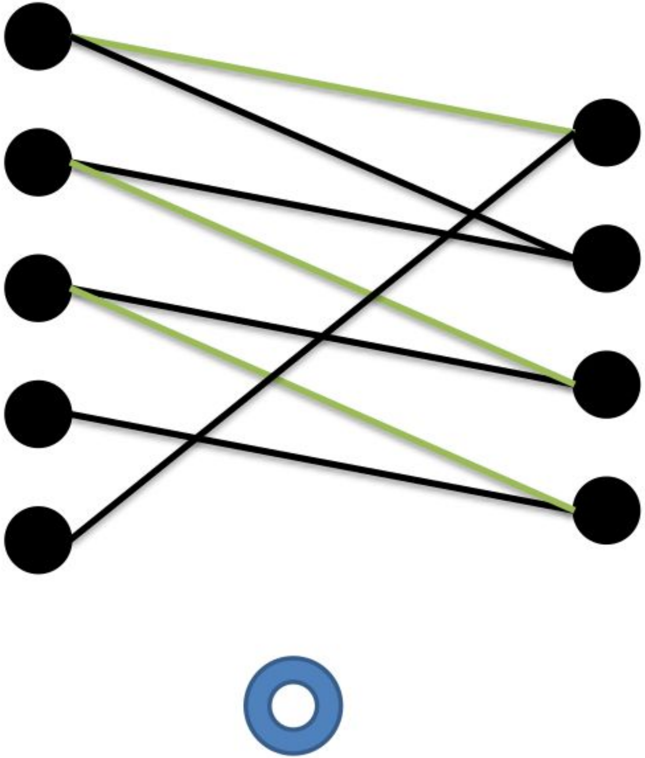
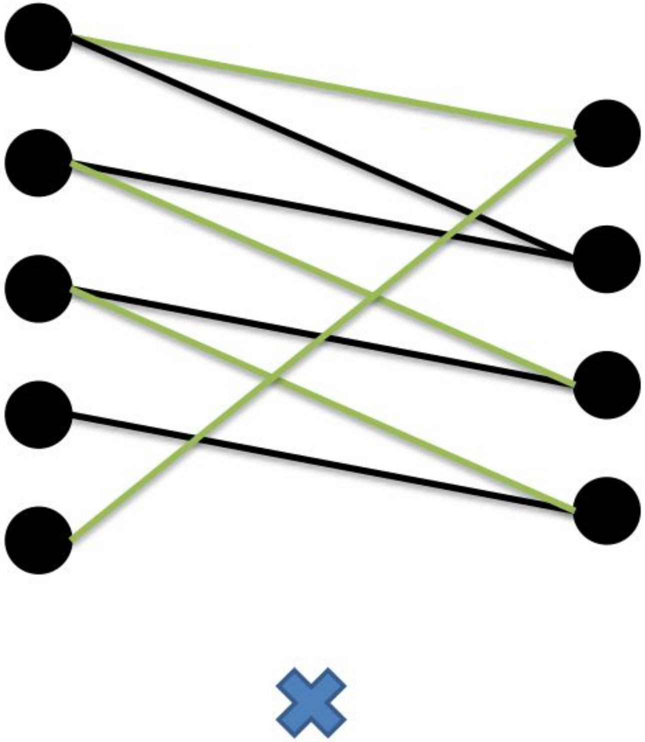
قضیه کونیگ

