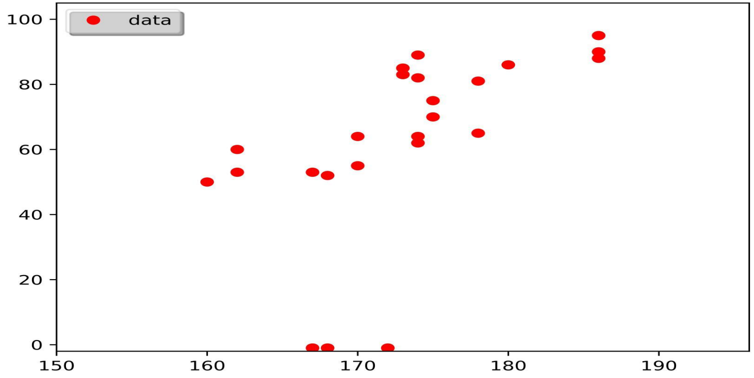
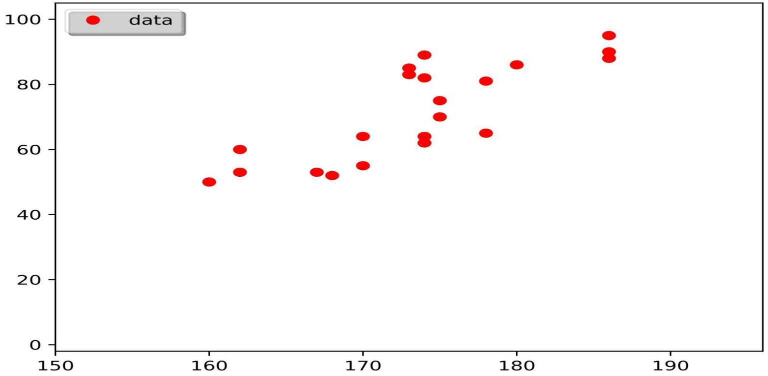
Pattern Recognition Project1

Warm-up

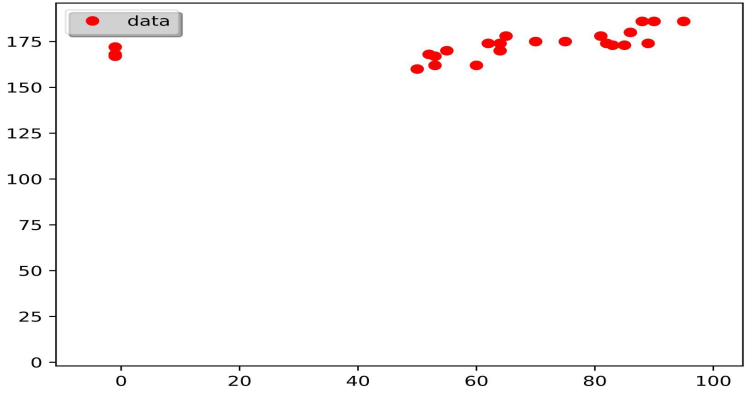
Task 1: Plots



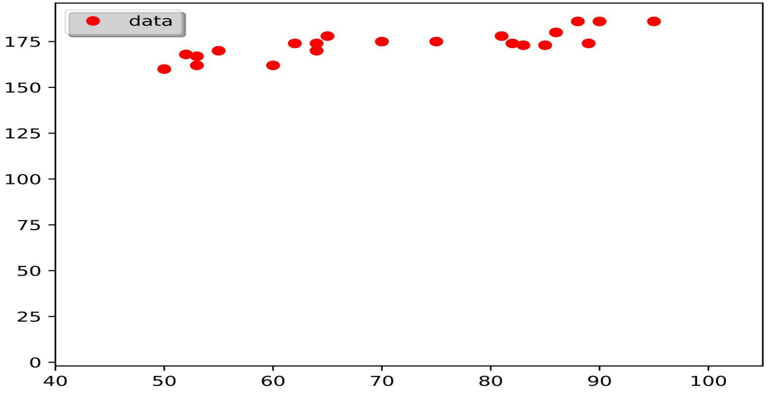
Plot height vs weight



Plot height vs weight(non-negative)

