Market Entry and Plant Location in Multi-Product Firms

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Motivation

- Multinational firms are often multi-product, with each product manufactured and sold in many (but not all) countries.
- Decisions on where to produce and where to sell a product are often interdependent.
- Production and sales decisions across products may also be interdependent.
- Nontrivial implications of policies/shocks: a product- and country-specific production or consumption subsidy may induce changes in production and sales decisions for all products in the firm's portfolio.
- Focus on the global car industry: firms are multi-product, multi-plant, & multi-market, and industry has recently been the target of large industrial policies .

This Paper

- Static model with firms deciding where to produce and sell each product in a portfolio.
  - Fixed selling costs: binary choice of whether to sell each product in a location.
  - Fixed production costs: binary choice of whether to produce each product in a location.
  - Nested CES demand: cannibalization across a firm's products sold in a location.
  - Export platforms: cannibalization across a firm's production locations for each product & complementarities between selling and production location choices.
  - No strategic interactions between firms; monopolistically competitive model.
- Provide an algorithm to bound the solution to the firm's problem: solution method for single-agent CDCPs featuring both pairwise complementarities and substitutabilities.
- Use bounds on the firm's solution & moment inequalities to estimate model.
- Use bounds on the firm's solution to predict changes in global car industry in reaction to changes both in national consumption and production subsidies and in tariffs.

Literature

1. Methodological literature on combinatorial discrete-choice problems (CDCPs):
   - Solution: Jia (2008), Arkolakis et al. (2023), Alfaro-Ureña et al. (2023), Castro-Vincenzi (2024), Sabal (2025), Head et al. (2025).
   - Estimation (moment inequalities): Holmes (2011), Morales et al. (2019), Fan and Yang (2023).
2. Literature on interdependencies in plant location choices:
   - Helpman et al. (2004), Tintelnot (2017), Oberfield et al. (2023), Castro-Vincenzi (2024).
3. Literature on interdependencies in within-market product-entry choices:
   - Eckel and Neary (2010), Dhingra (2013), Hottman et al. (2016), Head and Mayer (2019), Arkolakis et al. (2021).
4. Literature on interdependencies in input sourcing decisions:
   - Antràs et al. (2017), Arkolakis et al. (2023).
5. Literature on interdependencies in cross-market entry choices:
   - Morales et al. (2019), Antràs et al. (2021), Alfaro-Ureña et al. (2023).
6. Literature on interdependencies between choices across multiple dimensions.
   - Bernard et al. (2018), Helpman and Niswonger (2022), Antràs et al. (2023), Head et al. (2025).

Quantitative Model

Setting

- Static equilibrium model for a global industry (car segment).
- Continuum of firms (brands) indexed by i.
- The firm operates an exogenous & finite portfolio of models indexed by $j = 1, \ldots, J_i$.
- The firm simultaneously decides where to produce and sell each model in its portfolio.
- Index production locations by $o = 1, \ldots, N$ and destinations by $n = 1, \ldots, N$.

Demand

- Nested CES demand in every destination:

$$C_n = \left(\int_{i \in \Omega} (\psi_{in} C_{in})^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

$$C_{in} = \left(\sum_{j=1}^{J_i} I_{ijn} (\psi_{ijn} C_{ijn})^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}},$$

  with $\rho \geq \eta > 1$.

- Given an exogenous shifter A_n, the demand equation for a model ij equals:

$$C_{ijn} = A_n (\psi_{in})^{\eta-1} (\psi_{ijn})^{\rho-1} (P_{in})^{\rho-\eta} (P_{ijn})^{-\rho}.$$

  In empirics, ψ_{in} and ψ_{ijn} depend on firm and model effects, respectively, and covariates.

Marginal Costs and Market Structure

- Marginal production costs:

$$c_{ijo} = \phi_i \phi_o \phi_{oh(i)} \phi_{ij}.$$

  In empirics, both $\phi_{oh(i)}$ and ϕ_{ij} are functions of observed characteristics.

- Iceberg trade costs:

$$\tau_{on} = \kappa_n \kappa_{on}.$$

  In empirics, κ_{on} is a function of covariates.

Fixed Costs of Product Entry and Production Location

- Fixed costs of selling product *ij* in destination *n*:

$$F^e_{ijn} = \gamma_{nh(i)} + \nu^e_{ijn}, \qquad \text{with} \qquad \nu^e_{ijn} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_e).$$

  In empirics, $\gamma_{nh(i)}$ is a function of covariates.

- Fixed costs of producing model *ij* in origin *o*:

$$F^p_{ijo} = (1 - d_{ijo})(\alpha_{oh(i),1} + \nu^p_{ijo}), \quad \text{with} \quad \nu^p_{ijo} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_p),$$

  and d_{ijo} a categorical variable with

$$P(d_{ijo} = 1) = \frac{\exp(\alpha_{oh(i),2})}{1 + \exp(\alpha_{oh(i),2})}.$$

  In empirics, $\alpha_{oh(i),1}$ and $\alpha_{oh(i),2}$ are functions of covariates.

Pricing Equation and Variable Gross Profits

- Potential price of model ij in destination n is:

$$P_{ijn}(D_{ij}) = \frac{\eta}{\eta - 1} \times \min_{o:D_{ijo}=1} \{\tau_{on} c_{ijo}\},$$

  with $D_{ij} = \{D_{ijo}\}_o$ and $D_{ijo} = 1$ if model ij is produced in o (and zero otherwise).

- Variable potential gross profits of selling model ij in market n are:

$$\pi_{ijn}(I_{in}, D_i) = A_{ijn}(P_{in}(I_{in}, D_i))^{\rho-\eta}(P_{ijn}(D_{ij}))^{1-\rho},$$

$$P_{in}(I_{in}, D_i) = \left(\sum_{j'=1}^{J_i} I_{ij'n}\left(\frac{P_{ij'n}(D_{ij'})}{\psi_{ij'n}}\right)^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

  with $D_i = \{D_{ijo}\}_{j,o}$, $I_{in} = \{I_{ijn}\}_j$ and $I_{ijn} = 1$ if model ij is sold in n (and zero otherwise).

Optimal Product Entry and Production Location Decisions

- Each firm chooses $I_i = \{I_{ijn}\}_{j,n}$ and $D_i = \{D_{ijo}\}_{j,o}$ to maximize profits:

$$\Pi_i(I_i, D_i) = \sum_{n=1}^{N} \sum_{j=1}^{J_i} I_{ijn}(\pi_{ijn}(I_{in}, D_i) - F^e_{ijn}) - \sum_{o=1}^{N} \sum_{j=1}^{J_i} D_{ijo} F^p_{ijo}.$$

- The following pairs have weakly negative cross partials:
  1. D_{ijo} and $D_{ij'o'}$ for $j \neq j'$ or $o \neq o'$,
  2. I_{ijn} and $I_{ij'n}$ for $j \neq j'$,
  3. D_{ijo} and $I_{ij'n}$ for $j \neq j'$,

  and the following pairs have weakly positive cross partials:
  1. D_{ijo} and I_{ijn} for all (o, n).

Solution Algorithm For “General” CDCP

Marginal Value & Cross-Partial

- The firm's problem is of the form

$$C^* = \text{argmax}_{C \in \mathcal{B}^K} \pi(C)$$

  with $\mathcal{B}^K$ the set of 2^K K-dimensional binary vectors: the K-dimensional Boolean set.
- For any $k = \{1, \ldots, K\}$, define the marginal value of k under C as

$$\Delta_k(\pi(C)) := \pi(C^{k \to 1}) - \pi(C^{k \to 0}),$$

  where, for example, $C^{k \to 1}$ is the vector C with the kth coordinate set to one.
- Thus, $\Delta_k(\pi(C))$ is the change in $\pi(C)$ when changing the kth coordinate from zero to one holding all other coordinates constant.
- For any $k, l = \{1, \ldots, K\}$, define the cross partial of k and l under C as

$$\Delta_{kl}(\pi(C)) := \Delta_k(\Delta_l(\pi(C))) \; (= \Delta_l(\Delta_k(\pi(C)))).$$

Complementarities and Substitutabilities

Assumption

For all $C \in \mathcal{B}^K$, and all $k, l \in \{1, \ldots, J\}$, the sign of $\Delta_{kl}(\pi(C))$ is known and independent of C.

- We do not require $\pi(C)$ to be either submodular or supermodular, or to exhibit single crossing differences in choices (Jia, 2008; Arkolakis et al., 2023).
- We define a dummy s_{kl} that equals 1 if $\Delta_{kl}(\pi(C)) \geq 0$ for all C (and 0 otherwise).

Vocabulary

- Meet: Let A be a set with a partial order $\leq$, and let $x, y \in A$. An element m of A is the meet (or the greatest lower bound or infimum) of x and y and is denoted $a \wedge b$ if the following two conditions are satisfied:
  1. $m \leq x$ and $m \leq y$.
  2. For any $w \in A$, if $w \leq x$ and $w \leq y$, then $w \leq m$.

  The meet may not exist, but, if it does, it must be unique.

- Join: Let A be a set with a partial order $\leq$, and let $x, y \in A$. An element j of A is called the join (or the smallest upper bound or supremum) of x and y and is denoted $a \vee b$ if the following two conditions are satisfied:
  1. $j \geq x$ and $j \geq y$.
  2. For any $w \in A$, if $w \geq x$ and $w \geq y$, then $w \geq j$ (that is, j is smaller than or equal to any other upper bound of x and y).

  The join may not exist, but, if it does, it must be unique.

Vocabulary

- If the power set $\mathfrak{P}(X)$ of some set X is partially ordered by the partial order $\subseteq$, then the meet of any two elements of $\mathfrak{P}(X)$ is their intersection and the join is their union; that is, for any $a, b \in \mathfrak{P}(X)$, we have $a \wedge b = a \cap b$ and $a \vee b = a \cup b$.

- Consider the set $\mathcal{F} = \{\{1\}, \{2\}, \{1, 2, 3\}, \mathbb{R}\}$ partially ordered by $\subseteq$. Then, $\{1\} \vee \{2\} = \{1, 2, 3\}$ but the meet, $\{1\} \wedge \{2\}$, does not exist. However, the meet $\{1\} \wedge \{1, 2, 3\}$ exists and equals $\{1\}$. Thus, join and meet may exist for two elements of a partially ordered set but not for a different pair of elements of the set.

