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Computer Simulation

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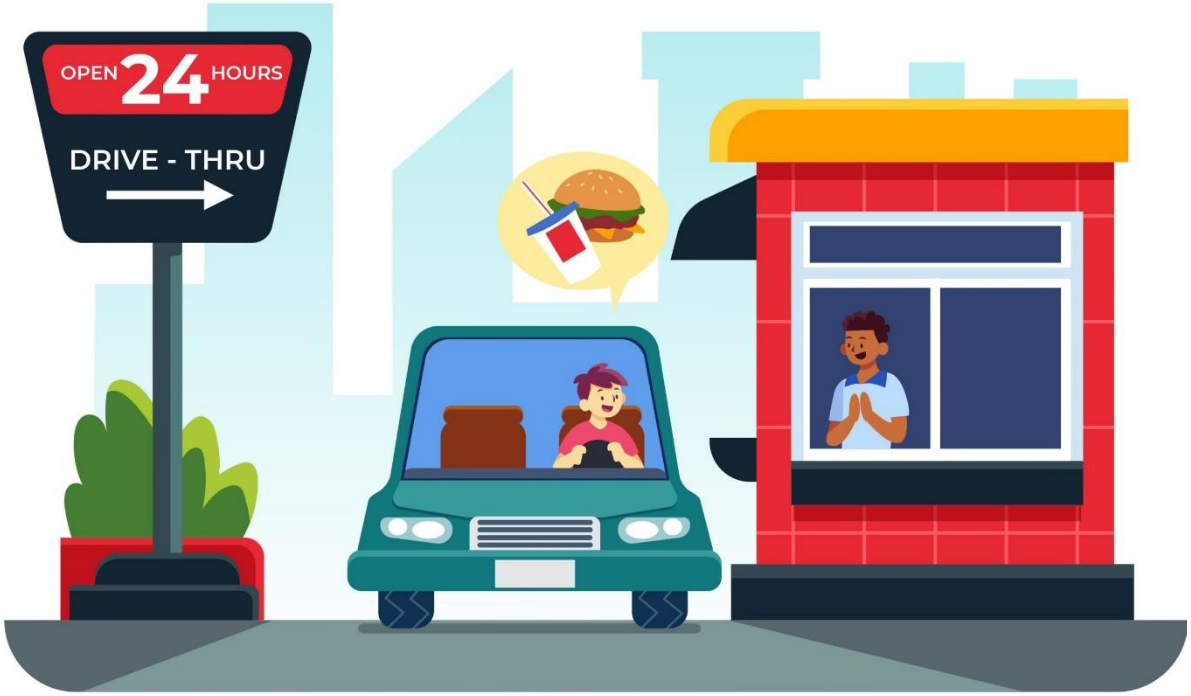
Chapter Two: Simulation Examples



Steps Required for a Successful Simulation

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- Three stages of simulations:
  - Determining the **characteristics of the inputs**
    - Typically, inputs are modeled as **probability distributions**
      - Either continuous or discrete
  - Constructing a **simulation table**
    - Based on the problem at hand, a different simulation table is developed
  - For every repetition i:
    - Generate a value for each of the p inputs, and evaluate the function, which gives the value of the response Y_i
      - The input values may be computed by **sampling values** from the distributions determined in stage 1
- A response typically depends on the inputs and one or more previous responses

Simulation Table

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- Simulation table is a representation tool for monitoring the simulated system during the simulation period
- It mainly contains:
  - Round of replication i
  - Inputs X_{ij}
  - Outputs (or response) Y_i

	Inputs						Response
Repetitions	X_{i1}	X_{i2}	…	X_{ij}	…	X_{ip}	y_i
1							
2							
⋮							
n							

Simulation of Queuing Systems

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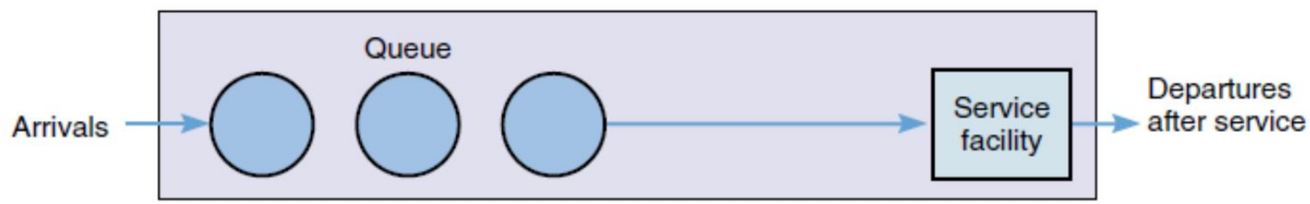
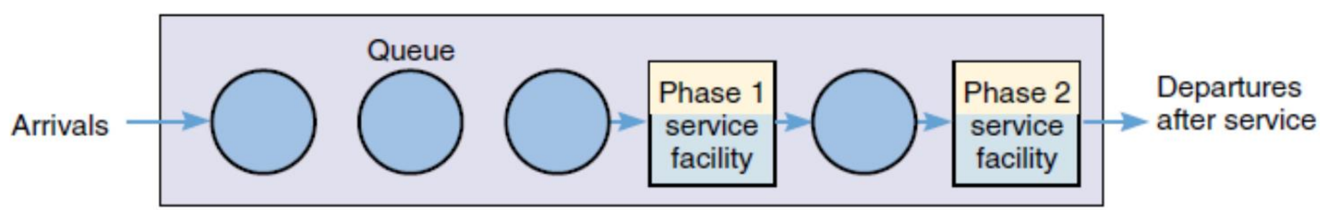
- Why Queues?
  - Queues are the best known model for discrete systems
  - We do not have a queue-less system
- A queuing system is described by:
  - Its calling population
  - The nature of the arrival
    - Indicated with **rate** of arrival
  - The service mechanism
    - Includes **rate** of service, number of servers, servers configuration
  - The system capacity
    - Number of customers can wait + Number of customers can get service
  - The queuing discipline
    - How the customers are selected for getting service
      - Example: First come, First serve (FIFO)