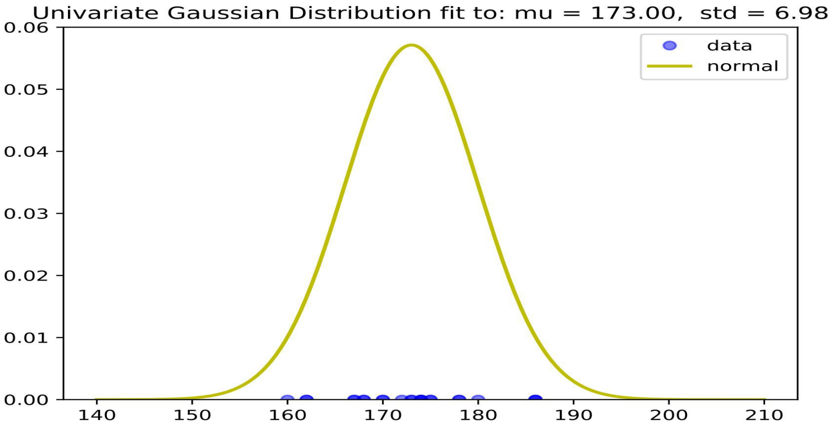


Plot weight vs height



Plot weight vs height(non-negative)

Task 2: Plot a Normal distribution



Mean: 173.00
Standard Deviation: 6.98

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Task 3: Fitting a Weibull Distribution

2 parameter Weibull distribution:

$$f(x|\kappa, \alpha) = \frac{\kappa}{\alpha}\left(\frac{x}{\alpha}\right) e^{-\left(\frac{x}{\alpha}\right)^{\kappa}}$$

where κ and α are the shape and scale parameters respectively.

Task: Fit Weibull distribution to the interest in the word 'myspace' over time using Maximum Likelihood Estimate.

Log-likelihood of the Weibull Distribution (as given):

$$L(\alpha, \kappa \mid D) = N\left(\log \kappa - \kappa \log \alpha\right) + (\kappa - 1)\sum_i \log d_i - \sum_i (d_i/\alpha)^{\kappa}.$$

Task 3: Fitting a Weibull Distribution

Newton's formula for simultaneous equations:

$$\begin{bmatrix} \kappa^{\text{new}} \\ \alpha^{\text{new}} \end{bmatrix} = \begin{bmatrix} \kappa \\ \alpha \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 L}{\partial \kappa^2} & \frac{\partial^2 L}{\partial \kappa \partial \alpha} \\ \frac{\partial^2 L}{\partial \kappa \partial \alpha} & \frac{\partial^2 L}{\partial \alpha^2} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\partial L}{\partial \kappa} \\ -\frac{\partial L}{\partial \alpha} \end{bmatrix}$$

D **: Set of *N* data samples**

Components of partial derivatives with *D*

$$\sum_{i=1}^{N} \log(d_i)$$

$$\sum_{i=1}^{N} \left(\frac{d_i}{\alpha}\right)^{\kappa} \log\left(\frac{d_i}{\alpha}\right)$$

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$$\sum_{i=1}^{N} \left(\frac{d_i}{\alpha}\right)^{\kappa} \left(\log\left(\frac{d_i}{\alpha}\right)\right)^{2}$$

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Task: Fit to *h* instead of D

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h* : Set of *n* entries in histogram of *D

<u>Components of partial derivatives with *h*</u>

$$\sum_{i=1}^{n} h_i \log(i)$$

$$\sum_{i=1}^{n} h_i \left(\frac{i}{\alpha}\right)^{\kappa} \log\left(\frac{i}{\alpha}\right)$$