- In set $\mathcal{F} = \{\{1\}, \{2\}, \{1, 2, 3\}, \{0, 1, 2\}, \mathbb{R}\}$ partially ordered by $\subseteq$, the join $\{1\} \vee \{2\}$ does not exist. Note that $\{1\} \subseteq \{1, 2, 3\}$ and $\{2\} \subseteq \{1, 2, 3\}$, and $\{1\} \subseteq \{0, 1, 2\}$ and $\{2\} \subseteq \{0, 1, 2\}$, but $\{1, 2, 3\} \not\subseteq \{0, 1, 2\}$ and $\{0, 1, 2\} \not\subseteq \{1, 2, 3\}$.

- Consider $\mathcal{F} = \{\{1\}, \{2\}, \{0, 2, 3\}, \{0, 1, 3\}\}$ partially ordered by $\subseteq$. Then, $\{1\} \vee \{2\}$ does not exist as there is no element $w \in \mathcal{F}$ such that $\{1\} \subseteq w$ and $\{2\} \subseteq w$.

Vocabulary

- Note that meet and join may be defined for more than two elements. For example, let A be a set with a partial order $\leq$, and let $x, y, z \in A$. An element m of A is the meet (or the greatest lower bound or infimum) of x, y, and z and is denoted $\wedge\{x, y, z\}$ if:
  1. $m \leq x$, $m \leq y$, and $m \leq z$.
  2. For any $w \in A$, if $w \leq x$, $w \leq y$, and $w \leq z$, then $w \leq m$.

  We can similarly define the join of $x, y, z \in A$.
- Consider the set $\mathcal{F} = \{\{1\}, \{2\}, \{1,2,3\}, \{0,1,2\}, \mathbb{R}\}$ partially ordered by $\subseteq$. Then, $\{1,2,3\} \wedge \{0,1,2\}$ does not exist. Note that, $\{1\} \subseteq \{1,2,3\}$ and $\{1\} \subseteq \{0,1,2\}$, and $\{2\} \subseteq \{1,2,3\}$ and $\{2\} \subseteq \{0,1,2\}$, but $\{1\} \not\subseteq \{2\}$ and $\{2\} \not\subseteq \{1\}$. Conversely, meet $\wedge\{\{2\}, \{1,2,3\}, \{0,1,2\}\}$ exists and is equal to $\{2\}$. The join $\{1,2,3\} \vee \{0,1,2\}$ also exists and is equal to $\mathbb{R}$.

Vocabulary

- Lattice: a partially ordered set $(L, \leq)$ such that for any $\{a, b\} \subseteq L$, meet and join exist.
- Sublattice: non-empty subset of a lattice that is itself a lattice by the partial order of the larger lattice. That is, for any $\{a, b\}$ in the sublattice, considering exclusively the elements of the sublattice itself, meet and join exist.
- Complete lattice: partially ordered set $(L, \leq)$ such that for every subset $S \subseteq L$, meet and join exist.
- Complete sublattice: a subset of a complete lattice that is itself a complete lattice by the partial order of the larger complete lattice. That is, for any subset of the complete sublattice, considering exclusively the elements of the complete sublattice itself, meet and join exist.
- Any finite and non-empty lattice is complete.
- The empty set is a lattice but not a complete lattice. As the empty set is a subset of every set and is a lattice, it is then a sublattice of every lattice.

Vocabulary

- Mapping $=$ function.
- A function is monotone increasing (equivalently, montonically increasing, increasing, weakly increasing, or non-decreasing) if for all x and y such that $x \leq y$, it is the case that $f(x) \leq f(y)$, so f preserves the order.
- A function is strictly increasing if for all x and y such that $x < y$, one has $f(x) < f(y)$.
- One can similarly define monotone decreasing and strictly decreasing functions.

Tarski's Theorem

Theorem (Tarski 1955)

Let $(L, \leq)$ be a complete lattice. Suppose $F : L \to L$ is monotone increasing; i.e. for all $x, y \in L$, $x \leq y \Rightarrow F(x) \leq F(y)$. The set of fixed points of F is then a complete lattice with respect to $\leq$.

- Implication of 1 of Tarski (1955): the set of fixed points of F is non-empty. *Proof*: the empty set is not a complete lattice.
- Implication of 2 of Tarski (1955): by starting from the join of L and applying iteratively the mapping F, one converges to the join of the set of fixed points in a finite number of iterations. *Proof*: denote as $\bar{L}$ the join of L; if $F(\bar{L}) = \bar{L}$, the $\bar{L}$ is the join of the set of fixed points of F; if $F(\bar{L}) = A$ with $A \neq \bar{L}$, then it must be that $A < \bar{L}$; if $F(A) = A$, A must the join of the set of fixed points of F; if $F(A) = B$ with $B \neq A$, then it must be that $B < A$, and so on.

Our Approach: Complete Lattice

- Denote as $S(\mathcal{B}^K)$ the set of all sublattices of the partially ordered set $\{\mathcal{B}^K, \leq\}$.
- If $K = 2$, then $\mathcal{B}^K = \{(0,0),(1,0),(0,1),(1,1)\}$ and

$$
\begin{aligned}
S(\mathcal{B}^K) = \{&\emptyset, \{(0,0)\}, \{(1,0)\}, \{(0,1)\}, \{(1,1)\}, \{(0,0),(1,0)\}, \{(0,0),(0,1)\}\\
&\{(0,0),(1,1)\}, \{(1,0),(1,1)\}, \{(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\},\\
&\{(0,0),(0,1),(1,1)\}, \{(0,0),(1,0),(0,1),(1,1)\}\}.
\end{aligned}
$$

- Note $S(\mathcal{B}^K) \neq \mathfrak{P}(\mathcal{B}^K)$ as, e.g., $\{(0,1),(1,0)\} \notin S(\mathcal{B}^K)$. This subset of $\mathcal{B}^K$ has neither meet nor join with respect to $\leq$ and, thus, is not a sublattice of $\{\mathcal{B}^K, \leq\}$.
- The partially ordered set $(S(\mathcal{B}^K), \subseteq)$ is a lattice. As it is non-empty and finite, it is also a complete lattice.
- Thus, Tarski (1955) applies to any $F\colon S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ that is monotone increasing.

Our Approach: Monotone Increasing Mapping

- As a preliminary step, for any $\boldsymbol{C} \in S(\mathcal{B}^K)$, define

$$\overline{\Omega}(\boldsymbol{C}) = \{k = 1, \ldots, K \colon \Delta_k(\pi(\sup_k(\boldsymbol{C}))) < 0\},$$
$$\underline{\Omega}(\boldsymbol{C}) = \{k = 1, \ldots, K \colon \Delta_k(\pi(\inf_k(\boldsymbol{C}))) \geq 0\},$$

  where, for each $k = 1, \ldots, J$ and $l \neq k$,

$$[\sup_k(\boldsymbol{C})]_l := s_{kl}[\sup(\boldsymbol{C})]_l + (1 - s_{kl})[\inf(\boldsymbol{C})]_j,$$
$$[\inf_k(\boldsymbol{C})]_l := s_{kl}[\inf(\boldsymbol{C})]_l + (1 - s_{kl})[\sup(\boldsymbol{C})]_l,$$

  with $[\inf(\boldsymbol{C})]_l$ and $[\sup(\boldsymbol{C})]_l$ the lth element of the meet and join, respectively, of the partially ordered set $\{\boldsymbol{C}, \leq\}$.
- In words, $\overline{\Omega}(\boldsymbol{C})$ incorporates a coordinate $k = 1, \ldots, J$ if and only if its marginal value is negative when evaluated at the vector that maximizes such marginal value among all consistent with those included in the set $\boldsymbol{C}$; i.e., when evaluated at $\sup_k(\boldsymbol{C})$.

Our Approach: Monotone Increasing Mapping

- Then, define the mapping $F\colon S(\mathcal{B}^K) \to S(\mathcal{B}^K)$ as

$$F(\boldsymbol{C}) = \{C \in \boldsymbol{C}\colon C_k = 0 \text{ for all } k \in \overline{\Omega}(\boldsymbol{C}) \text{ and } C_k = 1 \text{ for all } k \in \underline{\Omega}(\boldsymbol{C})\}.$$

- Mapping F is monotone increasing; i.e., $\boldsymbol{C} \subseteq \boldsymbol{C}' \Rightarrow F(\boldsymbol{C}) \subseteq F(\boldsymbol{C}')$. Intuition:

$$\boldsymbol{C} \subseteq \boldsymbol{C}' \Rightarrow \forall k \;\; \Delta_k(\pi(\sup_k(\boldsymbol{C}))) \leq \Delta_k(\pi(\sup_k(\boldsymbol{C}'))) \Rightarrow \overline{\Omega}(\boldsymbol{C}') \subseteq \overline{\Omega}(\boldsymbol{C})$$
$$\boldsymbol{C} \subseteq \boldsymbol{C}' \Rightarrow \forall k \;\; \Delta_k(\pi(\inf_k(\boldsymbol{C}))) \geq \Delta_k(\pi(\inf_k(\boldsymbol{C}'))) \Rightarrow \underline{\Omega}(\boldsymbol{C}') \subseteq \underline{\Omega}(\boldsymbol{C})$$

  and

$$\left.\begin{array}{l} \overline{\Omega}(\boldsymbol{C}') \subseteq \overline{\Omega}(\boldsymbol{C}) \\ \underline{\Omega}(\boldsymbol{C}') \subseteq \underline{\Omega}(\boldsymbol{C}) \end{array}\right\} \Rightarrow F(\boldsymbol{C}) \subseteq F(\boldsymbol{C}').$$

- As $S(\mathcal{B}^K)$ is a complete lattice and F is monotone increasing, Tarski (1955) implies the set of fixed points of $F\colon S(\mathcal{B}^K) \to S(\mathcal{B}^K)$ is a complete lattice with respect to $\subseteq$.