Servers Configuration (1)

- Based on the number of service phases, and service channels we have 4 types of configurations:
  - Single-channel/Single-phase system



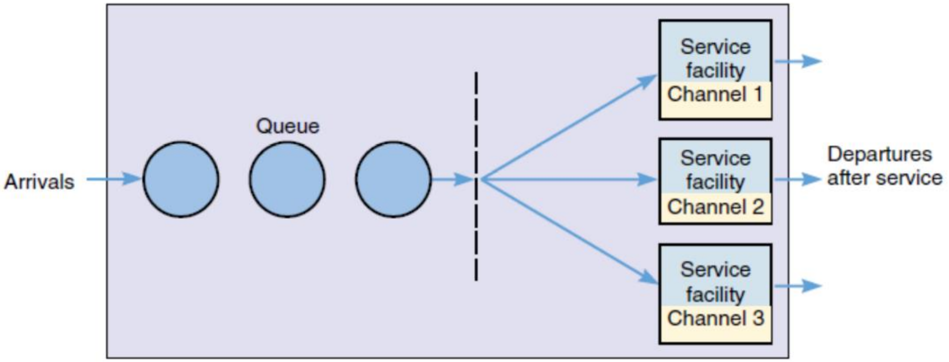
  - Single-channel/Multi-phase system



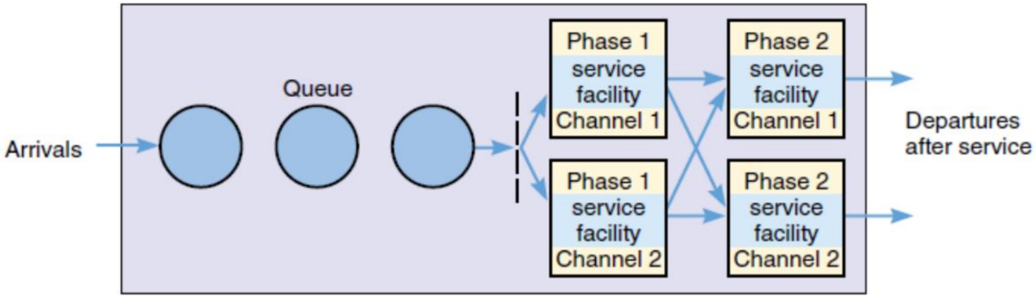
Single-channel, multiphase system

Servers Configuration (2)

- Based on the number of service phases, and service channels we have 4 types of configurations:
  - Multi-channel/Single-phase system



  - Multi-channel/Multi-phase system



Single-channel Queue Assumptions (1)

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- The calling population is **infinite**
  - If a customer leaves the calling population and joins the waiting line or enters service, there is no change in the arrival rate of other customers that may join the queue
  - This simplifies our study and reduces calculations
- Arrivals for service occur in a random fashion
- Once a customer joins the waiting line, it will be eventually served (If we have self-service process)
- Service times are of some random length according to a probability distribution
  - Which does not change over time

Single-channel Queue Assumptions (2)

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- The system capacity has no limit
  - Meaning that any number of customers can wait in line
- Customers are served in the order of their arrival
  - Often called FIFO: First In, First out
  - By a single server or channel
- Arrivals are defined by the distribution of the time between arrivals
  - Interarrival time
    - Inverse of arrival rate
- Services are defined by the distribution of service times

Single-channel Queue Assumptions (3)

- For any simple single (or multi) channel queue, the overall effective arrival rate must be less than the total service rate
  - Otherwise, the waiting line will grow without bound
  - System turns into unstable state
- In some systems, the condition about arrival rate being less than service rate may not guarantee stability
  - Other factors are also involved

Simulation of Queuing Systems (1)

