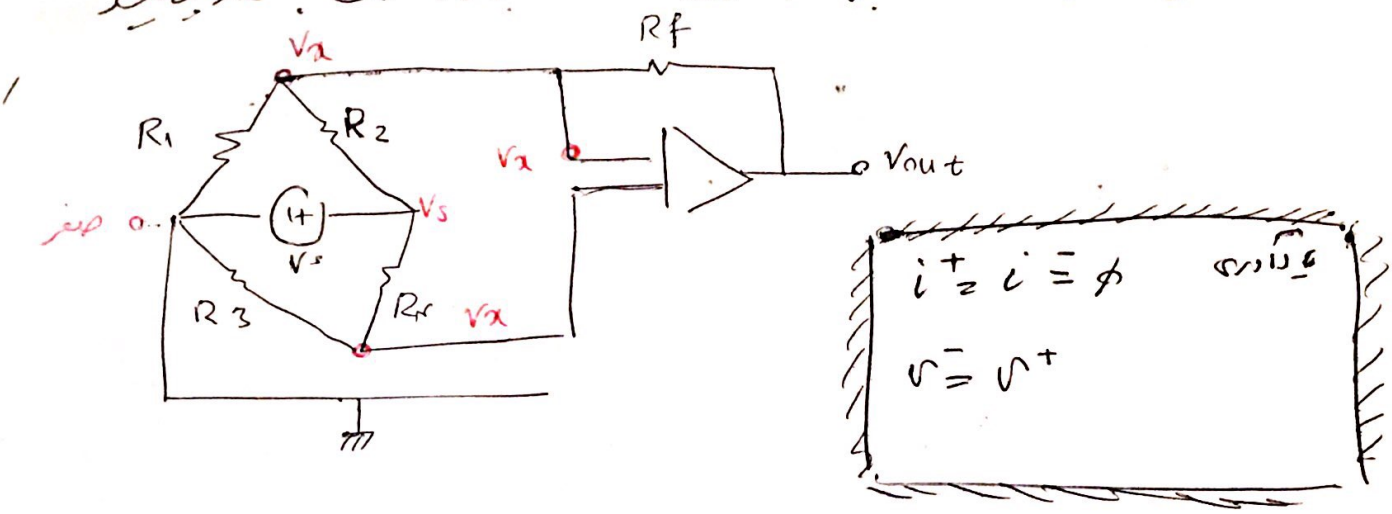
Midterm exam

① در مدار شکل زیر رابطه ولتاژ خروجی به صورت $V_o = k V_s$ است. k را بیابید



KCL / $\frac{0 - V_x}{R_1} = \frac{V_x - V_s}{R_2} + \frac{V_x - V_{out}}{R_f}$

$\rightarrow \frac{V_{out}}{R_f} = V_x \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_f}\right) - \frac{V_s}{R_2}$ I

KCL / $\frac{0 - V_x}{R_3} = \frac{V_x - V_s}{R_4} \rightarrow \frac{V_s}{R_4} = \left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_x$

$\Rightarrow \frac{V_s}{R_4} = \left(\frac{R_4 + R_3}{R_3 R_4}\right) V_x \rightarrow V_s = \frac{R_4 + R_3}{R_3} V_x$

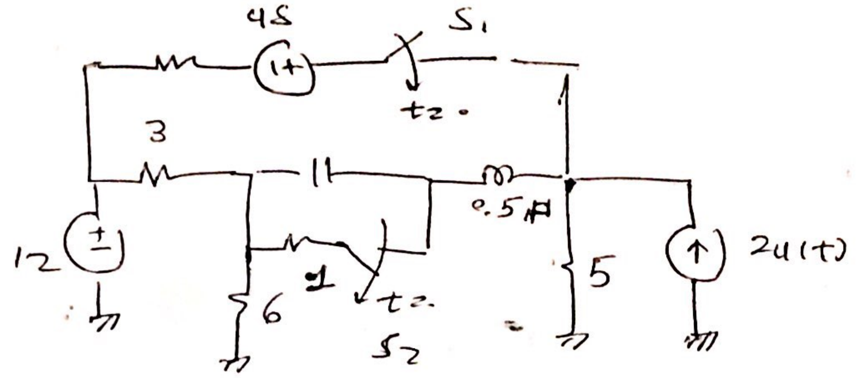
$V_x = \frac{R_3}{R_3 + R_4} V_s$ → replacing in I