Our Approach: Learning about C^*

- Given the definition of C^* as

$$C^* = \text{argmax}_{C \in \mathcal{B}^K} \pi(C)$$

  we know that, for all $k = 1, \dots, J$, it holds

$$\begin{aligned} C_k^* = 0 &\iff \Delta_k(\pi(C^*)) < 0 \iff k \in \overline{\Omega}(\boldsymbol{C}^*) \iff [F(\boldsymbol{C}^*)]_k = 0; \\ C_k^* = 1 &\iff \Delta_k(\pi(C^*)) \geq 0 \iff k \in \underline{\Omega}(\boldsymbol{C}^*) \iff [F(\boldsymbol{C}^*)]_k = 1, \end{aligned}$$

  where $\boldsymbol{C}^*$ denotes the element of $S(\mathcal{B}^K)$ whose only element is equal to C^*.
- Therefore, $F(\boldsymbol{C}^*) = \boldsymbol{C}^*$ and, thus, $\boldsymbol{C}^*$ is a fixed point of F.
- Moreover, as we start the algorithm from $\boldsymbol{C}_0 = \mathcal{B}^K$ and the algorithm converges to a set $\boldsymbol{C}^f$ that is the supremum (given order $\subseteq$) of the fixed points of F, then it must be the case that $\boldsymbol{C}^* \subseteq \boldsymbol{C}^f$ and, thus, $C^* \in \boldsymbol{C}^f$.

Algorithm: Example

- Consider a setting with $K = 3$ and with $s_{12} = 1$, $s_{13} = 0$, and $s_{23} = 0$.
- As $\boldsymbol{C}_0 = \mathcal{B}^K$, then

$$\mathcal{B}^K = \{(0,0,0),(1,0,0),(0,1,0),(0,0,1),(1,1,0),(1,0,1),(0,1,1),(1,1,1)\}.$$

- As a result, $\sup(\boldsymbol{C}_0) = (1,1,1)$ and $\inf(\boldsymbol{C}_0) = (0,0,0)$, and

$$\begin{aligned}
\sup_1(\boldsymbol{C}_0) &= [\cdot, [\sup(\boldsymbol{C}_0)]_2, [\inf(\boldsymbol{C}_0)]_3] = [\cdot, 1, 0], \\
\sup_2(\boldsymbol{C}_0) &= [[\sup(\boldsymbol{C}_0)]_1, \cdot, [\inf(\boldsymbol{C}_0)]_3] = [1, \cdot, 0], \\
\sup_3(\boldsymbol{C}_0) &= [[\inf(\boldsymbol{C}_0)]_1, [\inf(\boldsymbol{C}_0)]_2, \cdot] = [0, 0, \cdot],
\end{aligned}$$

  and similarly

$$\inf_1(\boldsymbol{C}_0) = [\cdot, 0, 1] \qquad \inf_2(\boldsymbol{C}_0) = [0, \cdot, 1] \qquad \inf_3(\boldsymbol{C}_0) = [1, 1, \cdot].$$

Algorithm: Example (Cont.)

- Suppose

$$\{\Delta_1(\pi(\sup_1(\boldsymbol{C}_0))) < 0, \Delta_2(\pi(\sup_2(\boldsymbol{C}_0))) \geq 0, \Delta_3(\pi(\sup_3(\boldsymbol{C}_0))) \geq 0\}$$
$$\{\Delta_1(\pi(\inf_1(\boldsymbol{C}_0))) < 0, \Delta_2(\pi(\inf_2(\boldsymbol{C}_0))) < 0, \Delta_3(\pi(\inf_3(\boldsymbol{C}_0))) \geq 0\}$$

then

$$\overline{\Omega}(\boldsymbol{C}_0) = 1 \qquad \text{and} \qquad \underline{\Omega}(\boldsymbol{C}_0) = 3.$$

- Applying mapping F, we obtain:

$$F(\boldsymbol{C}_0) = \{C \in \boldsymbol{C}_0 : C_1 = 0, C_3 = 1\} = \{(0, 1, 1), (0, 0, 1)\}.$$

Algorithm: Example (Cont.)

- As $\boldsymbol{C}_1 = F(\boldsymbol{C}_0)$, then

$$\boldsymbol{C}_1 = \{(0, 1, 1), (0, 0, 1)\}.$$

- As a result, $\sup(\boldsymbol{C}_1) = (0, 1, 1)$, and $\inf(\boldsymbol{C}_1) = (0, 0, 1)$, and

$$\begin{aligned}
\sup_1(\boldsymbol{C}_1) &= [\cdot, [\sup(\boldsymbol{C}_1)]_2, [\inf(\boldsymbol{C}_1)]_3] = [\cdot, 1, 0], \\
\sup_2(\boldsymbol{C}_1) &= [[\sup(\boldsymbol{C}_1)]_1, \cdot, [\inf(\boldsymbol{C}_1)]_3] = [0, \cdot, 1], \\
\sup_3(\boldsymbol{C}_1) &= [[\inf(\boldsymbol{C}_1)]_1, [\inf(\boldsymbol{C}_1)]_2, \cdot] = [0, 0, \cdot],
\end{aligned}$$

and similarly

$$\inf_1(\boldsymbol{C}_1) = [\cdot, 0, 1] \qquad \inf_2(\boldsymbol{C}_1) = [0, \cdot, 1] \qquad \inf_3(\boldsymbol{C}_1) = [0, 1, \cdot].$$

Algorithm: Example (Cont.)

- Consider two possible cases.
- Case 1. Suppose

$$\{\Delta_1(\pi(\text{sup}_1(\boldsymbol{C}_1))) < 0, \Delta_2(\pi(\text{sup}_2(\boldsymbol{C}_1))) < 0, \Delta_3(\pi(\text{sup}_3(\boldsymbol{C}_1))) \geq 0\}$$
$$\{\Delta_1(\pi(\text{inf}_1(\boldsymbol{C}_1))) < 0, \Delta_2(\pi(\text{inf}_2(\boldsymbol{C}_1))) < 0, \Delta_3(\pi(\text{inf}_3(\boldsymbol{C}_1))) \geq 0\}$$

then

$$\overline{\Omega}(\boldsymbol{C}_1) = \{1, 2\} \qquad \text{and} \qquad \underline{\Omega}(\boldsymbol{C}_1) = 3.$$

- Applying mapping F, we obtain:

$$F(\boldsymbol{C}_1) = \{C \in \boldsymbol{C}_1 : C_1 = 0, C_2 = 0, C_3 = 1\} = (0, 0, 1)$$

and we can conclude that $C^* = (0, 0, 1)$.

Algorithm: Example (Cont.)

- Case 2. Suppose

$$\{\Delta_1(\pi(\mathrm{sup}_1(\boldsymbol{C}_1))) < 0, \Delta_2(\pi(\mathrm{sup}_2(\boldsymbol{C}_1))) \geq 0, \Delta_3(\pi(\mathrm{sup}_3(\boldsymbol{C}_1))) \geq 0\}$$
$$\{\Delta_1(\pi(\mathrm{inf}_1(\boldsymbol{C}_1))) < 0, \Delta_2(\pi(\mathrm{inf}_2(\boldsymbol{C}_1))) < 0, \Delta_3(\pi(\mathrm{inf}_3(\boldsymbol{C}_1))) \geq 0\}$$

then

$$\overline{\Omega}(\boldsymbol{C}_1) = 1 \qquad \text{and} \qquad \underline{\Omega}(\boldsymbol{C}_1) = 3.$$

- Applying mapping F, we obtain:

$$F(\boldsymbol{C}_1) = \{C \in \boldsymbol{C}_1 : C_1 = 0, C_3 = 1\} = \{(0,1,1),(0,0,1)\} = \boldsymbol{C}_1.$$

Thus, $\boldsymbol{C}_1$ is a fixed point of F and we can conclude that $C^* \in \{(0,1,1),(0,0,1)\}$.

Applying Solution Algorithm To Our Model

Solving the Model: Mapping 1

- Fix $K = 2 \times N \times J_i$ and define the lattice $(S(\mathcal{B}^K), \subseteq)$.
- We can then use the mapping $F\colon S(\mathcal{B}^K) \to S(\mathcal{B}^K)$ described in slides 20 and 21 to find a sublattice of $\mathcal{B}^K$ that includes the vector (I_i^*, D_i^*), defined as

$$(I_i^*, D_i^*) = \text{argmax}_{(I_i, D_i) \in \mathcal{B}^K} \Pi_i(I_i, D_i).$$

- Only assumption needed to apply our algorithm is that in slide 12: the sign of all cross-partials is known and independent of the vector (I_i, D_i) at which those are evaluated.
- See slide 10 for a description of the sign of all cross-partials in our model.

Solving the Model: Mapping 2

- To introduce this second mapping, consider first a firm i that is single-product. Then, $D_i = \{D_{in}\}_n$ and $I_i = \{I_{in}\}_n$, and we can write

$$D_i^* = \text{argmax}_{D_i \in \mathcal{B}^N} V_i(D_i) \qquad \text{with} \qquad V_i(D_i) = \max_{I_i \in \mathcal{B}^N} \Pi_i(I_i, D_i),$$

- Crucially, it is computationally feasible to solve exactly for the optimal I_i given D_i:

$$V_i(D_i) = \sum_{n=1}^{N} \max_{I_{in} \in \{0,1\}} I_{in}(\pi_{in}(D_i) - F_{in}^e) - \sum_{o=1}^{N} D_{io} F_{io}^p$$

- As $V_i(D_i)$ has negative cross-partials between any two coordinates o and o', we can use our algorithm (or that in Arkolakis et al., 2024) to obtain bounds $\overline{D}_i$ and $\underline{D}_i$ on D_i^*.
- Defining $\bar{I}_i := \text{argmax}_{I_i \in \mathcal{B}^N} \Pi_i(I_i, \overline{D}_i)$, as $\Pi_i(I_i, D_i)$ has positive cross-partials between any two coordinates o and n, it holds $\bar{I}_i \geq I_i^*$. Similarly, if $\underline{I}_i := \text{argmax}_{I_i \in \mathcal{B}^N} \Pi_i(I_i, \underline{D}_i)$, we have $\underline{I}_i \leq I_i^*$. Then, for single-product firms, mapping 2 also yields bounds on I_i^*.