$$\sum_{i=1}^{n} h_i \left(\frac{i}{\alpha}\right)^{\kappa}$$

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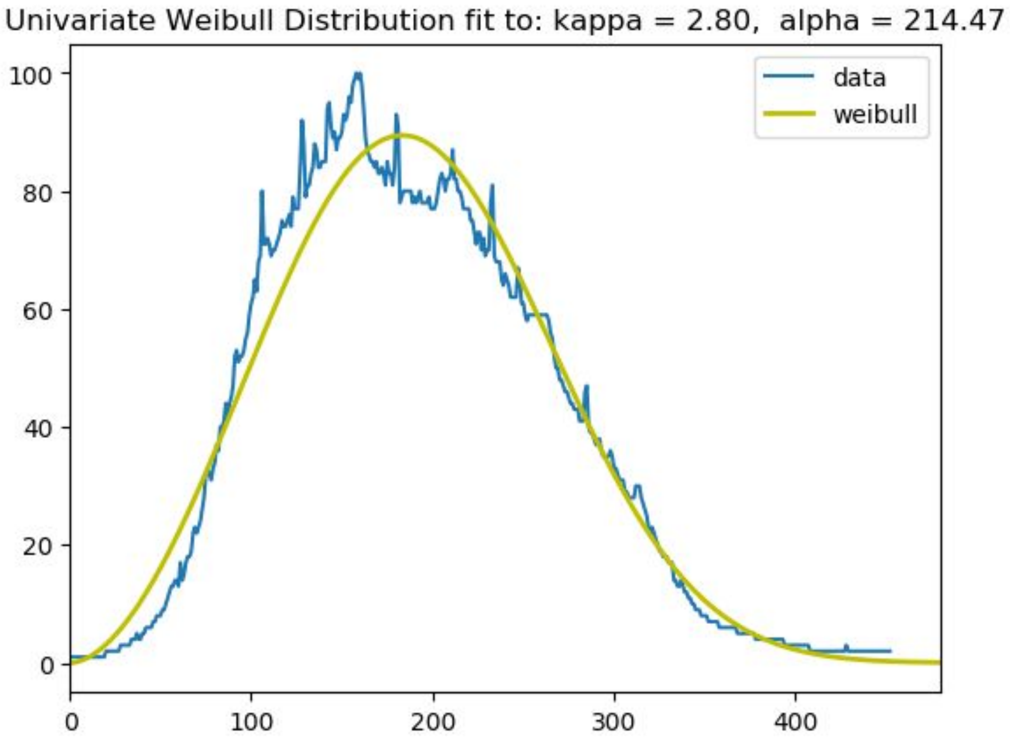
Task 3: Fitting a Weibull Distribution

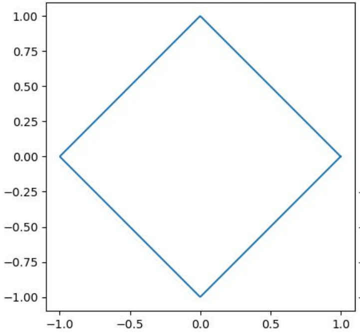


Interpretation:

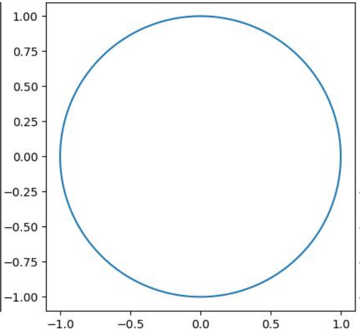
1. κ equal to 2 indicates linear growth in interest while that between 3 and 4 indicates a symmetric distribution. 2.8 might mean a faster rate in growth of interest and a slower fall.

2. α = 214.47 indicates that 63.2% of the data (time instances from start) has falls within 214.47 time instances.

