1.2: Special Right Triangles

Learning Objectives

- Recognize Special Right Triangles.
- Use the special right triangle rations to solve special right triangles.

30-60-90 Right Triangles

Hypotenuse equals twice the smallest leg, while the larger leg is $\sqrt{3}$ times the smallest.

One of the two special right triangles is called a **30-60-90 triangle**, after its three angles.

30-60-90 Theorem: If a triangle has angle measures 30°, 60° and 90°, then the sides are in the ratio $x : x\sqrt{3} : 2x$.

The shorter leg is always x, the longer leg is always $x\sqrt{3}$, and the **hypotenuse** is always $2x$. If you ever forget these theorems, you can still use the **Pythagorean Theorem**.

What if you were given a 30-60-90 right triangle and the length of one of its side? How could you figure out the lengths of its other sides?

Example 1.2.1

Find the value of x and y.

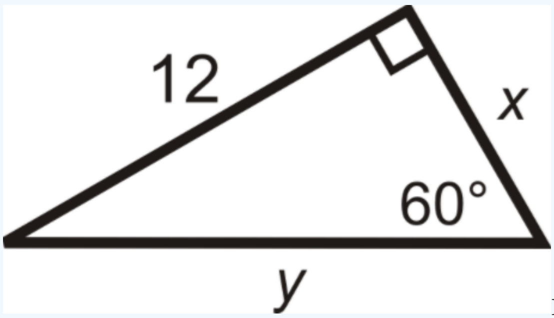


Figure 1.2.1

Solution

We are given the longer leg.

$$x\sqrt{3} = 12$$

$$x = 12\sqrt{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 12\frac{\sqrt{3}}{3} = 4\sqrt{3}$$

The hypotenuse is

$$y = 2(4\sqrt{3}) = 8\sqrt{3}$$

Example 1.2.2

Find the value of x and y.

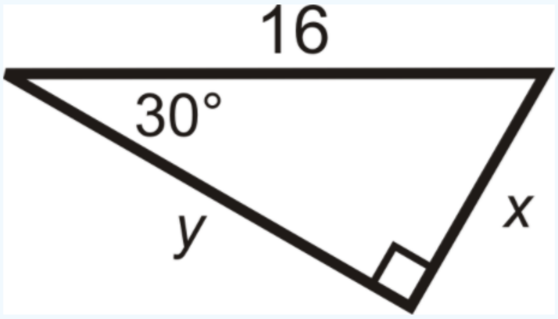


Figure 1.2.2

Solution

We are given the hypotenuse.

$$2x = 16$$
$$x = 8$$

The longer leg is

$$y = 8 \cdot \sqrt{3} \qquad = 8\sqrt{3}$$

Example 1.2.3

Find the length of the missing sides.

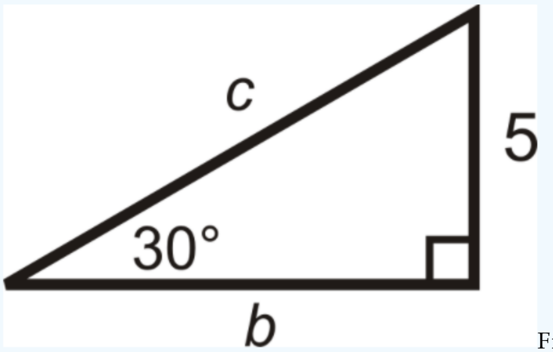


Figure 1.2.3

Solution

We are given the shorter leg. If $x = 5$, then the longer leg, $b = 5\sqrt{3}$, and the hypotenuse, $c = 2(5) = 10$.

Example 1.2.4

Find the length of the missing sides.

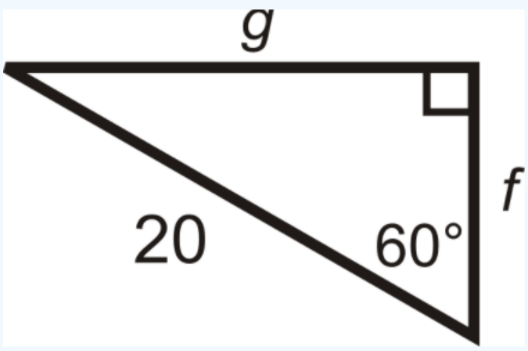


Figure 1.2.4

Solution

We are given the hypotenuse. $2x = 20$, so the shorter leg, $f = \frac{20}{2} = 10$, and the longer leg, $g = 10\sqrt{3}$.

