**L-3/T-2/CSE** **Date: 22/03/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2019-2020

Sub: **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Morpheus has one pill in his palm: it is either blue or red with equal probability. He puts another red pill in his palm (Now there are two pills in his palm) and randomly takes one out. If its color is red, what is the probability that the color of the other pill is also red? **(10)**

(b) Let X be a random (uniform) number x; $0 < x < 1$ and let $Y = X^2$. Find the covariance of X and Y. Also, specify if X and Y are positively or negatively correlated, or uncorrelated? **(10)**

(c) At a party n men take off their hats. The hats are then mixed up and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of at most k matches? Assume that n is sufficiently large for the ease of calculation. **(15)**

2. (a) The number of traffic accidents on any given day is a Poisson random variable with mean 2, and these random variables for different days are independent. **(8)**

(i) What is the probability that there is a total of six accidents over two days?

(ii) Take some 5 days, for instance, SAT-WED this week. What is the probability that at least three of these five days each have exactly two accidents?

(b) A miner is trapped in a mine (M1) containing three doors. The first door (D1) leads to a tunnel that takes him to safety after 4 hours of travel. The second door (D2) leads to a tunnel that returns him to the mine after 2 hours of travel. However, the third door (D3) leads to another mine (M2) after 3 hours. Now, the second mine (M2) has two other doors. First one (D4) leads to safety after 1 hour of travel. But the second door (D5) leads to a tunnel that returns him to the same mine (M2) after 2 hours of travel. Assume that the miner is at all times equally likely to choose any one of the doors (in M1, the miner can choose between D1, D2, D3, and in M2, the miner can choose between D3, D4, D5 ). What is the expected duration of time until the miner reaches safety? **(15)**

Contd …… P/2

**CSE 301**

**Contd... Q. No. 2**

(c) Let N be a Poisson random variable with mean $\lambda$. Given N, let X be another binomial random variable with parameters N and p (p being the probability).
Let $Y = N - X$ be a random variable and given that Y is non-negative. Prove that Y is a Poisson random variable with mean $\lambda(1-p)$. **(12)**

3. (a) Three players play a game in which they take turns and draw cards from an ordinary deck of 52 cards, successively, at random, and with replacement. Player-I draws cards until an ace is drawn. Then Player-II draws cards until a diamond is drawn. Next, Player-III draws cards until a face card is drawn. At that point, the deck is returned to Player-I and the game continues. **(15)**

i. Draw a state diagram with transition probability identifying the relevant states.
ii. Determine the long-term proportion of cards drawn by each player out of the total number of cards drawn by the three players.
(There are 4 aces, 13 diamond cards, and 12 face cards in a standard deck).

(b) Consider a population of individuals each of whom possesses two genes that can be either type A or type a. Suppose that in outward appearance type A is dominant and type a is recessive. (That is, an individual will have only the characteristics of the recessive gene if its pair is aa.) Suppose that the population has stabilized, and the percentages of individuals having respective gene pairs AA, aa, and Aa are p, q, and r. Call an individual dominant or recessive depending on the outward characteristics it exhibits. Let $S_{11}$ denote the probability that an offspring of two dominant parents will be recessive; and let $S_{10}$ denote the probability that the offspring of one dominant and one recessive parent will be recessive. Compute $S_{11}$ and $S_{10}$ in terms of p, q, r to show that $S_{11} = S_{10}^2$. **(10)**

(c) Define counting process and illustrate its properties. When do we consider a counting process as a Poisson process? **(6)**

(d) Prove that Exponential distribution is memoryless. **(4)**

4. (a) Customers arrive at a bank at a Poisson rate $\lambda$. Suppose three customers arrived during the first hour. What is the probability that exactly two customers arrived during the first 20 minutes? **(8)**

(b) Consider a shoe shop consisting of two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus, for instance, a potential customer might enter even if there is a customer in chair 2.)

Contd ...... P/3

## CSE 301

**Contd... Q. No. 4(b)**

Suppose that potential customers arrive in accordance with a Poisson process at rate $\lambda$, and that the service times for the two chairs are independent and have respective exponential rates of $\mu_1$ and $\mu_2$. **(15)**

i. Draw a state diagram of the system and write down balance equations for each state.

ii. What proportion of potential customers enters the system?

iii. What is the mean number of customers in the system?

iv. What is the average amount of time that an entering customer spends in the system?

v. What proportion of entering customers are blockers?

(c) Consider a network of three stations with a single server at each station. Customers arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 6, 8 and 8. The service times at the three stations are exponential with respective rates 12, 60 and 120. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3, or (iii) leave the system. A customer departing service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to either go to station 2 or leave the system. **(12)**

i. What is the probability that at a certain time, station 1 is empty, there are 2 customers in station 2, and 1 customer in station 3.

ii. What is the average number of customers in the system (consisting of all three stations)?

iii. What is the average time a customer spends in the system?

### SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) E(n) denotes the maximum number of regions that can be formed on a plane by drawing n intersecting ellipses. **(10+3=13)**

i. Find the recurrence formula for E(n).

ii. Derive a closed form of E(n).

(b) Given that, **(12)**

$$x^4 = x^{\underline{4}} + 6\, x^{\underline{3}} + 7\, x^{\underline{2}} + + x^{\underline{1}}$$

and,

$$\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k-1} \right\} + k \left\{ {n-1 \atop k} \right\}$$

**CSE 301**

**Contd... Q. No. 5(b)**

Evaluate the following sum,

$$S = \sum_{0 \le k \le n} k^5$$

(c) Let $f(x)$ be any continuous, monotonically strictly increasing function with the property that, **(10)**

If $f(x)$ is an integer, then x is an integer.

Prove that,

$$\lceil f(\lceil x \rceil) \rceil = \lceil f(x) \rceil$$

6. (a) We have an ant and a worm on the tip of two different horizontal rubber strings. Both strings are of length 100 cm. The ant moves 1 cm per second and the worm moves 2 cm per second. The rubber string of the ant is stretched 100cm each second. On the other hand, the rubber string of the worm is stretched 100 cm each 0.5 second. During the stretching operation the ant and the worm maintain their relative positions on their respective strings. **(15)**

Prove or disprove that, after n seconds (where $n \ge 1$ and $n \in \mathbb{N}$),

*(The fraction travelled by worm – the fraction travelled by ant) < 2%*

(b) Prove that, there are at least n – 1 composite integers between n! and n! + n, where n is a positive integer. **(10)**

(c) Prove the recurrence of the Eulerian Numbers stated below **(10)**

$$\left\langle {n \atop k} \right\rangle = (k+1)\left\langle {n-1 \atop k} \right\rangle + (n-k)\left\langle {n-1 \atop k-1} \right\rangle$$

7. (a) A Double Tower of Hanoi contains 2n disks of n different sizes, two of each size. We are required to move only one disk at a time, without putting a larger one over a smaller one. **(5+10=15)**

i. How many moves does it take to transfer a double tower from one peg to another, if disks of equal size are indistinguishable from each other?

ii. What if we are required to reproduce the original top-to-bottom order of all the equal-size disks in the final arrangement? How many moves does it take?

(b) Prove that, for any $m, k \in \mathbb{N}$ **(10)**

$$\phi(m^k) = \phi(m) \cdot m^{k-1}$$

Where $\phi$ is the Euler's totient function.

**CSE 301**

**<u>Contd... Q. No. 7</u>**

(c) Evaluate $B_1$ and $B_2$ **(5+5=10)**

$$B_1 = \sum_{k=0}^{n} 2^k \binom{n}{k}$$

$$B_2 = \sum_{k=0}^{n} 3^{k-n} \binom{n}{k}$$

8. (a) Evaluate the following sum, **(13)**

$$S = \sum_{1 \le j \le k < n} \frac{1}{j(k+1)(k+2)}$$

(b) How many integers n are there such that, **(12)**

$\left\lfloor \sqrt[3]{n} \right\rfloor$ divides n and $1 \le n \le 2000$?

