Computer Architecture

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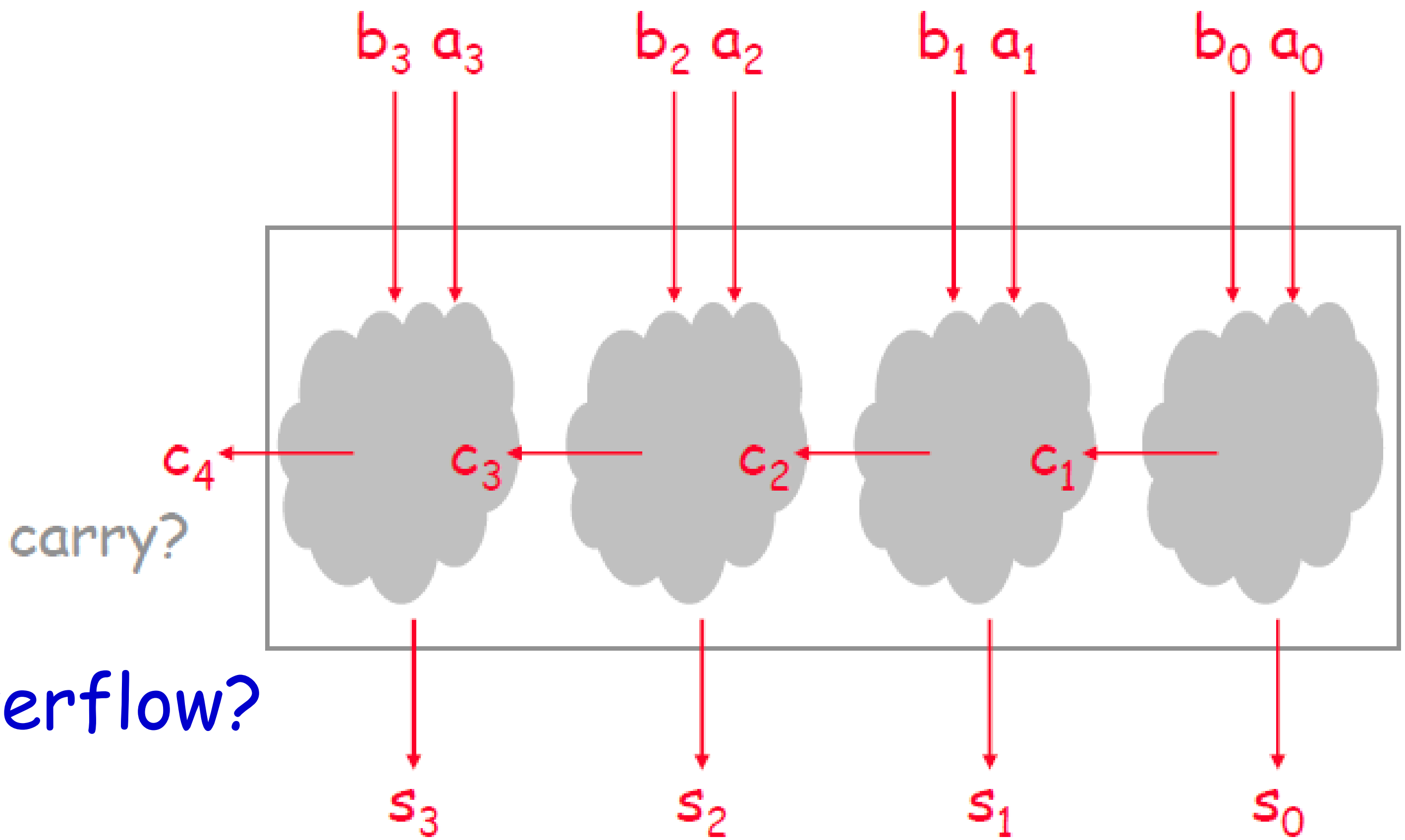
Today's Topics

- Adders
  - Ripple carry
  - Carry select adder
  - Carry-look-ahead adder
- Multipliers
  - Combinational multiplier
  - Sequential multiplier
  - Booth multiplier

Copyright Notice

- Parts (text & figures) of this lecture adopted from:
  - Computer Organization & Design, The Hardware/Software Interface, 3rd Edition, by D. Patterson and J. Hennessey, MK publishing, 2005.
  - "Intro to Computer Architecture" handouts, by Prof. Hoe, CMU, Spring 2009.
  - "Computer Architecture & Engineering" handouts, by Prof. Kubiatowicz, UC Berkeley, Spring 2004.
  - "Intro to Computer Architecture" handouts, by Prof. Hoe, UWisc, Spring 2021.
  - "Computer Arch I" handouts, by Prof. Garzarán, UIUC, Spring 2009.
  - "Intro to Computer Organization" handouts, by Prof. Mahlke & Prof. Narayanasamy, Winter 2008.