$\frac{V_{out}}{R_f} = \frac{1}{R_1 \| R_2 \| R_f} * \frac{R_3}{R_3 + R_4} V_s - \frac{V_s}{R_2}$

$\rightarrow \frac{V_o}{V_s} = k = R_f \left[\frac{R_3}{(R_3 + R_4)(R_1 \| R_2 \| R_f)} - \frac{1}{R_2}\right]$

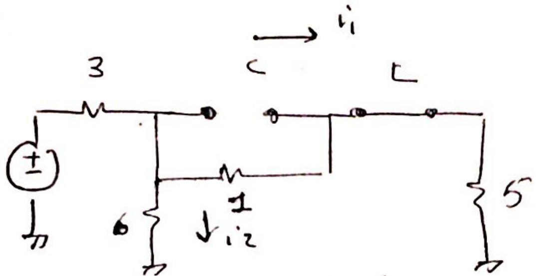
② کلید ۱ که برای مدت طولانی باز و کلید ۲ که برای مدت طولانی بسته بوده اند. در $t=0$ وضعیت عوض میشود. $v(t)=?$

$C=\frac{1}{16}F$



$-\infty<t<0^-$

C → open
L → short

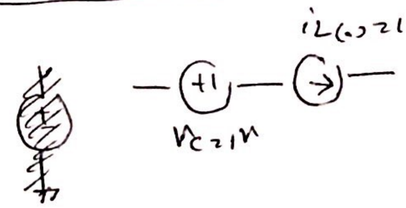


$R=3+6\|(1+5)=3+6\|6=6$

$i=\frac{12}{6}=2A \rightarrow i_1=i_2=\frac{1}{2}\times 2=1^A$

$V_c(0^-)=9^v \quad i_L(0^-)=1^A \quad i=c\frac{dv}{dt} \rightarrow \dot{V_c}=?$

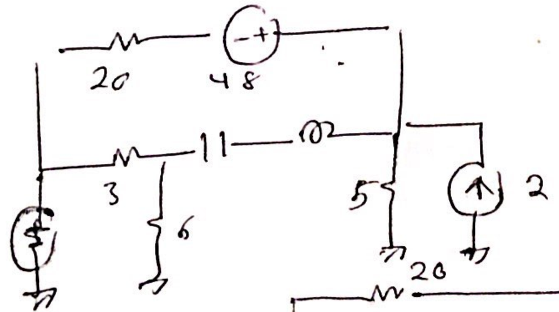
$t=0^+$



$i_L(0)=1A \rightarrow 1A=\frac{1}{18}\dot{V_c} \rightarrow \dot{V_c}=18$

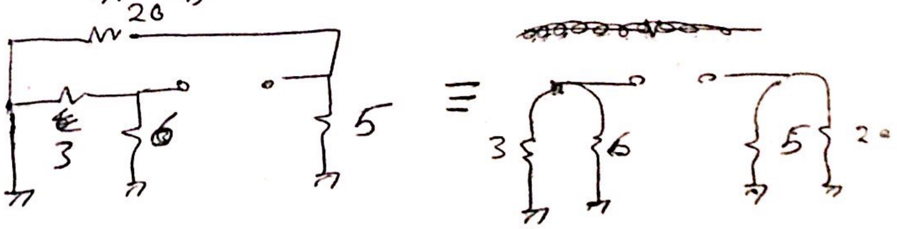
در اصل، تحلیل چند شکلی میکنیم.