Solving the Model: Mapping 2

- Consider now on a firm i that is multi-product and, for any $j \in M_i$, define

$$V_{ij}(D_{ij}, I_{i(-j)}, D_{i(-j)}) :=$$

$$\sum_{n=1}^{N} \max_{I_{ijn}\in\{0,1\}} I_{ijn}(\Delta_j \pi_{in}(D_{ij}, I_{i(-j)}, D_{i(-j)}) - F^e_{ijn}) + \mathcal{K}(D_{ij}, I_{i(-j)}, D_{i(-j)}),$$

  with $\Delta_j \pi_{in}(\cdot)$ firm i's change in profits in country n depending on whether product j is offered, and $\mathcal{K}(D_{ij}, I_{i(-j)}, D_{i(-j)})$ the firm's profits if $I_{ijn} = 0$ for all $j = 1, \dots, J_i$.
- Proposition 1: $D^*_{ij} = \text{argmax}_{D_{ij}\in\mathcal{B}^N} V_{ij}(D_{ij}, I^*_{i(-j)}, D^*_{i(-j)})$.
- Proposition 2: For any $(D_{ij}, I_{i(-j)}, D_{i(-j)})$, the function $V_{ij}(D_{ij}, I_{i(-j)}, D_{i(-j)})$ has negative cross partials between coordinates ijo and ijo' for any $o \neq o'$.
- Proposition 3: For any $(D_{ij}, I_{i(-j)}, D_{i(-j)})$, the function $V_{ij}(D_{ij}, I_{i(-j)}, D_{i(-j)})$ has negative cross partials between coordinates ijo and $ij'o'$ for any $j \neq j'$ and between ijo and $ij'n$ for any $j \neq j'$.

Solving the Model: Mapping 2

- Given any $(I_{i(-j)}, D_{i(-j)})$, Proposition 2 implies we can use our algorithm to bound

$$D^*_{ij}(I_{i(-j)}, D_{i(-j)}) := \text{argmax}_{D_{ij} \in \mathcal{B}^N} V_{ij}(D_{ij}, I_{i(-j)}, D_{i(-j)}).$$

- Propositions 1 and 3 imply that

$$D^*_{ij}(\overline{I}_{i(-j)}, \overline{D}_{i(-j)}) \leq D^*_{ij} \leq D^*_{ij}(\underline{I}_{i(-j)}, \underline{D}_{i(-j)}),$$

for any $\overline{I}_{i(-j)}$, $\overline{D}_{i(-j)}$, $\underline{I}_{i(-j)}$, and $\underline{D}_{i(-j)}$ such that

$$\underline{I}_{i(-j)} \leq I^*_{i(-j)} \leq \overline{I}_{i(-j)},$$
$$\underline{D}_{i(-j)} \leq D^*_{i(-j)} \leq \overline{D}_{i(-j)}.$$

- Even for multi-product firms, Mapping 2 helps us find bounds on D^*_{ij} for any ij.

Solving the Model: Combining Mappings 1 and 2

1. Fix $(\underline{I}, \underline{D}) = (\mathbf{0}, \mathbf{0})$ and $(\bar{I}, \overline{D}) = (\mathbf{1}, \mathbf{1})$.
2. Use Mapping 2 to obtain bounds on D^*_{i1}; i.e., production locations for model $j = 1$.
3. Use Mapping 1 to obtain bounds on I^*_{i1} given bounds on D^*_{i1}, $(\underline{I}_{i(-1)}, \underline{D}_{i(-1)}) = (\mathbf{0}, \mathbf{0})$ and $(\bar{I}_{i(-1)}, \overline{D}_{i(-1)}) = (\mathbf{1}, \mathbf{1})$.
4. Use Mapping 2 to obtain bounds on D^*_{i2} given bounds on D^*_{i1} and I^*_{i1} from steps 2 and 3, and given bounds $(\underline{I}_{i(-\{1,2\})}, \underline{D}_{i(-\{1,2\})})) = (\mathbf{0}, \mathbf{0})$ and $(\bar{I}_{i(-\{1,2\})}, \overline{D}_{i(-\{1,2\})})) = (\mathbf{1}, \mathbf{1})$.
5. Use Mapping 1 to obtain bounds on I^*_2 given previous bounds on (D^*_1, I^*_1, D^*_2) and given bounds $(\underline{I}_{-\{1,2\}}, \underline{D}_{-\{1,2\}}) = (\mathbf{0}, \mathbf{0})$ and $(\bar{I}_{-\{1,2\}}, \overline{D}_{-\{1,2\}}) = (\mathbf{1}, \mathbf{1})$.
6. Loop over all models $j = 1, \ldots, M_{ij}$.
7. Iterate steps 2 to 6 until convergence of D^*_{ij} for all $j = 1, \ldots, M_{ij}$.

Multinational Firms in Global Car Industry: Summary Statistics

Data

- Data on global car industry:
    - Source: IHS Markit (Cosar et al. 2018, Head and Mayer 2019, Alcott et al., 2024).
    - Year: 2019.
    - New car registrations: information by model (1,245) on brand-segment (375), production (assembly) country (53) and registration country (77).
    - Model price, quantity sold, and characteristics for Australia, Brazil, China, Spain, France, Germany, UK, India, Italy, Japan, Mexico, and the US.
- Other sources of data:
    - CEPII: geographical distance between countries.
    - MacMap: car tariffs.
    - World Bank: Income per capita and population per country.

Summary Statistics

MNEs are multi-product, with each product produced & sold in several countries.

Number of:	Mean (unweigh.)	Mean (weight.)	p25	p50	p75	p90	Max
Models (per brand-segment)	3.5	10.2	1	2	5	8	23
Sales countries (per model)	12.1	30.7	1	2	17	43	75
Production countries (per model)	1.5	3.1	1	1	1	3	12

Other Statistics

Number of:	Mean (unweigh.)	Mean (weight.)	p25	p50	p75	p90	Max
Models sold (per country)	41.4	137.9	11	31	65	102	387
Sales countries (per brand-segment)	18.9	49.8	1	5	40	62	77
Production countries (per brand-segment)	2.4	8.1	1	1	2	5	19
Share exported (by model-prod. loc.)	38.7%	12.9%	0%	6.4%	94%	100%	100%
Share produced in HQs (by brand-segment)	80.2%	44.1%	63.2%	100%	100%	100%	100%
Share sold in HQs (by brand-segment)	61.2%	30.9%	19.9%	78.7%	100%	100%	100%

Estimation

Estimation Strategy

- Trade costs and marginal production costs parameters.
  - Price equation.
- Demand function parameters.
  - Revenue share equations: model within brand-segment, brand-segment within country.
- Fixed cost parameters.
  - Moment inequalities.

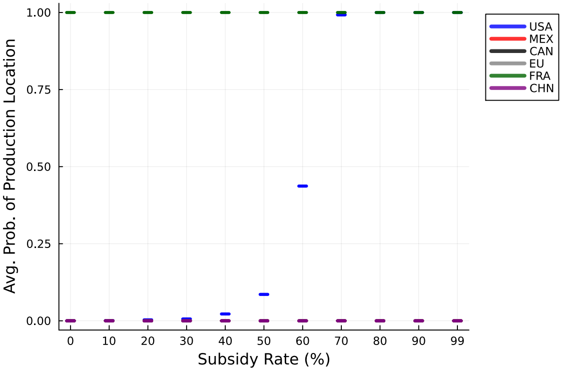
Quantification: 2-Product Firms Comparative Statics

Quantification: 2-Product Firm

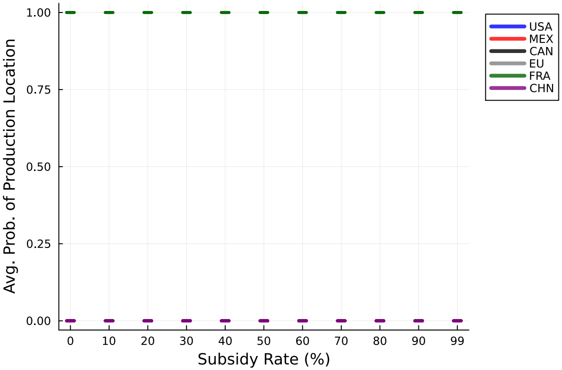
- One brand-segment: Peugeot-wagon.
- Two models: Peugeot 308 (non-electric) and Peugeot 508 (electric).
- Initial firm choices:
  - Both models produced only in France.
  - Neither model sold in the US.
- Match initial production choices and explore impact of US policies as a function of initial sales choices.
- US policies:
  - Production subsidy (to marginal costs) for electric model in the US.
  - Consumption subsidy for electric model in the US.

