مثال ۱

$$KCL_A: \quad i_1 - i_2 = 0 \qquad ①$$

$$KVL: \quad -V_s + V_1 + V_2 = 0 \qquad ②$$

$$R_1 \text{ مشخصه}: \quad V_1 = R_1 i_1 \qquad ③$$

$$R_2 \text{ مشخصه}: \quad V_2 = R_2 i_2 \qquad ④$$

$$① \Rightarrow i_1 = i_2 \qquad\qquad\qquad\longrightarrow (R_1 + R_2) i_1 = V_s$$

$$②,③,④ \Longrightarrow -V_s + R_1 i_1 + R_2 i_2 = 0$$

$$\Rightarrow i_1 = \frac{V_s}{R_1 + R_2} = i_2 \longrightarrow \begin{cases} V_1 = \frac{R_1}{R_1 + R_2} V_s \\ V_2 = \frac{R_2}{R_1 + R_2} V_s \end{cases}$$
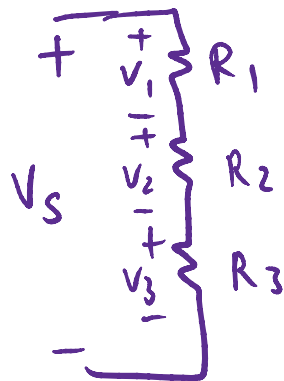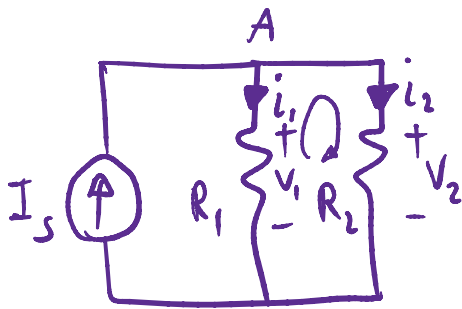
$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V_s$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_s$$
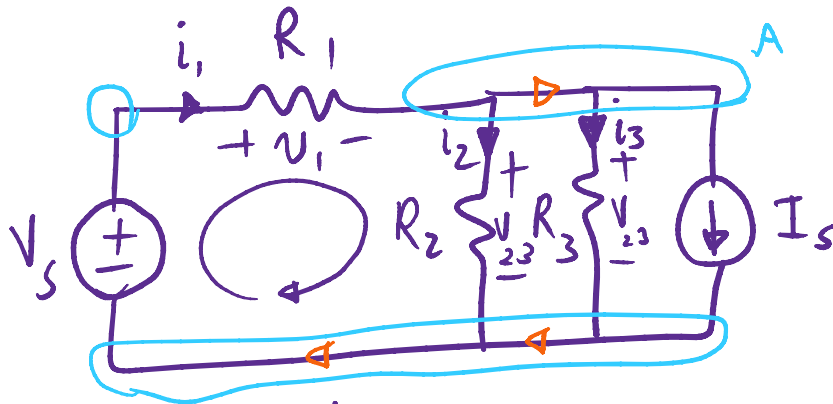
$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V_s$$

$$\begin{cases} KCLA: \quad I_s - i_1 - i_2 = 0 \quad ① \\ KVL: \quad -V_1 + V_2 = 0 \quad ② \\ R_1 \text{ قانون}: \quad i_1 = \dfrac{V_1}{R_1} \quad ③ \\ R_2 \text{ قانون}: \quad i_2 = \dfrac{V_2}{R_2} \quad ④ \end{cases}$$

$② \Rightarrow \quad V_1 = V_2$

$①,③,④ \Rightarrow \quad I_s = \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} \quad \rightrightarrows \quad \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)V_1 = I_s$

$\Rightarrow \quad V_1 = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I_s = \dfrac{R_1 R_2}{R_1 + R_2} I_s = V_2$

$\rightarrow \begin{cases} i_1 = \dfrac{1/R_1}{\frac{1}{R_1} + \frac{1}{R_2}} I_s = \dfrac{G_1}{G_1 + G_2} I_s \\[4mm] i_2 = \dfrac{1/R_2}{\frac{1}{R_1} + \frac{1}{R_2}} I_s = \dfrac{G_2}{G_1 + G_2} I_s \end{cases}$

$$R_1 \text{ منفصل} \quad v_1 = R_1 i_1$$

$$R_2 \quad '' \quad v_{23} = R_2 i_2$$

$$R_3 \quad '' \quad v_{23} = R_3 i_3 \quad \Rightarrow \quad i_3 = \frac{R_2}{R_3} i_2$$

$$\text{KCL A} \quad i_1 - i_2 - i_3 - I_s = 0 \quad \star$$

$$\text{KVL} \quad -V_s + v_1 + v_{23} = 0 \quad \longrightarrow \quad V_s = R_1 i_1 + R_2 i_2$$