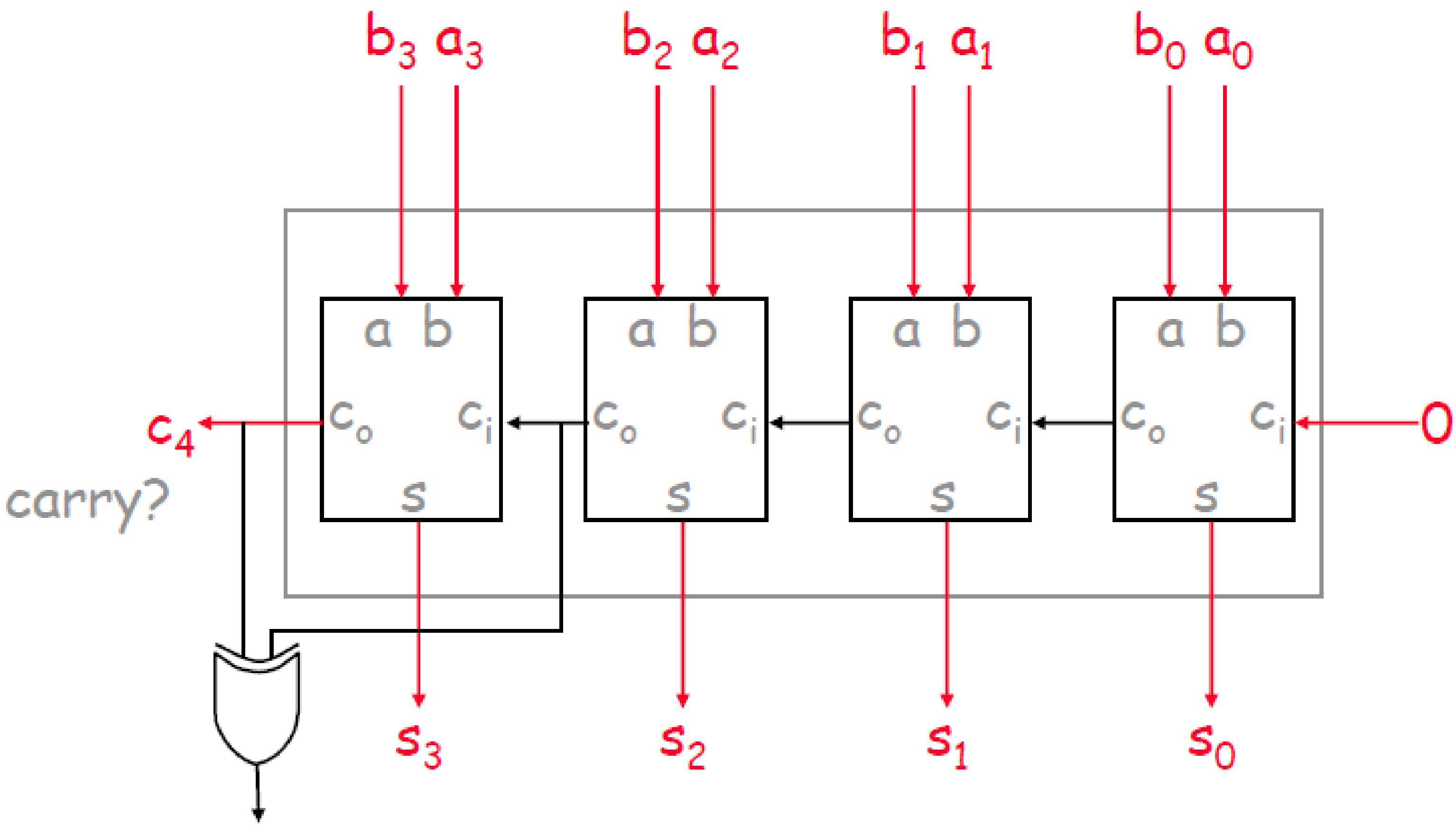
Unsigned Binary Addition



Overflow?

