Product Entry in the Global Automobile Industry

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Abstract

Changes in product offerings are important for understanding changes in market outcomes in the automobile industry. In a global industry, national subsidies affect global market outcomes through firms' product portfolio decisions. This paper proposes a new model to study product entry in a multi-market setting with product differentiation and heterogeneous consumer preferences within and across countries. Methodologically, the contribution of this paper is to provide a method to estimate and solve entry games with multiple asymmetric firms, each making multiple discrete choices. Using data on global passenger vehicle sales, prices, and product characteristics, I estimate large overhead product line costs, which imply that firms have strong incentives to offer the same product in different markets to achieve sufficient scale. I quantify the effects of discriminatory production and consumption subsidies favoring US brands in the United States. I find that global product portfolio changes in response to these policies cause profit shifting towards US brands worldwide and that changes in consumer surplus vary greatly across markets due to heterogeneity in preferences.

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1 Introduction

The automobile sector consists of a relatively stable set of firms offering an evolving mix of products throughout the globe. The number of active automobile firms in the United States barely changed between 1980 and 2018, while the number of products offered doubled (Grieco et al. 2023). Given the recent resurgence of trade and industrial policy, an important topic in policy discussions is the extent to which the large consumer and producer subsidies outlined in policy packages like the Inflation Reduction Act or the European Green Deal will affect the structure of the industry, and thus which firms and consumers will benefit throughout the world. Yet, despite the apparent importance of product entry in an industry subject to significant government interventions, little is known about its role in determining the effect of national policies on global market outcomes.

This paper studies how national government policies affect firms' global product offerings and consumer and firm-level outcomes throughout the world. In the model, firms choose their global product portfolios to maximize profits, given heterogeneous consumer preferences and policies across countries. Thus, national policies affect global market outcomes through firms' product portfolio decisions. In this setting, firms face multiple interdependent choices. First, because developing differentiated products is costly, firms have an incentive to offer them in multiple countries so as to ensure sufficient scale. Second, firms know that offering an additional product in a country can reduce or cannibalize demand for its other products. To deal with a model incorporating these features tractably, I derive inequalities that bound the probabilities of firms' entry decisions and show how to use them to estimate the model and solve it under different policy scenarios.

The framework I propose allows for heterogeneity in preferences across product characteristics as in Berry et al. (1995) and incorporates both product portfolio and market entry decisions. The model features a finite set of firms, each endowed with a set of potential products. Firms make portfolio and market entry decisions in a sequential game with three stages. In the first stage, firms choose which subset of their potential products to include in their global product portfolio, subject to a fixed cost per product line. In the second stage, firms choose which subset of their portfolio to offer in each market, subject to market entry fixed costs. In the third and final stage, firms set prices for each product in each market.

Methodologically, the key contribution of this paper is to show how to estimate fixed costs and compute the impact of counterfactual policies in entry games with multiple asymmetric firms, each making multiple discrete choices. Two main challenges arise in such settings. First, the existence and uniqueness of Nash equilibria are not guaranteed. Second, solving for the equilibria of the model may be computationally infeasible, particularly in settings with a large number of heterogeneous players, each with a large set of potential choices, as in the automobile industry.

I estimate fixed costs and solve for counterfactual outcomes using novel inequalities that bound the probabilities of firms' portfolio and market entry decisions. Firms' portfolio and market entry fixed costs depend on an observed and an unobserved component to the researcher; I refer to the latter as fixed cost shocks. The inequalities are derived under two assumptions: unobserved rival fixed cost shocks and submodularity of variable profits with respect to product offerings. A benefit of the first property is that it guarantees the existence of a pure strategy Nash equilibrium, which is not ensured with complete information. The second property implies that the change in variable profits from offering a product in a market declines with the set of offered products.[1] I apply submodularity to derive bounds on the gains from offering a product in a market, which I use to derive necessary conditions for entry that hold across all equilibria. Fan and Yang (2024) also employ submodularity to derive necessary conditions for entry, though in a complete rather than incomplete-information framework. Such conditions permit integrating the unobserved component of the fixed cost shocks, under any assumed distribution, to obtain bounds on firms' choice probabilities that depend on the fixed cost parameters and firms' expectations over rivals' actions.[2]

For estimation, I further bound firms' choice probabilities to derive moment inequalities that depend only on fixed cost parameters and observed data. My moment inequalities are valid under equilibrium multiplicity, as are those in Ciliberto and Tamer (2009). Compared to their approach, my method does not require computing the equilibria of the model under any set of parameters, making it computationally feasible to implement even in games with a large number of discrete choices.[3] I follow two additional steps to derive moment inequalities. First, I use convex upper and concave lower bounds on the fixed-cost CDF. This relates to Dickstein and Morales (2018), Dickstein et al. (2024), and Porcher et al. (2024), who use convex odds functions or linear approximations to derive moment inequalities in a single-agent setting, though my approach directly bounds choice probabilities in a game. Then, I apply Jensen's inequality to construct moments that depend on the observed realization of rivals' offerings. This extends the insights from Pakes (2010) and Dickstein and Morales (2018) to an incomplete-information game, where firms' expectations are over rivals' endogenous entry decisions rather than over exogenous ex-post realized variables.

My moment inequalities have two features that are desirable for estimation in entry games. First, they identify both the mean and variance of the distributions of fixed costs, which is sufficient to characterize the distribution fully under commonly used specifications.[4] This is an advantage relative to common approaches in multi-product entry games that only identify the mean and not the full fixed cost distribution and, therefore, complicate the simulation of counterfactual outcomes. Second, assuming that firms do not observe their rivals' fixed cost shocks allows using the observed realization of firms' entry choices for estimation, which I show renders the moment inequalities informative about the fixed cost parameters both in simulations and in practice.

To evaluate the effects of policies given estimated parameters, I develop a novel solution algorithm for multi-product entry games. My solution algorithm has two main advantages relative to existing approaches that assume complete information on firms' fixed cost shocks. First, it provides bounds on the equilibrium distribution of entry decisions even in settings with multiple asymmetric

[1]The change in variable profits from offering a product in a given market when the set of products offered in the market is Ω is smaller than when the set of products offered in that market is $\Omega' \subseteq \Omega$.

[2]Submodularity is stronger than required for estimation, but plays a more critical role in the method for solving the model.

[3]With only 10 firms each making 10 discrete choices, there are $2^{100} \approx 10^{30}$ possible entry configurations. It is computationally infeasible to compute all of them.

[4]For instance, commonly used specifications include the log-normal, normal, or logistic distributions.

oligopolistic firms.[5] Second, my solution approach does not rely on approximation methods or equilibrium selection assumptions and bounds, in a computationally feasible manner, any equilibrium of the entry game. I find informative bounds on counterfactual outcomes in practice.

The solution method operates as follows. I start by evaluating changes in profits for each product at the most competitive conditions (all potential products are simultaneously offered) and least competitive conditions (no other potential product is offered). This yields weak upper and lower bounds on the probability that any product is offered in a market. I then show how to simulate tighter upper bounds by evaluating the change in expected profits from offering a product using the initialized *lower* bounds on rival product offering probabilities. Similarly, I simulate tighter lower bounds by evaluating the change in expected profits using the initialized *upper* bound rival offering probabilities. I prove that iterating on this procedure yields monotonically tighter bounds on the joint equilibrium probability distribution of offerings in each country, which I then use to compute bounds on market outcomes of interest such as consumer surplus.[6]

I estimate the model with 2019 *IHS Markit* data on the universe of new passenger vehicle registrations in a representative set of countries. I complement this main dataset with gravity variables from CEPII; PPP and Gini data from the World Bank; and the MRI Simmons 2019 US Crosstab Report. The latter provides information relating vehicle characteristics to buyers' demographics. With these cross-sectional data, I estimate a heterogeneous agent demand model similar to Berry et al. (1995) for passenger vehicles using micro-moments as in Petrin (2002).

Demand and marginal cost estimates yield average own-price elasticities and markups consistent with previous estimates in the automotive industry (i.e., Berry et al. 1995, Grieco et al. 2023). I find substantial heterogeneity in distaste for high prices across the income distribution and a considerable home market bias, as in Coşar et al. (2017); consumers are willing to pay on average over $1500 to purchase from a local brand, all else equal. My marginal cost estimates reveal a positive relationship between cost and distance to the brand's headquarters country, as well as larger costs of producing vehicles that are bigger or have greater horsepower.

Using my method, I obtain fixed cost estimates that reveal important economies of scale at the product level. The estimates imply that, with 95% confidence, the median fixed cost of maintaining a product line is between $138-548 million, while the median fixed market entry cost for a product is $8-15 million.[7] The large quantitative difference between the product and market entry costs implies important interdependence across markets arising from scale economies. High product level costs mean that firms have an incentive to offer similar bundles of products across markets to ensure sufficient scale. I estimate additional parameters that describe the variance of the distributions of fixed costs. These estimates reveal significant unobserved heterogeneity in market and product fixed

[5] Jia (2008) also uses a procedure to bound the Nash equilibria in a duopolistic entry game, but the procedure in that paper is not valid when there are more than two players.

[6] The solution method bounds the equilibrium distribution of product offerings in each market in the sense of first-order stochastic dominance. The approach is reminiscent of solutions methods that iteratively eliminate dominated strategies, albeit in a setting of incomplete rather than complete information.

[7] My product fixed cost estimates align with industry estimates of product development costs in the automobile industry provided by IHS Global.

costs, which confirm the importance of allowing for terms observed by firms but not by researchers.

With fixed cost estimates in hand, I use my solution algorithm to explore the effects of national policies on global consumer and firm-level outcomes. I conduct two experiments relevant to the current policy landscape. First, recognizing recent large policy packages such as the Inflation Reduction Act that disproportionately favor production of domestic over foreign brands, I study the effect of a policy that reduces the marginal cost for US brands by 20%.[8] Second, given the rise of protectionist consumer-side subsidies favoring domestic over foreign brands, I study the effects of a 50% consumption subsidy favoring US brands in the United States. Large consumer-side subsidies of similar magnitudes have been implemented in several countries to favor some vehicle types over others.[9] These policy experiments showcase the importance of product entry in determining the global effects of currently discussed policies in the automotive industry, particularly when the policy is implemented in large markets like the United States.

The first policy, which reduces American brands' costs by 20%, affects global market outcomes both through the intensive margin (prices/quantities) and the extensive margin (products offered). The policy increases American brand dominance throughout the globe, with American brands' market shares rising by at least 3 percentage points in Japan or over 13 percentage points in the UK. American brands' variable profits also rise in all markets, with near threefold increases in multiple countries. Meanwhile, the effect on non-American brands is the opposite. Increased competition from cheaper American products tends to reduce their profits and market shares worldwide.

A key finding is that endogenous product portfolio adjustments amplify the increase in American brand dominance following the policy. Intuitively, American brands expand their product portfolios in anticipation of greater profits, while non-American brands downsize their product offerings, knowing they will be relatively less competitive. I find that across many jurisdictions, ignoring endogenous product portfolio adjustments would lead to significantly underestimating the rise in US brand shares and profits. For instance, across most markets, product entry accounts for over 25% of the increase in the lower and upper bound of US-brand market shares. This shows that accounting for product portfolio adjustments following the policy is crucial for understanding changes in firm-level outcomes.

I find that consumers throughout the world unambiguously benefit from access to now-cheaper American products, so detrimental product exit by non-US brands induced by the policy does not outweigh the consumer gains. Endogenous portfolio decisions do not amplify increases in consumer surplus very significantly due to such offsetting effects on the extensive margin. The gains in consumer surplus are heterogeneous across countries. Consumers in countries like Brazil or Mexico, who are poorer and particularly value low prices, benefit more, with raises in consumer surplus of over 10%. Likewise, US consumers benefit more than consumers in other rich nations due to home bias; Americans value cheap American cars more than Europeans or Australians.

[8]Since the focus of the paper is not on production location decisions, I reduce the marginal cost of all products produced by brands headquartered in the United States.

[9]For instance, in China, consumer subsidies on electric vehicles peaked at around 40-60% in 2014, according to the Environmental Energy Study Institute (Lu 2018).

Contrary to the production subsidy, the 50% consumption subsidy on US-branded products in the United States does not affect potential products' cost or preference fundamentals overseas. I find that such a policy significantly increases consumer surplus in the United States by over 42% and shifts US profits and shares towards US brands, while the effects on other markets are small. Both in the United States and in other markets, the consumer subsidy induces entry of US-branded product offerings. However, I find that the marginal new product introduced by a US brand is relatively unpopular and, therefore, does not cause significant shifts in market shares in countries other than the United States.

Three main takeaways arise from these exercises. First, both policies benefit consumers so that harmful exit by brands that face discrimination does not dominate the beneficial entry of additional products by favored brands. Second, product entry amplifies profit shifting towards brands that benefit from the policies. Finally, a national US consumer subsidy favoring US brands results in profit shifting towards US brands only in the United States because the newly introduced products are relatively unattractive to unsubsidized consumers abroad. Meanwhile, production subsidies that make American cars cheaper lead to significant profit shifting towards US brands worldwide.

This paper relates to previous work studying the effects of government policies in the automobile industry. My framework contributes to Berry et al. (1995), Goldberg (1995), Petrin (2002), Coşar et al. (2017), Grieco et al. (2023), and Allcott et al. (2024) by providing a model of product entry in the international automobile industry to study the cross-market effects of national policies on global consumer and firm-level outcomes. While such papers focus on pricing behavior and market power in the passenger vehicle industry, I study international product entry. My methodological contribution to this literature is to show how to estimate and solve a product entry game while accounting for horizontal and vertical product differentiation, heterogeneous consumer preferences, and strategic pricing.

I develop a multi-product entry model with features similar to those in Eizenberg (2014), Wollmann (2018), Fan and Yang (2024), and Montag (2024). These papers also define a set of potential products that firms can offer and use moment inequalities to estimate fixed costs. However, my paper differs in two key dimensions. First, I model product entry in a global setting, allowing firms to develop products and leverage scale economies by selling them in multiple markets. Thus, my aim is to understand the role of scale economies at the product level in shaping cross-country outcomes. Second, my methods rely on the assumption of unobserved rival fixed cost shocks, while such papers assume complete information. As such, their approach makes it challenging to compute counterfactual outcomes, except in low-dimensional settings or otherwise using approximation methods or equilibrium selection rules. In contrast to Eizenberg (2014) and Wollmann (2018), but similarly to Fan and Yang (2024), my moment inequalities do not impose support restrictions on firms' fixed cost shocks and identify both the mean and variance of the fixed cost distributions, but require a distributional assumption on the fixed costs.

This paper also contributes to the international trade literature studying interdependent global firm decisions, including Tintelnot (2016), Antràs et al. (2017), Morales et al. (2019), Head and

Mayer (2019), Alfaro-Urena et al. (2023), Castro-Vincenzi (2024), Castro-Vincenzi et al. (2024), and Head et al. (2024). There are two key differences relative to this literature. First, I focus on global product portfolio decisions rather than production location, sourcing, or dynamic market entry decisions. Second, in my model, firms behave strategically and are not atomistic. Moreover, my model has features that have been studied in previous theoretical and empirical work in international trade, including scale economies (as in Krugman 1980, Thomas 2011, Costinot et al. 2019), oligopolies (as in Atkeson and Burstein 2008) and multi-product firms (as in Bernard et al. 2011 and Mayer et al. 2021). My contribution to this literature is to provide a quantitative framework suitable for studying policy questions in settings with multi-product firms, strategic behavior, and endogenous global product entry. A key distinction relative to many papers in the international trade literature is that while I allow for heterogeneous consumers and products, flexible substitution patterns in demand, and strategic behavior, I hold fixed general equilibrium variables such as wages and focus on an industry equilibrium.

Methodologically, this paper contributes to the literature on solution methods in settings with multiple interdependent discrete choices. In single-agent settings, Arkolakis et al. (2023), Alfaro-Urena et al. (2023), and Castro-Vincenzi et al. (2024) provide solution algorithms that exploit knowledge of complementarity or substitutability across discrete choices. In a strategic setting, Seim (2006) solves an incomplete-information game with ex-ante symmetric firms. Jia (2008) provides an algorithm to solve a complete-information entry game with two asymmetric firms making multiple entry decisions. My contribution is to provide a method to solve entry games featuring multiple asymmetric firms, each making multiple discrete choices, in an incomplete-information setting.

The remainder of the paper is organized as follows. Section 2 overviews the data and the industry setting. Section 3 develops a model of multi-product and multi-market entry. Section 4 provides bounds on firms' choice probabilities and explains how to use them to partially identify the fixed cost parameters. Section 5 reports estimation results. Section 6 develops a solution algorithm to bound the equilibrium distribution of product offerings in each market. Section 7 evaluates the effects of US policies favoring domestic brands on global market outcomes. Section 8 concludes.

2 Data and Industry Setting

My primary source of data is information on the universe of new passenger vehicle registrations in the year 2019 in 12 countries: Australia, Brazil, China, France, Germany, India, Italy, Japan, Mexico, Spain, the United Kingdom, and the United States. I obtain these data from *IHS Markit*.[10] The data contain manufacturer-suggested retail prices (MSRP) in USD, units sold, and other characteristics such as fuel type, body type, horsepower, length, height, width, and weight. The data is at the quarterly-trim-country level and include 177 different brands and 73 different parent companies.[11] I merge this dataset with the brand's headquarters country and with information from

[10] Previous literature has used various versions of these data, e.g., Coşar et al. (2017) and Head and Mayer (2019).

[11] A trim is a definition used by manufacturers to identify a vehicle's special features and level of equipment at a finer level than a nameplate. An example of a nameplate is a Toyota Corolla. Within the nameplate, there is

CEPII on the distance between the destination country and the brand's headquarters country.

To obtain product market shares at the year-country level, I divide units sold in 2019 by market size. I follow Grieco et al. (2023) and define market size as the product of the number of households in a country in 2019 and the average number of vehicles per household in such a country, divided by the average tenure of car ownership, which is assumed to be 5 years.

I also obtain micro-moments relating the income of vehicle buyers to the prices of vehicles they purchase from the MRI-Simmons 2019 Crosstab Report for the United States. I obtain market-level data on PPP income per capita and Gini coefficients from the World Bank. Assuming income follows a log-normal distribution in each country, income per capita and the Gini coefficient are sufficient to back out the location and scale parameters of the income distributions.

I refer to parent companies (e.g., Ford) as firms. Each firm has a set of brands (e.g., Ford, Lincoln). I define a product as a brand-body type-fuel type combination. The set of possible body types includes Cars, SUVs, wagons, multi-purpose vehicles (MPVs), and convertibles.[12] Possible fuel types are internal combustion engine (ICE), (plug-in) electric, or hybrid. For instance, a potential product is a Lincoln SUV with an internal combustion engine.[13]

I aggregate the remaining characteristics of these products (e.g., size or horsepower) by taking a quantity-weighted average across all trims within this category. In Appendix A, I discuss the procedure used to impute product characteristics for potential products that I do not observe in my sample. After dropping brands specializing in luxury products, there are 49 parent companies, 130 brands, and 1530 potential products.[14]

Figure 1 reports the fraction of potential products that firms sell in at least one market and how it varies across product categories (Panel A) and across firms (Panel B). Firms offer fewer than 30% of their potential products across all markets. Across the firm distribution, no firm offers more than 50% of its potential products.

Figure 1 also shows that global product offerings vary substantially across product categories, which suggests a response to demand conditions that vary across product types. While more than 40% of potential SUV products are offered in at least one market, fewer than 10% of potential hybrid or wagon products are offered.

Figures 2 and 3 condition on the set of products sold in at least one market and show that across all body types and fuel types, a substantial fraction of products are sold in a single market. For instance, around 60% of SUVs or Cars are sold in a single market. Still, a considerable fraction of such products (for these categories, around 20%) is sold in over 9 markets. There is also significant

typically differentiation across trims, for instance, the Toyota Corolla ZR or the Toyota Corolla Ascent, which may have different characteristics like horsepower.

[12]As stated in Appendix A, I define a Car to be a sedan, hatchback, or coupe.

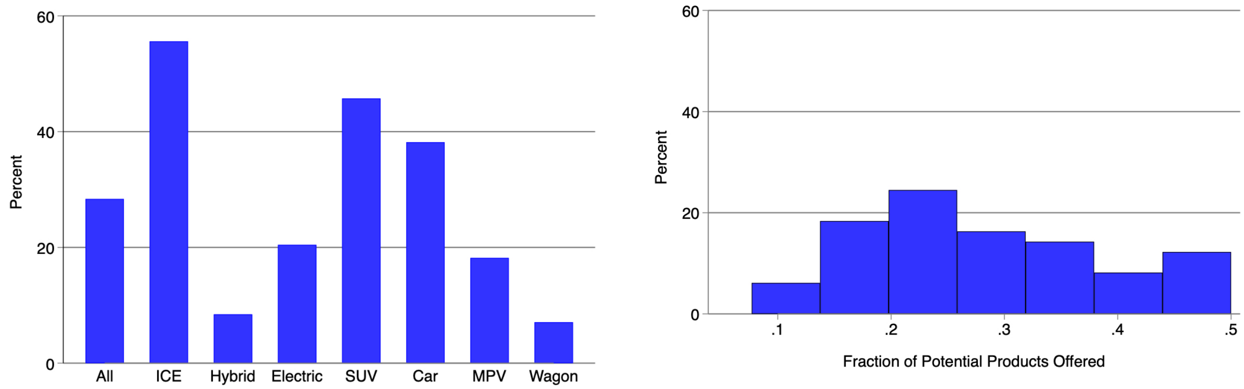
[13]This is a different level of aggregation than in most of the literature on automobiles, which usually aggregates across trims at the brand-nameplate level (e.g., Head and Mayer 2019, Grieco et al. 2023), e.g., Toyota Corolla. My product definition permits the extrapolation of the full set of product characteristics to all potential products, whereas it is not possible to take a stance on a brand's potential nameplates.

[14]Brands like Ferrari, Maserati, Lamborghini, or Rolls-Royce have small quantity shares and substantially higher prices across markets. Thus, I excluded them from the estimation sample to avoid issues with small market shares and because they belong to a substantially different segment of the market.

Figure 1: Fraction of Potential Products Offered Globally

Panel A: Across Product Categories

Panel B: Across Firms

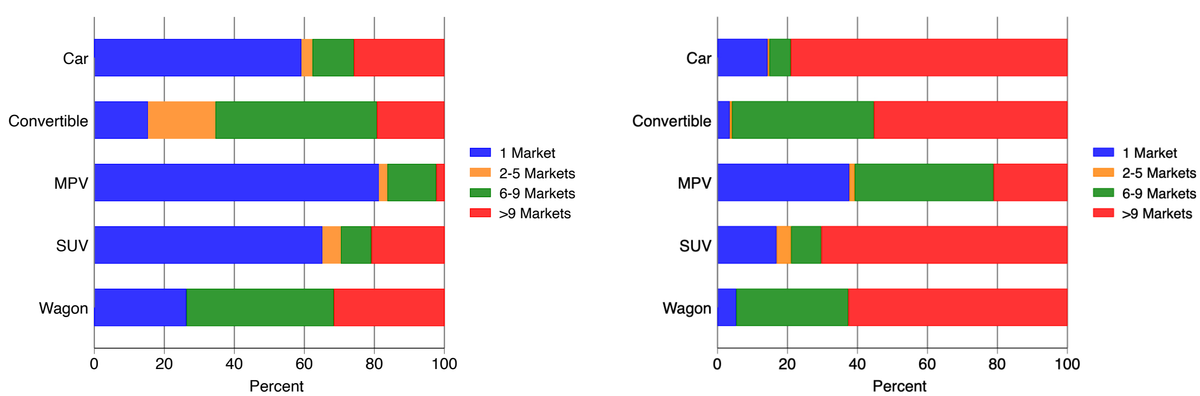


Notes: In Panel A, the "All" category reports the percentage of potential products (potential brand-body type-fuel type combinations) that are sold in at least one market. All other categories report the fraction of potential products of that category that are sold in at least one market. "ICE" stands for internal combustion engine and "MPV" stands for multi-purpose vehicle. Panel B reports the distribution of the fraction of potential products offered in at least one market across parent companies (e.g., Ford).

Figure 2: Number of Markets Offered Conditional on Portfolio, by Body Type

Panel A: Not Quantity Weighted

Panel B: Quantity Weighted



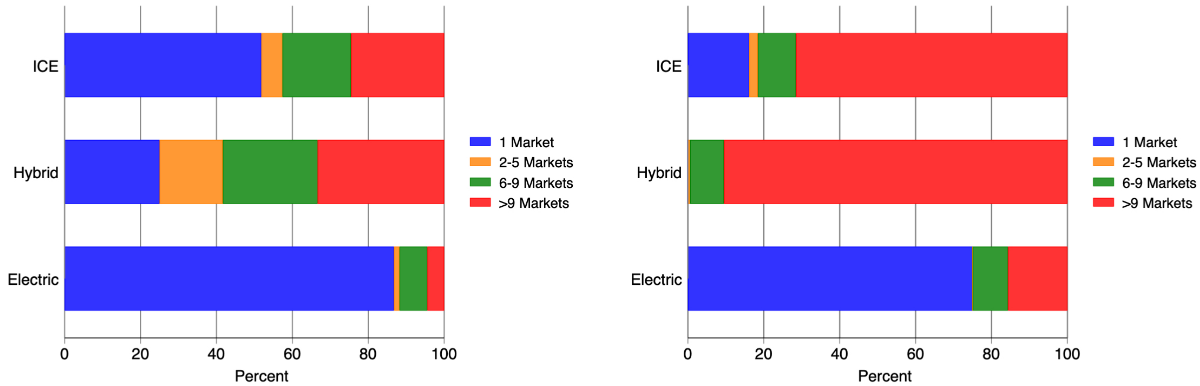
Notes: Both Panel A and Panel B condition on the products that are observed to be offered in at least one market in the data. For each body type, Panel A reports the percentage of products within that body type that are offered in different number-of-market categories. Panel B reports the quantity share of each number-of-market category across body type categories.

heterogeneity in entry patterns across product categories, with the most extreme example being electric vehicles, the majority of which are sold in a single market. The right panel in Figure 2 shows that weighting by quantity changes these statistics. Products offered in over 9 markets account for the vast majority of units sold (70-80% for Cars and SUVs). This shows that product market shares differ substantially, with some capturing a large share of world demand.

Figure 3: Number of Markets Offered Conditional on Portfolio, by Fuel Type

Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

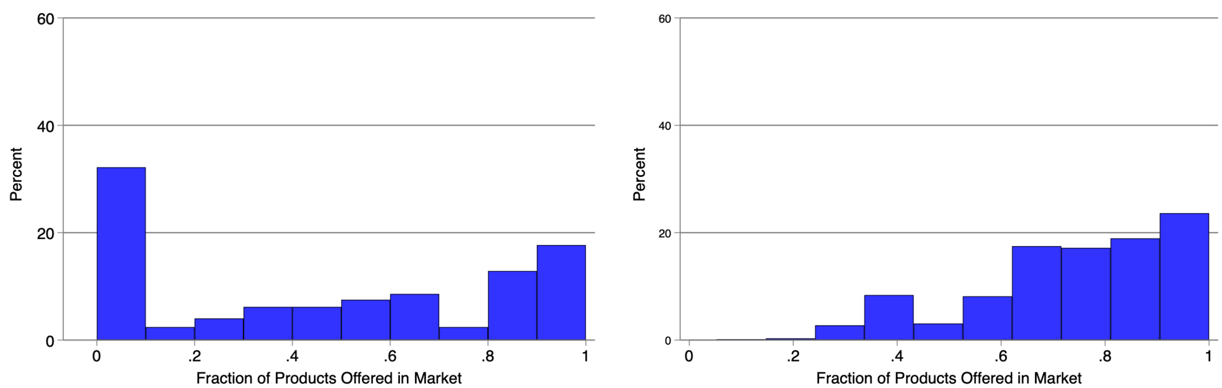


Notes: Both Panel A and Panel B condition on the products that are observed to be offered in at least one market in the data. For each fuel type, Panel A reports the percentage of products within that fuel type that are offered in different number-of-market categories. Panel B reports the quantity share of each number-of-market category across fuel type categories.

Figure 4: Fraction of Products Offered

Panel A: Not Quantity Weighted

Panel B: Quantity Weighted



Notes: Panel A plots the distribution of the fraction of products offered across firm-market pairs. Panel B weights by the total units sold by the firm in the market.

Finally, automotive firms not only serve multiple markets but also offer a portfolio of multiple products. Both Panel A and Panel B in Figure 4 show that, in most cases, firms sell a strict subset of their products in each market. Panel A shows that only in around 30% of firm-markets there is near-zero entry. Moreover, only in around 17% (22% if quantity-weighted) of the cases do firms sell all their products in a market.

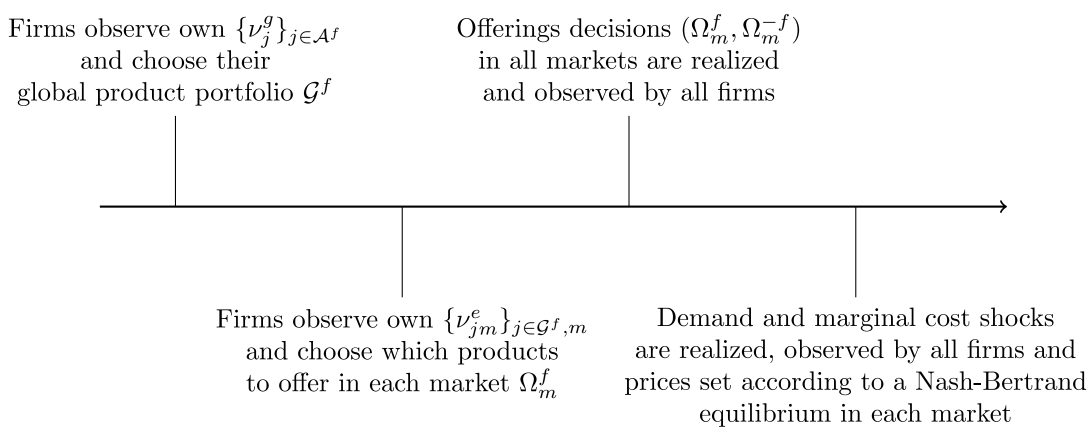
3 Model of Strategic Global Multi-Product Entry

In this section, I develop a model of strategic product portfolio choice and market entry. A set $\mathcal{F} = \{1, ..., f, ..., F\}$ of firms competes in a set $\mathcal{M} = \{1, ..., m, ..., M\}$ of markets.[15] A given firm f is endowed with a set of potential products. I denote a firm's potential set of products as $\mathcal{A}^f$. I denote the union of all potential products across firms by $\mathcal{A}$.

I model firms' portfolio and market entry choices in three stages. In Stage 1, each firm f realizes a set of product portfolio fixed cost shocks $\{\nu_j^g\}_{j \in \mathcal{A}^f}$ for each of their potential products, and upon observing these, chooses which products to introduce in its global product portfolio, $\mathcal{G}^f$. In Stage 2, each firm f realizes a set of market entry fixed cost shocks $\{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}$ and chooses which subset of products $\Omega_m^f \subseteq \mathcal{G}^f$ in its portfolio to offer in each potential market m.[16] Finally, in Stage 3, each firm realizes a set of demand and marginal cost shocks for each product offered in each market and chooses which prices to charge. Firms solve the model by backward induction. Figure 5 illustrates the timing of the game.

A key assumption is that throughout Stages 1 and 2, firms do not observe their rivals' fixed cost shocks when making their entry decisions. Thus, in Stages 1 and 2, firms play a Bayesian Nash equilibrium in product entry decisions.

Figure 5: Timing of the Game



Assumption 1 (Unobservability of Rival Fixed Cost Shocks) *Firms' fixed cost shocks $\{\nu_j^g\}_{j \in \mathcal{A}^f}$ and $\{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}$ are private information at the time of making product portfolio and market entry decisions throughout Stages 1 and 2 of the game.*

I summarize the key informational assumption in Assumption 1 for future reference. I now describe the three stages of the game in reverse order.

[15] I hold the set of firms fixed due to evidence that product entry has been more important than firm entry in leading to changes in market structure in this industry from 1980-2018 (see Grieco et al. 2023). However, the model can be extended to allow for firm or brand entry provided their potential products' characteristics are defined.

[16] I lighten notation by referring to collections of variables across markets $\{Y_m\}_{m \in \mathcal{M}}$ as $\{Y_m\}_m$.

3.1 Stage 3

3.1.1 Demand and Marginal Costs

The demand side of the model follows a mixed logit specification, as in Berry et al. (1995). Buyer i in market m chooses to purchase a new passenger vehicle j from the set of offerings Ω_m or the outside option so as to maximize utility. Within a market, buyers have idiosyncratic demographic characteristics, which determine how much they value the different attributes of each good, as well as their distaste for prices. The indirect utility that buyer i in market m derives from product j is given by,

$$U_{ijm} = \beta_m + \beta_{b(j)} - \alpha_i p_{jm} + \beta^x \boldsymbol{X}_{jm} + \xi_{jm} + \varepsilon_{ijm} \tag{1}$$

$$= \tilde{\delta}_{jm} - \alpha_i p_{jm} + \xi_{jm} + \varepsilon_{ijm}. \tag{2}$$

In equation (1), $\boldsymbol{X}_{jm}$ denotes a set of non-price attributes of product $j \in \Omega_m$ or brand-market characteristics. I allow buyers to have different preferences across markets over (a subset of) these characteristics. $\boldsymbol{X}_{jm}$ includes market identifiers interacted with (i) a dummy denoting whether the product is electric or hybrid, (ii) a set of dummies for different body type categories, and (iii) size. It also includes horsepower/weight, horsepower, and a home market dummy denoting whether the brand's headquarters are located in market m.[17] $\beta_{b(j)}$ is a brand fixed effect, and ξ_{jm} is a product-market demand shock that is realized at this stage of the game and, subsequently, perfectly observed by all buyers and firms. $\tilde{\delta}_{jm}$ denotes the mean non-price utility of product j in market m net of the demand shock ξ_{jm}. Buyer i has a marginal utility of income,

$$\alpha_i = \exp(\alpha_0 + \alpha_1 \log(income_i) + \alpha_2 China_i + \sigma^y u_i)$$

where u_i are i.i.d. normal shocks. I allow distaste for prices to be different in China, conditional on income, in light of the unique policy environment that characterizes China during my sample period, which makes consumers seemingly less price sensitive. Finally, ε_{ijm} denotes an i.i.d. Type 1 Extreme Value distributed preference shock that is buyer-product-market specific. The mean utility of the outside good (good 0) is normalized to zero, such that $U_{i0m} = \varepsilon_{i0m}$.

The above specification of indirect utility yields buyer-specific logit probabilities that can then be integrated over the joint distribution of market-specific buyer demographics and taste shocks to obtain market shares s_{jm}. Then, the quantity of product j sold in market m can be obtained by multiplying the market share s_{jm} by the market size M_m.

The marginal cost of supplying a unit of product j in market m is given by,

$$\begin{aligned}\log(c_{jm}) &= \gamma_m + \gamma_{b(j)} + \gamma_{fueltype(j)} + \gamma_1 \log(hp_j) + \gamma_2 \log(hpw_j) \\ &\quad + \gamma_3 \log(size_j) + \gamma_4 \log(dist_{jm}) + \omega_{jm} \\ &= \tilde{c}_{jm} + \omega_{jm}\end{aligned} \tag{3}$$

[17] Horsepower/weight proxies inversely for fuel efficiency, which is an omitted variable in my dataset.

where hp and hpw denote horsepower and horsepower/weight, respectively, and $dist_{jm}$ denotes the distance between market m and the headquarters country of brand b. γ_m is a market fixed effect and $\gamma_{b(j)}$ is a brand fixed effect. $\tilde{c}_{jm}$ denotes the mean (log) marginal cost of product j in market m net of the product-market marginal cost shock ω_{jm}. Marginal costs, therefore, depend on observed characteristics and a marginal cost shock ω_{jm} that is realized at this stage of the game.

Demand and marginal cost shocks $\{\xi_{jm}, \omega_{jm}\}_{j\in\Omega_m,m}$ are i.i.d. and I assume they follow a bivariate normal distribution.[18]

3.1.2 Pricing

Firms set prices according to a complete-information information, Nash-Bertrand pricing game in each market m, as in Berry et al. (1995).

When firms choose prices, demand and marginal cost shocks $\{\xi_{jm}, \omega_{jm}\}_{j\in\Omega_m}$ are known by all firms for all products Ω_m offered in market m. Each firm chooses its prices to maximize its variable profits, given by,

$$\begin{aligned}\Pi_m^{f,3} &= \sum_{j\in\Omega_m^f} (p_{jm} - c_{jm}) M_m s_{jm}(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{p}_m^f, \boldsymbol{p}_m^{-f}, \boldsymbol{\xi}_m) \\ &= \sum_{j\in\Omega_m^f} \pi_{jm}(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{p}_m^f, \boldsymbol{p}_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m).\end{aligned}$$

I use boldface notation to denote vectors of variables. $\boldsymbol{p}_m^f$ is the vector of prices in market m across products Ω_m^f offered by firm f, $\boldsymbol{p}_m^{-f}$ denotes the prices charged for products Ω_m^{-f} offered by firms other than firm f, and $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ stand for the demand and marginal cost shocks for all products Ω_m offered in market m.

I denote equilibrium variable profits given demand and marginal cost shocks, and product offerings Ω_m, by,

$$\pi_{jm}^*(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m) := \pi_{jm}(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{p}_m^{f,*}, \boldsymbol{p}_m^{-f,*}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$$

where $\boldsymbol{p}_m^{f,*}$ denotes the equilibrium price vector that emerges in market m at $(\Omega_m^f, \Omega_m^{-f})$ given demand and marginal cost shocks.

