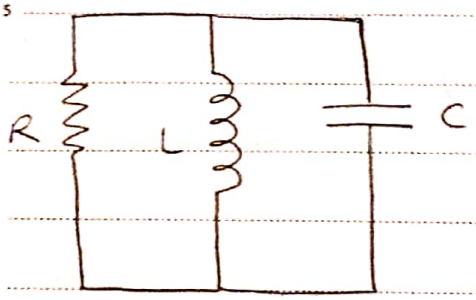
Subject:
Year. Month. Date.

مبانی مدارهای الکتریکی و الکترونیکی

جلسه 13ام

مدارهای مرتبهٔ 2



$$\frac{d^2 i_L}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha}\frac{di_L}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} i_L = 0$$

$$\rightarrow p^2 + 2\alpha p + \omega_0^2 = 0 \quad \rightarrow p_1, p_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

1) $\alpha > \omega_0 \longrightarrow p_1, p_2$: حقیقی

$$\rightarrow i_L = k_1 e^{p_1 t} + k_2 e^{p_2 t}$$

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② $\alpha = \omega_0 \quad p_1 = p_2$

$$\rightarrow i_L = k_1 e^{p_1 t} + k_2 t e^{p_2 t}$$

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③ $\alpha < \omega_0 \quad p_1, p_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ (منفی)

$\rightarrow i_L = k_1 e^{p_1 t} + k_2 e^{p_2 t}$ مختلط اند! p_1, p_2

مثلا $\rightarrow i_L = k_1 e^{(-1+2j)t} + k_2 e^{(-1-2j)t}$ $\quad j = \sqrt{-1}$

$$e^{jt} = \frac{\cos t + \sin t}{2}$$

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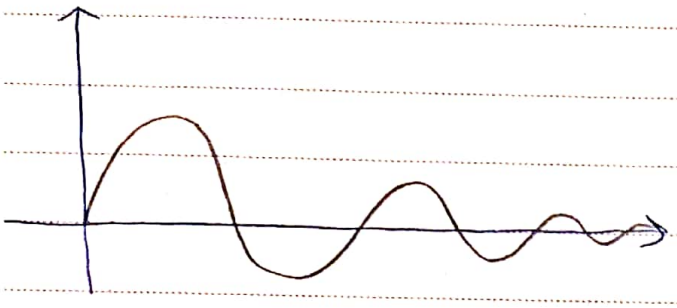
$$\rightarrow i_L = k_1 e^{-t} \times \left(\frac{\sin(2t)+\cos(2t)}{2}\right) + k_2 e^{-t}\left(\frac{\sin(-2t)+\cos(-2t)}{2}\right)$$

$$\alpha < \omega_0 \Longrightarrow i_L = k_1 e^{-\alpha t}\cos\left(\underbrace{\sqrt{\alpha^2-\omega_0^2}}_{\omega_d} t\right) + k_2 e^{-\alpha t}\sin\left(\sqrt{\alpha^2-\omega_0^2}\, t\right)$$

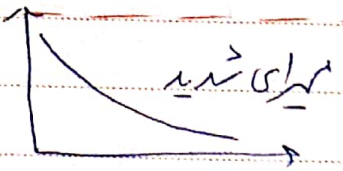
$$\rightarrow i_L = k_1 e^{-\alpha t}\cos(\omega_d t) + k_2 e^{-\alpha t}\sin(\omega_d t)$$



