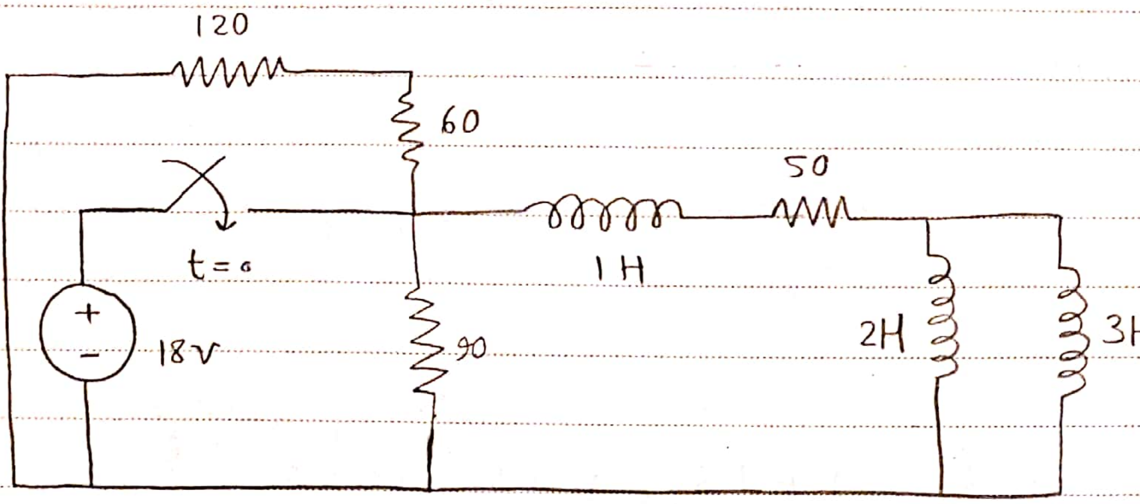
مبانی مدارهای الکتریکی و الکترونیکی

جلسه 12ام

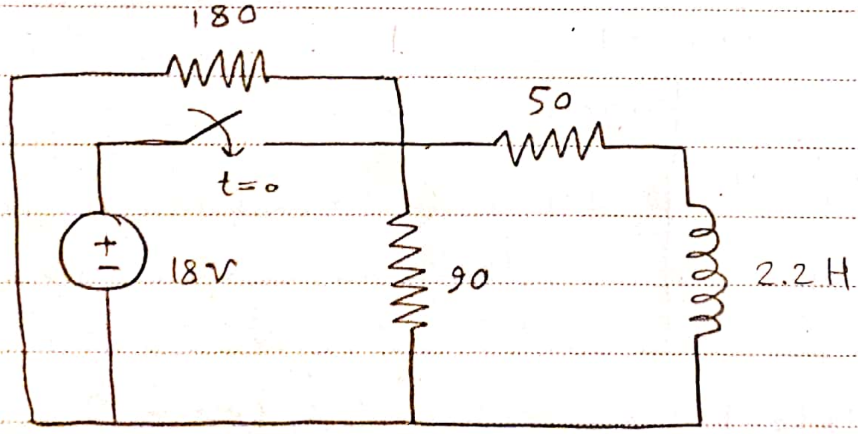
مثال ۱)



