

سوال

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$$\begin{cases} x^r y'' + c x y' - 18 y = 0 \rightarrow x^r y'' + c/r x y' - 18/c y = 0 \\ y(1) = 0 \\ y'(1) = 1 \end{cases}$$

$$h = 0.0$$

$$y(1,1) = ?$$

$$\begin{aligned} x^r \frac{d^2 y}{dx^2} + a m \frac{dy}{dx} + b y &= 0 \rightarrow m^r + (a-1)m + b = 0 \\ \downarrow m(m-1)x^{m-1} &\rightarrow m x^{m-1} \end{aligned} \quad \left. \begin{aligned} x^r y'' + r/c x y' - 18/c y &= 0 \\ m^r + (r/c)m - 18/c &= 0 \end{aligned} \right\}$$

$$\Rightarrow (rm - 18)(m + 1) = 0 \rightarrow \begin{cases} 18/c \\ -1 \end{cases}$$

$$\begin{aligned} y &= C_1 x^{18/c} - C_2 x^{-1} \rightarrow y(1) = 0 = C_1 + C_2 \rightarrow C_1 = 1/11 \\ y'(1) &= 1 = 18/c C_1 - C_2 \rightarrow C_2 = -9/11 \end{aligned}$$

$$y = \frac{1}{11} x^{18/c} - \frac{9}{11} x^{-1}$$

$$y' = -rxy^r$$

$$y(0) = 1$$

$$h = 0.1 \quad y(0.1)$$

4th) $f(x, y) = -rxy^r$

$$k_1 = 0.1 f(x_0, y_0) = 0.1 f(0, 1) = 0$$

$$k_2 = 0.1 f(x_0 + h/2, y_0 + k_1/2) = 0.1 f(0.05, 1) = -0.1 \cdot 0.5$$

$$k_3 = 0.1 f(x_0 + h, y_0 + k_2) = 0.1 f(0.1, 0.975) = -0.1 \cdot 0.975 \cdot 0.5$$

$$k_4 = 0.1 f(x_0 + 3h/4, y_0 + 3k_3/4) = -0.1 \cdot 0.9414 \cdot 0.5$$

$$y_1 = y_0 + 1/4 (k_1 + 2k_2 + 2k_3 + k_4) = 1 + 1/4 (0 - 0.1 \cdot 0.5 - 0.1 \cdot 0.975 \cdot 0.5 - 0.1 \cdot 0.9414 \cdot 0.5) = 0.9414$$

third) $k_1 = h f(x_0, y_0) = 0$

$$k_2 = h f(x_0 + h/2, y_0 + k_1/2) = -0.1 \cdot 0.5$$

$$k_3 = h f(x_0 + h, y_0 + k_2) = 0.1 f(0.1, 1 + (-0.1 \cdot 0.5)) = -0.1 \cdot 0.975 \cdot 0.5$$

$$y_{i+1} = y_i + 1/4 (k_1 + 2k_2 + 2k_3 + k_4) = 1 + 1/4 (0 + 2(-0.1 \cdot 0.5) + 2(-0.1 \cdot 0.975 \cdot 0.5) + 0) = 0.9414$$

2nd) $k_1 = h f(x_0, y_0) = 0$

$$k_2 = h f(x_0 + h/2, y_0 + k_1/2) = 0.1 f(0.05, 1) = -0.1 \cdot 0.5$$

$$y_{i+1} = y_i + 1/2 (k_1 + k_2) = 1 + 1/2 (0 + (-0.1 \cdot 0.5)) = 0.975$$

\rightarrow

$$4th) \quad k_1 = .12 P(.12, .1948V) = -.11729$$

$$k_2 = .12 P(.12, .1948) = -.11091$$

$$k_3 = .12 P(.12, .1948) = -.11091$$

$$k_4 = .12 P(.12, .1948V) = -.11091$$

$$y = y - \frac{1}{4} (-.11729 - 2(-.11091) + 2(-.11091) - .11091) = .11729$$

$$third) \quad k_1 = .12 P(.12, 1.239) = -.1191$$

$$k_2 = .12 P(.12, 1.239 + \frac{1.1}{.12}) = -.119$$

$$k_3 = .12 P(.12, 1.239 - .12 + .1191) = .1192V$$

$$y = 1.239 + \frac{1}{4} (-.1191 + 4 \times .119 - .1192V) = .1192$$

2nd)