(c) Compute the value of $7^{10010}$ mod 11. **(10)**

--------------------------

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

**L-3/T-1** B. Sc. Engineering Examinations (January 2020 Term)

Sub: **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 180 Section Marks: 90 Time: 2 Hours (Sections A + B)

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Find integers a, b, and c such that for all m, the following equation is satisfied. 15

$$m^3 = a\binom{m}{3} + b\binom{m}{2} + c\binom{m}{1}$$

Also, find out the sum of the series $1^3 + 2^3 + 3^3 + ... + n^3$ by using this equation.

(b) Use combinatorial reasoning to prove the following identity, 10

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

(c) What is the coefficient of $x_1^3 x_2^3 x_3\, x_4^2$ in the expansion of $(x_1 - x_2 + 2x_3 - 2x_4)^9$? 5

2. (a) Prove that the derangement number $D_n$ is an even number if and only if $n$ is an odd number. 10

(b) Suppose a class of $n$ boys takes a walk every day. The boys walk in a line so that every boy except the first is preceded by another. In order that a boy can not see the same person in front of him, on the second day the boys decide to switch positions so that no boy is preceded by the same boy who preceded him on the first day. Let $Q_n$ denote the number of how many ways they can switch positions. Find an expression for $Q_n$. Also, prove that $Q_n = D_n + D_{n-1}$ ($n \geq 2$), where $D_n$ is the derangement number. 20

3. (a) What is the number of ways to place six non-attacking rooks on the 6 × 6 board with forbidden positions as shown? 10

| | | | | | |
|---|---|---|---|---|---|
| x | x | | | | |
| x | x | | | | |
| | | x | x | | |
| | | x | x | | |
| | | | | x | x |
| | | | | x | x |

(b) Suppose the $n$ men and the $n$ women at the party check their hats before they dance. At the end of the party, their hats are returned randomly. In how many ways can they be returned if each man gets a male hat and each woman gets a female hat, but no one gets the hat he or she checked? 10

(c) Assume that there are four communication devices $A$, $B$, $C$, and $D$ in the network. The total bandwidth of the communication channel available to these four devices is known and it is a function of the physical channel capacity. A bandwidth manager is used to configure and limit the traffic from these devices which determines the upper bound. There is a need to keep certain service levels for users which determines the lower bound. How many solutions are possible if only integer solutions are accepted as valid for the following five equations which represent the scenario? 10

$$X_A + X_B + X_C + X_D = 20,\ 1 \leq X_A \leq 6,\ 0 \leq X_B \leq 7,\ 4 \leq X_C \leq 8,\ 2 \leq X_D \leq 6$$

4. (a) Express the number $h_n$ of regions that are created by $n$ mutually overlapping circles in general positions in the plane as a recurrence relation and find a formula for $h_n$ in terms of $n$. 10

(b) Formulate a combinatorial problem that leads to the following generating function: 10

$$(1+x+x^2)(1+x^2+x^4+x^6)(1+x^2+x^4+...)(x+x^2+x^3+...)$$

(c) Show that the sums of the entries along the diagonals of Pascal's triangle running upward from the left are Fibonacci numbers. 10

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

**L-3/T-2** B. Sc. Engineering Examinations 2018-2019

Sub: **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 180 Section Marks: 90 Time: 2 Hours (Sections A + B)

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Twenty tickets are sold in a lottery, numbered 1 to 20, inclusive. Five tickets are drawn for prizes. Find out the probability that two of the five winning tickets are in the range between 1 to 5, two are in the range between 6 to 10, and one is in the range between11 to 20. (15)

(b) Consider the following bus ridership model: (15)

- Suppose the bus is empty when it arrives at its first stop.
- Assume the bus is so large that it never becomes full, so the new passengers can always get on.
- At each stop, each passenger gets down from the bus, independently, with a probability of 0.2.
- Either 0, 1 or 2 new passengers get on the bus, with probabilities 0.5, 0.4 and 0.1, respectively. Passengers at successive stops are independent.
- Suppose the bus company charges Tk.15/- for each passenger who boards (gets on) at the first stop, but charges Tk.10/- for a passenger who joins at the second stop, regardless of the destination.

Find the expected total revenue from the first two stops.

6. (a) Consider the numbers 1, 2, . . . ,N written around a ring. Consider a Markov Chain $X_n$ that at any point jumps with equal probability to one of the two adjacent numbers. (15)

   (i) What is the expected number of steps that $X_n$ will take to return to its starting position?

   (ii) What is the probability that $X_n$ will visit all the other states before returning to its starting position?

(b) A taxicab driver moves between the airport A and two hotels B and C according to the following rules: if the taxicab is at the airport, go to one of the hotel with equal probability, and if the taxicab is at one hotel, go to the airport with probability 1/4, and to the other hotel with probability 3/4. (15)

   (i) Find out the transition matrix.

   (ii) Suppose the driver starts (time 1) at the airport. Find the probability for each of the three possible locations at time 2. Find the probability that the taxicab is at the airport at time 3.

7. (a) Each element in a sequence of binary data is either 1 with probability $p$ or 0 with probability $1-p$. A maximal subsequence of consecutive values having identical outcomes is called a run. For instance, if the outcome sequence is 1, 1, 0, 1, 1, 1, 0, the first run is of length 2, the second is of length 1, and the third is of length 3. (15)
   (i) Find the expected length of the first run.
   (ii) Find the expected length of the second run.

   (b) Suppose we are given a set of $n$ elements, numbered 1 through $n$, which are to be arranged in some ordered list. At each unit of time a request is made to retrieve one of these elements, element $i$ being requested (independently of the past) with probability $P_i$. After being requested, the element then is moved one closer to the front of the list; for instance, if the present list ordering is 1, 3, 4, 2, 5 and element 2 is requested, then the new ordering becomes 1, 3, 2, 4, 5. Find the long-run average position of the element requested. (15)

8. (a) The financial condition of a store is affected significantly during pandemic situation. During non-pandemic times, customers arrive at a certain single-server queueing system in accordance with a Poisson process with rate $\lambda_1$, and during the pandemic times, they arrive in accordance with a Poisson process with rate $\lambda_2$. A non-pandemic time period lasts for an exponentially distributed time with rate $\alpha_1$, and a pandemic time period lasts for an exponential time with rate $\alpha_2$. An arriving customer will only enter the queueing system if the server is free both in pandemic and non-pandemic cases; an arrival finding the server busy goes away. All service times are exponential with rate $\mu$. (20)
   (i) Define the states so as to be able to analyze this system.
   (ii) Give a set of linear equations whose solution will yield the long-run proportion of time the system is in each state.
   (iii) What proportion of time is the system empty?
   (iv) What is the average rate at which customers enter the system?

   (b) 100 items are simultaneously put on a lifetime duration test. Suppose the lifetimes of the individual items are independent exponential random variables with mean 200 hours. The test will end when there have been a total of 5 failures. Find out the expected time at which the test ends. (10)

L-3/T-2/CSE Date : 25/03/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A certain word is equally likely to be in any one of three different files. Let, $p_i$ be the probability that you will find the word upon making a quick examination of file i, if the word is, in fact, in file i, for i = 1,2,3, Given $p_1 = 1/2$, $p_2=1/3$, $p_3 = 1/4$. Suppose you look into file 2 and do not find the word. What is the probability that the word is in file 2? **(10)**

(b) Two chips are drawn at random without replacement from a box that contains five chips numbered 1 through 5. If the sum of chips drawn is even, the random variable X equals 5; if the sum of chips drawn is odd, then X = -3. **(10)**

(i) Find the moment-generating function (mgf) for X.

(ii) Use the mgf to find the first and second moments.

(iii) Find the expected value and variance of X.

(c) It is known that DVDs produced by a certain company will be defective with probability 0.01, independent of each other. The company sells the DVDs in packages of size 10 and offers a money-back guarantee that if at least 1 of the 10 DVDs in a package is defective, money will be returned. If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them? **(15)**

2. (a) Each element in a sequence of binary data is either 1 with probability p or 0 with probability 1 - p. A maximal subsequence of consecutive values having identical outcomes is called a run. For instance, if the outcome sequence is 1, 1, 0, 1, 1, 1, 0, the first run is of length 2, the second is of length 1, and the third is of length 3. **(10)**

(i) Find the expected length of the first run.