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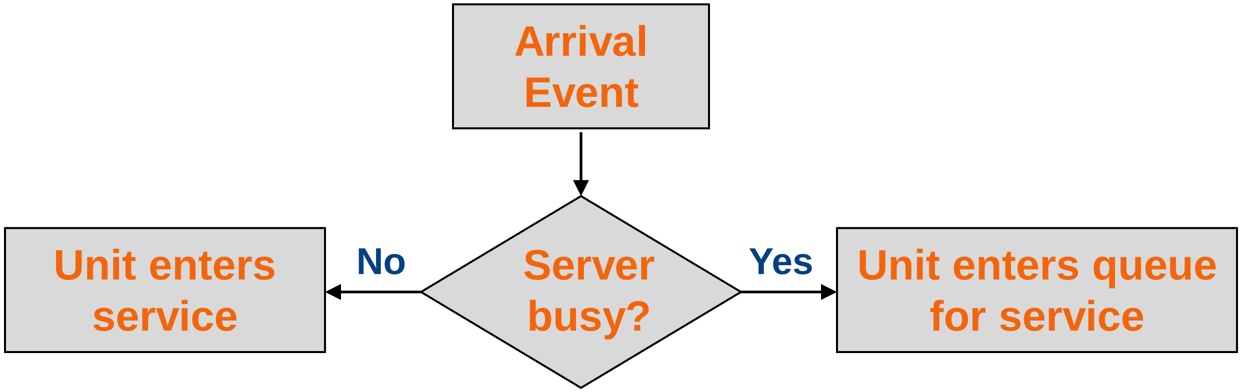
- System state in our simple queue model is the number of units in the system along with the status of the server
  - Status of the server could be busy or idle
- Recall:
  - Event is a set of circumstances that cause an instantaneous change in the state of the system
- In a single-channel queuing system there are only two possible events that can affect the state of the system:
    - Arrival event: the entry of a unit into the system
    - Departure event: the completion of service on a unit
- An important element in simulations is the **clock**
  - Clock is used to monitor the system during the simulation period

Simulation of Queuing Systems (2)

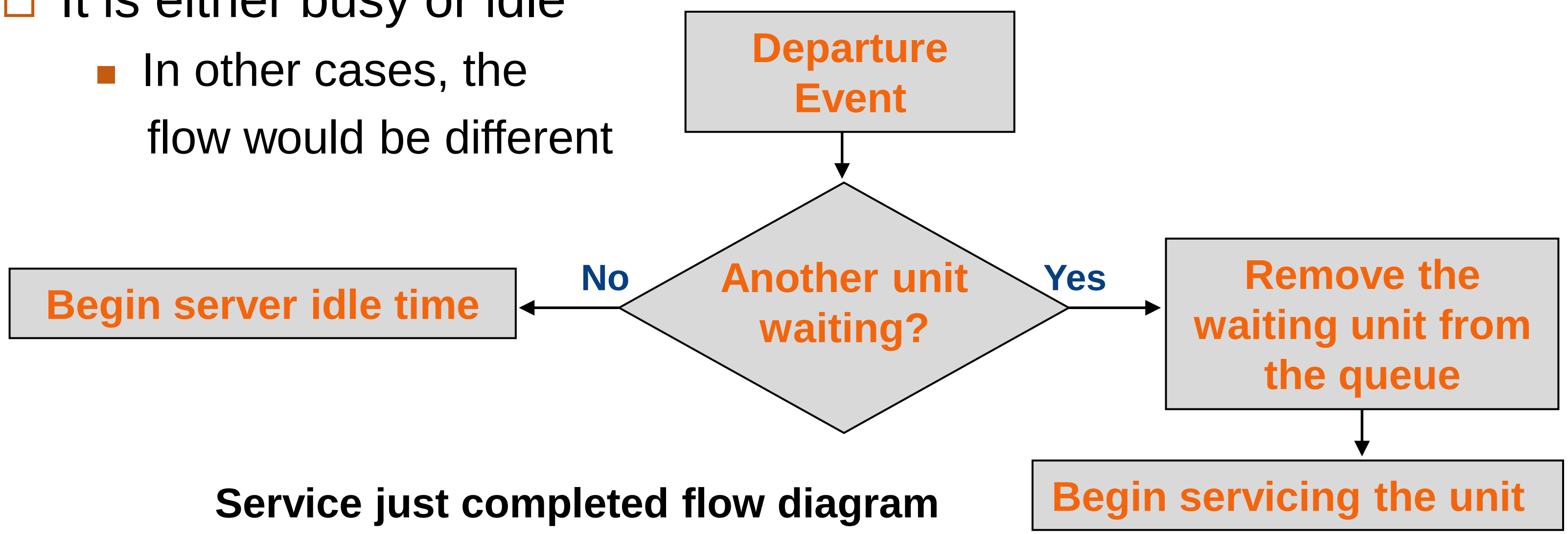
- The arrival event occurs when a unit enters the system
- The unit may find the server either idle or busy
  - Idle: the unit begins service immediately
  - Busy: the unit enters the queue for the server