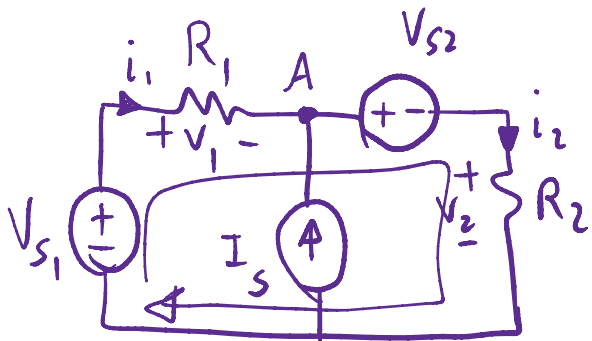$$\Rightarrow i_2 = \frac{V_s - R_1 i_1}{R_2}$$

$$\star \Rightarrow i_1 - \left(\frac{V_s - R_1 i_1}{R_2} \right) - \left(\frac{R_2}{R_3} \left(\frac{V_s - R_1 i_1}{R_2} \right) \right) - I_s = 0$$

$$\Rightarrow \left[1 - \left(1 + \frac{R_2}{R_3} \right) \left(-\frac{R_1}{R_2} \right) \right] i_1 - \left(1 + \frac{R_2}{R_3} \right) \left(\frac{V_s}{R_2} \right) - I_s = 0$$

$$\Rightarrow i_1 = \frac{\frac{R_2 + R_3}{R_2 R_3} V_s + I_s}{1 + \left(\frac{R_1}{R_2} + \frac{R_1}{R_3} \right)} = \frac{\frac{R_2 + R_3}{R_2 R_3} V_s + I_s}{\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2 R_3}}$$

$$\boxed{ i_1 = \frac{(R_2 + R_3) V_s + R_2 R_3 I_s}{R_1 R_2 + R_2 R_3 + R_3 R_1} } \Rightarrow \begin{cases} i_2 \\ i_3 \\ v_1 \\ v_{23} \end{cases}$$