$$k_1 = .12 (.12, 1.1) = -.1182$$

$$k_2 = .12 (.12 + .1182, 1.1 + \frac{1.1}{.12} \times .1182) = -.1194V$$

$$y = 1.1 + \frac{1}{2} (-.1182 + 2 \times -.1194V) = .1194$$

در ۲ سترین (در ۴ سترین است)

$$t \in [0, 1]$$

$$h = 0.2$$

$$\begin{cases} \frac{dy}{dt} = -y + t^2 \\ y(0) = 1 \end{cases}$$

المسألة الأولى: حل المعادلة التفاضلية

بـ: حل المعادلة التفاضلية

$$\text{Predictor} \rightarrow y_{i+1}^* = y_i + h y_i'$$

$$\text{Corrector} \rightarrow y_{i+1} = y_i + \frac{h}{2} (y_i' + y_{i+1}^{'*})$$

المسألة

t	y _i	y _i '
0	1	-2
0.2	0.8458	-0.178
0.4	0.5712	0.458

$$I) y_1^* = y_0 + h y_0' = 1 + 0.2(-2) = 0.6$$

$$y_1^{'*} = -y_1^* + t_1^2 = -0.6 + 0.04 = -0.56$$

$$y_1 = y_0 + \frac{h}{2} (y_0' + y_1^{'*}) = 1 + 0.1(-2 - 0.56) = 0.8458$$

$$y_1' = -y_1 + t_1^2 = -0.8458 + 0.04 = -0.8058$$

$$II) y_2^* = y_1 + h y_1' = 0.8458 + 0.2(-0.8058) = 0.2938$$

$$y_2^{'*} = -y_2^* + t_2^2 = -0.2938 + 0.16 = -0.1338$$

$$y_2 = y_1 + \frac{h}{2} (y_1' + y_2^{'*}) = 0.8458 + 0.1(-0.8058 - 0.1338) = 0.7312$$

$$y_2' = -y_2 + t_2^2 = -0.7312 + 0.16 = -0.5712$$

t	y	y'
0	1	-2
0.18	0.18514	0.18002
1	0.8	.

$$y_{i+1} = y + 1/2 k_1 + 1/2 k_2$$

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$$k_1 = h y'$$

$$k_2 = h p(x_i + 1/2 h, y + 1/2 k_1)$$

$$I) k_1 = 0.18 \times -2 = -0.36$$

$$k_2 = 0.18 p(0.18 + 1/2 \times 0.18, 1 + 1/2 \times -0.36) = -0.1514$$

$$y_1 = 1 + 1/2 \times -0.36 + 1/2 \times (-0.1514) = 0.8007$$

$$y'_1 = -2(0.8007) + 0.18 = -1.4214$$

$$II) k_1 = 0.18(-1.4214) = -0.2558$$

$$k_2 = 0.18 p(0.18 + 1/2 \times 0.18, 0.8007 + 1/2 \times -0.2558) = -0.1500$$

$$y_2 = 0.8007 + 1/2(-0.2558) + 1/2(-0.1500) = 0.7007$$

$$y'_2 = -2(0.7007) + 0.18 = -1.2214$$

t	y	y'
0	1	-2
0.18	0.8007	-1.4214
1	0.7007	-1.2214

$$y_{i+1} = y_i + k_i$$

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$$k_2 = h p(x_i + 1/2 h, y_i + 1/2 k_1)$$