- Overflow = c4

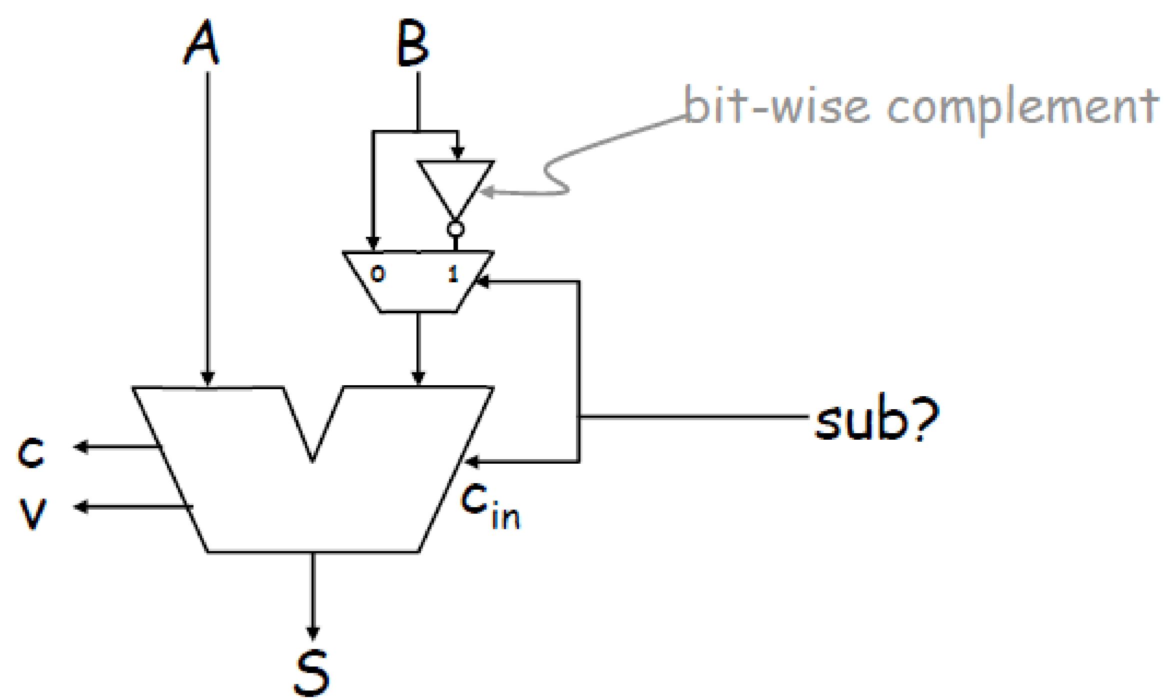
2's Complement Addition



- Overflow = (a3 xor b3) ? 0 : (a3 xor s3)

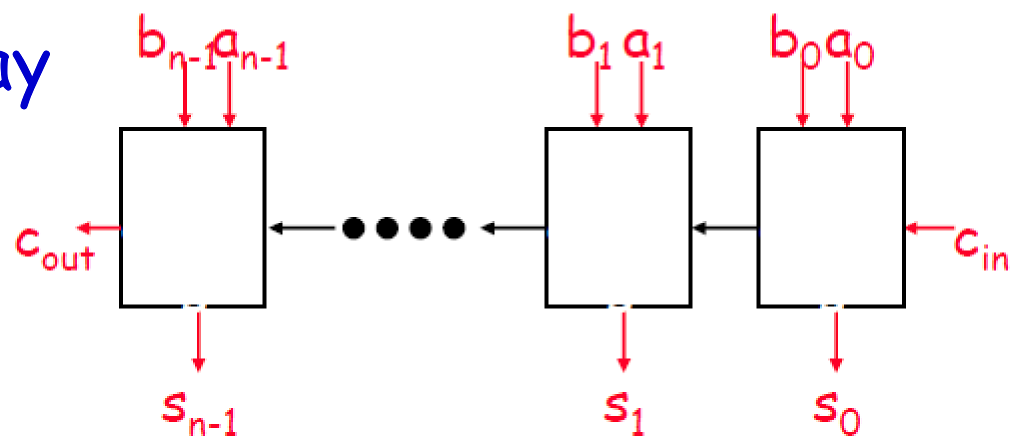
2's Complement Subtraction

- Subtraction
  - Similar to adding negative number



Analysis of an "n-bit" Ripple-Carry (RC) Adder

- Size/Complexity
  - n * SizeOf(Full Adder)
  - O(n)
- Critical Path Delay
  - n * DelayOf(Full Adder)
  - n * 2 * gate delay
  - O(n)