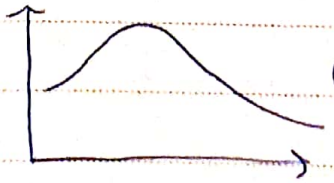
④ $\alpha = 0$ $\quad p_1, p_2 = -\alpha \pm \sqrt{\alpha^2-\omega_0^2}$

$$\rightarrow p_1, p_2 = \pm j\omega_0$$

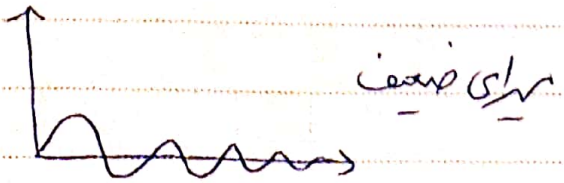
سینک ← ① $\alpha > \omega_0$



② $\alpha = \omega_0$



③ $\alpha < \omega_0$



④ $\alpha = 0$ $\quad$ ایده‌آل

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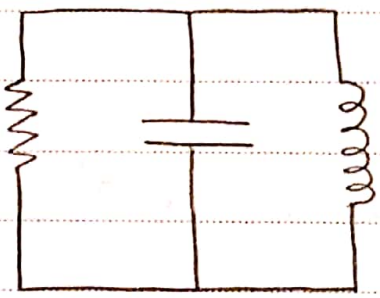
④ $R = \infty$ $\omega_0 = \sqrt{6}$

$$v_c(t) = \cos(\omega_0 t) + k_2 \sin(\omega_0 t)$$

$\longrightarrow$ $k_1 = 0$



i_L α

v_C ω_0

i_R

$$\alpha = \frac{1}{2RC} \quad , \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

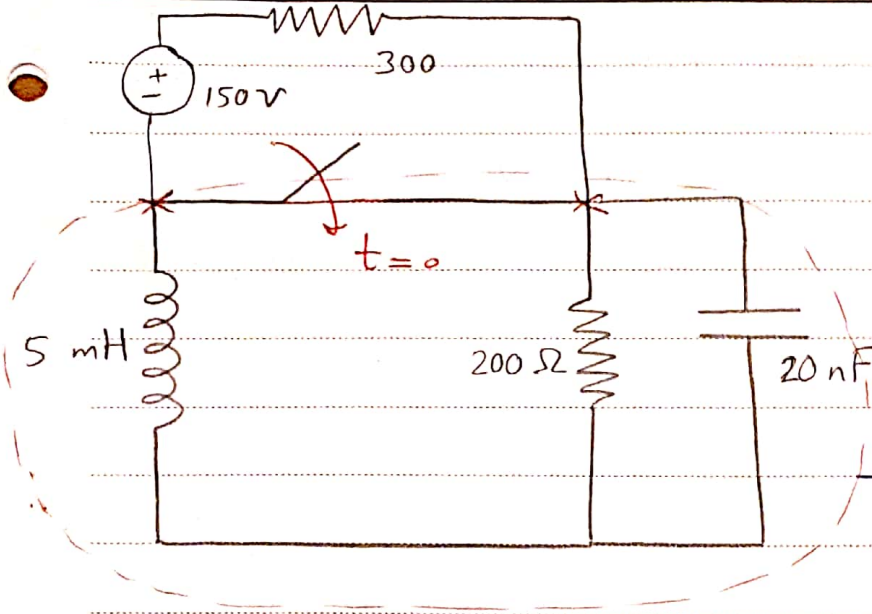
$\rightarrow K_1, K_2$ i_R $i_R(0^+)$ $\frac{d\, i_R(0^+)}{dt}$

$$\left.\begin{matrix} i_R \\ i_C \\ v_C \\ i_L \\ v_L \end{matrix}\right\} = k_1 e^{-3t} + k_2 e^{-4t}$$

مثلاً!

$\rightarrow$ شکل جواب برای همهٔ پارامترها یکسان است، امّا هرکدام K_1 و K_2 مخصوص خود با توجه به شرایط اولیهٔ شان، دارند.

مثال)



$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5$$

$$\alpha = \frac{1}{2RC} = \frac{1}{400 \times 20 \times 10^{-9}}$$

$$\alpha = 1.125 \times 10^5$$

$\alpha > \omega_0$ $\quad S_1 = -2 \times 10^5$, $S_2 = -\frac{1}{2} \times 10^5$

$V_C(0^+)$

$t=0^-$ ← سلف اتصال کوتاه
خازن مدار باز