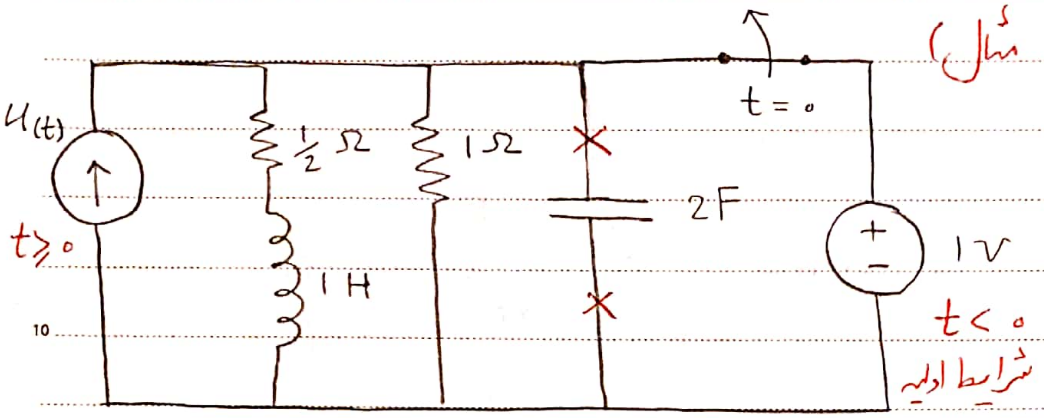
Subject:

Year. Month. Date.

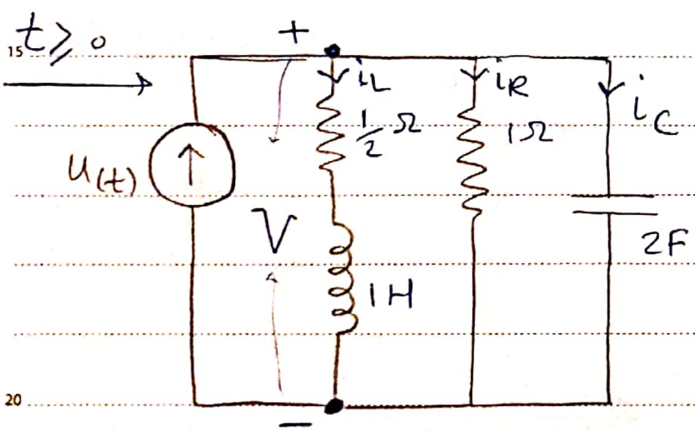
مبانی مدارهای الکتریکی و الکترونیکی

جلسه 14ام

مدار مرتبه 2 (مثال)



$$V_C(0^-) = 1V \qquad i_L(0^-) = 2A$$



$$\overset{V/R}{i_R} + i_L + \overset{C\frac{dv}{dt}}{i_C} = u(t) \qquad (*)$$

ولتاژ سلف ، ولتاژ مقاومت

$$V = \frac{di_L}{dt} + \frac{1}{2} i_L$$

ولتاژ دو سر $\longrightarrow \frac{di_L}{dt} + \frac{1}{2} i_L \longrightarrow$ ولتاژ دو سر شاخه‌های دیگر نیز همین است

$$(*) \longrightarrow 2\frac{d^2 i_L}{dt^2} + 2\frac{di_L}{dt} + \frac{3}{2} i_L = u(t)$$

$$\left(i_R = \frac{V}{\underset{1\Omega}{R}} = \frac{di_L}{dt} + \frac{1}{2} i_L \ , \ i_C = \overset{2}{C}\frac{dV}{dt} = 2\frac{d^2 i_L}{dt^2} + \frac{di_L}{dt} \right)$$

$$\frac{d^2 i_L}{dt^2} + \underbrace{(1)}_{2\alpha}\frac{di_L}{dt} + \underbrace{\frac{3}{4}}_{\omega_0^2} i_L = \frac{u(t)}{2}$$

$$\rightarrow \alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

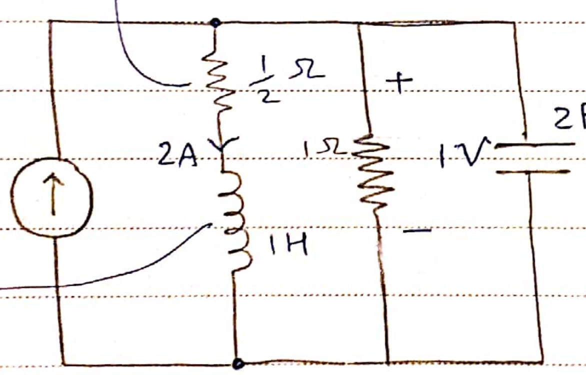
$$\alpha = \frac{1}{2} \qquad \omega_0 = \frac{\sqrt{3}}{2}$$