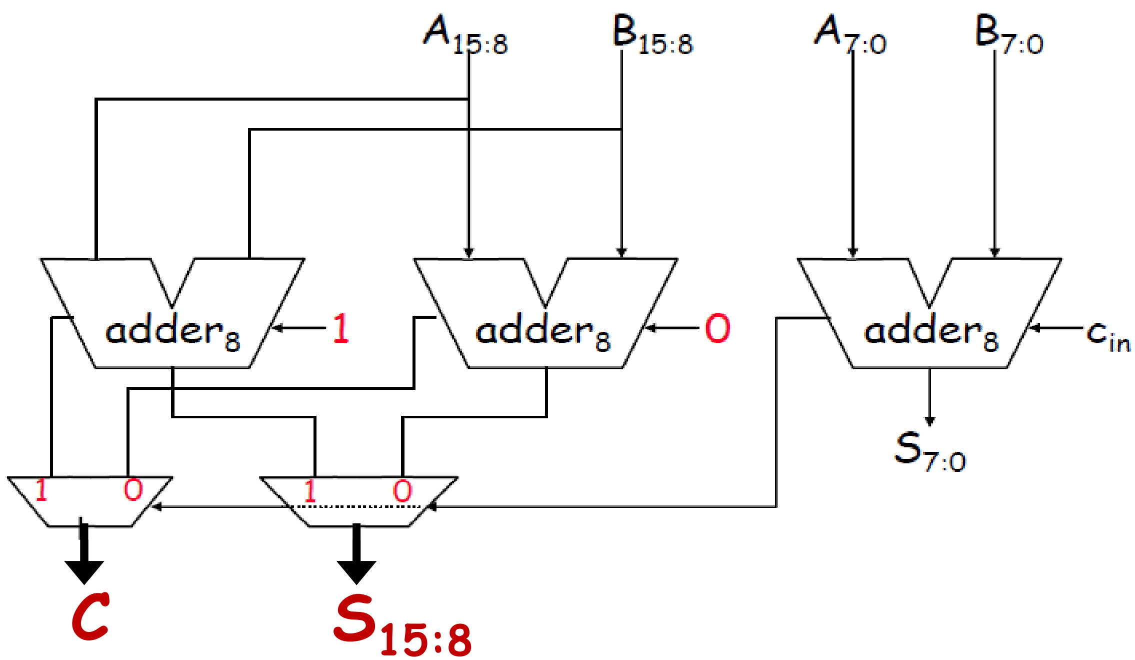


High-Performance Adders

- Question:
  - Any adder running faster than RC adder?
    - How to reduce carry propagation delay?
- Answer:
  - Compute intermediate carry signal
    - E.g., compute C1, C2, and C3 in parallel

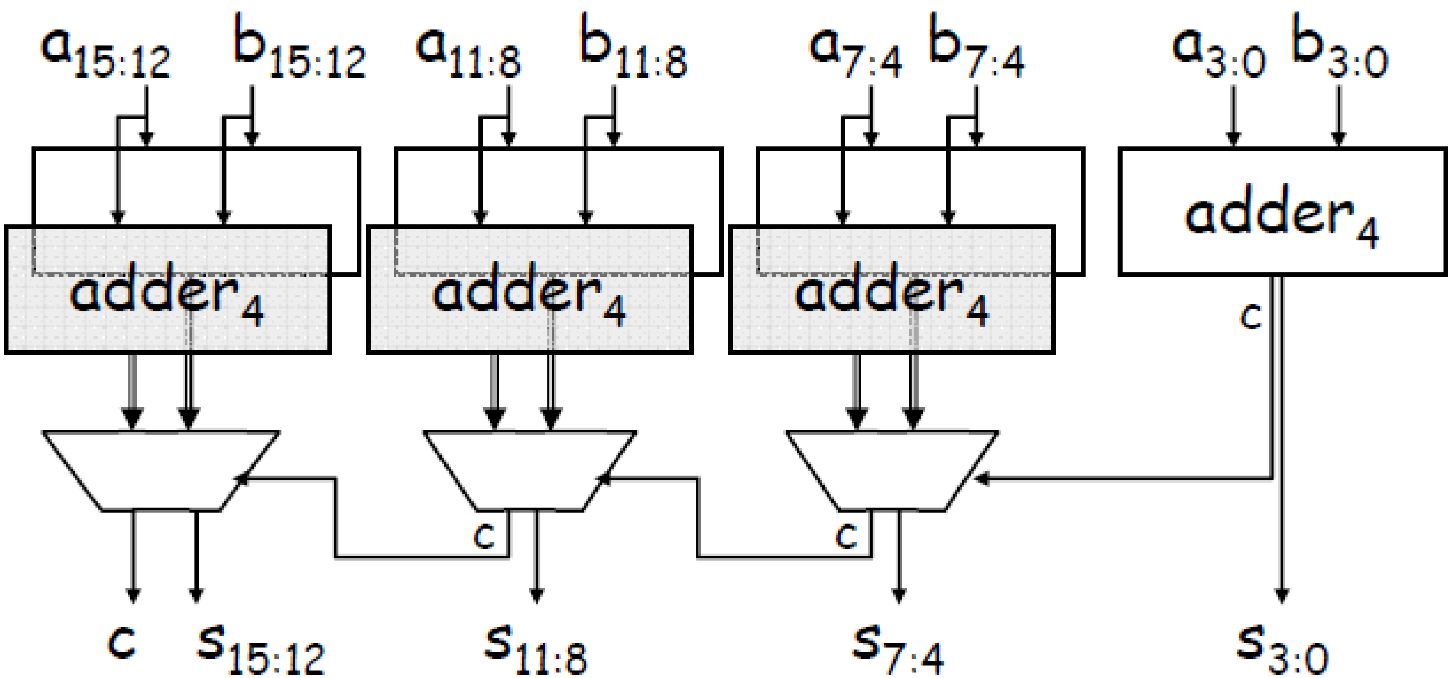
Carry-Select Adder (CSA)

- Delay
  - $8*D_{FA} + D_{mux}$
- Cost
  - $24*F$



Multi-Stage CSA

- Delay
  - $4*D_{FA} + 3*D_{mux}$
- Cost
  - 28*FA + 3*Mux



Multi-Stage CSA (cont.)

