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Computer Simulation
Assignment 2

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1. Question 1

   (a) photo

$$* E[X] = \sum_{x=0}^{\infty} (1 - P(X \leq x)) \quad ?$$

$$E(x) = \sum_i x_i \cdot P(X = x_i)$$

$$P(X \geq i) = P(X = i) + P(X = i+1) + \dots$$

$$\sum_i P(X \geq i) = P(X \geq 0) + P(X \geq 1) + P(X \geq 2) + \dots$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + \dots$$

$$= \cancel{0 \cdot P(X = 0)} + 1 \cdot P(X = 1) + 2P(X = 2) + \dots$$

$$= \sum_i i \cdot P(X = i)$$

   (b) photo

$$* \underbrace{Z}_{} = X + Y \quad \lambda = \lambda_1 + \lambda_2$$

$\xrightarrow{?}$ Poisson random var with parameter λ

$$P_Z(Z) = P(Z = z)$$

$$= \sum_{j=0}^{z} P(X = j \ \& \ Y = z - j) \quad \to X + Y = Z$$

$$= \sum_{j=0}^{z} P(X = j) \cdot P(Y = z - j) \quad \to \text{since } X, Y \text{ independent}$$

$$= \sum_{j=0}^{z} \frac{e^{-\lambda_1} \lambda_1^{j}}{j!} \frac{e^{-\lambda_2} \lambda_2^{z-j}}{(z-j)!} \quad \Big) \times \frac{z!}{z!}$$

$$= \sum_{j=0}^{z} \frac{1}{j!(z-j)!} \frac{e^{-\lambda_1} \lambda_1^{j} e^{-\lambda_2} \lambda_2^{z-j}}{z!}$$

$$= \sum_{j=0}^{z} \binom{z}{j} \frac{e^{-\lambda_1} \lambda_1^{j} e^{-\lambda_2} \lambda_2^{z-j}}{z!} \quad \Big) \ e^{-\lambda_1} e^{-\lambda_2} = e^{-\lambda}$$

$$= \frac{e^{-\lambda}}{z!} \sum_{j=0}^{z} \binom{z}{j} \lambda_1^{j} \lambda_2^{z-j}$$

$$= \frac{e^{-\lambda}}{z!} (\lambda_1 + \lambda_2)^{z}$$

$$= \frac{e^{-\lambda} \lambda^{z}}{z!} \quad \rightsquigarrow P_Z(z) = \frac{e^{-\lambda} \lambda^{z}}{z!}$$

$\Rightarrow Z = X + Y$ Poisson with λ

2. Question 2

   $1/2! = 1/2$

   Only the correct seat or the seat assigned to the first person will be available when the last person arrives. If the seat assigned for the xth person to sit in is empty when the last passenger arrives, it was empty when the xth person entered and she would have taken it at that time,which is contradictory. The same contradiction holds true for every passenger that arrives after the first.

The first and last seats must both be available whenever a passenger makes a random selection. For example, contrary to what was stated in the preceding sentence, there is a non-zero likelihood that a passenger who coms after one of these unique seats has been claimed will choose the other of the two seats at random.

These two expressions would have to be the same because every time a random choice is made, the probability of the first person's seat being chosen is equal to the probability of the last person's seat being chosen. (A) The probability that the first person's seat is taken before the last person's seat, and (B) for the probability that the last person's seat is taken before the first person's seat. Since A=B, which accounts for all scenarios, they must both equal 1/2.

3. Question 3

   (a) jupiter file

   (b) calculated in jupiter file
   (server busy time / t)*100 busy time is sum of all customers in serevr service time

4. Question 4

   (a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$\text{generator matrix} \Rightarrow \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \\ 3/2 & 3/2 & -3 \end{bmatrix}$$

   (b)

$$\pi G = \begin{bmatrix} \pi 1 & \pi 2 & \pi 3 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \\ 3/2 & 3/2 & -3 \end{bmatrix} = 0$$

$$\pi 1 + \pi 2 + \pi 3 = 1$$

$$\pi 1 = 3/19$$

$$\pi 2 = 12/19$$

$$\pi 3 = 4/19$$

5. Question 5 (practical)
   excel file in zip

   (a) p(0) = I in excel with orange, transition marix in excel with pink

   (b) tables for ten weeks in excel with green =>

$$\begin{bmatrix} previousweek \end{bmatrix} \begin{bmatrix} transitionmatrix \end{bmatrix} = \begin{bmatrix} thisweek \end{bmatrix}$$

   (c) charts in excel

   (d) 100 customers after 10 weeks in excel with blue