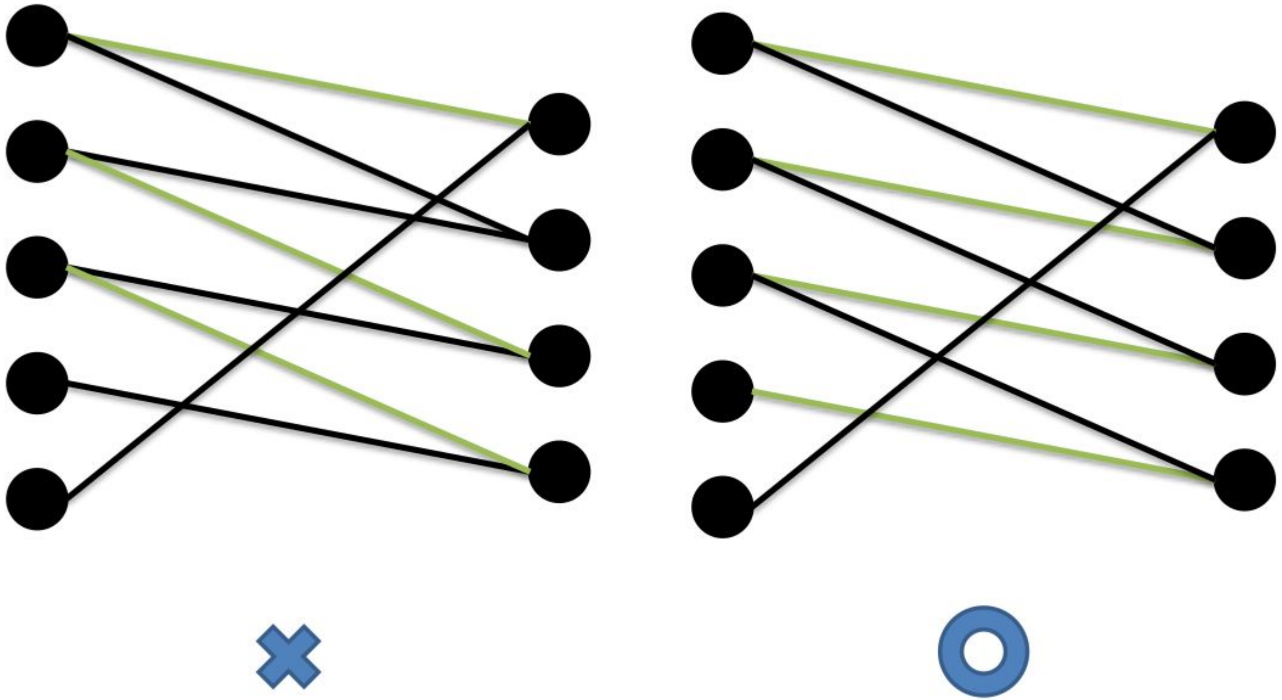
König's theorem

Matching

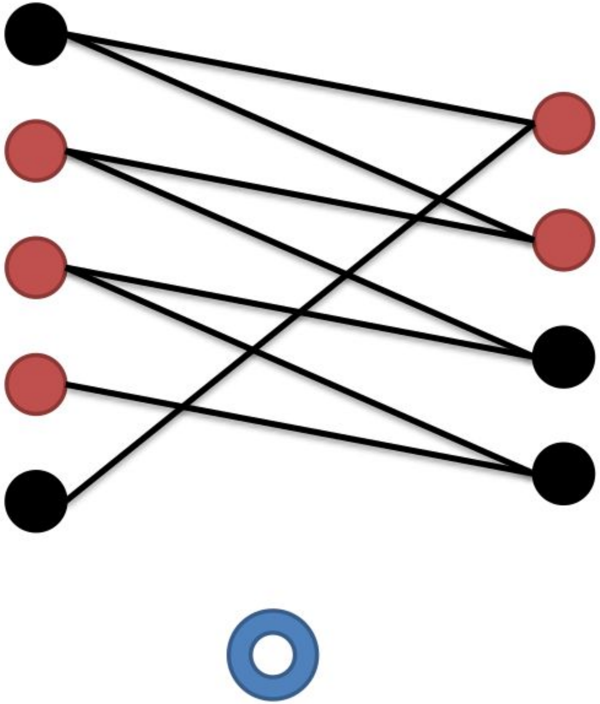


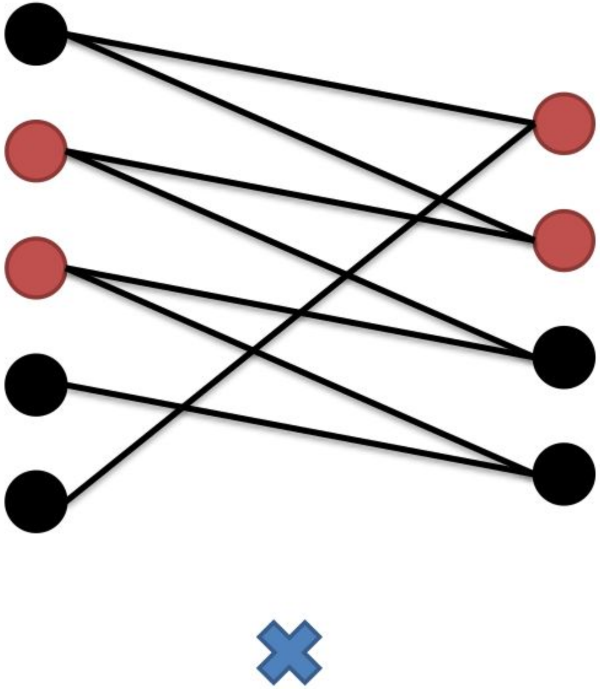


Maximum matching

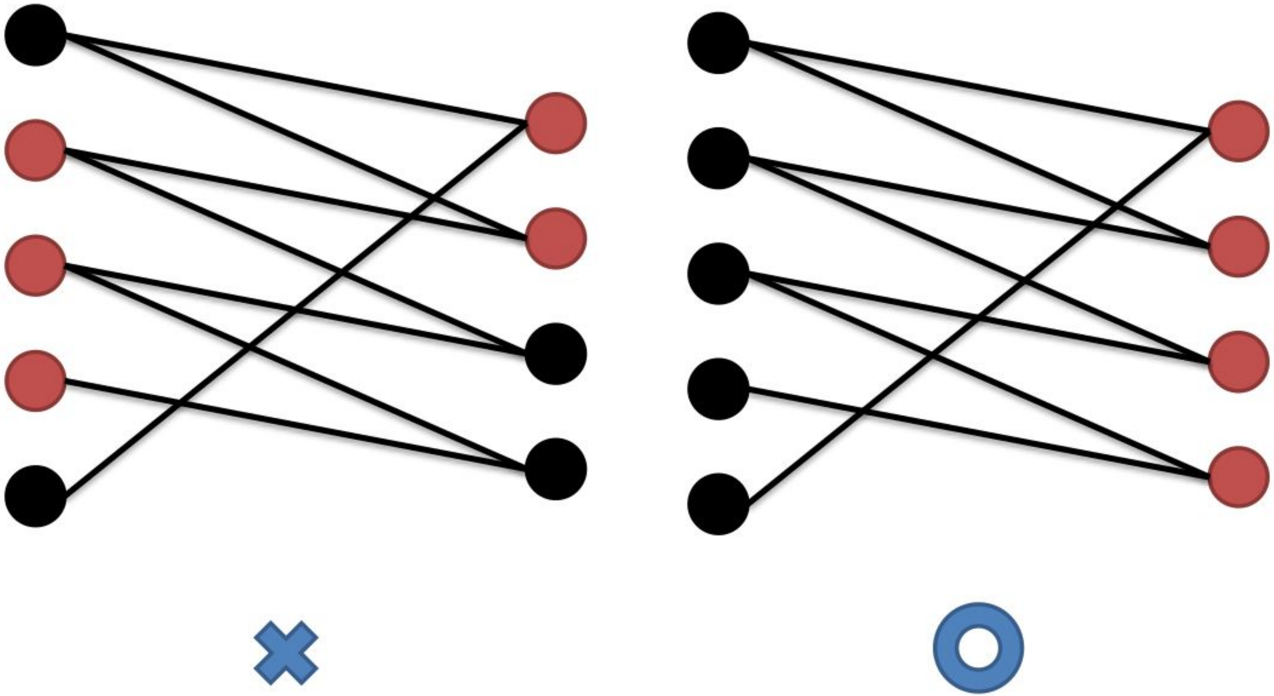


Vertex cover





Minimum vertex cover



The relationship between minimum vertex cover and maximum matching