Example 1.2.5

A rectangle has sides 4 and $4\sqrt{3}$. What is the length of the diagonal?

Solution

If you are not given a picture, draw one.

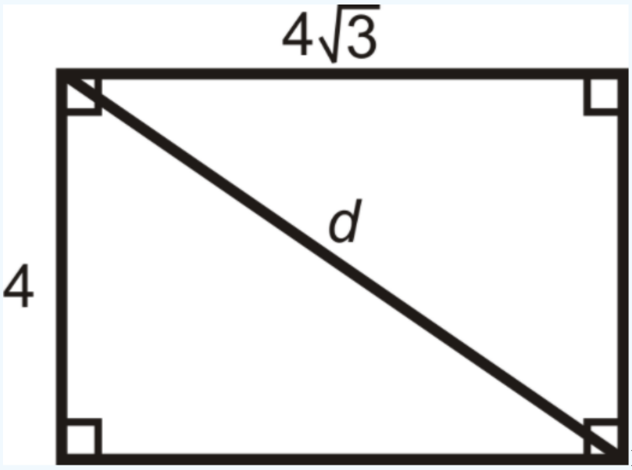


Figure 1.2.5

The two lengths are x, $x\sqrt{3}$, so the diagonal would be $2x$, or $2(4) = 8$.

If you did not recognize this is a 30-60-90 triangle, you can use the Pythagorean Theorem too.

$$4^2 + (4\sqrt{3})^2 = d^2$$
$$16 + 48 = d^2$$
$$d = \sqrt{64} = 8$$

Review

1. In a 30-60-90 triangle, if the shorter leg is 5, then the longer leg is ___________ and the hypotenuse is ____________.
2. In a 30-60-90 triangle, if the shorter leg is x, then the longer leg is ___________ and the hypotenuse is ____________.
3. A rectangle has sides of length 6 and $6\sqrt{3}$. What is the length of the diagonal?
4. Two (opposite) sides of a rectangle are 10 and the diagonal is 20. What is the length of the other two sides

45-45-90 Right Triangles

A right triangle with congruent legs and acute angles is an **Isosceles Right Triangle**. This triangle is also called a **45-45-90 triangle** (named after the angle measures).

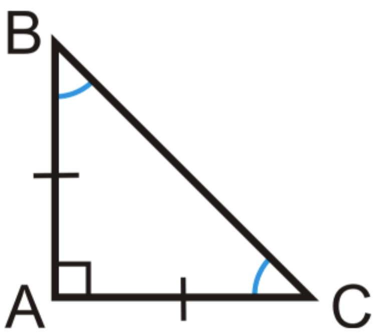


Figure 1.2.6

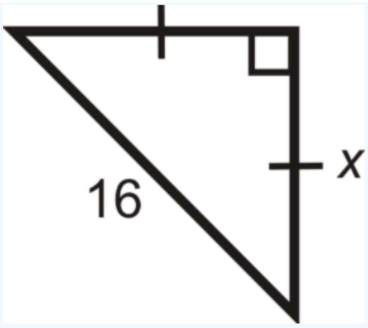
ΔABC is a right triangle with $m\angle A = 90^\circ$, $\overline{AB} \cong \overline{AC}$ and $m\angle B = m\angle C = 45^\circ$.

45-45-90 Theorem: If a right triangle is isosceles, then its sides are in the ratio $x : x : x\sqrt{2}$. For any isosceles right triangle, the legs are x and the **hypotenuse** is always $x\sqrt{2}$.

What if you were given an isosceles right triangle and the length of one of its sides? How could you figure out the lengths of its other sides?

Example 1.2.6

Find the length of x.



Solution

Use the $x : x : x\sqrt{2}$ ratio.

Here, we are given the hypotenuse. Solve for x in the ratio.

$$x\sqrt{2} = 16$$
$$x = 16\sqrt{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$$

Example 1.2.7

Find the length of x, where x is the hypotenuse of a 45-45-90 triangle with leg lengths of $5\sqrt{3}$.

Solution

Use the $x : x : x\sqrt{2}$ ratio.

$x = 5\sqrt{3} \cdot \sqrt{2} = 5\sqrt{6}$

Example 1.2.8

Find the length of the missing side.