3.2 Stage 2: Market Entry Decisions

At this stage, firms have chosen their global product portfolio $\mathcal{G}^f$ and realize a set of market entry fixed cost shocks $\{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m}$ for each product in their portfolio in each market. Firms make market entry decisions conditional on what they know; the information set of firm f in Stage 2 is,

$$\mathcal{I}^{f,2} := (\mathcal{G}^f, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m}, \mathcal{I}),$$

[18] Other papers have made alternative assumptions on the distribution of these shocks. For instance, Wollmann (2018) draws from the empirical distribution of these shocks when computing counterfactual experiments.

where set $\mathcal{I}$ denotes the component of the information set that is common knowledge to all firms and is given by,

$$\mathcal{I} := (\{\tilde{\delta}_{jm}\}_{j\in\mathcal{A},m}, \{\tilde{c}_{jm}\}_{j\in\mathcal{A},m}, \mathcal{A}^f, \mathcal{A}^{-f}).$$

Set $\mathcal{I}$ also includes knowledge of the distributions of unobserved demand, marginal cost, and fixed cost shocks, policies in the counterfactual, and the equilibrium strategies that firms play.[19]

Given this information, each firm f chooses offerings in each market Ω_m^f so as to maximize expected profits. That is, it solves,

$$\Pi_m^{f,2}(\mathcal{I}^{f,2}) = \max_{\Omega_m^f \subseteq \mathcal{G}^f} \sum_{j\in\Omega_m^f} O_{jm}[\mathbb{E}[\pi_{jm}^*(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)|\mathcal{I}] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e)] \tag{4}$$

where

$$O_{jm} = 1 \iff j \in \Omega_m^f.$$

The variable $F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e)$ denotes the market entry fixed cost of offering product j in market m. Assumption 2 makes a parametric assumption regarding these fixed costs.

Assumption 2 (Market Entry Fixed Costs) *The fixed cost of offering product $j \in \mathcal{G}^f$ in market m is given by,*

$$F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) = \exp(Z_{jm}'\theta_e + \sigma_e \nu_{jm}^e)$$

where Z_{jm} is $\mathcal{I}$-measurable and $\nu_{jm}^e|\mathcal{I} \sim_{i.i.d.} Normal(0,1)$.

In the empirical implementation, I assume that Z_{jm} is a constant. Note that this specification rules out economies and diseconomies of scope, as the fixed costs are independent of the firm's decisions to offer other products. In Appendix E, I explain how the model and method could be extended to allow for such interdependencies through fixed costs and why it is difficult to estimate the corresponding parameters given my data.

Firms choose which subset of their portfolio to offer in each market while best responding to other firms' product entry strategies and taking into account cannibalization across their products. The expectation in equation (4) is with respect to the distribution of $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ that are realized in Stage 3 and the distributions of $\boldsymbol{\nu}^{-f,e}$ and $\boldsymbol{\nu}^{-f,g}$ determining rival firms' offerings. Due to Assumption 1, only the information in the common part of the information set $\mathcal{I}$ is useful to predict variable profits, so I remove conditioning on the private information component in the conditional expectation in equation (4). When a firm chooses which products to offer in a market, it takes expectations over the market structure against which it will be competing and the markups it will be able to charge for each product it potentially sells in such a market. Such expectations are

[19] In summary, $\mathcal{I}$ denotes knowledge of the data-generating process.

conditional on firms' knowledge of the policy environment subsumed in $\mathcal{I}$, which firms anticipate affects rival firms' offerings in equilibrium.

3.3 Stage 1: Global Portfolio Decision

In the first stage of the game, firms realize a set of product portfolio fixed cost shocks $\{\nu_j^g\}_{j\in\mathcal{A}^f}$ for each of their potential products. The information set of firm f is therefore,

$$\mathcal{I}^{f,1} := (\{\nu_j^g\}_{j\in\mathcal{A}^f}, \mathcal{I}).$$

Each firm f chooses its product portfolio by maximizing expected profits,

$$\Pi^{f,1}(\mathcal{I}^{f,1}) = \max_{\mathcal{G}^f \subseteq \mathcal{A}^f} \mathbb{E}\Big[\sum_m \Pi_m^{f,2}(\mathcal{G}^f, \boldsymbol{\nu}_m^{f,e}, \mathcal{I})\big|\mathcal{I}\Big] - \sum_{j\in\mathcal{A}^f} G_j F_j^g(\nu_j^g; \theta_g, \sigma_g), \tag{5}$$

where

$$G_j = 1 \iff j \in \mathcal{G}^f.$$

The variable $F_j^g(\nu_j^g; \theta_g, \sigma_g)$ denotes the fixed cost of introducing product j into the firm's global product portfolio. Assumption 3 makes a parametric assumption regarding these fixed costs.

Assumption 3 (Product Portfolio Fixed Costs) *The fixed cost of introducing product $j \in \mathcal{A}^f$ into firm f's global product portfolio is given by,*

$$F_j^g(\nu_j^g; \theta_g, \sigma_g) = \exp(Z_j'\theta_g + \sigma_g \nu_j^g)$$

where Z_j is $\mathcal{I}$-measurable and $\nu_j^g|\mathcal{I} \sim_{i.i.d.} Normal(0, 1)$.

In the empirical implementation, I assume that Z_j is a constant. As in Assumption 2, this specification rules out economies of scope.

Firms choose their portfolio so as to best respond to other firms' portfolio and market entry strategies. The portfolio decision affects the expected profits of firm f in all potential markets since including a product in its portfolio gives the firm the option to sell it in any country in the second stage. As in Stage 2, firm f's expectation is over other firms' portfolio and market entry shocks $\boldsymbol{\nu}^{-f,e}$ and $\boldsymbol{\nu}^{-f,g}$ determining rival firms' offerings in each market, and ex-post demand and marginal cost shocks. At this stage of the game, firm f also takes expectations over its own market entry fixed cost shocks $\boldsymbol{\nu}^{f,e}$, which it does not realize until Stage 2. Assumption 1 implies that only information contained in $\mathcal{I}$ is useful to predict market-level profits $\Pi_m^{f,2}$, so I remove conditioning on the private information component in the conditional expectation in equation (5). The product-specific fixed cost F_j^g induces a complementarity across markets. Greater expected profitability for product j in market m can lead to the introduction of product j into the firm's global product portfolio and its offering in other markets in Stage 2.

3.4 Marginal Value and Submodularity

I introduce the concept of the marginal value of offering a product into a given market.

Definition 1 *The marginal value of introducing product j in market m at market structure Ω_m under demand and marginal cost shocks $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ is given by,*

$$
\begin{aligned}
MV_{jm}(\Omega_m^f, \Omega_m^{-f}; \boldsymbol{\xi}_m, \boldsymbol{\omega}_m) := & \underbrace{\pi^*_{jm}(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)}_{\text{variable profits from } j \text{ in } m} \\
& + \underbrace{\sum_{j' \neq j, j' \in \Omega_m^f} \left[\pi^*_{j'm}(\Omega_m^f, \Omega_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m) - \pi^*_{j'm}(\Omega_m^f \setminus \{j\}, \Omega_m^{-f}, \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)\right]}_{\text{cannibalization: change in variable profits for other products when } j \text{ is offered}}.
\end{aligned} \tag{6}
$$

It is the change in variable profits from offering product j in market m for firm f when the initial market structure is given by $\Omega_m^f \setminus \{j\}$ and under demand and marginal cost shocks in market m given by $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$.

Throughout the remainder of the paper, I will leverage the following property.

Assumption 4 (Submodularity) *$MV_{jm}(\Omega_m^f, \Omega_m^{-f}; \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ is decreasing in Ω_m^f and Ω_m^{-f} at any demand and marginal cost shocks $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$.*

While ideally, this property should be formally proven, doing so requires comparing profits across Nash-Bertrand pricing equilibria, which is beyond the scope of this paper.[20] However, I find that at estimated parameters and under thousands of $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ draws, there is no instance in which it is violated, which is consistent with the economic forces present in the model.

3.5 Discussion of Modeling Assumptions

While Stage 3 of the model is standard, Assumption 1 is a departure from previous papers such as Jia (2008), Ciliberto and Tamer (2009), Eizenberg (2014), or Wollmann (2018), which assume that firms perfectly observe other firms' fixed cost shocks when making their entry decisions.

As shown in Appendix F, under Assumption 1, Milgrom and Weber (1985) and Balder (1988) guarantee pure strategy equilibrium existence in both Stage 1 and Stage 2 of the game. This is an advantage relative to complete information, where the existence of a pure strategy equilibrium is not guaranteed in games featuring strategic substitutes with more than two players.[21]

Assumption 4 is more likely to hold when (i) marginal costs are constant and when (ii) the unobserved shocks $(\boldsymbol{\xi}, \boldsymbol{\omega})$ are independent of firms' portfolio and market entry decisions. The latter

[20] In practice, I solve for prices by using the contraction mapping from Morrow and Skerlos (2011) to compute a unique "resting point." Conlon and Gortmaker (2020) show that this resting point reliably finds an equilibrium and is computationally 3-12 faster than Newton-type approaches. This is, in practice, an equilibrium selection assumption that uniquely determines the MV_{jm} mapping as a function only of offerings and demand and marginal cost shocks.

[21] In Appendix F.1, I provide an example of a complete information game with strategic substitutes and 3 players where no pure strategy Nash equilibrium exists.

assumption is guaranteed due to the timing assumption on the demand and marginal cost shocks, which are realized in Stage 3. This timing assumption allows one to use the standard techniques (i.e., Berry et al. 1995) to estimate demand and marginal costs.

In Appendix E, I show how the model and methods can be extended to cases where the variable profit function exhibits complementarities across product offerings within a market. For instance, if marginal cost synergies are large, two products could be net complements rather than substitutes from the firm's perspective. With knowledge of the sign of the interdependence across any pair of choices, I show that my moment inequality approach discussed in Section 4 can be used to estimate the model's fixed cost parameters.

4 Moment Inequality Estimation

There are two main challenges in estimating fixed costs in settings where firms have multiple discrete choices and behave strategically. First, it is computationally infeasible to solve the model fully under each vector of fixed cost parameters $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. With only 20 discrete choices per firm and 10 firms, this gives $2^{200} \approx 10^{60}$ possible entry configurations that can be realized in equilibrium. Evaluating all such configurations once is already computationally infeasible, let alone evaluating them many times as would be required for parameter estimation with standard techniques. Second, while the existence of a pure strategy Nash equilibrium is guaranteed in my model, uniqueness is not. Thus, even if solving the model fully and quickly were possible, one would have to make an equilibrium selection assumption or use moments robust to multiplicity to employ standard point identification techniques such as MLE or GMM. These two reasons suggest using moment inequalities, which do not require computing entry equilibria under different parameter vectors and remain valid irrespective of the Bayesian Nash equilibrium that generates the data.

An additional challenge in estimating fixed cost parameters in product entry games has been to allow for fixed cost shocks that are observable to firms when they make their entry decisions but unobservable to researchers. I develop a new approach that allows for (i) multiple discrete choices per agent, (ii) selection on unobserved (to the researcher) fixed cost shocks, and (iii) strategic entry.

In this section, I first show how to derive bounds on firms' choice probabilities. Then, I show how to use these bounds to derive moment inequalities that partially identify the fixed cost parameters. Throughout, I assume that all demand and marginal cost parameters have been estimated. Together with the Nash-Bertrand pricing assumption in Stage 3, such parameters are sufficient to obtain variable profits for each product-market pair given any demand and marginal cost shock realizations $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ and market structures Ω_m. I then compute the marginal value of product j in market m, $MV_{jm}(\Omega_m^f, \Omega_m^{-f}; \boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ under any offerings and demand and marginal cost shocks.

4.1 Bounds on Choice Probabilities

The methodological insights in this paper rely on computing bounds on firms' market entry and product portfolio choice probabilities. In this section, I first provide intuition on the derivation of the bounds on the probability of market entry. In Appendix B, I provide the formal derivations of these bounds and the corresponding bounds on the probability of portfolio introduction. Bounding firms' choice probabilities is the first step toward deriving moment inequalities for estimation.

First, consider a firm f in Stage 2 of the game that has chosen a singleton product portfolio, $\{j\}$, in Stage 1. In this case, the optimal choice in market m is straightforward to characterize as,

$$O_{jm} = 1 \iff \mathbb{E}[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \nu_{jm}^e, \mathcal{G}^f = \{j\}] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0. \tag{7}$$

That is, firm f offers product j in market m if and only if the expected profits from doing so are weakly positive. When making this choice, firm f conditions on what it knows, which includes the common information set $\mathcal{I}$, and the firm's private information, ν_{jm}^e and its portfolio, $\mathcal{G}^f = \{j\}$. A key insight is that due to Assumption 1, firm f's private information is independent of rival firms' chosen offerings, Ω_m^{-f}, conditional on $\mathcal{I}$. Therefore, I remove conditioning on ν_{jm}^e in the expectation in condition (7) and obtain,

$$O_{jm} = 1 \iff \mathbb{E}[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f = \{j\}] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0.$$

As such, and because ν_{jm}^e is realized after $\mathcal{G}^f$ is chosen, I use the known distribution of $\nu_{jm}^e|\mathcal{I}$ given by Assumption 2 to obtain,

$$\mathbb{P}(O_{jm} = 1|\mathcal{I}, \mathcal{G}^f = \{j\}) = \Gamma_{jm}(\mathbb{E}[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f = \{j\}]; \theta_e, \sigma_e), \tag{8}$$

where Γ_{jm} is the CDF of a log-normal distribution with location $Z_{jm}'\theta_e$ and scale σ_e.

Equation (8) demonstrates the tractability that Assumption 1 provides. Despite strategic behavior, it is possible to obtain an expression for the probability that the firm chooses to enter market m, similar to that in a binary and single-agent probit or logit model. Under complete information, equation (8) is not valid because rival firms' offerings in market m would be correlated with ν_{jm}^e (even conditional on $\mathcal{I}$), which makes it impossible to integrate ν_{jm}^e without solving the model fully. As discussed earlier, with complete information, challenges arise regarding equilibrium existence, multiplicity, and computational feasibility.

In my setting, I also need to address an additional difficulty: firms are, in general, multi-product. This feature implies that the decision to offer product j in market m is not independent of the decision to offer product $j' \neq j$. That is, firms' best-response bundle Ω_m^f in market m depends on all market entry fixed cost shocks it realizes $\{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}$ and in particular on ν_{jm}^e. Thus, necessary conditions for entry of product j conditioning on the observed bundle would lead to a selection problem for characterizing the probability that such a product is offered; the researcher does not know the distribution of ν_{jm}^e conditional on the observed bundle Ω_m^f.[22]

In Appendix B, inspired by Fan and Yang (2024), I leverage Assumption 4 to deal with this selection problem and derive upper bound inequality,

[22] Eizenberg (2014) and Wollmann (2018) deal with this issue by imposing support restrictions on fixed cost shocks, which allows them to generate moment inequalities that average out the unobserved fixed cost shocks.

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \geq 0\} \geq O_{jm}, \tag{9}$$

and lower bound inequality,

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\mathcal{G}^f,\Omega_m^{-f})|\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) < 0\} \geq 1 - O_{jm}. \tag{10}$$

Inequalities (9) and (10) hold for *any* entry opportunity for product $j \in \mathcal{G}^f$ in market m, thus overcoming the selection problem.[23] Inequality (9) says that if product j is offered in market m ($O_{jm} = 1$), necessarily an upper bound on the expected profit gain from doing so must be weakly positive. If it is not offered, such an upper bound could be positive or negative. Under Assumption 4, I obtain an upper bound on the expected profit gain by evaluating the marginal value as if product j were the only product to be offered by the firm in the market, i.e., at bundle $\{j\}$. At this extreme bundle, there can be no cannibalization. Inequality (10) says that if j is not offered in market m ($O_{jm} = 0$), necessarily a lower bound on the expected gain from this choice must be negative. If it is offered, then this lower bound could be positive or negative. Assumption 4 provides the extreme bundle that minimizes such possible gains and calls for evaluating the marginal value as if all products in the firm's portfolio, $\mathcal{G}^f$, were simultaneously offered in market m, which maximizes cannibalization within the firm.[24] Importantly, these bundles are not chosen optimally by the firm given its private information $\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}$, which eliminates the selection problem generated by not knowing the distribution of ν^e_{jm} conditional on the observed and optimal bundle Ω^f_m.

The key insight now is that in inequalities (9) and (10), the unobserved fixed cost shock is independent of the expectation operators inside the indicator functions; that is, independent of $\mathcal{I}$ and $\mathcal{G}^f$. For firm f, knowing ν^e_{jm} is not useful for predicting the bounds on its expected gain in variable profits. Thus, I take expectations conditional on $\mathcal{I}$ on both sides of inequalities (9)-(10) and use the known distribution of $\nu^e_{jm}|\mathcal{I}$ to obtain bounds on the probability that any product j in the firm's portfolio is offered in market m,

$$\begin{aligned}\Gamma_{jm}(\mathbb{E}[MV_{jm}(\mathcal{G}^f,\Omega_m^{-f})|\mathcal{I},\mathcal{G}^f];\theta_e,\sigma_e) &\leq \mathbb{P}(O_{jm}=1|\mathcal{I},\mathcal{G}^f) \\ &\leq \Gamma_{jm}(\mathbb{E}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I},\mathcal{G}^f];\theta_e,\sigma_e)\end{aligned} \tag{11}$$

The inequalities in (11) provide bounds on the ex-ante probability that product $j \in \mathcal{G}^f$ is offered in market m. The lower bound is the probability that the market entry fixed cost is smaller than the change in variable profits from offering product j in market m obtained under maximal cannibalization. The upper bound is the probability that the market entry fixed cost is smaller than the change in variable profits for product j in market m obtained under minimal cannibalization.

[23] Fan and Yang (2024) deal with the selection problem in a multiple discrete choice setting by bounding the probability of entry below by the probability that entry is dominant and above by the probability that entry is not dominated. They similarly implement their inequalities using submodularity of variable profits, though in a setting of complete rather than incomplete information.

[24] In Appendix E, I show that for estimation, Assumption 4 is stronger than required. What is needed for estimation is the existence of product-market-specific within-firm bundles that bound the variable profit gains from offering any product in a market.

In Appendix B.2, I derive bounds on the probability, $\mathbb{P}(G_j = 1|\mathcal{I})$, that product j is introduced in the firm's global product portfolio using similar methodological insights as in the derivations of inequalities (9) and (10). Instead of bounding the marginal value of offering a product in a market, the derivations bound the change in value from introducing a product into the firm's portfolio. An additional complication arises, which is that I have to deal with subgame perfection: firms make portfolio decisions in Stage 1, anticipating that they will make optimal market entry decisions in Stage 2. In Appendix B.2 I show how to deal with this issue and that,

$$\tilde{\Lambda}_j\Big(\big\{\mathbb{E}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big) \leq \mathbb{P}(G_j = 1|\mathcal{I}) \leq \tilde{\Lambda}_j\Big(\big\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big), \qquad (12)$$

where $\tilde{\Lambda}_j$ is an increasing function in each of its M arguments and depends both on the distribution of portfolio fixed costs Λ_j and the distributions of market entry fixed costs across markets $\{\Gamma_{jm}\}_m$. Intuitively, bounds on the probability of portfolio introduction depend on (i) bounds on the additional variable profits that the product can earn in each market, (ii) the distribution of portfolio fixed costs determining how costly it is to include the product in the firm's portfolio, and (iii) the distributions of market entry fixed costs which determine how costly it is to offer the product in each market conditional on being included in the firm's portfolio.

In summary, I showed how to derive bounds on firms' choice probabilities in a strategic setting with multiple firms making multiple discrete choices under Assumptions 1 and 4.

4.2 Moment Inequalities using Convex/Concave Bounds of the Fixed Cost CDF

In this subsection, I show how to use the bounds on firms' market entry probabilities to derive moment inequalities that are useful for the estimation of (θ_e, σ_e). In Appendix B.2, I similarly show how to derive moment inequalities that partially identify (θ_g, σ_g) based on the bounds on the probability of portfolio introduction.

The inequalities in (11) provide bounds on the ex-ante probability of product offerings, but these depend on firms' expectations over rivals' product offerings Ω_m^{-f} in the equilibrium that generates the data. I develop moment inequalities that do not require solving the model and only depend on observed data and parameters.

To obtain an upper bound moment inequality for estimation of fixed cost parameters (θ_e, σ_e), I bound the term on the right-hand side of the inequalities in (11) with a convex upper bound of Γ_{jm}, which I denote by $\overline{\Gamma}_{jm}$.[25] This step relates to how Dickstein and Morales (2018), Dickstein et al. (2024), and Porcher et al. (2024) use a convex odds or linear approximation function to derive moment inequalities in a single-agent setting. Such a bound yields,

$$\overline{\Gamma}_{jm}(\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f]; \theta_e, \sigma_e) \geq \mathbb{E}[O_{jm}|\mathcal{I}, \mathcal{G}^f]. \qquad (13)$$

[25]There are many potential convex upper bounds of any CDF function. In Appendix B, I discuss the implementation in detail.

Since $\overline{\Gamma}_{jm}$ is convex, I apply Jensen's inequality to obtain,

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{j\}, \Omega_m^{-f}); \theta_e, \sigma_e) - O_{jm} | \mathcal{I}, \mathcal{G}^f] \geq 0. \tag{14}$$

Inequality (14) is a conditional moment inequality that depends on observed data, and in particular on the realization of rivals' offerings, Ω_m^{-f}. This relates to previous work that shows how to derive moment inequalities that depend on the ex-post realization of payoff-relevant variables (i.e., Pakes 2010, Pakes et al. 2015, Dickstein and Morales 2018); I extend the tricks developed in such papers to an incomplete-information game where expectations are over rivals' endogenous entry decisions. Due to the assumption that rivals' fixed cost shocks are unobserved when firms decide their product offerings, rival firms' entry decisions are independent (conditional on $\mathcal{I}$) of firm f's entry decisions, which permits using the realized set of entry decisions for estimation, while simultaneously allowing for selection on unobserved (own) fixed cost shocks. Under complete information, inequality (14) would be invalid due to violation of this independence result. The unobservability of rival fixed cost shocks effectively reduces the issue of selection on unobservables, yielding model-consistent bounds that are tighter than the analogous moment inequalities obtained under the assumption of complete information.[26] I also derive, using similar arguments but instead, a concave lower bound for the log-normal CDF, which I denote by $\underline{\Gamma}_{jm}$,

$$\mathbb{E}[\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f, \Omega_m^{-f}); \theta_e, \sigma_e) - O_{jm} | \mathcal{I}, \mathcal{G}^f] \leq 0. \tag{15}$$

Conditional moment inequalities (14) and (15) partially identify (θ_e, σ_e). I also prove the corresponding conditional moment inequalities that partially identify (θ_g, σ_g), which are based on bounds on the probability of product portfolio introduction, given by the inequalities in (12). The derivations in Appendix B.2 show how to use the bounds on the probability of portfolio introduction to obtain moment inequalities (A36)-(A37) using similar insights as used for inequalities (14)-(15). I summarize the partial identification result in Theorem 1.

Theorem 1 *The conditional moment inequalities in (14)-(15) and (A36)-(A37) partially identify* $(\theta_e, \sigma_e, \theta_g, \sigma_g)$.

Proof. See Appendix B. ■

My moment inequality approach is based on moments that bound firms' choice probabilities, which makes them suitable for computing counterfactual exercises. In my setting, Assumption

[26] Under complete information, one could obtain the analogous model-consistent inequality,

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{j\}, \emptyset); \theta_e, \sigma_e) - O_{jm} | \mathcal{I}] \geq 0.$$

Indeed, under Assumption 4, the largest gain from offering product j in market m is obtained when product j is the only product (other than the outside option) offered in market m. This extreme bundle is independent of all of the firms' fixed cost shock realizations. With complete information, such an extreme bound is required because it is the only way to obtain a valid bound that is independent of the (unobserved) information set of any given firm, which is what is required to integrate the unobserved fixed cost shocks. The bounds are less informative when applied to the same data because $MV_{jm}(\{j\}, \emptyset) \geq MV_{jm}(\{j\}, \Omega_m^{-f})$ for all $j \in \mathcal{A}$ and all $m \in \mathcal{M}$.

1 provides a model of firms' expectations over rivals' actions, which I exploit in Section 6 to provide a solution method. In other settings, one could use the moment inequalities directly to compute counterfactual exercises. For instance, the insights discussed above apply to multiple discrete-choice single-agent settings where expectations are over an exogenous, ex-post realized, payoff-relevant variable.

Though I assume that the distributions of both product portfolio and market entry fixed costs are log-normal, the arguments discussed in Section 4 apply under different assumptions on the fixed cost distributions.[27] I assume a log-normal specification because it is a natural benchmark in a setting with significant product-market heterogeneity.

4.2.1 Identification

My approach yields moment inequalities based on the derived bounds on product introduction probabilities. Excessive product portfolio costs can be rejected if they imply that the upper bound on the probability of portfolio introduction (see inequality (A36) in the Appendix) is smaller than the empirical probability of portfolio introduction. Insufficient portfolio costs can be rejected if they imply that the lower bound on the probability of portfolio introduction (see inequality (A37) in the Appendix) is larger than the empirical probability. Similarly, market entry fixed cost parameters can be rejected whenever the empirical probability of market entry (conditional on the observed portfolios) does not lie within the bounds given by inequalities (14) and (15). The product portfolio and market entry fixed costs rationalize the cross-sectional entry patterns discussed in Section 2: firms do not introduce all potential products in their portfolio and do not sell all products in their portfolio in all markets.

The inequalities identify not only the location parameters of the log-normal distributions but also the scale parameters, determining the variance of (log) fixed costs. Too large a scale parameter (holding fixed the location parameter) can be rejected because it would imply too low a probability of product introduction for highly profitable products. This is because a large scale parameter makes the tails of the fixed cost distributions fatter, therefore making the implied bounds on the probability of product introduction tend towards 0.5 irrespective of the magnitude of MV_{jm}. Similarly, scale parameters that are too small can be rejected because they would imply that the rate of product introduction tends to 1 for highly profitable deviations, which is not what is observed in the data. Intuitively, the variance of fixed costs must be large enough to rationalize market entry and portfolio deviations that are apparently highly profitable and yet are not undertaken by firms.

Identifying the second moment of the fixed cost distribution is a key advantage relative to previous approaches in the multi-product entry literature, such as Eizenberg (2014) or Wollmann (2018), which only identify the mean. By estimating all the parameters describing the fixed cost distributions, one can accurately integrate firms' fixed cost shocks, which is necessary to solve for

[27] More precisely, the log-normal assumption only matters for the construction of the convex/concave bound of the fixed cost CDFs. Under a different distributional assumption and, therefore, a different CDF, the convex upper bound and concave lower bound functions would differ, but all remaining arguments would still be valid.

model-consistent counterfactual outcomes.

To summarize, I have shown how to derive moment inequalities in a setting with multiple discrete choices in a computationally feasible manner robust to equilibrium non-uniqueness. The inequalities solely depend on changes in variable profits, which can be computed using the model for variable profits (Stage 3), observable entry decisions O_{jm} and Ω_m^{-f}, and fixed cost parameters.

4.3 Simulations to Determine the Informativeness of the Moment Inequalities

In Appendix G.2, I simulate a solvable version of the model to evaluate the tightness of the moment inequalities under various assumptions on the data-generating process. The simplified model has the exact same structure as described in Section 3 but relies on symmetry across firms and products to tractably obtain a full solution.[28] I simulate N symmetric firms with 3 symmetric potential products that they can offer across 12 heterogeneous markets. Firms earn variable profits in each market m according to

$$\Pi_m^f(N_m^f, N_m^{-f}) = A_m \frac{N_m^f}{(N_m^f)^{\kappa_o}(N_m^{-f})^{\kappa_r}},$$

where N_m^f denotes the number of products offered by firm f in market m and N_m^{-f} denotes the total number of products offered by rival firms in market m. A_m is a market profit shifter, and κ_o and κ_r regulate substitutability across products within and across firms, respectively. Firms pay fixed portfolio and market entry costs to develop products and offer them across markets, respectively.

In the simulations, I vary the number of firms, κ_o and κ_r, to understand how changes in such fundamentals affect the moment inequalities' informativeness of the global portfolio and market entry fixed cost parameters. These fundamentals are relevant to the informativeness of the moment inequalities, which depends on the loss from using "extreme" bundles to bound changes in variable profits (which depends on substitutability across products), as well as the loss from using convex/concave bounds together with Jensen's inequality to average out firms' expectational errors (which are affected by the size of firms relative to the overall market).

I find that both the number of firms and κ_o play an important role in determining the tightness of the bounds on both the portfolio and market entry fixed cost parameters. Greater degrees of substitutability across products within the firm (higher κ_o) render the moment inequalities less informative. If products are independent or there is low substitutability within the firm, the moment inequalities become extremely informative. Thus, my method is particularly useful for studying entry by firms with a single product. A larger number of firms renders them more informative, all else equal. Interestingly, substitutability across firms (κ_r) does not significantly affect the informativeness of the moment inequalities. While substitutability across firms can affect the level of equilibrium profits, it does not directly affect the informativeness of the moments used to inform the fixed cost parameters.

[28] Despite this simplicity, I attempt to calibrate the simulated model's parameters so as to match some observed features in the data, such as the share of potential products introduced in firms' portfolios.

4.4 Moments and Inference

To use the conditional moment inequalities from Theorem 1 for estimation, I first construct instrument functions that are positive-valued and exogenous to obtain unconditional moment inequalities.[29] My approach is similar to two-stage least squares: I project endogenous variables onto exogenous variables and then use the first-stage predicted values to compute instruments. I provide additional details about the implementation in Appendix D.2.

First, I simulate S draws of demand and marginal cost shocks $(\boldsymbol{\xi},\boldsymbol{\omega})$ using the fitted bivariate normal distribution. Second, I use the model for Stage 3 of the game to compute $\widehat{MV}_{jm}(\{j\},\Omega_m^{-f})$, $\widehat{MV}_{jm}(\mathcal{G}^f,\Omega_m^{-f})$, and $\widehat{MV}_{jm}(\mathcal{A}^f,\Omega_m^{-f})$, where the $\widehat{MV}$ notation denotes that such marginal values are averaged across the S simulation draws for $(\boldsymbol{\xi},\boldsymbol{\omega})$. Third, I project each of the endogenous $\widehat{MV}_{jm}(\{j\},\Omega_m^{-f})$, $\widehat{MV}_{jm}(\mathcal{G}^f,\Omega_m^{-f})$, and $\widehat{MV}_{jm}(\mathcal{A}^f,\Omega_m^{-f})$ on exogenous ($\mathcal{I}-$measurable) market size M_m, and interactions of market identifiers with $\tilde{\delta}_{jm}$, $\tilde{c}_{jm}$ and $\tilde{\delta}_{jm}\times\tilde{c}_{jm}$ using PPML, where $\tilde{\delta}_{jm}$ and $\tilde{c}_{jm}$ are defined in equations (2) and (3). I then use the predicted values from the PPML model to construct instruments. Introducing notation, let $\hat{\bar{x}}_{jm}$, $\hat{\underline{x}}^h_{jm}$, and $\hat{\underline{x}}^l_{jm}$ be the predicted values of $\widehat{MV}_{jm}(\{j\},\Omega_m^{-f})$, $\widehat{MV}_{jm}(\mathcal{G}^f,\Omega_m^{-f})$, and $\widehat{MV}_{jm}(\mathcal{A}^f,\Omega_m^{-f})$, respectively. For inequality (14), I build instruments based on percentile category bins of $\hat{\bar{x}}_{jm}$ i.e.,

$$\mathbb{1}\{\hat{\bar{x}}_{jm}\in Q_\tau(\hat{\bar{x}}_{jm})\}$$

where Q_τ denotes a percentile category bin τ.[30] Similarly, I construct instruments of inequality (15) according to $\mathbb{1}\{\hat{\underline{x}}^h_{jm}\in Q_\tau(\hat{\underline{x}}^h_{jm})\}$. For the Stage 1, upper and lower bound inequalities, I construct percentile category bin indicators, respectively, of the form,

$$\mathbb{1}\Big\{\sum_{m\in\mathcal{M}}\hat{\bar{x}}_{jm}\in Q_\tau\Big(\sum_{m\in\mathcal{M}}\hat{\bar{x}}_{jm}\Big)\Big\}\text{ and }\mathbb{1}\Big\{\sum_{m\in\mathcal{M}}\hat{\underline{x}}^l_{jm}\in Q_\tau\Big(\sum_{m\in\mathcal{M}}\hat{\underline{x}}^l_{jm}\Big)\Big\}.$$

These instruments are positive and exogenous ($\mathcal{I}$-measurable), given that they only depend on factors that firms know at the time of making their product entry decisions.[31] Interacting the moments with indicators for different levels of an exogenous measure of the profitability of the product (or product-market) provides information on the fixed cost parameters. On the one hand, interactions with products (or product-markets) that are apparently unprofitable provide an upper bound: fixed costs must be small enough to rationalize that some of them enter. On the other hand, interactions with seemingly profitable products provide a lower bound: fixed costs must be large enough to rationalize the empirical fact that not all such products enter. To implement the moment inequalities empirically, I compute the empirical counterparts of the moments in Theorem 1, interacted with the discussed instruments. I report such empirical analogs in Appendix D.3.

[29] More precisely, one requires instruments that are positive and $\mathcal{I}$-measurable.

[30] For instance, if we have data on X_i, and we construct $Q_1(X_i)$, $Q_2(X_i)$, and $Q_3(X_i)$, then $x_i\in Q_1(X_i)$ if and only if x_i lies below the 33th percentile of the empirical distribution of X_i.

[31] See Appendix D.2 for a detailed description of the construction of instruments. In the main specifications, I use 3 bins for the Stage 2 inequalities, 5 bins for the Stage 1 inequalities, and polynomials of the PPML-predicted values.

To conduct inference, I proceed sequentially by constructing confidence sets using Andrews and Soares (2010). First, I construct a confidence set for (θ_e, σ_e) using the empirical analogs of inequalities (14)-(15), interacted with the instruments. Then, I construct a confidence set for (θ_g, σ_g) using the empirical analogs of the portfolio introduction moment inequalities (see (A36)-(A37) in the Appendix) interacted with the instruments, and the confidence set obtained for (θ_e, σ_e) in the first step. Throughout, I perform a grid search and apply a Bonferroni correction to account for the multiple-testing issue. For the (θ_e, σ_e) confidence set, I rely on asymptotics as $JM \to \infty$. The (θ_g, σ_g) confidence set relies on asymptotics as $J \to \infty$.

An issue with inference in this setting is that, due to the strategic nature of the model, expectational errors are correlated, leading to the violation of statistical independence across observations. Ideally, one would observe many realizations of the global equilibrium and implement inference relying on asymptotics as the number of games goes to infinity, but this is not feasible in practice. Recent literature has begun to provide approaches to inference that rely on a unique realization of a Bayesian game and many players for asymptotics, much as in my setting. Menzel (2016) develops an asymptotic theory for discrete action games with many players. As the number of players goes to infinity, firms can more precisely forecast all payoff-relevant aspects of the market structure due to the law of large numbers.

In Appendix G.3, I address these potential inference issues by simulating a solvable version of the model and showing that the Andrews and Soares (2010) confidence sets perform well in terms of coverage even if a single cross-section is used to estimate the fixed costs. I replicate my estimation procedure with simulated data and show that undercoverage is very limited even when using a robust variance-covariance matrix, particularly when the number of firms is large.

5 Estimation Results

In this section, I report the estimates of model parameters. The timing of the model implies that estimation can be done in 3 steps. First, I estimate demand and marginal cost parameters. Then, I sequentially estimate $(\theta_e, \sigma_e, \theta_g, \sigma_g)$ in two steps, as described in Section 4.

5.1 Demand and Marginal Costs

Demand estimation follows Petrin (2002). I use Gandhi and Houde (2019) differentiation instruments and micro-moments that match the probability that buyers of different incomes purchase vehicles in different price ranges. The key identifying assumption leverages the assumption that demand and marginal cost shocks are realized after firms make their portfolio and market entry choices. This guarantees that $\mathbb{E}[(\xi_{jm}, \omega_{jm})|\mathcal{I}] = 0$, which makes standard Berry et al. (1995) or Gandhi and Houde (2019) instruments valid. Additional details about the implementation are included in Appendix D.1.

Table 1 reports the demand and marginal cost estimates. The signs and magnitudes of all estimates are consistent with previous estimates in the literature. Moreover, all of the key parameter

Table 1: Stage 3 Parameter Estimates

	Parameter estimate	**Standard error**
Demand		
Mean parameters		
price (α_0)	2.88	0.690
home market	1.01	0.153
horsepower (log)	5.00	2.46
horsepower/weight (log)	-2.19	1.44
Non-linear parameters (price coefficient)		
Income (α_1)	-0.790	0.117
China	-1.51	0.297
Shock Std (σ^y)	0.809	0.131
Marginal Costs (log)		
electric	0.340	0.051
hybrid	0.272	0.030
horsepower/weight (log)	-0.426	0.111
horsepower (log)	1.00	0.112
size (log)	0.251	0.209
distance to brand HQ (log)	0.062	0.007
Observations	1,414	
Mean Share-Weighted Implied Own Price Elasticity	-8.41	
Percent Implied Negative Marginal Costs	0	

Notes: The demand specification includes body-type and electric-hybrid dummies interacted with market dummies. It also includes size interacted with market dummies. Both specifications include brand and market fixed effects. Standard errors are clustered at the brand level.

estimates are significant at the 95% significance level. The mean share-weighted implied own-price elasticity is -8.41 across all countries, higher (in absolute value) than that obtained for the United States in Grieco et al. (2023) (they find it to be -6.37 in 2018). The coefficient of -0.790 for the interaction of each buyer's income with its price sensitivity is highly significant and in line with previous estimates of around -1 in the literature (e.g., Coşar et al. 2017 obtain a point estimate of -0.997). The coefficient on the home market dummy shows that consumers are willing to pay a substantial premium to purchase from a local brand. The coefficient of 1.01 implies that, on average, across the world income distribution, consumers' additional utility from a home brand is valued, all else equal, equivalently to a $1,518 US dollars decrease in price. Consumers significantly value vehicles with greater horsepower and vehicles with less horsepower/weight, which is negatively correlated with fuel efficiency (an omitted variable in my data set). Finally, interacting the price coefficient with a China dummy proves to be important. The coefficient of -1.51 is highly significant and shows that all else equal, Chinese consumers appear to be less price-sensitive. I hypothesize that this is due to aggressive and unobserved demand-side policies that render the MSRP an overestimate of consumer prices in China. I interpret this dummy as a control for such unobserved policies in China, which I will hold fixed in counterfactual policy experiments.

