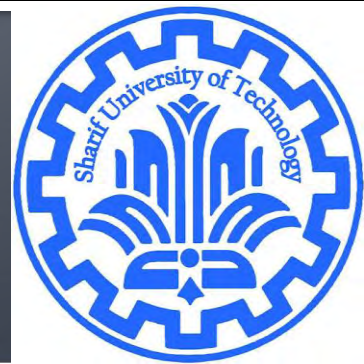


Introduction

CE40282-1: Linear Algebra
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Sharif University of Technology



Why are we here?

- Linear algebra is a main underlying tool for a lot of
 - Computer science applications
 - Machine learning
 - Robotics
 - Operations research
 - Theory
- You will learn the basic tools of Linear Algebra and how they apply to various aspects of computer science

Who we are?

- Instructors:
 - Hamid R. Rabiee: (rabiee@sharif.edu)
 - Maryam Ramezani: (maryam.ramezani@sharif.edu)
- Head TA:
 - Mina Rafiei: (mrafiei@ce.sharif.edu)
- Room: (Sunday & Tuesday: 13:30-15:00)
 - <https://vc.sharif.edu/ch/rabiee>
- Course Website:
 - <http://ce.sharif.edu/courses/00-01/1/ce282-1>

TA class

- TA class: (Sunday: 12-13:15- Start: 1400/07/11)
 - <https://vc.sharif.edu/ch/rabiee>
- Homework Upload:
 - <https://quera.ir/course/9131/>
- Exams:
 - <https://cw.sharif.edu/course/view.php?id=6870>