Production Subsidy for Electric Model in the US

Production Location



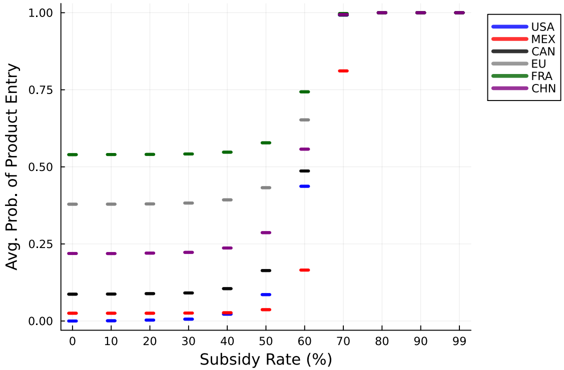
(a) Electric



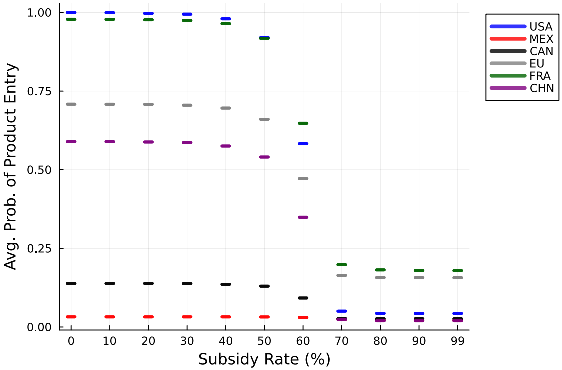
(b) Non-Electric

Production Subsidy for Electric Model in the US

Product Entry



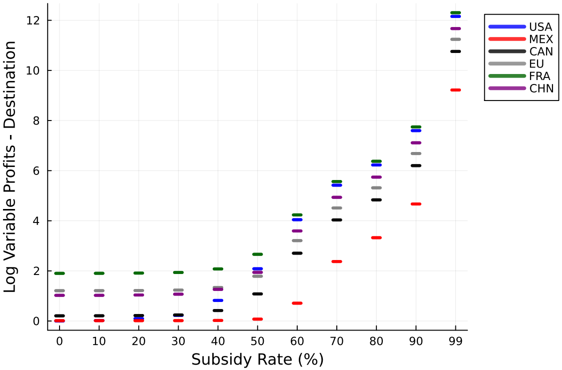
(a) Electric



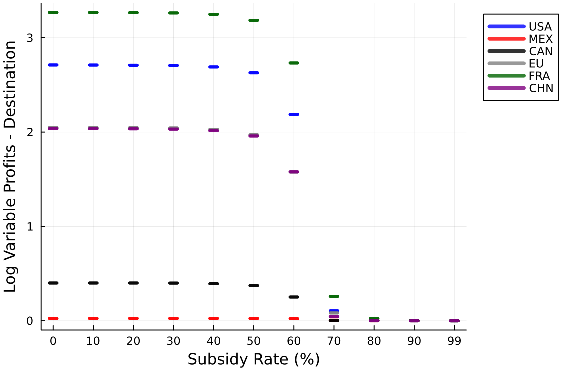
(b) Non-Electric

Production Subsidy for Electric Model in the US

Sales by Destination



(a) Electric



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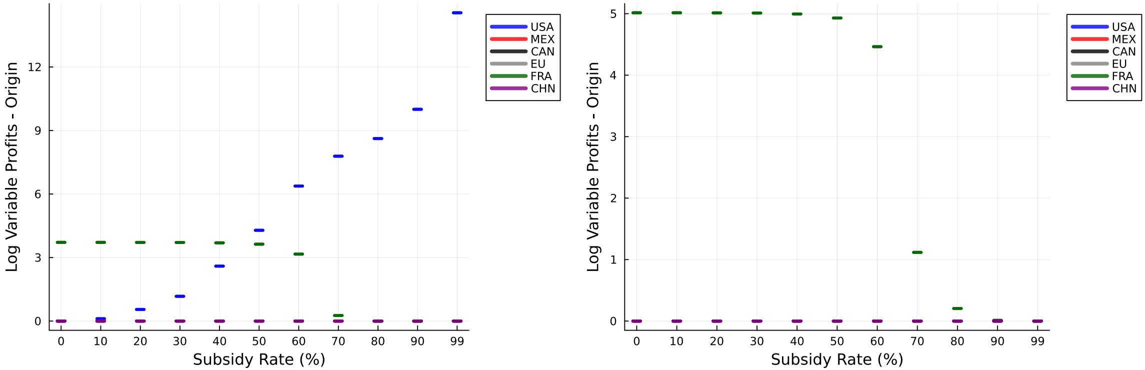
Production Subsidy for Electric Model in the US

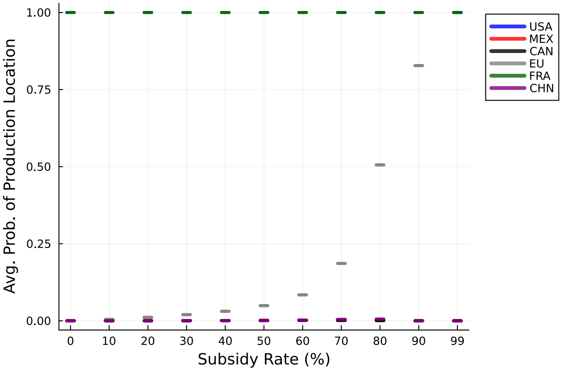
Sales by Origin



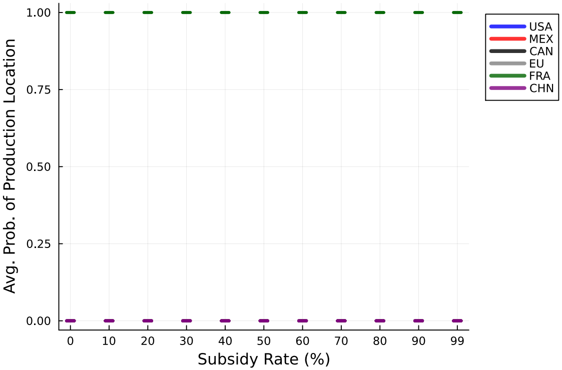
(a) Electric (b) Non-Electric

Consumption Subsidy for Electric Model in the US

Production Location



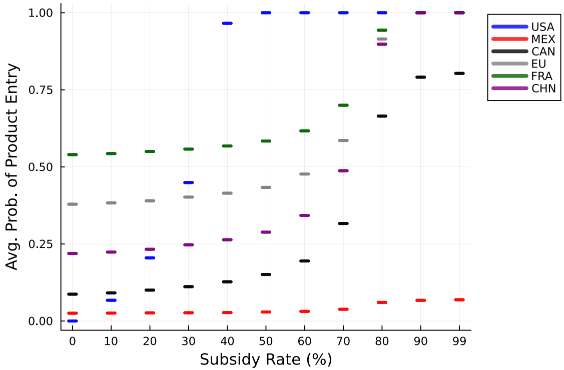
(a) Electric



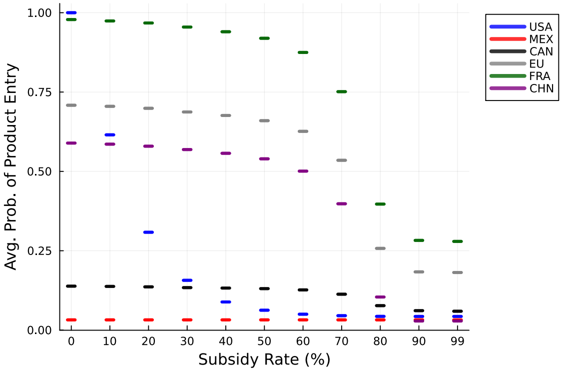
(b) Non-Electric

Consumption Subsidy for Electric Model in the US

Product Entry



(a) Electric



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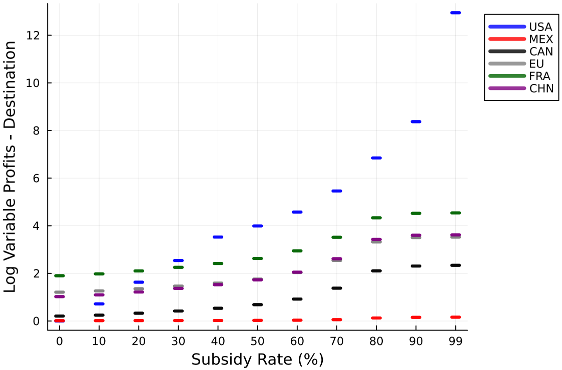
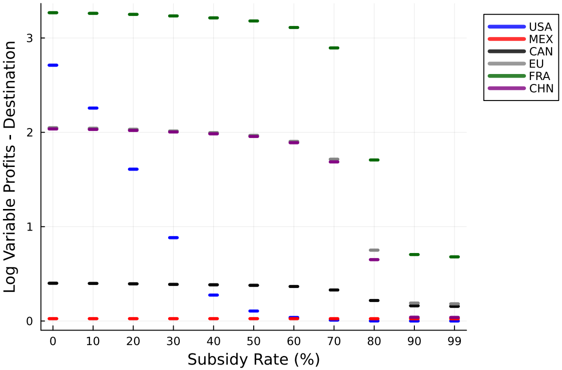
Consumption Subsidy for Electric Model in the US

Sales by Destination



(a) Electric



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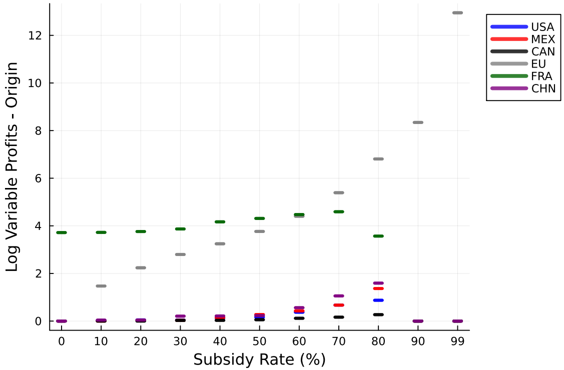
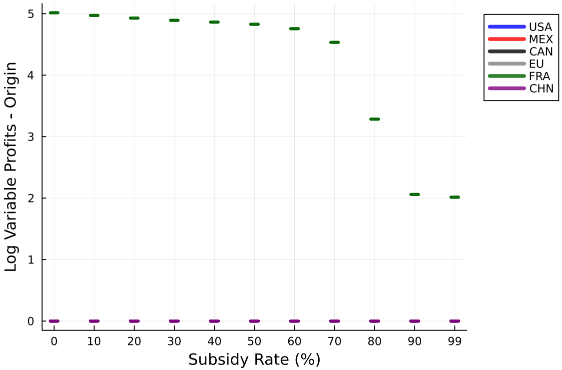
Consumption Subsidy for Electric Model in the US

Sales by Origin



(a) Electric



(b) Non-Electric

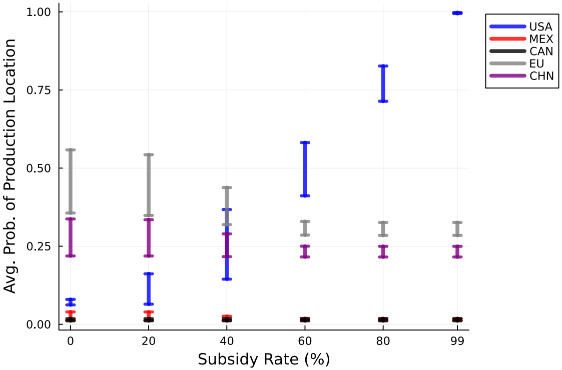
Quantification: Aggregate Policy Effects