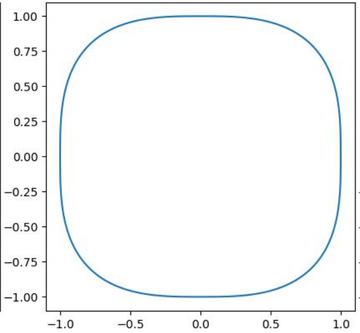
Task 4: p-Norm



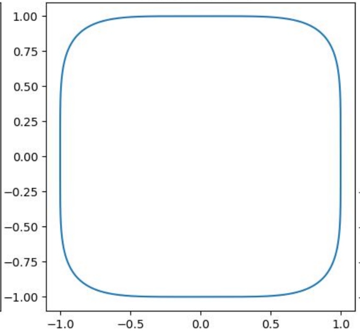
p = 1



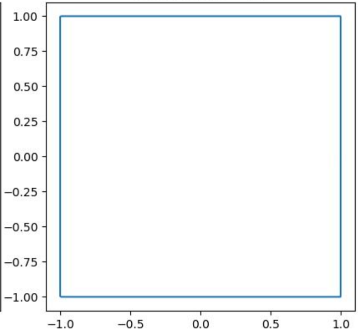
p = 2



p = 3

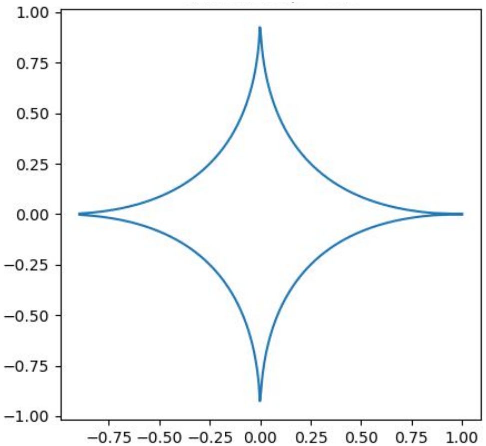


p = 5



p = inf

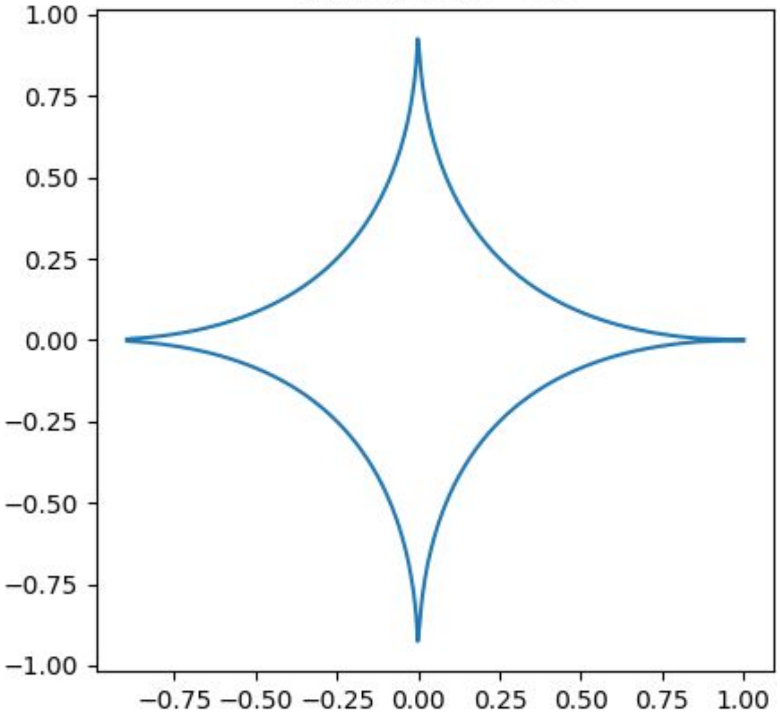
$$\|x\|_p := \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}$$



p = 1/2

Task 4: Is it actually norm with p = 1/2?

1. $d(\boldsymbol{x}, \boldsymbol{y}) \geqslant 0 \ \wedge \ d(\boldsymbol{x}, \boldsymbol{y}) = 0 \Leftrightarrow \boldsymbol{x} = \boldsymbol{y}$
2. $d(\boldsymbol{x}, \boldsymbol{y}) = d(\boldsymbol{y}, \boldsymbol{x})$
3. $d(\boldsymbol{x}, \boldsymbol{z}) \leqslant d(\boldsymbol{x}, \boldsymbol{y}) + d(\boldsymbol{y}, \boldsymbol{z})$



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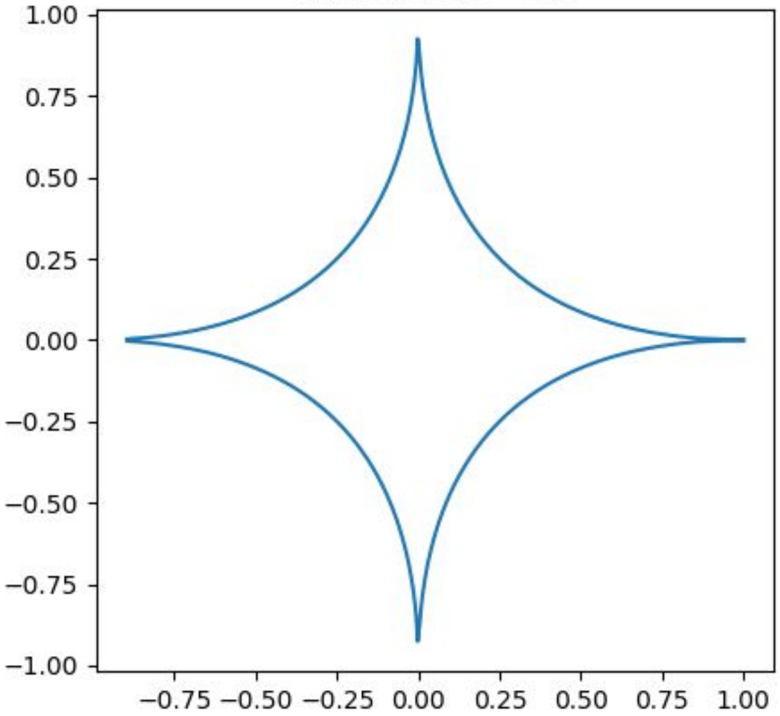