$\rightarrow \alpha < \omega_0 \longrightarrow$ شکل جواب : $K_1 e^{-\alpha t}\cos(\omega_d t) + K_2 e^{-\alpha t}\sin(\omega_d t)$

$$\omega_d = \sqrt{\alpha^2 - \omega_0^2} \rightarrow |\omega_d| = \frac{\sqrt{2}}{2}$$

$V_R = 1V$

در مدار $\rightarrow$



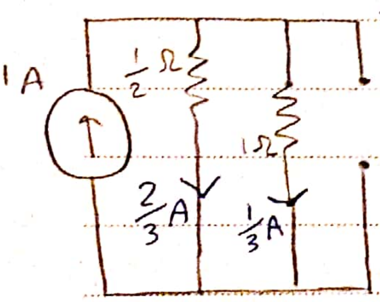
ولتاژ خازن نمی پره!

جریان سلف نمی پره!

$V_L = 1V - V_R = 0$

$$i_L(0^+) = i_L(0^-) = 2A \qquad \frac{di_L}{dt}(0^+) = V_L = 0$$

در $t = \infty \leftarrow$ خازن مدار باز، سلف اتصال کوتاه $\leftarrow$



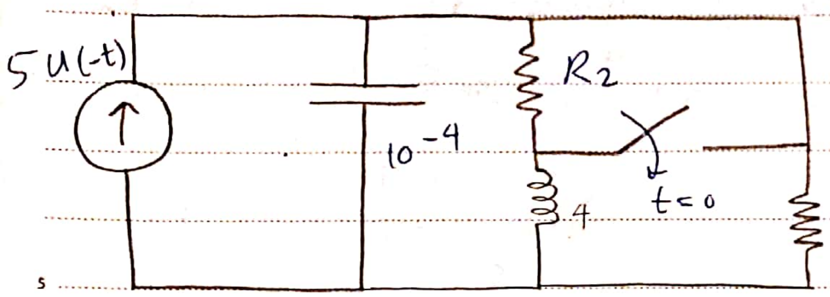
$$\rightarrow i_L(\infty) = \frac{2}{3} A$$

$$\rightarrow i_L = \frac{2}{3} + \overset{4/3}{K_1} e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{2}}{2}t\right) + \overset{\frac{2\sqrt{2}}{3}}{K_2} e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{2}}{2}t\right)$$

$$\left(i_L(0^+) = 2A \,,\, \frac{di_L}{dt}(0^+) = 0\right)$$

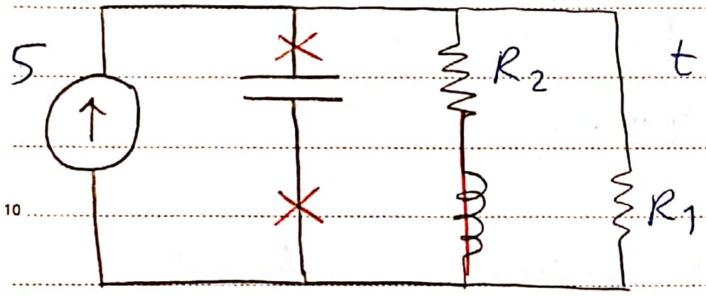
$$\longrightarrow \frac{2}{3} + K_1 = 2 \Rightarrow K_1 = \frac{4}{3}$$

مثال)



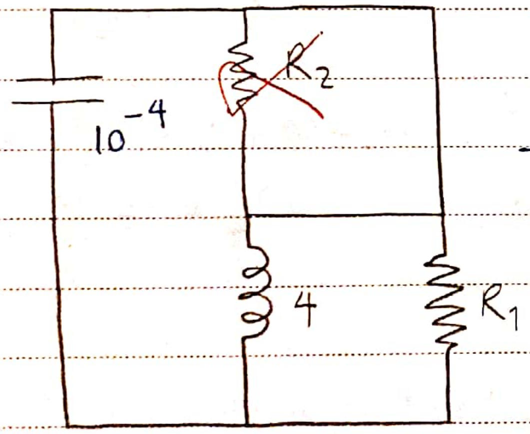
$V_C(0^+) = 2V \leftarrow V_C(0^-) = 2V$