Aggregate Counterfactual Policy Experiments

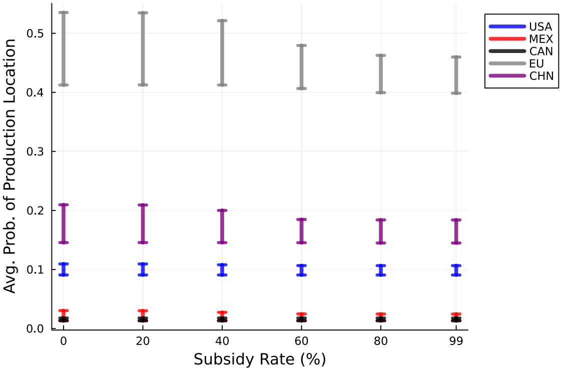
- 128 is drawn from the distributions of $\{\nu^e_{ijn}\}$, $\{\nu^p_{ijo}\}$, and $\{d_{ijo}\}$.
- Solve model for all segments and firms for each draw, each parameter in confidence set, and each value of the policy parameters we consider:
  - Production and Consumption Subsidies in the US favoring Electric Vehicles.
  - Tariffs to World and to the EU on Electric Vehicles.
- Report bounds by averaging over fixed cost and lottery draws.
- Bounds reflect:
  1. Parameter uncertainty (as reflected in confidence sets).
  2. Solution uncertainty

Production Subsidy on Electric Products in the US

Production Location



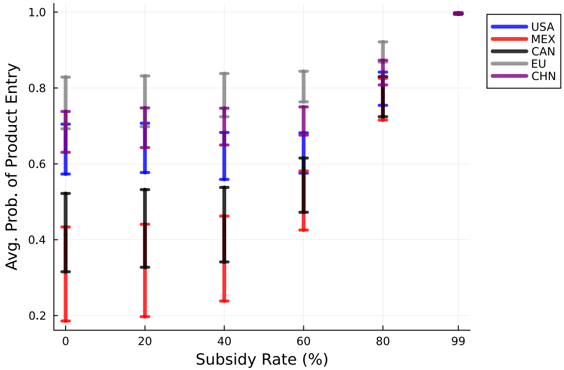
(a) Electric



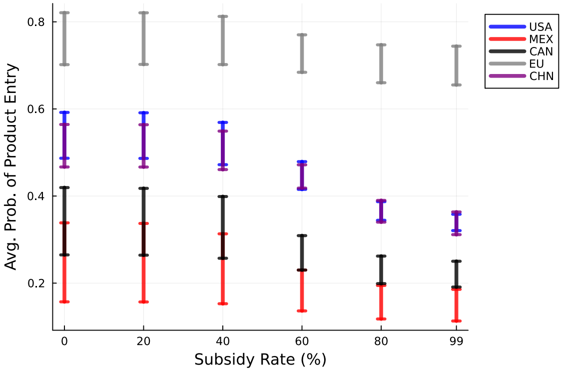
(b) Non-Electric

Production Subsidy on Electric Products in the US

Product Entry



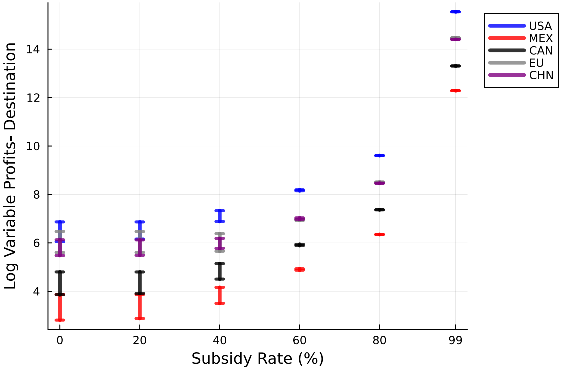
(a) Electric



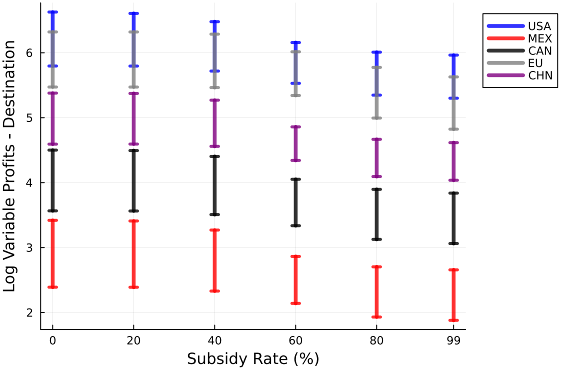
(b) Non-Electric

Production Subsidy on Electric Products in the US

Variable Profits - Destination



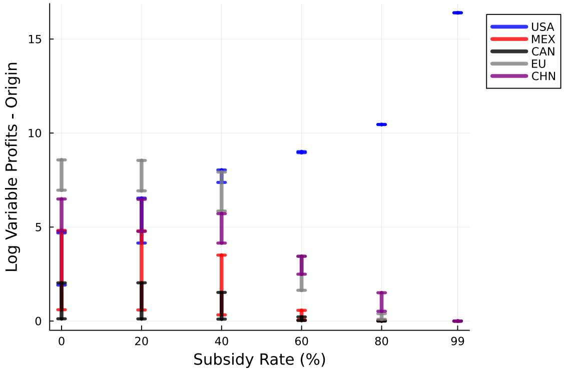
(a) Electric



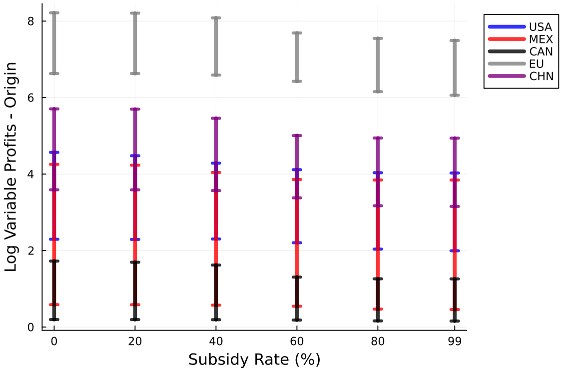
(b) Non-Electric

Production Subsidy on Electric Products in the US

Variable Profits - Origin



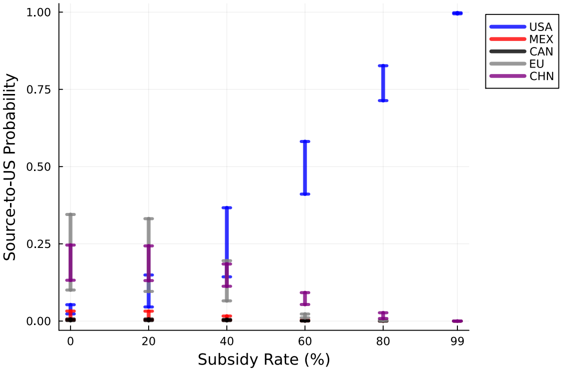
(a) Electric



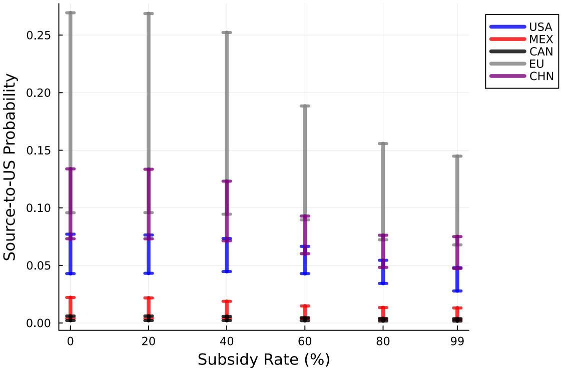
(b) Non-Electric

Production Subsidy on Electric Products in the US

Probability of Sourcing to the US



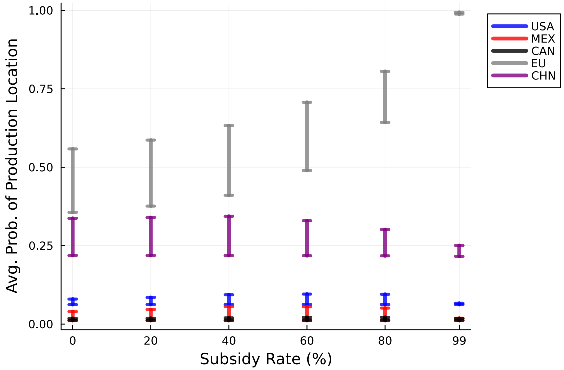
(a) Electric



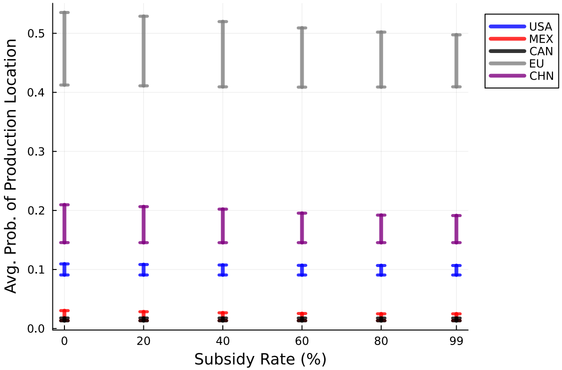
(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Production Location



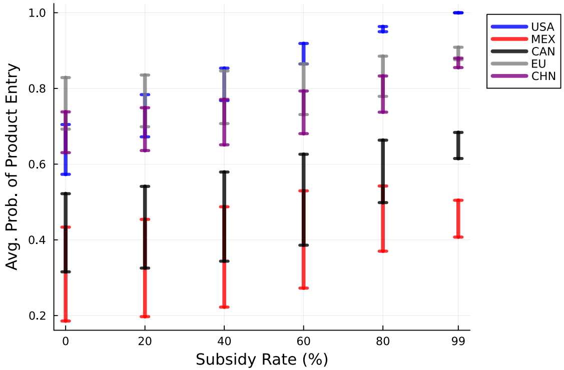
(a) Electric



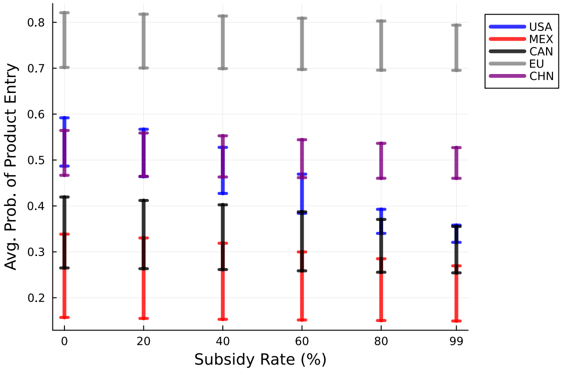
(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Product Entry



