HW3/ صبا فیری / 98101052

(1)

الف)

255	0	255	0	255	0	255	0
0	255	0	255	0	255	0	255
255	0	255	0	255	0	255	0
0	255	0	255	0	255	0	255
255	0	255	0	255	0	255	0
0	255	0	255	0	255	0	255
255	0	255	0	255	0	255	0
0	255	0	255	0	255	0	255

تصویر بعد از اعمال فیلتر ⟸

255	0	255	0	255	0	255	0
0	141.6	113.3	141.6	113.3	141.6	113.3	255
255	113.3	141.6	113.3	141.6	113.3	141.6	0
0	141.6	113.3	141.6	113.3	141.6	113.3	255
255	113.3	141.6	113.3	141.6	113.3	141.6	0
0	141.6	113.3	141.6	113.3	141.6	113.3	255
255	113.3	141.6	113.3	141.6	113.3	141.6	0
0	255	0	255	0	255	0	255

ب) برای blur کردن و یا smooth کردن تصویر بکار می‌رود. هم‌چنین برای کاهش noise استفاده می‌شود.

Input:

output dimension : $32 \times 32 \times 3$ n parameters : 0

$Conv_{3-10}(1,1)$:

۱۰ فیلتر $(3\times3)\times3$ با stride=1 , padding=1 داریم و تصویر $34\times34\times3$ است پس :

output dimension: $32 \times 32 \times 10$

n parameters: $\underbrace{3\times3}_{kernel} \times \underbrace{3}_{channell} \times \underbrace{10}_{filters} + \underbrace{10}_{bias/filter} = 280$

ReLU :

output dimension: $32 \times 32 \times 10$

n parameters: 0

Pool-2:

max-pooling با ابعاد 2x2 ، stride=2 , padding=0 داریم :

output dimension : $16 \times 16 \times 10$

n parameters : 0

Conv 3-20 (3,2) :

۲۰ فیلتر $(3\times3)\times10$ با $stride=3$ و $Padding=2$ داریم و تصویر $20\times20\times10$ است پس :

output dimension : $6\times6\times20$

n parameters : $\underbrace{3\times3}_{Kernel} \times \underbrace{10}_{Channel} \times \underbrace{20}_{filters} + \underbrace{20}_{bias/filter} = 1820$

Relu :

output dimension : $6\times6\times20$

n parameters : 0

Pool-2 :

output dimension : $3\times3\times20$

n parameters : 0

Flatten :

output dimension : 1×180

n parameters : 0

Fc-10 :

output dimension : 1×10

n parameters : $\underbrace{10}_{neurons}\times(\underbrace{180}_{weights}+\underbrace{1}_{bias}) = 1810$

جدول کامل به صورت زیر است:

layer	output dimension	# parameters
Input	32×32×3	0
Conv3-10	32×32×10	280
Relu	32×32×10	0
pool-2	16×16×10	0
conv3-20(3,2)	6×6×20	1820
Relu	6×6×20	0
pool-2	3×3×20	0
flatten	1×180	0
Fc-10	1×10	1810

(3)

الف) $k_1, k_2, k_3, k_4, b, w_1, w_2, a$ و x را متغیر شبکه هستند.

ب)

$$L = \frac{1}{2}(y-\hat{y})^2 = \frac{1}{2}(y - w_1v_1 - w_2v_2 - a)^2 \Rightarrow$$

$$\frac{\partial L}{\partial w_1} = (y - w_1v_1 - w_2v_2 - a) \times -v_1$$

$$\frac{\partial L}{\partial w_2} = (y - w_1v_1 - w_2v_2 - a) \times -v_2$$

$$\frac{\partial L}{\partial a} = -(y - w_1v_1 - w_2v_2 - a)$$

(ج)

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = 0 \;,\; \frac{\partial L}{\partial z_3} = 0$: $z_3, z_1, z_2 < 0$ ← 1 *

: $z_2 > 0 \;,\; z_2 > z_3 \;,\; z_1, z_2 < 0$ ← 1 *

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = 1 \times \delta_2 \;,\; \frac{\partial L}{\partial z_3} = 0$

: $z_3 > 0 \;,\; z_3 > z_2 \;,\; z_1, z_2 < 0$ ← 1 *

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = 0 \;,\; \frac{\partial L}{\partial z_3} = 1 \times \delta_2$

: $z_2, z_3 < 0 \;,\; z_1 > 0, z_1 > z_2$ ← 1 *

$\frac{\partial L}{\partial z_1} = \delta_1 \;,\; \frac{\partial L}{\partial z_2} = 0 \;,\; \frac{\partial L}{\partial z_3} = 0$

