



Computer Simulation  
Assignment 2

1. Question 1

(a) photo

$$\begin{aligned} * E[X] &= \sum_{n=0}^{\infty} (1 - P(X \leq n)) \quad ? \\ E(X) &= \sum i \cdot x_i \cdot P(X = x_i) \\ P(X \geq i) &= P(X = i) + P(X = i+1) + \dots \\ \sum_i P(X \geq i) &= P(X \geq 0) + P(X \geq 1) + P(X \geq 2) + \dots \\ &= P(X = 0) + P(X = 1) + P(X = 2) + \dots \\ &= 0 \cdot P(X = 0) + 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + \dots \\ &= \sum_i i \cdot P(X = i) \end{aligned}$$

(b) photo

$$\begin{aligned}
 * Z &= X + Y \quad d = d_1 + d_2 \\
 &\quad \text{Poisson random var. with parameter } d \\
 P_Z(z) &= P(Z=z) \\
 &= \sum_{j=0}^z P(X=j \text{ and } Y=z-j) \rightarrow X+Y=z \\
 &= \sum_{j=0}^z P(X=j) \cdot P(Y=z-j) \rightarrow \text{since } X, Y \text{ independent} \\
 &= \sum_{j=0}^z \frac{e^{-d_1} d_1^j}{j!} \cdot \frac{e^{-d_2} d_2^{z-j}}{(z-j)!} \\
 &= \sum_{j=0}^z \frac{1}{j!(z-j)!} \cdot \frac{e^{-d_1} d_1^j e^{-d_2} d_2^{z-j}}{z!} \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \times \frac{z!}{z!} \\
 &= \sum_{j=0}^z \binom{z}{j} \frac{e^{-d_1} d_1^j e^{-d_2} d_2^{z-j}}{z!} \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} e^{-d_1} e^{-d_2} = e^{-d} \\
 &= \frac{e^{-d}}{z!} \sum_{j=0}^z \binom{z}{j} d_1^j d_2^{z-j} \\
 &= \frac{e^{-d}}{z!} (d_1 + d_2)^z \\
 &= \frac{e^{-d} d^z}{z!} \quad \rightsquigarrow P_Z(z) = \frac{e^{-d} d^z}{z!} \\
 &\Rightarrow Z = X + Y \text{ Poisson with } d
 \end{aligned}$$

## 2. Question 2

$$1/2! = 1/2$$

Only the correct seat or the seat assigned to the first person will be available when the last person arrives. If the seat assigned for the  $x$ th person to sit in is empty when the last passenger arrives, it was empty when the  $x$ th person entered and she would have taken it at that time, which is contradictory. The same contradiction holds true for every passenger that arrives after the first.

The first and last seats must both be available whenever a passenger makes a random selection. For example, contrary to what was stated in the preceding sentence, there is a non-zero likelihood that a passenger who comes after one of these unique seats has been claimed will choose the other of the two seats at random.

These two expressions would have to be the same because every time a random choice is made, the probability of the first person's seat being chosen is equal to the probability of the last person's seat being chosen. (A) The probability that the first person's seat is taken before the last person's seat, and (B) for the probability that the last person's seat is taken before the first person's seat. Since  $A=B$ , which accounts for all scenarios, they must both equal  $1/2$ .

### 3. Question 3

(a) jupiter file

(b) calculated in jupiter file

(server busy time / t)\*100 busy time is sum of all customers in server service time

### 4. Question 4

next deadline

### 5. Question 5 (practical)

next deadline