(ii) Find the expected length of the second run.

(b) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom. Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (In this version, for instance, if the prisoner initially tries door 1, then when he returns to the cell, he will now select only from doors 2 and 3.) **(10)**

**CSE 301**

**Contd ... Q. No. 2**

(c) A computer receives requests for elements stored in its memory. Consider n elements $e_1, e_2 \ldots e_n$ are initially arranged in some ordered list. At each unit of time a request is made for one of these elements, $e_i$ independently of the past, with probability $P_i$. After being requested, the element is then moved to the front of the list. That is, for instance, if the present ordering is $e_1, e_2, e_3, e_4$ and $e_3$ is requested, then the next ordering is $e_3, e_1, e_2, e_4$. Determine the expected position of the element requested after this process has been in operation for a long time. **(15)**

3. (a) A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done will or badly. Let $p_i$ denote the probability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i, for i = 1, 2, 3? **(10)**

   (b) A particle moves among n + 1 vertices that are situated on a circle in the following manner. At each step it moves to the next vertex either in the clockwise direction with probability p or the counterclockwise direction with probability q = 1 - p. Starting at a specified vertex, call it vertex 0, let T be the time of the first return to vertex 0. Find the probability that all vertices have been visited by time T. **(10)**

   (c) Consider a Poisson process with rate $\lambda$, and let us denote the time of the first event by $T_1$. Further, for $n > 1$, let $T_n$ denote the elapsed time between the (n-1)st and the nth event. The sequence $\{T_n, n=1,2\ldots\}$ is called the sequence of inter-arrival times. Show that $T_n$, n=1,2 ..., are independent identically distributed exponential random variables having mean $1/\lambda$. **(7)**

   (d) Suppose that $X_1$ and $X_2$ are independent exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$; what is $P\{X_1 < X_2\}$? **(8)**

4. (a) Consider a shoe shine shop consisting of two chairs. Suppose that an entering customer will first go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this ship as long as chair 1 is empty. (Thus, for instance, a potential customer might enter even if there is a customer in chair 2.) Suppose that potential customers arrive in accordance with a Poisson process at rate $\lambda$, and that the service times for the two chairs are independent and have respective exponential rates of $\mu_1$ and $\mu_2$. **(10)**

**CSE 301**

**Contd ... Q. No. 4(a)**

(i) Draw a state diagram of the system and write down balance equations for each state.

(ii) What proportion of potential customers enters the system?

(iii) What is the mean number of customers in the system?

(iv) What is the average amount of time that an entering customer spends in the system?

(b) Suppose that customers arrive at a single-server service station in accordance with a Poisson process having rate $\lambda$. That is the times between successive arrivals are independent exponential random variables having mean $1/\lambda$. Each customer, upon arrival, goes directly into service if the server is free and, if not, the customer joins the queue. When the server finishes serving a customer, the customer leaves the system, and the next customer in line, if there is any, enters service. The successive service times are assumed to be independent exponential random variable having mean $1/\mu$. This system is called the M/M/1 queue. For the M/M/1 queuing system, compute **(10)**

(i) the average number of customers in the system,

(ii) the average time a customer spends in the system,

(iii) the average number of customers in the queue and

(iv) the average time a customer spends in the queue.

(c) Consider a network of three stations with a single server at each station. Customers arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 5, 10, and 15. The service times at the three stations are exponential with respective rates 10, 50, and 100. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3, or (iii) leave the station. A customer departing service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to either go to station 2 or leave the system. **(15)**

(i) What is the average number of customers in the system (consisting of all three station)?

(ii) What is the average time a customer spends in the system?

## SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Find the solution to the following recurrence relation. **(20)**

$$a_0 = 1$$
$$a_1 = 9$$
$$a_{n+2} - 17a_{n+1} + 70a_n = 6 \quad \text{where } (n \geq 2)$$

(b) What is spectrum (*Spec*( )) of a real number? Let n be an arbitrary positive integer. Let p be the number of elements of *Spec*($\sqrt{2}$) that are less than or equal to n. Let q be the number of elements of *Spec*($2+\sqrt{2}$) that are less than or equal to n. Prove that p + q = n. **(15)**

Contd .......... P/4

**CSE 301**

6. (a) Fine a closed form of the sum $\sum_{1 \le k \le n} k^2 c^{dk}$ where c and d are non-negative integer constants. **(20)**

   (b) Derive, with detailed reasoning, the **L** and **R** matrices for Stern-Brocot tree. Using these, write down the algorithm for representing any positive fraction a/b, with $a \perp b$, as a string of the letters L and R. **(15)**

7. (a) Find, through detailed steps, the number of ways a rooted ordered binary tree can be constructed from n vertices. **(15)**

   (b) Deduce the recurrence relation for the Stirling numbers of the first and second kind. **(10)**

   (c) State and prove the inversion formula. **(10)**

8. (a) Prove, with detailed reasoning, that the number of partitions of a positive integer into distinct summands is identical to the number of partitions of that integer into odd summands. **(20)**

   (b) A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of the signals have at least three white flags or no white flags at all? **(15)**

------------------------

**L-3/T-2/CSE** **Date : 18/02/2018**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2016-2017

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A man has 89 quarreling cows that he wants to separate with straight fencing. What is the minimum number of fences he needs. You must deduce any formula you use. **(15)**

(b) Show solution of Multi-Peg Tower of Hanoi problem with (n,p)=(347,7) using a binary tree as used in the class specifying values of $K_{max}$, $N_a(K_{max})$ and all the inequalities relevant quantities should satisfy. **(20)**

2. (a) Expand the binomial coefficient representing (m+n+1) things taken n at a time in a series with the help of combinatorial argument. **(15)**

(b) Deduce inversion formula. Use this formula for solving the following problem: n men throw their hats and pick them randomly. What is the probability that nobody will end up in his own hat? **(20)**

3. (a) A cricket team of 16 persons is supposed to be accommodated in 5 **distinguishable** rooms with no room remaining empty. In how many ways can we do it? **(15)**

(b) Elaborately explain your deduction of the size of the maximum overhang that n cards, each of length 2, can make when placed on a table. **(20)**

4. (a) Discuss Euler numbers and deduce recurrences they satisfy. **(15)**

(b) Compute in logarithmic time the value of $F_n$ for large n using recurrence relation. **(20)**

## SECTION-B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) It is known that DVDs produced by a certain company will be defective with probability 0.01, independent of each other. The company sells the DVDs in packages of size 10 and offers a money-back guarantee that if at least 1 of the 10 DVDs in a package is defective, money will be returned. If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them? **(13)**

(b) Two chips are drawn at random without replacement from a box that contains five chips numbered 1 through 5. If the sum of chips drawn is even, the random variable X equals 5; if the sum of chips drawn is odd, X = -3. **(10)**

(i) Find the moment-generating function (mgf) for X.

(ii) Use the mgf to find the first and second moments.

(iii) Find the expected value and variable of X.

Contd .......... P/2

# CSE 301

**Contd... Q. No. 5**

(c) Suppose that two teams are playing a series of games, each of which is independently won by team $A$ with probability $p$ and by team $B$ with probability $1- p$. The winner of the series is the first team to win $i$ games. Find the expected number of games that are played when (a) $i = 2$, and (b) $i = 3$. **(6+6)**

6. (a) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom. **(6+6)**

    (i) Assuming that the prisoner will always select doors 1, 2, and 3 with probabilities 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom?

    (ii) Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (In this version, for instance, if the prisoner initially tries door 1, then when he returns to the cell, he will now select only from doors 2 and 3.)

