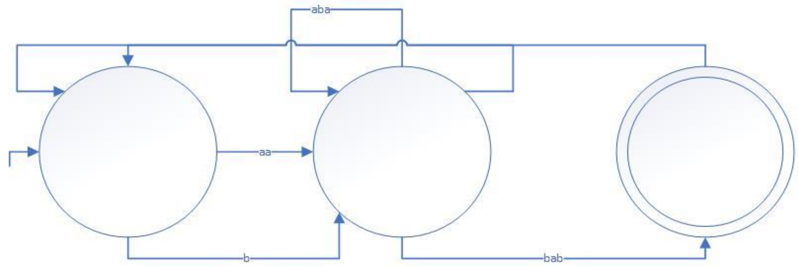
2.2

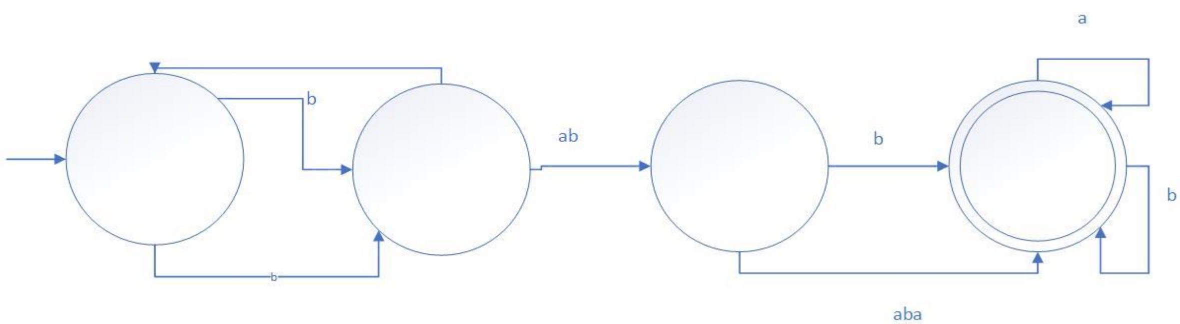
a) a(a+b*aba*b)b*aba*

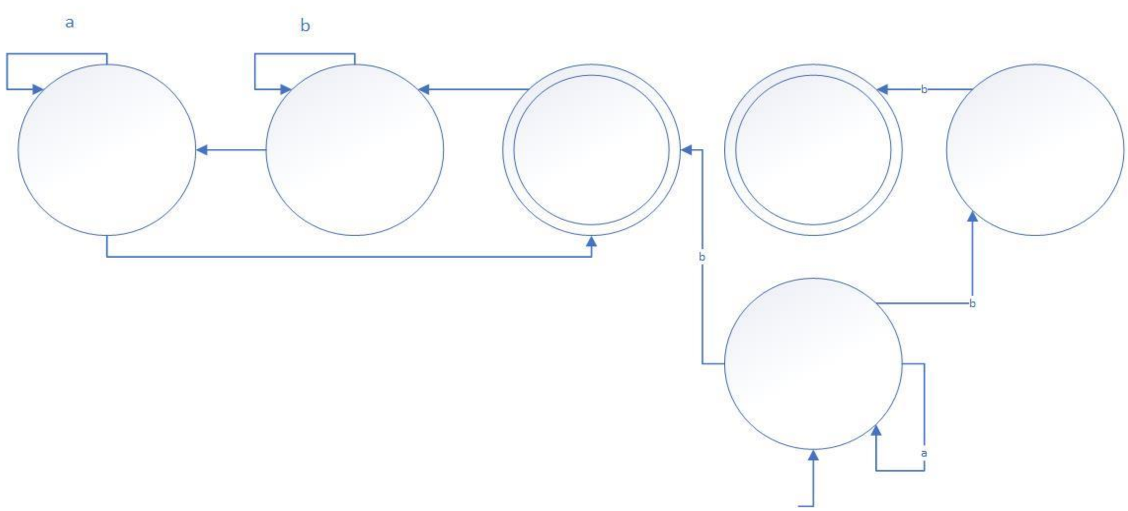
b) ab(ababb +abababaab)* + abab

c) b* + a(ba*bb)*aa

2.3







2.4

a) $(a \cup b) * = (a * \cup ba*) *$

$(a \cup b)* = (a \cup ba*)*$

$= (a* \cup ba*)*$

b) $b +(a*b*\cup\epsilon)b = b(b*a*\cup\epsilon)b+$

$b+(a*b* \cup \varepsilon)b = bb*(a*b* \cup \varepsilon)b$

$= b(b*a* \cup \varepsilon)b*b$

$= b(b*a* \cup \varepsilon)b+$

c) $(ba) +(a *b*\cup a*) = (ba)\ ba+(b*\cup \epsilon)$

$(ba)+(a*b* \cup a*) = (ba)*ba(a*b*\cup a*)$