Marginal cost estimates align with prior institutional knowledge of the industry. In my sample period, supplying an electric vehicle is substantially more costly than a hybrid or internal combustion engine (ICE) vehicle. The marginal cost of ICE vehicles is around 31% (40%) lower than that of a hybrid (electric) vehicle. It is also more costly to supply a vehicle with higher horsepower and size, though the latter coefficient is not statistically significant. It is less costly to supply vehicles with higher horsepower/weight. Finally, it is also substantially more expensive to supply markets that are distant from the brand's headquarters country. A 1% increase in distance from the headquarters country raises the marginal cost of supplying a given market by around 0.06%.

Marginal costs are recovered from prices given the Nash-Bertrand conduct assumption, so they contain information on optimal sourcing decisions from 2019 production locations. I treat these as fixed in counterfactual experiments and interpret my marginal cost parameters and fixed effects as capturing firms' sourcing possibilities.

Given demand estimates and the Nash-Bertrand pricing assumption, I compute $MV_{jm}(\mathcal{G}^f, \Omega_m^{-f})$, $MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})$, and $MV_{jm}(\{j\}, \Omega_m^{-f})$, which are the pieces of "data" required for my moment inequality procedure.[32]

5.2 Fixed Costs

Implementing the moment inequality procedure, I separately identify the parameters determining the distributions of global product portfolio and market entry fixed costs. In Section 2, I provided descriptive evidence that both portfolio and market entry costs matter for understanding cross-sectional product offering patterns. The goal now is to learn about the magnitude of the fixed costs through the lens of the structural model. Obtaining these estimates is a first step towards quantifying policy effects using my framework.

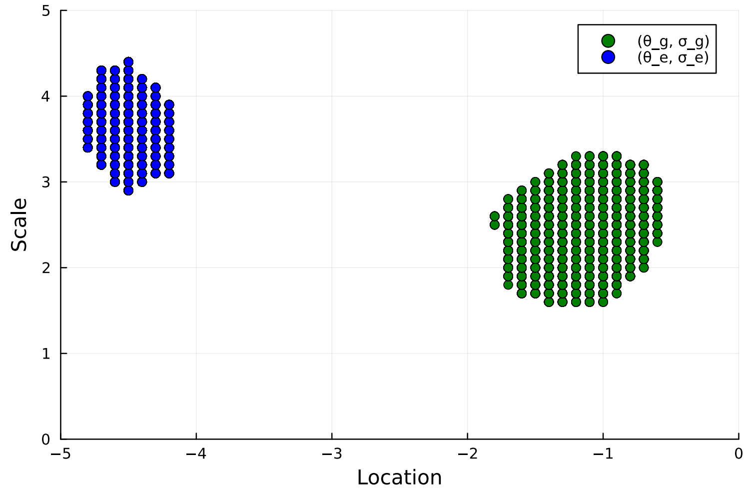
In a world with high global portfolio fixed costs and low product market entry costs, firms would sell similar (or the same) varieties across all countries. In such a setting, reduced globalization or policies limiting firms' ability to leverage international markets to scale up their operations are likely to significantly affect product offerings and market outcomes across countries. Instead, a world with low global portfolio fixed costs and large market entry costs is a world of independent markets, where global market integration plays a small role in shaping industry outcomes. In which world do we live in?

Figure 6 and Table 2 provide evidence on this question. Figure 6 projects the 4-dimensional 95% confidence set for $(\theta_e, \sigma_e, \theta_g, \sigma_g)$ into a 2-dimensional grid by plotting the (θ_e, σ_e) and (θ_g, σ_g) separately. Recall that these parameters determine the location and scale parameters of the log-normal fixed cost distributions. Table 2 reports the confidence set limits for all 4 parameters.

The estimates show that the distribution of product fixed costs has a higher median than the distribution of market entry fixed costs. The estimates imply that the median fixed cost of adding

[32] Ideally, one should account for the statistical uncertainty regarding demand and marginal cost estimates when computing standard errors for fixed cost parameters. One way of doing this is by bootstrapping the entire estimation procedure. Given the computational burden of this approach, I currently treat these marginal values as data and do not account for the statistical uncertainty regarding demand and marginal cost parameters when computing confidence sets for the fixed cost parameters.

Figure 6: 95% Confidence Set



Notes: This figure projects the 95% confidence set for the location and scale parameters describing the distributions of product portfolio and market entry fixed costs on a two-dimensional grid on the location-scale dimensions.

Table 2: Stages 1 and 2 Parameter Confidence Set Limits

	95% Confidence Set Limits
Stage 2: Market Entry Fixed Cost	
θ_e (Location)	[-4.8, -4.2]
σ_e (Scale)	[2.9, 4.4]
Stage 1: Product Fixed Cost	
θ_g (Location)	[-1.8, -0.6]
σ_g (Scale)	[1.6, 3.3]
Observations - Stage 2	3240
Observations - Stage 1	739

Notes: Confidence sets are computed using Andrews and Soares (2010). First, I implement a grid search to compute a 97.5% confidence set for parameters (θ_e, σ_e) using the Stage 2 moment inequalities. Then, I use the Stage 1 moment inequalities to compute a 97.5% confidence set for (θ_g, σ_g), evaluating the moments at the accepted values of (θ_e, σ_e). The Bonferroni correction yields a 95% confidence set for all 4 parameters. Marginal values are in billions of US dollars.

an additional product to a firm's portfolio is $138-549 million.[33] Meanwhile, the estimates imply that the median fixed cost of offering a product in a market is $8-15 million per product. This suggests that scale economies at the product level are important and are likely to generate strong

[33] A downside of this product definition is that it abstracts away from studying the entry of Toyota Corollas or Honda Civics, which is a more common product definition in this industry. An advantage of aggregating at a coarser rather than at a finer level is that it reduces the concern that product development costs are shared across product categories. For instance, developing a Toyota Corolla might be very inexpensive once the Camry has been designed, so assuming that this is a joint decision might be a decent approximation. Meanwhile, a Toyota SUV is a substantially different product line, with a different chassis or marketing campaign. For similar reasons, products of different fuel types are likely to face different development costs. I see my product definition as a compromise between respecting product differentiation on the dimensions that matter most for consumers while reducing the concern that the bulk of the product development cost is shared across different product categories.

interdependence in global market outcomes in the automobile industry.

Moreover, the estimates reveal sizeable scale parameters for both the global portfolio and market entry fixed cost (log-normal) distributions, which determine the variance of (log) fixed costs from the perspective of the researcher. Such scale parameters rationalize why some firms make some apparently unprofitable choices and why they do not make some seemingly profitable choices. Thus, these parameters reflect complexities that firms observe, and researchers do not.[34]

The magnitudes of the estimates accord well with other evidence in the literature. Wollmann (2018) estimates that, in the truck industry, it costs \$5-25 million on average to introduce a product into the United States. My median market entry costs of \$8-15 million per product lie within this range. While I am unaware of previous estimates of product portfolio costs in the automotive industry, I follow the approach in Wollmann (2018) to convert my cross-sectional estimates into "dynamic" estimates. Under a discount factor of 0.9 (a hurdle rate of 0.1), my estimates imply a median of \$1.4-5.5 billion per product.[35] Meanwhile, IHS Global reports that the cost of developing and maintaining product lines in the automotive industry is \$1-6 billion per product, which aligns with my estimates (Autoblog 2010).

A limitation of identifying fixed costs from cross-sectional product offerings is that it makes it impossible to distinguish fixed from sunk costs fully. My preferred interpretation of the estimates is that, through the lens of a static model, I capture "steady state" magnitudes that rationalize the observed product offerings throughout the globe. While incorporating dynamics is definitely an important direction for future research, my framework captures many important features of the automobile industry and is capable of separately identifying product and market entry fixed costs.

Finally, these results are highly robust to the choice of instruments. In Appendix D.4.1, I show that even with more instrument bins, it is not possible to reject the model by obtaining an empty confidence set. Much as in an over-identification test, the fact that I fail to reject the model with a greater number of moments increases my confidence in the results.

6 Solution Method for Product Entry Games

Computing the Bayesian Nash equilibria of the model is computationally challenging but is required to study counterfactual policy experiments. In these settings, it is desirable to have a solution method that is computationally feasible and robust to equilibrium multiplicity. In this section, I develop a method with these properties that bounds the distribution of firms' equilibrium product offerings in each market given any policy environment implied by the information set $\mathcal{I}$.

Given any information set $\mathcal{I}$, and implied equilibrium distribution of product offerings in market m, which I denote by $\boldsymbol{\mu}_m^*$, the algorithm yields bounds $\underline{\boldsymbol{\mu}}_m$ and $\overline{\boldsymbol{\mu}}_m$ such that,

[34] The estimated scale parameters could potentially also reflect omitted variables from the fixed cost specification. For instance, the specification for market entry costs does not allow for systematic variation with the distance to the brand's headquarters.

[35] This approach converts fixed costs estimated from a cross-section into sunk costs in a model that assumes that firms choose product offerings myopically without anticipating future changes in market structure.

$$\underline{\boldsymbol{\mu}}_m \leq_{FOSD} \boldsymbol{\mu}_m^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m. \tag{16}$$

This solution approach does not rely on approximation methods nor equilibrium selection rules (e.g., order of entry assumptions) and is computationally feasible to implement even in settings with many firms making many discrete choices.[36]

Throughout this section, I treat the set of fixed cost parameters $(\theta_e, \sigma_e, \theta_g, \sigma_g)$ determining the distributions of global portfolio and market entry fixed costs as known for ease of exposition. Recall that the CDF of the fixed cost of offering product j in market m is denoted by Γ_{jm}, while the CDF of the global portfolio fixed cost is denoted by Λ_j.

The solution method relies on the same baseline inequalities that bound choice probabilities discussed in Section 4.1. Given any policy environment $\mathcal{I}$, from inequalities (11), I obtain,

$$\begin{aligned}\Gamma_{jm}\big(\mathbb{E}_{\boldsymbol{\mu}_m^*}\big[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}\big]; \theta_e, \sigma_e\big) &\leq \mathbb{P}(O_{jm} = 1|\mathcal{I}, G_j = 1) \\ &\leq \Gamma_{jm}\big(\mathbb{E}_{\boldsymbol{\mu}_m^*}\big[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}\big]; \theta_e, \sigma_e\big).\end{aligned} \tag{17}$$

The inequalities in (17) provide bounds on the probability that product j is offered in market m, conditional on being in the firm's global portfolio.[37]

In Appendix B.2, I derive the corresponding inequalities that bound the probability that product j is introduced into the firm's product portfolio. Recall from inequalities (20) that,

$$\tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\boldsymbol{\mu}_m^*}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big) \leq \mathbb{P}(G_j = 1|\mathcal{I}) \leq \tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\boldsymbol{\mu}_m^*}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big), \tag{18}$$

where $\tilde{\Lambda}_j$ is increasing in the expected marginal values of offering product j across markets.

As discussed in Section 4, the inequalities in (17) and (18) follow from Assumptions 1 and 4. Upper bounds are obtained by evaluating expected changes in profits at minimal cannibalization (only that product is offered by firm f), while lower bounds are obtained by computing expected changes in profits at maximal cannibalization (the product is offered alongside all other firm f potential products).

In the model, firms care about the true distribution of rivals' offerings in each market, $\boldsymbol{\mu}_m^*$. In Appendix C, I show that provided fixed cost draws are independent conditional on $\mathcal{I}$, the product of the lower and the upper bounds in inequalities (17) and (18) yield bounds on the probability that product j is offered in market m, $\mathbb{P}(O_{jm} = 1|\mathcal{I})$.

In practice, neither inequalities (17) nor (18) are useful to solve the model because the expectation entering the fixed cost CDFs depend on $\boldsymbol{\mu}_m^*$ in each market m, which are the objects of

[36] Common heuristic approaches in the literature approximate Nash equilibria by iterating over deviations from a given action vector in payoff-improving directions, such as the "greedy" algorithm or simulated annealing.

[37] For notational simplicity I have omitted demand and marginal cost shocks $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$, but recall that the expectation operator inside the CDF function is not only with respect to the *true* distribution of rival entry decisions Ω_m^{-f} under $\boldsymbol{\mu}_m^*$, but also under demand and marginal cost shocks which will be realized in Stage 3 of the game. Also, note that the lower bound inequality differs from the lower bound in inequality (11) in that I now use all of the firm's potential products as the extreme bundle instead of conditioning on the firm's chosen portfolio $\mathcal{G}^f$. This helps to deal with subgame perfection, as shown in Appendix C.

interest in this section. To address this issue, I exploit Assumption 4 to initialize the algorithm. Under submodularity, I derive the weakest possible bounds on $\boldsymbol{\mu}_m^*$ by integrating over rival offerings according to $\overline{\boldsymbol{\mu}}_m^0$ and $\underline{\boldsymbol{\mu}}_m^0$, degenerate random vectors such that $\overline{\mu}_{jm}^0 = 1$ (product j is introduced in market m with probability 1) and $\underline{\mu}_{jm}^0 = 0$ (product j is introduced in market m with probability 0) for all $j \in \mathcal{A}$. That is,

$$\begin{aligned}\Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^0}\big[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}\big]; \theta_e, \sigma_e\big) &\leq \mathbb{P}(O_{jm} = 1|\mathcal{I}, G_j = 1) \\ &\leq \Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^0}\big[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}\big]; \theta_e, \sigma_e\big), \end{aligned} \tag{19}$$

and

$$\tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^0}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big) \leq \mathbb{P}(G_j = 1|\mathcal{I}) \leq \tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^0}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}]\big\}_m\Big). \tag{20}$$

Submodularity permits the computation of the weakest possible bounds on the marginal value of any product j in each market m. The highest marginal value of product j in each market is obtained by evaluating it at the least competitive conditions possible: only product j (and the outside option) is offered in market m. The lowest possible marginal value of product j in market m is obtained under the hypothesis that all potential products are simultaneously being offered, the most competitive conditions possible. Evaluating the marginal values at such extreme bundles in market m yields bounds on the probability that product j is offered in such a market conditional on being in the firm's portfolio, given by the inequalities in (19). Evaluating the marginal values at such extreme bundles across all markets yields bounds on the probability that product j is introduced in the firm's portfolio, given by the inequalities in (20). Computing the product of the two upper bounds and the two lower bounds in inequalities (19) and (20) gives an upper and a lower bound on the probability that product j is offered in market m: $\overline{\mu}_{jm}^1$ and $\underline{\mu}_{jm}^1$, respectively.

I implement this procedure for each potential product $j \in \mathcal{A}$ and each market $m \in \mathcal{M}$. In Appendix C, I show that,

$$\underline{\boldsymbol{\mu}}_m^1 \leq_{FOSD} \boldsymbol{\mu}_m^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^1,$$

where $\underline{\boldsymbol{\mu}}_m^1$ and $\overline{\boldsymbol{\mu}}_m^1$ are vectors of independent (conditional on $\mathcal{I}$) Bernoulli random variables. Thus, bounds on the marginal probabilities of offering each product, $\mathbb{P}(O_{jm} = 1|\mathcal{I})$, bound the *joint* distribution of product offerings in each market. Moreover, while any equilibrium distribution of offerings $\boldsymbol{\mu}_m^*$ in market m exhibits within-firm interdependence arising from cannibalization, the bounding random vectors $\underline{\boldsymbol{\mu}}_m^1$ and $\overline{\boldsymbol{\mu}}_m^1$ are instead vectors of independent random variables. This result holds because (i) the within-firm bounds eliminate any dependence within the firm arising from the multi-product problem, and (ii) firms' inability to observe rivals' fixed cost shocks means that conditional on $\mathcal{I}$, entry decisions across firms are independent.

Then, I show that one can tighten the initialized bounds by simulating,

$$\Gamma_{jm}\Big(\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^1}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}];\theta_e,\sigma_e\Big) \leq \mathbb{P}(O_{jm}=1|\mathcal{I},G_j=1) \leq \Gamma_{jm}\Big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^1}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}];\theta_e,\sigma_e\Big), \tag{21}$$

and

$$\tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^1}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}]\big\}_m\Big) \leq \mathbb{P}(G_j=1|\mathcal{I}) \leq \tilde{\Lambda}_j\Big(\big\{\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^1}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}]\big\}_m\Big), \tag{22}$$

where inequalities (21) and (22) use the initialized bounds $\underline{\boldsymbol{\mu}}_m^1$ and $\overline{\boldsymbol{\mu}}_m^1$ to compute the expectation over rival firms' offerings.[38]

As before, I multiply the upper bounds and lower bounds in inequalities (21) and (22) across all products $j \in \mathcal{A}$ to obtain vectors of conditionally independent Bernoulli random variables $\overline{\boldsymbol{\mu}}_m^2$ and $\underline{\boldsymbol{\mu}}_m^2$, respectively, which also bound the distribution of product offerings conditional on $\mathcal{I}$.

The intuition is that, due to substitution across firms arising from Assumption 4, "dominated" strategies can be iteratively eliminated. For instance, if the probability that a product is offered at the most competitive conditions is high, it must be weakly higher in any equilibrium. Similarly, if the probability that a product is offered at the least competitive conditions possible is low, it must be weakly lower in any equilibrium. All firms know how to compute these maximal and minimal probabilities conditional on $\mathcal{I}$, so iterating on this logic provides bounds on any equilibrium distribution of product offerings, as stated in Theorem 2.

Theorem 2 *Under Assumptions 1-4, the iterative algorithm converges monotonically to bounds, in the sense of first-order stochastic dominance, of any equilibrium distribution of product offerings in each market m given any information set $\mathcal{I}$. That is, for any iteration $k>0$ and any $m \in \mathcal{M}$,*

$$\underline{\boldsymbol{\mu}}_m^{k-1} \leq_{FOSD} \underline{\boldsymbol{\mu}}_m^k \leq_{FOSD} \boldsymbol{\mu}_m^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^k \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^{k-1}.$$

Proof. See Appendix C for the proof and formal arguments. ■

The proof leverages submodularity, which implies that MV_{jm} is a decreasing function, and the fact that a probability distribution X first-order stochastically dominates Y if and only if for every monotonically decreasing function f, $\mathbb{E}[f(X)] \leq \mathbb{E}[f(Y)]$.[39]

To summarize, the method provides a mapping from parameters and exogenous product characteristics and the policy environment to bounds on the joint distribution of product offerings implied by the Bayesian Nash conduct assumption. This is related to how, in Berry et al. (1995), exogenous variables and parameters, together with the Nash-Bertrand assumption, yield equilibrium markups.

[38] I simulate $T=100$ sets of $J \times M$ uniformly distributed draws to compute the expectation for each product j and each market m and for each of the bounds. Details on the implementation are provided in Appendix C.1.

[39] In Appendix E, I show how a similar algorithm can be derived if the primitive variable profit function exhibits global supermodularity rather than global submodularity within each market.

7 The Impact of National US Policies on Global Market Outcomes

In recent years, the world has witnessed the resurgence of aggressive industrial and trade policies. In the automotive industry, policies promoting clean vehicles have been implemented across jurisdictions. Such policies include highly generous production and consumption tax credits or subsidies as part of policy packages such as the Inflation Reduction Act in the United States or the European Green Deal. Aggressive policy has long been prevalent in China.

To assess the impact of subsidies implemented in a large market on market structures across the globe, I study the effects of marginal cost and consumer subsidies favoring US-headquartered brands in the United States. The counterfactual experiments disentangle the effects of such policies on market outcomes in other countries, both through the intensive margin (prices/quantities) and through the extensive margin (products offered) in a world of interdependent product offerings. They also demonstrate the quantitative importance of product entry in determining profit-shifting and consumer surplus effects in other markets.

I focus on the protectionist rather than the environmental consequences of these policies to reduce concerns about the lack of dynamics in my model. Focusing only on the entry of electric and hybrid vehicles is difficult through the lens of a model that holds 2019 market conditions (including preference and cost parameters) fixed. Moreover, while many of the policies implemented in the United States outline incentives contingent on the production location of the brand rather than the location of its headquarters, I consider a policy that favors only brands headquartered in the United States. According to publicly available data at the *Good Jobs First Subsidy Tracker* (Good Jobs First 2024), brands with US headquarters are the primary recipients of US federal and state subsidies. It is also a good approximation of policies that have been implemented in China, which have overwhelmingly benefited brands with local headquarters.

To assess the quantitative impact of national policies on global market outcomes while allowing for endogenous product portfolio responses, I use the iterative algorithm described in Section 6. For instance, because consumer surplus is increasing in the set of products offered, I bound expected consumer surplus given any policy environment $\mathcal{I}$ using,

$$\mathbb{E}_{\underline{\boldsymbol{\mu}}_m}[CS_m|\mathcal{I}] \leq \mathbb{E}_{\boldsymbol{\mu}_m^*}[CS_m|\mathcal{I}] \leq \mathbb{E}_{\overline{\boldsymbol{\mu}}_m}[CS_m|\mathcal{I}].$$

I follow a similar approach to bound other outcomes of interest, such as brand-level market shares and variable profits.[40]

Computing counterfactual experiments in a manner robust to equilibrium multiplicity is crucial, particularly if policymakers are interested in profit-shifting motives, where the identity of entrants

[40] To simulate a lower bound on brand-level shares and profits, I integrate other brands' entry choices using $\overline{\boldsymbol{\mu}}_m^{-b}$ and brand b's entry choices using $\underline{\boldsymbol{\mu}}_m^{b}$. To simulate an upper bound on brand-level shares and profits, I integrate other brands' entry choices using $\underline{\boldsymbol{\mu}}_m^{-b}$ and brand b's entry choices using $\overline{\boldsymbol{\mu}}_m^{b}$. I report bounds on counterfactual outcomes under a point in the confidence set: $(\theta_e, \sigma_e, \theta_g, \sigma_g) = (-4.5, 3.6, -1.2, 2.5)$. This point lies in the midpoint of the 95% confidence set limits. In counterfactual experiments, I compute expectations with respect to the solved bounds on the equilibrium distribution of offerings in each market and over demand and marginal cost shocks. In Appendix H, I show that the conclusions do not change significantly at different points in the confidence set.

is of interest. Suppose that there are two equilibria – one where it is more likely for Ford to introduce additional products and another where it is more likely for Geely (a Chinese brand) to introduce additional varieties. Whether the subsidy leads to one equilibrium or another could potentially have different effects on firms' profits and consumer surplus.

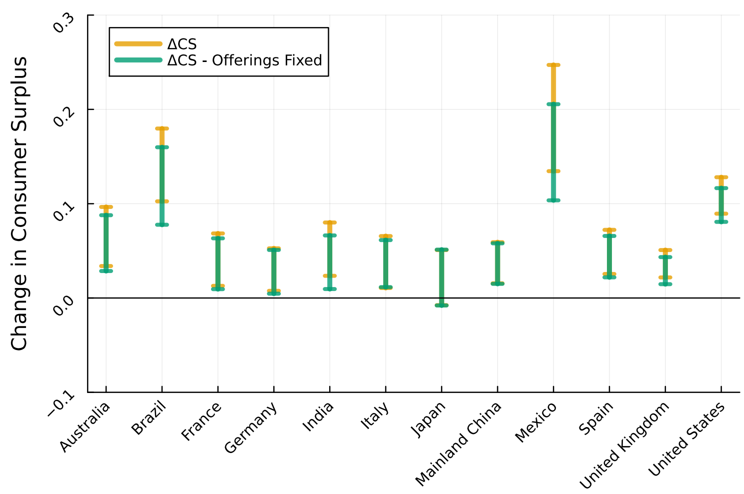
7.1 Effect of a 20% Production Subsidy on American Brands

I study the equilibrium effects of a policy that leads to a 20% reduction in the marginal cost c_{jm} of each product potentially offered by a brand with headquarters in the United States in any market. The American brands that receive the production subsidy include Buick, Cadillac, Chevrolet, Chrysler, Dodge, Ford, GMC, Jeep, Lincoln, and Tesla. The actual policy outlined in the Inflation Reduction Act targets clean and energy-efficient vehicles with 10% production subsidies for stages like critical mineral or electrode active material production, in addition to a subsidy of \$35 per kilowatt-hour for battery cell manufacturing and \$10 per kilowatt-hour for battery module assembly, according to the 45X production tax credit (Banks 2023). In addition to federal policies, there are a wide range of additional state-level production incentives (Good Jobs First 2024).

7.1.1 Consumer Surplus

First, I report in Figure 7 the effects of the policy on consumer surplus across countries. The intervals in orange report bounds on the overall effects, accounting for changes in prices and quantities (intensive margin) as well as changes in product offerings (extensive margin). The intervals

Figure 7: Change in Consumer Surplus



Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 20% marginal cost reduction for US brands. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented. The intervals in green only reflect the intensive margin response.

in green showcase the effects of the policy holding constant bounds on the distribution of product offerings at the pre-policy values.

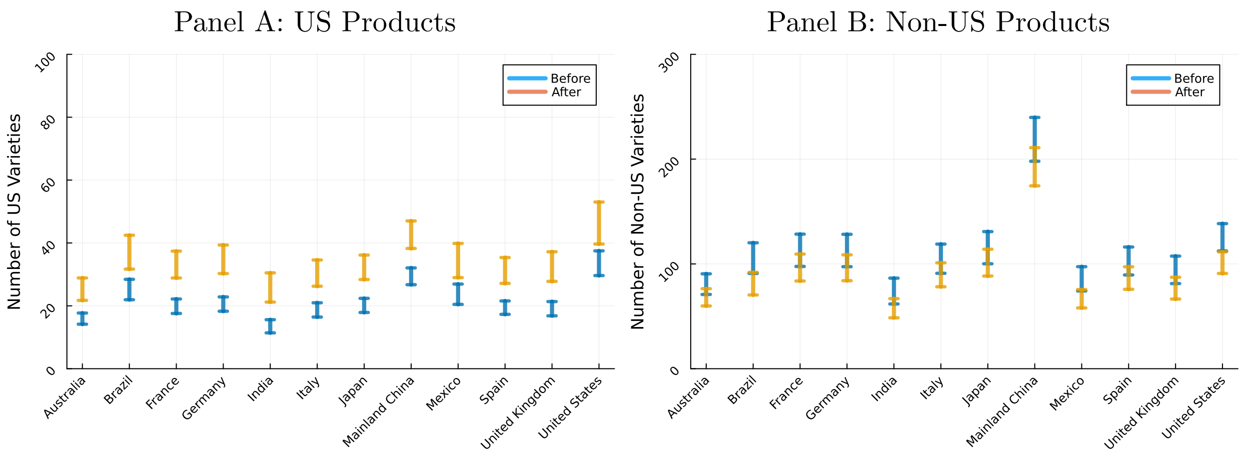
Figure 7 shows large minimum gains in consumer surplus throughout many countries, with gains of over 8% in the United States, Mexico, or Brazil, and minimum gains greater than 3% in India. The 20% decline in the marginal cost of producing US products has significantly larger effects in poorer nations like Brazil than in richer nations like Germany. This illustrates the importance of preference heterogeneity in understanding the global repercussions of national industrial policies. In this case, lower-income countries benefit more from access to cheaper US products than richer nations due to their stronger distaste for high prices. Moreover, the United States benefits more than other rich countries due to home bias – American consumers prefer American brands and thus benefit more from the decline in costs than Germans or Italians.

The effects of the policy on consumer surplus do not seem to reflect changes on the extensive margin. While the upper and lower bounds of the green intervals lie slightly below those of the orange intervals, such differences are small. This suggests either that (i) product entry does not change in response to the policy or (ii) there are offsetting changes in market structure from the point of view of consumers. To shed light on this, I now report the effects on brand-level outcomes.

7.1.2 Products Offered

Figure 8 shows the effect of the policy on the composition of products offered across countries. Panel A shows that the policy leads to an expansion of the sets of products offered by US brands throughout the globe. Lower marginal costs generate greater profitability in all markets, which leads to a greater number of product offerings. Panel B shows that because of competition from

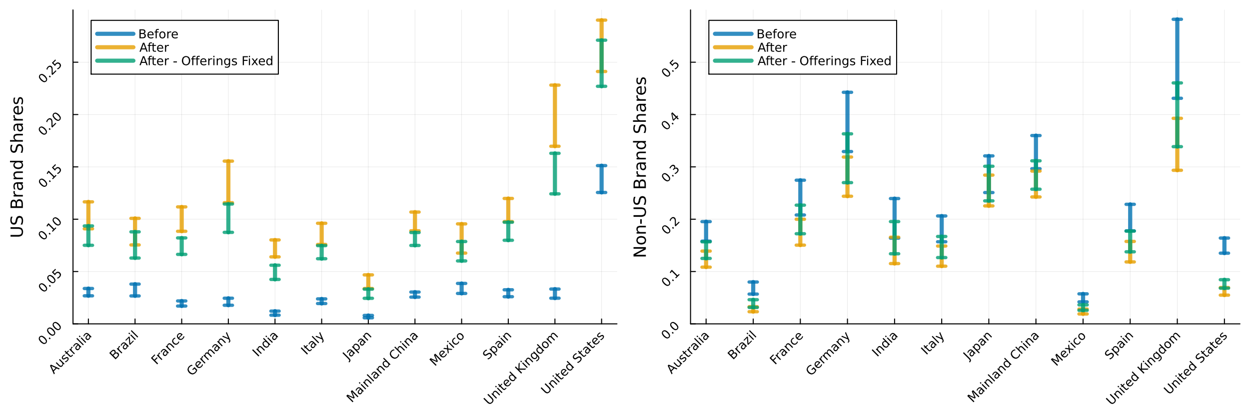
Figure 8: Number of Products Offered



Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

Figure 9: Market Shares

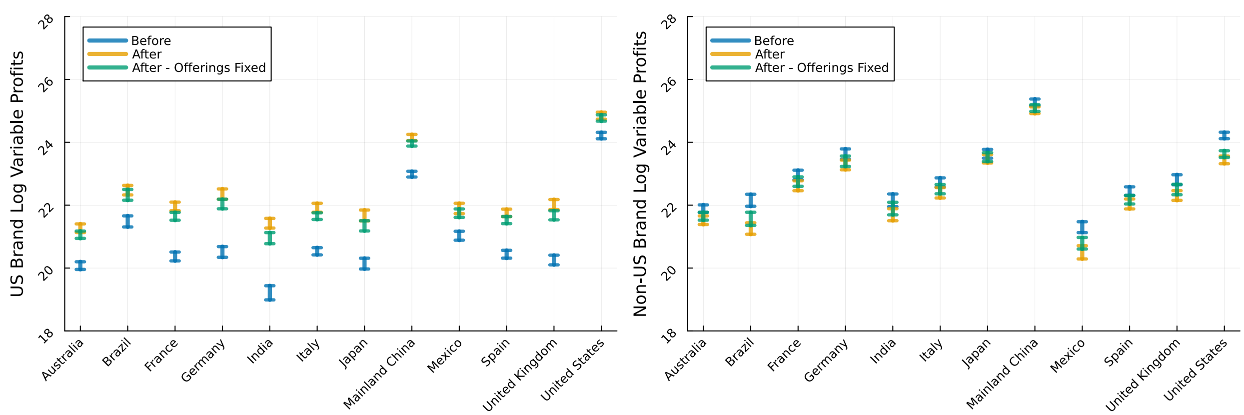
Panel A: US Products   Panel B: Non-US Products



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected total market share of non-US brands before and after the policy is implemented.

Figure 10: Variable Profits

Panel A: US Products   Panel B: Non-US Products



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.

US brands, non-US brands tend to downsize their product offerings across markets, with both the upper bound and lower bound number of non-US varieties declining across the globe.

7.1.3 Market Shares and Variable Profits

Figure 9 shows the overall effect of the policy on the market shares of US brands. Comparing the blue intervals to the orange intervals in Panel A, there are very significant and heterogeneous effects of the policy across markets. In the United Kingdom, the increase in the share of US brands is greater than 10 percentage points, whereas in Japan, only 3 to 4 percentage points. In the United States, there is also a substantial increase of at least 7 percentage points.

Importantly, product entry amplifies the increase in the share of US brands, as seen by comparing the green to the orange intervals in Panel A. Across most non-US jurisdictions, product entry accounts for over 25% of the rise in the lower and the upper bound of the US-brand market shares. Knowledge of the subsidy causes US brands to expect higher profitability in all markets, which enables them to strengthen their position by offering more products. Meanwhile, non-US brands anticipate reduced profitability due to their relatively greater cost of production. Their expected market share declines by more after accounting for the endogenous exit of non-US brands.

In Figure 10, I plot bounds on expected (log) total variable profits of US brands before and after the subsidy is implemented. The plot shows very substantial increases in profits of more than 1 log point (172%) in countries like the United Kingdom, India, or Germany and very sizeable increases in variable profits across most jurisdictions, with the smallest minimum increase in the United States. As with market shares, the rise in profits (comparing the green to the orange intervals) would be underestimated if one ignored endogenous product offering changes. A similar and reverse story can be told for non-US brands, who observe a decline in their expected variable profits across most markets, though the decline is not as large as the rise in US brands' profits.

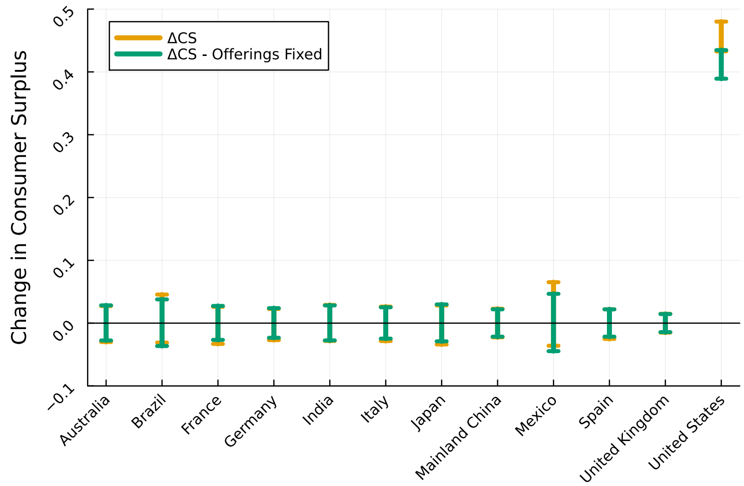
7.2 Effect of a 50% Consumption Subsidy on American Brands

In this section, I study the effect of a 50% US consumption subsidy on products offered by US brands. The motivation for studying the effect of a consumer-side subsidy is the large and generous consumer-side policies implemented throughout the world providing incentives to purchase clean vehicles. As with production-side subsidies, many of these policies have implicitly or explicitly favored local brands over foreign brands. In China, a highly protected automotive market since its inception, subsidies favoring Chinese-made electric vehicles covered around 40-60% of their price in 2014 (Lu 2018). In the United States, clean vehicles are heavily subsidized with tax credits of up to $7,500, with Buy American incentives contingent on local final assembly.

7.2.1 Consumer Surplus

First, I report the effects of the policy on consumer surplus worldwide. Figure 11 shows a large minimum increase in consumer surplus in the United States following the policy of over 42%. Moreover, ignoring product entry would lead to underestimating the rise in consumer surplus. In other countries, this large US subsidy has small effects on consumer surplus.

Figure 11: Change in Consumer Surplus



Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 50% consumer subsidy for US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented. The intervals in green only reflect the intensive margin response.

7.2.2 Products Offered

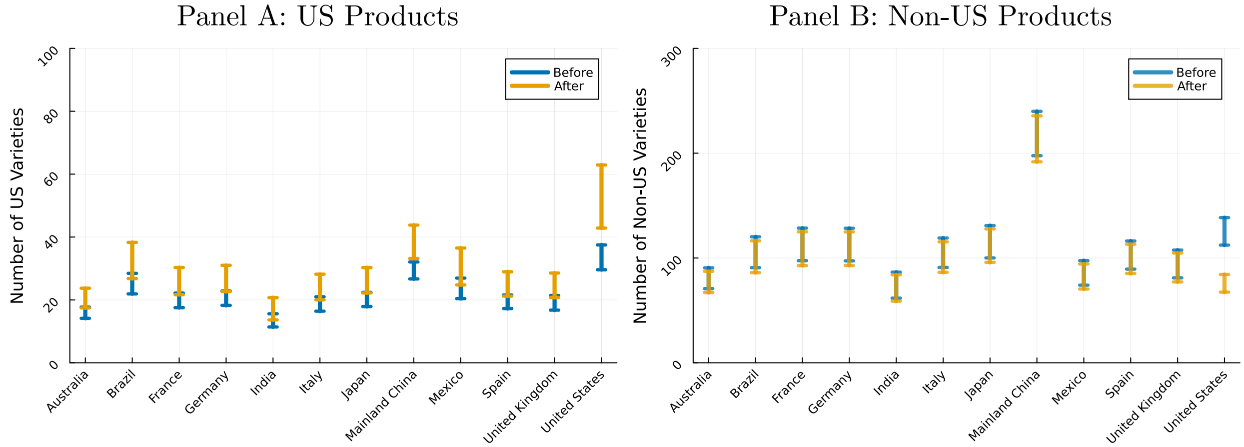
Figure 12 shows the effect of the policy on the composition of products offered across countries. Panel A shows that the policy leads to an expansion of the sets of products offered by US brands throughout the globe. The large consumer subsidy on US brands induces the entry of US-branded products into firms' portfolios, which firms can choose to offer overseas. In the United States, the policy leads to a contraction of varieties offered by foreign brands in light of their reduced profitability. Interestingly, non-US brands do not significantly contract their product offerings in other markets. This suggests that the decline in profits resulting from the discriminatory policy in the United States is not large enough to induce the global exit of many non-US products.

7.2.3 Market Shares and Variable Profits

Figure 13 shows the overall effect of the policy on market shares of US and non-US brands across markets. The policy has a very large effect on the market share of US brands in the United States, with a rise of over 23 percentage points. Non-US brands lose most of their market share in the United States, with a drop of at least 16 percentage points. Interestingly, the rise in the US-brand market share in the United States appears to be mostly driven by the intensive margin, and not by the introduction of new products in the United States. This shows that while the policy has the effect of expanding US brands' offerings, the marginal new product being introduced is relatively unpopular and captures a small market share.