    (b) A computer receives requests for elements stored in its memory. Consider $n$ elements $e_1, e_2, \ldots, e_n$ are initially arranged in some ordered list. At each unit of time a request is made for one of these elements — $e_i$ being requested, independently of the past, with probability, $P_i$. After being requested, the element is then moved to the front of the list. That is, for instance, if the present ordering is $e_1, e_2, e_3, e_4$ and $e_3$ is requested, then the next ordering is $e_3, e_1, e_2, e_4$. Determine the expected position of the element requested after this process has been in operation for a long time. **(13)**

    (c) The number of customers entering a store on a given day is Poisson distributed with mean $\lambda = 10$. The amount of money spent by a customer is uniformly distributed over (0, 100). Find the mean and variance of the amount of money that the store takes in on a given day. **(10)**

7. (a) A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let $p_i$ denote the probability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i, i = 1, 2, 3? **(10)**

    (b) Consider a gambler who at each play of the game has probability $p$ of winning one unit and probability $q = 1 - p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with $i$ units, the gambler's fortune will reach 0 before reaching N? **(10)**

    (c) Consider a Poisson process with rate $\lambda$, and let us denote the time of the first event by $T_1$. Further, for $n > 1$, let $T_n$ denote the elapsed time between the (n – 1)st and the nth event. The sequence $\{ T_n, n = 1,2,\ldots \}$ ia called the sequence of inter-arrival times. Show that $T_n, n = 1,2,\ldots$, are independent identically distributed exponential random variables having mean $1/\lambda$. **(7)**

Contd .......... P/3

**CSE 301**

**Contd... Q. No. 7**

(d) Suppose that $X_1$ and $X_2$ are independent exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$; what is $P\{X_1 < X_2\}$? **(8)**

8. (a) Consider a single-server exponential system in which ordinary customers arrive at a rate $\lambda$ and have service rate $\mu$. In addition, there is a special customer who has a service rate $\mu_1$. Whenever this special customer arrives, she goes directly into service (if anyone else is in service, then this person is bumped back into queue). When the special customer is not being serviced, she spends an exponential amount of time (with mean $1/\theta$) out of the system. **(10)**

   (i) What is the average arrival rate of the special customer?

   (ii) Define an appropriate state space and set up balance equations.

   (iii) Find the probability that an ordinary customer is bumped n times.

   (b) Potential customers arrive to a single-server hair salon according to a Poisson process with rate $\lambda$. A potential customer who finds the server free enters the system; a potential customer who finds the server busy goes away. Each potential customer is type i with probability $p_i$, where $p_1 + p_2 + p_3 = 1$. Type 1 customers have their hair washed by the server; type 2 customers have their hair cut by the server; and type 3 customers have their hair first washed and then cut by the server. The time that it takes the server to wash hair is exponentially distributed with rate $\mu_1$, and the time that it takes the server to cut hair is exponentially distributed with rate $\mu_2$. **(10)**

   (i) Explain how this system can be analyzed with four states.

   (ii) Give the equations whose solution yields the proportion of time the system is in each state.

   (iii) Find the proportion of time the server is cutting hair.

   (iv) Find the average arrival rate of entering customers.

   (c) Customers arrive at a two-server station in accordance with a Poisson process with a rate of two per hour. Arrivals finding server 1 free begin service with that server. Arrivals finding server 1 busy and server 2 free begin service with server 2. Arrivals finding both servers busy are lost. When a customer is served by server 1, she then either enters service with server 2 if 2 is free or departs the system if 2 is busy. A customer completing service at server 2 departs the system. The service times at server 1 and server 2 are exponential random variables with respective rates of four and six per hour. **(15)**

   (i) What fraction of customers do not enter the system?

   (ii) What is the average amount of time that an entering customer spends in the system?

   (iii) What fraction of entering customers receives services from server 1?

---

**L-3/T-2/CSE** **Date : 05/08/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CSE 301** (Mathematical Analysis for Computer Science )

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Establish all recurrence relations satisfied by J(n) of Josephus problem. **(10)**
   (b) Compute $J_5$ (10,000) where every 5th man is deleted. **(10)**
   (c) Construct a binary free sharing how multi-peg tower of Hanoi is solved using presumed optimal solution where p =7 and n = 489. **(15)**

2. (a) Deduce average case complexity of Quick sort algorithm using summation factor. **(10)**
   (b) Deduce $\sum x^2 H_x \delta_x$ by summation by parts. **(10)**
   (c) Use combinational argument to establish the value of $\sum_{k \le n} \binom{k}{m}$. **(15)**

3. (a) Find the multiplicity of 72 in 200!. **(10)**
   (b) Deduce recurrence relations satisfied by the Stirling numbers of the first and second kind. **(10)**
   (c) A group of n fans of the winning football team throw their hats high into the air. The hats came back randomly, one hat to each of the n fans. How many ways are there for all n fans not to end up in having their hats? Solve it using generating function. **(15)**

4. (a) Construct a generating function for Fibonacci numbers and find their values. **(10)**
   (b) Given n cards and a table what is the largest possible overhang by stacking the cards up over the tables edge? **(10)**
   (c) prove that $x^{\bar{n}} = \sum_k \left[ {n \atop k} \right] x^k$, integers $n \ge 0$. **(15)**

## SECTION-B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) It is known that DVDs produced by a certain company will be defective with probability 0.01, independent of each other. The company sells the DVDs in packages of size 10 and offers a money-back guarantee that if at least 1 of the 10 DVDs in a package is defective, money will be returned. if someone buys 3 packages, what is the probability that he or she will return exactly 1 of them? **(13)**

Contd .......... P/2

**CSE 301**

**Contd... Q. No. 5**

(b) An airline knows that *5* percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell *52* tickets for a flight that can hold only *50* passengers. What is the probability that there will be a seat available for every passenger who shows up? **(10)**

(c) Suppose that two teams are playing a series of games, each of which is independently won by team $A$ with probability $p$ and by team $B$ with probability $1$-$p$. The winner of the series is the first team to win $i$ games. Find the expected number of games that are played when (a) $i = 2$ and (b) $i = 3$. **(6+6)**

6. (a) At a party $n$ men take off their hats. The hats are then mixed up and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of no matches? What is the probability of exactly $k$ matches? **(8+5)**

(b) A computer receives requests for elements stored in its memory. Consider that n elements $e_1, e_2, \ldots, e_n$ are initially arranged in some ordered list. At each unit of time a request is made for one of these elements – $e_i$, being requested, independently of the past, with probability $P_i$. After being requested, the element is then moved to the front of the list. That is, for instance, if the present ordering is $e_1, e_2, e_3, e_4$ and $e_3$ is requested, then the next ordering is $e_3,\ e_1, e_2,\ e_4$. Determine the expected position of the element requested after this process has been in operation for a long time. **(12)**

(c) In an election, candidate A receives $n$ votes, and candidate B receives $m$ votes where $n>m$. Assuming that all orderings are equally likely, show that the probability that A is always ahead in the count of votes is $(n-m)/(n+m)$. **(10)**

7. (a) A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same next day, and if it changes then it is equally likely to be either of the other two possibilities. In the long run, what proportions of days are sunny? What proportions are cloudy? **(8)**

(b) Consider a gambler who at each play of the game has probability $p$ of winning one unit and probability $q = 1$-$p$ of losing one unit. Assume that successive plays of the game are independent, what is the probability that, starting with $i$ units, the gambler's fortune will reach $N$ before reaching $0$? **(12)**

(c) Define Markov chain. Derive the Chapman-Kolmogorov equations for computing n-step transition probabilities in a Markov chain. **(3+4)**

Contd ......... P/3

**CSE 301**

**Contd... Q. No. 7**

(d) Consider a large population of individuals, each of whom possesses a particular pair of genes, of which each individual gene is classified as being of type α or type β. Assume that the proportions of individuals whose gene pairs are αα, ββ, or αβ, respectively are $p_o, q_o$ and $r_o\left(p_o + q_o + r_o = 1\right)$ respectively. When two individuals mate, each contributes one of his or her genes, chosen at random, to the resultant offspring. Assuming that the mating occurs at random, in which each individual is equally likely to mate with other individual, determine the proportions p, q, and r of individuals in the next generation whose genes are αα, ββ, or αβ respectively. **(8)**

8. (a) If X and Y are independent Poisson random variables with respective means $\lambda_1$ and $\lambda_2$, calculate the conditional expected value of X given that $X + Y = n$. **(10)**

(b) Define moments of a Random Variable. How can we obtain them from moment generating function? Derive the moment generating function for the binomial distribution with parameters $n$ and $p$. If $X$ and $Y$ are independent binomial random variables with parameters $(n,p)$ and $(m,p)$, respectively, then what is the distribution of $X+Y$. **(2+3+4+4=13)**

(c) Two chips are drawn at random without replacement from a box that contains five chips numbered 1 through 5. If the sum of chips drawn is even, the random variable X equals 5; if the sum of chips drawn is odd, X = -3. **(12)**

(i) Find the moment-generating function (MGF) for X.