$t>0$



RLC سری

R_{th}:

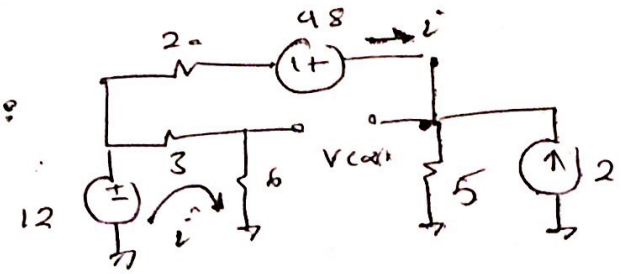


$R_{th}=3\|6+5\|20=2+4=6^{\Omega}$

RLC $\alpha = \frac{R+h}{2L} = \frac{6}{2*\frac{1}{2}} = 6$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}*\frac{1}{18}}} = \frac{1}{\sqrt{1/36}} = \frac{1}{\sqrt{1/26}} \to 6$

$\alpha = \omega_0 \to$ مساوی $v(t) = A + (B+Ct)e^{-\alpha t}$



1/ $A = v(\infty)$:

*/ $-12 + 20i - 48 + 5(i+2) = 0 \to -12 + 20i - 48 + 10 + 5i = 0$

$25i = 48 + 12 - 10 = 50$

$i = 2$ A

*/ $12 = 3\hat{i} + 6\hat{i} = 9\hat{i} \to \hat{i} = 4/3$ A

$\to 6(\frac{4}{3}) = v(\infty) + 5(2+2) \Rightarrow 8 = v(\infty) + 20$

$v(\infty) = -12 \to A = -12$

$v(t) = -12 + (B+Ct)e^{-6t}$

$v(0) = 1^{v} \to -12 + B = 1 \to B = 13$

$\frac{d}{dt}v = -18 \to C = 96$

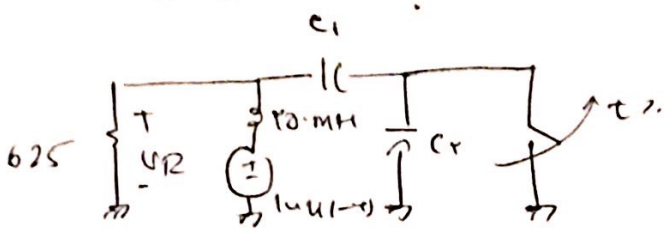
$$v(t) = -12 + (13+96t)e^{-6t} \quad t > 0.$$

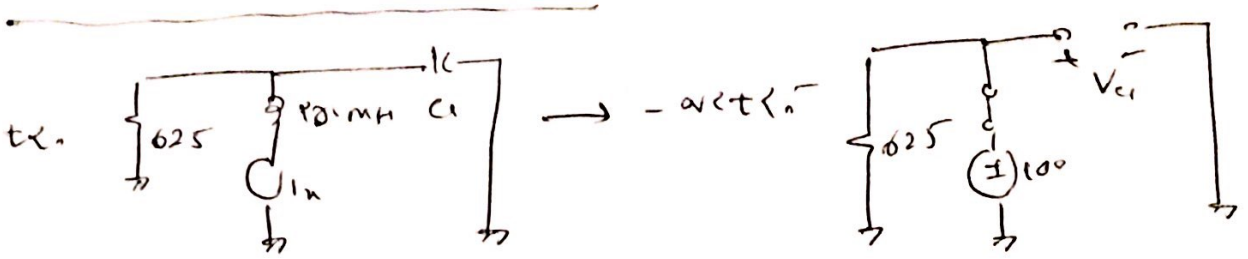
(۳) در مدار شکل زیر کلید برای مدت طولانی بسته بوده و در $t=0$ باز

$C_1 = 1\mu F$

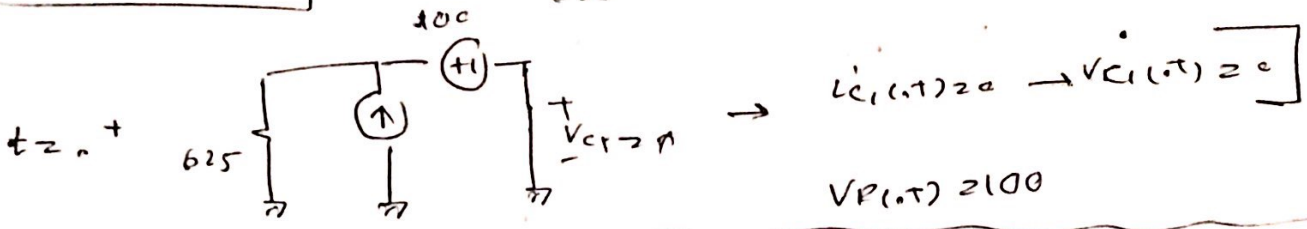
$C_2 = 0.25\mu F$

$V_R(t)$





$V_{c1}(0^-) = 100$ $\quad i_L(0^-) = \frac{-100}{625} = -0.16 \downarrow = 0.16 \uparrow$