$= (ba)*baa*(b* \cup \varepsilon)$

$= (ba)*ba+(b* \cup \varepsilon)$

3

3.1

a)

$S \rightarrow AAS \mid ab \mid aab$

$A \rightarrow ab \mid ba \mid \epsilon$

$S \rightarrow ababS|abS|abbaS|baS|baabS|ab|S|aab$

b)

$S \rightarrow AB$

$A \rightarrow aAa \mid bAb \mid a \mid b$

$B \rightarrow aB \mid bB \mid \epsilon$

$S \rightarrow aB|Bb$

$B \rightarrow aB \mid bB \mid \epsilon$

c)

S → AA | B

A → AAA | Ab | bA | a

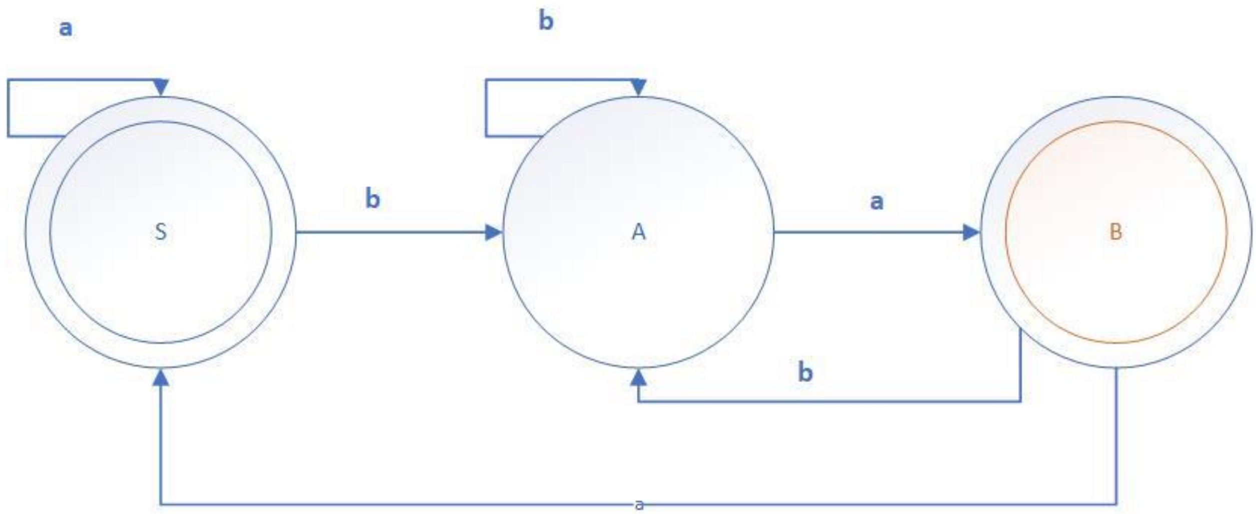
B → bB | ϵ

S →

3.2

First, we draw NFA and then reverse it (change arrows and initial state). Then we get reversed grammar from reversed NFA

a)