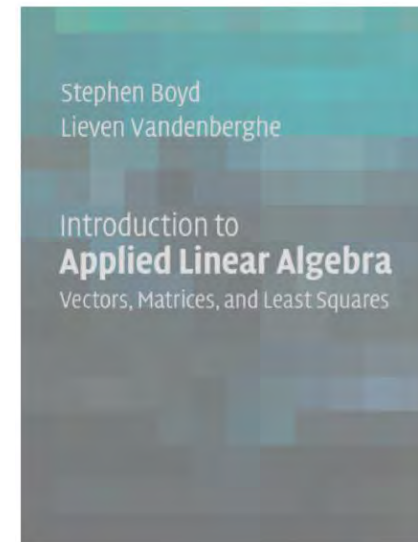
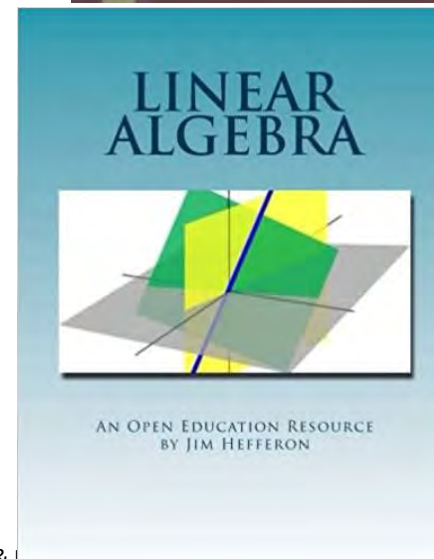
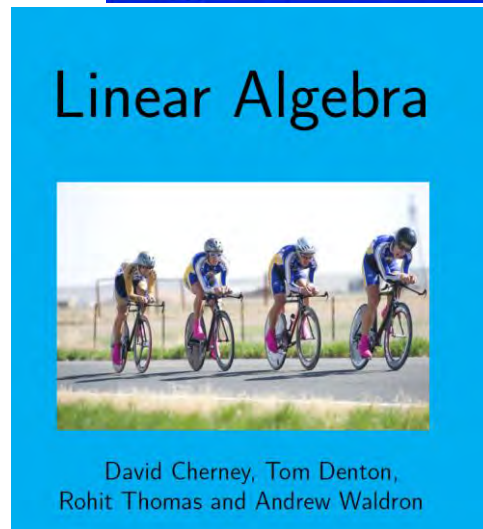
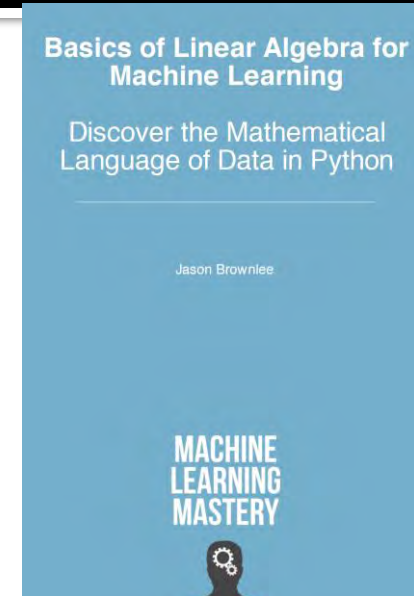
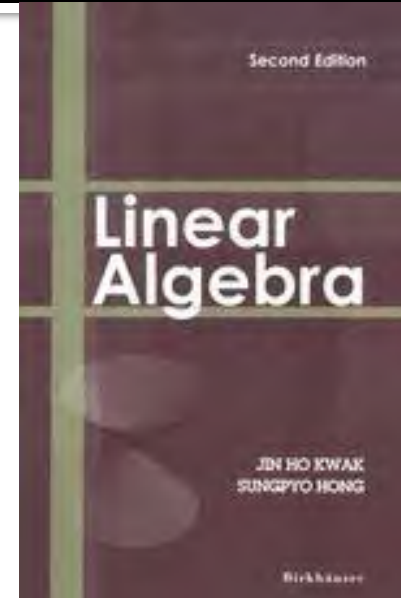
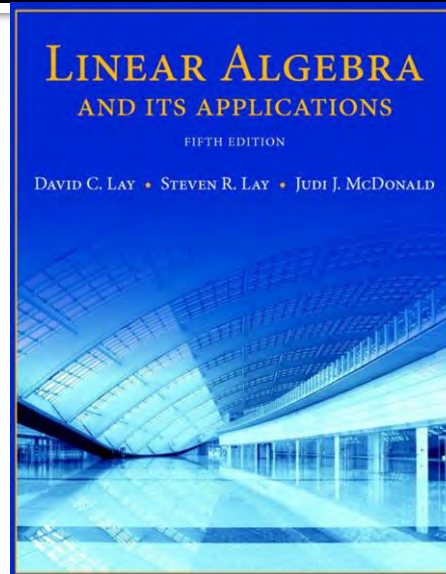
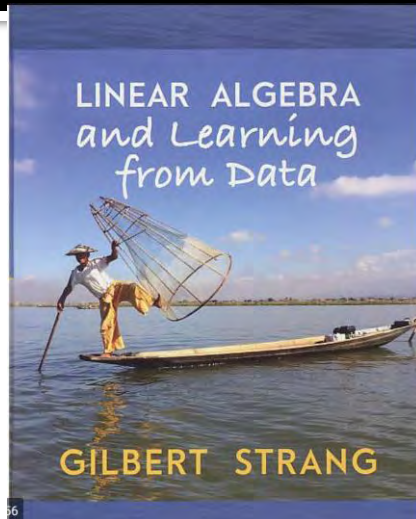
Homework

- 5-6 theory questions
- Basic Python for programming
- Basic NumPy for array manipulation
- Basic programming with Pytorch
- Learn linear algebra understanding and application of machine learning.

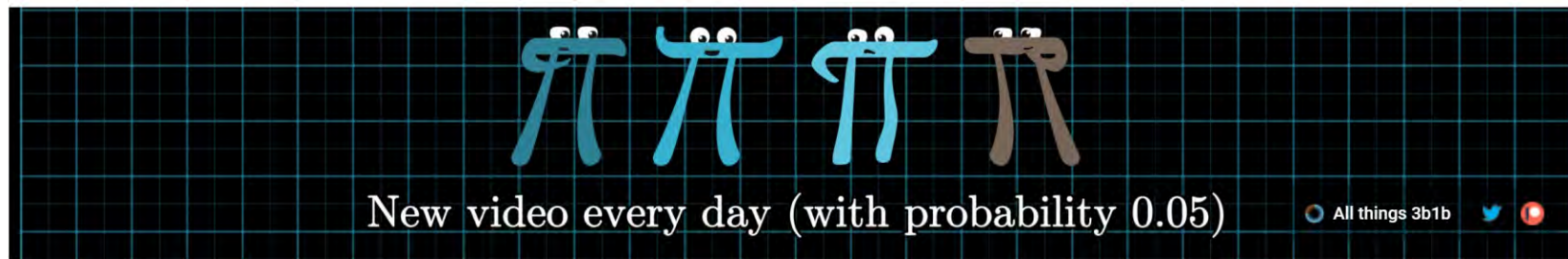
Grading Scheme

- Homework: 50% + 10%
- Mid-Term Exam: 20% (1400/09/02)
- Final Exam: 30% (Comprehensive) (1400/10/28)
- You have a total 11 days of allowed late submission (for both theoretical and code part).
- You can use your late submission before TA class. No submissions are accepted after TA class.
- After spending all your allowed late submission, you can submit your homework by 10% decrease in grade by each day.