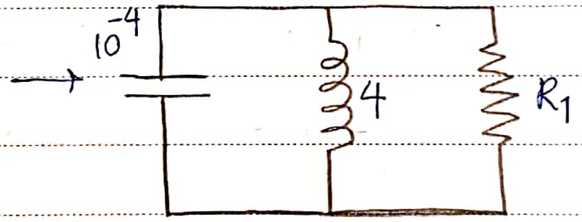
$\alpha = \omega_0$ میرای بحرانی



$t = 0^-$

$R_1 \| R_2 = 0.4\,\Omega$ $\left(\frac{2V}{5A}\right)$

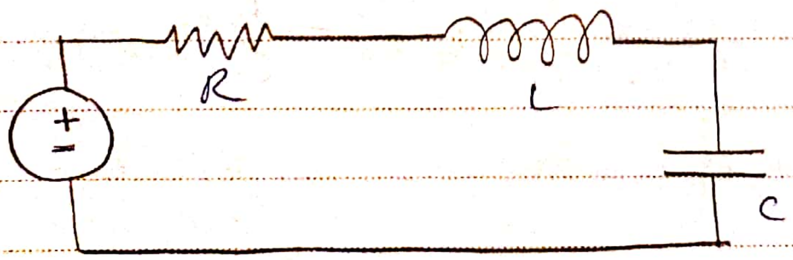




$$\alpha = \frac{1}{2R_1C} \quad , \quad \omega_0^2 = \frac{1}{LC}$$

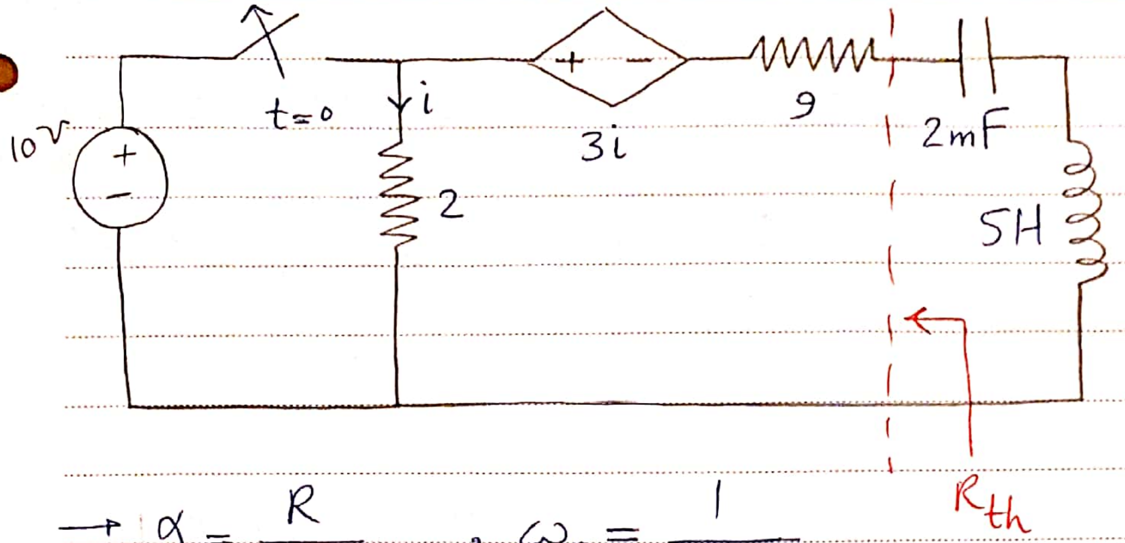
$\alpha = \omega_0 \longrightarrow R_1 = 100\,\Omega$

← سری RLC



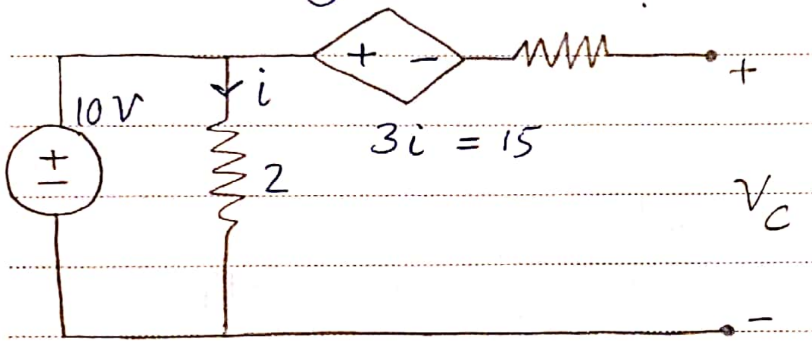
$$\alpha = \frac{R}{2L} \quad , \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

