

## 1.3: Basic Trigonometric Functions

### Learning Objectives

- Find the six trigonometric function values of an angle in a right triangle.

Sine, cosine, tangent, and other ratios of sides of a right triangle.

### Sine, Cosine, and Tangent

**Trigonometry** is the study of the relationships between the sides and angles of right triangles. The legs are called *adjacent* or *opposite* depending on which **acute angle** is being used.

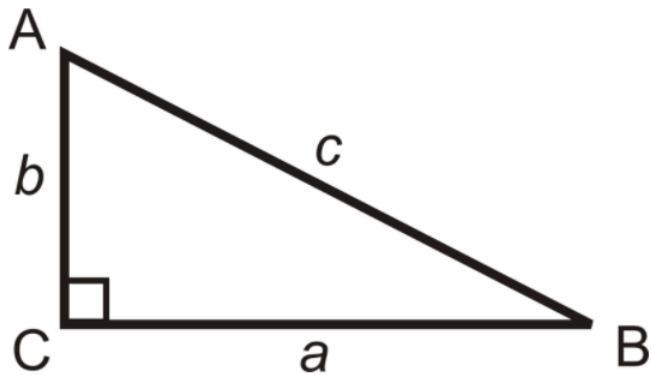


Figure 1.3.1

- |                               |                            |
|-------------------------------|----------------------------|
| $a$ is adjacent to $\angle B$ | $a$ is opposite $\angle A$ |
| $b$ is adjacent to $\angle A$ | $b$ is opposite $\angle B$ |
| $c$ is the hypotenuse         |                            |

The three basic **trigonometric ratios** are called sine, cosine and tangent. For right triangle  $\triangle ABC$ , we have:

$$\begin{array}{ll} \text{sine Ratio: } \frac{\text{opposite leg}}{\text{hypotenuse}} & \sin A = \frac{a}{c} \text{ or } \sin B = \frac{b}{c} \\ \text{cosine Ratio: } \frac{\text{adjacent leg}}{\text{hypotenuse}} & \cos A = \frac{b}{c} \text{ or } \cos B = \frac{a}{c} \\ \text{Tangent Ratio: } \frac{\text{opposite leg}}{\text{adjacent leg}} & \tan A = \frac{a}{b} \text{ or } \tan B = \frac{b}{a} \end{array}$$

An easy way to remember ratios is to use SOH-CAH-TOA.

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Figure 1.3.2

### A few important points:

- Always **reduce ratios** (fractions) when you can.
- Use the **Pythagorean Theorem** to find the missing side (if there is one).
- If there is a **radical** in the denominator, **rationalize the denominator**.

What if you were given a right triangle and told that its sides measure 3, 4, and 5 inches? How could you find the sine, cosine, and tangent of one of the triangle's non-right angles?



## Trigonometric Ratios: Lesson (Basic)



### Example 1.3.1

Find the sine, cosine and tangent ratios of  $\angle A$ .

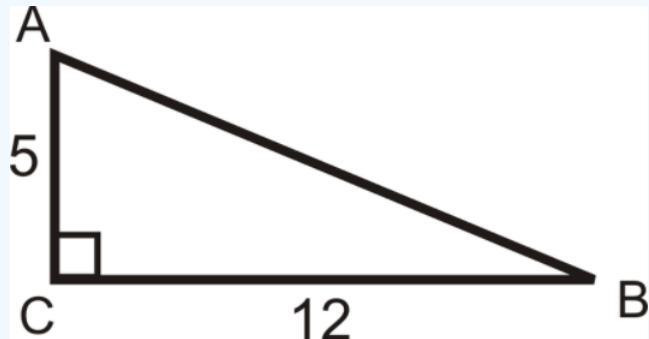


Figure 1.3.3

### Solution

First, we need to use the Pythagorean Theorem to find the length of the **hypotenuse**.

$$5^2 + 12^2 = c^2$$

$$13 = c$$

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{12}{13} \quad \cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{5}{13},$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{12}{5}$$

### Example 1.3.2

Find the sine, cosine, and tangent of  $\angle B$ .

