سارا آذرنوش ۹۸۱۷۰۴۹۸

$\lambda_1 + \lambda_2 + \dots + \lambda_n = trace(A)$ (۱ آ

$\lambda_1 \lambda_2 \dots \lambda_n = det(A)$ (ب

$A = Q\Lambda Q^T \Rightarrow trace(A) = trace(Q\Lambda Q^T) \overset{\text{I}}{\Rightarrow} trace(\Lambda \underbrace{Q Q^T}_{I})$ (آ

$= trace(\Lambda I) = trace(\Lambda) = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_2 + \dots + \lambda_n$

$\Rightarrow trace(A) = \lambda_1 + \lambda_2 + \dots + \lambda_n$

I $\;\; trace(\underset{m\times n}{B}\underset{n\times m}{C}) = trace(CB) = \sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij}c_{ji} = \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ji}b_{ij}$

$A = Q\Lambda Q^T \to det(A) = det(Q)\, det(\Lambda)\, det(Q^T) = det(\Lambda)$ (ب

$det(A) = det(\Lambda) = \prod_{i=1}^{n} \lambda_{ii} = \lambda_1 \lambda_2 \dots \lambda_n$

$\Rightarrow det(A) = \lambda_1 \lambda_2 \dots \lambda_n$

$f(x,y) = 4x^2 + 10y^2 \qquad x^2 + y^2 \leq 4 \qquad f_{max,\,min}$ (۲

$\lambda \geq 0 \to L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

$= 4x^2 + 10y^2 + \lambda(x^2 + y^2 - 4) = 4x^2 + 10y^2 + \lambda x^2 + \lambda y^2 - 4\lambda$

$L_x = 8x + 2\lambda x = 0 \to \lambda = -4$

$L_y = 20y + 2\lambda y = 0 \to \lambda = -10$

$L_\lambda = x^2 + y^2 - 4 = 0 \to (x,y,\lambda) = (\pm 2, 0, -4), (0, \pm 2, -10)$

$max \longrightarrow 40 \qquad x = 0 \qquad y = \pm 2$

$min \longrightarrow -4 \qquad x = 0 \qquad y = 0$

۳)

آ)

$$A = \sum_{1}^{r} \sigma_i u_i v_i^T$$

$$A^+ = \sum_{1}^{r} \frac{v_i u_i^T}{\sigma_i}$$

$$A^+A = \sum_{1}^{r} v_i v_i^T$$

ب)

$$(A^+A)^2 = A^+A$$

آ)

$$A^+A = \sum_{1}^{r} \frac{v_i u_i^T}{\sigma_i} \sum_{1}^{r} \sigma_i u_i v_i^T = \sum_{1}^{r} \frac{1}{\cancel{\sigma_i}} \cancel{\sigma_i} v_i v_i^T \cancel{u_i u_i^T} = \sum_{1}^{r} v_i v_i^T$$

$$A = U\Sigma V^T$$

$$A^+ = V \underbrace{\Sigma^+}_{\text{diag}} U^T$$

$$A^+A = V\Sigma^T \underbrace{U^T U}\Sigma V^T = V\underbrace{\Sigma^+\Sigma} V^T = VV^T$$

ب)

$$(A^+A)^2 = A^+AA^+A = V\Sigma^+ \cancel{U^T U}\Sigma \cancel{V^T V}\Sigma^+ \cancel{U^T U}\Sigma V^T = V\underbrace{\Sigma^+\Sigma}\underbrace{\Sigma^+\Sigma}V^T$$

$$= A^+A$$

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$A = [a_{ij}] \in M_n(\mathbb{C})$ $\quad R_i = \sum_{1 \le j \le n,\, i \ne j} |a_{ij}|$

الف) $R_i < |a_{ii}|$

ب) $\forall i : 1 \le i \le n \longrightarrow A$ وارون‌پذیر

ج) تعبیر هندسی؟ $\cup_{i=1}^{n} \{ z \in \mathbb{C} \mid |z - a_{ii}| \le R_i \}$

$\text{nullity}(A) = 0$

$\exists x \ne 0 \longrightarrow Ax = 0 \longrightarrow \forall i \in N, 1 \le i \le n \longrightarrow \sum_{j=1}^{n} a_{ij} c_j = 0$

از بین c_i ها c_i در نظر می‌گیریم $\longrightarrow |c_i'| \le c_i'$

$\sum_{j=1}^{n} c_{ij} c_j' = 0 = -c_{i i}' c_i'$

$\sum_{i \ne j, j=1}^{n} |c_{ij} c_j'| = |c_{ii} c_i'|$

$\sum_{i \ne j, j=1}^{n} |c_{ij}| |c_i'| \ge |c_{ii}| |c_i'|$

$\sum_{j=1, i \ne j}^{n} |c_{ij}| \ge |c_{ii}|$

$\Rightarrow \text{nullity}(A) = 0 \longrightarrow A$ fullrank $\longrightarrow A$ معکوس‌پذیر

ب)

از بزرگترین $V_i \longrightarrow |v_i| \le |V_i'|$

$v' = \frac{1}{v_i'} v$

$Av' = \lambda v'$ $\quad \sum_{j=1, j \ne i}^{n} |a_{ij} v_j'| = |\lambda - a_{ii}'|$

