به نام خدا

نظریه زبان‌ها و ماشین‌ها

تمرین 1

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فهرست

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a)

$$\mathbf{\neg (p \lor \neg q) \lor (\neg p \land \neg q) \equiv \neg p}$$

$$\neg (p \lor \neg q) \lor (\neg p \land \neg q) \equiv$$

$$(\neg p \land q) \lor (\neg p \land \neg q) \equiv$$

$$\neg p \land (q \lor \neg q) \equiv$$

$$\neg p \land T$$

$$\neg p$$

b)

$$\mathbf{\neg ((\neg p \land q) \lor (\neg p \land \neg q)) \lor (p \land q) \equiv p}$$

$$\neg (\neg p \land (q \lor \neg q)) \lor (p \land q) \equiv$$

$$P \lor (p \land q) \equiv$$

$$P$$

c)

$$\mathbf{p \lor q \lor (\neg p \land \neg q \land r) \equiv p \lor q \lor r}$$

$$p \lor ((q \lor \neg p) \land (q \lor \neg q) \land (q \lor r)) \equiv$$

$$p \lor ((q \lor \neg p) \land T \land (q \lor r)) \equiv$$

$$(p \lor q \lor \neg p) \land (p \lor q \lor r) \equiv$$

$$(T \lor q) \land (p \lor q \lor r) \equiv$$

$$T \land (p \lor q \lor r) \equiv$$

$$p \lor q \lor r$$

d)

$$\mathbf{(\neg p \lor \neg q) \rightarrow (p \land q \land r) \equiv p \land q}$$

$$(\neg p \lor \neg q) \rightarrow (p \land q \land r) \equiv$$

$$(p \land q) \lor (p \land q \land r) \equiv$$

$$(p \land q) \land (T \lor r) \equiv$$

$$(p \land q) \land T \equiv$$

$(p \wedge q)$

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(A1) $(\phi \to (\psi \to \phi))$

(A2) $(\phi \to (\psi \to \theta)) \to ((\phi \to \psi) \to (\phi \to \theta))$

(A3) $(((\neg\phi) \to (\neg\psi)) \to (\psi \to \phi))$

a)

$$\mathbf{\neg (\neg p) \to p}$$

$$\neg (\neg (\neg p)) \vee p \equiv$$

$$\neg p \vee p \equiv$$

$$T$$

b)

$$\mathbf{(p \to q \to r) \to p \to r}$$

c)

$$\mathbf{(\neg r \to \neg q \to \neg p) \to p \to q \to r}$$

d)

$$\mathbf{(p \to q) \to \neg q \to \neg p}$$

e)

$$\mathbf{p \to \neg q \to \neg (p \to q)}$$

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a)

$$\mathbf{p \to q \; \neg q \; \neg r}$$

$$\mathbf{\therefore \neg(p \vee r)}$$

$$p \to q \; \neg q$$

$$\therefore \neg p$$

$$\neg p \; \neg r$$

$$\therefore \neg p \wedge \neg r$$

$$\neg p \wedge \neg r \equiv \neg(p \vee r)$$

b)

$$\begin{array}{c} \mathbf{p \leftrightarrow q \quad q \rightarrow r \quad r \lor \neg s \quad \neg s \rightarrow q} \\ \hline \mathbf{\therefore s} \end{array}$$

$$\begin{array}{c} q \rightarrow r \quad r \lor \neg s \\ \hline \therefore q \end{array}$$

$$\begin{array}{c} q \quad p \leftrightarrow q \quad \neg s \rightarrow q \\ \hline \therefore \neg s \end{array}$$

False. We can see If s is false and others are true. right side is true and left side is false.

c)

$$\begin{array}{c} \mathbf{p \quad p \rightarrow r \quad p \rightarrow (q \lor \neg r) \quad \neg q \land \neg s} \\ \hline \mathbf{\therefore s} \end{array}$$

$$\begin{array}{c} p \quad p \rightarrow r \\ \hline \therefore r \end{array}$$

$$\begin{array}{c} p \quad p \rightarrow (q \lor \neg r) \\ \hline \therefore q \lor \neg r \end{array}$$

$$\begin{array}{c} r \quad q \lor \neg r \\ \hline \therefore q \end{array}$$

$$\begin{array}{c} q \quad \neg q \lor \neg s \\ \hline \therefore s \end{array}$$

d)

$$\begin{array}{c} \mathbf{(\neg p \lor q) \rightarrow r \quad r \rightarrow (s \lor t) \quad \neg s \land \neg u \quad \neg u \rightarrow \neg t} \\ \hline \mathbf{p} \end{array}$$

$$\begin{array}{c} \neg s \land \neg u \quad \neg u \rightarrow \neg t \\ \hline \therefore \neg t \end{array}$$

$$\begin{array}{c} \neg s \land \neg u \quad \neg t \\ \hline \therefore \neg s \land \neg t \end{array}$$