$\to L_{eq} = 2.2\ H$

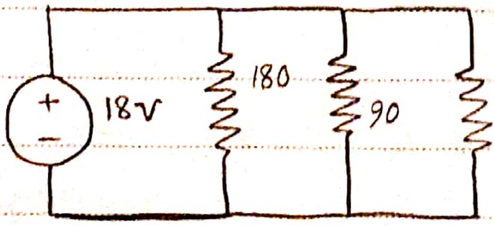
1.2 H

معادل →



$i_L(0^-) = 0 = i_L(0^+)$

برای i_∞ →

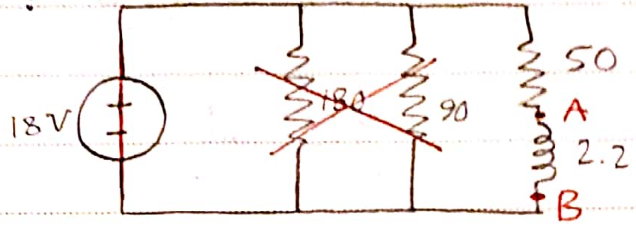


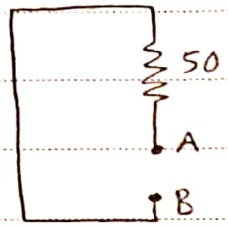
در $t = \infty$، سلف مانند اتصال کوتاه عمل می کند.

$$i_L(\infty) = \underbrace{i_{R50}} = \frac{18}{50} = 0.36\ A$$

جریان عبوری از مقاومت 50

مقاومت معادل دوسر سلف :



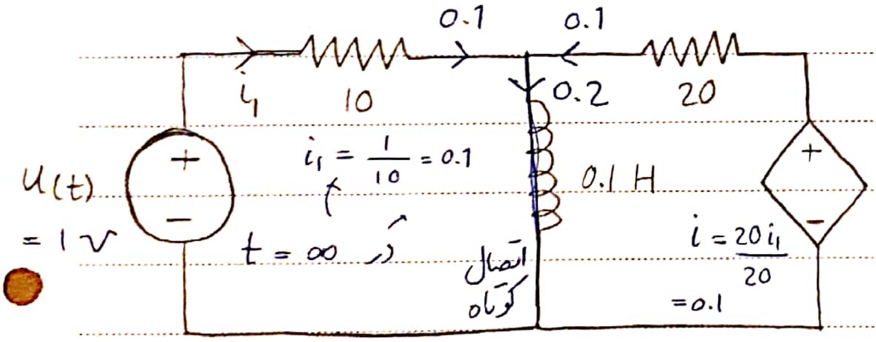


$\rightarrow R = 50 \rightarrow \tau = \frac{L}{R} = \frac{2.2}{50}$

$$\rightarrow i_{(t)} = i_L(\infty) + (i_{L(0^+)} - i_L(\infty)]e^{-t/\tau}$$

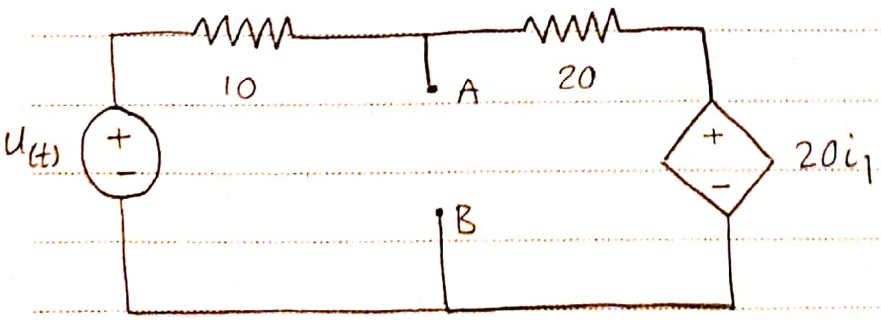
$$\rightarrow i_{(t)} = 0.36 + (0 - 0.36) \times e^{-\frac{t}{0.044}} \quad t \geq 0$$

مثال



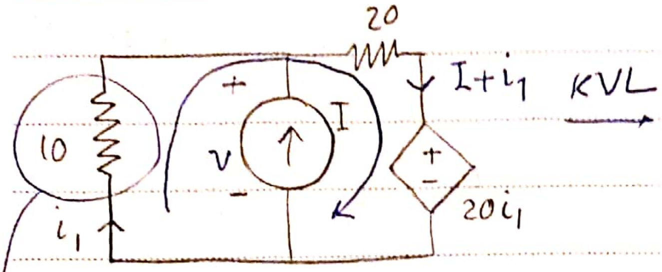
$i_{L(0^-)} = i_L(0^+) = 0 \quad \hookrightarrow i_L(\infty) = 0.2$