$i_{C_1}(0^+) = 0 \rightarrow V_{C_1}(0^+) = 0$

$V_R(0^+) = 100$

روش اول

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2 * 625 (1^M \| 0.25M)} = 4000$$

روش دوم

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow \omega^2 = 20 * 10^6 \rightarrow \omega_d = \sqrt{\omega^2 - \alpha^2} = 2000$$

$$V_R = e^{-\alpha t}[A\cos\omega_d t + B\sin\omega_d t]$$

$$= e^{-4000t}[A\cos 2000t + B\sin 2000t]$$

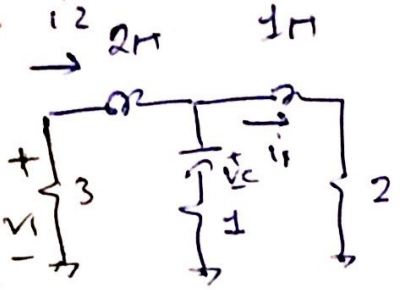
$V_R(0) = 100 \rightarrow A = 100$

$\dot{V}_R(0^+) = ?$

$$\begin{cases} V_R(t) = V_2(t) = V_{C1} + V_{C2} \\ \dot{V}_R(t) = \dot{V}_{C1}(0^+) + \dot{V}_{C2}(0^+) \end{cases}$$

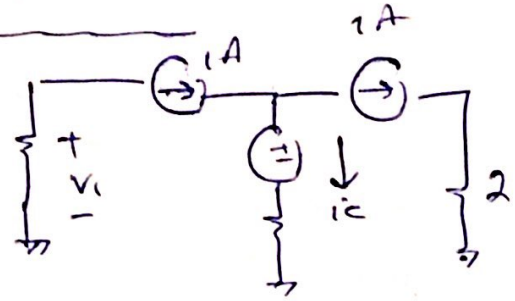
$$\dot{V}_R(0^+) = \frac{i_{C1}(0^+)}{C_1} + \frac{i_{C2}(0^+)}{C_2} \rightarrow \phi + \phi \rightarrow \phi$$

$$\Rightarrow \dot{V}_R(0) = -\alpha A + B\omega_d = 0 \rightarrow B = 200$$



$\frac{dV_1(0^+)}{dt} = ?$ ، $1A = i_1(0^-) = i_2(0^-)$ (ک)

$V_c(0^-) = 1$ (م)



$i_c(0^+) = C = C\,dV_c(0^+)/dt$

$\boxed{V_c(0^+) = 0}$

از روی $1^V = V_c(0^+) = V_{L_1}(0^+) + 2*1 \rightarrow V_{L_1}(0^+) = -1^V = L_1 \frac{di_{L_1}(0^+)}{dt}$

$\boxed{i_{L_1}(0^+) = -1\ A/s}$

از روی: $-3 = V_{L2}(0^+) + 1 \rightarrow V_{L_2}(0^+) = -4 = 2\, i_{L_2}(0^+)$

$\boxed{i_{L_2}(0^+) = -2\ A/s}$

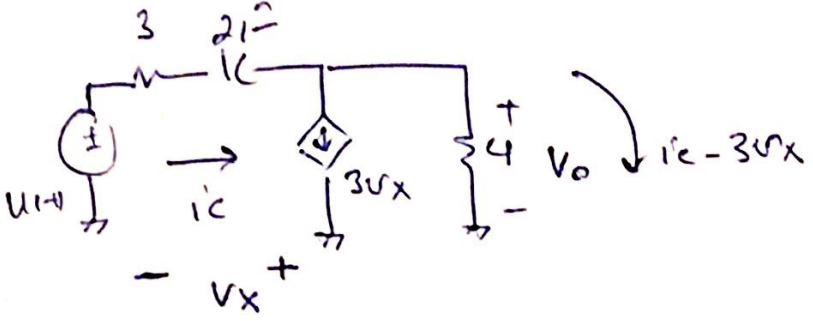
$\frac{dV_1(0^+)}{dt} = \frac{d}{dt}(1^V X - i_{L_2}(0^+))$ ### → maybe you make a mistake

از سمت راست

$\frac{dV_1}{dt}(0^+) = 3 * \frac{d}{dt}(-i_{L_2}(0^+)) = -3\,\frac{d}{dt} i_{L_2}(t)$

$\Rightarrow t = 0^+ \rightarrow \frac{dV_1}{dt}(0^+) = -3 * 2 = -6\ V/s$

(a) $v_o(t)$ را بیابید.