(ii) Use the MGF to find the first and second moments.

(iii) Find the expected value and variance of X.

---------------------------------

**L-3/T-2/CSE** **Date : 23/07/2016**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2014-2015

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Deduce the maximum number of regions n planes can divide the 3-D space. **(15)**

   (b) Let us have Josephus problem where among n persons every 9th person is deleted, and let $J_q(n)$ be the ultimate survivor. Deduce $J_q(n)$ and computed $J_8(1000)$. **(20)**

2. (a) Discuss multiple Tower of Hanoi problem. Deduce the properties of the Presumed Optimal Solution. Solve the problem for n = 378 and p = 8 showing the solution in a binary tree of level 2. **(15)**

   (b) Discuss the properties of finite calculus with examples. Compute the value of $\sum_{0\le k<n} k^2 H_k$. **(20)**

3. (a) Using combinatorial arguments establish the identities **(15)**

$$\sum_{0\le k \le n}\binom{k}{m}=\binom{n+1}{m+1}$$

$$\binom{n-1}{k}+\binom{n-1}{k-1}=\binom{n}{k}$$

   (b) If n men throw their hats and randomly picks up hats. In how many ways can they end up in receiving wrong hats? Use inversion formula to deduce the results. **(20)**

4. (a) Discuss Stirling numbers in detail and deduce recurrence relations they satisfy. **(15)**

   (b)(i) Discuss Harmonic numbers with their properties. Calculate maximum overhang that can be created by placing n cards on n table. **(20)**

   (ii) Discuss Euler number and establish recurrence relations it satisfies.

## SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Suppose that two teams are playing a series of games, each of which is independently won by team $A$ with probability $p$ and by team $B$ with probability $1-p$. The winner of the series is the first team to win $i$ games. Find the expected number of games that are played with (i) $i = 2$, and (ii) i = 3. **(5+5)**

   (b) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin? Suppose that he flips the same coin a second time and this time it shows tail. Now what is the probability that it is the fair coin? **(5+5)**

   (c) Define moments of a Random Variable. How can we obtain them from moment generating function? Derive the moment generating function for the Poisson distribution with parameter $\lambda$? If $X$ and $Y$ are independent Poisson random variables with parameters $\lambda_1$ and $\lambda_2$, respectively, then what is the distribution of $X+Y$? **(2+3+5+5)**

6. (a) Each element in a sequence of binary data is either 1 with probability $p$ or $0$ with probability $1-p$. A maximal subsequence of consecutive values having identical outcomes is called a run. For instance, if the outcome sequence is 1, 1, 0, 1, 1, 1, 0, the first run is of length 2, the second is of length 1, and the third is of length 3. **(6+6)**
   (i) Find the expected length of the first run.
   (ii) Find the expected length of the second run.

   (b) In an election, candidate $A$ receives $n$ votes, and candidate $B$ receives m votes where $n > m$. Assuming that all orderings are equally likely, show that the probability that $A$ is always ahead in the count of votes is $(n-m)/(n+m)$. **(11)**

   (c) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom. **(6+6)**
   (i) Assuming that the prisoner will always select doors 1, 2, and 3 with probabilities 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom?
   (ii) Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (In this version, for instance, if the prisoner initially tries door 1, then when he returns to the cell, he will now select only from doors 2 and 3.)

Contd .......... P/3

7. (a) A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same the next day, and if it changes then it is equally likely to be either of the other two possibilities. In the long run, what proportion of days are sunny? What proportion are cloudy? **(8)**

(b) Consider a large population of individuals, each of whom possesses a particular pair of genes, of which each individual gene is classified as being of type $\alpha$ or type $\beta$. Assume that the proportions of individuals whose gene pairs are $\alpha\alpha$, $\beta\beta$ or $\alpha\beta$ are respectively $p_0, q_0$, and $r_0\left(p_0+q_0+r_0=1\right)$. When two individuals mate, each contributes one of his or her genes, chosen at random, to the resultant offspring. Assuming that the mating occurs at random, in that each individual is equally likely to mate with any other individual, determine the proportions p, q, and r of individuals in the next generation whose genes are $\alpha\alpha$, $\beta\beta$, or $\alpha\beta$ respectively. **(12)**

(c) Consider a Poisson process with rate $\lambda$, and let us denote the time of the first event by $T1$. Further, for $n>1$, let $T_n$ denote the elapsed time between the $(n-1)$st and the $n$th event. The sequence $\{T_n, n=1, 2, \ldots,\}$ is called the sequence of inter-arrival times. Show that $T_n, n=1, 2, \ldots,$ are independent identically distributed exponential random variables having mean $1/\lambda$. **(7)**

(d) Suppose that $X_1$ and $X_2$ are independent exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$; what is $P\{X_1 < X_2\}$? **(8)**

8. (a) Customers arrive at a single-server station in accordance with a Poisson process with rate $\lambda$. All arrivals that find the server free immediately enter service. All service times are exponential distributed with rate $\mu$. An arrival that finds the server busy will leave the system and roam around "in orbit" for an exponential time with rate $\theta$ at which time it will then return. If the server is busy when an orbiting customer returns, then that customer returns to orbit for another exponential time with rate $\theta$ before returning again. An arrival that finds the server busy and $N$ other customers in orbit will depart and not return. That is, $N$ is the maximum number of customers in orbit. **(4+3+3+3)**

(i) Define states.

(ii) Give the balance equations.

In terms of the solution of the balance equations, find

(iii) the proportion of all customers that are eventually served;

(iv) the average time that a served customer spends waiting in orbit.

Contd .......... P/4

**CSE 301**

**Contd... Q. No. 8**

(b) Consider a two-server system in which customers arrive at a Poisson rate $\lambda$ at server 1. After being served by server 1 they then join the queue in front of server 2. We suppose there is infinite waiting space at both servers. Each server serves one customer at a time with server $i$ taking an exponential time with rate $\mu_i$ for a service, $i = 1, 2$. Such a system is called a tandem or sequential system. For the tandem queuing system, compute **(12)**

(i) the average number of customers in the system,

(ii) the average time a customer spends in the system,

(c) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates of 1 and 2 are, respectively, 8 and 10. A customer upon completion of service at server 1 is equally likely to go to server 2 or to leave the system (i.e., $P_{11} = 0, P_{12} = 1/2$), whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise (i.e., $P_{21} = 1/4, P_{22} = 0$). Determine **(10)**

(i) the probability that there are $n$ customers at server 1 and $m$ customers at server 2.

(ii) the average number of customers in the system $L$, and

(iii) the average time a customer spends in the system $W$.