مسأله ۴

$$\begin{cases} V_1 = R_1 i_1 \\ V_2 = R_2 i_2 \\ \text{KCLA} \quad i_1 + I_s - i_2 = 0 \Rightarrow i_1 = i_2 - I_s \; ** \\ \text{KVL} \quad -V_{s_1} + V_1 + V_{s_2} + V_2 = 0 \; * \end{cases}$$
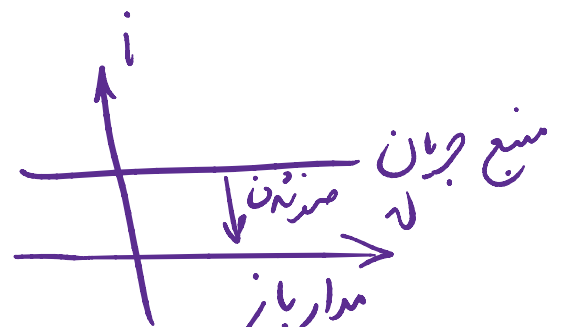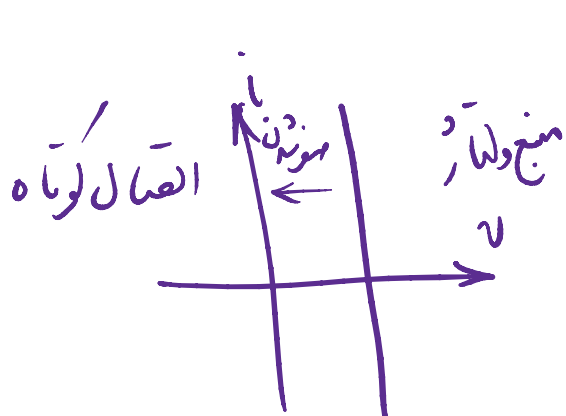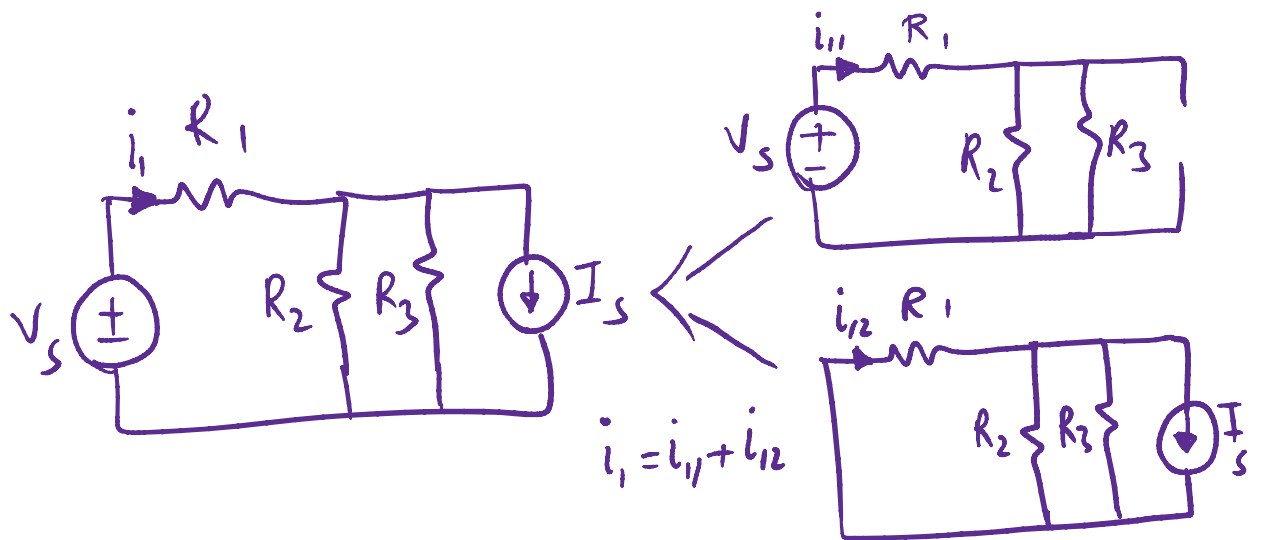
$$* \Rightarrow -V_{s_1} + R_1 i_1 + V_{s_2} + R_2 i_2 = 0$$

$$** \Rightarrow -V_{s_1} + R_1(i_2 - I_s) + V_{s_2} + R_2 i_2 = 0$$

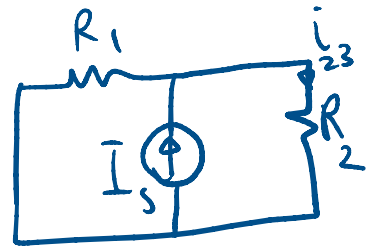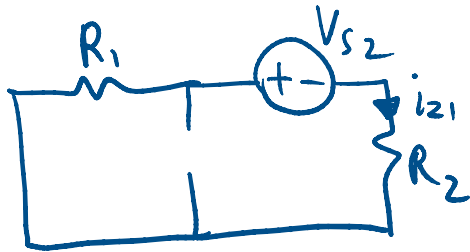$$\Rightarrow \boxed{i_2 = \frac{V_{s_1} - V_{s_2} + R_1 I_s}{R_1 + R_2}} \Rightarrow \begin{cases} i_1 \\ V_1 \\ V_2 \end{cases}$$

قضیه برهم نهی (جمع آثار)

در هر مدار خطی، پاسخ به تمامی منابع موجود برابر مجموع پاسخ به تک تک منابع به تنهایی است.