Unit entering the system flow diagram

Simulation of Queuing Systems (3)

- If a unit has just completed service
  - The simulation proceeds in the manner shown in the flowing diagram
- Note that we have assumed that the server has only two possible states
  - It is either busy or idle
    - In other cases, the flow would be different



Service just completed flow diagram

Simulation of Queuing Systems (4)

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- Based on the queue and server status, the following system states are expected in the **instance** a unit enters or leaves the system

Potential unit actions upon arrival

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

Server outcomes after service completion

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Simulation of Queuing Systems (4)

- Simulation of queuing systems generally requires an event list
  - To determine what will happen next
- Simulation clock is used to determine the exact times of arrival and departure events
  - Exact times are calculated based on interarrival and service times
- Events usually occur at random times
  - Randomness provides uncertainty as in real life
  - Random numbers are distributed uniformly and independently on the interval (0,1)
  - One simple approach for generating random numbers is to randomly select digits, which are uniformly distributed on the set {0, 1, 2, … , 9}, and concatenate them
    - The proper number of digits is dictated by the accuracy of the data being used for input purposes

Simulation of Queuing Systems (5)

- Pseudo-random numbers should be generated using a procedure
  - Detailed in Chapters 6 and 7
  - What is the difference between true and pseudo random number?
- Consider a shop, where 6 customers arrive randomly
  - Assume that the **Interarrival times** for customers are generated by rolling a die five times and recording the up face

Customer	Interarrival Time	Arrival Time on Clock
1	–	0
2	2	2
3	4	6
4	1	7
5	2	9
6	6	15

Simulation of Queuing Systems (6)

- Assume that there are four possible values for **Service Times** $\in \{1,2,3,4\}$
  - All have equal probabilities to occur
  - These values could have been generated by writing down the numbers on chips and drawing the chips from a hat with replacement
    - Be sure to record the numbers selected ☺

Customer	*Service Time*
1	2
2	1
3	3
4	2
5	1
6	4

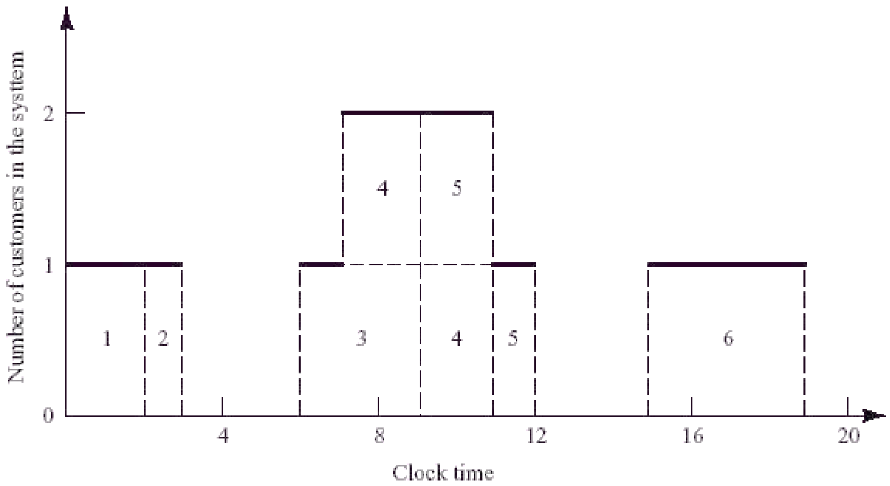
Simulation of Queuing Systems (7)

- In order to analyze our simulated single-channel queuing system, the Interarrival times and service times must be integrated
  - Two previous tables must be merged
- The following table is specifically designed for a single-channel queue, which serves customers based on FIFO
- Events are indicated based on the clock time

A Customer Number	B Arrival Time (Clock)	C Time Service Begins (Clock)	D Service Time (Duration)	E Time Service Ends (Clock)
1	0	0	2	2
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

Simulation of Queuing Systems (8)

- Event table:
  - Unlike simulation table, entries are chronically ordered based on the occurrence of events (not the customers)
  - Occurrence of two types of events (arrival and departure) in their chronological order is shown in the following table
  - Based on this table, the number of customers in every instance of time could be represented as in the figure



Event Type	Customer Number	Clock Time
Arrival	1	0
Departure	1	2
Arrival	2	2
Departure	2	3
Arrival	3	6
Arrival	4	7
Departure	3	9
Arrival	5	9
Departure	4	11
Departure	5	12
Arrival	6	15
Departure	6	19

Simulation of Queuing in a Grocery Store (1)

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- Assumptions in this example:
  - Only one checkout counter (only one server)
  - Customers arrive at this checkout counter at random from 1 to 8 minutes apart
  - Each possible value of inter-arrival time has the **same probability** of occurrence, as shown in next slide
  - The service times vary from 1 to 6 minutes with **different probabilities** shown in the next slide
- The problem is to analyze the system by simulating the arrival and service of 20 customers