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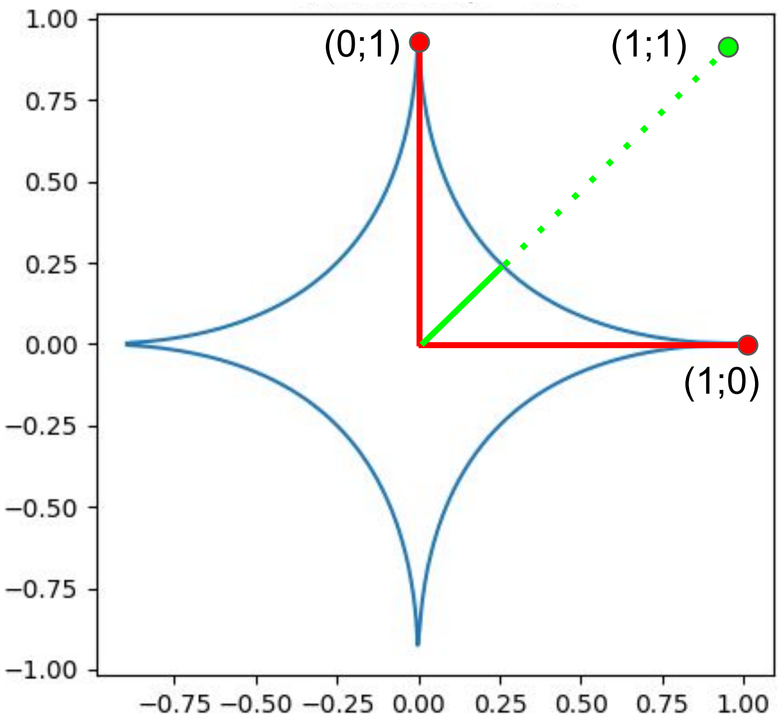

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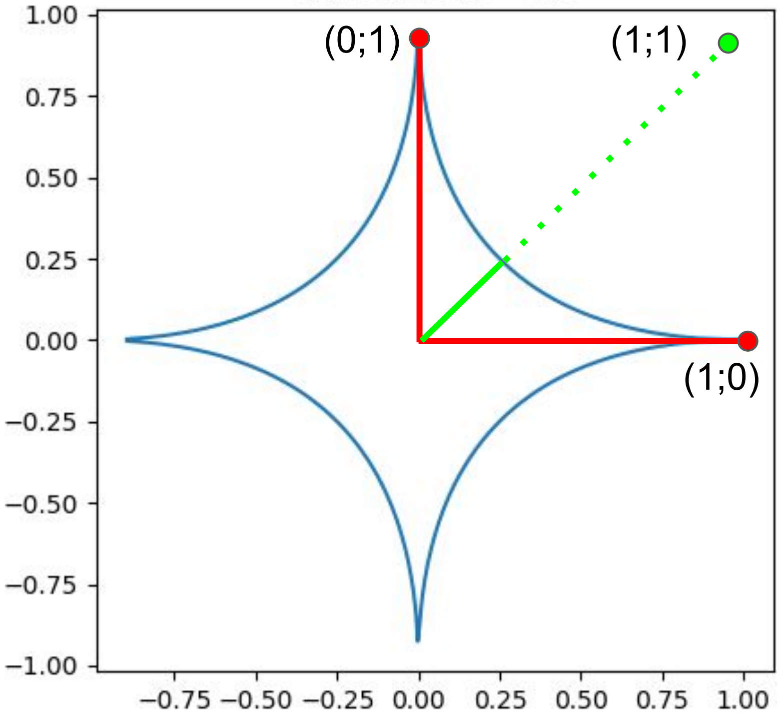
$$\|(1;1)\| \leq \|(1;0) + (0;1)\|$$