$$i_1 = i_{11} + i_{12}$$

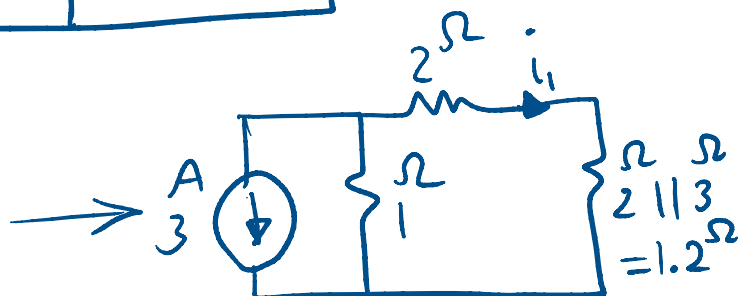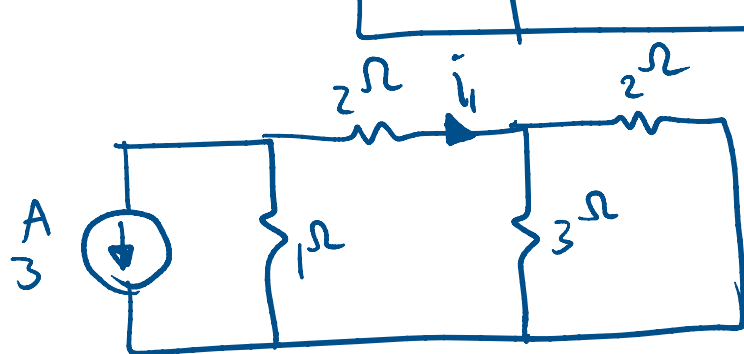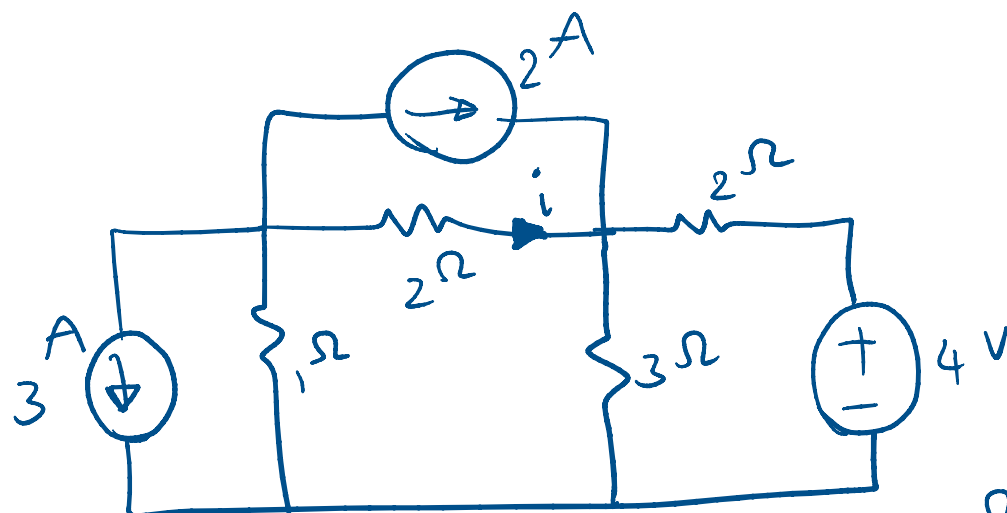$$i_{21} = -\frac{V_{S2}}{R_1 + R_2}$$