S → aS | bA

A → bA | aB

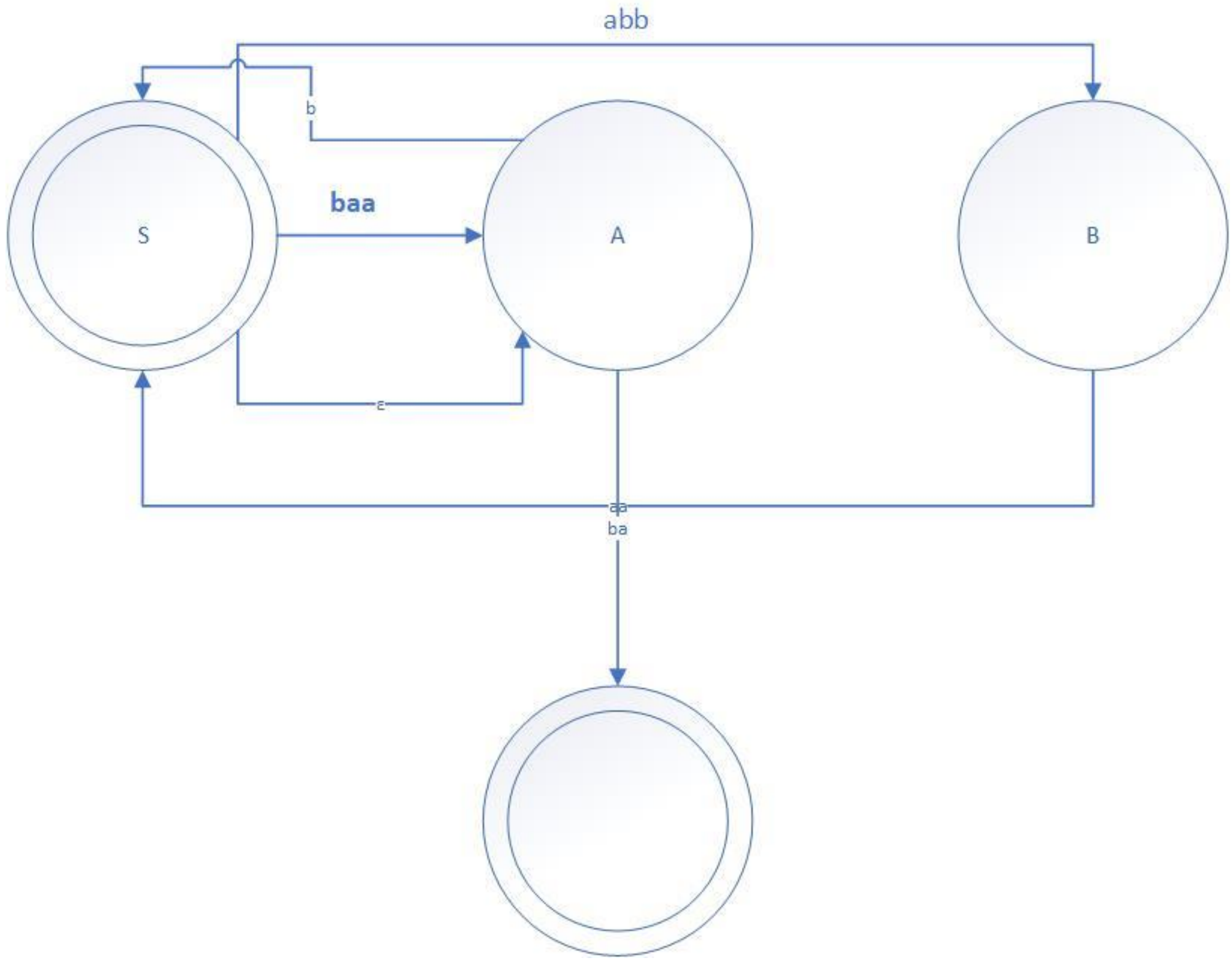
B → bA | aS | ϵ

S → aS | aB

A → bA | bS | bB

B → aA | S

b)



S → baaA | abbB | A | ϵ

A → bS | ba

B → aaS

S → bA | aaB

A → aabS | S

B → bbaS

Z → abA

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4.1

4.2