$$\|(1;1)\| = 2^{1/p} = 4$$

$$\|(1;0)\| = \|(0;1)\| = 1$$



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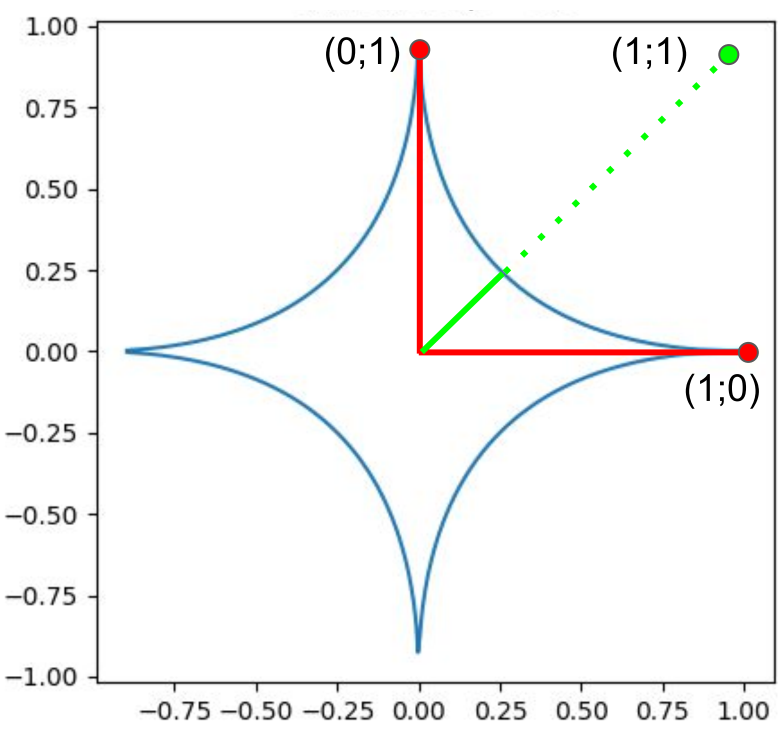

Task 5: How to identify the “foreground”

-Apply two Gaussian filters with different variance on a picture and subtract the results from each other.

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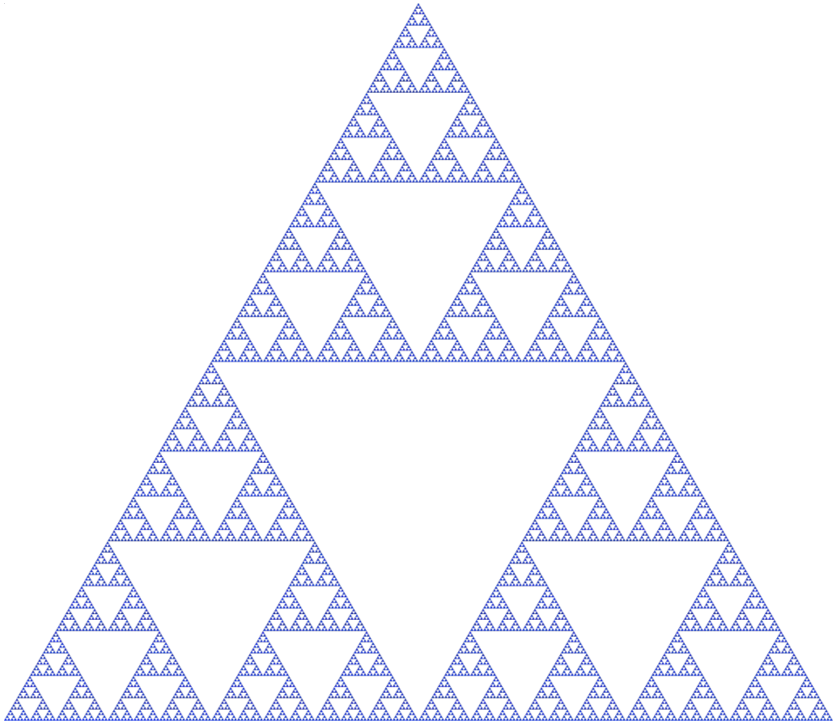
-Use the closing operator to fill small holes in the result.