----------------------------------

**L-3/T-2/CSE** **Date : 23/01/2016**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A certain letter is equally likely to be in any one of three different files. Let, $\alpha_i$ be the probability that you will find the letter upon making a quick examination of file $i$, if the letter is, in fact, in file $i$, $i = 1,2,3$. Given $\alpha_1 = 1/2$, $\alpha_2 = 1/3$, $\alpha_3 = 1/4$. Suppose you look into file 1 and do not find the letter after quick examination. What is the probability that the letter is in file 1? **(10)**

(b) An airline knows that 5 percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up? **(10)**

(c) Define moments of a Random Variable. How can we obtain them from moment generating function? Derive the moment generating function for the binomial distribution with parameters $n$ and $p$? If $X$ and $Y$ are independent binomial random variables with parameters $(n,p)$ and $(m,p)$, respectively, then what is the distribution of $X+Y$? **(2+3+5+5)**

2. (a) At a party $n$ men take off their hats. The hats are then mixed up and each man randomly selects one. We say that a match occurs if a man selects his own hat. What is the probability of no matches? What is the probability of exactly $k$ matches? **(8+5)**

(b) A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety? **(10)**

(c) A computer receives requests for elements stored in its memory. Consider $n$ elements $e_1, e_2, \ldots, e_n$ are initially arranged in some ordered list. At each unit of time of request is made for one of these elements — $e_i$ being requested, independently of the past, with probability $P_i$. After being requested, the element is then moved to the front of the list. That is, for instance, if the present ordering is $e_1, e_2, e_3, e_4$ and $e_3$ is requested, then the next ordering is $e_3, e_1, e_2, e_4$. Determine the expected position of the element requested after this process has been in operation for a long time. **(12)**

Contd .......... P/2

## CSE 301

3. (a) An organization has $N$ employees where $N$ is a large number. Each employee has one of three possible job classifications and changes classifications (independently) according to a Markov chain with transition probabilities. **(8)**

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

In the long run, what percentages of employees are in each classification?

(b) Consider a gambler who at each play of the game has probability $p$ of winning one unit and probability $q = 1-p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with $i$ units, the gambler's fortune will reach $N$ before reaching $0$? **(12)**

(c) Define counting process and illustrate its properties. When do we regard a counting process as a Poisson process? **(4+3)**

(d) Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day. **(4+4)**

(i) What is the expected time until the tenth immigrant arrives?

(ii) What is the probability that the elapsed time between the tenth and the eleventh arrival exceeds two days?

4. (a) Consider a shoe shop consisting of two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus, for instance, a potential customer might enter even if there is a customer in chair 2.) Suppose that potential customers arrive in accordance with a Poisson process at rate $\lambda$, and that the service times for the two chairs are independent and have respective exponential rates of $\mu_1$ and $\mu_2$. **(7+2+2+2)**

(i) Draw a state diagram of the system and write down balance equations for each state.

(ii) What proportion of potential customers enters the system?

(iii) What is the mean number of customers in the system?

(iv) What is the average amount of time that an entering customer spends in they system?

(b) Suppose that customers arrive at a single-server service station in accordance with a Poisson process having rate $\lambda$. That is, the times between successive arrivals are independent exponential random variables having mean $1/\lambda$. Each customer, upon

## CSE 301

**Contd... Q. No. 4(b)**

arrival, goes directly into service if the server is free and, if not, the customer joins the queue. When the server finishes serving a customer, the customer leaves the system, and the next customer in line, if there is any, enters service. The successive service times are assumed to be independent exponential random variables having mean $1/\mu$. This system is called the *M/M/1* queue. For the *M/M/1* queuing system, compute **(12)**

(i) the average number of customers in the system,
(ii) the average time a customer spends in the system,
(iii) the average number of customers in the queue and
(iv) the average time a customer spends in the queue.

(c) Consider a network of three stations with a single server at each station. Customers arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 5, 10, and 15. The service times at the three stations are exponential with respective rates 10, 50, and 100. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3, or (iii) leave the system. A customer departing service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to either go to station 2 or leave the system. **(10)**

(i) What is the average number of customers in the system (consisting of all three stations)?
(ii) What is the average time a customer spends in the system?

### SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Use repertoire method to solve the following general four-parameter recurrence: **(10)**

$$R(0)=\alpha;$$
$$R(n)=R(n-1)+\beta n^2+\gamma n+\delta, \qquad \textit{for } n>0$$

(b) Prove that, each fraction that appears in the Stern-Brocot tree is in lowest terms. **(8)**

(c) Prove that, $E_2(n!)=n-V_2(n)$, for $n>0$. Here, $E_2(n!)$ is the exponent of 2 in the unique prime factorization of $n!$ and $V_2(n)$ is the number of 1's in the binary representation of $n$. **(9)**

(d) Alice writes a radix 4 number in a paper and sends it to Bob. But Bob thinks that the received number is a decimal number. For example, if Alice wants to send $(57)_{10}$, he writes down 321 (which is the radix 4 representation of $(57)_{10}$) and sends it to Bob. Bob thinks the number is $(321)_{10}$. Suppose $F$ is the function that converts Alice's number to Bob's number. For example, $F((57)_{10})=(321)_{10}$. Provide a recursive definition of $F$. **(8)**

Contd .......... P/4

## CSE 301

6. (a) Let $Z_n$ denote the maximum number of regions that can be created by placing $n$ zigs in a plane. Determine the closed form of $Z_n$ with appropriate explanation. **(9)**

(b) Prove that, $\ln n < H_n < \ln n + 1$, for $n > 1$, where $H_n = \sum_{k=1}^{n} \frac{1}{k}$. **(9)**

(c) Let $J(n)$ be the number of the survivor among $n$ people in the Josephus problem. There are infinitely many $n$'s for which $J(n) = n/2$. Write down the first three such numbers. **(7)**

(d) In how many ways, can you select five non-consecutive integers from the set $\{1,2,3,\cdots,20\}$? **(10)**

7. (a) Use generating functions to prove that, $\sum_{k=0}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$. **(12)**

(b) State and explain the recurrence relation for "*n subset k*". There are $m^n$ functions from a set of $n$ elements into a set of $m$ elements. How many of them range over exactly $k$ different function values? **(6+6)**

(c) Consider the following two problems. The first problem is to find the number of rooted binary trees consisting of $n$ nodes. The second problem is to find the number of valid parenthesizations formed by $n$ pair of brackets. **(6+5)**

(i) There exists a bijection between the above two problems. Describe a rule of converting a rooted binary tree into a unique valid parenthesization and vice versa.

(ii) There are five rooted binary trees with three nodes and five valid parenthesizations can be formed using three pair of brackets. For all five valid parenthesization, draw the corresponding rooted binary tree according to the rule of conversion you mentioned.

8. (a) How many integers $n$ are there such that, $\lfloor \sqrt[3]{n} \rfloor \backslash n$ and $1 \leq n \leq 2000$. **(13)**

(b) State Vandermonde's convolution formula. Give a combinatorial proof of Vandermonde's convolution formula for non-negative integers. Then use polynomial argument to prove the general case. **(2+5+5)**

(c) Give a combinatorial proof that, the alternating sum of the $n^{th}$ row of Pascal's triangle is zero. In other words, $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$, for $n > 0$. **(10)**

-----------------------------------

L-3/T-2/CSE Date : 15/12/2014

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2012-2013

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

---

## SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Discuss the Josephus problem. Deduce the recurrence relations of the ultimate survivor among n people satisfies. Find J(23456). **(20)**

   (b) Discuss the problem of planes in the space. Find the maximum number of regions that n planes can divide a 3-D space. **(15)**

2. (a) Discuss the strategy of the presumed optimal solution for the multi-peg tower of Hanoi problem. For (n,p) = (300,7), find the number of disks to be shifted to the intermediate peg. **(20)**

   (b) Evaluate the sum of the terms $K^2H_K$, where K varies from 0 to n. **(15)**

3. (a) The concrete mathematics club has a casino in which there is a roulette wheel with 10,000 slots, numbered 1 to 10,000. If the number n that comes up on a spin is divisible by the floor of its cube root, then it is a winner and the house pays Tk 5; otherwise it is a looser and we must pay Tk. 1 Can you expect to make money by playing this game? **(20)**

   (b) Using combinatorial argument, prove that K things taken n at a time summing up over all K <= m equals (m = 1) things taken (n + 1) at a time. **(15)**

   (m+1)

4. (a) Using inversion formula find an expression for number of permutations of n things in which no element is in its own place. **(20)**

   (b) Define Stirling number of the first and second kinds. Find the maximum overhang that can be made by placing n cards each of length 2 on a table. **(15)**

## SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Three prisoners are informed by their jailer that one of them has been chosen at random to be executed, and the other two are to be freed. Prisoner *A* asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information, since he already knows that at least one will go free. The jailer refuses to answer this question, pointing out that if *A* knew which of his fellows were to be set free, then his own probability of being executed would rise from 1/3 to 1/2, since he would then be one of two prisoners. What do you think of the jailer's reasoning? **(9)**

Contd .......... P/2

## CSE 301

**Contd ... Q. No. 5**

(b) $A$ and $B$ play until one has 2 more points than the other. Assuming that each point is independently won by $A$ with probability $p$, what is the probability that they will play a total of $2n$ points? What is the probability that $A$ will win? **(6+7)**

(c) A total of $r$ keys are to be put, one at a time, in $k$ boxes, with each key independently being put in box $i$ with probability $p_i$, where $\sum_{i=1}^{k} p_i = 1$. Each time a key is put in a nonempty box, we say that a collision occurs. Find the expected number of collisions. **(13)**

6. (a) Alice wants to choose a point randomly on a disk of radius 2 centered at the origin. She comes up with two strategies. Strategy *I* involves determining the Polar coordinate of the point by choosing the modulus ($r$) uniformly from the range [0, 2] and the argument ($\theta$) uniformly from the range $\cancel{[0, 2\pi]}$ $[0, 2\pi)$. Strategy II involves determining the Cartesian coordinate of the point by choosing both the x-coordinate (x) and y-coordinate (y) uniformly ~~form~~ from the range $[-2, 2]$ until $x^2 + y^2 \le 4$. What is the probability that the randomly chosen point lies inside a disk of radius 1 centered at the origin (i) when Alice uses Strategy *I* and (ii) when Alice uses Strategy *II*. **(5+5)**

   (b) An urn contains $b$ black balls and $r$ red balls. One of the balls is drawn at random, but when it is put back in the urn $c$ additional balls of the same color are put in with it. Now suppose that we draw another ball. Find the probability that the first ball drawn was black given that the second ball drawn was red. **(10)**

   (c) A television store owner figures that *50* percent of the customers entering his store will purchase an ordinary television set, *20* percent will purchase a color television set, and *30* percent will just be browsing. If five customers enter his store on a certain day (i) What is the probability that two customers purchase color sets, one customer purchases an ordinary set, and two customers purchase nothing? (ii) What is the probability that our store owner sells three or more televisions on that day? **(8+7)**

7. (a) Let $X$ be a Poisson random variable with parameter $\lambda$. Show that, $P\{X = i\}$ increases monotonically and then decreases monotonically as $i$ increases, reaching its maximum when $i$ is the largest integer not exceeding $\lambda$. **(8)**

   (b) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns his to his cell after three days of travel. The third door leads immediately to freedom. (i) Assuming that the prisoner will always select doors *1*, *2* and *3* with probabilities *0.5*, *0.3*, *0.2*, what is the expected number of days until he reaches freedom?

Contd .......... P/3

**Contd ... Q. No. 7(b)**

(ii) Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (In this version, for instance, if the prisoner initially tries door *1*, then when he returns to the cell, he will now select only from doors *2* and *3*.) **(7+8)**

(c) A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let $p_i$ denote the probability that the class does well on a type *i* exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type *1*. What proportion of exams are type *i* in the long run, $i = 1, 2, 3$? **(12)**

8. (a) Specify the classes of the following Markov chains, and determine whether they are transient or recurrent: **(8)**

(i)

$$\begin{vmatrix} 1/2 & 0 & 1/2 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{vmatrix}$$

(ii)

$$\begin{vmatrix} 1/4 & 3/4 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{vmatrix}$$

(b) Prove that, exponentially distributed random variables are memory-less. **(7)**

(c) A small barbershop, operated by a single barber, has room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and the successive service times are independent exponential random variables with mean 1/4 hour. (i) What is the average number of customers in the shop? (ii) What is the proportion of potential customers that enter the shop? (iii) If the barber could work twice as fast, how much more business would he do? **(9+5+6)**

-----------------------------------

L-3/T-2/CSE **Date : 09/06/2014**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2011-2012

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Deduce the recurrence relation satisfied by the ultimate survivor J(n). Compute J(54321). **(15)**

   (b) Deduce the formula for maximum number of regions created by n zigs. **(10)**

   (c) Solve the recurrence relation for minimum number of moves required by Tower of Hanoi problem. **(10)**

2. (a) Calculate $\sum_{0\le k<m} k.2^k$ using the method of finite calculus. **(15)**

   (b) Find multiplicity of 96 in 1000! **(10)**

   (c) Give a combinatorial argument to the identity $\sum_{0\le k<m}\binom{k}{r}=\binom{m+1}{r+1}$ for integer $r, n \ge 0$. **(10)**

3. (a) Use inversion formula to calculate number of derangements $n!$ **(15)**

   (b) Prove that $\sum_{k\le n}\binom{n+r}{k}x^k y^{n-k} = \sum_{k\le n}\binom{-r}{k}(-k)^k(x+y)^{n-k}$, where $n$ is an integer. **(10)**

   (c) Deduce and prove recurrence relations satisfied by the stirling numbers of the 1st and 2nd kinds. **(10)**

4. (a) Establish the number of time units required by the worm to reach the other end of the superelastic rubber band. **(15)**

   (b) Establish recurrences satisfied by the Euler number. **(10)**

   (c) Let $P_n$, $N_n$, $D_n$, $Q_n$ and $C_n$ be the number of ways to pay $n$ cents using coins worth at most 1, 5, 10, 25 and 50 cents, respectively. Deduce recurrence relations for these expressions. **(10)**

19

~~18~~

Contd .......... P/2

**CSE 301**

## SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

In all cases, use correct symbols and notations.

5. (a) Show that for all random variables $X$ and $Y$, $E[x] = E[E[X \mid Y]]$. Assume that X and Y are discrete. **(10)**

(b) For a Geometric random variable $X$ with success probability $p$, show that $P\{X > i\} = (1-p)^i$. **(10)**

(c) (**Best Prize Problem**) Suppose that we are to be presented with $n$ distinct prizes in sequence. After being presented with a prize, we must immediately decide whether to accept it or reject it and consider the next prize. The only information we are given when deciding whether to accept a prize is the relative rank of that prize compared to ones already seen. Let a strategy reject the first $l$ prizes and then accepts the first one that is better than all of those first $l$. Let $P_l(best)$ denote the probability that the best prize is selected when this strategy is applied. Compute $P_l(best)$. **(15)**

6. (a) Let $X$ be a random variable with probability density **(15)**

$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & otherwise \end{cases}$$

(i) What is the value of $c$?

(ii) Compute $Vat(X)$.

(b) A dice with six faces got some problem with the paint that has been used to inscribe the dots on its faces. In particular, due to defective material the face containing more dots got heavier and the corresponding face rolls up more likely. To be specific, probability of rolling up face $i$ is proportional to $i$. If so, compute the following— **(10)**

(i) The probability that face 1 rolls up at least once in three successive rolls.

(ii) The expected number of rolls required to get a six.

(c) What is meant by "conditioning" in computing probability of events? In reference to Question 6(b), suppose two friends, Kamal and Jamal, are playing a game where they both roll the mentioned biased dice. If both the dice roll up the same face, Kamal wins; otherwise Jamal wins. By conditioning, deduce who has the higher chances of winning. **(10)**

7. (a) (**Gambler's Ruin**) Consider a gambler who at each play of the game has probability $p$ of winning one unit and probability $q = 1 - p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with $n$ units, the gambler's fortune will reach $N$ before reaching 0? Show that if $p \leq \frac{1}{2}$, the gambler will, with probability 1, go broke against an infinitely rich adversary. **(15)**

Contd .......... P/3

**Contd... Q. No. 7**

(b) In sociology, the transitions among various social classes, namely upper, middle and lower, are modeled as a Markov chain assuming that the occupation of a child depends only on its parent's occupation. Let these classes be 0, 1 and 2 respectively. Hence, a state transition probability matrix can be given as follows: **(7+8+5=20)**

$$P = \begin{Vmatrix} 0.45 & 0.48 & 0.07 \\ 0.05 & 0.70 & 0.25 \\ 0.01 & 0.50 & 0.49 \end{Vmatrix}$$

For instance, the underlined value (0.05) denotes the probability that a child of middle class parent takes an upper class occupation. Now, answer the following—

(i) Given that 25 and 65 out of 100 people live in lower and middle class respectively in this generation, how many of them would remain so in the following generation?