Textbooks



Some Useful Links



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3.94M subscribers

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Description

3Blue1Brown, by Grant Sanderson, is some combination of math and entertainment, depending on your disposition. The goal is for explanations to be driven by animations and for difficult problems to be made simple with changes in perspective.

For more information, other projects, FAQs, and inquiries see the website:
<https://www.3blue1brown.com>

Stats

Joined Mar 4, 2015

217,063,334 views

Details

Location: United States

What we'll cover

- Vector space
- Linear functions
- Linear independence
- Matrices
- Matrix operations
- Derivatives of matrices
- Factorization
- Linear equations
- Matrix calculus
- Matrix application
- Linear programming
- Tensor
- Linear algebra in ML applications

What is linear algebra?

- Linear algebra is a branch of mathematics, but the truth of it is that linear algebra is the mathematics of data.
- Matrices and vectors are the language of data
- Linear algebra is the study of lines and planes, vector spaces and mappings that are required for linear transforms.
- Linear algebra is the study of lines and planes, vector spaces and mappings that are required for linear transforms.

$$y = 0.1 \times x_1 + 0.4 \times x_2$$

$$y = 0.3 \times x_1 + 0.9 \times x_2$$

$$y = 0.2 \times x_1 + 0.3 \times x_2$$

...



$$y = A \cdot b$$

Numerical Linear Algebra

- The application of linear algebra in computers is often called **numerical linear algebra**
- Computers are good at performing linear algebra calculations, and much of the dependence on Graphical Processing Units (GPUs) by modern machine learning methods such as deep learning is because of their ability to compute linear algebra operations fast.
- Three popular open source numerical linear algebra libraries that implement these functions are:
 - Linear Algebra Package, or LAPACK.
 - Basic Linear Algebra Subprograms, or BLAS (a standard for linear algebra libraries).
 - Automatically Tuned Linear Algebra Software, or ATLAS.

Linear Algebra and Statistics

- Linear algebra is a valuable tool in other branches of mathematics, especially statistics.
- Some clear fingerprints of linear algebra on statistics and statistical methods include:
 - Use of vector and matrix notation, especially with multivariate statistics.
 - Solutions to least squares and weighted least squares, such as for linear regression.
 - Estimates of mean and variance of data matrices.
 - The covariance matrix that plays a key role in multinomial Gaussian distributions.
 - Principal component analysis for data reduction that draws many of these elements together.

Applications of Linear Algebra

- Matrices in Engineering
- Graphs and Networks, such as analyzing networks
- Markov Matrices, Population, and Economics, such as population growth
- Linear Programming, the simplex optimization method
- Fourier Series: Linear Algebra for functions, used widely in signal processing
- Linear Algebra for statistics and probability, such as least squares for regression
- Computer Graphics, such as the various translation, rescaling and rotation of images

Linear Algebra in Machine Learning

■ Dataset and Data Files

- In machine learning, you fit a model on a dataset.
- This is the table-like set of numbers where each row represents an observation and each column represents a feature of the observation.

Attribute Information:

1	5.1,3.5,1.4,0.2,Iris-setosa
2	4.9,3.0,1.4,0.2,Iris-setosa
3	4.7,3.2,1.3,0.2,Iris-setosa
4	4.6,3.1,1.5,0.2,Iris-setosa
5	5.0,3.6,1.4,0.2,Iris-setosa

1. sepal length in cm
2. sepal width in cm
3. petal length in cm
4. petal width in cm
5. class:
 - Iris Setosa
 - Iris Versicolour
 - Iris Virginica

- This data is in fact a matrix: a key data structure in linear algebra.

Linear Algebra in Machine Learning

■ Images and Photographs

- Each image that you work with is itself a table structure with a width and height and one pixel value in each cell for black and white images or 3 pixel values in each cell for a color image.
- A photo is yet another example of a matrix from linear algebra.
- Operations on the image, such as cropping, scaling, shearing, and so on are all described using the notation and operations of linear algebra.