1/ $-v_x = 3i_c + v_c$

2/ $u(t) = 4i_c + v_c + 4(i_c - 3v_x)$

$= 4i_c + v_c + 4i_c + 12(3i_c + v_c)$

$= 4i_c + v_c + 4i_c + 36i_c + 12v_c$

$= 6\dot{v}_c + v_c + 8\dot{v}_c + 72\dot{v}_c + 12v_c = 86\dot{v}_c + 13v_c$

$1 = 86\dot{v}_c + 13v_c \Rightarrow v_c(t) = \left(\frac{1}{13} + Ae^{-13/86\,t}\right)u(t)$

$v_c(0) = 0 \longrightarrow A = -1/13$

$$\begin{cases} v_c(t) = \frac{1}{13}(1 - e^{-\alpha t})u(t), \quad \alpha = -13/86 \\ -v_x = 4i_c + v_c = 6\dot{v}_c + v_c \end{cases}$$

$\dot{v}_c = \frac{1}{13}\left[\delta(t)(1 - e^{-\alpha t}) + \alpha e^{-\alpha t}u(t)\right] = \frac{\alpha}{13}e^{-\alpha t}u(t)$

$\Rightarrow -v_x = 6\dot{v}_c + v_c = \left[\frac{6\alpha}{13}e^{-\alpha t} + \frac{1}{13} - \frac{1}{13}e^{-\alpha t}\right]u(t)$

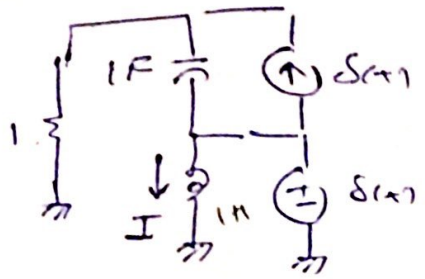
$= \left[\frac{6\alpha - 1}{13}e^{-\alpha t} + \frac{1}{13}\right]u(t)$

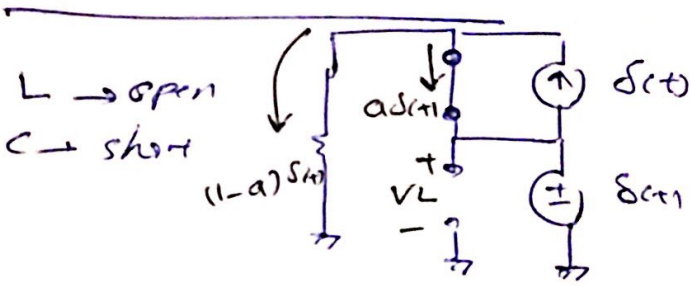
$v_o = 4i_c - 12v_x$ ✓

panic !!

$q(0^+) = ?$ ← $V_c(0^-) = 0$ ⑥
$i_L(0^-) = 0$

$I(0^+) = ?$





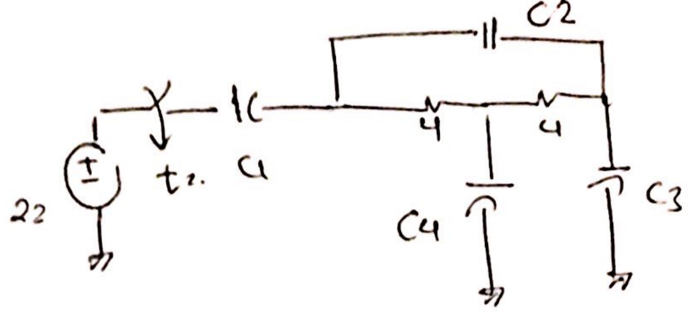
1/ $V_L = \delta(t)$ → $(1-a)\delta(t) = \delta(t)$