- K-Stage n-bit CSA
- Delay
  - $(n/k)*D_{FA} + (k-1)*D_{mux}$
- Cost
  - $N*\{(2k-1)/k\}*FA + (k-1)*Mux$

Carry Generate & Propagate

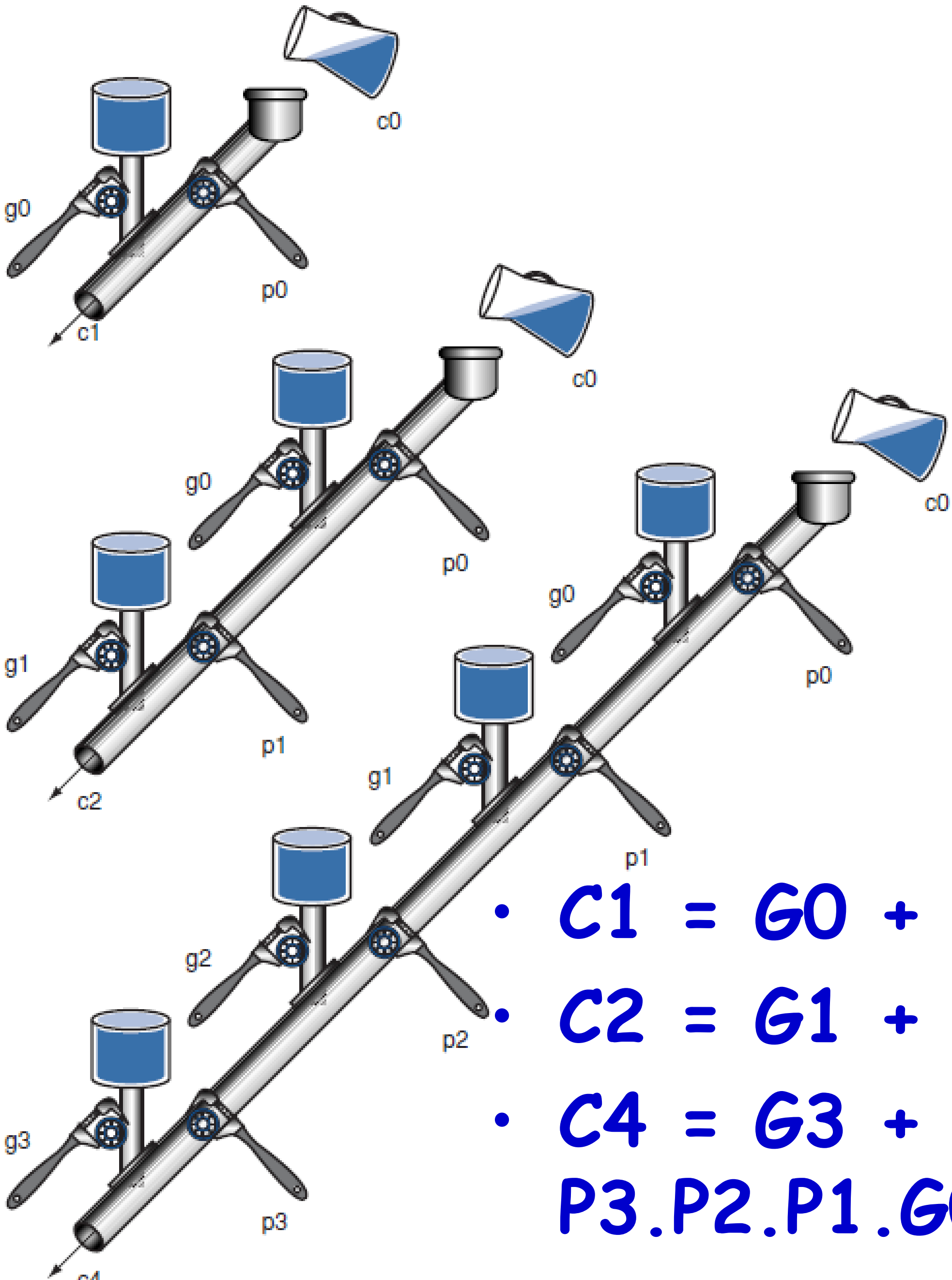
- If a.b = 1 → Cout = 1 regardless of C_{in}
  - Carry generate
- If a xor b = 1 → $C_{out} = C_{in}$
  - Carry propagate
- We define
  - $Gi = a_i.b_i$ (Generate)
  - $Pi = a_i$ xor b_i (Propagate)
  - → $C_{i+1} = G_i + P_i.C_i$

Carry Look-Ahead Adder

- $C1 = G0 + P0.C0$
- $C2 = G1 + P1.C1 = G1 + P1.(G0+P0.C0)$

  $= G1 + P1.G0 + P1.P0.C0$
- $C3 = G2 + P2.G1 + P2.P1.G0 + P2.P1.P0.C0$
- $C4 = G3 + P3.G2 + P3.P2.G1 + P3.P2.P1.G0$

  $+ P3.P2.P1.P0.C0$



- $C1 = G0 + P0.C0$
- $C2 = G1 + P1.G0 + P1.P0.C0$
- $C4 = G3 + P3.G2 + P3.P2.G1 + P3.P2.P1.G0 + P3.P2.P1.P0.C0$

Carry Look-Ahead Adder (cont.)