← مقاومت معادل دوسر سلف :



→ برای بدست آوردن $R_{\text{معادل}}$ ، یک منبع مستقل فرضی بین A و B می‌گذاریم و نسبت $\frac{V}{I}$ را بدست می‌آوریم ← $R_{\text{معادل}}$

Scanned by CamScanner



$$10i_1 + 20(i_1+I) + 20i_1 = 0 \rightarrow i_1 = -\frac{2}{5}I$$

$$V = -10i_1 = 4I \rightarrow R_{th} = \frac{V}{I} = 4$$

* به جای منبع جریان، می توانستیم منبع ولتاژ قرار بدیم و مشکلی پیش نمی آمد.

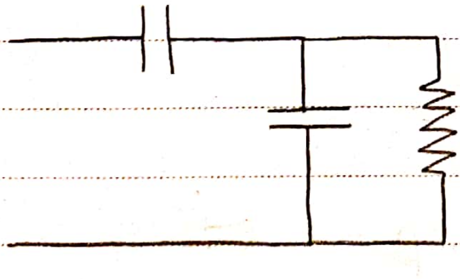
$$L \rightarrow i_{(t)} = i_L(\infty) + (i_{L(0+)} - i_{L(\infty)})\, e^{-t/\tau}$$

$$i_L(0) = 0 \ , \ i_L(\infty) = 0.2A \ , \ \tau = \frac{L}{R} = \frac{0.1}{4} = 0.025$$

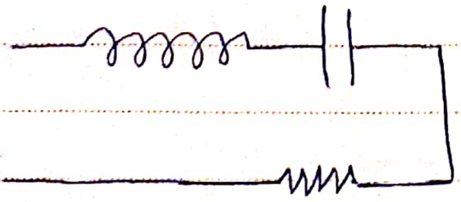
$$\rightarrow i_{(t)} = 0.2 + (0 - 0.2)\, e^{-\frac{t}{0.025}}$$

مدارهای مرتبهٔ دوم

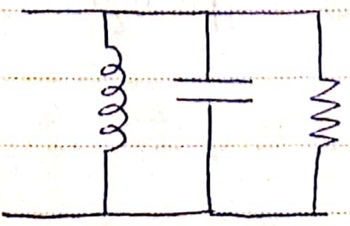
مثال ←



دو حالت خاص:

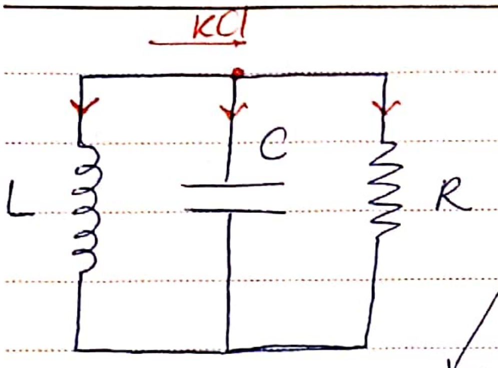


RLC سری



RLC موازی

RLC موازی :



$$i_L + i_C + i_R = 0$$

$i_L \rightarrow \frac{1}{L}\int V dt$, $i_C \rightarrow C\frac{dV}{dt}$, $i_R \rightarrow \frac{V}{R}$

مشتق بگیری

$$\xrightarrow{} \frac{1}{L}V + C\frac{d^2V}{dt} + \frac{1}{R}\frac{dV}{dt}$$

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V = 0$$

$$\div C \quad \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$$

معادله مشخصه

$$p^2 + \underbrace{\frac{1}{RC}}_{2\alpha}p + \underbrace{\frac{1}{LC}}_{\omega_0^2} = 0$$

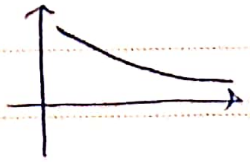
$$\rightarrow p^2 + 2\alpha p + \omega_0^2 = 0 \rightarrow S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\Rightarrow V_{(t)} = k_1 e^{S_1 t} + k_2 t e^{S_2 t}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