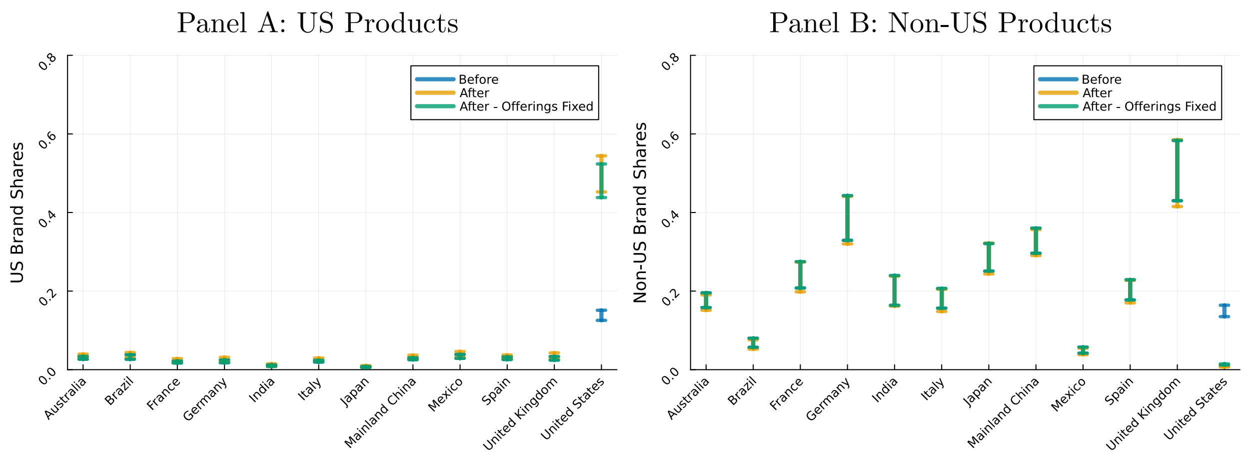
For this reason, the policy does not lead to very significant effects on US brand dominance

Figure 12: Number of Products Offered



Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

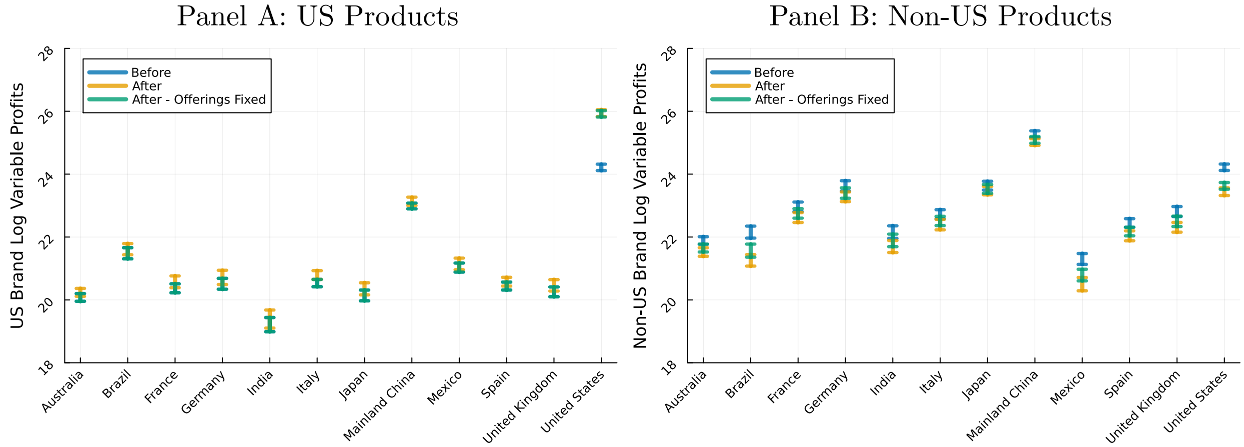
Figure 13: Market Shares



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays similar bounds on the expected total market share of non-US brands before and after the policy is implemented.

outside of the United States. While US brands significantly expand their product offerings in other markets (as shown in Figure 12), the effect on their dominance is small due to small effects on the intensive margin. Consumers are relatively unimpressed by the availability of these new products, which leads to small effects of their offering on brand-level market shares overseas. This contrasts

Figure 14: Variable Profits



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. Panel B displays similar bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.

with the previous exercise, where cheaper US products do threaten foreign brands' positions in markets other than the United States.

Figure 14 shows the effects on variable profits. The effects reported in Panel A and Panel B mirror the previously discussed effects on market shares. While the national consumer subsidy favoring US brands has large effects on US brand dominance in the United States, there is little profit shifting in other markets.

Relative to the production subsidy, the consumer subsidy in the United States can only affect foreign outcomes through product entry and not directly through the intensive margin. This is a major difference between the two policies, which contributes to making the US consumer subsidy have a smaller effect on global market outcomes than the US production subsidy.

8 Conclusion

Changes in product offerings are a major force affecting market structure and outcomes in the automobile industry. Due to its size and effects on the environment, national governments implement ambitious policies that aim to affect which products firms offer. In global industries, national policies affect global market outcomes through firms' product portfolio decisions. Studying changes in global product offerings in response to policy is challenging because of the many interdependent possibilities that firms face.

The key methodological contribution of this paper is to develop a tractable method both to estimate and solve an incomplete-information entry game with multiple asymmetric firms, each making multiple discrete choices. Entry games with asymmetric firms are challenging because (i) there may exist multiple equilibria, each with potentially different policy implications, and (ii) with multiple interdependent and discrete choices, it may be computationally infeasible to find the equilibria of the game. The method overcomes these challenges using novel inequalities that bound the ex-ante equilibrium probabilities of firms' portfolio and market entry choices before the unobserved components (to the researcher) of firms' fixed costs are realized. Such inequalities are the basis for both estimating and solving the model.

First, I estimate large product portfolio fixed costs, which imply that firms have an incentive to offer similar bundles of products across markets. Then, using my solution method, I study the effects of a production and a consumption subsidy favoring US brands. Three key findings emerge: (i) both policies induce entry of additional US products globally, (ii) consumers benefit heterogeneously from the additional entry of US products despite the exit of non-US varieties, (iii) product portfolio choices cause profit shifting towards US brands worldwide under the production subsidy, but not under a consumption subsidy that does not benefit unsubsidized consumers abroad, since marginally introduced products are relatively unpopular. Such findings illuminate the potential effects of currently proposed policies in many different jurisdictions.

With the novel methods developed in this paper, researchers can study the effects of different policies on global market structures or the role of product entry in other industries. International mergers, bans on specific products, or subsidies to electric and hybrid vehicles are examples of different policies that could be quantitatively evaluated with these insights. As in the automobile industry, many other sectors consist of firms that offer differentiated products to consumers with heterogeneous preferences. Additionally, the methods developed in this paper could potentially be extended to other discrete choice models featuring interdependencies. Of course, the framework has some limitations. Incorporating dynamics is definitely an important direction for future work. Finally, allowing for richer interdependencies through fixed or sunk costs is potentially important, particularly in settings where platform-sharing is common.

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Appendices

A Data Cleaning & Imputation of Product Characteristics

In this section, I explain the procedure to obtain my estimation sample from the raw data. I first describe preliminary data cleaning procedures. Then, I describe in detail how I aggregate and impute product characteristics to all potential products $\mathcal{A}$.

A.1 Preliminary Cleaning

The *IHS Markit* 2019 sample contains information on the universe of passenger vehicle registrations for 12 countries: Australia, Brazil, France, Germany, India, Italy, Japan, Mainland China, Mexico, Spain, the United Kingdom, and the United States. The data is at the quarterly-trim-country level.

Fuel type categories: I define 3 fuel type categories using information on fuel type in the data set. First, I define "Electric" vehicles as any vehicle that fits in a plug-in category. For instance, I observe Plug-In Electric or Plug-in (Petrol or Diesel) vehicles. Any such fuel type will be considered part of the"Electric" category. Second, I define a "Hybrid" vehicle as any non-plug-in vehicle with a fuel type in the hybrid category. Finally, I define an internal combustion engine vehicle ("ICE") as any vehicle that is neither hybrid nor plug-in, be it Petrol or Diesel.

Body type categories: The data set does not contain the vast majority of large pickups or vans, so I drop the small number of instances of such items from my sample. I use the SUV and Wagon categories in the data to define the respective body types. I define the Convertible body type as including convertibles, retractable hardtops, and roadsters. Finally, I define the Car category as including either sedans, hatchbacks, or coupes. This gives 5 different possible body types.

Drop the most high-end vehicles: In the sample, I also observe certain products in extreme price ranges. Since the goal of this paper is, in part, to estimate product development fixed costs, I drop the most luxurious products, which are likely designed and produced with very different technologies than the typical passenger vehicle. I drop all products with observed prices higher than \$150,000 or pertaining to the following brands: Lamborghini, McLaren, Bentley, Ferrari, Aston Martin, Maserati, Bugatti, and Rolls-Royce.

A.2 Demand Estimation Sample

I aggregate the data at the product-year level, where the product definition is a brand-body type-fuel type combination. I obtain total units sold by summing across all quarters and trims corresponding to my product definition. I aggregate the remaining characteristics (including horsepower, weight, length, width, and height) by taking a quantity-weighted average of such characteristics across all quarter-trim observations within my product definition. I merge this data set with information from CEPII containing the distance between any two countries (population-weighted), which gives the distance between the headquarters country of the brand and the destination country. Market size is defined using the formula,

$$M_m = \{\text{Num. of HH's in } m\} \times \{\text{Avg Num. of Vehicles per HH in } m\}/\{\text{Avg Tenure of Car Ownership}\},$$

where I assume the average tenure of car ownership is 5 years, as in Grieco et al. (2023). The number of households in each country in 2019 and the average number of vehicles per household are obtained from the World Population Review and other publicly available sources, respectively. For instance, the number of vehicles per household in the United States is readily available on the Department of Transportation website.

I obtain market shares for each product in each market by dividing the total units sold by market size. As typical in the automobile literature that estimates mixed logit demand models, I drop very small market shares. More precisely, I drop all products with shares smaller than 0.00001 within a market. I only do this for demand and marginal cost estimation. In the estimation of fixed costs, I will treat these products as part of the set of products in firms' global portfolios as well as in the set of product offerings in such markets.

A.3 Fixed Cost Estimation Sample & Product Characteristic Imputation

For the fixed cost estimation sample, I require a data set at the (potential) product-market level that contains the observed characteristics of each product-market pair. This is what is required to use the demand and marginal cost model to predict variable profits under any arbitrary market structure for each possible entry opportunity. Given that I defined the set of potential products as the set of all brand-body type-fuel type combinations, I need to impute all observed characteristics that enter the demand and marginal cost specification for all potential products. These characteristics include horsepower, horsepower/weight, and size (length $\times$ width). To do this, I use the characteristics of the products that are observed in the data. Below, I describe how I impute each of the characteristics for all products that are not observed in the data.

Imputing size: I impute size for all products not observed in the *IHS Markit* sample according to the following sequential procedure: (1) using the mean size of observed products of the same body type sold by the same brand; (2) if there are no such products, I use the mean size of observed products of the same body type sold by the same parent company; (3) if there are no such products, I use the mean size across observed products of said body type.

Imputing horsepower and horsepower/weight: I impute horsepower (and horsepower/weight) for all products not observed in the *IHS Markit* sample according to the following procedure: (1) using the mean horsepower (horsepower/weight) of observed products sold of the same fuel type and body type sold by the same parent company; (2) if there are no such products, I use the mean horsepower (horsepower/weight) of observed products with the same body type and fuel type offered in that country; (3) if there are no such products, I use the mean horsepower (horsepower/weight) of all products with the same fuel type and body type.

After imputing these characteristics, I use the estimated demand and marginal cost systems (together with the retrieved brand and market fixed effects) to predict variable profits given any market structure and realization of demand and marginal cost shocks.

Observed product offerings: I treat a product as offered in a market ($O_{jm} = 1$) if and only if at least one unit is sold in that market. This is standard in previous papers studying product entry.

Observed product portfolios: I define a product as being a firm's product portfolio ($G_j = 1$) if and only if it is observed to be sold in at least one market in my sample. This definition assumes that there are no products that are only offered in markets that are not included in my sample. This is a decent assumption, given that the markets in my sample account for most of the global demand. An additional caveat is that, through the lens of my model, it could be that a firm introduces a product in its portfolio and then draws market entry costs that are large enough in all markets so that the product is ultimately not offered anywhere. If this were the case, setting $G_j = 0$ for such products when actually $G_j = 1$ would lead to an underestimate of the fraction of products introduced in the firms' portfolios and would likely lead to overestimates of product portfolio fixed costs. Such bias is likely to be very small because, with large enough product portfolio costs and independent market entry cost shocks, it is highly unlikely that a firm would introduce a product in its portfolio and then optimally choose not to sell it anywhere.

B Proof of Theorem 1: Moment Inequality Derivations

In this section, I derive the key results that I use for the estimation of $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. In Section B.1, I derive moment inequalities that partially identify (θ_e, σ_e). In Section B.2, I derive moment inequalities that partially identify $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. For ease of notation, let,

$$\Gamma_{jm}(x;\theta_e,\sigma_e) := \mathbb{P}(F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \leq x)$$

where

$$F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) = \exp(Z'_{jm}\theta_e + \sigma_e\nu^e_{jm}),$$

and

$$\Lambda_j(x;\theta_g,\sigma_g) := \mathbb{P}(F^g_j(\nu^g_j;\theta_g,\sigma_g) \leq x)$$

where

$$F^g_j(\nu^g_j;\theta_g,\sigma_g) = \exp(Z'_j\theta_g + \sigma_g\nu^g_j).$$

Also for notation, let,

$$\mathbb{1}_{\mathcal{C}^f} = \mathbb{1}\{\mathcal{C}^f \text{ is chosen by firm } f\}.$$

where $\mathcal{C}^f$ is a bundle product offerings chosen by firm f.

B.1 Stage 2: Market Entry Inequalities

Upper bound inequality: At this stage, firms have already chosen their product portfolios $\mathcal{G}^f$. By revealed preference and best response, I write for any $j \in \mathcal{G}^f$,

$$(\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}\{\mathbb{E}[MV_{jm}(\Omega^f_m,\Omega^{-f}_m)|\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m},\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \geq 0\} = (\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}_{\Omega^f_m}. \quad \text{(A1)}$$

Equation (A1) says that conditional on firm f choosing Ω^f_m or $\Omega^f_m \setminus \{j\}$, the firm chooses Ω^f_m if and only if it is weakly preferred to $\Omega^f_m \setminus \{j\}$. At this point in the game, firm f knows both $\mathcal{I}$, its ex-ante chosen portfolio choices $\mathcal{G}^f$, and the realization of the market entry fixed cost shocks $\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}$.

Under Assumption 4, the largest possible change in expected profits from introducing product j in market m can be obtained by offering no product other than j in market m. Thus, I obtain inequality,

$$(\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\},\Omega^{-f}_m)|\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m},\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \geq 0\} \geq (\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}_{\Omega^f_m}. \quad \text{(A2)}$$

In inequality (A2), inside the indicator function, the expected marginal value is evaluated at a set of entry decisions that is not optimal and is therefore independent of $\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}$ conditional on $\mathcal{I}$. Also, due to Assumption 1, rivals' product offerings Ω^{-f}_m are independent of $\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}$ conditional on $\mathcal{I}$. This implies that,

$$(\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\},\Omega^{-f}_m)|\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \geq 0\} \geq (\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m\setminus\{j\}})\mathbb{1}_{\Omega^f_m}. \quad \text{(A3)}$$

Note that inequality (A3) holds for all bundles Ω^f_m containing product j, and that

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\},\Omega^{-f}_m)|\mathcal{I},\mathcal{G}^f] - F^e_{jm}(\nu^e_{jm};\theta_e,\sigma_e) \geq 0\}$$

is now independent of both Ω^f_m and $\Omega^f_m \setminus \{j\}$.

Therefore, realizing that,

$$O_{jm} = \sum_{\Omega_m^f : j \in \Omega_m^f} \mathbb{1}_{\Omega_m^f}$$

and

$$1 - O_{jm} = \sum_{\Omega_m^f : j \in \Omega_m^f} \mathbb{1}_{\Omega_m^f \setminus \{j\}},$$

I sum inequality (A3) across all such mutually exclusive bundles to obtain,

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0\} \geq O_{jm}. \tag{A4}$$

That is, if product j is offered in market m, the largest possible change in profits from offering product j in market m must be weakly positive. The term $\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f]$ is independent of ν_{jm}^e, so I take expectations conditional on $\mathcal{I}$ and $\mathcal{G}^f$ to derive,

$$\Gamma_{jm}\big(\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f]; \theta_e, \sigma_e\big) \geq \mathbb{E}[O_{jm}|\mathcal{I}, \mathcal{G}^f], \tag{A5}$$

where recall that Γ_{jm} denotes the CDF of F_{jm}^e given θ_e and σ_e. Inequality (A5) gives an upper bound on the probability that product j is offered in market m, conditional on $\mathcal{I}$ and $\mathcal{G}^f$. To make further progress towards constructing a moment inequality, I now use a convex upper bound of CDF Γ_{jm}. For instance, if F_{jm}^e is log-normal, it has inflection point $\check{x}_{jm}(\theta_e, \sigma_e) = \exp(Z_{jm}'\theta_e - \sigma_e^2)$, so one can define,

$$\begin{aligned}\overline{\Gamma}_{jm}^1(x; \theta_e, \sigma_e) := {}& \Gamma_{jm}(x; \theta_e, \sigma_e)\mathbb{1}\{x < \check{x}_{jm}(\theta_e, \sigma_e)\} \\ & + [\Gamma_{jm}(\check{x}; \theta_e, \sigma_e) + \gamma_{jm}(\check{x}_{jm}; \theta_e, \sigma_e)(x - \check{x}_{jm}(\theta_e, \sigma_e))]\mathbb{1}\{x \geq \check{x}_{jm}(\theta_e, \sigma_e)\}\end{aligned} \tag{A6}$$

where γ_{jm} denotes the PDF of the corresponding log-normal distribution. In the empirical implementation, I use,

$$\begin{aligned}\overline{\Gamma}_{jm}(x, \hat{x}; \theta_e, \sigma_e) := {}& \overline{\Gamma}_{jm}^1(x; \theta_e, \sigma_e)\mathbb{1}\{\hat{x} < \check{x}_{jm}(\theta_e, \sigma_e)\} \\ & + \max\{\Gamma_{jm}(\hat{x}; \theta_e, \sigma_e) + \gamma_{jm}(\hat{x}; \theta_e, \sigma_e)(x - \hat{x}), \Gamma_{jm}(x; \theta_e, \sigma_e)\}\mathbb{1}\{\hat{x} \geq \check{x}_{jm}(\theta_e, \sigma_e)\},\end{aligned} \tag{A7}$$

where $\hat{x}$ is a $\mathcal{I}$-measurable approximation point.

That is, if the $\mathcal{I}$-measurable approximation point lies below the inflection point, I use the convex upper bound given by equation (A6). Otherwise, I use a linear approximation at the approximation point (which is on the concave part of the CDF), bounded below by the CDF, Γ_{jm}. Figure 15 illustrates the convex upper bounds.

Thus, given convex upper bound $\overline{\Gamma}_{jm}(x, \hat{x}_{jm}; \theta_e, \sigma_e)$, I derive,

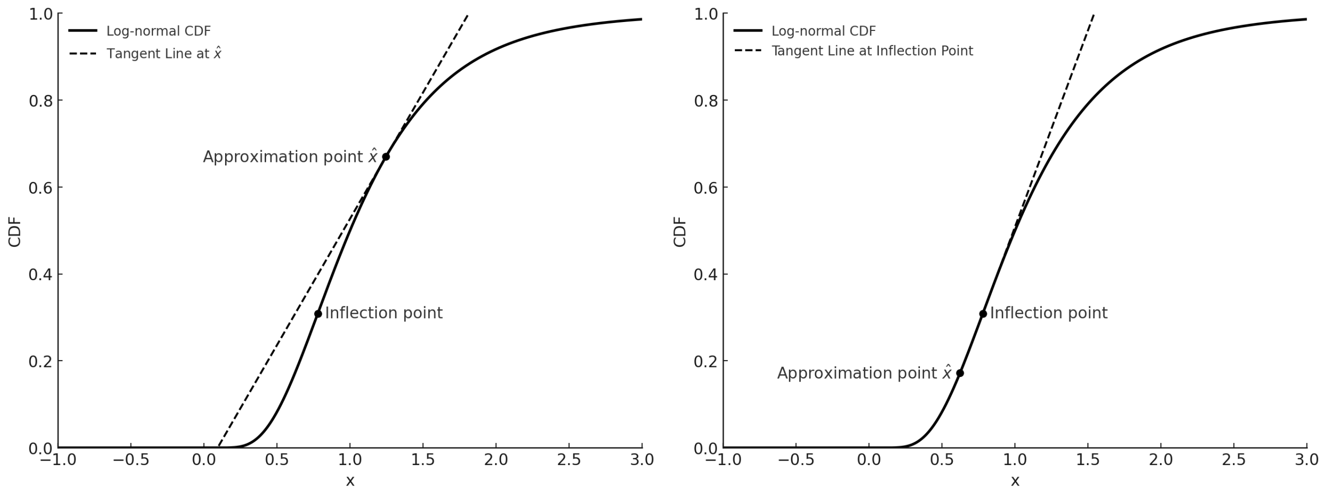
$$\overline{\Gamma}_{jm}\big(\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f], \hat{x}_{jm}; \theta_e, \sigma_e\big) \geq \mathbb{E}[O_{jm}|\mathcal{I}, \mathcal{G}^f].$$

$\overline{\Gamma}_{jm}$ is convex, so I now apply Jensen's inequality and obtain,

$$\mathbb{E}[\overline{\Gamma}_{jm}\big(\pi_{jm}(\{j\}, \Omega_m^{-f}), \hat{x}_{jm}; \theta_e, \sigma_e\big) - O_{jm}|\mathcal{I}, \mathcal{G}^f] \geq 0 \tag{A8}$$

Inequality (A8) is a conditional moment inequality that can be used for estimation. Given this conditional moment inequality, unconditional moment inequalities can be derived using positive functions of $\mathcal{I}$. Moreover,

Figure 15: Convex Upper Bounds of a Log-Normal CDF



provided the approximation points $\hat{x}_{jm}$ are $\mathcal{I}$-measurable, one can use observation-specific upper bound functions $\overline{\Gamma}_{jm}$.[41]

Lower bound inequality: I start in a manner similar to equation (A1), by writing,

$$(\mathbb{1}_{\Omega^f_m \cup \{j\}} + \mathbb{1}_{\Omega^f_m})\mathbb{1}\{\mathbb{E}[MV_{jm}(\Omega^f_m \cup \{j\}, \Omega^{-f}_m) - F^e_{jm}(\nu^e_{jm}; \theta_e, \sigma_e) \leq 0\} = (\mathbb{1}_{\Omega^f_m \cup \{j\}} + \mathbb{1}_{\Omega^f_m})\mathbb{1}_{\Omega^f_m}. \tag{A9}$$

The above equation says that conditional on firm f choosing either $\Omega^f_m \cup \{j\}$ or Ω^f_m, it will choose Ω^f_m if and only if it is preferred to $\Omega^f_m \cup \{j\}$. Equation (A9) holds for all bundles Ω^f_m such that $j \notin \Omega^f_m$.

Then, submodularity implies that the lowest possible expected change in profits from offering product j in market m is obtained whenever the firm is offering all products in its portfolio $\mathcal{G}^f$ in market m, which implies that,

$$\begin{aligned}(\mathbb{1}_{\Omega^f_m \cup \{j\}} + \mathbb{1}_{\Omega^f_m})\mathbb{1}\{\mathbb{E}[MV_{jm}(\mathcal{G}^f, \Omega^{-f}_m)|\{\nu^e_{jm}\}_{j \in \mathcal{G}^f, m}, \mathcal{I}, \mathcal{G}^f] - F^e_{jm}(\nu^e_{jm}; \theta_e, \sigma_e) \leq 0\} \\ \geq (\mathbb{1}_{\Omega^f_m \cup \{j\}} + \mathbb{1}_{\Omega^f_m})\mathbb{1}_{\Omega^f_m}.\end{aligned} \tag{A10}$$

As with the upper bound inequality, I remove conditioning on $\{\nu^e_{jm}\}_{j \in \mathcal{G}^f, m}$ in the expectation in inequality (A10) due to Assumption 1. Realizing that,

$$\sum_{\Omega^f_m : j \notin \Omega^f_m} \mathbb{1}_{\Omega^f_m} = 1 - O_{jm}$$

and

$$\sum_{\Omega^f_m : j \notin \Omega^f_m} \mathbb{1}_{\Omega^f_m \cup \{j\}} = O_{jm},$$

I sum inequality (A10) over all bundles Ω^f_m with $j \notin \Omega^f_m$ to obtain,

[41] Porcher et al. (2024) also employ observation-specific linear approximations to derive moment inequalities, though in a single-agent setting and using odds-based inequalities rather than bounding choice probabilities using convex upper and concave lower bounds of the CDF of the unobserved shock.

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\mathcal{G}^f, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \le 0\} \ge 1 - O_{jm}. \tag{A11}$$

Taking expectations conditional on $\mathcal{I}$ and $\mathcal{G}^f$ on both sides of inequality (A11) yields,

$$\Gamma_{jm}\big(\mathbb{E}[MV_{jm}(\mathcal{G}^f, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f]; \theta_e, \sigma_e\big) \le \mathbb{E}[O_{jm}|\mathcal{I}, \mathcal{G}^f]. \tag{A12}$$

Inequality (A12) provides a lower bound on the probability that product j is offered in market m, conditional on $\mathcal{I}$. I now follow a very similar logic as with the upper bound and use a *concave lower bound* for Γ_{jm} before applying Jensen's inequality. A family of such concave lower bounds is given by,

$$\begin{aligned} \underline{\Gamma}_{jm}^1(x; \theta_e, \sigma_e) := {} & \Gamma(x; \theta_e, \sigma_e)\mathbb{1}\{x \ge \check{x}_{jm}(\theta_e, \sigma_e)\} \\ & + [\Gamma_{jm}(\check{x}; \theta_e, \sigma_e) + \gamma_{jm}(\check{x}_{jm}; \theta_e, \sigma_e)(x - \check{x}_{jm}(\theta_e, \sigma_e))]\mathbb{1}\{x < \check{x}_{jm}(\theta_e, \sigma_e)\} \end{aligned} \tag{A13}$$

where, as before, $\check{x}_{jm}(\theta_e, \sigma_e) = \exp(Z_{jm}'\theta_e - \sigma_e^2)$ denotes the inflection point of the distribution of market entry fixed costs given θ_e and σ_e. In the empirical implementation, I use,

$$\begin{aligned} \underline{\Gamma}_{jm}(x, \hat{x}; \theta_e, \sigma_e) := {} & \underline{\Gamma}_{jm}^1(x; \theta_e, \sigma_e)\mathbb{1}\{\hat{x} \ge \check{x}_{jm}(\theta_e, \sigma_e)\} \\ & + \min\{\Gamma_{jm}(\hat{x}; \theta_e, \sigma_e) + \gamma_{jm}(\hat{x}; \theta_e, \sigma_e)(x - \hat{x}), \Gamma_{jm}(x; \theta_e, \sigma_e)\}\mathbb{1}\{\hat{x} < \check{x}_{jm}(\theta_e, \sigma_e)\}. \end{aligned} \tag{A14}$$

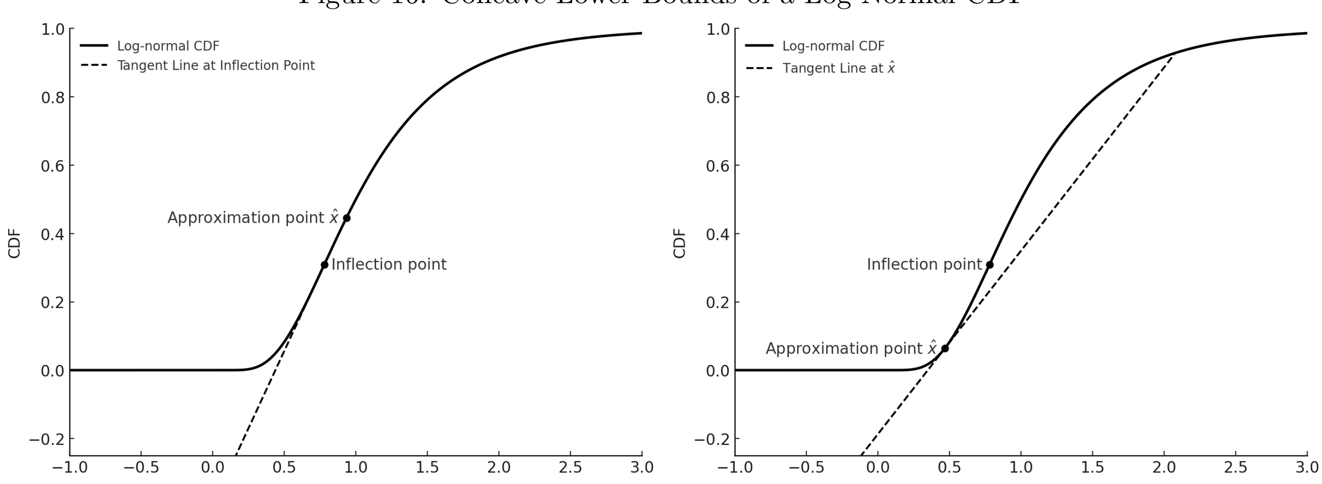
That is, if the approximation point lies above the inflection point, I use the concave lower bound given in equation (A13). Otherwise, I use a linear approximation at the approximation point (which is in the convex part of the CDF), bounded above by the CDF, Γ_{jm}. Figure 16 illustrates the concave lower bound functions.

Given such a concave lower bound, together with Jensen's inequality, I obtain

$$\mathbb{E}[\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f, \Omega_m^{-f}), \hat{x}_{jm}; \theta_e, \sigma_e) - O_{jm}|\mathcal{I}, \mathcal{G}^f] \le 0, \tag{A15}$$

As with the upper bound, conditional moment inequality (A15) can now be used for estimation given positive-valued functions of $\mathcal{I}$, which yield unconditional moment inequalities. Also, as before, provided $\hat{x}_{jm}$ is $\mathcal{I}$-measurable, one can use observation-dependent lower bound functions $\underline{\Gamma}_{jm}$.

Figure 16: Concave Lower Bounds of a Log-Normal CDF



Importantly, note that because by assumption, market entry fixed cost shocks $\{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}$ are unobserved at the time of choosing $\mathcal{G}^f$, I condition on the observed product portfolios when using conditional moment inequalities (A8) and (A15) for estimation.

This concludes the derivation of the inequalities used to estimate bounds on (θ_e, σ_e). In the next subsection, I derive inequalities that provide bounds on (θ_g, σ_g).

B.2 Stage 1: Global Portfolio Choice Inequalities

At this stage, firms choose their global product portfolios $\mathcal{G}^f$. The derivation of the inequalities at this stage follows a similar logic as the derivations in Stage 2. For the derivations that follow, I will use the following result:

Proposition 1 *The function $g : \mathbb{R} \to \mathbb{R}$ given by,*

$$g(y) = \mathbb{E}_X[\mathbb{1}(X \leq y)(y - X)] = \mathbb{E}_X[y - X|X \leq y]\mathbb{P}_X(X \leq y)$$

is convex in y for any continuous random variable X provided $F_X(y) := \mathbb{P}_X(X \leq y) > 0$ i.e., the conditional expectation is well defined.

Proof. Differentiating with respect to y, I obtain,

$$\begin{aligned} g'(y) &= F_X(y)\left[1 - \frac{f_X(y)}{F_X(y)}(y - \mathbb{E}[X|X \leq y])\right] + \mathbb{E}[y - X|X \leq y]f_X(y) \\ &= F_X(y) \end{aligned}$$

where f_X denotes the density of random variable X. But then, clearly, $g''(y) = f(y) > 0$, which proves convexity. ■

To derive the inequalities at this stage, I first define the value of a given portfolio for a firm f as,

$$\mathcal{V}_f(\mathcal{G}^f, \{\nu^g_j\}_{j\in\mathcal{A}^f}, \mathcal{I}) = \mathbb{E}\left[\sum_{m\in\mathcal{M}} \max_{\{\Omega^f_m \subseteq \mathcal{G}^f\}} \Pi^f_m(\Omega^f_m; \mathcal{G}^f, \{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}, \mathcal{I})\middle|\mathcal{I}\right] - \sum_{j\in\mathcal{A}^f} G_j F^g_j(\nu^g_j; \theta_g, \sigma_g) \tag{A16}$$

where

$$\Pi^f_m(\Omega^f_m; \mathcal{G}^f, \{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}, \mathcal{I}) = \mathbb{E}\left[\sum_{j\in\Omega^f_m} [\pi_{jm}(\Omega^f_m, \Omega^{-f}_m) - F^e_{jm}(\nu^e_{jm}; \theta_e, \sigma_e)]\middle|\mathcal{G}^f, \{\nu^e_{jm}\}_{j\in\mathcal{G}^f,m}, \mathcal{I}\right].$$

That is, given any chosen product portfolio $\mathcal{G}^f$ and realized global portfolio fixed cost shocks, the firm can compute its Stage 1 value as the expected maximal profits it will obtain once it realizes its market entry fixed cost shocks for each product and chooses offerings in each market optimally. Importantly, the expectation in equation (A16) is only conditional on $\mathcal{I}$, so the firm must also integrate over its own market entry fixed costs shocks. At Stage 2, firm f's optimal entry decisions depend on the portfolio it chooses in Stage 1 and the market entry fixed cost shocks it realizes in Stage 2. Thus, I write the decision rule in each market as $\Omega^f_m(\mathcal{G}^f, \{\nu^e_{jm}\}_{j\in\mathcal{G}^f}, \mathcal{I})$.

Henceforth, I will abuse notation and denote by $\Omega^{\mathcal{G}^f}_m$ the decision rule under portfolio $\mathcal{G}^f$ (at given market entry fixed cost shocks and at $\mathcal{I}$) and $O^{\mathcal{G}^f}_{jm}$ will likewise denote the optimal entry decision rule for product j in market m under $\mathcal{G}^f$ corresponding to $\Omega^{\mathcal{G}^f}_m$.

Upper bound inequality: First, I proceed similarly to how I derived the second-stage inequalities and write,

$$(\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}\Big\{\mathcal{V}_f(\mathcal{G}^f, \{\nu_j^g\}_{j\in\mathcal{A}^f}, \mathcal{I}) - \mathcal{V}_f(\mathcal{G}^f \setminus \{j\}, \{\nu_j^g\}_{j\in\mathcal{A}^f}, \mathcal{I}) \geq 0\Big\} = (\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}_{\mathcal{G}^f}. \tag{A17}$$

That is, conditional on choosing either portfolio $\mathcal{G}^f$ or portfolio $\mathcal{G}^f \setminus \{j\}$, the firm chooses $\mathcal{G}^f$ if and only if it is preferred to $\mathcal{G}^f \setminus \{j\}$. Next, notice that a lower bound of $\mathcal{V}_f(\mathcal{G}^f \setminus \{j\}, \{\nu_j^g\}_{j\in\mathcal{A}^f}, \mathcal{I})$ can be obtained by using the fact that the entry decisions in all markets for products $j \in \mathcal{G}^f \setminus \{j\}$ must weakly dominate the optimal-under-$\mathcal{G}^f$ entry decision rules for such products. This means that the equality (A17) implies the following inequality,

$$(\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \big(\mathbb{E}[\Pi_m^f(\Omega_m^{\mathcal{G}^f}) - \Pi_m^f(\Omega_m^{\mathcal{G}^f} \setminus \{j\})|\mathcal{I}]\big) - F_j^g(\nu_j^g; \theta_g, \sigma_g) \geq 0\Big\} \geq (\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}_{\mathcal{G}^f}. \tag{A18}$$

That is, the change in value from including j into the portfolio starting from a suboptimal entry decision rule in the second stage of the game – which mandates choosing the *same* entry decisions in each market under $\mathcal{G}^f \setminus \{j\}$ as under $\mathcal{G}^f$ for all products $j' \neq j$ – must be higher than the actual change in expected value from introducing product j into the firm's portfolio at the best response. I re-write inequality (A18) as,

$$\begin{aligned}
&(\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\times \\
&\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \big(\mathbb{E}\big[O_{jm}^{\mathcal{G}^f}\big[\mathbb{E}\big[MV_{jm}(\Omega_m^{\mathcal{G}^f} \setminus \{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f, m}, \mathcal{G}^f\big] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e)\big|\mathcal{I}\big]\big) - F_j^g(\nu_j^g; \theta_g, \sigma_g) \geq 0\Big\} \\
&\geq (\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}_{\mathcal{G}^f}
\end{aligned} \tag{A19}$$

where MV_{jm} is defined in Definition 1. Inequality (A19) shows that if product j is introduced in firm f's global portfolio, it must be that the change in profits from offering it in each market in which firm f chooses to offer product j (evaluated at the optimal entry decisions under $\mathcal{G}^f$) is weakly positive net of the portfolio fixed cost. The next step is to again use submodularity of variable profits to bound the expression inside the indicator function by above and make it independent of the optimal portfolio choice. Under Assumption 4, inequality (A19) implies,

$$\begin{aligned}
&(\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\times \\
&\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \big(\mathbb{E}\big[O_{jm}^{\mathcal{G}^f}\big[\mathbb{E}\big[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f, m}, \mathcal{G}^f\big] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e)\big|\mathcal{I}\big]\big) - F_j^g(\nu_j^g; \theta_g, \sigma_g) \geq 0\Big\} \\
&\geq (\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}_{\mathcal{G}^f}.
\end{aligned} \tag{A20}$$

I therefore obtain,

$$\begin{aligned}
&(\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \big(\mathbb{E}\big[O_{jm}^{\{j\}}\big[\mathbb{E}\big[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e)\big|\mathcal{I}\big]\big) - F_j^g(\nu_j^g; \theta_g, \sigma_g) \geq 0\Big\} \\
&\geq (\mathbb{1}_{\mathcal{G}^f} + \mathbb{1}_{\mathcal{G}^f \setminus \{j\}})\mathbb{1}_{\mathcal{G}^f}
\end{aligned} \tag{A21}$$

where

$$O_{jm}^{\{j\}} = \mathbb{1}\big\{\mathbb{E}[\pi_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0\big\}.$$

That is, the contribution of product j into firm f's Stage 1 value has to be smaller than the contribution if it were the only product the firm sold in each market, conditional on positive profits from product j in each market net of the fixed market entry cost.