- Delay Complexity
  - $O(\log n)$
- Size Complexity
  - $O(n^2)$
- Manageable for Small n's
- Can be used in Two-Level CLA Adders
  - 16-bit adder using 4-bit CLA modules

Carry Look-Ahead Adder (cont.)

- Dg: Delay of a Single Gate
- At each Stage
  - Dg for generating all Pi and Gi
  - 2*Dg for generating all Ci (2-level gate)
  - 2*Dg for generating all Si (2-level gate)
- Total Delay: 5*Dg (independent of n)
  - Issue?
    - D(gate with fainin=32) >> D(gate w fainin=2)
  - [Copyright I. Kormen, umass, spring'08]

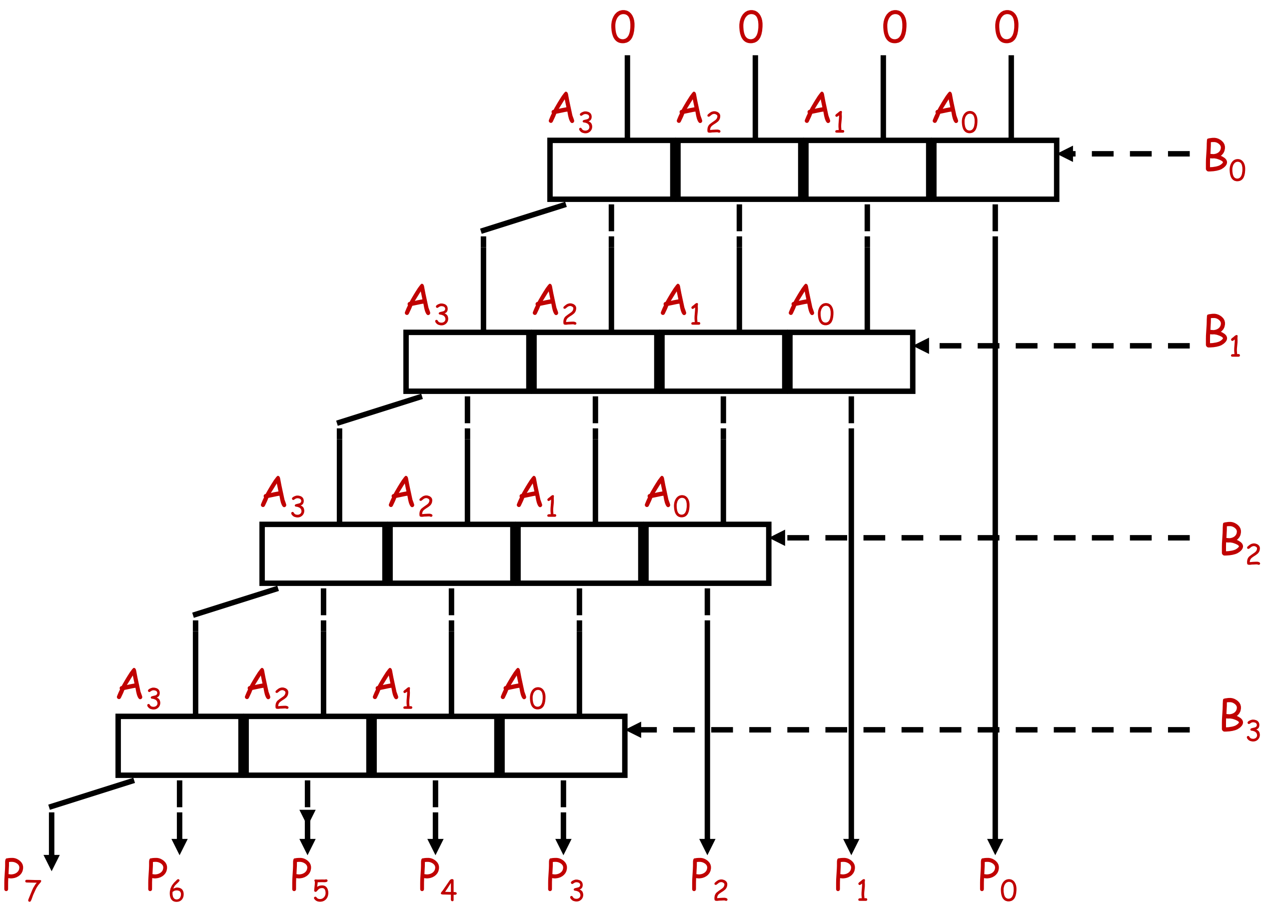
Carry Look-Ahead Adder (cont.)