For any graph G=(V, E):

The Cardinality of any vertex cover is equal to or greater than the cardinality of any matching.

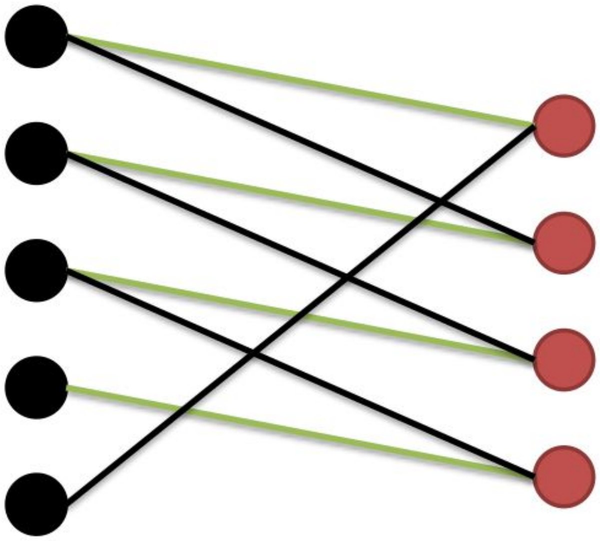
Conclusion:

The cardinality of minimum vertex cover is equal to or greater than the cardinality of maximum matching.