Summing all inequalities of the form of (A21) across all bundles containing product j, I obtain,

$$\mathbb{1}\Big\{\sum_{m\in\mathcal{M}}\big(\mathbb{E}\big[O_{jm}^{\{j\}}\big[\mathbb{E}\big[\pi_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-F_{jm}^e(\nu_{jm}^e;\theta_e,\sigma_e)\big|\mathcal{I}\big]\big)-F_j^g(\nu_j^g;\theta_g,\sigma_g)\geq 0\Big\}\geq G_j \tag{A22}$$

The expectation inside the indicator function in inequality (A22) is an expectation such as that in Proposition 1.[42] Applying Jensen's inequality, I obtain,

$$\begin{aligned}\mathbb{1}\Big\{\mathbb{E}\Big[\sum_{m\in\mathcal{M}}\Gamma_{jm}\big(\pi_{jm}(\{j\},\Omega_m^{-f})\big)\big[\pi_{jm}(\{j\},\Omega_m^{-f})-\mathbb{E}[F_{jm}^e(\nu_{jm}^e)|\mathcal{I},F_{jm}^e(\nu_{jm}^e)\leq\pi_{jm}(\{j\},\Omega_m^{-f})]\big]\big|\mathcal{I}\Big]\\ -F_j^g(\nu_j^g;\theta_g,\sigma_g)\geq 0\Big\}\geq G_j\end{aligned} \tag{A23}$$

Henceforth, the analysis is identical to the derivation of the Stage 2 inequalities. As before, ν_j^g is now independent of the expectation inside the indicator function in inequality (A23). I take expectations on both sides conditional on $\mathcal{I}$ to obtain,

$$\begin{aligned}&\Lambda_j\Big(\mathbb{E}\Big[\sum_{m\in\mathcal{M}}\Gamma_{jm}\big(\pi_{jm}(\{j\},\Omega_m^{-f})\big)\big[\pi_{jm}(\{j\},\Omega_m^{-f})-\mathbb{E}[F_{jm}^e(\nu_{jm}^e)|\mathcal{I},F_{jm}^e(\nu_{jm}^e)\leq\pi_{jm}(\{j\},\Omega_m^{-f})]\big]\big|\mathcal{I}\Big];\theta_g,\sigma_g\Big)\\ &\geq\mathbb{E}[G_j|\mathcal{I}].\end{aligned} \tag{A24}$$

As with the Stage 2 inequalities, I use the convex upper bound of Λ_j of the form of that in equation (A7), at an $\mathcal{I}$-measurable approximation point $\hat{x}_j$, such that, applying Jensen's inequality,

$$\begin{aligned}&\mathbb{E}\Big[\overline{\Lambda}_j\Big(\sum_{m\in\mathcal{M}}\Gamma_{jm}\big(\pi_{jm}(\{j\},\Omega_m^{-f})\big)\big[\pi_{jm}(\{j\},\Omega_m^{-f})-\mathbb{E}[F_{jm}^e(\nu_{jm}^e)|\mathcal{I},F_{jm}^e(\nu_{jm}^e)\leq\pi_{jm}(\{j\},\Omega_m^{-f})]\big],\hat{x}_j;\theta_g,\sigma_g\Big)-G_j|\mathcal{I}\Big]\\ &\geq 0.\end{aligned} \tag{A25}$$

Inequality (A25) can now be used to estimate θ_g and σ_g. Note that

$$\mathbb{E}\big[F_{jm}^e(\nu_{jm}^e)|\mathcal{I},F_{jm}^e(\nu_{jm}^e)\leq\pi_{jm}(\{j\},\Omega_m^{-f})\big]$$

can be computed using numerical integration or Gaussian quadrature. I use the QuadGK Julia package to evaluate this conditional mean.

Lower bound inequality: I now derive a lower bound conditional moment inequality starting from the fact that the following equality must hold at the best response for any $\mathcal{G}^f$ with $j\notin\mathcal{G}^f$,

$$(\mathbb{1}_{\mathcal{G}^f\cup\{j\}}+\mathbb{1}_{\mathcal{G}^f})\mathbb{1}\big\{\mathcal{V}_f(\mathcal{G}^f\cup\{j\},\{\nu_j^g\}_{j\in\mathcal{A}^f},\mathcal{I})-\mathcal{V}_f(\mathcal{G}^f,\{\nu_j^g\}_{j\in\mathcal{A}^f},\mathcal{I})\leq 0\big\}=(\mathbb{1}_{\mathcal{G}^f\cup\{j\}}+\mathbb{1}_{\mathcal{G}^f})\mathbb{1}_{\mathcal{G}^f}. \tag{A26}$$

The above equation says that conditional on firm f choosing either $\mathcal{G}^f\cup\{j\}$ or $\mathcal{G}^f$, it chooses $\mathcal{G}^f$ if and only if $\mathcal{G}^f$ is preferred to $\mathcal{G}^f\cup\{j\}$. Consider now the following sub-optimal second-stage decision rules under portfolio $\mathcal{G}^f\cup\{j\}$ in all markets m,

$$\underline{\Omega}_m^{\mathcal{G}^f\cup\{j\}}=\begin{cases}O_{j'm}^{\mathcal{G}^f}, & j'\neq j\\ \underline{O}_{j'm}, & j'=j.\end{cases}$$

That is, the firm chooses the same decision rule as under $\mathcal{G}^f$ under $\mathcal{G}^f\cup\{j\}$ for all non-j products and uses the

[42] I lighten notation by removing the (θ_e,σ_e) arguments from Γ_{jm} and F_{jm}^e.

$\underline{O}_{jm}$ decision rule for product j, where for now this decision rule is any arbitrary decision rule that must depend on $\mathcal{I}$ and any set of realized market entry fixed cost realizations. I therefore obtain,

$$(\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\Pi_m^f\big(\underline{\Omega}_m^{\mathcal{G}^f\cup\{j\}}\big) - \Pi_m^f\big(\Omega_m^{\mathcal{G}^f}\big)|\mathcal{I}\big] - F_j^g(\nu_j^g;\theta_g,\sigma_g) < 0\Big\} \geq (\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}_{\mathcal{G}^f}$$

or

$$\begin{aligned}(\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}_{jm}\big[\mathbb{E}\big[MV_{jm}(\Omega_m^{\mathcal{G}^f},\Omega_m^{-f})|\mathcal{I},\{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m},\mathcal{G}^f\big] - F_{jm}^e(\nu_{jm}^e)|\mathcal{I}\big] - F_j^g(\nu_j^g;\theta_g,\sigma_g) < 0\Big\}\\ \geq (\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}_{\mathcal{G}^f}.\end{aligned} \tag{A27}$$

As with the upper bound, I use submodularity of variable profits to further bound the expression inside the indicator function in inequality (A27). In particular,

$$\begin{aligned}(\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}_{jm}\big[\mathbb{E}\big[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)|\mathcal{I}\big] - F_j^g(\nu_j^g;\theta_g,\sigma_g) < 0\Big\}\\ \geq (\mathbb{1}_{\mathcal{G}^f\cup\{j\}} + \mathbb{1}_{\mathcal{G}^f})\mathbb{1}_{\mathcal{G}^f}.\end{aligned} \tag{A28}$$

Inequality (A28) holds for all bundles $\mathcal{G}^f$ that do not contain product j. Thus, all such inequalities across all bundles $\mathcal{G}^f$ that do not contain product j yields,

$$\mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}_{jm}\big[\mathbb{E}\big[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)|\mathcal{I}\big] - F_j^g(\nu_j^g;\theta_g,\sigma_g) < 0\Big\} \geq 1 - G_j \tag{A29}$$

Next, I let,

$$\underline{O}_{jm} = \mathbb{1}\big\{\mathbb{E}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}] - F_{jm}^e(\nu_{jm}^e) \geq 0\big\}.$$

Under Proposition 1, and letting $y := MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})$, a lower bound of

$$g(\mathbb{E}(y|\mathcal{I})) := \mathbb{E}[\mathbb{1}\{\mathbb{E}(y|\mathcal{I}) - F_{jm}^e(\nu_{jm}^e) \geq 0\}[\mathbb{E}(y|\mathcal{I}) - F_{jm}^e(\nu_{jm}^e)]|\mathcal{I}]$$

can be obtained by taking a first-order approximation around some $\mathcal{I}$-measurable point $\hat{x}_{jm}$. This yields,

$$\begin{aligned}g(\mathbb{E}(y|\mathcal{I})) &\geq \Gamma_{jm}(\hat{x}_{jm})[\mathbb{E}[y|\mathcal{I}] - \mathbb{E}[F_{jm}^e(\nu_{jm}^e)|F_{jm}^e \leq \hat{x}_{jm},\mathcal{I}]]\\ &= \Gamma_{jm}(\hat{x}_{jm})[\mathbb{E}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}] - \mathbb{E}[F_{jm}^e(\nu_{jm}^e)|F_{jm}^e(\nu_{jm}^e) \leq \hat{x}_{jm},\mathcal{I}]].\end{aligned} \tag{A30}$$

Plugging inequality (A30) into inequality (A29), I obtain,

$$\mathbb{1}\Big\{\mathbb{E}\Big[\sum_{m\in\mathcal{M}} \Gamma_{jm}\big(\hat{x}_{jm}\big)\big[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f}) - \mathbb{E}[F_{jm}^e(\nu_{jm}^e)|F_{jm}^e(\nu_{jm}^e) \leq \hat{x}_{jm},\mathcal{I}]\big]\big|\mathcal{I}\Big] - F_j^g(\nu_j^g;\theta_g,\sigma_g) \geq 0\Big\} \leq G_j \tag{A31}$$

where recall Γ_{jm} was the CDF of F_{jm}^e (given θ_e and σ_e). Given that the expectation term is now independent of ν_j^g conditional on $\mathcal{I}$, I take expectations conditional on $\mathcal{I}$ and obtain,

$$\Lambda_j\Big(\mathbb{E}\Big[\sum_{m\in\mathcal{M}} \Gamma_{jm}\big(\hat{x}_{jm}\big)\big[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f}) - \mathbb{E}[F_{jm}^e(\nu_{jm}^e)|F_{jm}^e(\nu_{jm}^e) \leq \hat{x}_{jm},\mathcal{I}]\big]\big|\mathcal{I}\Big];\theta_g,\sigma_g\Big) \leq \mathbb{E}[G_j|\mathcal{I}]. \tag{A32}$$

Finally, I use the concave lower bound (as in equation (A14)) $\underline{\Lambda}_j$ at an $\mathcal{I}$-measurable approximation point $\hat{x}_j$ and apply Jensen's inequality to obtain,

$$\mathbb{E}\Big[\underline{\Lambda}_j\Big(\sum_{m\in\mathcal{M}}\Gamma_{jm}(\hat{x}_{jm})\big[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})-\mathbb{E}[F^e_{jm}(\nu^e_{jm})|F^e_{jm}(\nu^e_{jm})\leq\hat{x}_{jm},\mathcal{I}]\big],\hat{x}_j;\theta_g,\sigma_g\Big)-G_j\big|\mathcal{I}\Big]\leq 0. \tag{A33}$$

I now have shown how to derive the inequalities that can be used for estimation of parameters θ_g and σ_g. These are given by inequalities (A25) and (A33).

In sum, I have proven Theorem 1, which I formally restate below.

Theorem 1 *The following conditional moment inequalities partially identify the true fixed cost parameters (θ_e,σ_e) and (θ_g,σ_g):*

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{j\},\Omega_m^{-f});\theta_e,\sigma_e)-O_{jm}|\mathcal{I},\mathcal{G}^f]\geq 0, \tag{A34}$$

$$\mathbb{E}[\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f,\Omega_m^{-f});\theta_e,\sigma_e)-O_{jm}|\mathcal{I},\mathcal{G}^f]\leq 0, \tag{A35}$$

$$\begin{aligned}\mathbb{E}\Big[\overline{\Lambda}_j\Big(&\sum_{m\in\mathcal{M}}\Gamma_{jm}(\pi_{jm}(\{j\},\Omega_m^{-f});\theta_e,\sigma_e)\\ &\times[\pi_{jm}(\{j\},\Omega_m^{-f})-\mathbb{E}[F^e_{jm}(\nu^e_{jm})|\mathcal{I},F^e_{jm}(\nu^e_{jm})\leq\pi_{jm}(\{j\},\Omega_m^{-f})]];\theta_g,\sigma_g\Big)-G_j\big|\mathcal{I}\Big]\geq 0,\end{aligned} \tag{A36}$$

$$\mathbb{E}\Big[\underline{\Lambda}_j\Big(\sum_{m\in\mathcal{M}}\Gamma_{jm}(\hat{\underline{x}}^l_{jm};\theta_e,\sigma_e)[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})-\mathbb{E}[F^e_{jm}(\nu^e_{jm})|F^e_{jm}(\nu^e_{jm})\leq\hat{\underline{x}}^l_{jm},\mathcal{I}]];\theta_g,\sigma_g\Big)-G_j\big|\mathcal{I}\Big]\leq 0, \tag{A37}$$

where $\hat{\underline{x}}^l_{jm}$ is an $\mathcal{I}-$measurable approximation of $MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})$ and $\overline{\Lambda}_j$ and $\underline{\Lambda}_j$ are convex and concave upper and lower bounds of the CDF of F_j^g, respectively.

C Solution Method Based on Inequalities

In this section, I provide a method to bound the equilibrium distribution of product offerings in each market. The method relies on first-order stochastic dominance for multivariate random vectors. Before defining first-order stochastic dominance, I define a partial order in $\mathbb{R}^n$. Throughout, I say that $\boldsymbol{x}\geq\boldsymbol{y}$ if and only if $x_i\geq y_i$ for all $i\in\{1,...,n\}$. An upper set in $\mathbb{R}^n$ is any set U of the form $U(\boldsymbol{y})=\{\boldsymbol{x}:\boldsymbol{x}\geq\boldsymbol{y}\}$.

Definition 2 *Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be two random vectors in $\mathbb{R}^n$ such that,*

$$\mathbb{P}(\boldsymbol{X}\in U)\geq\mathbb{P}(\boldsymbol{Y}\in U)\qquad \textit{for all upper sets } U\subseteq\mathbb{R}^n.$$

Then $\boldsymbol{X}$ is said to first-order stochastically dominate $\boldsymbol{Y}$.

In my context, I will be dealing with Bernoulli random vectors, and thus $\boldsymbol{X}$ first order stochastically dominating $\boldsymbol{Y}$ amounts to,

$$\mathbb{P}\Big(\bigcap_{i\in\Omega}\{X_i=1\}\Big)\geq\mathbb{P}\Big(\bigcap_{i\in\Omega}\{Y_i=1\}\Big)\qquad\text{for any }\Omega\subseteq\{1,...,n\}.$$

I will also employ the following result.

Theorem 3 (Shaked 2007) *$\boldsymbol{X}$ FOSD $\boldsymbol{Y}$ if and only if $\mathbb{E}[u(\boldsymbol{X})]\geq\mathbb{E}[u(\boldsymbol{Y})]$ for all non-decreasing functions u for which the expectations exist.*

In counterfactual exercises, I am interested in learning about changes in the equilibrium market structure and product offerings in response to policies subsumed in $\mathcal{I}$. Notice that a change in $\mathcal{I}$ could be a subsidy, a tax, a change in the ownership structure of firms, or any other change in market conditions that is known at the time of making portfolio and market offerings choices. To obtain counterfactual bounds to these entry probabilities, I use Bayes' rule to write,

$$\mathbb{P}(O_{jm} = 1|\mathcal{I}) = \mathbb{P}(O_{jm} = 1|\mathcal{I}, G_j = 1)\mathbb{P}(G_j = 1|\mathcal{I}).$$

From the moment inequality sections, recall that from inequalities (A4) and (A11), I derived that conditional on product $j \in \mathcal{G}^f$,

$$\underline{O}^*_{jm} \leq O^*_{jm} \leq \overline{O}^*_{jm},$$

where

$$\overline{O}^*_{jm} := \mathbb{1}\{\mathbb{E}_{\boldsymbol{\mu}^*_m}[MV_{jm}(\{j\}, \Omega^{-f}_m)|\mathcal{I}] - F^e_{jm}(\nu^e_{jm}) \geq 0\} \tag{A38}$$

$$\underline{O}^*_{jm} := \mathbb{1}\{\mathbb{E}_{\boldsymbol{\mu}^*_m}[MV_{jm}(\mathcal{A}^f, \Omega^{-f}_m)|\mathcal{I}] - F^e_{jm}(\nu^e_{jm}) \geq 0\}, \tag{A39}$$

Moreover, recall from (A22) and (A29) that,

$$\underline{G}^*_j \leq G^*_j \leq \overline{G}^*_j$$

where

$$\overline{G}^*_j := \mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\overline{O}^*_{jm}[\mathbb{E}_{\boldsymbol{\mu}^*_m}[MV_{jm}(\{j\}, \Omega^{-f}_m)|\mathcal{I}] - F^e_{jm}(\nu^e_{jm})|\mathcal{I}\big] - F^g_j(\nu^g_j) \geq 0\Big\} \tag{A40}$$

$$\underline{G}^*_j := \mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}^*_{jm}\big[\mathbb{E}_{\boldsymbol{\mu}^*_m}[MV_{jm}(\mathcal{A}^f, \Omega^{-f}_m)|\mathcal{I}] - F^e_{jm}(\nu^e_{jm})|\mathcal{I}\big] - F^g_j(\nu^g_j) \geq 0\Big\}. \tag{A41}$$

In equations (A38)-(A41), $\boldsymbol{\mu}^*_m$ denotes the distribution of firms' offerings decisions in market m at whichever equilibrium emerges under $\mathcal{I}$.[43]

Independence: Under Assumptions 2-3, for each market m, the sequences $\{\underline{O}^*_{jm}\}_{j\in\mathcal{A}}$, $\{\overline{O}^*_{jm}\}_{j\in\mathcal{A}}$, $\{\underline{G}^*_j\}_{j\in\mathcal{A}}$, and $\{\overline{G}^*_j\}_{j\in\mathcal{A}}$ are all sequences of independent random variables conditional on $\mathcal{I}$ bounding the equilibrium binary decisions $\{O^*_{jm}\}_{j\in\mathcal{A}}$ and $\{G^*_j\}_{j\in\mathcal{A}}$.

Importantly, the true equilibrium sequences are *not* independent sequences of random variables given that firms' product introduction choices are correlated due to interdependencies coming from cannibalization. The asterisks in the definitions of the bounding random variables in (A38)-(A41) denote that firms' expectations are with respect to the *true* joint distribution of entry decisions made by other firms. Firms care about which products are ultimately sold in each market and, therefore, about the joint distribution (across products and markets) of random variables $\{V^*_{jm}\}_{j\in\mathcal{A}}$, where,

$$V^*_{jm} = O^*_{jm}G^*_j = O^*_{jm}, \tag{A42}$$

that is, the event that product j is offered in market m. The latter equality holds because, in equilibrium, $O^*_{jm} = 1$

[43] Firm f only integrates over rival firms' offerings decisions. I simplify notation by using a $\boldsymbol{\mu}^*_m$ subscript, rather than a $\boldsymbol{\mu}^{*;-f}_m$ subscript.

implies $G_j^* = 1$. I define auxiliary random variables V_{jm}^* for ease of exposition in the derivations that follow. It follows that,

$$\underline{V}_{jm}^* \leq V_{jm}^* \leq \overline{V}_{jm}^*, \tag{A43}$$

where

$$\underline{V}_{jm}^* := \underline{O}_{jm}^* \underline{G}_j^*,$$
$$\overline{V}_{jm}^* := \overline{O}_{jm}^* \overline{G}_j^*.$$

By construction, $\overline{O}_{jm}^*$ and $\overline{G}_j^*$ as well as $\underline{O}_{jm}^*$ and $\underline{G}_j^*$, are independent conditional on $\mathcal{I}$. Also, within each market m, $\{\underline{V}_{jm}^*\}_{j\in\mathcal{A}}$ and $\{\overline{V}_{jm}^*\}_{j\in\mathcal{A}}$ are independent sequences of random variables conditional on $\mathcal{I}$.

From equation (A43), I derive by taking expectations conditional on $\mathcal{I}$,

$$\begin{aligned}\mathbb{P}(\underline{O}_{jm}^* = 1|\mathcal{I})\mathbb{P}(\underline{G}_j^* = 1|\mathcal{I}) = \mathbb{P}(\underline{V}_{jm}^* = 1|\mathcal{I}) &\leq \mathbb{P}(O_{jm}^* = 1|\mathcal{I}) \\ &\leq \mathbb{P}(\overline{V}_{jm}^* = 1|\mathcal{I}) = \mathbb{P}(\overline{O}_{jm}^* = 1|\mathcal{I})\mathbb{P}(\overline{G}_j^* = 1|\mathcal{I}).\end{aligned} \tag{A44}$$

By Assumption 4, it also follows that,

$$\mathbb{P}(\underline{O}_{jm}^k = 1|\mathcal{I})\mathbb{P}(\underline{G}_j^k = 1|\mathcal{I}) = \mathbb{P}(\underline{V}_{jm}^k = 1|\mathcal{I}) \leq \mathbb{P}(O_{jm}^* = 1|\mathcal{I}) \leq \mathbb{P}(\overline{V}_{jm}^k = 1|\mathcal{I}) = \mathbb{P}(\overline{O}_{jm}^k = 1|\mathcal{I})\mathbb{P}(\overline{G}_j^k = 1|\mathcal{I}),$$

where

$$\overline{O}_{jm}^k := \mathbb{1}\{\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^{k-1}}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}] - F_{jm}^e(\nu_{jm}^e) \geq 0\}, \tag{A45}$$
$$\underline{O}_{jm}^k := \mathbb{1}\{\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^{k-1}}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}] - F_{jm}^e(\nu_{jm}^e) \geq 0\}, \tag{A46}$$

$$\overline{G}_j^k := \mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\overline{O}_{jm}^k\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^{k-1}}\big[MV_{jm}(\{j\}, \Omega_m^{-f})\big|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)\big|\mathcal{I}\big] - F_j^g(\nu_j^g) \geq 0\Big\}, \tag{A47}$$
$$\underline{G}_j^k := \mathbb{1}\Big\{\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}_{jm}^k\big[\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^{k-1}}\big[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})\big|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)\big|\mathcal{I}\big] - F_j^g(\nu_j^g) \geq 0\Big\}, \tag{A48}$$

provided

$$\underline{\boldsymbol{\mu}}_m^{k-1} \leq_{FOSD} \boldsymbol{\mu}^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^{k-1}.$$

This is an immediate consequence of submodularity implying that MV_{jm} is a decreasing function together with Theorem 3. I let,

$$\underline{\mu}_{jm,k}^{offer} = \mathbb{P}(\underline{O}_{jm}^k = 1|\mathcal{I}) = \Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^{k-1}}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}]\big), \tag{A49}$$
$$\overline{\mu}_{jm,k}^{offer} = \mathbb{P}(\overline{O}_{jm}^k = 1|\mathcal{I}) = \Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^{k-1}}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}]\big), \tag{A50}$$
$$\underline{\mu}_{j,k}^{portfolio} = \mathbb{P}(\underline{G}_j^k = 1|\mathcal{I}) = \Lambda_j\Big(\sum_{m\in\mathcal{M}} \mathbb{E}\big[\underline{O}_{jm}^k[\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^{k-1}}\big[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)]|\mathcal{I}\big]\Big), \tag{A51}$$
$$\overline{\mu}_{j,k}^{portfolio} = \mathbb{P}(\overline{G}_j^k = 1|\mathcal{I}) = \Lambda_j\Big(\sum_{m\in\mathcal{M}} \mathbb{E}\big[\overline{O}_{jm}^k\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^{k-1}}\big[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}\big] - F_{jm}^e(\nu_{jm}^e)\big]\,|\mathcal{I}\big]\Big), \tag{A52}$$

$$\underline{\mu}_{jm,k} = \underline{\mu}^{offer}_{jm,k}\,\underline{\mu}^{portfolio}_{j,k},$$
$$\overline{\mu}_{jm,k} = \overline{\mu}^{offer}_{jm,k}\,\overline{\mu}^{portfolio}_{j,k}.$$

Based on these iterative objects, I devise an algorithm that converges to bounds – in the sense of first-order stochastic dominance – of any distribution of equilibrium entry probabilities under $\mathcal{I}$.

Algorithm 4 *1. At iteration $k=0$, set the unconditional probabilities of entry as $\underline{\mu}_{jm,0}=0$ and $\overline{\mu}_{jm,0}=1$ for all products j and all markets m. Then, compute $\underline{\mu}^{offer}_{jm,1}$, $\overline{\mu}^{offer}_{jm,1}$, $\underline{\mu}^{portfolio}_{j,1}$, $\overline{\mu}^{portfolio}_{j,1}$ using (A49)-(A52) and obtain,*

$$\underline{\mu}_{jm,1} = \underline{\mu}^{offer}_{jm,1}\,\underline{\mu}^{portfolio}_{j,1}$$
$$\overline{\mu}_{jm,1} = \overline{\mu}^{offer}_{jm,1}\,\overline{\mu}^{portfolio}_{j,1}$$

2. Using $\underline{\boldsymbol{\mu}}^1$ and $\overline{\boldsymbol{\mu}}^1$ together with formulas (A49)-(A52), obtain updated offering probability bounds and portfolio probability bounds. This yields,

$$\underline{\mu}_{jm,2} = \underline{\mu}^{offer}_{jm,2}\,\underline{\mu}^{portfolio}_{j,2}$$
$$\overline{\mu}_{jm,2} = \overline{\mu}^{offer}_{jm,2}\,\overline{\mu}^{portfolio}_{j,2}$$

3. Repeat until convergence to obtain the tightest possible bounds on the distribution of product offerings in each market.

Theorem 2 *Under Assumptions 1-4, Algorithm 4 converges monotonically to bounds, in the sense of first-order stochastic dominance, of any equilibrium distribution of product offering decisions in each market m given any information set $\mathcal{I}$. That is, for any $k>0$ and any $m \in \mathcal{M}$, and any equilibrium distribution of product offerings $\boldsymbol{\mu}^*_m$ under $\mathcal{I}$, $\underline{\boldsymbol{\mu}}^{k-1}_m \leq_{FOSD} \underline{\boldsymbol{\mu}}^{k}_m \leq_{FOSD} \boldsymbol{\mu}^*_m \leq_{FOSD} \overline{\boldsymbol{\mu}}^{k}_m \leq_{FOSD} \overline{\boldsymbol{\mu}}^{k-1}_m$.*

Proof. I first prove that for any product j and any market m, $\{\overline{\mu}_{jm,k}\}_{k=0}^{\infty}$ is a decreasing sequence and $\{\underline{\mu}_{jm,k}\}_{k=0}^{\infty}$ is an increasing sequence. I prove this by induction. By construction $\underline{\mu}_{jm,1} > 0 = \underline{\mu}_{jm,0}$ and $\overline{\mu}_{jm,1} < 1 = \overline{\mu}_{jm,0}$. Assume that the hypothesis is true up to index $K>0$. I now show that (i) $\underline{\mu}_{jm,K+1} \geq \underline{\mu}_{jm,K}$ and (ii) $\overline{\mu}_{jm,K+1} \leq \overline{\mu}_{jm,K}$. To prove (i), note that by definition,

$$\underline{\mu}_{jm,K+1} = \Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}^K_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{Q}^{K+1}_{jm}[\mathbb{E}_{\overline{\boldsymbol{\mu}}^K_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]-F^e_{jm}(\nu^e_{jm})]|\mathcal{I}\big]\Big).$$

By the inductive hypothesis, $\overline{\mu}_{jm,K} \leq \overline{\mu}_{jm,K-1}$ for each j and m. By definition of first-order stochastic dominance, and due to independence, this implies that $\overline{\boldsymbol{\mu}}^K_m \leq_{FOSD} \overline{\boldsymbol{\mu}}^{K-1}_m$. It follows from Assumption 4 and Theorem 3 that,

$$\begin{aligned}
\underline{\mu}_{jm,K+1} &= \Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}^K_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{Q}^{K+1}_{jm}\big[\mathbb{E}_{\overline{\boldsymbol{\mu}}^K_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]-F^e_{jm}(\nu^e_{jm})\big]|\mathcal{I}\big]\Big)\\
&\geq \Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}^{K-1}_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{Q}^{K}_{jm}\big[\mathbb{E}_{\overline{\boldsymbol{\mu}}^{K-1}_m}[MV_{jm}(\mathcal{A}^f,\Omega^{-f}_m)|\mathcal{I}]-F^e_{jm}(\nu^e_{jm})\big]|\mathcal{I}\big]\Big)\\
&= \underline{\mu}_{jm,K}.
\end{aligned}$$

I have therefore proven that for each product j and each market m, the sequence of lower bound probabilities is increasing. Again, due to independence, this also implies that $\underline{\boldsymbol{\mu}}_m^{K+1} \geq_{FOSD} \underline{\boldsymbol{\mu}}_m^{K}$ for each K and each market m. I analogously prove (ii). Indeed,

$$\overline{\mu}_{jm,K+1} = \Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{O}_{jm}^{K+1}\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-F_{jm}^e(\nu_{jm}^e)\big]\big|\mathcal{I}\big]\Big).$$

By the inductive hypothesis, $\underline{\mu}_{jm,K} \geq \underline{\mu}_{jm,K-1}$ for each j and in each m. By definition of first-order stochastic dominance, and due to independence, this implies that $\underline{\boldsymbol{\mu}}_m^{K} \geq_{FOSD} \underline{\boldsymbol{\mu}}_m^{K-1}$. It follows from submodularity and Theorem 3 that,

$$\begin{aligned}
\overline{\mu}_{jm,K+1} &= \Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\overline{O}_{jm}^{K+1}\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-F_{jm}^e(\nu_{jm}^e)\big]\big|\mathcal{I}\big]\Big)\\
&\leq \Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^{K-1}}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\overline{O}_{jm}^{K}\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-F_{jm}^e(\nu_{jm}^e)\big]\big|\mathcal{I}\big]\Big)\\
&= \overline{\mu}_{jm,K}.
\end{aligned}$$

Next, I prove that for all k, it must be that,

$$\underline{\boldsymbol{\mu}}_m^k \leq_{FOSD} \boldsymbol{\mu}_m^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^k \tag{A53}$$

where asterisks denote any equilibrium distribution of product offering decisions under any arbitrary information set $\mathcal{I}$, for each $m \in \mathcal{M}$. I also prove this by induction. Inequalities (A53) clearly hold for $k = 0$. Assume they hold for arbitrary $K \in \mathbb{N}$. I now prove that it must hold for $K + 1$. To do so, take any arbitrary subset $\mathcal{W} \subseteq \mathcal{A}$. Note that $\mathcal{W} = \mathcal{W}^1 \cup \mathcal{W}^2 \cup ... \cup \mathcal{W}^f$, a partition across firms. Then,

$$\mathbb{P}\Big(\bigcap_{j\in\mathcal{W}}\{\overline{V}_{jm}^{K+1}=1\}|\mathcal{I}\Big) = \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\overline{V}_{jm}^{K+1}=1|\mathcal{I}) \tag{A54}$$

$$= \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\Gamma_{jm}\big(\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\overline{O}_{jm}^{K+1}\big[\mathbb{E}_{\underline{\boldsymbol{\mu}}_m^K}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]\big]-F_{jm}^e(\nu_{jm}^e)\big]\big|\mathcal{I}\big]\Big)$$

$$\geq \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\Gamma_{jm}\big(\mathbb{E}_{\boldsymbol{\mu}_m^*}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\overline{O}_{jm}^{*}\big[\mathbb{E}_{\boldsymbol{\mu}_m^*}\big[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-F_{jm}^e(\nu_{jm}^e)\big]\big|\mathcal{I}\big]\Big) \tag{A55}$$

$$= \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\overline{O}_{jm}^*=1|\mathcal{I})\mathbb{P}(\overline{G}_j^*=1|\mathcal{I}) = \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\overline{V}_{jm}^*=1|\mathcal{I}) = \prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{E}(\overline{V}_{jm}^*|\mathcal{I})$$

$$= \mathbb{E}\Big[\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\overline{V}_{jm}^*|\mathcal{I}\Big] \tag{A56}$$

$$\geq \mathbb{E}\Big[\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}V_{jm}^*|\mathcal{I}\Big] = \mathbb{P}\Big(\bigcap_{j\in\mathcal{W}}\{V_{jm}^*=1\}|\mathcal{I}\Big),$$

where (A54) and (A56) follow from the (conditional) independence of the upper bounds and (A55) follows from the inductive hypothesis. This proves $\overline{\boldsymbol{\mu}}^{K+1} \geq_{FOSD} \boldsymbol{\mu}^*$. I analogously prove the other direction. Indeed,

$$
\begin{aligned}
&\mathbb{P}\Big(\bigcap_{j\in\mathcal{W}}\{\underline{V}_{jm}^{K+1}=1\}|\mathcal{I}\Big)=\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\underline{V}_{jm}^{K+1}=1|\mathcal{I}) && \text{(A57)}\\
&=\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\Gamma_{jm}\big(\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^K}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{O}_{jm}^{K+1}\big[\mathbb{E}_{\overline{\boldsymbol{\mu}}_m^K}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}]-F_{jm}^e(\nu_{jm}^e)\big]|\mathcal{I}\big]\Big)\\
&\leq\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\Gamma_{jm}\big(\mathbb{E}_{\boldsymbol{\mu}_m^*}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}]\big)\Lambda_j\Big(\sum_{m\in\mathcal{M}}\mathbb{E}\big[\underline{O}_{jm}^{*}\big[\mathbb{E}_{\boldsymbol{\mu}_m^*}[MV_{jm}(\mathcal{A}^f,\Omega_m^{-f})|\mathcal{I}]-F_{jm}^e(\nu_{jm}^e)\big]|\mathcal{I}\big]\Big) && \text{(A58)}\\
&=\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\underline{O}_{jm}^*=1|\mathcal{I})\mathbb{P}(\underline{G}_j^*=1|\mathcal{I})=\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{P}(\underline{V}_{jm}^*=1|\mathcal{I})=\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\mathbb{E}(\underline{V}_{jm}^*|\mathcal{I})\\
&=\mathbb{E}\Big[\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}\underline{V}_{jm}^*|\mathcal{I}\Big] && \text{(A59)}\\
&\leq\mathbb{E}\Big[\prod_{f=1}^{F}\prod_{j\in\mathcal{W}^f}V_{jm}^*|\mathcal{I}\Big]=\mathbb{P}\Big(\bigcap_{j\in\mathcal{W}}\{V_{jm}^*=1\}|\mathcal{I}\Big),
\end{aligned}
$$

where (A57) and (A59) follow from (conditional) independence of the lower bounds and (A58) follows from the inductive hypothesis.

This concludes the proof. ■

Notice that this proof only relies on Assumption 1, Assumption 4, and conditional independence of fixed cost shocks. It does not rely on the log-normality assumption.

C.1 Implementation

To implement Algorithm 4 in practice, I need to compute expectations $\mathbb{E}_{\boldsymbol{\mu}_m}$, which are over the distribution of rival firms' offerings decisions given $\boldsymbol{\mu}_m$ and over demand and marginal cost shocks. Thus, I first take 100 draws from the joint distribution of $(\boldsymbol{\xi},\boldsymbol{\omega})$ across 9766 product markets, yielding a 9766×100 matrix of demand shocks and a similar matrix of marginal cost shocks. Second, I draw a 9766×100 matrix U^{offer} and a 1530×100 matrix U^{port} of uniformly distributed draws on $(0,1)$. I hold all draws fixed throughout all iterations.

The U^{port} matrix yields thresholds for the portfolio decisions given by probabilities $\mu_{j,k}^{portfolio}$ (both the upper and lower bound). If a given product has a draw smaller than $\mu_{j,k}^{portfolio}$, then that product is introduced in the firm's portfolio under that draw at iteration k. Otherwise, at iteration k and under such a draw, that product is not introduced in the firm's global product portfolio. The U^{offer} matrix yields thresholds for the offering choices given by probabilities $\mu_{jm,k}^{offer}$ (both the upper and lower bounds). For instance, if for product-market the U^{offer} draw is less than the $\mu_{jm,k}^{offer}$ probability at iteration k, then that draw corresponds to such a product being offered in such a market at iteration k (conditional on being in the firm's portfolio). Otherwise, that product is not offered in market m under iteration k and that draw. For a product to be offered in a market, it must satisfy both the offering and portfolio thresholds.

It follows that under each iteration k, I need to compute 9766×100 marginal values across all product-market pairs. Then, I average them across the 100 draws to obtain simulated values of the expected marginal values at the product-market level. This averaging yields the expectations that enter the fixed cost CDFs in equations (A49)-(A52). To compute the marginal values under any draw, I use the Morrow and Skerlos (2011) contraction mapping for the pricing equilibria with 500 income and normal draws for each market.

I run Algorithm 4 for 6 iterations under each parameter vector $(\theta_e,\sigma_e,\theta_g,\sigma_g)$ and each policy counterfactual. I find that after $k>6$, the additional gains in informativeness are small in my application.

D Estimation Implementation

D.1 Demand and Marginal Cost Estimation

Demand estimation follows Petrin (2002). The assumption that $(\boldsymbol{\xi}, \boldsymbol{\omega})$ are realized after firms make offerings choices implies that $\mathbb{E}[(\xi_{jm}, \omega_{jm})|\mathcal{I}] = 0$. Under this assumption, standard Berry et al. (1995) or Gandhi and Houde (2019) instruments are valid. I use size, horsepower, and horsepower/weight to build Gandhi and Houde (2019) differentiation instruments. These instruments are constructed by using the characteristics of products that are "close" in the characteristics space. The intuition is that these characteristics will have a larger impact on the price set on product j (p_{jm}) while being, by assumption, uncorrelated with the demand and marginal cost shocks. I use the PyBLP Python package to construct the differentiation instruments (Conlon and Gortmaker 2020). For each of the three characteristics, I build two instruments. Let $x_{jm\ell}$ denote the value of any such characteristic, indexed by ℓ. I construct, for each $\ell \in \{\text{size, horsepower, horsepower/weight}\}$:

$$z_{jm\ell}^{other} = \sum_{k \in \Omega_m^f \setminus \{j\}} \mathbb{1}\{|d_{jkm\ell}| < SD_\ell\}$$

$$z_{jm\ell}^{rival} = \sum_{k \in \Omega_m^{-f}} \mathbb{1}\{|d_{jkm\ell}| < SD_\ell\}$$

where $d_{jkm\ell} = x_{jm\ell} - x_{km\ell}$ and SD_ℓ denotes the standard deviation of all such pairwise differences computed across all markets. This yields 6 differentiation instruments.