$\longrightarrow |\lambda - a_{ii}'| \le R_i'$

$\sum_{j=1, j \ne i}^{n} |a_{ij} v_j'| \le R_i'$

ج) مقادیر ویژه در دایره‌هایی به مرکز اعداد روی قطر اصلی و شعاع‌های R_iها قرار گرفته $\quad |z - a_{ii}| \le R$

۱۵

$$A=\begin{bmatrix}1 & 9 & 2\\ 2 & 12 & 5\\ -1 & -3 & -1\end{bmatrix}$$

آ) A تجزیه LU ندارد

ب) سطرها جابجا کنید ماتریس دارای LU و تجزیه ماتریس؟

ج)

$$x_1+9x_2+2x_3=9$$
$$2x_1+12x_2+5x_3=-4$$
$$x_1+3x_2+x_3=-17$$

$$\begin{bmatrix}1 & 9 & 2\\ 2 & 12 & 5\\ -1 & -3 & -1\end{bmatrix}=\begin{bmatrix}l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$$a_{1i}\begin{cases}l_{11}u_{11}=1\\ l_{11}u_{12}=9\\ l_{11}u_{13}=2\end{cases}\qquad a_{2i}\begin{cases}l_{21}u_{11}=2\\ l_{21}u_{12}+l_{22}u_{22}=12\\ l_{21}u_{13}+l_{22}u_{23}=5\end{cases}\qquad a_{3i}\begin{cases}l_{31}u_{11}=-1\\ l_{31}u_{12}+l_{32}u_{22}=-3\\ l_{31}u_{13}+l_{32}u_{23}+l_{33}u_{33}=-1\end{cases}$$

$$l_{11}=u_{11}^{-1}=-l_{31}$$
$$u_{12}=9u_{11}$$
$$u_{13}=2u_{11}$$
$$l_{21}=2u_{11}^{-1}$$

$$\longrightarrow\begin{cases}l_{22}u_{22}=0\\ l_{22}u_{23}=1\end{cases}\longrightarrow u_{22}=0 \longrightarrow \times$$

تجزیه LU ندارد

$$l_{32}\cdot u_{22}=3 \longrightarrow u_{22}\neq 0$$

ب)

$$\begin{bmatrix}1 & 0 & 0\\ -1 & \boxed{1} & 0\\ -2 & \boxed{0} & \boxed{1}\end{bmatrix}\begin{bmatrix}1 & 9 & 2\\ 0 & \boxed{3} & \boxed{1}\\ 0 & 0 & \boxed{1}\end{bmatrix}=\begin{bmatrix}1 & 9 & 2\\ -1 & -3 & -1\\ 2 & 12 & 5\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\ 1/2 & 1 & 0\\ -1/2 & 3 & +3\end{bmatrix}\begin{bmatrix}2 & 12 & 5\\ 0 & 1 & -0.5\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 12 & 5\\ 1 & 9 & 2\\ -1 & -3 & -1\end{bmatrix}$$

۱۶

$$x_1+9x_2+2x_3=9$$
$$2x_1+12x_2+5x_3=-4$$
$$x_1+3x_2+x_3=-17$$

$$\longrightarrow\begin{cases}x_1=D_{x_1}\div D\\ x_2=D_{x_2}\div D\\ x_3=D_{x_3}\div D\end{cases},\quad D=\begin{vmatrix}1 & 9 & 2\\ 2 & 12 & 5\\ 1 & 3 & 1\end{vmatrix}=3$$

$$D_{x_1}=\begin{vmatrix}9 & 9 & 2\\ -4 & 12 & 5\\ -17 & 3 & 1\end{vmatrix}=-129 \longrightarrow x_1=-129/3=-43$$

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$$D_{x_2} = \begin{vmatrix} 1 & 9 & 2 \\ 2 & -6 & 8 \\ 1 & -17 & 1 \end{vmatrix} = 48 \longrightarrow x_2 = 48/3 = 16$$

$$D_{x_3} = \begin{vmatrix} 1 & 9 & 9 \\ 2 & 12 & -6 \\ 1 & 10 & -17 \end{vmatrix} = -96 \longrightarrow x_3 = -96/3 = -32$$

۱۶) L و U به شکل قطری

A $\quad a_{ij} = 0 \quad for \quad |i-j| > 1$

$$A = \begin{bmatrix} a_1 & b_1 & 0 & \cdots & 0 \\ c_1 & a_2 & b_2 & \cdots & 0 \\ 0 & c_2 & a_3 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & a_n \end{bmatrix}$$

اعداد روی قطر اصلی ماتریس U همین هستند

" " " L با یک جمع می شوند

$u_1 = a_1$

$l_2 = \frac{b_2}{a_1} \longrightarrow u_2 = a_2 - u_1 l_2$

$l_n = \frac{b_n}{u_{n-1}} \longrightarrow u_n = a_n - c_{n-1} l_n$

قطر زیر قطر اصلی = ۰

$$U = \begin{bmatrix} u_{11} & u_{12} & 0 & 0 & \cdots & 0 \\ & u_{22} & u_{23} & & & \vdots \\ \vdots & & \ddots & & & \\ & & & & & u_{n-1\,n} \\ & & & & u_{nn} & \\ 0 & \cdots & & & & \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ l_1 & 1 & & \vdots \\ \vdots & l_2 & \ddots & \\ 0 & & l_n & 1 \end{bmatrix}$$