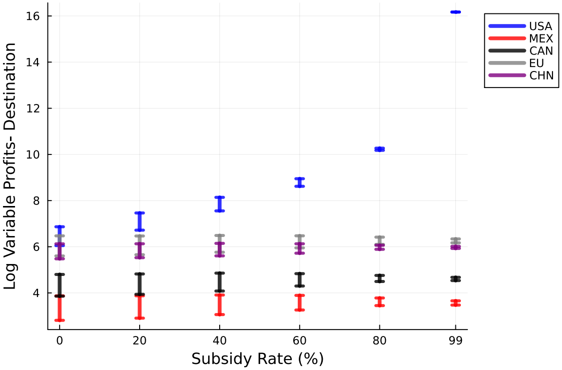
(a) Electric



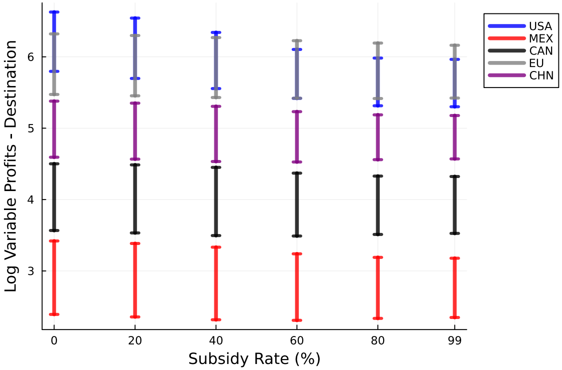
(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Variable Profits - Destination



(a) Electric



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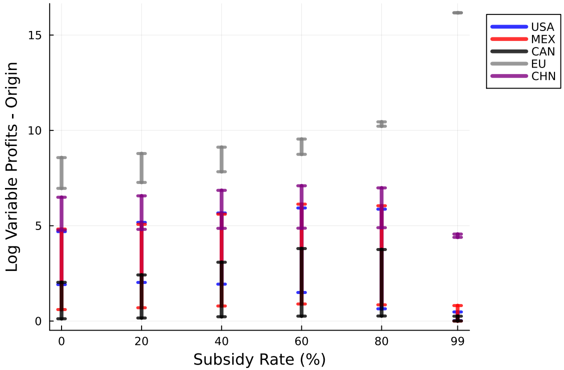
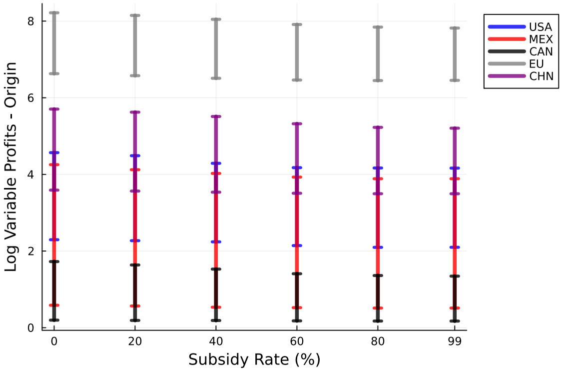
Consumption Subsidy on Electric Products in the US

Variable Profits - Origin



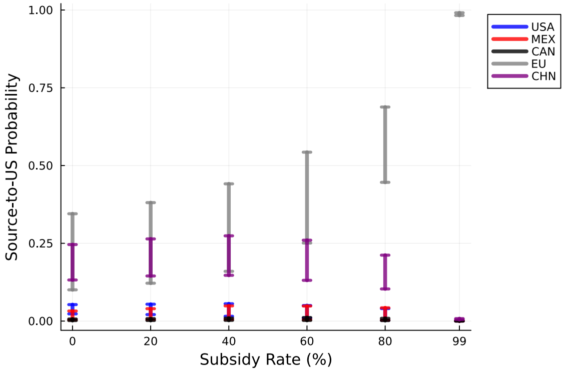
(a) Electric



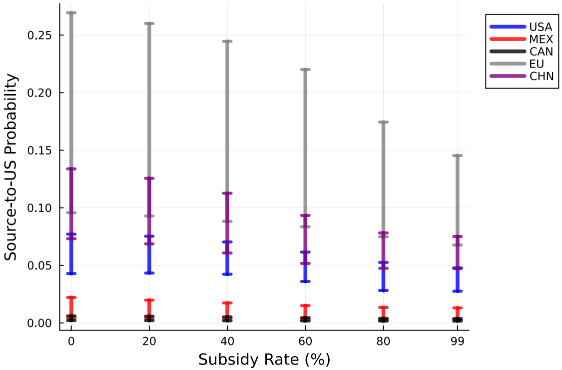
(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Probability of Sourcing to the US



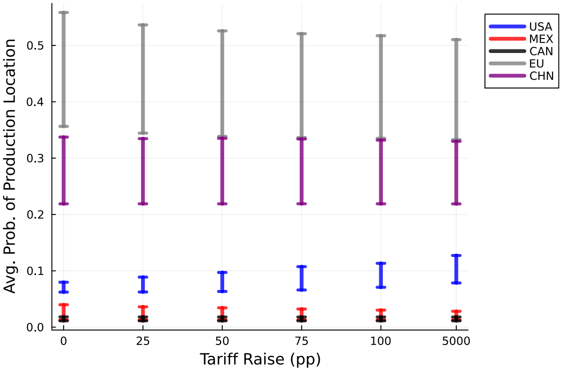
(a) Electric



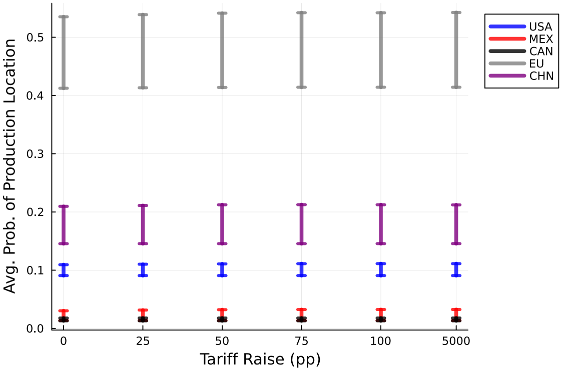
(b) Non-Electric

Tariffs on Electric Products Not Produced in the US

Production Location



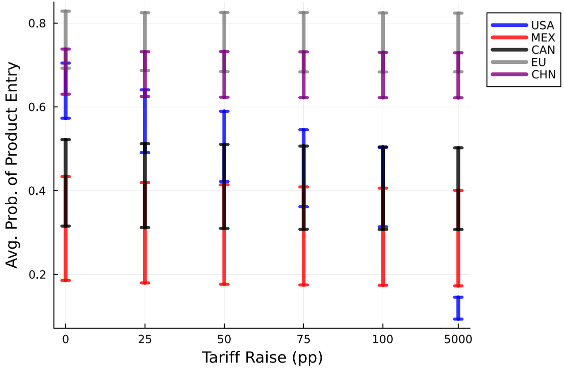
(a) Electric



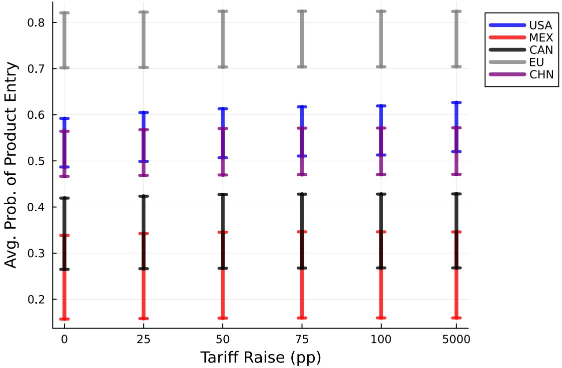
(b) Non-Electric

Tariffs on Electric Products Not Produced in the US

Product Entry



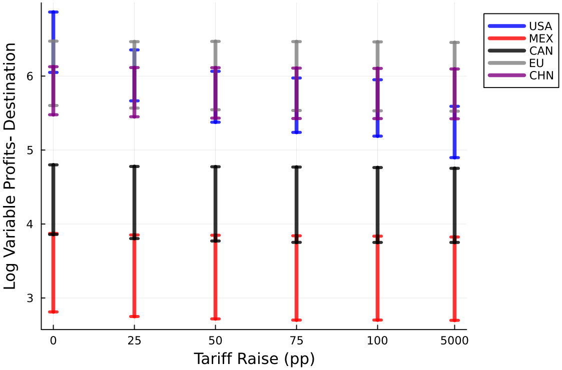
(a) Electric



(b) Non-Electric

Tariffs on Electric Products Not Produced in the US

Variable Profits - Destination



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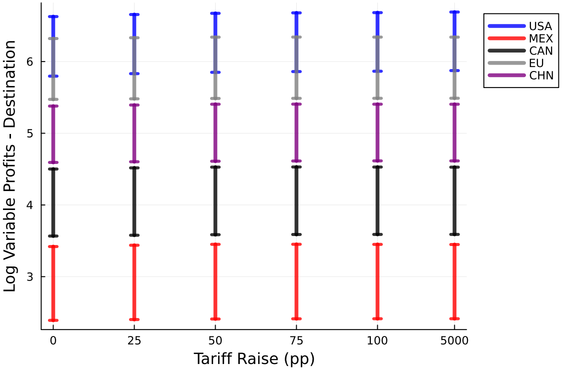

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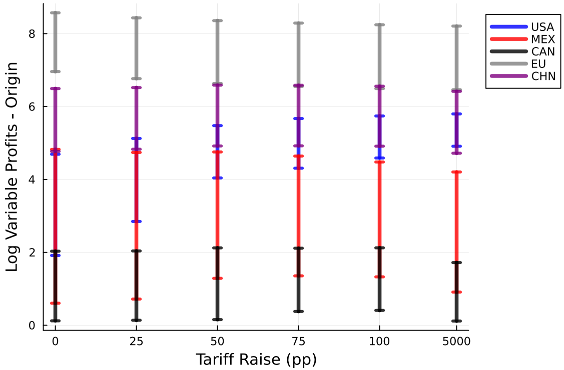
Tariffs on Electric Products Not Produced in the US