$\neg s \land \neg t \equiv \neg(s \lor t)$

$$\begin{array}{c} r \rightarrow (s \lor t) \quad \neg(s \lor t) \\ \hline \end{array}$$

$\therefore \neg r$

$(\neg p \vee q) \rightarrow r \quad \neg r$

$\therefore \neg(\neg p \vee q)$

$\neg(\neg p \vee q) \equiv p \wedge \neg q$

$p \wedge \neg q$

$\therefore p$

e)

$\mathbf{\neg p \vee q \rightarrow r \quad s \vee \neg q \quad \neg t \quad p \rightarrow t \quad \neg p \wedge r \rightarrow \neg s}$

$\mathbf{\therefore \neg q}$

$\neg p \vee q \rightarrow r \quad \neg t \quad p \rightarrow t$

$\therefore r$

$\neg t \quad p \rightarrow t$

$\therefore \neg p$

$\neg p \wedge r \rightarrow \neg s \quad r \quad \neg p$

$\therefore \neg s$

$s \vee \neg q \quad \neg s$

$\therefore \neg q$

f)

$\mathbf{p \vee q \quad q \rightarrow r \quad p \wedge s \rightarrow t \quad \neg r \quad \neg q \rightarrow u \wedge s}$

$\mathbf{\therefore t}$

$p \vee q \quad q \rightarrow r \quad \neg r$

$\therefore p$

$\neg q \rightarrow u \wedge s \quad q \rightarrow r \quad \neg r$

$\therefore u \wedge s$

$u \wedge s$

$\therefore u \quad s$

$p \quad s \quad p \wedge s \rightarrow t$

$\therefore t$

۲. خواص مجموعه ها، كارديناليتي، مجموعه هاي شمارا و ناشمارا

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a)

$\forall$ x, y $\in$ R , x R y $\equiv$ x $\geq$ y

Reflexive = yes. $x \geq x$

Symmetric = no. $x \geq y$ is anti-symmetric

Transitive = yes. $x \geq y$ and $y \geq z$ then $x \geq z$

Equivalence or Partial Order = Reflexive and anti-symmetric so it's partial order

b)

$\forall$ x, y $\in$ R , x R y $\equiv$ $x^2 + y^2 = 1$

Reflexive = no. 1+1=2

Symmetric = yes. x+y = y=x

Transitive = no. $x^2 + y^2 = 1$ and $z^2 + y^2 = 1$ then $x^2 + z^2 \neq 1$ (could be equal but not for all of the possibilities)

Equivalence or Partial Order = none

c)

$\forall$ x, y $\in$ Z+ , x R y $\equiv$ x | y

Reflexive = yes. $x|x$

Symmetric = no. $\forall$ x, y $\in$ Z+ its anti-symmetric.

Transitive = yes. $x|y$ and $y|z$ then $x|z$

Equivalence or Partial Order = Reflexive and anti-symmetric so it's partial order

d)

$\forall$ x, y $\in$ R , x R y $\equiv$ |x| = |y|

Reflexive = yes. $|x| = |x|$

Symmetric = yes. $|x| = |y|$ equal $|y| = |x|$

Transitive = yes. $|x| = |y|$ and $|y| = |z|$ then $|x| = |z|$

Equivalence or Partial Order = Reflexive, Transitive and symmetric so it's Equivalence

e)

$\forall$ x, y $\in$ S , x R y $\equiv$ x and y begin with the same ten characters

Reflexive = no. it may be less than 10 char

Symmetric = yes. we can change them

Transitive = yes. x,y,z are string if x = y and y = z then x = z

Equivalence or Partial Order = none.

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الف)

$(x_n)_{n\geqslant 1}$ is an infinite sequence of distinct elements of (0,1).

$X=\{x_n|n\geqslant 1\}$, $X\subset(0,1)$ $x_0=1$.

for every $n\geqslant 0$ $f(x_n)=x_{n+1}$

for every x in $(0,1)\setminus X$ $f(x)=x$.

$f:(0,1]\to(0,1)$ is bijective.

To sum up, one extracts a copy of N from (0,1) and one uses the fact that the map $n\mapsto n+1$ is a bijection between $N\cup\{0\}$ and N

ب)

$a \in A$, $B_a = \{(a, b) \in A \times B \mid b \in B\}$.

B is countable, each B_a is countable. $\cup_{a\in A}B_a$ is the countable union of countable sets. $A \times B = \cup_{a\in A}B_a$, we have that $A \times B$ is countable.