To improve the precision of my estimates, I include 4 additional moments. First, I include (i) the log of distance to the brand's HQ country, which enters my marginal cost specification, and (ii) the average price of the same product in other markets / the average price of products of the same parent company in other markets (when a given product is only sold in one market). The latter set of instruments is reminiscent of the Hausman (1996) instruments, which are valid provided demand and marginal cost shocks $(\boldsymbol{\xi}_m, \boldsymbol{\omega}_m)$ are uncorrelated across markets.

Second, I use micro-moments, similarly to Petrin (2002). I use micro-data from the 2019 MRI-Simmons Crosstab report to help pin down the heterogeneity in preferences for prices within countries. I match the probabilities that a consumer in the United States is within a given income group conditional on purchasing a vehicle in a given price range. More specifically, I match the following two moments:

(a) $\mathbb{E}[income_i > \$100,000 | price_{jm} > \$50,000, m = \text{United States}]$

(b) $\mathbb{E}[income_i \in [\$60,000, \$100,000] | price_{jm} > \$50,000, m = \text{United States}]$.

I jointly estimate demand and marginal costs using Python's PyBLP package. To integrate over the distribution of income in each market/country, I take 20,000 simulation draws in each market m from a log-normal distribution with scale

$$\sigma_m = \sqrt{2}\Phi^{-1}\left(\frac{Gini_m + 1}{2}\right),$$

and location,

$$\mu_m = \log(GDP\_per\_capita\_PPP_m) - \sigma_m^2/2.$$

I obtain the Gini coefficient and PPP GDP per capita in each market m from the World Bank. The above parametrization ensures that income is drawn from a log-normal distribution with mean and Gini coefficients equal to the observed values.

The main results from the estimation are in the main text, Table 1. Below, I report the matched micro-moments:

Moment	Observed	Estimated	Difference
a	0.631	0.612	0.0188
b	0.212	0.245	-0.0329

D.1.1 Distribution of $(\boldsymbol{\xi}, \boldsymbol{\omega})$

The Berry et al. (1995) contraction mapping yields product-market specific demand and marginal cost shocks (ξ_{jm}, ω_{jm}) as a by-product of demand and marginal cost estimation. I fit a bivariate normal distribution for the distribution of demand and marginal cost shocks, which I assume firms know at the time of making product portfolio and product offerings decisions. Figure 17 shows the marginal distributions of ξ and ω. The estimated variance-covariance matrix is reported in Table 3.[44]

Table 3: Bivariate Normal Parameters for Joint Distribution of (ξ, ω)

	ξ	ω
ξ	2.36	
ω	0.081	0.024.

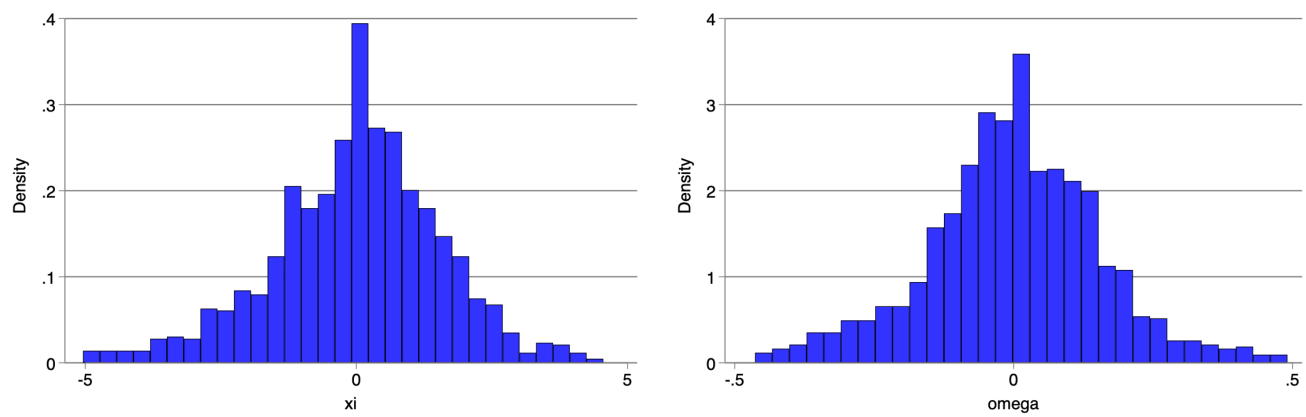


Figure 17: Empirical Distribution of (ξ, ω) - Marginals

D.2 Fixed Cost Estimation - Instruments

To estimate fixed costs, I implement the moment inequalities derived in Sections B.1 and B.2. As described in the main text, I first construct instruments using an approach similar to two-stage least squares. I then use these instruments to build unconditional moment inequalities. I use the empirical counterparts of these moment inequalities to estimate $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. The steps are described in the main text, Section 4.

To construct the instruments, I use the Stage 3 model for variable profits, together with the fitted distribution of $(\boldsymbol{\xi}, \boldsymbol{\omega})$ to compute the ingredients necessary to implement the inequalities in Theorem 1. I simulate $S = 200$

[44] I do not report standard errors for these estimates. Doing so would require bootstrapping the demand and marginal cost estimation procedure.

draws $\{\xi^s_{jm}, \omega^s_{jm}\}^{200}_{s=1, j\in\mathcal{A}, m\in\mathcal{M}}$ from the fitted bivariate normal distribution and construct, for each product j and each market m,

$$\widehat{MV}_{jm}(\{j\}, \Omega^{-f}_m) = \frac{1}{S}\sum_{s=1}^{S} MV_{jm}(\{j\}, \Omega^{-f}_m; \xi^s_{jm}, \omega^s_{jm}) \tag{A60}$$

$$\widehat{MV}_{jm}(\mathcal{G}^f, \Omega^{-f}_m) = \frac{1}{S}\sum_{s=1}^{S} MV_{jm}(\mathcal{G}^f, \Omega^{-f}_m; \xi^s_{jm}, \omega^s_{jm}) \tag{A61}$$

$$\widehat{MV}_{jm}(\mathcal{A}^f, \Omega^{-f}_m) = \frac{1}{S}\sum_{s=1}^{S} MV_{jm}(\mathcal{A}^f, \Omega^{-f}_m; \xi^s_{jm}, \omega^s_{jm}). \tag{A62}$$

To obtain an exogenous ($\mathcal{I}$- measurable) predictor of the above marginal values, I project the log of the predicted values in (A60)-(A62) on the following objects: (i) the log of $\tilde{\delta}_{jm} = \beta_m + \beta_{b(j)} + \beta^x \boldsymbol{X}_{jm}$, the non-price mean utility of product j in market m net of unobserved heterogeneity ξ_{jm}, (ii) $\tilde{c}_{jm} = \log(c_{jm}) - \omega_{jm}$, the (log) marginal cost of supplying market m with product j net of the unobserved heterogeneity ω_{jm}, and (iii) $\log(\tilde{\delta}_{jm}) \times \tilde{c}_{jm}$. To carry out the projection, I estimate 4 PPML specifications. I first project (A60)-(A61), conditional on $j \in \mathcal{G}^f$, using,

$$\hat{y}_{jm} = \exp(\kappa^1 M_m + \kappa^2_m \tilde{\delta}_{jm} + \kappa^3_m \tilde{c}_{jm} + \kappa^4_m \tilde{c}_{jm} \times \tilde{\delta}_{jm}) + \varepsilon_{jm}. \tag{A63}$$

I then use the same specification to project (A60) and (A62), using the full sample of potential products.

The results from these PPML regressions are reported in Table 4. As described in the main text, I then use predicted values from these regressions to construct the unconditional moments that can be used for estimation.

To implement the Stage 2 inequalities that partially identify (θ_e, σ_e), I use tercile bin indicators for the predicted values in the PPML regressions (1)-(2) from Table 4, $\hat{\bar{x}}^g_{jm}$ and $\hat{\underline{x}}^h_{jm}$, respectively. I also use the squares of such (positive) predicted values interacted with an indicator function denoting whether the log of the predicted value (in billions of USD) is greater than -2, which selects more profitable products. This helps to provide bounds on the scale parameter σ_e, as argued in Section 4.

To implement the Stage 1 inequalities that partially identify (θ_g, σ_g), I use quintile bin indicators (5 bins) for the *sum* across markets of the predicted values arising from PPML regressions (3)-(4) in Table 4 – $\hat{\bar{x}}^a_{jm}$ and $\hat{\underline{x}}^l_{jm}$, respectively –, as well as squares of such sums of predicted values, interacted with an indicator function for whether the log of the sum (in billions of USD) is greater than 0.

D.2.1 Approximation Points Used for Convex and Convex CDF Bounds

To implement the moment inequality procedure, I use observation-specific upper and lower bounds of the fixed-cost CDFs. I use the convex and concave families of functions in equations (A7) and (A14), respectively. The approximation points I use for the Stage 2 upper and lower bound inequalities are $\hat{\bar{x}}^g_{jm}$ and $\hat{\underline{x}}^h_{jm}$, respectively. I use $\sum_m \hat{\bar{x}}^a_{jm}$ and $\sum_m \hat{\underline{x}}^l_{jm}$, respectively, for the Stage 1 upper and lower bound inequalities.

D.3 Fixed Cost Estimation - Empirical Analogues of Moments in Theorem 1

As described in the main text, I construct positive-valued instruments and interact them with the conditional moments in Theorem 1 to construct empirical unconditional moment inequalities. These take the form,

$$\frac{1}{J_g M} \sum_{j\in\mathcal{G}, m\in\mathcal{M}} [\overline{\Gamma}_{jm}(MV_{jm}(\{j\}, \Omega^{-f}_m); \theta_e, \sigma_e) - O_{jm}] \mathbb{1}\{\hat{\bar{x}}^a_{jm} \in Q_\tau(\hat{\bar{x}}^g_{jm})] \geq 0, \tag{A64}$$

Table 4: PPML Estimates

VARIABLES	(1) Max MV - Portfolio	(2) Min MV - Portfolio	(3) Min MV - All Potential	(4) Max MV - All Potential
Mkt Size	-0.000***	-0.000***	-0.000***	-0.000***
	[0.000]	[0.000]	[0.000]	[0.000]
AUS $\times\tilde{\delta}$	1.142***	1.186***	1.142***	1.101***
	[0.130]	[0.117]	[0.100]	[0.124]
BRA $\times\tilde{\delta}$	1.573***	1.451***	1.409***	1.428***
	[0.156]	[0.111]	[0.107]	[0.125]
FRA $\times\tilde{\delta}$	1.017***	0.934***	0.914***	0.999***
	[0.137]	[0.119]	[0.108]	[0.141]
DEU $\times\tilde{\delta}$	1.188***	1.049***	1.036***	1.089***
	[0.195]	[0.145]	[0.147]	[0.223]
IND $\times\tilde{\delta}$	1.412***	1.444***	1.307***	1.387***
	[0.084]	[0.127]	[0.104]	[0.073]
ITA $\times\tilde{\delta}$	1.348***	1.350***	1.278***	1.294***
	[0.182]	[0.152]	[0.117]	[0.154]
JPN $\times\tilde{\delta}$	0.911***	0.381***	0.001	0.708***
	[0.135]	[0.109]	[0.115]	[0.139]
CHN $\times\tilde{\delta}$	1.759***	1.698***	1.104***	1.200***
	[0.114]	[0.121]	[0.090]	[0.082]
MEX $\times\tilde{\delta}$	1.906***	1.746***	1.007***	1.132***
	[0.458]	[0.350]	[0.247]	[0.328]
ESP $\times\tilde{\delta}$	1.248***	1.061***	1.188***	1.298***
	[0.161]	[0.122]	[0.115]	[0.152]
GBR $\times\tilde{\delta}$	0.969***	1.161***	1.230***	1.082***
	[0.145]	[0.150]	[0.109]	[0.124]
USA $\times\tilde{\delta}$	2.025***	1.827***	1.287***	1.585***
	[0.175]	[0.175]	[0.102]	[0.094]
AUS $\times\tilde{c}$	-6.571***	-6.485***	-6.494***	-6.474***
	[0.185]	[0.170]	[0.142]	[0.189]
BRA $\times\tilde{c}$	-6.124***	-6.038***	-6.463***	-6.399***
	[0.128]	[0.117]	[0.099]	[0.129]
FRA $\times\tilde{c}$	-6.368***	-6.287***	-6.428***	-6.397***
	[0.159]	[0.145]	[0.124]	[0.166]
DEU $\times\tilde{c}$	-6.443***	-6.337***	-6.517***	-6.501***
	[0.150]	[0.136]	[0.117]	[0.155]
IND $\times\tilde{c}$	-7.177***	-7.102***	-7.398***	-7.405***
	[0.152]	[0.148]	[0.125]	[0.154]
ITA $\times\tilde{c}$	-6.380***	-6.296***	-6.429***	-6.399***
	[0.159]	[0.145]	[0.124]	[0.166]
JPN $\times\tilde{c}$	-6.246***	-6.119***	-6.377***	-6.413***
	[0.133]	[0.122]	[0.100]	[0.138]
CHN $\times\tilde{c}$	-4.319***	-4.234***	-5.382***	-5.291***
	[0.227]	[0.233]	[0.083]	[0.072]
MEX $\times\tilde{c}$	-6.286***	-6.224***	-6.394***	-6.314***
	[0.162]	[0.146]	[0.128]	[0.174]
ESP $\times\tilde{c}$	-6.570***	-6.475***	-6.562***	-6.543***
	[0.171]	[0.156]	[0.132]	[0.176]
GBR $\times\tilde{c}$	-7.142***	-7.148***	-7.273***	-7.108***
	[0.179]	[0.165]	[0.137]	[0.181]
USA $\times\tilde{c}$	-4.665***	-4.577***	-5.667***	-5.581***
	[0.202]	[0.207]	[0.077]	[0.066]
AUS $\times\tilde{\delta}\times\tilde{c}$	-0.013	-0.019	-0.015	-0.011
	[0.013]	[0.012]	[0.010]	[0.013]
BRA $\times\tilde{\delta}\times\tilde{c}$	-0.067***	-0.058***	-0.050***	-0.051***
	[0.015]	[0.010]	[0.010]	[0.012]
FRA $\times\tilde{\delta}\times\tilde{c}$	-0.013	-0.007	-0.003	-0.010
	[0.013]	[0.011]	[0.010]	[0.014]
DEU $\times\tilde{\delta}\times\tilde{c}$	-0.036**	-0.027**	-0.024*	-0.026
	[0.017]	[0.013]	[0.013]	[0.020]
IND $\times\tilde{\delta}\times\tilde{c}$	-0.075***	-0.079***	-0.065***	-0.069***
	[0.008]	[0.011]	[0.009]	[0.007]
ITA $\times\tilde{\delta}\times\tilde{c}$	-0.041**	-0.042***	-0.035***	-0.035**
	[0.017]	[0.015]	[0.011]	[0.015]
JPN $\times\tilde{\delta}\times\tilde{c}$	0.011	0.054***	0.088***	0.031**
	[0.013]	[0.010]	[0.010]	[0.013]
CHN $\times\tilde{\delta}\times\tilde{c}$	-0.080***	-0.077***	-0.012	-0.020***
	[0.012]	[0.013]	[0.008]	[0.008]
MEX $\times\tilde{\delta}\times\tilde{c}$	-0.093**	-0.080**	-0.007	-0.018
	[0.045]	[0.034]	[0.025]	[0.034]
ESP $\times\tilde{\delta}\times\tilde{c}$	-0.034**	-0.019	-0.029***	-0.038***
	[0.015]	[0.012]	[0.011]	[0.015]
GBR $\times\tilde{\delta}\times\tilde{c}$	-0.023*	-0.038***	-0.041***	-0.033***
	[0.013]	[0.013]	[0.010]	[0.011]
USA $\times\tilde{\delta}\times\tilde{c}$	-0.117***	-0.101***	-0.041***	-0.068***
	[0.017]	[0.017]	[0.010]	[0.009]
Observations	3,240	3,240	8,868	8,868
Portfolio Only	Yes	Yes	No	No

$$\frac{1}{J_g M} \sum_{j \in \mathcal{G}, m \in \mathcal{M}} [\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f, \Omega_m^{-f}); \theta_e, \sigma_e) - O_{jm}] \mathbb{1}\{\underline{\hat{x}}_{jm}^h \in Q_\tau(\underline{\hat{x}}_{jm}^h)] \leq 0, \tag{A65}$$

$$\begin{aligned} \frac{1}{J} \sum_{j \in \mathcal{A}} \Big[\overline{\Lambda}_j \Big(& \sum_{m \in \mathcal{M}} \Gamma_{jm}\big(\pi_{jm}(\{j\}, \Omega_m^{-f})\big) \big[\pi_{jm}(\{j\}, \Omega_m^{-f}) \\ & - \mathbb{E}\big[F_{jm}^e(\nu_{jm}^e) | \mathcal{I}, F_{jm}^e(\nu_{jm}^e) \leq \pi_{jm}(\{j\}, \Omega_m^{-f})\big]\big]; \theta_g, \sigma_g \Big) - G_j \Big] \\ & \times \mathbb{1}\Big\{ \sum_{m \in \mathcal{M}} \hat{\overline{x}}_{jm}^a \in Q_\tau \Big(\sum_{m \in \mathcal{M}} \hat{\overline{x}}_{jm}^a \Big) \Big\} \geq 0, \end{aligned} \tag{A66}$$

$$\begin{aligned} \frac{1}{J} \sum_{j \in \mathcal{A}} \Big[\underline{\Lambda}_j \Big(& \sum_{m \in \mathcal{M}} \Gamma_{jm}\big(\underline{\hat{x}}_{jm}^l\big) \big[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f}) - \mathbb{E}\big[F_{jm}^e(\nu_{jm}^e) | F_{jm}^e(\nu_{jm}^e) \leq \underline{\hat{x}}_{jm}^l, \mathcal{I}\big]\big]; \theta_g, \sigma_g \Big) - G_j \Big] \\ & \times \mathbb{1}\Big\{ \sum_{m \in \mathcal{M}} \underline{\hat{x}}_{jm}^l \in Q_\tau \Big(\sum_{m \in \mathcal{M}} \underline{\hat{x}}_{jm}^l \Big) \Big\} \leq 0, \end{aligned} \tag{A67}$$

where the expectation of the truncated fixed costs in inequalities (A66)-(A67) are computed using the QuadGK Julia package (Gaussian quadrature). J_g denotes the number of products in the sample that are offered in at least one market.

D.4 Fixed Cost Estimation - Robustness

D.4.1 More Instrument Bins

I report the estimation results using more instrument bins for both Stage 2 and Stage 1 inequalities. I construct 10 bins for the Stage 2 inequalities and 8 bins for the Stage 1 inequalities using the regressions from Section D.2. Recall that the baseline specification used 3 bins for Stage 2 and 5 bins for Stage 1. I keep the remaining polynomial-based instruments as they are in the main specification.

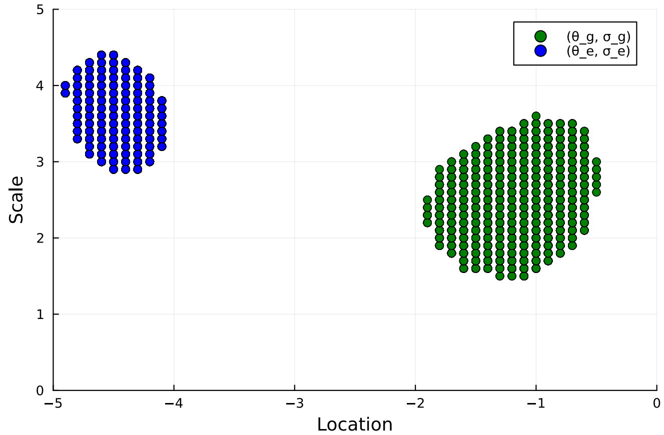
I do not find that including more instrument bins leads to model rejection. Table 5 and Figure 18 show that the confidence sets do not change very significantly under more instrument bins. And, if anything, the limits of the confidence sets become wider. This is due to increased covariance across more instruments.

Table 5: Stages 1 and 2 Parameter Confidence Set Limits

	95% Confidence Set Limits
Stage 2: Market Entry Fixed Cost	
θ_e (Location)	[-4.9, -4.1]
σ_e (Scale)	[2.9, 4.4]
Stage 1: Product Fixed Cost	
θ_g (Location)	[-1.9, -0.5]
σ_g (Scale)	[1.5, 3.6]
Observations - Stage 2	3240
Observations - Stage 1	739

Notes: Confidence sets computed using Andrews and Soares (2010). First, I implement a grid search to compute a 97.5% confidence set for parameters (θ_e, σ_e) using the Stage 2 moment inequalities. Then, I use the Stage 1 moment inequalities to compute a 97.5% confidence set for (θ_g, σ_g), evaluating the moments at the accepted values of (θ_e, σ_e). The Bonferroni correction yields a 95% confidence set for all 4 parameters $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. Marginal values are in billions of US dollars.

Figure 18: 95% Confidence Set



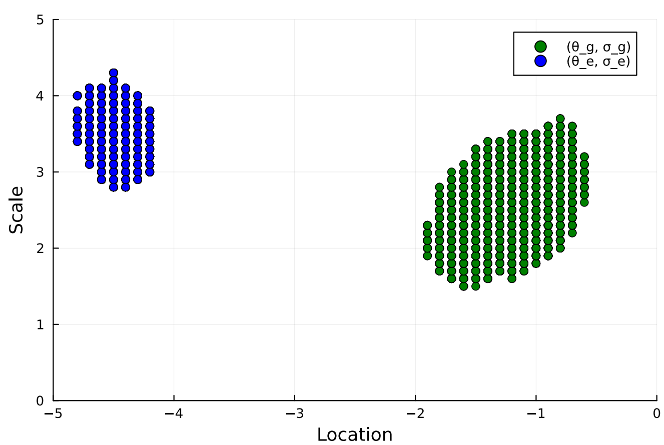
Notes: This figure projects the 95% confidence set for the location and scale parameters describing the distributions of product portfolio and market entry fixed costs on a two-dimensional grid on the location-scale dimensions.

Table 6: Stages 1 and 2 Parameter Confidence Set Limits

	95% Confidence Set Limits
Stage 2: Market Entry Fixed Cost	
θ_e (Location)	[-4.8, -4.2]
σ_e (Scale)	[2.8, 4.3]
Stage 1: Product Fixed Cost	
θ_g (Location)	[-1.9, -0.6]
σ_g (Scale)	[1.5, 3.7]
Observations - Stage 2	3240
Observations - Stage 1	739

Notes: Confidence sets computed using Andrews and Soares (2010). First, I implement a grid search to compute a 97.5% confidence set for parameters (θ_e, σ_e) using the Stage 2 moment inequalities. Then, I use the Stage 1 moment inequalities to compute a 97.5% confidence set for (θ_g, σ_g), evaluating the moments at the accepted values of (θ_e, σ_e). The Bonferroni correction yields a 95% confidence set for all 4 parameters $(\theta_e, \sigma_e, \theta_g, \sigma_g)$. Marginal values are in billions of US dollars.

Figure 19: 95% Confidence Set



Notes: This figure projects the 95% confidence set for the location and scale parameters describing the distributions of product portfolio and market entry fixed costs on a two-dimensional grid on the location-scale dimensions.

D.4.2 Fewer Instrument Bins

Due to the results from Section D.4.1, which show that under more instrument bins, the confidence sets are larger, I also report the confidence sets under fewer instrument bins. I now report the results with 2 bins for both Stage 2 and Stage 1. Again, all remaining instruments are left unchanged.

Figure 19 and Table 6 show that the confidence sets do not change much relative to the main specification. This increases my confidence in the results and shows that they are robust to the number of instrument bins.

E Discussion of Model Extensions

In this section, I discuss possible model extensions and the relaxation of several assumptions.

E.1 Submodularity of Variable Profits

First, I discuss the role that submodularity plays in the moment inequalities I derive for estimation. I then discuss its relevance for the iterative solution method.

E.1.1 Moment Inequalities and Submodularity

To show how submodularity assumption could potentially be relaxed, I discuss the derivation of the upper bound market entry fixed cost moment inequality analyzed in section B.1. Notice that equation (A1) still holds. Recall that this equation is,

$$(\mathbb{1}_{\Omega_m^f} + \mathbb{1}_{\Omega_m^f \setminus \{j\}}) \times [\mathbb{1}\{\underbrace{\mathbb{E}[MV_{jm}(\Omega_m^f, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m \in \mathcal{M}}, \mathcal{G}^f] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0}_{\Omega_m^f \text{ is preferred to } \Omega_m^f \setminus \{j\}}\} - \mathbb{1}_{\Omega_m^f}] = 0$$

It says that if either bundle Ω_m^f or $\Omega_m^f \setminus \{j\}$ is chosen, then bundle Ω_m^f is chosen if and only if Ω_m^f is preferred to $\Omega_m^f \setminus \{j\}$. Thus, this equality does not use submodularity. Submodularity does arise in the next step when I use it to find an upper bound for $\mathbb{E}[MV_{jm}(\Omega_m^f, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m \in \mathcal{M}}, \mathcal{G}^f]$ that is independent of private information $\{\nu_{jm}^e\}_{j \in \mathcal{G}^f}$. For the derivation of the moment inequalities, this is the only role that submodularity plays. What is really needed for estimation is not that the demand and pricing model implies that MV_{jm} is submodular, but that for any $j \in \mathcal{A}$, any $m \in \mathcal{M}$, and any Ω_m^{-f}, there exists known product bundles $\overline{\mathcal{B}}_{jm}^f$ and $\underline{\mathcal{B}}_{jm}^f$, independent of the fixed cost shock realizations $\{\nu_{jm}^e\}$ such that,

$$\mathbb{E}[MV_{jm}(\underline{\mathcal{B}}_{jm}^f, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f] \leq \mathbb{E}[MV_{jm}(\Omega_m^f, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m \in \mathcal{M}}, \mathcal{G}^f] \leq \mathbb{E}[MV_{jm}(\overline{\mathcal{B}}_{jm}^f, \Omega_m^{-f})|\mathcal{I}, \mathcal{G}^f]. \quad \text{(A68)}$$

For instance, suppose that there are two firms competing in market m. Firm 1 has 2 products in its portfolio, and firm 2 has only 1 product. For firm 2, the inequality follows trivially, as there is only one potential deviation it can undertake and, therefore, no interdependence across products. For firm 1, suppose that marginal cost synergies are very large between the two products so that independently of whether firm 2 is offering its product or not, it is the case that, for products $j \in \{1, 2\}$,

$$\mathbb{E}[MV_{jm}(\{1,2\}, \Omega_m^2)|\mathcal{I}, \mathcal{G}^f] > \mathbb{E}[MV_{jm}(\{j\}, \Omega_m^2)|\mathcal{I}, \mathcal{G}^f].$$

That is, the expected marginal value of offering a given product is always higher when the other product is also sold due to high enough marginal cost synergies between both products. In this case, one can derive the upper bound inequality for firm 1 as,

$$(\mathbb{1}_{\Omega^f_m} + \mathbb{1}_{\Omega^f_m \setminus \{j\}}) \times [\mathbb{1}\{\mathbb{E}[MV_{jm}(\{1,2\}, \Omega^{-f}_m)|\mathcal{I}, \mathcal{G}^f] - F^e_{jm}(\nu^e_{jm}; \theta_e, \sigma_e) \geq 0\} - \mathbb{1}_{\Omega^f_m}] \geq 0.$$

Summing, as in the main text, across bundles with $j \in \Omega^f_m$, I obtain inequality,

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\{1,2\}, \Omega^{-f}_m)|\mathcal{I}, \mathcal{G}^f] - F^e_{jm}(\nu^e_{jm}; \theta_e, \sigma_e) \geq 0\} - O_{jm} \geq 0.$$

Taking expectations conditional on $\mathcal{I}$ and $\mathcal{G}^f$ yields,

$$\Gamma_{jm}(\mathbb{E}[MV_{jm}(\{1,2\}, \Omega^{-f}_m)|\mathcal{I}, \mathcal{G}^f]; \theta_e, \sigma_e) \geq \mathbb{E}[O_{jm}|\mathcal{I}, \mathcal{G}^f].$$

Finally, using a convex upper bound for Γ_{jm} and applying Jensen's inequality yields:

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{1,2\}, \Omega^{-f}_m); \theta_e, \sigma_e) - O_{jm}|\mathcal{I}, \mathcal{G}^f] \geq 0$$

or

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f, \Omega^{-f}_m); \theta_e, \sigma_e) - O_{jm}|\mathcal{I}, \mathcal{G}^f] \geq 0.$$

So, in the case in which marginal cost synergies can be proven to dominate consumer substitution across products, such an alternative upper bound moment inequality can be derived. In this simple case, the two products of firm 1 exhibit complementarities rather than substitutabilities.

Thus, for estimation, neither supermodularity nor submodularity is needed. Condition (A68) is more general but requires knowledge of model-consistent bounding bundles $\overline{\mathcal{B}}^f_{jm}$ and $\underline{\mathcal{B}}^f_{jm}$. For instance, Castro-Vincenzi et al. (2024) provides a non-strategic model where any pair of a firm's discrete choices can be proven to be either complements or substitutes in the increasing (decreasing) differences sense. Properties like these could potentially be exploited to construct such bounds.

E.1.2 Solution Algorithm and Submodularity

The solution algorithm relies more heavily on submodularity, given that I need to apply Theorem 3 to prove the monotonicity and convergence properties of the iterative algorithm, Theorem 2. An analogous iterative algorithm works in the case that variable profits are globally supermodular in product offerings. In this case, the upper bound inequalities (both for market entry and product portfolio) are based on

$$\mathbb{E}[MV_{jm}(\mathcal{A}^f, \Omega^{-f}_m)|\mathcal{I}]$$

while the lower bound inequalities use

$$\mathbb{E}[MV_{jm}(\{j\}, \Omega^{-f}_m)|\mathcal{I}].$$

The algorithm can be modified to exploit supermodularity rather than submodularity. To initialize the modified algorithm under complementarity, one now uses $\underline{\boldsymbol{\mu}}^0_m = \mathbf{0}$ and $\overline{\boldsymbol{\mu}}^0_m = \mathbf{1}$ to construct the lower bound and upper bound probabilities of product offerings in each market, respectively. The iteration then continues exactly in the same fashion as in Algorithm 4, but by using the updated $\underline{\boldsymbol{\mu}}^1_m$ and $\overline{\boldsymbol{\mu}}^1_m$ to construct updated *lower bounds* and *upper bounds*, respectively. The algorithm iterates on this mapping and converges to bounds, in the sense of FOSD, of the equilibrium distribution of product offerings market by market, assuming that all discrete choices are complements rather than substitutes. Under supermodularity, convergence and monotonicity follow immediately using Theorem

3, given that the MV_{jm} function is now monotonically increasing.

E.2 Moment Inequalities and Economies of Scope

The model described in the main text assumes that portfolio fixed costs and market entry fixed costs are independent of the firms' chosen portfolio and offerings bundles. In this subsection, I show how this assumption can be relaxed. In particular, I allow market entry fixed costs to exhibit a version of economies of scope. Similar arguments can also be used to allow for economies of scope in global portfolio fixed costs.

Suppose that the total market entry fixed costs paid by firm f if it offers products Ω_m^f are,

$$F_m^{e,f} = \theta_0 \mathbb{1}\{|\Omega_m^f| \geq 1\} + \sum_{j \in \Omega_m^f} F_{jm}^e.$$

That is, firms pay a constant amount θ_0 to enter the market, and then market entry fixed costs with a specification as in the main text, given by Assumption 2. Throughout, I assume that a natural lower bound on θ_0 is 0 to be consistent with the fact that fixed costs are positive under the log-normal assumption. Then, I apply similar arguments to those in the main text to derive a lower and an upper bound inequality. As before, I start with revealed-preference equality,

$$\begin{aligned}&(\mathbb{1}_{\Omega_m^f} + \mathbb{1}_{\Omega_m^f \setminus \{j\}})\\ &\times [\mathbb{1}\{\underbrace{\mathbb{E}[MV_{jm}(\Omega_m^f, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}, \mathcal{G}^f] - \theta_0 \mathbb{1}\{\Omega_m^f = \{j\}\} - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0}_{\Omega_m^f \text{ is preferred to } \Omega_m^f \setminus \{j\}}\} - \mathbb{1}_{\Omega_m^f}] = 0,\end{aligned}$$

where I have now allowed the fixed cost to be higher if j is the first product to be introduced into market m. Under submodularity and $\theta_0 \geq 0$, one can derive upper bound inequality,

$$(\mathbb{1}_{\Omega_m^f} + \mathbb{1}_{\Omega_m^f \setminus \{j\}}) \times [\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}, \mathcal{G}^f] - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0\} - \mathbb{1}_{\Omega_m^f}] \geq 0.$$

Under $\theta_0 \geq 0$, one can always derive an upper bound inequality by ignoring θ_0, which maximizes the marginal value net of marginal fixed cost. For the upper bound, I then follow the same arguments as in the main text to derive upper bound inequality,

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{j\}, \Omega_m^{-f}); \theta_e, \sigma_e) - O_{jm}|\mathcal{I}, \mathcal{G}^f] \geq 0. \tag{A69}$$

Inequality (A69) does not provide any bound on θ_0. For the lower bound, I proceed in a similar fashion. Minimizing MV_{jm} and maximizing marginal fixed costs, I derive inequality,

$$(\mathbb{1}_{\Omega_m^f} + \mathbb{1}_{\Omega_m^f \setminus \{j\}}) \times [\mathbb{1}\{\mathbb{E}[MV_{jm}(\mathcal{G}^f, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j \in \mathcal{G}^f, m}, \mathcal{G}^f] - \theta_0 - F_{jm}^e(\nu_{jm}^e; \theta_e, \sigma_e) \geq 0\} - \mathbb{1}_{\Omega_m^f}] \leq 0.$$

Following similar steps as in section B.1, I obtain conditional moment inequality,

$$\mathbb{E}[\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^f, \Omega_m^{-f}) - \theta_0; \theta_e, \sigma_e) - O_{jm}|\mathcal{I}] \leq 0.$$

This inequality provides a lower bound on θ_0. Therefore, the inequalities discussed so far only identify a lower bound and not an upper bound of the new parameter of interest θ_0. In the following subsection, I discuss additional inequalities that partially identify θ_0.

E.2.1 Additional Inequalities

To derive additional inequalities, I explore what happens if I condition on either a single product or no product being offered in market m. That is, I now start with,

$$(\mathbb{1}_{\{j\}_m} + \mathbb{1}_{\emptyset_m}) \times [\mathbb{1}\{\underbrace{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m}, \mathcal{G}^f] - \theta_0 - F_{jm}^e(\nu_{jm}^e;\theta_e,\sigma_e) \geq 0}_{\{j\}_m \text{ is preferred to } \emptyset_m}\} - \mathbb{1}_{\{j\}_m}] = 0.$$

To deal with the issue of selection, I first re-write the above inequality as,

$$(\mathbb{1}_{\{j\}_m} + \mathbb{1}_{\emptyset_m}) \times \mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m}, \mathcal{G}^f] - \theta_0 - F_{jm}^e(\nu_{jm}^e;\theta_e,\sigma_e) \geq 0\} = \mathbb{1}_{\{j\}_m}.$$

Because $(\mathbb{1}_{\{j\}_m} + \mathbb{1}_{\emptyset_m}) \leq 1$, the following upper bound inequality holds,

$$\mathbb{1}\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}, \{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m\in\mathcal{M}}, \mathcal{G}^f] - \theta_0 - F_{jm}^e(\nu_{jm}^e;\theta_e,\sigma_e) \geq 0\} - \mathbb{1}_{\{j\}_m} \geq 0.$$

The above inequality says that if only a single product j is offered in market m, then the change in profits from doing so must necessarily be weakly positive. Letting O_{jm}^1 denote the event that only product j is sold in market m by firm f, I follow steps similar to those in the main text to derive inequality,

$$\mathbb{E}[\overline{\Gamma}_{jm}(MV_{jm}(\{j\}, \Omega_m^{-f}) - \theta_0;\theta_e,\sigma_e) - O_{jm}^1|\mathcal{I}] \geq 0.$$

To implement this moment inequality in practice, the empirical counterpart of $\mathbb{E}[O_{jm}^1|\mathcal{I}]$ averages across firm-markets a dummy equal to 1 if a given firm introduces a singleton bundle in a given market. Intuitively, an upper bound on θ_0 is identified because a value of θ_0 that is too large would imply too few observations of singleton product entry across markets.

In practice, this is difficult to implement in my data because of a lack of statistical power. I do not observe sufficient instances of singleton product entry to obtain informative bounds on θ_0.

F Equilibrium Existence in the Global Entry Game

To prove this result, I use Theorem 3.1 from Balder (1988). This paper provides general existence results for equilibria in Bayesian games. More precisely, the paper shows that provided,

1. The payoff function is measurable on the product set of actions and types,
2. The payoff function is continuous on actions given types,
3. The payoff function is bounded by some $\mathcal{L}_1$ function,
4. The measure of types is absolutely continuous,
5. The action set for each player is a compact set,

then a Nash equilibrium in *behavioral* strategies exists. In my setting, action sets are finite. This trivially implies that condition 5 holds true. Moreover, because any convergent sequence in a finite set is eventually constant, condition 2 also holds. Condition 3 holds because payoffs are bounded below by 0 (firms can always choose to exit all markets and earn 0 profits) and bounded above by monopoly profits. In addition, the measure of types is absolutely continuous in my setting due to my assumption of independent and log-normal private types. Finally, the payoff function is measurable on the product set of actions and types. This holds because the payoff function is continuous on own and rivals' actions (since actions are a finite set), own types, and also other firms' types since other firms' types only enter a firm's payoff function through actions. Thus, due to Balder (1988), a Nash equilibrium in behavioral strategies exists.