Simulation of Queuing in a Grocery Store (2)

- The distribution of interarrival and service times are shown below
  - Based on the probability distribution, and their **cumulation**, the two following issues would be determined:
    - The precision required for random number generation
      - Number of decimal points
    - An interval for selecting a specific value for interarrival or service time

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.125	0.125	001 – 125
2	0.125	0.250	126 – 250
3	0.125	0.375	251 – 375
4	0.125	0.500	376 – 500
5	0.125	0.625	501 – 625
6	0.125	0.750	626 – 750
7	0.125	0.875	751 – 875
8	0.125	1.000	876 – 000

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.10	0.10	01 – 10
2	0.20	0.30	11 – 30
3	0.30	0.60	31 – 60
4	0.25	0.85	61 – 85
5	0.10	0.95	86 – 95
6	0.05	1.00	96 – 00

Simulation of Queuing in a Grocery Store (3)

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- Simulating a grocery store that starts with an empty system is **not realistic**
  - Unless, the goal is to model the system from startup until the **steady-state**
- Based on the required accuracy for interarrival and service times, we must generate a random number
  - In order to determine the value of interarrival and service time parameters for every customer, we must match the generated numbers with the intervals
    - The rightmost two columns
  - Random numbers have the following properties:
    - They are uniformly distributed between 0 and 1
    - Successive random numbers are independent
  - Use RAND() function in Excel or rand function in Matlab

Simulation of Queuing in a Grocery Store (4)

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- Example: Determining an interarrival time for the 2nd customer
  - Since there were 8 values for interarrival times with uniform distribution, each will have 0.125 probability
    - Cumulation of these values and turning them into integer intervals requires three point decimal precision for the random number
      - The generated number will be then multiplied by 1000
  - Assume that the first random digits is **913**
    - To obtain the corresponding interarrival time, enter the fourth column of the table and read **8 minutes** from the first column of the table

Customer	Random Digits	Time between Arrivals (Minutes)	Customer	Random Digits	Time between Arrivals (Minutes)
1	–	–	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

Simulation of Queuing in a Grocery Store (5)

- Determining the service time:
  - Unlike interarrival time, service time is also specified for the 1st customer
  - In the following table, the first customer's service time is 4 minutes, because the generated random digits 84 fall in the range 61-85

Customer	*Random Digits*	*Service Time (Minutes)*	*Customer*	*Random Digits*	*Service Time (Minutes)*
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	79	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

Simulation of Queuing in a Grocery Store (6)

- As we observed, the fundamental tool for a **manual simulation** is the simulation table
- One of the most important reasons for using a simulation table is to obtain **statistical measures of performance** for the intended system
- Statistical measures of performance in this example are:
  - Every **customer's time** in the system
  - The **server's idle time**
- In order to compute the required statistics, totals are formed as shown in the next table
  - These totals are calculated for:
    - **Service times**
    - **Time customers spend in the system**
    - **Idle time of the server**
    - **Customers waiting time in the queue**

Simulation of Queuing in a Grocery Store (7)

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A Customer	B *Time Since Last Arrival (Minutes)*	C *Arrival Time*	D *Service Time (Minutes)*	E *Time Service Begins*	F *Time Customer Waits in Queue (Minutes)*	G *Time Service Ends*	H *Time Customer Spends in System (Minutes)*	I *Idle Time of Server (Minutes)*
1	–	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
			68		56		124	18

Performance Analysis of Grocery Store (1)

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- Based on the achieved totals, the statistical performance measures could be calculated for analysis:
  - The **average waiting time** for a customer (minutes):

$$\text{Average waiting time} = \frac{\text{Total time customers wait in Queue}}{\text{Total number of customers}} = \frac{56}{20} = 2.8 \text{ minute}$$

  - The probability that a customer has to **wait in the queue:**

$$\text{Waiting probability} = \frac{\text{Number of customers who wait}}{\text{Total number of customers}} = \frac{13}{20} = 0.65$$

  - The percentage of **idle time** for the server:

$$\text{Probability of idle server} = \frac{\text{Total idle time of server}}{\text{Total run (time of simulation)}} = \frac{18}{86} = 0.21$$

  - The probability of the **server being busy:**

$$\text{Probability of busy server} = 1 - \text{idle probability} = 1 - 0.21 = 0.79$$

Performance Analysis of Grocery Store (2)

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- Calculations (cont'd):
  - The **average (mean) service time** (minute):

$$Average\ service\ time = \frac{Total\ service\ time}{Total\ number\ of\ customers} = \frac{68}{20} = 3.4\ \text{minute}$$

  - Note: we could have calculated the average service time without using the simulation table
    - It could be obtained by using the probability distribution table for service time, and the expected value equation

$$\text{Expected Value} = \text{E(S)} = \sum_{s=0}^{\infty} \text{S} \times \text{P(S)} \qquad \text{[Eq. 1]}$$

$$= 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(1.10) + 6(0.05) = 3.2\ \text{minute}$$

