Paul's Online Notes

Home / Algebra / Preliminaries / Rational Exponents

Section 1.2 : Rational Exponents

Now that we have looked at integer exponents we need to start looking at more complicated exponents. In this section we are going to be looking at rational exponents. That is exponents in the form

$$b^{\frac{m}{n}}$$

where both m and n are integers.

We will start simple by looking at the following special case,

$$b^{\frac{1}{n}}$$

where n is an integer. Once we have this figured out the more general case given above will actually be pretty easy to deal with.

Let's first define just what we mean by exponents of this form.

$$a = b^{\frac{1}{n}} \qquad \text{is equivalent to} \qquad a^n = b$$

In other words, when evaluating $b^{\frac{1}{n}}$ we are really asking what number (in this case a) did we raise to the n to get b. Often $b^{\frac{1}{n}}$ is called the **n^{th} root of b**.

Let's do a couple of evaluations.

Example 1 Evaluate each of the following.

(a) $25^{\frac{1}{2}}$

(b) $32^{\frac{1}{5}}$

(c) $81^{\frac{1}{4}}$

(d) $(-8)^{\frac{1}{3}}$

(e) $(-16)^{\frac{1}{4}}$

(f) $-16^{\frac{1}{4}}$

Show Discussion

(a) $25^{\frac{1}{2}}$ ***Show Solution***

(b) $32^{\frac{1}{5}}$ ***Show Solution***

(c) $81^{\frac{1}{4}}$ ***Show Solution***

(d) $(-8)^{\frac{1}{3}}$ ***Show Solution***

(e) $(-16)^{\frac{1}{4}}$ ***Show Solution***

(f) $-16^{\frac{1}{4}}$ ***Show Solution***

As the last two parts of the previous example has once again shown, we really need to be careful with parenthesis. In this case parenthesis makes the difference between being able to get an answer or not.

Also, don't be worried if you didn't know some of these powers off the top of your head. They are usually fairly simple to determine if you don't know them right away. For instance, in the part b we needed to determine what number raised to the 5 will give 32. If you can't see the power right off the top of your head simply start taking powers until you find the correct one. In other words compute 2^5, 3^5, 4^5 until you reach the correct value. Of course, in this case we wouldn't need to go past the first computation.

The next thing that we should acknowledge is that all of the **properties for exponents** that we gave in the previous section are still valid for all rational exponents. This includes the more general rational exponent that we haven't looked at yet.

Now that we know that the properties are still valid we can see how to deal with the more general rational exponent. There are in fact two different ways of dealing with them as we'll see. Both methods involve using property 2 from the previous section. For reference purposes this property is,

$$\left(a^n\right)^m = a^{nm}$$

So, let's see how to deal with a general rational exponent. We will first rewrite the exponent as follows.

$$b^{\frac{m}{n}} = b^{\left(\frac{1}{n}\right)(m)}$$

In other words, we can think of the exponent as a product of two numbers. Now we will use the exponent property shown above. However, we will be using it in the opposite direction than what we did in the previous section. Also, there are two ways to do it. Here they are,

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m \qquad \text{OR} \qquad b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}}$$

Using either of these forms we can now evaluate some more complicated expressions

Example 2 Evaluate each of the following.

(a) $8^{\frac{2}{3}}$

(b) $625^{\frac{3}{4}}$

(c) $\left(\frac{243}{32}\right)^{\frac{4}{5}}$

Show Discussion ▸

(a) $8^{\frac{2}{3}}$ ***Show Solution*** ▸

(b) $625^{\frac{3}{4}}$ ***Show Solution*** ▸

(c) $\left(\frac{243}{32}\right)^{\frac{4}{5}}$ ***Show Solution*** ▸

We can also do some of the simplification type problems with rational exponents that we saw in the previous section.

Example 3 Simplify each of the following and write the answers with only positive exponents.

(a) $\left(\frac{w^{-2}}{16v^{\frac{1}{2}}}\right)^{\frac{1}{4}}$

(b) $\left(\dfrac{x^2 y^{-\frac{2}{3}}}{x^{-\frac{1}{2}} y^{-3}}\right)^{-\frac{1}{7}}$

(a) $\left(\dfrac{w^{-2}}{16v^{\frac{1}{2}}}\right)^{\frac{1}{4}}$ **Show Solution**

(b) $\left(\dfrac{x^2 y^{-\frac{2}{3}}}{x^{-\frac{1}{2}} y^{-3}}\right)^{-\frac{1}{7}}$ **Show Solution**

We will leave this section with a warning about a common mistake that students make in regard to negative exponents and rational exponents. Be careful not to confuse the two as they are totally separate topics.

In other words,

$$b^{-n} = \frac{1}{b^n}$$

and NOT

$$b^{-n} \neq b^{\frac{1}{n}}$$

This is a very common mistake when students first learn exponent rules.

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