König's theorem

In bipartite graphs:

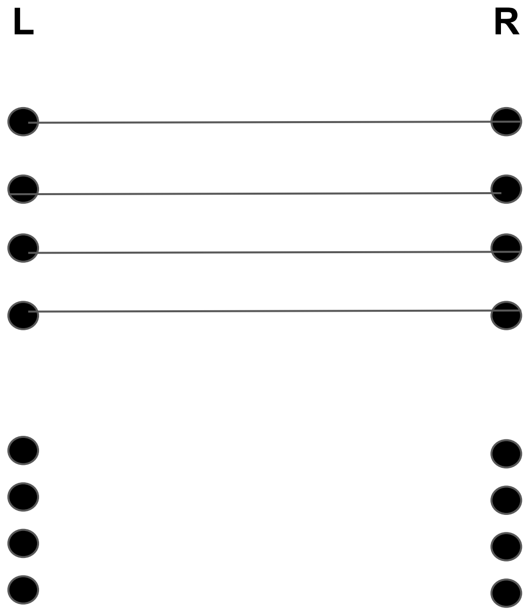
The cardinality of minimum vertex cover
is equal
to the cardinality of maximum matching



König's theorem proof

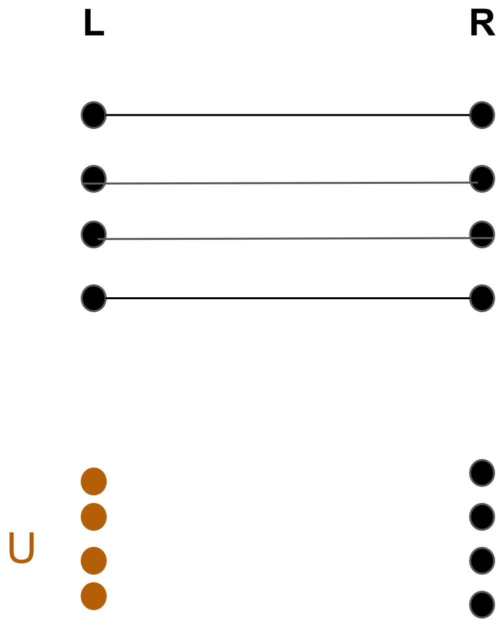
G=(V, E):

Bipartite graph with
maximum matching



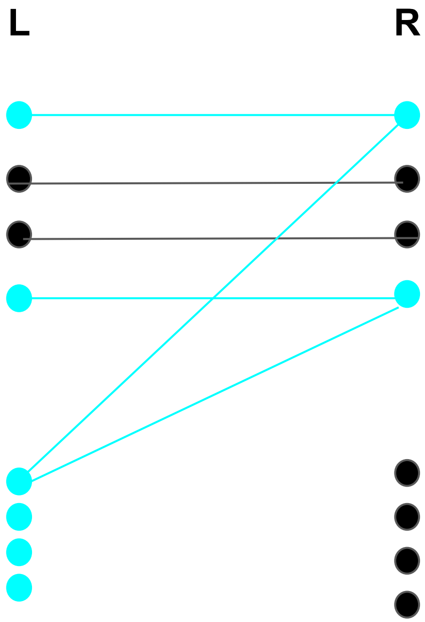
König's theorem proof

U = The set of unmatched vertices in **L**



König's theorem proof

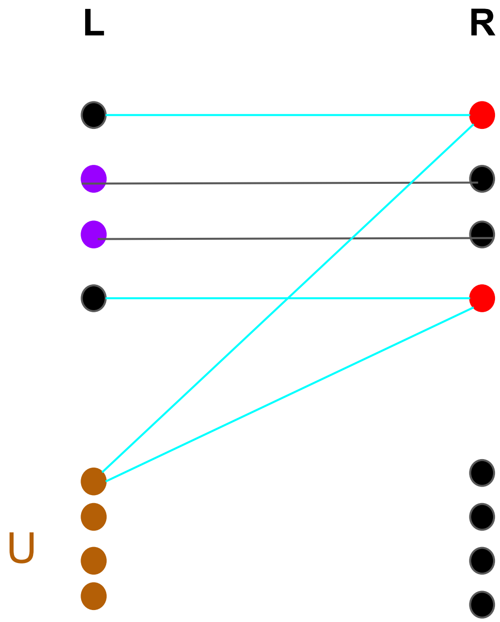
Z =The set of vertices that are either in **U** or are connected to **U** by alternating paths.