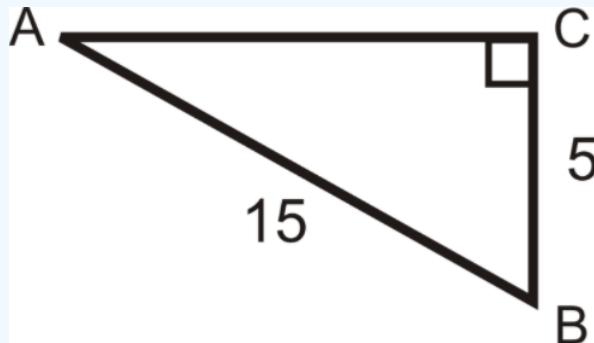


Figure 1.3.4

Find the length of the missing side.

### Solution

$$AC^2 + 5^2 = 15^2$$

$$AC^2 = 200$$

$$AC = 10\sqrt{2}$$

$$\sin B = \frac{10\sqrt{2}}{15} = \frac{2\sqrt{2}}{3} \quad \cos B = \frac{5}{15} = \frac{1}{3} \quad \tan B = \frac{10\sqrt{2}}{5} = 2\sqrt{2}$$

**Example 1.3.3**

Find the sine, cosine and tangent of  $30^\circ$ .

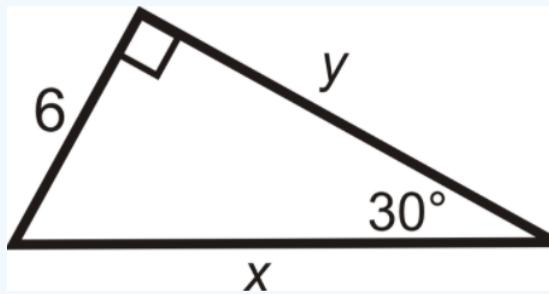


Figure 1.3.5

**Solution**

This is a 30-60-90 triangle. The short leg is 6,  $y = 6\sqrt{3}$  and  $x = 12$ .

$$\sin 30^\circ = \frac{6}{12} = \frac{1}{2} \quad \cos 30^\circ = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

**Example 1.3.4**

Answer the questions about the following image. Reduce all fractions.

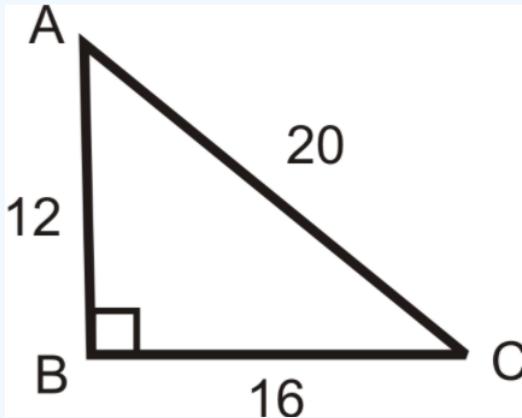


Figure 1.3.6

What is  $\sin A$ ,  $\cos A$ , and  $\tan A$ ?

**Solution**

$$\sin A = \frac{16}{20} = \frac{4}{5}$$

$$\cos A = \frac{12}{20} = \frac{3}{5}$$

$$\tan A = \frac{16}{12} = \frac{4}{3}$$

## Resources



duction to Trigonometric Functions ...



## Vocabulary

Term	Definition
<b>Acute Angle</b>	An acute angle is an angle with a measure of less than 90 degrees.
<b>Adjacent Angles</b>	Two angles are adjacent if they share a side and vertex. The word 'adjacent' means 'beside' or 'next-to'.
<b>Hypotenuse</b>	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
<b>Legs of a Right Triangle</b>	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
<b>opposite</b>	The opposite of a number $x$ is $-x$ . A number and its opposite always sum to zero.
<b>Pythagorean Theorem</b>	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$ , where $a$ and $b$ are legs of the triangle and $c$ is the hypotenuse of the triangle.
<b>Radical</b>	The $\sqrt{\phantom{x}}$ or square root, sign.
<b>sine</b>	The sine of an angle in a right triangle is a value found by dividing the length of the side opposite the given angle by the length of the hypotenuse.
<b>Trigonometric Ratios</b>	Ratios that help us to understand the relationships between sides and angles of right triangles.

## Additional Resources

Video: Introduction to Trigonometric Functions Using Triangles

Activities: Sine, Cosine, Tangent Discussion Questions

Study Aids: Trigonometric Ratios Study Guide

Practice: Right Triangle Trigonometry

Real World: Sine Cosine Tangent

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