Task 5: The fractal dimension

- Idea to determine the dimension of fractals, like the Sierpiński Triangle



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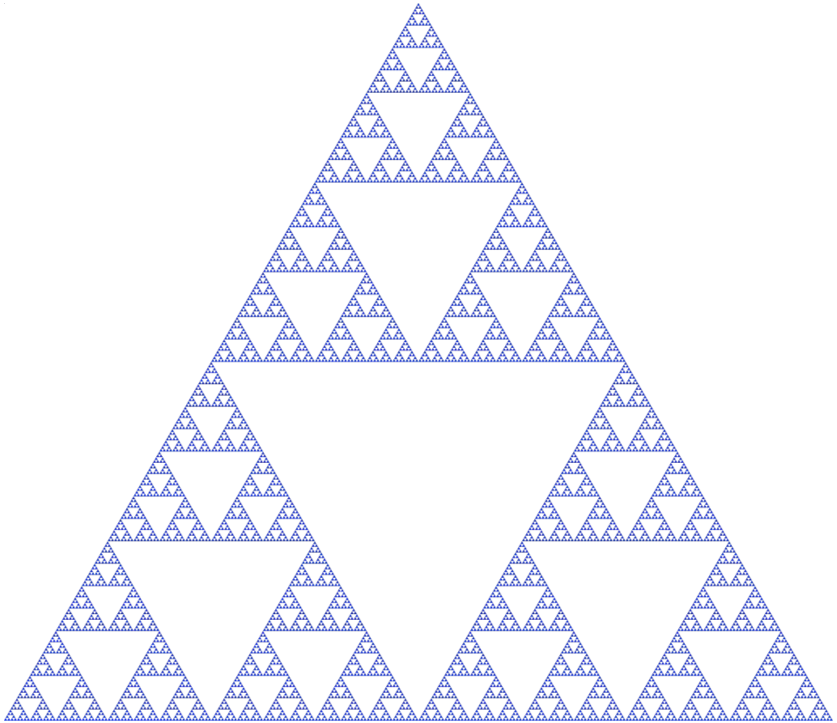
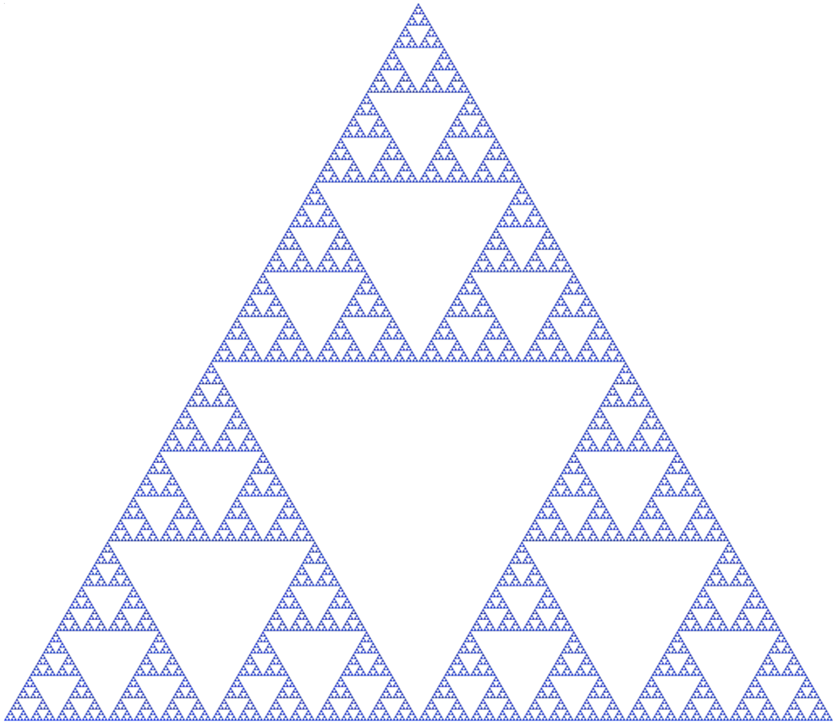
- Idea to determine the dimension of fractals, like the Sierpiński Triangle
- Can also be applied to define the dimension of objects in a picture.



Task 5: The fractal dimension

- Idea to determine the dimension of fractals, like the Sierpiński Triangle
- Can also be applied to define the dimension of objects in a picture.
    - via box counting



Task 5: Results

Fractal Dimension: 1.590

Fractal Dimension: 1.867