- n Stages Divided into Groups
  - Separate CLA in each group
  - Group interconnected by RC
- Example: Group Size =4
  - Dg for generating all Pi and Gi
  - 2*Dg to propagate carry through a group
  - (n/4)*2*Dg to propagate carry using RC
  - 2*Dg to generate Si
  - Total: [2(n/4)+3]Dg = [n/2+3]*Dg ➔
  - 75% reduction compared to full RC

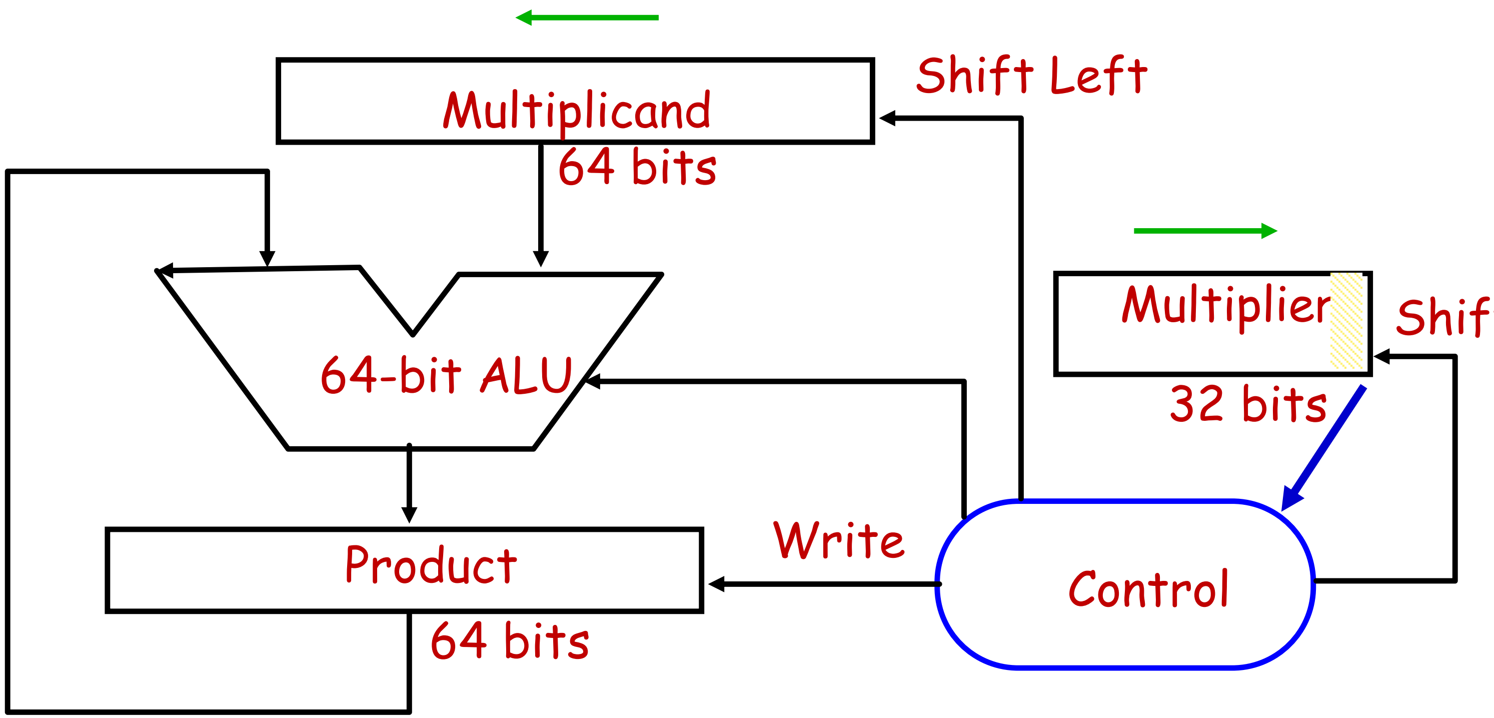
Multiplier

- Combinational Multiplier
  - Also called array multiplier
- Shift-Add Multiplier
- Booth Mulitplier

Combinational Multiplier



Shift-Add Multiplier



Booth's Algorithm

- Example 2 x 6 = 0010 x 0110:

```
            0010
x           0110
   ________________
+           0000     shift (0 in multiplier)
+          0010      add (1 in multiplier)
+         0010       add (1 in multiplier)
+        0000        shift (0 in multiplier)
   ________________
        00001100
```

- *ALU with add or subtract gets same result in more than one way:*

```
6      = – 2 + 8
0110   = – 00010 + 01000 = 11110 + 01000
```

- For example
-

```
            0010
x           0110
________________
            0000 shift (0 in multiplier)
-          0010  sub (first 1 in multply)
          0000  shift (mid string of 1s)
+        0010    add (prior step had last 1)
________________
        00001100
```

Booth's Algorithm (cont.)