مثال)



→ $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

در $t=0^-$: ← خازن مدار باز ، سلف اتصال کوتاه



→ $i = 5 = \frac{10}{2}$

→ $-10+15+V_C=0$

→ $V_C=-5\ V$

* هر شرط اولیه باید برای $t=0^+$ بدست بیاید ← $V, i_L, \frac{di_L}{dt}, \frac{dV}{dt}$

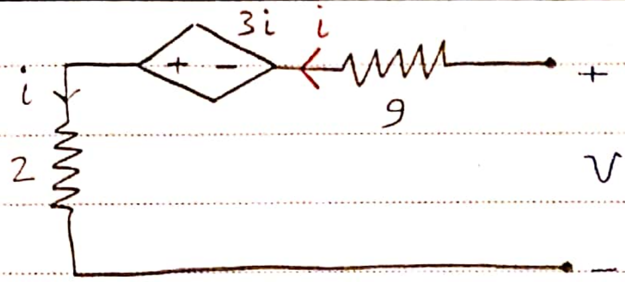
$V_C(0^+) = V_C(0^-) = -5\ V$

$$\frac{dV_C}{dt}(0^+) = \frac{1}{C} i_C(0^+) = \frac{1}{C} i_L(0^+) = \frac{1}{C} i_L(0^-)$$

$i_L(0^-) = 0$ → $\frac{dV_C}{dt}(0^+) = 0$

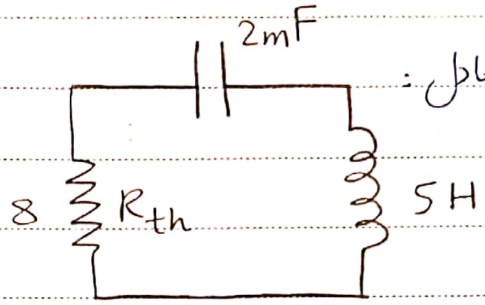
← R_{th} را باید بدست آوریم ←

$R_{th} \rightarrow$



$\rightarrow 9i - 3i + 2i = V \quad \rightarrow V = 8i \quad \rightarrow R_{th} = 8$

مدار معادل :



$\Rightarrow \alpha = \frac{R}{2L} = 0.8$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10$

$\alpha < \omega_0 \longrightarrow$ شکل جواب : $K_1 e^{-0.8t} \cos(\omega_d t) + K_2 e^{-0.8t} \sin(\omega_d t)$

مثال) ورودی از جنس دیراک باشد :

$$\frac{d^2V}{dt^2} + 4\frac{dV}{dt} + 3V(t) = 3\delta(t)$$

شکل جواب : $K_1 e^{-t} + K_2 e^{-3t} +$ جواب خصوصی

$2\alpha = 4 \;,\; \omega_0 = \sqrt{3} \quad \left(\alpha > \omega_0 \rightarrow P_1, P_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}\right)$

معادله کلی : $\frac{d^n V(t)}{dt^n} + \cdots + a_n V(t) = \frac{d^m \delta(t)}{dt^m} + \cdots + b_m \delta(t)$

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① اگر $n>m$: در جواب خصوصی، $\delta(t)$ وجود ندارد.

جواب خصوصی $= 0$

② اگر $n=m$: جواب خصوصی $= k\delta(t)$

③ اگر $n<m$: جواب خصوصی $= k_1\delta(t) + k_2\delta'(t) + \dots + k_{n-m+1}\delta^{(n-m)}(t)$

$$\frac{d^2v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 3v(t) = 3\delta(t)$$ ← ادامهٔ مثال قبل:

$\to$ جواب $= \left[k_1 e^{-t} + k_2 e^{-3t}\right]u(t)$ ← جواب خصوصی $= 0$

$\to = \left[k_1 e^{-t} + k_2 e^{-3t}\right]u(t)$ $\delta(t) = u'(t)$

$$\frac{dv(t)}{dt} = \left(-k_1 e^{-t} + (-3)k_2 e^{-3t}\right)u(t) + \underbrace{\left(k_1 e^{-t} + k_2 e^{-3t}\right)}_{k_1+k_2}\delta(t)$$

$$\frac{d^2v(t)}{dt^2} = \left(k_1 e^{-t} + 9k_2 e^{-3t}\right)u(t) + \left(-k_1 - 3k_2\right)\delta(t) + (k_1+k_2)\delta'(t)$$

← چون تنها در $t=0$، $\delta(t)$ مقدار غیرصفر دارد.

PAPCO