    - The expected service time is slightly lower than the average service time in the simulation
      - The longer the simulation, the closer the average will be to

Performance Analysis of Grocery Store (3)

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- Calculations (cont'd):
  - The average of interarrival times (minute):

$$\text{Average of interarrival times} = \frac{\text{Sum of interrrivals}}{\text{Number of customers} - 1} = \frac{82}{19} = 4.3 \text{ minute}$$

  - This result can be compared to the expected time for the uniformly distributed interarrivals
    - This could be calculated by finding the mean of the **discrete uniform distribution** whose endpoints are a=1 and b=8

$$\mathrm{E(A)} = \frac{\mathrm{a+b}}{2} = \frac{1+8}{2} = 4.5 \text{ minute}$$

    - Again, the longer the simulation, the closer the average will be
  - The **average waiting time** of those who wait:

$$\text{Average waiting time of those who wait} = \frac{\text{Total time customers wait in queue}}{\text{Total numbers of customers who wait}} = \frac{56}{13} = 4.3 \text{ minute}$$

Performance Analysis of Grocery Store (4)

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- Calculations (cont'd):
  - The **average time a customer spends in the system**:

$$\text{Average time customers spends in the system} = \frac{\text{Total time customers spend in the system}}{\text{Total number of customers}}$$

$$= \frac{124}{20} = 6.2 \ \text{minute}$$

  - As it could be seen:

Average time customer spends in the system = **Average time customer spends waiting in the queue** + **Average time customer spends for getting service**

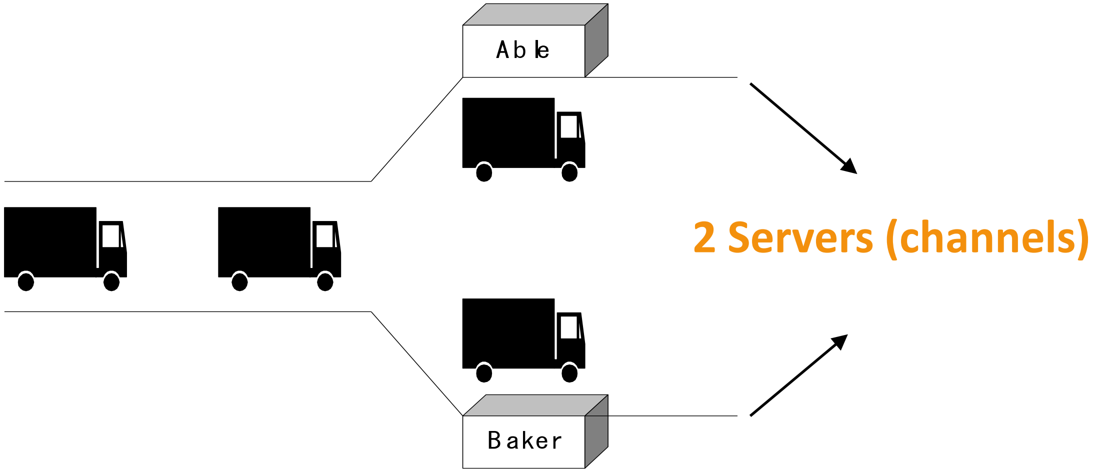
- Average time a customer spends in the system = 2.8 + 3.4 = 6.2 minute

Simulation of Able Baker Carhop Problem (1)

- In fast food restaurants, a drive-in facility has been considered
  - Customers can order and receive their food while driving
  - The person who gets the job done is called carhop
- In our example, two carhops are employed for both receiving orders, payment, and delivery simultaneously
  - Hence, the system is two-channel/single-phase
  - But the system could have been more complicated if the drivers had to order in one window, and get their food in another window, a few meters ahead
    - The system would be two-channel/two-phase

Simulation of Able Baker Carhop Problem (1)

- Assumptions:
  - Customers (cars) arrive in the manner shown in next slide
  - Our carhops have different attributes:
    - Able does the job a bit faster than Baker
    - The distribution of their service times is shown in next slide
      - There would be two different tables



A drive-in restaurant where carhops take orders and bring food to the car

Simulation of Able Baker Carhop Problem (2)

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- Tables for interarrivals and service times are illustrated below
- Rules of the problem:
  - If both carhops are idle, Able gets the customer
  - If both are busy, the customer begins service with the first server to become free
  - To analyze the system performance, a 1-hour simulation operation is made
  - The problem is to find how well the current arrangement is working?