I now show that I can focus without loss of generality on pure strategy Nash equilibria. To prove this, I use Theorem 4 from Milgrom and Weber (1985). This theorem states that under some conditions, any mixed strategy equilibrium has a "purification", i.e., a pure strategy equilibrium at which each player has the same expected payoff and distribution of observable behavior as at the mixed strategy equilibrium in each of the informational states. The conditions that suffice in my setting are that,

1. Players' types are independent,
2. Players' types are atomless,
3. Each player's payoff depends only on its own type and the list of actions,
4. The action set is finite for each player.

Note that all of these conditions hold in my setting. Therefore, I have proven the following proposition.

Proposition 2 *Under Assumptions 1-4, there exists a pure strategy Bayesian Nash equilibrium of the global product introduction game.*

F.1 Counterexample: No PSNE under Complete Information with More than 2 Players and Strategic Substitutes

Consider the following game:

P3 plays 1

P1/ P2	1	0
1	$(-5,-5,-2)$	$(-4,0,1)$
0	$(0,1,-1)$	$(0,0,2)$

P3 plays 0

P1 / P2	1	0
1	$(1,-1,0)$	$(2,0,0)$
0	$(0,1,0)$	$(0,0,0)$

This is a static binary choice complete information entry game. Each player's payoff from entering is weakly decreasing in the set of entry decisions chosen by other players. No pure strategy Nash equilibrium exists.

G Simulating the Method

In this section, I use simulation to:

1. Test the behavior of the moment inequalities proposed in Section B under varying data-generating processes (DGPs),
2. Test the performance of the inference methods used in this paper, based on Andrews and Soares (2010), when a single realization of the product entry game is observed.

To simulate data, I need to solve the model fully. Thus, I simulate a solvable version of my product entry model, which I describe in Section G.1.

G.1 Fully Solvable Version of Global Product Entry Game

The solvable model features N symmetric J-product firms competing in M markets. I set $J = 3$ so that firms have 3 potential products that they can introduce in their global product portfolios and across markets. Both firms and products are symmetric in their profit shifters, but markets are allowed to be heterogeneous.

Profits for firm f from selling N_m^f products in market m take the form,

$$\Pi_m^f(N_m^f, N_m^{-f}) = A_m \frac{N_m^f}{(N_m^f)^{\kappa_o}(N_m^{-f})^{\kappa_r}},$$

where $\kappa_o \in (0,1)$ regulates substitutability across the firm's own products, and $\kappa_r \in (0,1)$ regulates substitution across rival firms' products. A_m is an exogenous market-level profit shifter.

Fixed costs: I assume that firms have log-normal fixed costs with

$$\begin{aligned} F_{jm}^e &= \exp(\theta_e + \sigma_e \nu_{jm}^e) \\ F_j^g &= \exp(\theta_g + \sigma_g \nu_j^g), \end{aligned}$$

where F_j^g denotes the cost of introducing product j in the firm's product portfolio and F_{jm}^e denotes the cost of offering product j in market m. I abuse notation and index all firms' potential products as 1, 2, or 3, even though firms draw independent fixed-cost shock realizations.

Timing: In the first stage of the game, each firm f draws private fixed portfolio cost shocks $\{\nu_j^g\}$ for each of their 3 potential products. Upon observing this private information, firms choose which products to introduce in their portfolio. Next, they draw private fixed market entry shocks $\{\nu_{jm}^e\}_{j\in\mathcal{G}^f,m}$ for each product and choose how many of the products in their portfolio to offer in each market.

Best response: In the second stage of the game, firms know their portfolio $\mathcal{G}^f$ and must choose which products to offer in each market. Due to the fact that products are symmetric, firms' best response in the second stage is simple to characterize. Letting N_p^f denote the number of products in the firms' portfolio in Stage 2, the decision rule in market m is as follows.

If $N_p^f = 1$, the firm chooses to sell the product in market m if and only if

$$\mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - F_{1m}^e(\nu_{1m}^e) \geq 0.$$

If $N_p^f = 2$, the firm chooses to introduce 1 product only if

$$\mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \min\{F_{1m}^e(\nu_{1m}^e), F_{1m}^e(\nu_{2m}^e)\} \geq 0$$

and

$$\mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \max\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e)\} < 0,$$

and all 2 products if and only if,

$$\mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \max\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e)\} \geq 0.$$

If $N_p^f = 3$, the firm chooses to offer all 3 products if,

$$\mathbb{E}[\Pi_m^f(3, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \max\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e), F_{3m}^e(\nu_{3m}^e)\} \geq 0;$$

2 products if,

$$\mathbb{E}[\Pi_m^f(3, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \max\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e), F_{3m}^e(\nu_{3m}^e)\} < 0$$

and

$$\mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \text{med}\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e), F_{3m}^e(\nu_{3m}^e)\} \geq 0;$$

and 1 product if,

$$\mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \text{med}\{F_{1m}^e(\nu_{1m}^e), F_{2m}^e(\nu_{2m}^e), F_{3m}^e(\nu_{3m}^e)\} < 0,$$

and

$$\mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}] - \min\{F_{1m}^e(\nu_{jm}^e), F_{2m}^e(\nu_{2m}^e), F_{3m}^e(\nu_{3m}^e)\} \geq 0.$$

In Stage 1, I define the value of a portfolio as,

$$\mathcal{V}_f(N_p^f) = \mathbb{E}\Big[\sum_m \max_{N_m^f \leq N_p^f} \mathbb{E}[\overline{\Pi}_m^f(N_m^f, N_m^{-f})|\mathcal{I}]\big|\mathcal{I}\Big]$$

where $\overline{\Pi}_m^f$ denotes profits net of market entry costs. The firm takes expectations not only over rival firms' entry choices but also over its own offering decisions given N_p^f. The best response is to develop 3 products if,

$$\mathcal{V}_f(3) - \mathcal{V}_f(2) - \max\{F_1^g(\nu_1^g), F_2^g(\nu_2^g), F_3^g(\nu_3^g)\} \geq 0;$$

2 products if

$$\mathcal{V}_f(3) - \mathcal{V}_f(2) - \max\{F_1^g(\nu_1^g), F_2^g(\nu_2^g), F_3^g(\nu_3^g)\} < 0,$$

and

$$\mathcal{V}_f(2) - \mathcal{V}_f(1) - \text{med}\{F_1^g(\nu_1^g), F_2^g(\nu_2^g), F_3^g(\nu_3^g)\} \geq 0;$$

and 1 product if

$$\mathcal{V}_f(2) - \mathcal{V}_f(1) - \text{med}\{F_1^g(\nu_1^g), F_2^g(\nu_2^g), F_3^g(\nu_3^g)\} < 0,$$

and

$$\mathcal{V}_f(1) - \min\{F_1^g(\nu_1^g), F_2^g(\nu_2^g), F_3^g(\nu_3^g)\} \geq 0.$$

Solution: The solution of the product entry game consists of 3 thresholds for each market,

$$t_{1m}^e = \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}], \tag{A70}$$
$$t_{2m}^e = \mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(1, N_m^{-f})|\mathcal{I}], \tag{A71}$$
$$t_{3m}^e = \mathbb{E}[\Pi_m^f(3, N_m^{-f})|\mathcal{I}] - \mathbb{E}[\Pi_m^f(2, N_m^{-f})|\mathcal{I}], \tag{A72}$$

and 3 thresholds for product development,

$$t_1^g = \mathcal{V}_f(1), \tag{A73}$$
$$t_2^g = \mathcal{V}_f(2) - \mathcal{V}_f(1), \tag{A74}$$
$$t_3^g = \mathcal{V}_f(3) - \mathcal{V}_f(2). \tag{A75}$$

$\mathcal{V}_f$ integrates over other firms' product offerings as well as firm f's own product offerings, which the firm anticipates will be determined by $(t_{1m}^e, t_{2m}^e, t_{3m}^e)$ in equilibrium.

Computing equilibrium profits given threshold strategies: Let $FE_n^{(k)}$ denote the k^{th} order statistic in a sample of size n for the market entry fixed cost and $FD^{(k)}$ denote the k^{th} order statistic of the product portfolio fixed cost in a sample of size 3. Then, the probabilities that any firm offers n products, for $n \in \{1, 2, 3\}$, given the portfolio and market entry strategies, are given by,

$$\begin{aligned}
p_1(\boldsymbol{t}^e, \boldsymbol{t}^g) &= \mathbb{P}(N_p^f = 3)P(N_m^f = 1|N_p^f = 3) + \mathbb{P}(N_p^f = 2)P(N_m^f = 1|N_p^f = 2) + \mathbb{P}(N_p^f = 1)P(N_m^f = 1|N_p^f = 1) \\
&= \mathbb{P}(FD^{(3)} \leq t_3^g)[\mathbb{P}(FE_3^{(1)} \leq t_1^e) - \mathbb{P}(FE_3^{(2)} \leq t_2^e)] \qquad \text{(A76)} \\
&+ [\mathbb{P}(FD^{(2)} \leq t_2^g) - \mathbb{P}(FD^{(3)} \leq t_3^g)][\mathbb{P}(FE_2^{(2)} \leq t_2^e) - \mathbb{P}(FE_2^{(1)} \leq t_1^e)] \\
&+ [\mathbb{P}(FD^{(1)} \leq t_1^g) - \mathbb{P}(FD^{(2)} \leq t_2^g)][\mathbb{P}(FE_1^{(1)} \leq t_1^e)], \\
p_2(\boldsymbol{t}^e, \boldsymbol{t}^g) &= \mathbb{P}(N_p^f = 3)\mathbb{P}(N_m^f = 2|N_p^f = 3) + \mathbb{P}(N_p^f = 2)\mathbb{P}(N_m^f = 2|N_p^f = 2) \qquad \text{(A77)} \\
&= \mathbb{P}(FD^{(3)} \leq t_3^g)[\mathbb{P}(FE_3^{(2)} \leq t_2^e) - \mathbb{P}(FE_3^{(3)} \leq t_3^e)] + [\mathbb{P}(FD^{(2)} \leq t_2^g) - \mathbb{P}(FD^{(3)} \leq t_3^g)]\mathbb{P}[FE_2^{(2)} \leq t_2^e], \\
p_3(\boldsymbol{t}^e, \boldsymbol{t}^g) &= \mathbb{P}(N_p^f = 3)\mathbb{P}(N_m^f = 3|N_p^f = 3) \qquad \text{(A78)} \\
&= \mathbb{P}(FD^{(3)} \leq t_3^g)\mathbb{P}(FE_3^{(3)} \leq t_3^e).
\end{aligned}$$

To compute expected profits in the first stage given any N_p^f, first realize that $\mathcal{V}_f$ takes the following form,

$$\mathcal{V}_f(N_p^f) = \sum_m \sum_{i=1}^{N_p^f} \mathbb{P}(N_m^f = i|N_p^f)[\mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}] - i \times \mathbb{E}[F_{jm}^e|F_{jm}^e \leq \mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}]]].$$

Equations (A76)-(A78) characterize $\mathbb{P}(N_m^f = i|N_p^f)$ as a function of $(\boldsymbol{t}^e, \boldsymbol{t}^p)$. Given $\mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}]$, the term $\mathbb{E}[F_{jm}^e|F_{jm}^e \leq \mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}]]$ is simple to compute numerically using Gaussian quadrature. I use the QuadGK package in Julia to compute this expectation.

To compute $\mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}]$ for each i given $(\boldsymbol{t}^e, \boldsymbol{t}^g)$, I use the DSP package in Julia to perform a convolution which gives the probability distribution of the number of rival product offerings given the number of firms N and $p_n(\boldsymbol{t}^e, \boldsymbol{t}^g)$ for $n \in \{1, 2, 3\}$.

Solving for $(\boldsymbol{t}^e, \boldsymbol{t}^g)$: Given $(\boldsymbol{t}^e, \boldsymbol{t}^g)$, I showed how to evaluate $\mathcal{V}_f$ and $\mathbb{E}[\Pi_m^f(i, N_m^{-f})|\mathcal{I}]$ for any $i \in \{1, 2, 3\}$. Thus, I solve for $(\boldsymbol{t}^e, \boldsymbol{t}^g)$ by solving the non-linear system of equations given by equations (A70)-(A75). To do so, I use the NLsolve Julia package.

G.2 Behavior of Moment Inequalities Across DGPs

In this section, I study how the tightness or informativeness of the moment inequalities varies with parameters κ_o and κ_r and with the number of firms N. Intuitively, the informativeness of the inequalities should depend on the loss from bounding the marginal value of introducing a product by evaluating it at extreme bundles and on the loss from applying Jensen's inequality to average out firms' expectational errors.

Implementation: I simulate $S = 500$ different realizations for each of $T = 12$ different "types" of global product and market entry game described in the previous section. In each game (s, t), there are N firms competing in 12 markets. I hold fixed the market-level profit shifters and set them at $A_m^{(s,t)} = 0.2mt$ for $m \in \{1, 2, ..., 12\}$, $t \in \{1, 2, ..., 12\}$, and $s \in \{1, 2, ..., 500\}$. Different values of t generate variation in profitability across different game types. In this subsection, I perform valid asymptotics as $ST \to \infty$. With $ST = 6000$, simulation noise is small, so I report identified sets, thus ignoring sampling uncertainty (i.e., without computing confidence sets).

True parameters: I set the true parameters to be $(\theta_g, \sigma_g) = (3, 1)$ and $(\theta_e, \sigma_e) = (1, 1)$.

Instruments: For the Stage 2 inequalities (market entry), I condition on the realized set of portfolio decisions and construct instruments following the PPML procedure described in the main text. More precisely, I project the minimal (maximal) marginal values at the product-market level – evaluated at all (no) products in the portfolio being introduced in the market and at the realized set of rival offerings decisions – on a specification of the form $\gamma_0 \exp(\gamma_m^t A_m^{(s,t)})$. I then compute the predicted values of both regressions, sort them, and define 4 percentile categories and bins associated with these categories. This gives 8 instruments in total – 4 for the upper bound inequality and 4 for the lower bound inequality.

For the Stage 1 inequalities, I implement an equivalent procedure as with the Stage 2 inequalities, but without conditioning on the realized product portfolios. I project the realized maximal and minimal (where now the minimal marginal value does not condition on the observed portfolio, so is evaluated at all 3 products being introduced in each market) marginal values on a specification of the form $\gamma_0 \exp(\gamma_m^t A_m^{(s,t)})$. I then compute the predicted values of both regressions and sum them across markets. Then, I sort the sums across markets and define 4 percentile categories for both the lower and the upper bound and define 4 bins associated with these categories, yielding 8 instrument bins - 4 for the lower bound and 4 for the upper bound.

G.2.1 Baseline: $N = 10$, $\kappa_o = 0.1$, $\kappa_r = 0.1$

In this baseline case, I set the number of firms to $N = 10$ and set $\kappa_o = 0.1$ and $\kappa_r = 0.1$. In this case, I obtain the following contour plots and identified sets for (θ_e, σ_e) and (θ_g, σ_g).

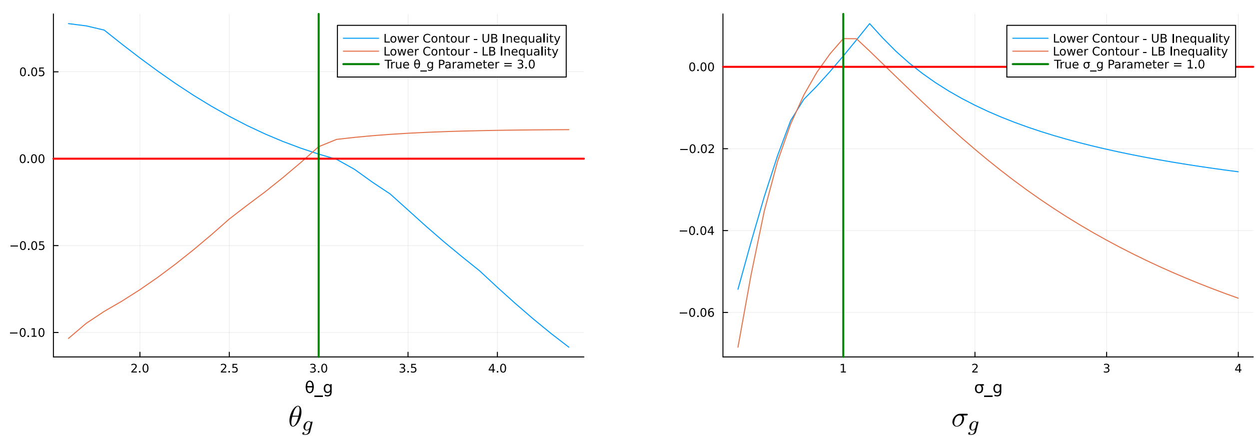
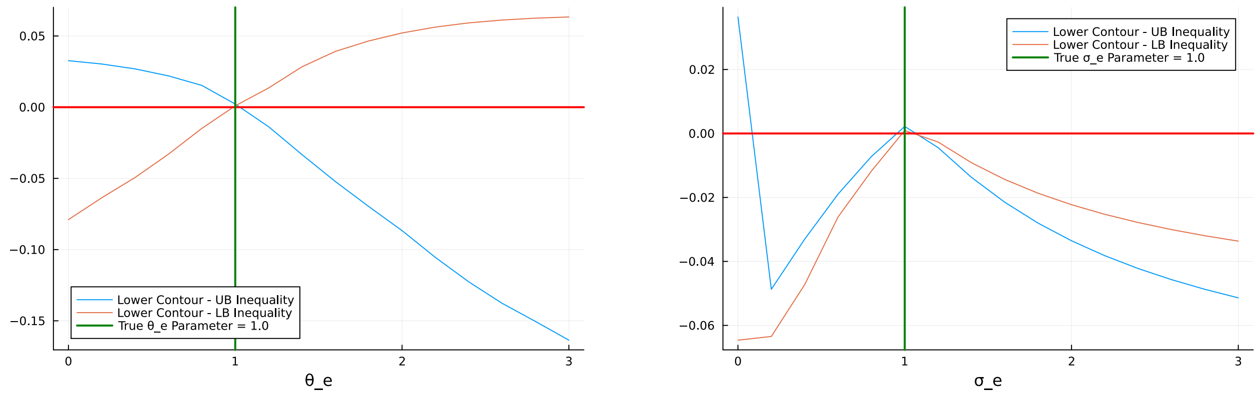


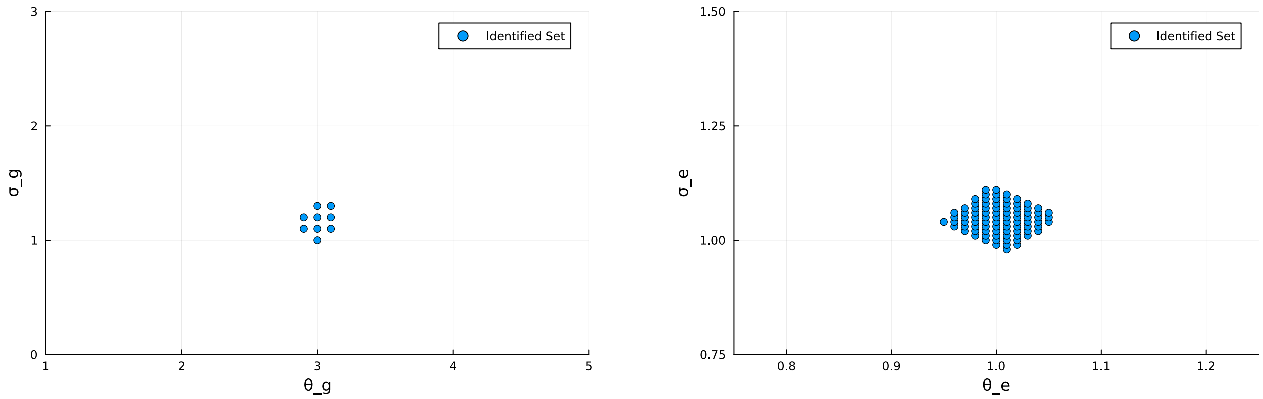
Figure 20: Lower Contours

Figure 20 shows the smallest value that the upper bound and lower bound inequalities take (across each of the 4 instrument bins) as I change the value of θ_g and σ_g, holding the other constant at the true value. The smallest values (or the lower contours) are the most informative because the inequalities are written so that positive values imply acceptance, while negative values imply rejection. For instance, the left panel in Figure 20 shows that all 4 unconditional lower bound moment inequalities (in orange) and all 4 unconditional upper bound moment



θ_e σ_e

Figure 21: Lower Contours



Stage 1 Stage 2

Figure 22: Identified Sets

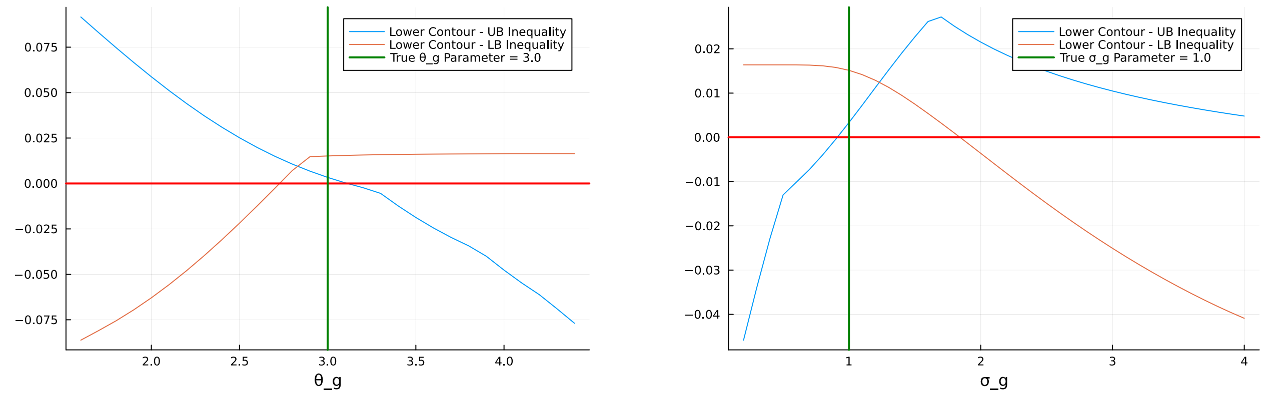
inequalities (in blue) are positive conditional on $\sigma_g = 1$ at the region where *both* the orange and blue lines are positive, above the red line indicating that the smallest value of the moments is 0.

Figure 22 shows the identified sets for (θ_g, σ_g) and (θ_e, σ_e), which I obtained via grid search. As seen in this figure, the identified set is quite informative both for the Stage 1 and the Stage 2 fixed cost parameters (note: the scale is not the same for the Stage 1 and the Stage 2 parameters).

G.2.2 High Substitutability Within the Firm: $N = 10$, $\kappa_o = 0.25$, $\kappa_r = 0.1$

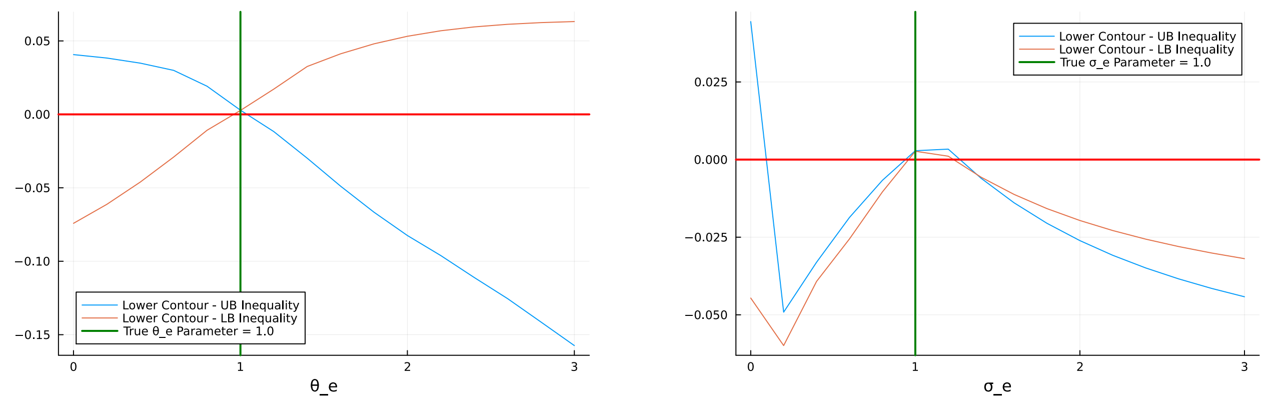
I now study a deviation from the baseline case in which the parameter determining substitution within the firm is larger and all else is as in the baseline case.

Figures 23-25 show the results. Compared to the baseline cases, high substitution within the firm reduces the informativeness of the fixed cost parameter bounds, both for the Stage 1 and for the Stage 2 fixed cost parameters. This is expected given that the loss from bounding the marginal value of introducing a product at extreme bundles (all other products vs. no other products offered) is greater when such products are highly substitutable. That is, greater cannibalization within the firm makes the moment inequalities less informative about the true parameters.



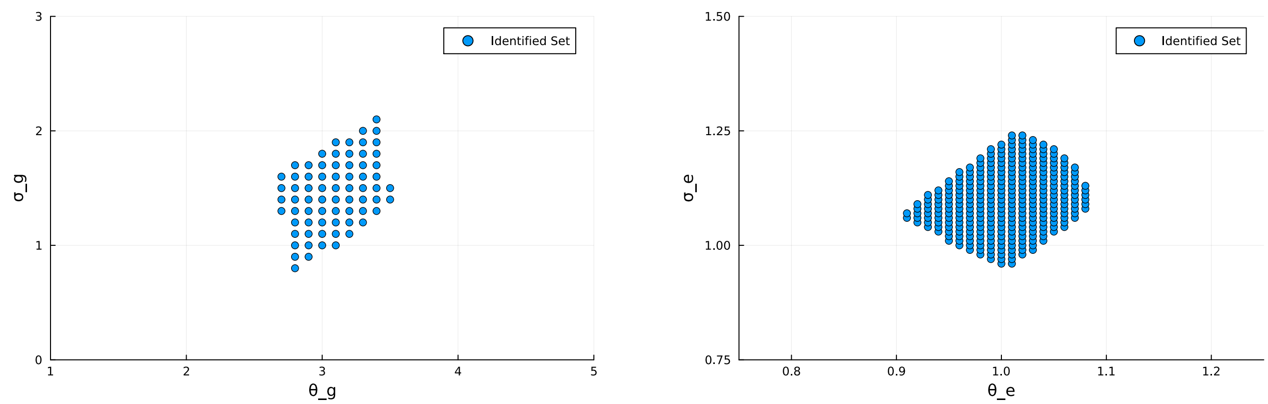
θ_g σ_g

Figure 23: Lower Contours



θ_e σ_e

Figure 24: Lower Contours



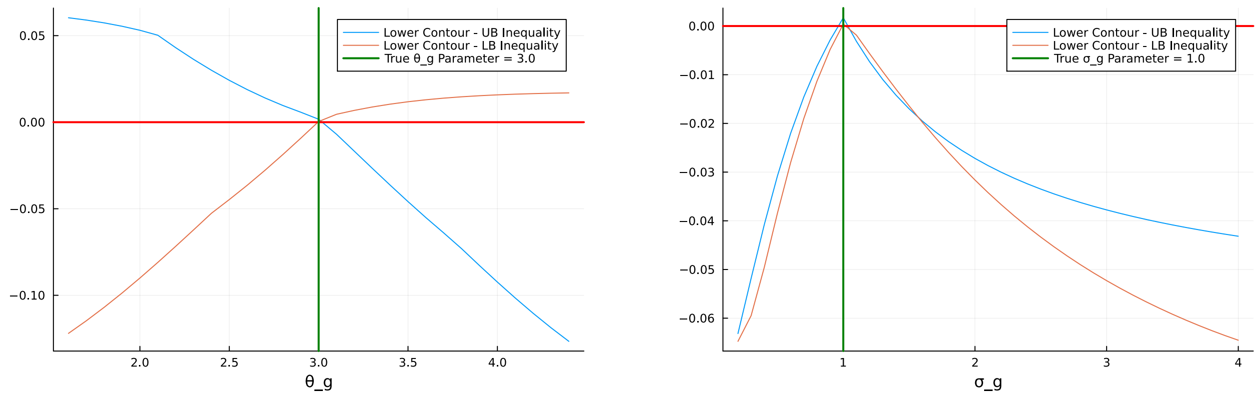
Stage 1 Stage 2

Figure 25: Identified Sets

G.2.3 Low Substitutability within the Firm: $N = 10$, $\kappa_o = 0.01$, $\kappa_r = 0.1$

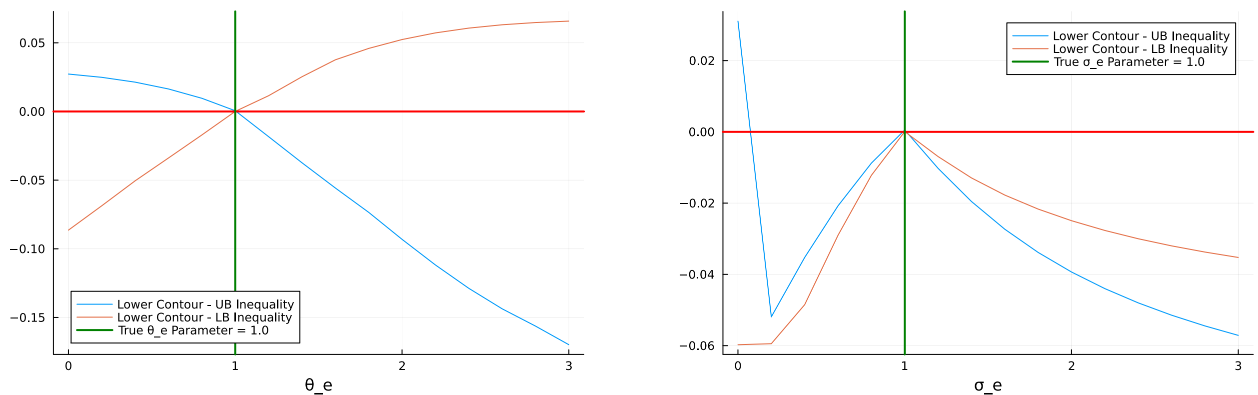
Figures 26-28 show what happens when there is very low substitutability across the firms' own products. Interestingly, the tightness of the inequalities increases greatly so that only the true fixed parameters both in Stage 1 and Stage 2 are accepted (given my grid).

This result is of special significance for anyone wanting to use this estimation approach in a setting where firms are single-product (rather than multi-product). Indeed, Figure 28 shows that in the absence of any interdependence within the firm, my estimation procedure is highly informative, and there is very little to no loss from using my moment inequalities.



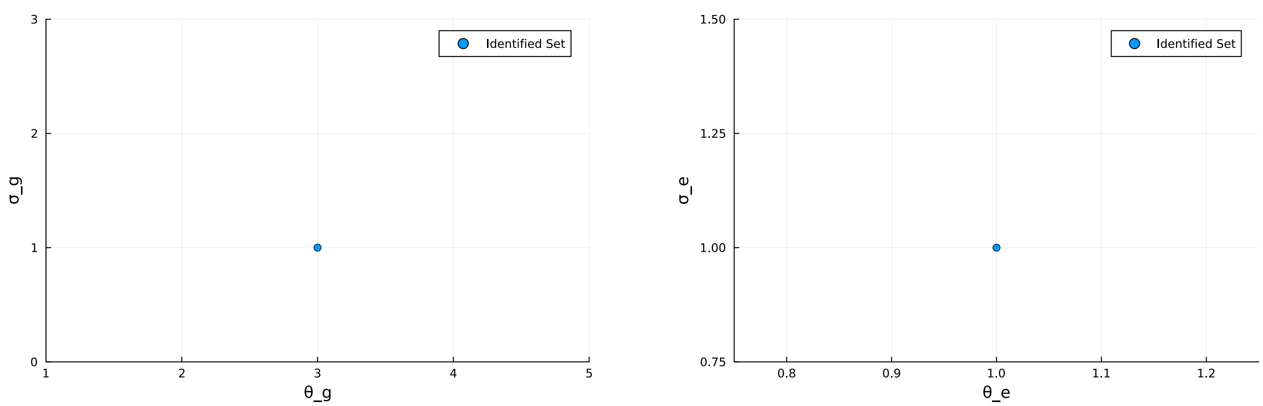
θ_g σ_g

Figure 26: Lower Contours



θ_e σ_e

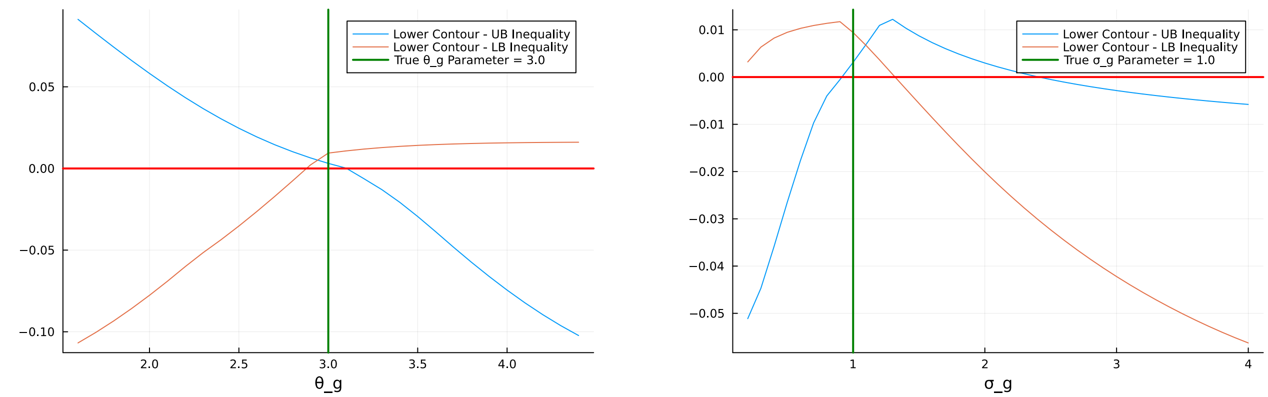
Figure 27: Lower Contours



Stage 1 Stage 2

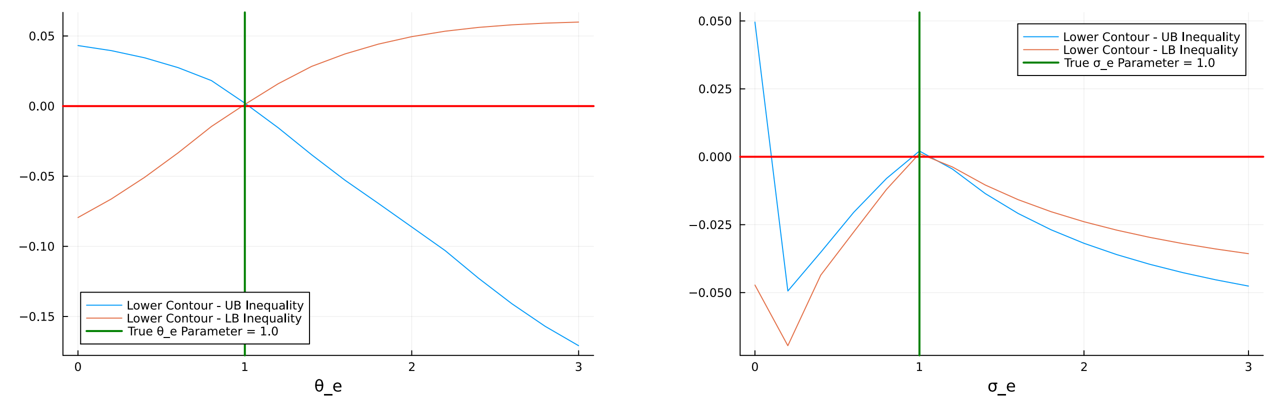
Figure 28: Identified Sets

G.2.4 High Substitutability Across Firms: $N = 10$, $\kappa_o = 0.1, \kappa_r = 0.25$



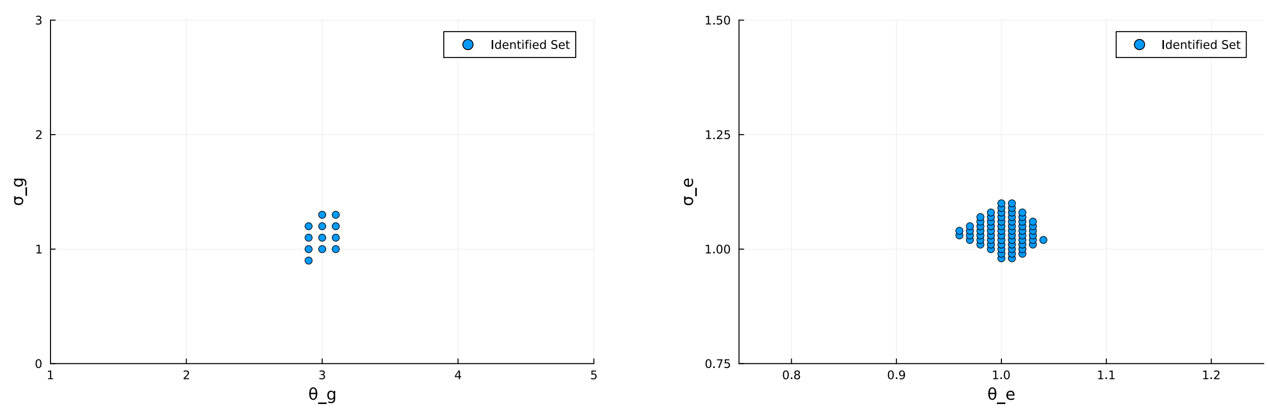
θ_g σ_g

Figure 29: Lower Contours



θ_e σ_e

Figure 30: Lower Contours



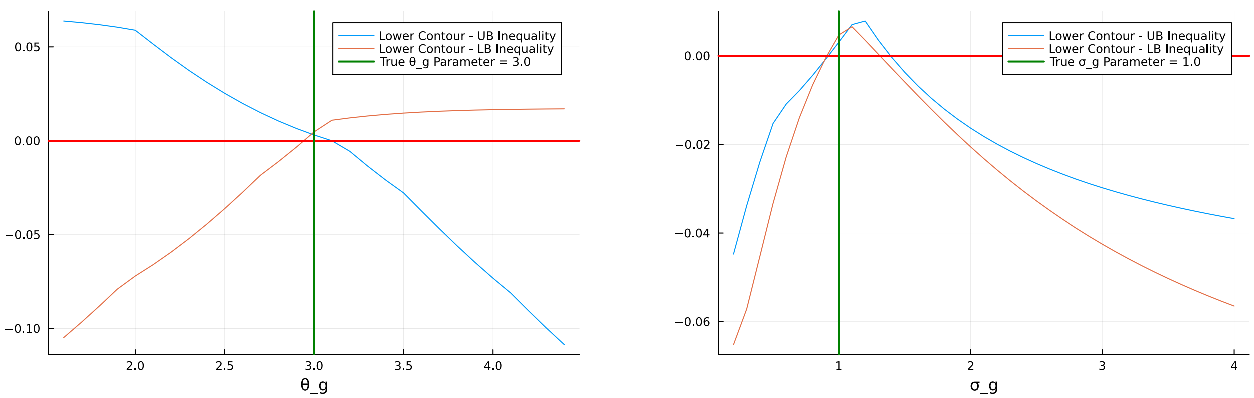
Stage 1 Stage 2

Figure 31: Identified Sets

Interestingly, high substitution across firms does not seem to change the tightness of the inequalities much relative to the baseline case. While it seems to slightly reduce the informativeness of the Stage 1 inequalities, it does not seem to lead to reduced informativeness in Stage 2. If anything, in this case, the Stage 2 inequalities yield tighter bounds on θ_e. The impact of higher κ_r on the informativeness of the inequalities is therefore not as quantitatively important as the impact of κ_o. This makes sense because, by virtue of the unobservability of rivals' fixed cost shocks, I showed that one can use the realized set of rival offerings decisions to construct the moment inequalities. Greater substitution across firms reduces the rate of product introduction in equilibrium, but firms expect this, and the informativeness of the moment inequalities is not significantly affected.