$$i_{22} = \frac{V_{S1}}{R_1 + R_2}$$

$$i_{23} = \frac{R_1}{R_1 + R_2} I_S$$

$$i_2 = -\frac{V_{S2}}{R_1 + R_2} + \frac{V_{S1}}{R_1 + R_2} + \frac{R_1 I_S}{R_1 + R_2}$$
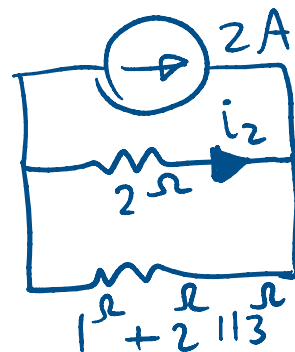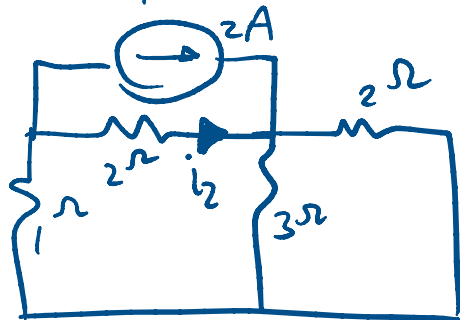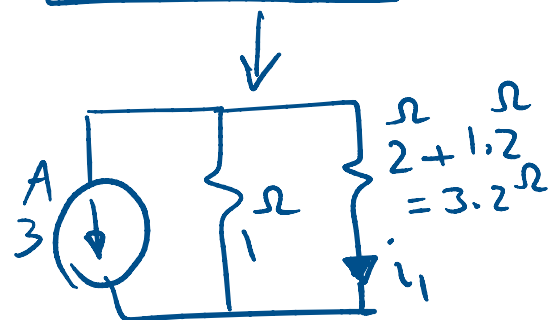
$$\boxed{i_2 = \frac{-V_{S2} + V_{S1} + R_1 I_S}{R_1 + R_2}} \quad \checkmark$$
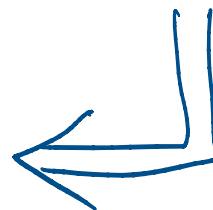
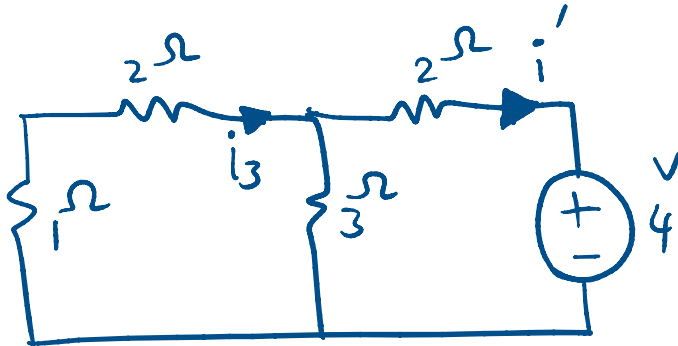$$i_1 = -\frac{1^\Omega}{1^\Omega + 3.2^\Omega} \times 3^A$$

$$= -\frac{3^A}{4.2} = -0.71^A$$

$$i_2 = -\frac{2.2^\Omega}{2^\Omega + 2.2^\Omega} \times 2^A$$

$$2^{\small JL} + 2.\ddot{\imath}$$

$$= -\frac{4.4^A}{4.2} = -1.05^A$$

$\Longleftarrow$

$$i' = -\frac{4^V}{2^{\Omega} + 3^{\Omega} \| (1+2)^{\Omega}}$$

$$= -\frac{4^V}{3.5^{\Omega}} = -1.14^A$$

$$i_3 = \frac{3^{\Omega}}{3^{\Omega} + (1^{\Omega} + 2^{\Omega})} \, i' = 0.5 i' = -0.57^A$$

$$\boxed{i = i_1 + i_2 + i_3 = -0.71^A - 1.05^A - 0.57^A = -2.33^A}$$
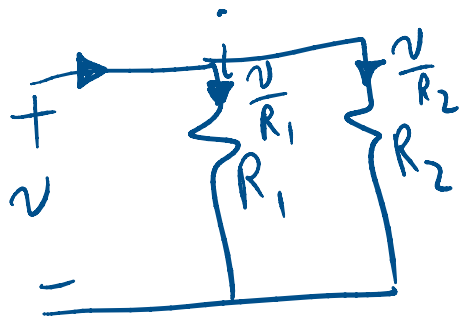
چیدمان کلی ربر
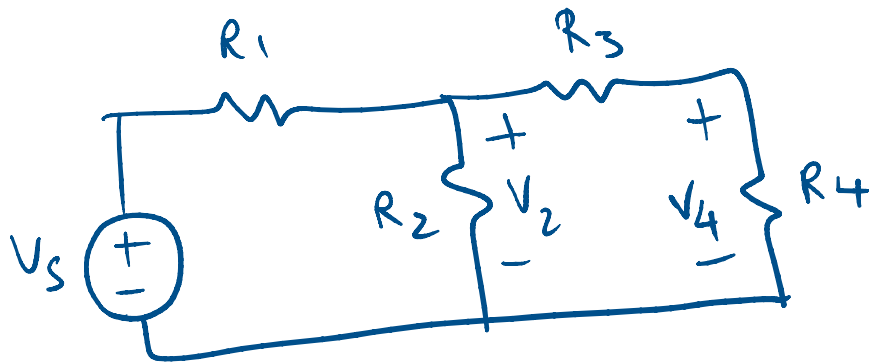
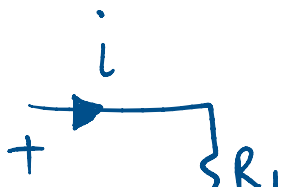$$V_2 = \frac{R_2 \| (R_3 + R_4)}{R_1 + R_2 \| (R_3 + R_4)} V_S$$
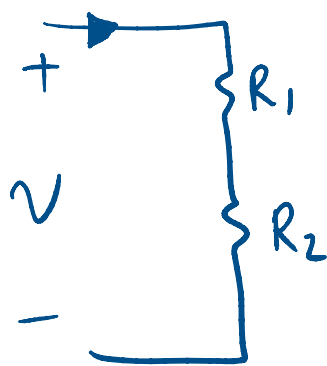
$$V_4 = \frac{R_4}{R_3 + R_4} V_2$$