König's theorem proof

$$K = (L \setminus Z) \cup (R \cap Z).$$

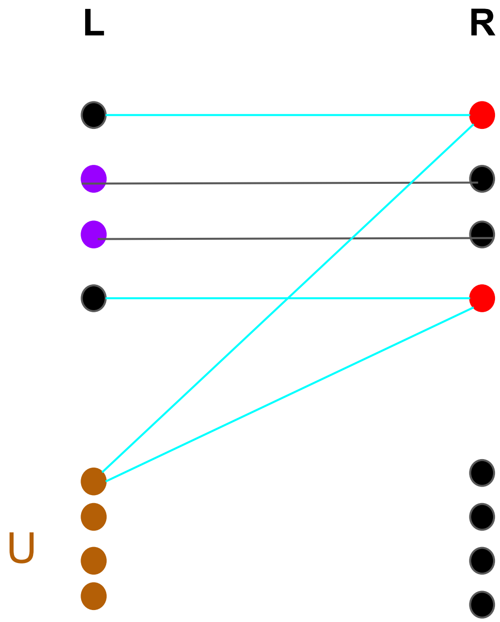
We prove that **K** is a vertex cover.



König's theorem proof

Each e in E is either:

1. Belongs to an alternating path. So it has a right endpoint in **K.**
2. It is a matching edge. So it has a left endpoint in **K**.
3. It is not a matching edge. So it has a left endpoint in **K**.



König's theorem proof

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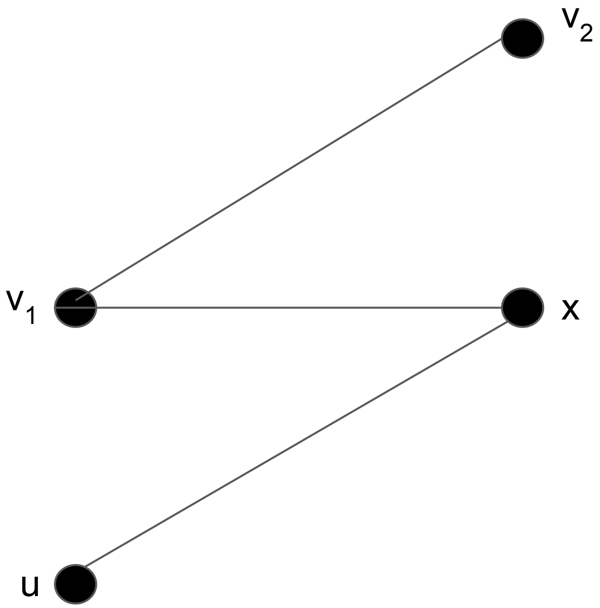
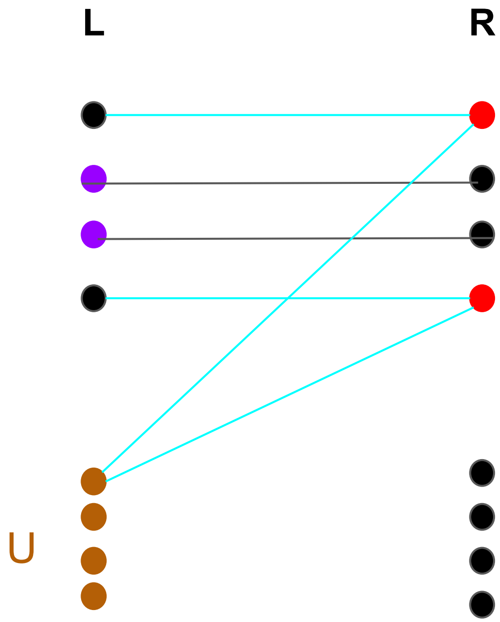


König's theorem proof

So **K** covers every edge.

Now, what is the cardinality of **K**?

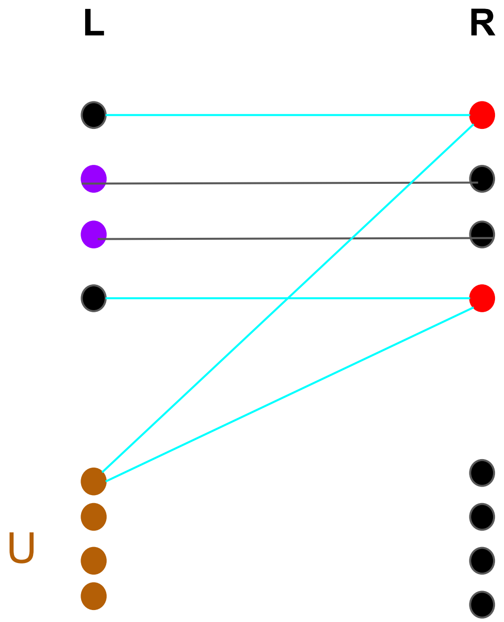
$$K = (L \setminus Z) \cup (R \cap Z).$$



König's theorem proof

Each vertex in **K** is an endpoint to a distinct matching edge.

So the cardinality of **K** is equal to the cardinality of matching.



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