2/ $V_L = 1^{-2} * (1-a)\delta(t)$ → $\boxed{a=0}$

$$\rightarrow i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} 1\cdot\delta(t) = 0 + 1 = 1$$

$$i_C(0^+) = a\delta(t) = 0 \rightarrow V_C(0^+) = 0$$

۱۵ آذر ⑦ ولتاژ خازن ها در $t=0^+$



$C_1 = 1F \qquad C_3 = 3F$

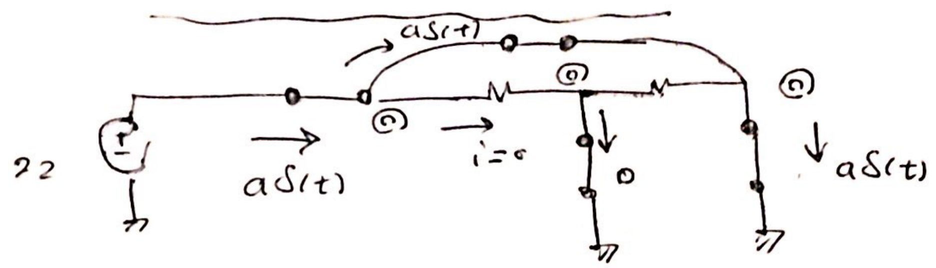
$C_2 = 2F \qquad C_4 = 4F$

ضربه ! در $t=0^+$ ،

if $V_{C_i}(0^-) = V_{C_i}(0^+)$, $i = 1:4$

$t=0^+ \qquad 22 = V_{C_1} + V_{C_2} + V_{C_3}$

$22 = 0 \quad$ ✕



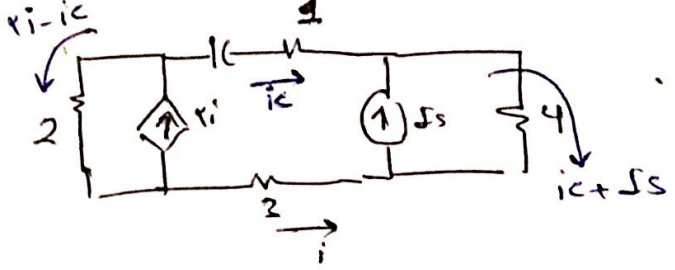
$V_{C_1}(0^+) = \frac{a}{1} = a$

$V_{C_2}(0^+) = \frac{a}{2}$

$V_{C_3}(0^+) = a/3$

$a + \frac{a}{2} + \frac{a}{3} = 11/6\,a = 22 \rightarrow a = 12$

$V_{C_1}(0^+) = 12^v \quad V_{C_2}(0^+) = 6^v \quad V_{C_3}(0^+) = 4^v \quad V_{C_4}(0^+) = 0$



⑧ اگر $I_s(t) = u(t) - \delta(t)$ باشد، $V_c(t)$ را بیابید.

1/ $ic = -i$

KVL: $2(2i - ic) = V_c + \dot{ic} + 4(ic + Is) + 3(-i = ic)$

$\Rightarrow 2(-2ic - \dot{ic}) = V_c + \dot{ic} + 4ic + 4Is + 3ic$

$\Rightarrow -6\dot{ic} = V_c + 8ic + 4Is$

$\Rightarrow -14\dot{ic} - V_c = 4u(t) - 12\delta(t)$

$-14\dot{V_c} - V_c = 4 - 12\delta(t)$

$$\dot{V_c} + \frac{1}{14}V_c = \frac{-4}{14} + \frac{6}{7}\delta(t)$$

$$\rightarrow \int_{0^-}^{0^+} / \left[V_c(0^+) - V_c(0^-)\right] + 0 = 6/7$$

$$\boxed{V_c(0^+) = 2 + 6/7 = 20/7} \quad (*)$$

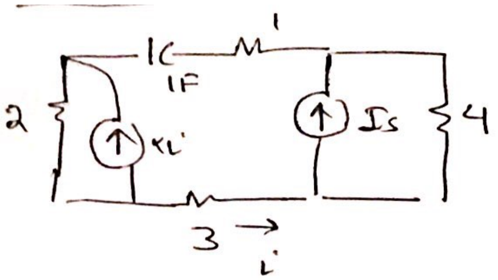
$$V_c(t) = -4 + A e^{-1/14 t} \quad / \quad A - 4 = 20/7 \rightarrow A = 48/7$$