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.25	0.25	01 – 25
2	0.40	0.65	26 – 65
3	0.20	0.85	66 – 85
4	0.15	1.00	86 – 00

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01 – 30
3	0.28	0.58	31 – 58
4	0.25	0.83	59 – 83
5	0.17	1.00	84 – 00

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.35	0.35	01 – 35
4	0.25	0.60	36 – 60
5	0.20	0.80	61 – 80
6	0.20	1.00	81 – 00

Simulation of Able Baker Carhop Problem (3)

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- Manual simulation data has been recorded as follows:

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

Simulation of Able Baker Carhop Problem (4)

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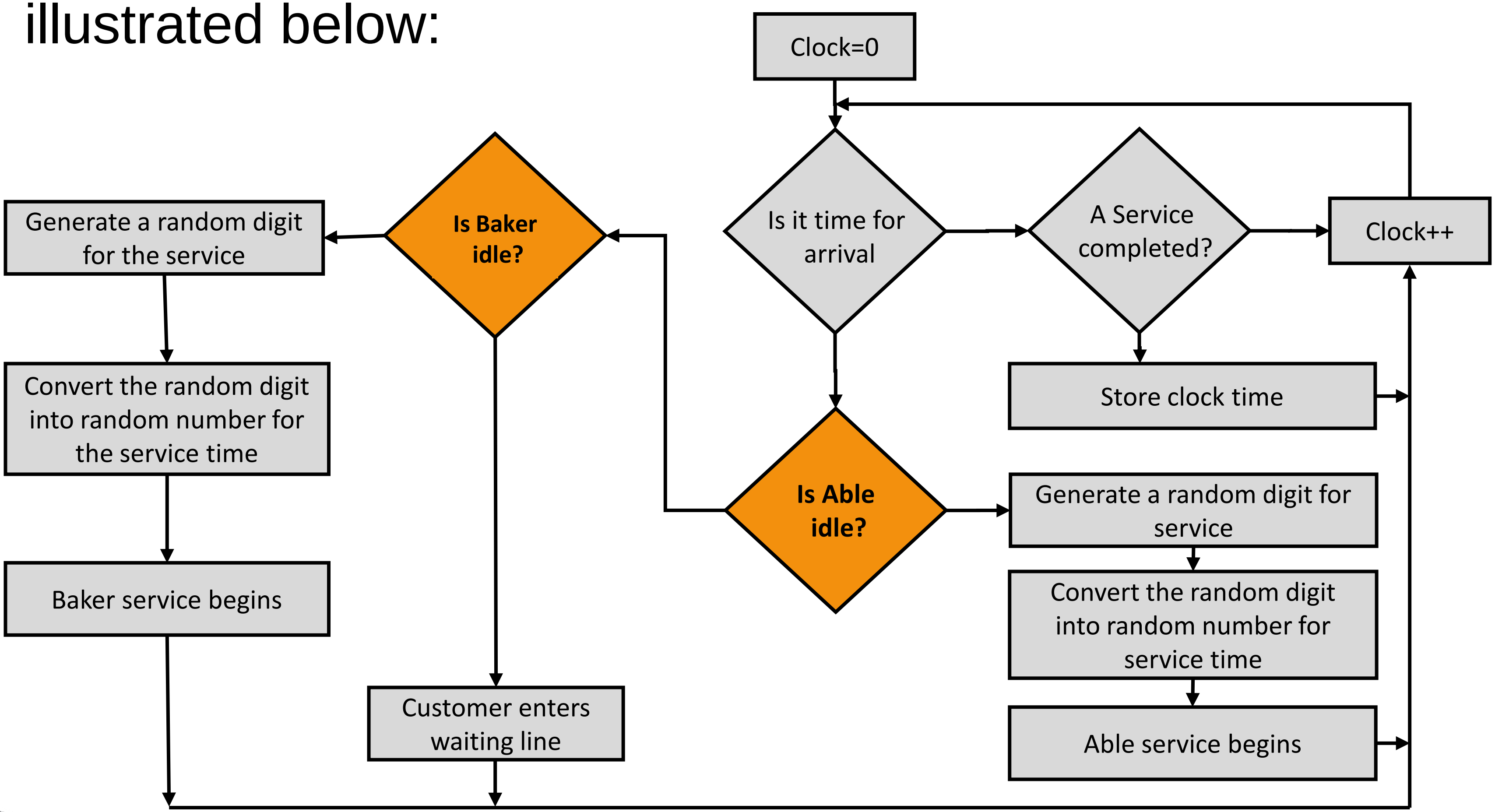
- In many cases, the pen & paper technique for obtaining the simulation table is **infeasible**
  - Especially when the number of customers and events is high and the configuration of the system is complicated
    - It takes significant time to generate the table ☹
  - Solution: Excel or Matlab
    - The row for the first customer is filled in manually
      - To generate random numbers, use **RAND()** function in case of Excel or another random function replacing the random digits
    - After the first customer, the rows for other customers must be filled based on logic and formulas
      - Example: the “Clock Time of Arrival” (column D) in the row for the second customer is computed as follows:
        - $D_i = D_{i-1} + C_i$
      - Add the arriving time for the previous customer, and interarrival time for the current customer

Simulation of Able Baker Carhop Problem (4)