Linear Algebra in Machine Learning

■ One Hot Encoding

- Sometimes you work with categorical data in machine learning.
- It is common to encode categorical variables to make them easier to work with and learn by some techniques. A popular encoding for categorical variables is the one hot encoding.
- A one hot encoding is:

For example, the color variable with the 3 rows:

1	red
2	green
3	blue
4	...

Might be encoded as:

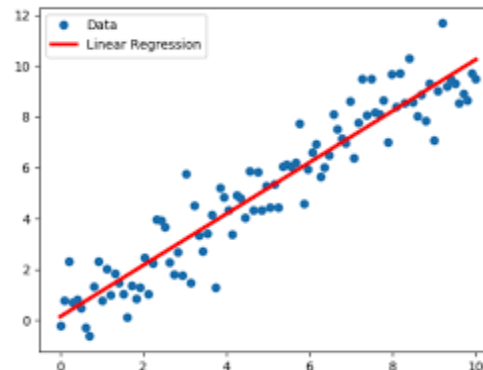
1	red, green, blue
2	1, 0, 0
3	0, 1, 0
4	0, 0, 1
5	...

Linear Algebra in Machine Learning

■ Linear Regression

- Describing the relationships between variables.
- Predicting numerical values in simpler regression problems.
- The most common way of solving linear regression is via a least squares optimization that is solved using matrix factorization methods from linear regression, such as an LU decomposition or a singular-value decomposition, or SVD.

$$y = A \cdot b$$



Linear Algebra in Machine Learning

■ Principal Component Analysis

- Often, a dataset has many columns, perhaps tens, hundreds, thousands, or more.
- It is hard to know which features of the data are relevant and which are not.
- Methods for automatically reducing the number of columns of a dataset are called dimensionality reduction, and perhaps the most popular method is called the principal component analysis, or PCA for short
- The core of the PCA method is a matrix factorization method from linear algebra.

Linear Algebra in Machine Learning

■ Singular-Value Decomposition

- It is a matrix factorization method from the field of linear algebra.
- It has wide use in linear algebra and can be used directly in applications such as feature selection, visualization, noise reduction, and more.

Linear Algebra in Machine Learning

■ Latent Semantic Analysis

- In the sub-field of machine learning for working with text data called natural language processing, it is common to represent documents as large matrices of word occurrences
- For example, the columns of the matrix may be the known words in the vocabulary and rows may be sentences, paragraphs, pages, or documents of text with cells in the matrix marked as the count or frequency of the number of times the word occurred.
- Matrix factorization methods, such as the singular-value decomposition can be applied to this sparse matrix.
- Documents processed in this way are much easier to compare, query, and use as the basis for a supervised machine learning model.
- This form of data preparation is called Latent Semantic Analysis, or LSA for short, and is also known by the name Latent Semantic Indexing, or LSI.

Linear Algebra in Machine Learning

■ Recommender Systems

- Predictive modeling problems that involve the recommendation of products are called recommender systems, a sub-field of machine learning.
- The development of recommender systems is primarily concerned with linear algebra methods. A simple example is in the calculation of the similarity between sparse customer behavior vectors using distance measures such as Euclidean distance or dot products.
- Matrix factorization methods like the singular-value decomposition are used widely in recommender systems to distill item and user data to their essence for querying and searching and comparison.

Linear Algebra in Machine Learning

■ Deep Learning

- Artificial neural networks are nonlinear machine learning algorithms that are inspired by elements of the information processing in the brain and have proven effective at a range of problems, not the least of which is predictive modeling.
- Deep learning is the recent resurgence in the use of artificial neural networks with newer methods and faster hardware that allow for the development and training of larger and deeper (more layers) networks on very large datasets.
- At their core, the execution of neural networks involves linear algebra data structures multiplied and added together. Scaled up to multiple dimensions, deep learning methods work with vectors, matrices, and even tensors of inputs and coefficients, where a tensor is a matrix with more than two dimensions.
- Linear algebra is central to the description of deep learning methods via matrix notation to the implementation of deep learning methods such as Google's TensorFlow Python library that has the word "tensor" in its name.

Next Session

- Vectors
 - Background (who knows what a vector is?)

Reference

Basics of Linear Algebra for Machine Learning

Discover the Mathematical
Language of Data in Python

Jason Brownlee

**MACHINE
LEARNING
MASTERY**

