Assignment 1:

Discuss various ‘Statistics’ used in occupancy problem. Elaborate your discussion with

examples.

**The occupancy problem**

The occupancy problem is a classic problem in probability. The setting of the problem is that balls are randomly distributed into cells (or boxes or other containers) one at a time. After all the balls are distributed into the cells, we examine how many of the cells are occupied with balls. In this post, we examine the occupancy problem using Markov chains.



One ball is thrown into the 6 boxes for a total of 8 balls. In Figure 1, a total of four cells are occupied. In the current discussion, the focus is on the number of occupied cells and not on which of the cells are occupied. Since Figure 1 has 6 cells, we can also view the throwing a ball into the 6 boxes as rolling a fair die (the occupied cell would be the face value that turns up). Thus we can view the experiment in Figure 1 a rolling a die 8 times. Then at the end, we look at the number faces that appear



he top of Figure 2 shows the 8 rolls of the die. Four faces turn up in these 8 rolls – faces 2, 4, 5 and 6, the same result described in Figure 1. Another way to look at the occupancy problem is that of random sampling. Each time a ball is thrown into n cells, the action can be viewed as randomly selecting one of the n cells. Thus the occupancy problem can be viewed as randomly selecting k balls (one at a time with replacement) from an urn with n balls labeled 1,2,\cdots,n. The following diagram shows an urn with 6 balls labeled 1 through 6. Then Figure 1 and Figure 2 would be equivalent to randomly selecting 8 balls with replacement from this urn.



Question 2:

Suppose that each of n sticks is broken into one long and one short part. The 2n parts are

arranged into n pairs from which new sticks are formed. Find the probability that

a) the parts will be joined in any order b) all long parts are paired with short parts.

the parts will be joined into their original form;

**SOLUTION**

The total number of ways to get *n* pairs is (2*n*)! . There is only one way to pair them

(2*n*)!

up into their original form. Then the probability is 1 .

2*nn*!

all long parts are paired with short parts;

**SOLUTION**

The total number of ways to pair them up (a long part and a short part are paired) is *n*!. Then the probability is *n*! .

2*nn*!

Question3:

An elevator starts from ground floor with seven persons and stops at ten floors. What is the

probability that a) no two persons leave at the same floor? b) Three persons leave together at

a certain floor, two other persons at another and last two at another floor?

**A Part**

Total ways in which one passenger can stop =10

Total ways in which 5 passengers can stop =10∗10∗10∗10∗10

=10 ^7

We will select 5 floors from 10 floors and assign each individual to each floor to keep everyone isolated from each other

No. of ways in which no two persons stop at the same floor =10C 7∗7!

=10P 7

​⇒P(E)=10P 7/10^7

**B part**

**sample space**

**n(s)=(floor)^passenger=(10)^7**

**1)(3,2,2)**

**2)7**

**3)(6,1)**

**4)(4,1,1,1)**

**( 7! ) ( 10! )**

3!\*2!\*2! 7!\*2!\*1! = 0.0075

10^7

**( 7! ) ( 10! )**

7! 9!\*1! = 0.00001

10^7

**( 7! ) ( 10! )**

6!\*1! 8!\*1!\*1! = 0.00006

10^7

**( 7! ) ( 10! )**

4!\*1!\*1! 6!\*8!\*1! = 0.0176

10^7

Question No 4:

A closet contains n pair of shoes. If 2r shoes are chosen at random (with 2r<n), what is the

probability that there will be

i) no complete pair ii) exactly one complete pair iii) exactly two complete pairs among them.

**SOLUTION**

**n(s)=(2n**

**2r)**

P(Ea)= **( n ) ( n )**

0 2 r 2^2r

(2n

2r)

P(Eb)= **( n ) ( n )**

1 2 r - 1 2^r-1

(2n

2r)

P(Ec)= **( n ) ( n )**

2 2 r -3 2^r-4

(2n

2r)

QUESTION No 6:

An elevator starts from ground floor with seven persons and stops at ten floors. What is the

probability that a) no two persons leave at the same floor? b) Three persons leave together at

a certain floor, two other persons at another and last two at another floor?

**A part**

Total ways in which one passenger can stop =10

Total ways in which 5 passengers can stop =10∗10∗10∗10∗10

=10 ^7

We will select 5 floors from 10 floors and assign each individual to each floor to keep everyone isolated from each other

No. of ways in which no two persons stop at the same floor =10C 7∗7!

=10P 7

​⇒P(E)=10P 7/10^7

**B part**

**sample space**

**n(s)=(floor)^passenger=(10)^7**

**1)(3,2,2)**

**2)7**

**3)(6,1)**

**4)(4,1,1,1)**

**( 7! ) ( 10! )**

3!\*2!\*2! 7!\*2!\*1! = 0.0075

10^7

**( 7! ) ( 10! )**

7! 9!\*1! = 0.00001

10^7

**( 7! ) ( 10! )**

6!\*1! 8!\*1!\*1! = 0.00006

10^7

**( 7! ) ( 10! )**

4!\*1!\*1! 6!\*8!\*1! = 0.0176

10^7