Variable Profits - Origin



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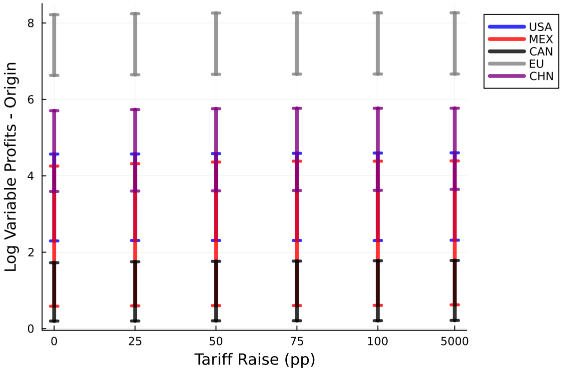
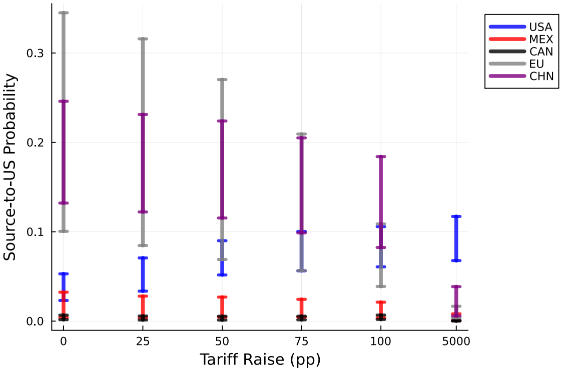


(b) Non-Electric

Tariffs on Electric Products Not Produced in the US

Probability of Sourcing to the US



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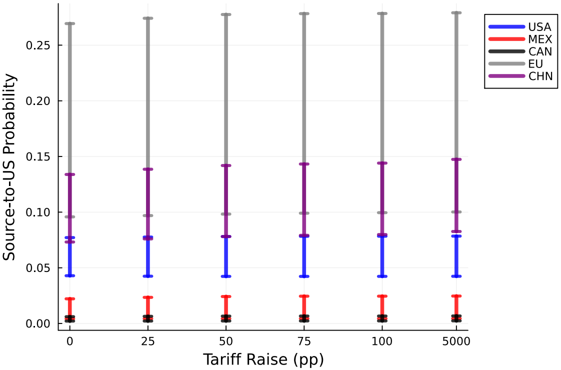
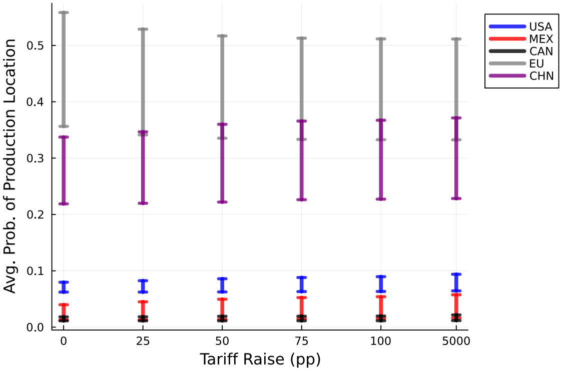


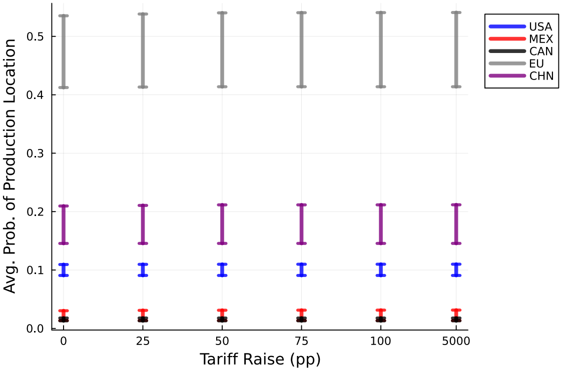
(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Production Location



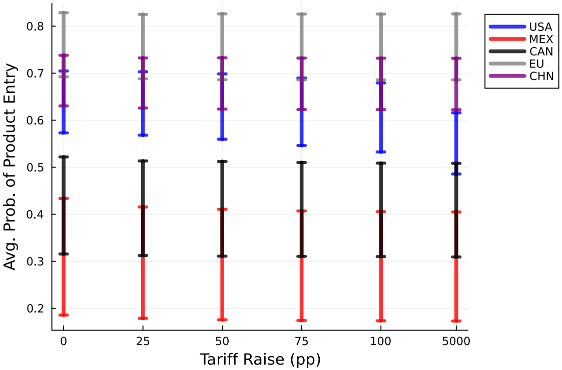
(a) Electric



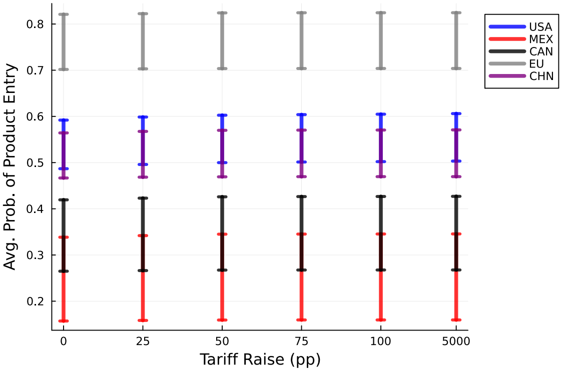
(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Product Entry



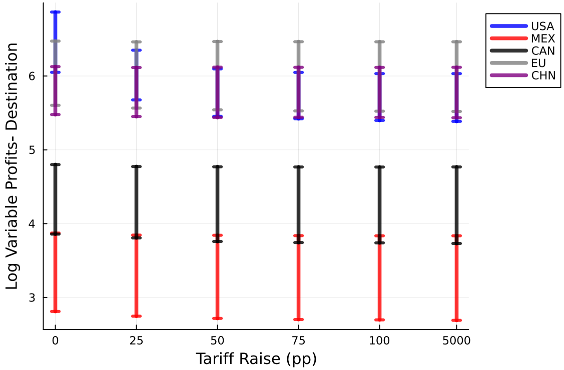
(a) Electric



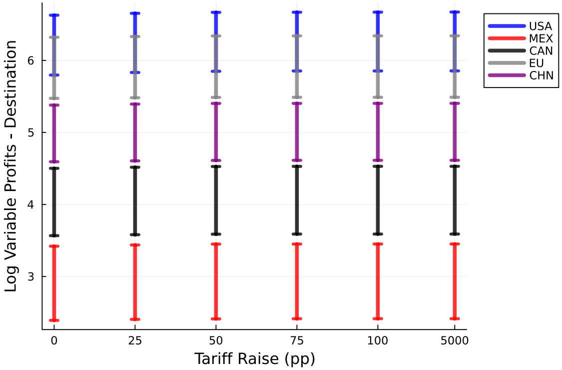
(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Variable Profits - Destination



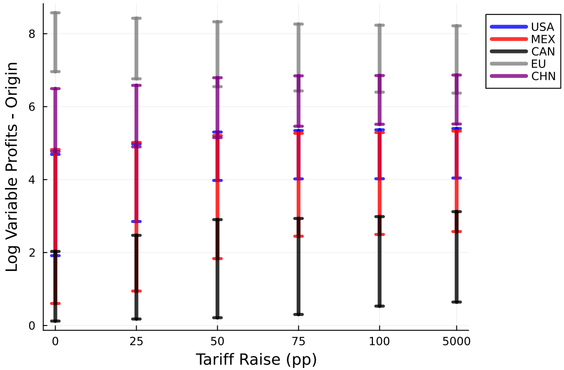
(a) Electric



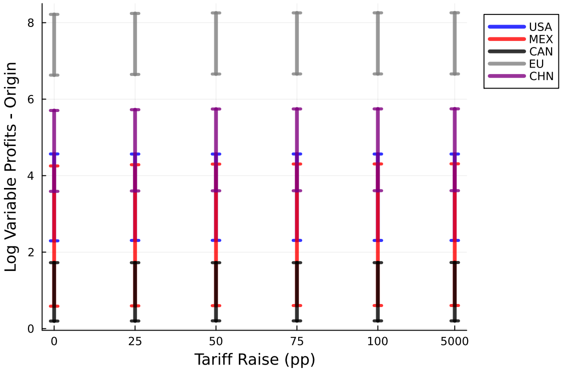
(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Variable Profits - Origin



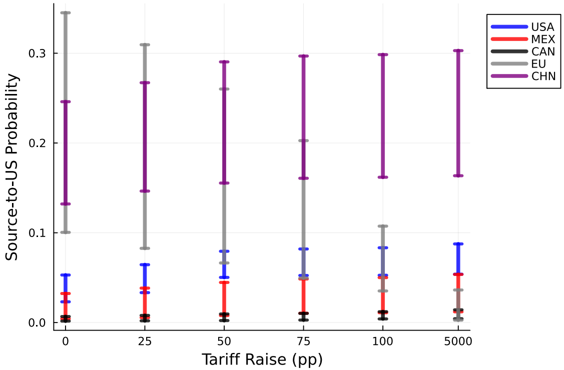
(a) Electric



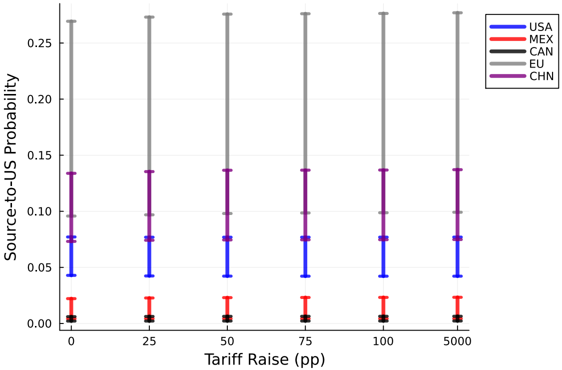
(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Probability of Sourcing to the US



(a) Electric



(b) Non-Electric

Conclusion

- Model of multi-product, multi-plant, and multi-market firms.
- Novel algorithm for CDCPs with complementarities and substitutabilities.
- Algorithm requires that, for any two coordinates, the sign of the cross-partial is known and independent of third choices.
- Moment inequalities to use algorithm in estimation.
- Evaluate firm-level responses to consumption and production subsidies, and tariffs.