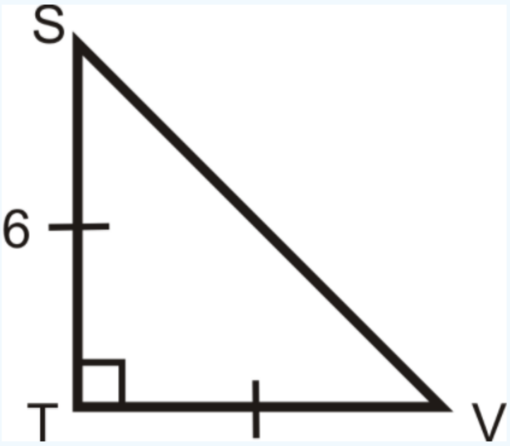


Figure 1.2.8

Solution

Use the $x : x : x\sqrt{2}$ ratio. $TV = 6$ because it is equal to ST. So, $SV = 6 \cdot \sqrt{2} = 6\sqrt{2}$.

Example 1.2.9

Find the length of the missing side.

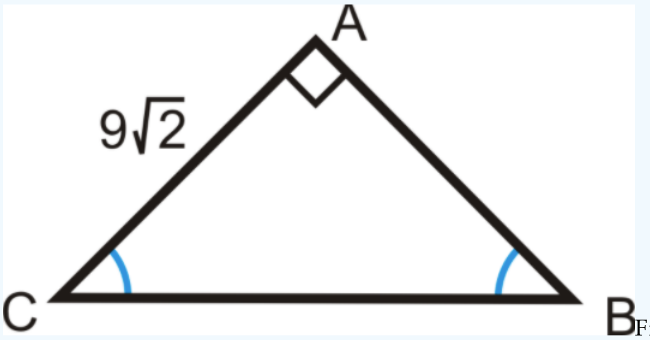


Figure 1.2.9

Solution

Use the $x : x : x\sqrt{2}$ ratio. $AB = 9\sqrt{2}$ because it is equal to AC. So, $BC = 9\sqrt{2} \cdot \sqrt{2} = 9 \cdot 2 = 18$.

Example 1.2.10

A square has a diagonal with length 10, what are the lengths of the sides?

Solution

Draw a picture.

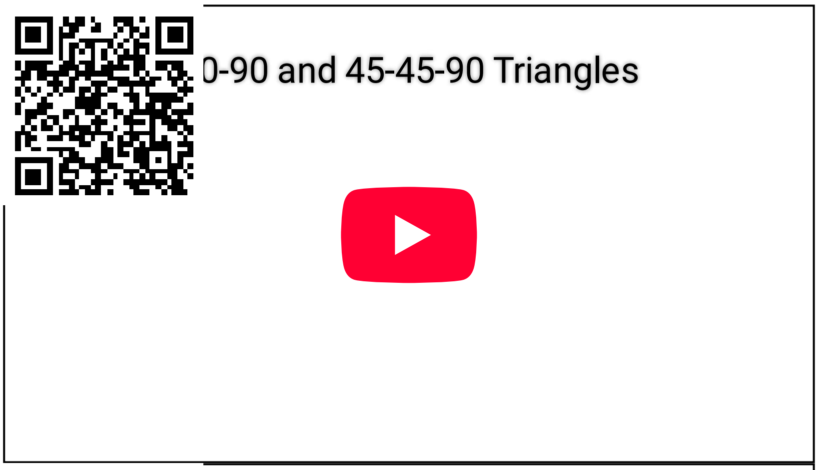
We know half of a square is a 45-45-90 triangle, so $10 = s\sqrt{2}$.

$$s\sqrt{2} = 10$$
$$s = 10\sqrt{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Review

1. In an isosceles right triangle, if a leg is 4, then the hypotenuse is __________.
2. In an isosceles right triangle, if a leg is x, then the hypotenuse is __________.
3. A square has sides of length 15. What is the length of the diagonal?
4. A square's diagonal is 22. What is the length of each side?

Resources



Vocabulary

Term	Definition
30-60-90 Theorem	If a triangle has angle measures of 30, 60, and 90 degrees, then the sides are in the ratio $x : x\sqrt{3} : 2x$
45-45-90 Theorem	For any isosceles right triangle, if the legs are x units long, the hypotenuse is always $x\sqrt{2}$.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Radical	The $\sqrt{}$, or square root, sign.

Additional Resources

Video: Solving Special Right Triangles

Activities: 30-60-90 Right Triangles Discussion Questions

Study Aids: Special Right Triangles Study Guide

Practice: 30-60-90 Right Triangles 45-45-90 Right Triangles

Real World: Fighting the War on Drugs Using Geometry and Special Triangles