$$i - \frac{v}{R_1} - \frac{v}{R_2} = 0$$

$$\Rightarrow R_{eq} = \frac{v}{i} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

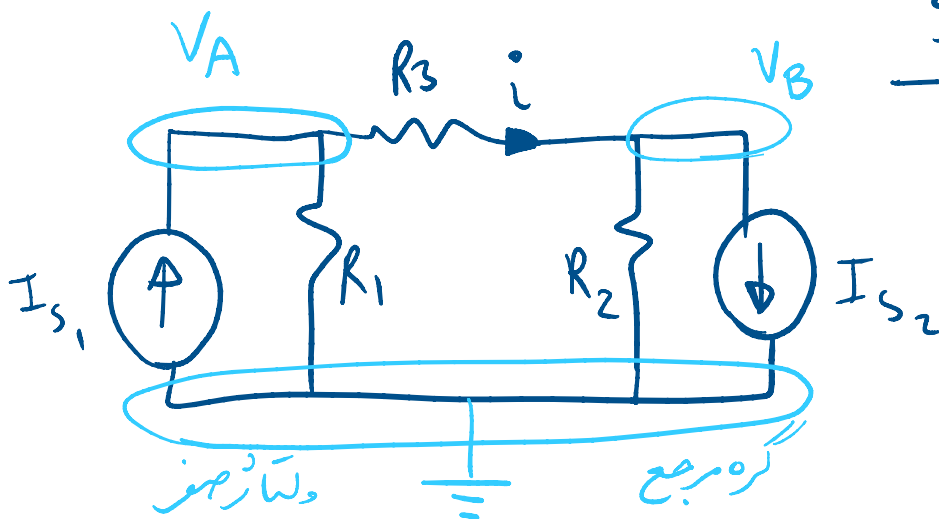$$\frac{1}{R_{eq}} = G_{eq} = G_1 + G_2$$

$$-v + R_1 i + R_2 i = 0$$

$$-v + R_1 i + R_2 i = 0$$

$$\Rightarrow \quad R_{eq} = \frac{v}{i} = R_1 + R_2$$

روش تحلیل گره

شکل ۴

V_A R_3 i V_B

I_{s_1} R_1 R_2 I_{s_2}

ولتاژ صفر   گره مرجع

$$KCLA: \quad I_{s_1} - \frac{V_A}{R_1} - \frac{V_A - V_B}{R_3} = 0$$

$$KCLB: \quad -I_{s_2} - \frac{V_B}{R_2} + \frac{V_A - V_B}{R_3} = 0$$

$$\begin{cases} \left(\frac{1}{R_1} + \frac{1}{R_3}\right)V_A - \frac{1}{R_3}V_B = I_{s_1} \\ \frac{1}{R_3}V_A - \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_B = I_{s_2} \end{cases} \Rightarrow \begin{cases} V_A = ? \\ V_B = ? \end{cases}$$

$$\Rightarrow \quad i = \frac{V_A - V_B}{}$$

$$\Rightarrow \quad i = \frac{V_A - V_B}{R_3}$$

$$R_1 = 1^{\Omega}, \quad R_2 = 2^{\Omega}, \quad R_3 = 3^{\Omega}$$

$$I_{S1} = 1^A, \quad I_{S2} = 2^A$$

$$\begin{cases} \frac{4}{3}V_A - \frac{1}{3}V_B = 1 \\ \frac{1}{3}V_A - \frac{5}{6}V_B = 2 \end{cases} \Rightarrow \begin{cases} V_A = \frac{1}{6}^V \\ V_B = -\frac{7}{3}^V \end{cases}$$