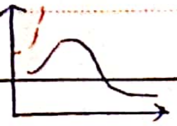
$$\alpha = \frac{1}{2RC}, \quad \omega_0^2 = \frac{1}{LC}$$

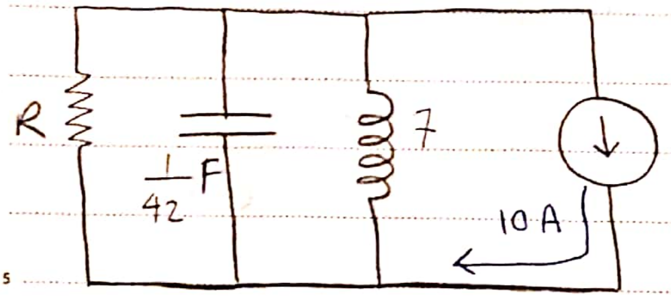
① $\alpha^2 > \omega_0^2$ S_1 و S_2 دو عدد حقیقی منفی



میرای شدید

② $\alpha^2 = \omega_0^2$ $S_1, S_2 = -\alpha$





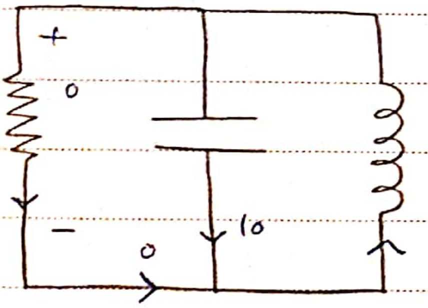
$10u(-t)$ $t \leq 0$

$t<0:$ $i_L(0^-) = 10A$

$V_C(0^-) = 0$

$V_C(0^+) = 0, \quad \frac{dV_C(0^+)}{dt}$

2 شرط اولیه نیاز داریم :



$\frac{dV_c}{dt} = 420$

$i_L(0^+) = 10$

$V_C(0^+) = 0$

$i_C = C\frac{dV_c}{dt} = 10$

① $R=6$ $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7 \times \frac{1}{42}}} = \sqrt{6}$

$\alpha = \frac{1}{2 \times 6 \times \frac{1}{42}} = \frac{7}{2} \longrightarrow \alpha > \omega_0$

$\Rightarrow P_1, P_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$$\rightarrow \qquad V_{C(t)} = k_1 e^{-t} + k_2 e^{-6t}$$

$$V_C(0^+) = 0 \quad , \quad \frac{dV_C}{dt}(0^+) = 420$$

$$\rightarrow \begin{cases} k_1 + k_2 = 0 \\ -k_1 - 6k_2 = 420 \end{cases} \Rightarrow \quad \begin{matrix} k_1 = 84 \\ k_2 = -84 \end{matrix}$$

② $R = \dfrac{7\sqrt{6}}{2} \quad \rightarrow \alpha = \dfrac{1}{2RC} = \sqrt{6}$

$$\rightarrow \alpha = \omega_0 \Rightarrow V_{C(t)} = k_1 e^{-\sqrt{6}t} + k_2 t e^{-\sqrt{6}t}$$

$$\rightarrow \quad k_1 = 0 \quad , \quad k_2 = 420$$

③ $R = \dfrac{21}{2} \qquad \alpha = \dfrac{1}{2RC} = \dfrac{1}{2\left(\frac{21}{2}\right)\left(\frac{1}{42}\right)} = 2$

$$\alpha < \omega_0 \rightarrow V_{C(t)} = k_1 e^{-2t} \cos(\underbrace{\sqrt{2}}_{\sqrt{\alpha^2 - \omega_0^2} = \sqrt{4-6}} t) + k_2 e^{-2t} \sin(\sqrt{2}t)$$

$$\rightarrow \begin{matrix} k_1 = 0 \\ k_2 = \dfrac{420}{\sqrt{2}} \end{matrix} \Rightarrow \quad V_C = \frac{420}{\sqrt{2}} e^{-2t} \sin(\sqrt{2}t)$$