- It is possible to use conditional logic for the customers in Excel
  - We can use the macro function **IF()**, which returns one of two values depending on whether a condition is true or false:
    - **IF (condition, value if true, value if false)**
  - This macro has three arguments:
    - The condition to be checked
    - The returned value in case of a true condition
    - The returned value in case of a false condition

Simulation of Able Baker Carhop Problem (5)

- Flowchart of the Able/Baker problem algorithm has been illustrated below:



Simulation of Able Baker Carhop Problem (6)

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BOOK

- How to compute when Able and Baker will become free?
  - It also helps us to calculate servers busy time
  - It is necessary to find the beginning of service for a customer
    - Let's say, customer 10 (If it gets service from Able = F10)
    - We use the built-in Excel function for maximum over a range, MAX()
    - $\mathrm{F10} = \mathrm{IF}\Big(\mathrm{D10} > \mathrm{MAX(H\$1{:}\,H9)}, \mathrm{D10}, \mathrm{IF}\Big(\mathrm{D10} > \mathrm{MAX(K\$1{:}\,K9)}, \text{""}$

      $,\ \mathrm{MIN}\big(\mathrm{MAX(H\$1{:}\,H9)}, \mathrm{MAX(K\$1{:}\,K9)}\big)\Big)\Big)$
- How the logic works?
  - D10 is the arrival of customer 10
  - H$1:H9 represents the finishing time of service for last customers who got service from Able
  - K$1:K9 represents the finishing time of service for last customers who got service from Baker

Simulation of Able Baker Carhop Problem (7)

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- If the first condition (Able idle when customer 10 arrives) is true, then the customer begins immediately at the arrival time in D10
- Otherwise, a second IF() function is evaluated, which says if Baker is idle, put nothing in the cell
- Otherwise, the function returns the time that Able or Baker becomes idle
  - Whichever is first [the minimum or MIN() of their respective completion times]
- Is this correct?
  - $$\text{F10} = \text{IF}\Big(\text{D10} > \text{MAX(H\$1:H9)}, \text{D10}, \text{IF}\Big(\text{D10} > \text{MAX(K\$1:K9)}, \text{""}, \text{MIN}\big(\text{MAX(H\$1:H9)}, \text{MAX(K\$1:K9)}\big)\Big)\Big)$$

Simulation of Able Baker Carhop Problem (8)

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- The mentioned code assumes that there is only a single column indicating the beginning of service for every customer
  - Regardless of having two servers
    - Each **must** have a column for beginning of service for their own
  - This assumption is not with accordance to principles of the problem
- We must indicate the beginning of service for a specific customer on the related column, only if the corresponding server handles the job
  - Just assume MAX(H)>MAX(K)
  - Baker beginning will be written on Able, which is **incorrect**
  - $\mathbf{IF(MAX(H\$1:H9) < MAX(K\$1:K9), MAX(H\$1:H9), "")}$

Simulation of Able Baker Carhop Problem (9)

- For **service times** for Able, you could use another IF() function to make the cell blank or have a value:
  - $\mathrm{G10} = \mathrm{IF}(\mathbf{F10} > \mathbf{0},\ \ \text{new service time}, "")$
  - F10>0 means customer 10 was assigned to Able
    - Therefore, we check the Able service time table and assign a service time to customer 10
- As you can see, service time value could be only indicated for a server, if it has a corresponding start time
  - Based on what we discussed in previous slide
- To obtain service ending time:
  - $\mathrm{H10} = \mathrm{IF}(\mathrm{F10}\ > 0, \mathrm{F10} + \mathrm{G10}, "")$

Simulation of Able Baker Carhop Problem (9)

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- Let's have a discussion about performance of this queuing system:
- The analysis of our simulation table results in the following:
  - Busy time of Able (%):
    - $\frac{\text{Total time of Able service time}}{\text{Total simulation time}} = \frac{56}{62} = 90\%$
    - Over the 62-minute period Able was busy 90% of the time.
  - Busy time of Baker (%):
    - $\frac{\text{Total time of Baker service time}}{\text{Total simulation time}} = \frac{43}{62} = 69\%$
    - Baker was busy only 69% of the time
    - The seniority rule keeps Baker less busy (and gives Able more tips)
    - Why?

Simulation of Able Baker Carhop Problem (10)

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- Nine of the 26 arrivals (about 35%) had to wait in the queue
- The average waiting time for all customers was only about 0.42 minute (25 seconds), which is very small
  - $\text{Average queue time} = \frac{11}{26} = 0.42\ \text{Minute} = 25\ \text{Seconds}$
- Those nine who did have to wait only waited an average of 1.22 min, which is quite low
  - $\text{Average queue time for those who waited} = \frac{11}{9} = 1.2\ \text{Minute}$

Simulation of Able Baker Carhop Problem (11)

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- In summary, this system seems to be well balanced
- One server cannot handle all the diners
- Three servers would probably be too many
- Adding an additional server would surely reduce the waiting time to nearly zero
  - However, the cost of waiting would have to be quite high to justify an additional server