: $z_2 > 0 \;,\; z_2 > z_3 \;,\; z_1 > 0 \;,\; z_1 > z_2$ ← 1 *

$\frac{\partial L}{\partial z_1} = \delta_1 \;,\; \frac{\partial L}{\partial z_2} = \delta_2 \;,\; \frac{\partial L}{\partial z_3} = 0$

: $z_3 > 0 \;,\; z_3 > z_2 \;,\; z_1 > 0 \;,\; z_1 > z_2$ ← 1 *

$\frac{\partial L}{\partial z_1} = \delta_1 \;,\; \frac{\partial L}{\partial z_2} = 0 \;,\; \frac{\partial L}{\partial z_3} = \delta_2$

: $z_2, z_3 < 0 \;,\; z_2 > 0 \;,\; z_2 > z_1$ ← 1 *

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = 1 \times \delta_1 \;,\; \frac{\partial L}{\partial z_3} = 0$

$z_2 > 0 \;,\; z_2 > z_3 \;,\; z_2 > 0 \;,\; z_2 > z_1$ ← 1 *

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = \delta_1 + \delta_2 \;,\; \frac{\partial L}{\partial z_3} = 0$

: $z_3 > 0 \;,\; z_3 > z_2 \;,\; z_2 > 0 \;,\; z_2 > z_1$ ← 1 *

$\frac{\partial L}{\partial z_1} = 0 \;,\; \frac{\partial L}{\partial z_2} = 1 \times \delta_1 \;,\; \frac{\partial L}{\partial z_3} = 1 \times \delta_2$

به طور خلاصه روابط به صورت زیر خواهند بود:

$$\frac{\partial L}{\partial z_1} = u(z_1 - z_2)\ \max\left(u(z_1), u(z_2)\right)\ \delta_1$$

$$\frac{\partial L}{\partial z_2} = u(z_2 - z_1)\ \max\left(u(z_1), u(z_2)\right)\ \delta_1 + u(z_2 - z_3)\ \max\left(u(z_2), u(z_3)\right)\ \delta_2$$

$$\frac{\partial L}{\partial z_3} = u(z_3 - z_2)\ \max\left(u(z_2), u(z_3)\right)\ \delta_2$$

منظور از $u(z)$، step function(z) است.

$$\frac{\partial L}{\partial k_1} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial k_1} + \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial k_1} + \frac{\partial L}{\partial z_3} \cdot \frac{\partial z_3}{\partial k_1} =$$

$$\alpha_1 \cdot x_1 + \alpha_2 \cdot x_2 + \alpha_3 \cdot x_3$$

$$\frac{\partial L}{\partial k_2} = \frac{\partial L}{\partial z_1} \frac{\partial z_1}{\partial k_2} + \frac{\partial L}{\partial z_2} \frac{\partial z_2}{\partial k_2} + \frac{\partial L}{\partial z_3} \frac{\partial z_3}{\partial k_2} =$$

$$\alpha_1 \cdot x_2 + \alpha_2 \cdot x_3 + \alpha_3 \cdot x_4$$

$$\frac{\partial L}{\partial k_3} = \frac{\partial L}{\partial z_1} \frac{\partial z_1}{\partial k_3} + \frac{\partial L}{\partial z_2} \frac{\partial z_2}{\partial k_3} + \frac{\partial L}{\partial z_3} \frac{\partial z_3}{\partial k_3} =$$

$$\alpha_1 \cdot x_3 + \alpha_2 \cdot x_4 + \alpha_3 \cdot x_5$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial z_1} \frac{\partial z_1}{\partial b} + \frac{\partial L}{\partial z_2} \frac{\partial z_2}{\partial b} + \frac{\partial L}{\partial z_3} \frac{\partial z_3}{\partial b} =$$

$$\alpha_1 + \alpha_2 + \alpha_3$$

$$\frac{\partial L}{\partial k_j} = \sum_{i=1}^{m} \frac{\partial L}{\partial z_i} \frac{\partial z_i}{\partial k_j} = \sum_{i=1}^{m} \alpha_i \alpha_{(i+j-1)} \quad (-5$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^{m} \alpha_i$$

(4)

الف)

$$n + 3 \times 65 = 205 \rightarrow n = 10$$

96 تا فیلتر $10 \times (10 \times 10)$ باید داشته باشیم.

ب)

$$\underbrace{10 \times 10}_{\text{kernel}} \times \underbrace{10}_{\text{channel}} \times \underbrace{96}_{\text{filters}} + \underbrace{96}_{\text{bias/filter}} = 96096$$

ج)

$$(66 \times 66) \times (10 \times 10 \times 10) \times 96 = 418.176 \times 10^6$$