$$I) y_1 = y_0 + k_1$$

$$k_1 = h y'_0 = 0.18 \times -2 = -0.36$$

$$k_2 = 0.18 p(0.18 + 1/2 \times 0.18, 1 + 1/2 \times -0.36) = -0.1514$$

$$y_1 = 1 - 0.36 = 0.64 \rightarrow y'_1 = -2(0.64) + 0.18 = -1.10$$

$$II) y_2 = y_1 + k_2$$

$$k_1 = h y'_1 = 0.18 \times -1.10 = -0.198$$

$$k_2 = 0.18 p(0.18 + 1/2 \times 0.18, 0.64 - 1/2 \times 0.198) = -0.1500$$

$$y_2 = 0.64 - 0.198 = 0.442 \rightarrow y'_2 = -2 \times (0.442) + 0.18 = -0.704$$

Predictor-corrector

$y(0.1)$

$$y' = y - x^2$$

Predictor: $y_{i+1}^* = y_i + h/r_2 (22p_i - 24p_{i-1} + 12p_{i-2} - 4p_{i-3})$

x	y	y'
0	1	1
0.1	1.121	1.121
0.2	1.221	1.142
0.3	1.321	1.163

Corrector: $y_{i+1} = y_i + h/r_2 (4p_{i+1}^* + 11p_i - 2p_{i-1} + p_{i-2})$

$$y_{0.1}^* = y_{0.0} + h/r_2 (22y'_{0.0} - 24y'_{0.1} + 12y'_{0.2} - 4y'_{0.3}) = 1.121 + \frac{0.1}{2} ($$

$$22 \times 1.163 - 24 \times 1.142 + 12 \times 1.121 - 4 \times 1) \rightarrow$$

$$y_{0.1}^* = 1.121 + 0.1 \times 1.163 = 1.142 \rightarrow y_{0.1}^* = 1.142$$

$$y' = y - x^2 \rightarrow y'_{0.1} = y_{0.1}^* - 0.1^2 = 1.142 - 0.1^2 = 1.132 \rightarrow y'_{0.1} = 1.132$$

Corrector $\rightarrow y_{0.1} = y_{0.0} + h/r_2 (4y'_{0.1}^* + 14y'_{0.0} - 2y'_{0.1} + y'_{0.0}) =$

$$1.121 + \frac{0.1}{2} (4 \times 1.132 + 14 \times 1.163 - 2 \times 1.142 + 1.121) \rightarrow$$

$$y_{0.1} = 1.121 + 0.1 \times 1.163 = 1.142 \rightarrow y_{0.1} = 1.142$$

$$y(0.1) = 1.142$$

١٥ عربيات

$$\ln y' + u = \sin x$$

$$\int \bar{\omega}_\mu u = -\ln y' + \sin x$$

الف

$$y(x) = \int e^{\int \bar{\omega}_\mu(x) dx} dx - c_1$$

$$y''y - ry' - ry' = 0 \rightarrow y'' = \frac{1}{y} (ry' + ry')$$

ب

$$u_1 = y \rightarrow u'_1 = y'$$

$$u_2 = y' \rightarrow u'_2 = y''$$

$$\frac{du_1}{dx} = u_2$$

$$\frac{du_2}{dx} = \frac{1}{u_1} (ry' + ry')$$

$$y'' = ry' / \sqrt{1 - y'^2} = 0$$

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$$u'_1 = y' / u'_2 = y'' / u'_2 = y''' / u'_2 = y^{(4)}$$

$$\frac{du_1}{dx} = u_2$$

$$\frac{du_2}{dx} = u_3$$

$$\frac{du_3}{dx} = u_4$$

$$\frac{du_4}{dx} = \epsilon u_2 \sqrt{1 - u_1'^2}$$

$$(y'')' = |ry' - y'|$$

$$u_1 = y \rightarrow u'_1 = y'$$

$$u'_2 = y'' = \sqrt{|ry' - y'|}$$

$$\frac{du_1}{dx} = u_2$$

$$\frac{du_2}{dx} = \sqrt{|ry' - y'|}$$

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