Current Bit	Bit to Right	Explanation	Example	Op
1	0	Begins run of 1s	0001111000	sub
1	1	Middle of run of 1s	0001111000	none
0	1	End of run of 1s	0001111000	add
0	0	Middle of run of 0s	0001111000	none

- Originally developed for Speed
  - When shift was faster than add

Booth Encoding: Example

1	**0**	**0**	**1**	**1**	**1**	**1**	**0**	**1**	**0**
-1	0	1	0	0	0	-1	1	-1	0

Booth's Algorithm: Example

- Example
  - multiply (**-5) x 2**
- Let's use 5-bit 2's complement:

  A: -5 is 11011 (multiplier)

  B: 2 is 00010 (multiplicand)

Beginning Product

- Multiplier is:

  11011

- Add 5 leading zeros to **multiplier** to get **beginning product**:

  00000 11011

Step 1 for each pass

- Use **LSB** (least significant bit) and **previous LSB** to determine arithmetic action
  - If it is FIRST pass, use **0** as previous LSB
- Possible arithmetic actions:
  - **00** → no arithmetic operation
  - **01** → add multiplicand to left half of product
  - **10** → subtract multiplicand from left half of product
  - **11** → no arithmetic operation

Step 2 for Each Pass

- Perform an **arithmetic right shift** (ASR) on entire product
- Note:
  - For X-bit operands, Booth's algorithm requires X passes

Example

- Let's continue with our example of multiplying **(-5) x 2**
- Remember:
  - – -5 is 11011 (multiplier)
  - – 2 is 00010 (multiplicand)

- And we added 5 leading zeros to **multiplier** to get **beginning product**:

00000 11011

Example

- Initial Product and previous LSB

00000 11011 0

(Note: Since this is first pass, we use 0 for previous LSB)

- Pass 1, Step 1: Examine last 2 bits

00000 11011 0

Last two bits are 10, so we need to:

subtract **multiplicand** from left half of product

Example: Pass 1 (cont.)

- Pass 1, Step 1: Arithmetic action

```
(1)  00000     (left half of product)
    -00010     (mulitplicand)
    ------
     11110     (uses a phantom borrow)
```

- Place result into **left half** of product

11110 11011 0

Example: Pass 1 (cont.)

- Pass 1, Step 2: ASR (arithmetic shift right)
  - Before ASR

    `11110 11011 0`

  - After ASR

    `11111 01101 1`

  (left-most bit was 1, so a 1 was shifted in on the left)

- Pass 1 is complete

Example: Pass 2

- Current Product and previous LSB

11111 01101 1

- Pass 2, Step 1: Examine last 2 bits

11111 01101 1

Last two bits are 11, so we do NOT need to perform an arithmetic action --

just proceed to step 2.

Example: Pass 2 (cont.)

- Pass 2, Step 2: ASR (arithmetic shift right)
  - Before ASR

    `11111 01101 1`
  - After ASR

    `11111 10110 1`

  (left-most bit was 1, so a 1 was shifted in on left)

- Pass 2 is complete

Example: Pass 3

- Current Product and previous LSB

11111 10110 1

- Pass 3, Step 1: Examine last 2 bits

11111 10110 1

Last two bits are 01, so we need to:

add **multiplicand** to left half of product

Example: Pass 3 (cont.)

- Pass 3, Step 1: Arithmetic action

```
~~(1)~~ 11111      (left half of product)
       +00010      (mulitplicand)
        00001      (drop the leftmost carry)
```

- Place result into **left half** of product

00001 10110 1

Example: Pass 3 (cont.)

- Pass 3, Step 2: ASR (arithmetic shift right)
  - Before ASR

    00001 10110 1

  - After ASR

    00000 11011 0

  (left-most bit was 0, so a 0 was shifted in on left)

- Pass 3 is complete

Example: Pass 4

- Current Product and previous LSB

00000 11011 0

- Pass 4, Step 1: Examine last 2 bits

00000 11011 0

Last two bits are 10, so we need to:

subtract **multiplicand** from left half of product

Example: Pass 4 (cont.)

- Pass 4, Step 1: Arithmetic action

```
(1)   00000     (left half of product)
     -00010     (mulitplicand)
     ------
      11110     (uses a phantom borrow)
```

- Place result into **left half** of product

11110 11011 0

Example: Pass 4 (cont.)

- Pass 4, Step 2: ASR (arithmetic shift right)
  - Before ASR

    11110 11011 0

  - After ASR

    11111 01101 1

  (left-most bit was 1, so a 1 was shifted in on left)

- Pass 4 is complete

Example: Pass 5

- Current Product and previous LSB

11111 01101 1

- Pass 5, Step 1: Examine last 2 bits

11111 01101 1

The last two bits are 11, so we do NOT need to perform an arithmetic action --

just proceed to step 2.

Example: Pass 5 (cont.)

- Pass 5, Step 2: ASR (arithmetic shift right)
  - Before ASR

    `11111 01101 1`

  - After ASR

    `11111 10110 1`

  (left-most bit was 1, so a 1 was shifted in on left)

- Pass 5 is complete

Final Product

- We have completed 5 passes on 5-bit operands, so we are done.

- Dropping the previous LSB, resulting **final product** is:

  11111 10110

Verification

- To confirm we have correct answer, convert the 2's complement **final product** back to decimal
- Final product: **11111 10110**
- Decimal value: **-10**

which is CORRECT product of:

(-5) x 2

Backup