$$V_c(t) = \left[\frac{48}{7} e^{-\frac{1}{14}t} - 4\right] u(t)$$

الف/

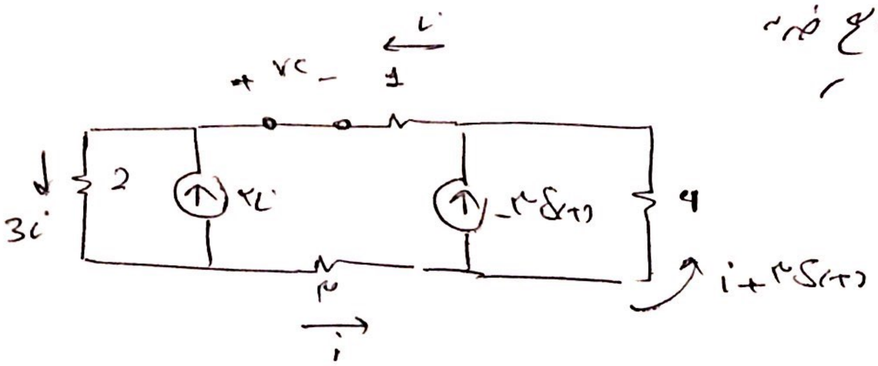
$v_c(0^-) = 2$ ⑧

$I_s(t) = u(t) - 3\frac{d}{dt}u(t)$



روش حل / ۱) تابع ضربه ... ۲) KVL KCL

مشکل/



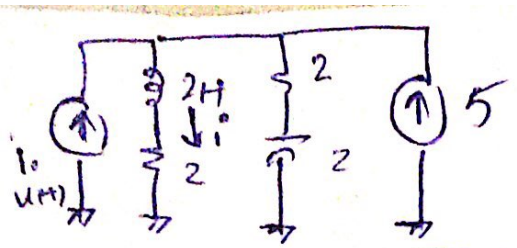
KVL/

$* / \text{KVL} / \quad 2(3i) + 3(i) + 4(i + 3\delta(t)) + i = 0$

$6i + 3i + 4i + 12\delta(t) + i = 0 \rightarrow 14i + 12\delta(t) = 0$

$i = -6/7\,\delta(t)$

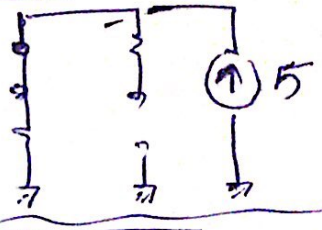
$i_c = -i = +6/7\,\delta(t) \longrightarrow V_c(0^+) = 2 + 6/7 = \boxed{20/7}$!



$\dot{v}_c(0^+)$, ? $\frac{d}{dt} i(0^+)$, $v(0^+)$ ⑨

o/c ω

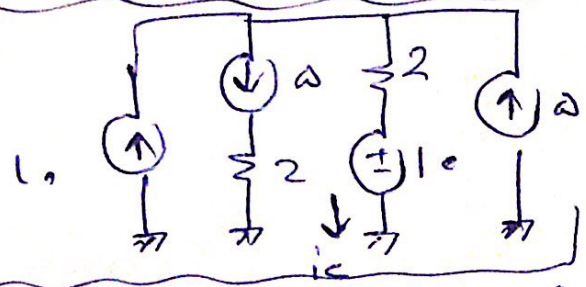
$-\infty < t < 0^-$ L = short
C open



$i_L(0^-) = 0A$

$v_c(0^-) = 10V$

$t = 0^+$



$1_0 + 0 = 0 + i_c \rightarrow i_c = 1.A$

$1_0 = C\dot{v}_c(0^+) \rightarrow \dot{v}_c(0^+) = 0\ V/s$

KVL: $Ri_c + 1_0 = (V_{i=0} = v_L(0^+) + 0 \times r \rightarrow v_{0^+} + 1_0 = 1_0 + v_L(0^+)$

$v_L(0^+) = v_0 = 5\, i_L^{\cdot} \rightarrow i_L^{\cdot} = 20\ A/s$