(ii) In the long run, what fraction of people belongs to lower class?

(iii) At what rates, people move out of their middle class status (move to upper or lower)?

8. (a) Define Poisson counting process. Suppose you are waiting for a ride in a street. Buses fly at a Poisson rate of 2 per hour whereas taxis show up at a Poisson rate of 5 per hour. Assuming that you catch the first vehicle you get, what is the expected amount of time you need to wait to avail a ride? Suppose you already waited for 20 minutes without any luck. What is the probability that you manage some ride in next 5 minutes? **(15)**

(b) A building has two switches that independently remain either ON or OFF during a day at a *variable* length, both following an exponential distribution with mean $1/\lambda$ (ON) and $1/\mu$ (OFF) respectively. Argue that the scenario can be modeled as a continuous-time Markov chain. Hence, construct a Markov chain with states 0, 1, and 2 where the state denotes the number of switches that are ON in any particular time. Also, compute what fraction of time, in the long run, at least one switch is ON. **(20)**

---------------------------------

A. S. M. Sohidul Islam

**L-3/T-2/CSE** **Date : 24/12/2012**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-3/T-2 B. Sc. Engineering Examinations 2010-2011

Sub : **CSE 301** (Mathematical Analysis for Computer Science)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

---

## SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Argue in favour of the recurrence relations Josephus number J(n) satisfies. Compute J(25321). **(10+5=15)**

   (b) Write down an algorithm for solving ~~Muti~~ Multi Peg Tower of Hanoi (MPTOH) problem based upon presumed optimal solution strategy. Show the solution of the problem for (n, p) = (361, 8) in a binary tree. **(10+10=20)**

2. (a) There is a roulette wheel with 10,000 slots, numbered 1 to 10000. Assume that if the number that comes up on a spin is divisible by the floor of its 4th root then it is a winner and the house pays \$ 7 and otherwise it is a loser and the player must pay \$ 1. Can we expect to make money if we play this game? **(15)**

   (b) Discuss how analysis of average performance of quicksort can be done using summation factor. **(20)**

3. (a) Deduce how many 0s are there in decimal representation of 600!? **(15)**

   (b) Construct combinatorial arguments in favour of the equations **(10+10=20)**

$$\sum_{0\le k\le m}\binom{k}{n}=\binom{m+1}{n+1} \quad \text{and} \quad \sum_{k\le m}\binom{n+k}{k}=\binom{n+m+1}{n+1}$$

4. (a) Prove the inversion formula. Use this formula to compute number of derangements on n distinct objects. **(5+10=15)**

   (b) Argue in favour of the recurrence Stirling's numbers of the first and second kinds satisfy. Deduce how large an overhang can be made placing n cards, each of length 2, on a table. **(10+10=20)**

## SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A football team, Cactus, wins a match with probability 0.75 irrespective of its opponents. What is the probability that the team wins 4 matches out of 5 matches? **(5+5+5=15)**
In a knockout tournament, Cactus faces a series of opponents until it loses to someone. How many matches does Cactus expect to play before it gets eliminated from the tournament? If Cactus is already in Quarter final, what is the probability that it's going to play the Final and becomes Champion?

Contd .......... P/2

**CSE 301**

**Contd ... Q. No. 5**

(b) A laboratory blood test is 96% effective in detecting a certain disease when it is "in fact" present. However, the test also yields a "false" positive result for 5% of the healthy persons tested (that is, a healthy person, with 0.05 probability, would be detected to have the disease!). If 30% of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive? **(15)**

(c) Bangladesh Cricket team lost tosses for the last 5 games. As we know, tosses are done by a fair coin at each game; we can argue that in the next game there is a high chance that BD team is going to win the toss. Frankly speaking, if you lose consecutively, your chance of winning on the next attempt increases. What is the fallacy of this argument? **(5)**

6. (a) A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door also does the same like the second door, but takes five hours in the tunnel. Assuming that the miner is all time equally likely to choose any of one of the doors, what is the expected length of time until the miner reaches safety?

   What happens when the miner can remember which door he took earlier and does not take the same door again in the next attempt? What is the expected exit time then? **(10+10=20)**

   (b) Let us consider a "simplified" version of a medium access scheme by a set of wireless hosts (the full version is widely known as IEEE 802.11 protocol). In principle, in a wireless medium only one host can transmit at a time. Each host senses the medium before it attempts to transmit a packet. If the channel is sensed free (none is transmitting), the packet is sent to the air immediately. Otherwise (i.e., if the channel is busy), the host waits for a while. In that, the host sets a counter to $k$ (some constant) and then decrements the counter by 1 in each slot time (say, 100 ms) until the counter becomes zero. When the counter becomes zero, the host tries to access the channel again and the process continues. **(15)**

   Let $p$ be the probability that the channel is sensed busy at any time. Now, construct a Markov chain to analyze this system. Define Markov states and associated transition probabilities. You can consider defining Markov states based on the number of slots the host needs to wait before attempting the medium access.

Contd .......... P/3

**CSE 301**

7. (a) Given the following transition probabilities for a Markov chain with three states; A, B and C – **(10)**

$$\begin{array}{c|ccc|} & A & B & C \\ A & 0.5 & 0.5 & 0 \\ B & 0.2 & 0.8 & 0 \\ C & 0 & 0.6 & 0.4 \end{array}$$

Answer the following:

(i) Is there any transient state in this chain? If any, which one?

(ii) Is this Markov chain Ergodic? Why?

(iii) Try to compute long-run state probabilities for this chain. Can you argue the results?

(b) A software component for an embedded device is written with four code blocks–A, B, C and D (the flow is shown in Figure for Question 7(b)). Based on inputs and other conditions, the execution takes branching, and the associated branching probabilities are shown on the corresponding edges. Due to some implementation issues, the program produces "faulty" outputs when it executes code in block C. In other blocks, outputs are "good". **(10+15=25)**

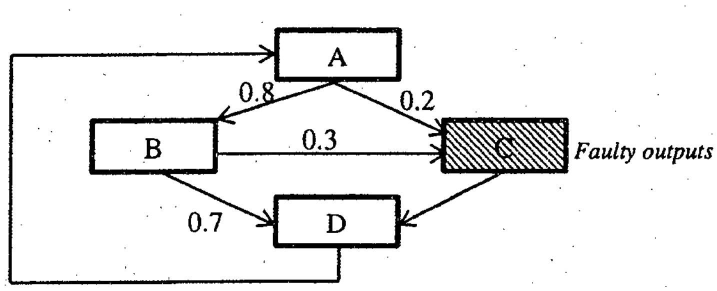


Figure for Question 7 (b)

Assuming that each code block executes 1 unit of time and the program executes in a continuous loop forever, construct the Markov chain associated with this software process along with the corresponding transition probability matrix. Hence, compute the following–

(i) The rate at which faulty outputs are produced.

(ii) The expected length of time the system produces consecutive good results.

## CSE 301

8. (a) What is meant to be the Markovian property of a continuous-time Markov chain? **(8)**

(b) For a pure birth process (also known as Yule process) with an individual birth rate $\lambda$, argue that for a population with *n* individuals, the effective birth rate becomes $n\lambda$. **(12)**

(c) What do symbols in M/M/s queuing system stand for? For an M/M/1 queuing system (of infinite queue capacity) with arrival rate $\lambda$ and service rate $\mu$, compute the limiting probabilities $P_n$, for $n = 0, 1, 2, \ldots\ldots$, where: **(15)**

$P_n$ = steady-state probability that the system has exactly *n* customers

Hence, compute –

(i) L, the average number of customers in the system

(ii) $L_Q$, the average number of clusters waiting in the queue

---