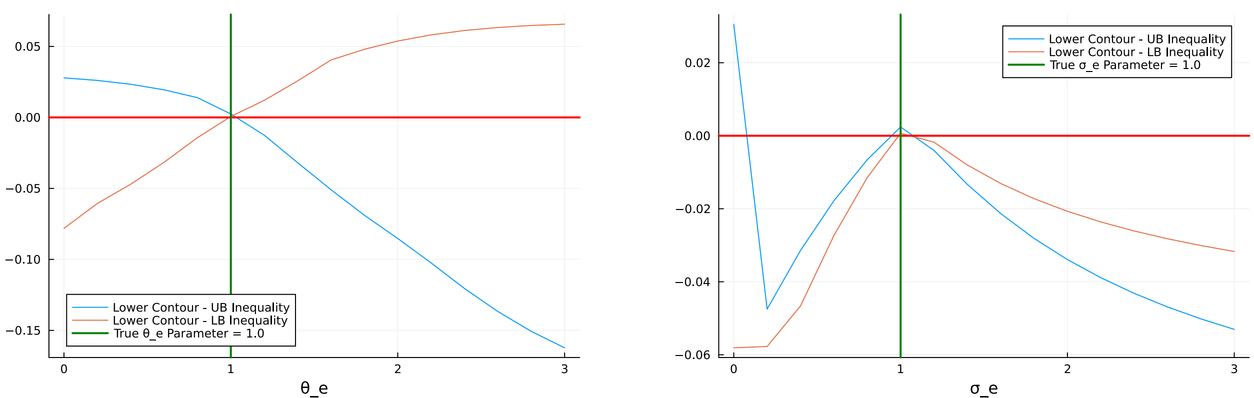
G.2.5 Low Substitutability Across Firms: $N = 10$, $\kappa_o = 0.1, \kappa_r = 0.01$

For completeness, I report the identified sets and lower contours for the case in which there is low substitution across rival firms' products. As expected, there are no substantial differences in the informativeness of the moment inequalities relative to the baseline case for the same reasons as in Section G.2.4.



θ_g σ_g

Figure 32: Lower Contours

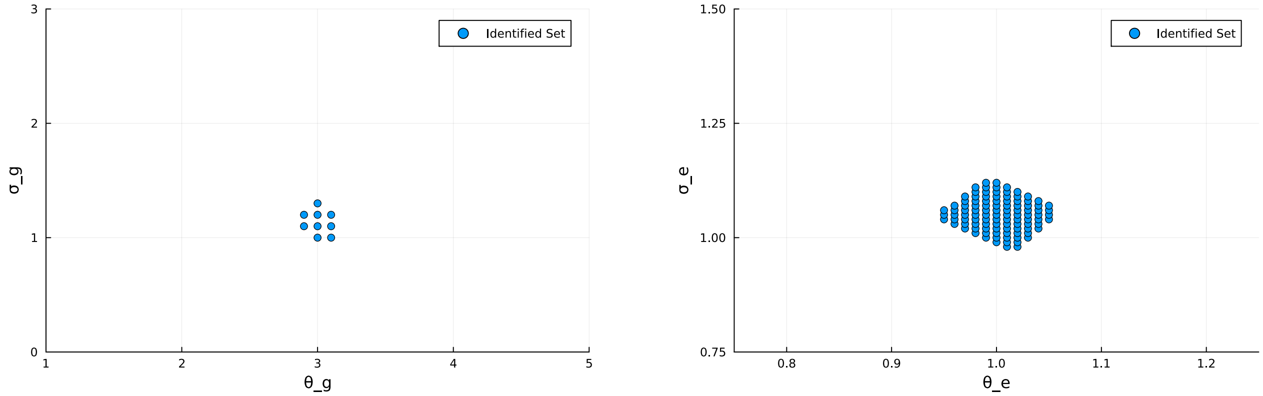


θ_e σ_e

Figure 33: Lower Contours

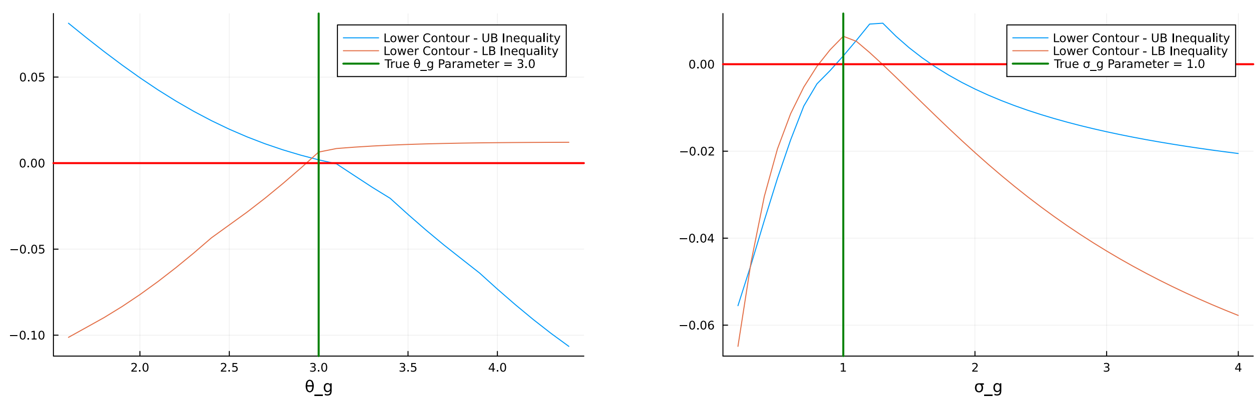
G.2.6 Large Number of Firms: $N = 20$, $\kappa_o = 0.1$, $\kappa_r = 0.1$

Figures 35-37 illustrate the effect of having more firms participating in the global entry game. The identified set becomes slightly smaller when there are more firms. This is due to two effects. First, when there are more firms, each firm makes smaller expectational errors. Second, when there are more firms, more products may be offered on average, which reduces the loss from bounding marginal values with extreme bundles due to submodularity.



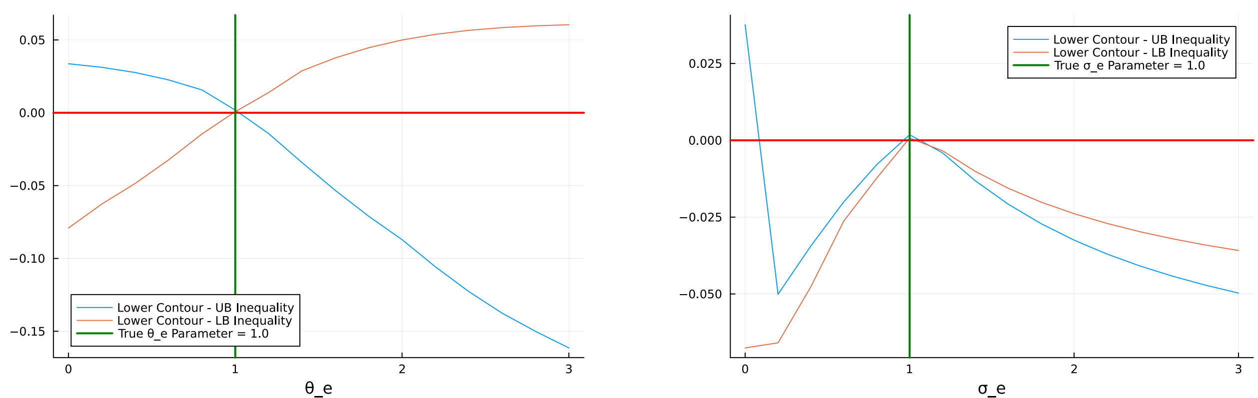
Stage 1 Stage 2

Figure 34: Identified Sets



θ_g σ_g

Figure 35: Lower Contours



θ_e σ_e

Figure 36: Lower Contours

G.2.7 Small Number of Firms: $N = 2$, $\kappa_o = 0.1$, $\kappa_r = 0.1$

Figures 38-40 illustrate the effect of having only 2 firms competing globally. In this case, the inequalities clearly become less informative for the same reasons as argued in Section G.2.6.

G.2.8 Main Takeaways

While the simulations abstract away from heterogeneity across products and across firms, they are still useful for understanding some of the key properties of the moment inequalities. I have established that much of the informativeness of the moment inequalities relies on the extent to which products *within* the firm are substitutable. High degrees of substitutability within the firm render the moment inequalities less informative, while lower degrees of substitutability make them tighter. Moreover, the number of firms (relative to the number of products per firm) matters. With smaller firms relative to the overall market, expectational errors are smaller. If the set of potential products of a firm is small relative to the set of products that are offered in the market, the loss from bounding a product's marginal value with extreme bundles within the firm is smaller. Finally, I showed that all else equal, substitution across firms does not have much of an effect on the tightness of the moment inequalities.

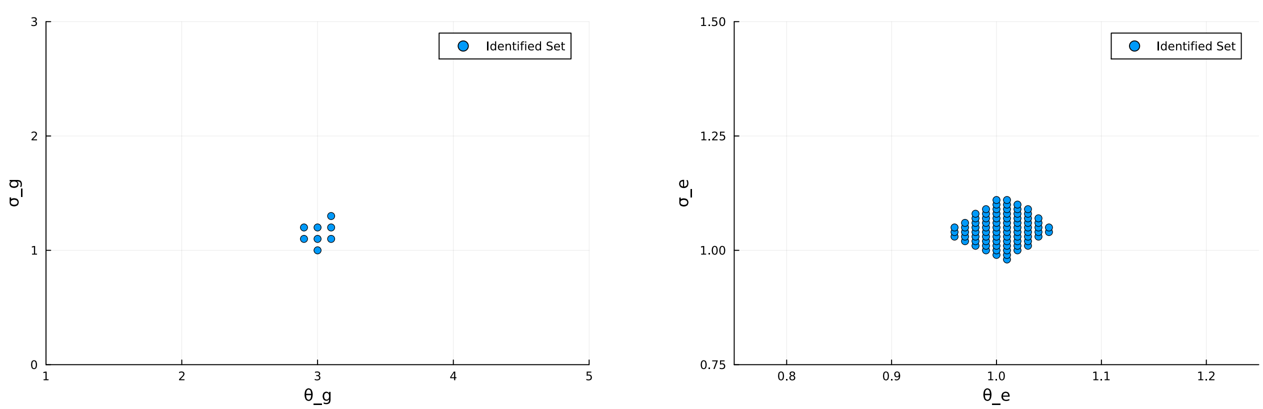
G.3 Inference Under a Single Realization of Global Product Entry Game

In this section, I use the fully solvable version of the model to assess the properties of Andrews and Soares (2010) confidence sets in my setting. I simulate $S = 100$ realizations for each of $T = 12$ different "types" of global product and market entry games, just as in Section G.2. In each game (s,t), there are N firms competing in 12 markets. I hold fixed the market-level profit shifters and set them at $A_m^{(s,t)} = 0.2mt$ for $m \in \{1, 2, ..., 12\}$, $t \in \{1, 2, ..., 12\}$ and $s \in \{1, 2, ..., 100\}$. Different values of t generate variation in profitability across different game types.

True parameters: I set the true parameters to be $(\theta_g, \sigma_g) = (3, 1)$ and $(\theta_e, \sigma_e) = (1, 1)$.

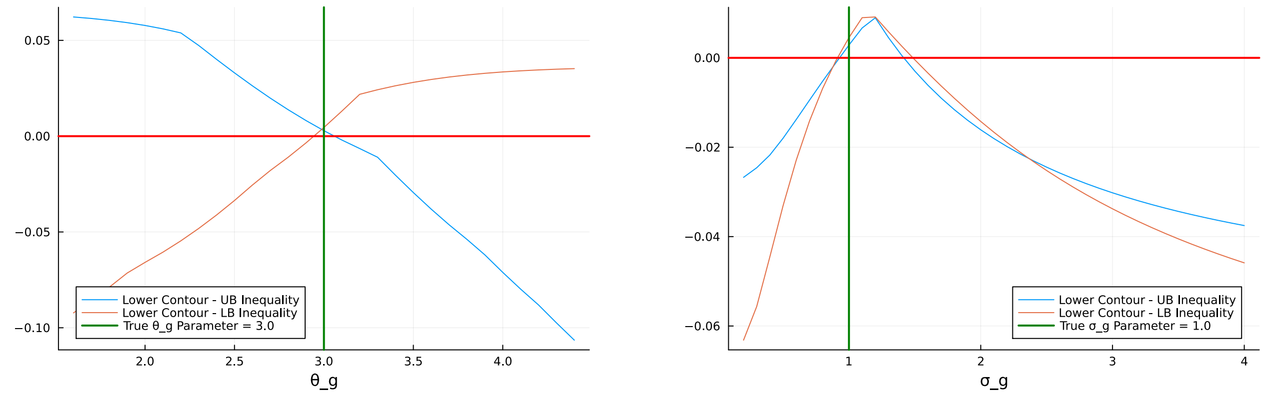
Instruments: I construct the instruments in the same way as described in Section G.2. However, I run the PPML regressions at the (s,t)-level to mimic the actual implementation in the main text, where I only observe a single realization of the cross-section and use only this information to construct instruments. Thus, the variation used to construct confidence sets in this section is across product-market pairs.

For each (s,t) pair, I use the simulated data to construct confidence sets for parameters (θ_e, σ_e) using the procedure in Andrews and Soares (2010). For each of the $S \times T$ 95% confidence sets, I record: (i) whether the true values $(1, 1)$ are included in the confidence set (coverage), (ii) the length of the confidence set along the θ_e dimension



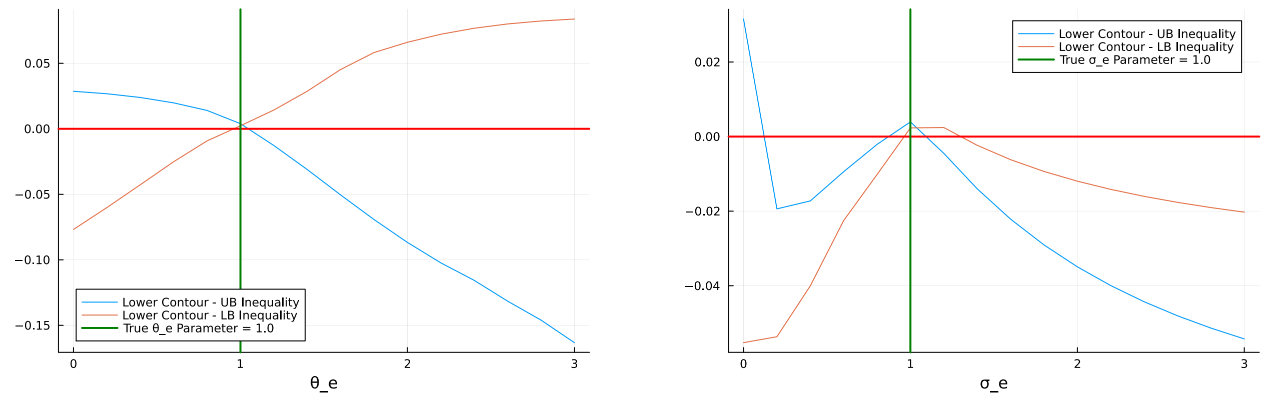
Stage 1 Stage 2

Figure 37: Identified Sets



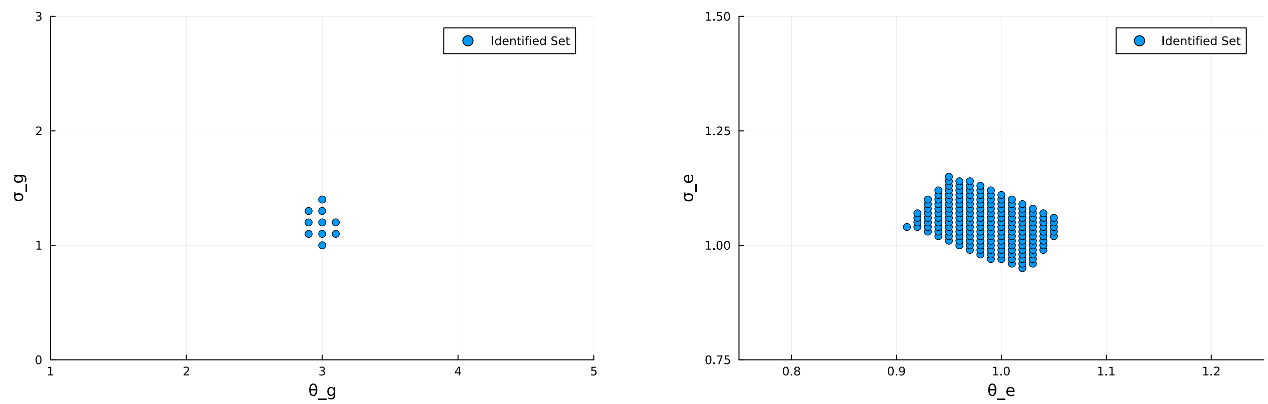
θ_g σ_g

Figure 38: Lower Contours



θ_e σ_e

Figure 39: Lower Contours



Stage 1 Stage 2

Figure 40: Identified Sets

(holding $\sigma_e = 1$ at the truth), (iii) the length of the confidence set along the σ_e dimension (holding $\theta_e = 1$ at the truth). In Table 7, I report the average coverage of the confidence set across all $S \times T$ realizations of the global product entry game. I do this both under a robust variance-covariance matrix and a clustered variance-covariance matrix with clustering at the market level.

Table 7 shows that undercoverage can occur, particularly when the number of firms is relatively small. As the

Table 7: Coverage of True (θ_e, σ_e) Parameters (%)

SE Type	$N = 5$	$N = 25$	$N = 50$	$N = 75$
Robust	92.9	93.8	94.1	93.7
Clustered	92.4	94.1	94.1	95.0

Notes: This table reports the average coverage across all $S \times T$ simulations of the confidence sets. Such confidence sets are computed at the (s, t)-level, as in my empirical application.

number of firms increases, the coverage of the Andrews and Soares (2010) confidence sets tends to increase. Note that clustering does not necessarily yield higher coverage, though this is the case when the number of firms is large ($N = 75$).

I now report the median length of the confidence set along each of the dimensions of (θ_e, σ_e), conditional on the confidence set not being empty.

Table 8: Median Length of Confidence Set Along (θ_e, σ_e)

SE Type	$N = 5$	$N = 25$	$N = 50$	$N = 75$
Robust	(0.8, 2.8)	(0.4, 0.9)	(0.2, 0.6)	(0.2, 0.5)
Clustered	(0.9, 4.4)	(0.4, 1.6)	(0.3, 1.1)	(0.3, 0.9)

Notes: This table reports across all $S \times T$ simulations the median length of the confidence set along each of the two dimensions of the parameter vector (θ_e, σ_e). The first coordinate reports the median length of the θ_e dimension of the confidence set, conditional on σ_e being at the true value. The second coordinate reports the median length of the σ_e dimension, conditional on θ_e being at the true value. Such confidence sets are computed at the (s, t)-level, as in my empirical application.

Table 8 shows that conditional on accepting the truth, the confidence sets are smaller whenever there are more firms and whenever I do not cluster. In the empirical implementation, I report confidence sets using a robust variance-covariance matrix because (i) Table 7 shows that it does not seem to suffer from large undercoverage and is not strictly dominated in terms of coverage by a clustered variance-covariance matrix, and (ii) Table 8 shows that using a clustered variance-covariance matrix can lead to significantly larger confidence sets.

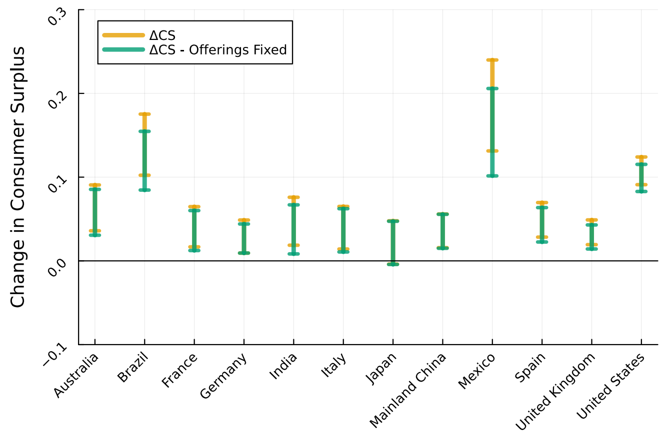
An important caveat in this exercise is that the simulations require symmetry across firms and products within a (s, t) pair. While at Stage 2, even in the simulations, firms are not fully symmetric due to variation in how many products they have in their global product portfolio, greater symmetry relative to reality should worsen correlation across expectational errors since the realization of the market structure affects firms in (almost) the exact same way. However, greater symmetry also means that there are no "large" firms in the sample. Large firms with a lot of market power can potentially deteriorate the asymptotic properties of the estimator.

H Counterfactual Exercises: Robustness

In this section, I report the effects of the policies studied in Section 7 under two additional points in the confidence set: $(\theta_e, \sigma_e, \theta_g, \sigma_g) = (-4.2, 3.6, -1.8, 2.6)$ (LH) and $(\theta_e, \sigma_e, \theta_g, \sigma_g) = (-4.8, 3.6, -0.6, 2.5)$ (HL). Under the first point, the distribution of market entry fixed costs has a location parameter at the uppermost extreme of the confidence set, while the location parameter of the product portfolio fixed cost is at the lowermost extreme. Under the second point, the location parameter of the market entry fixed cost is at the lowermost extreme, while that of the product fixed cost is at the uppermost extreme. While it is infeasible to compute the counterfactual outcomes under all points, these two extremes are economically meaningful. High product portfolio fixed costs and small market entry fixed costs imply that firms have an incentive to offer very similar bundles across countries, while low product portfolio and high market entry fixed costs increase incentives to tailor varieties to local preferences.

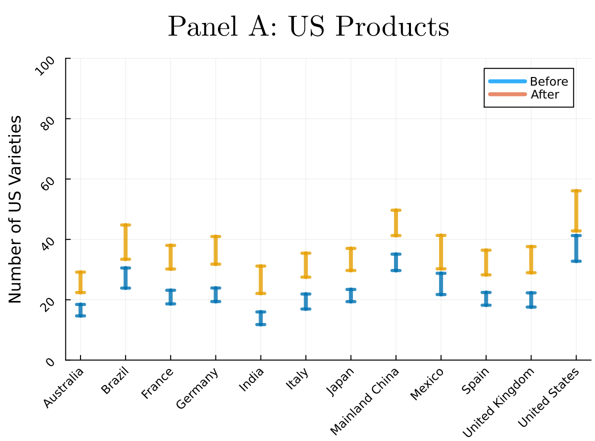
20% Marginal Cost Subsidy Under Point LH

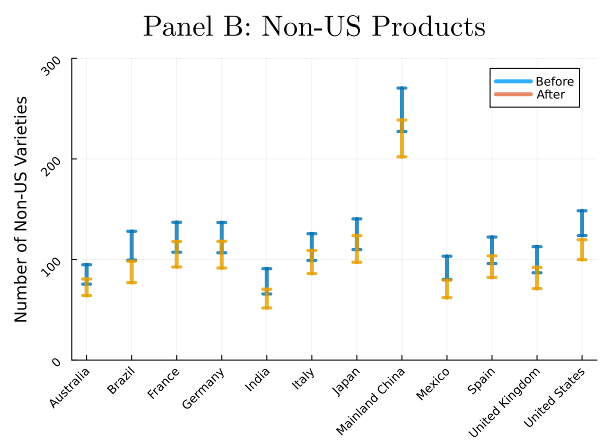
Figure 41: Change in Consumer Surplus



Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 20% marginal cost reduction for US brands. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented.

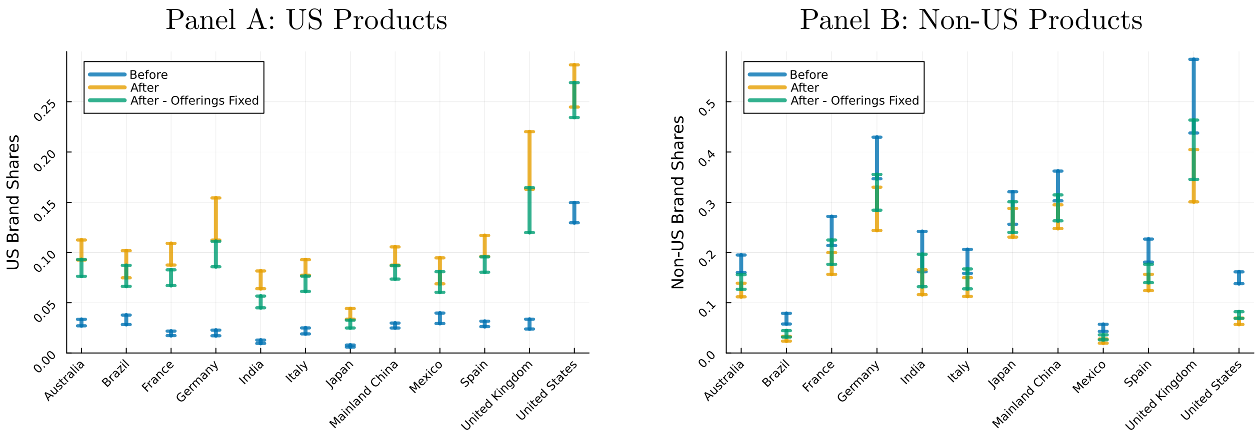
Figure 42: Number of Products Offered





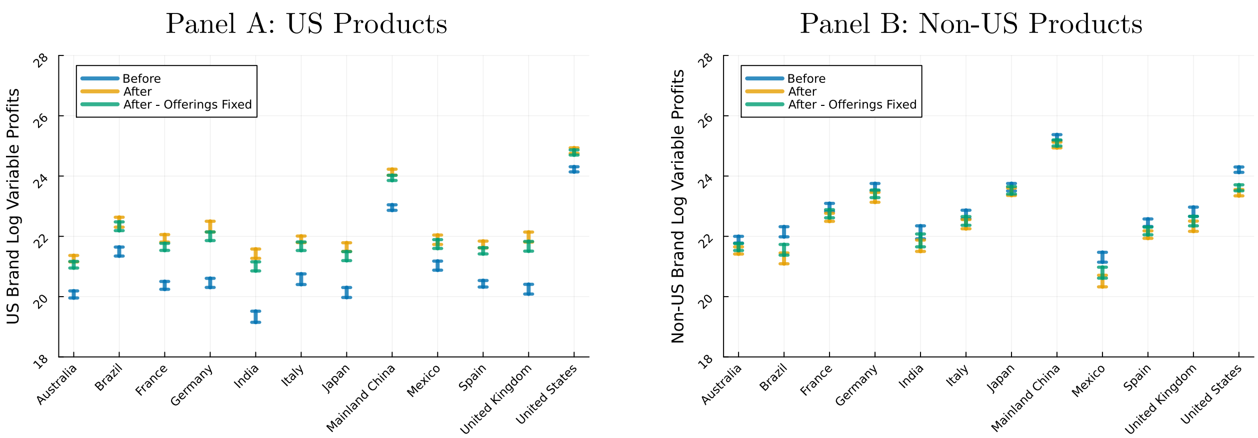
Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

Figure 43: Market Shares



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays similar bounds on the expected total market share of non-US brands before and after the policy is implemented.

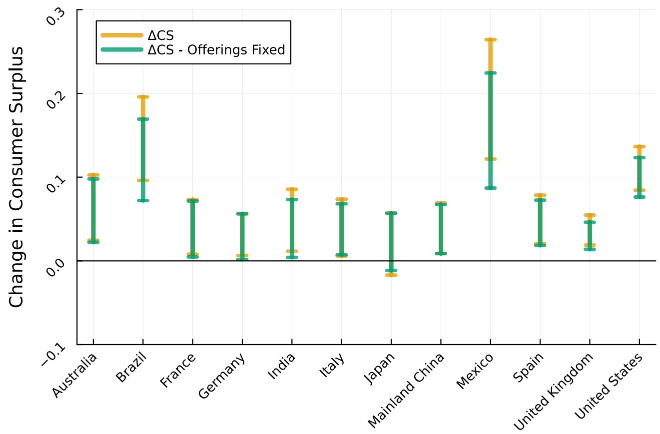
Figure 44: Variable Profits



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.

20% Marginal Cost Subsidy Under Point HL

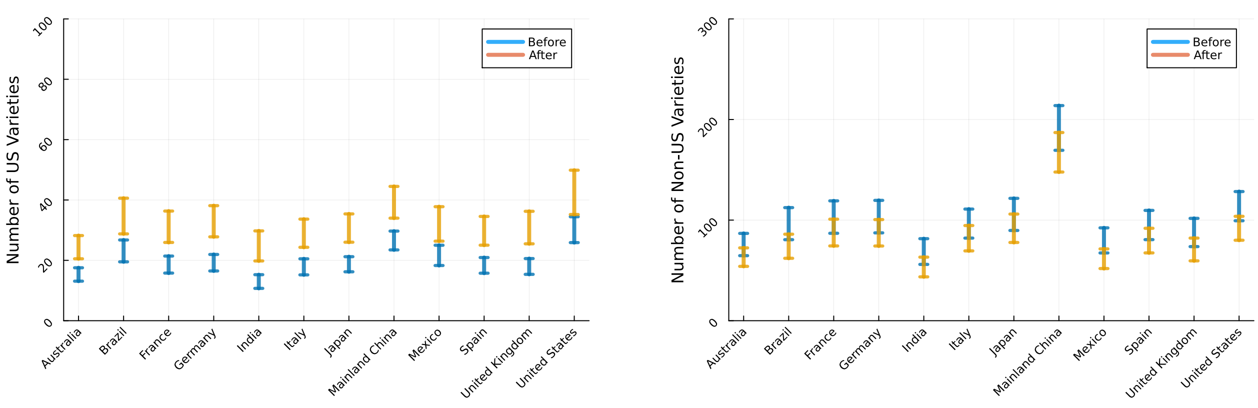
Figure 45: Change in Consumer Surplus



Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 20% marginal cost reduction for US brands. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented.

Figure 46: Number of Products Offered

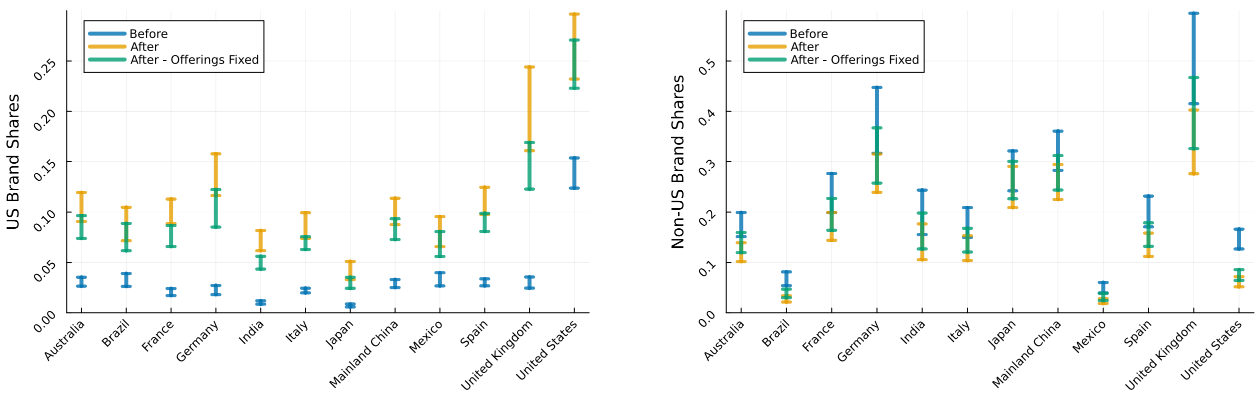
Panel A: US Products | Panel B: Non-US Products



Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

Figure 47: Market Shares

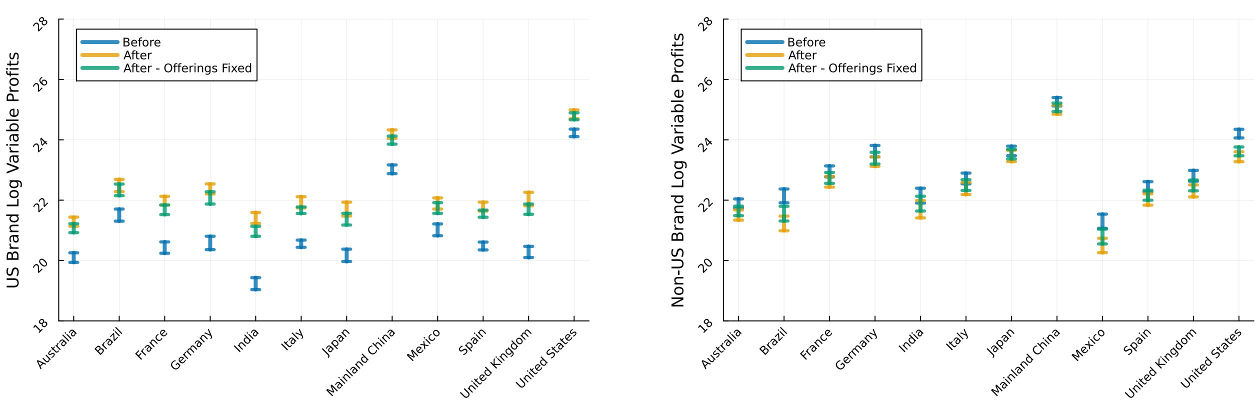
Panel A: US Products   Panel B: Non-US Products



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays similar bounds on the expected total market share of non-US brands before and after the policy is implemented.

Figure 48: Variable Profits

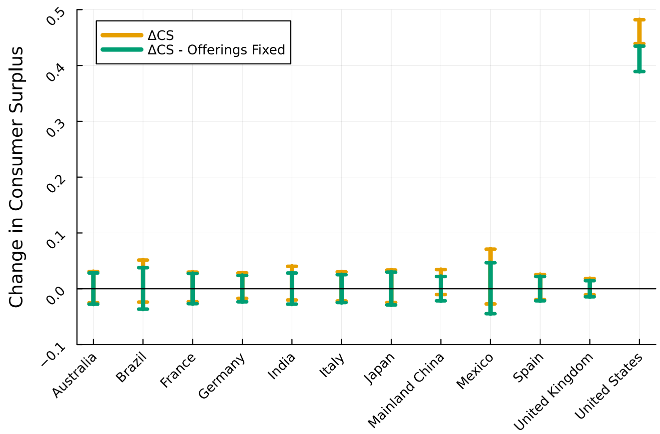
Panel A: US Products   Panel B: Non-US Products



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.

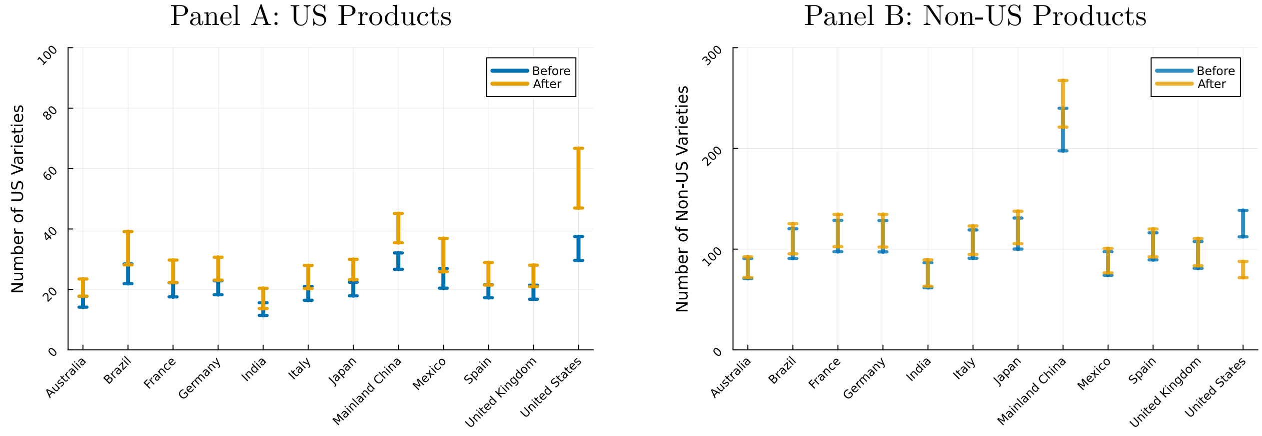
50% Consumption Subsidy Under Point LH

Figure 49: Change in Consumer Surplus



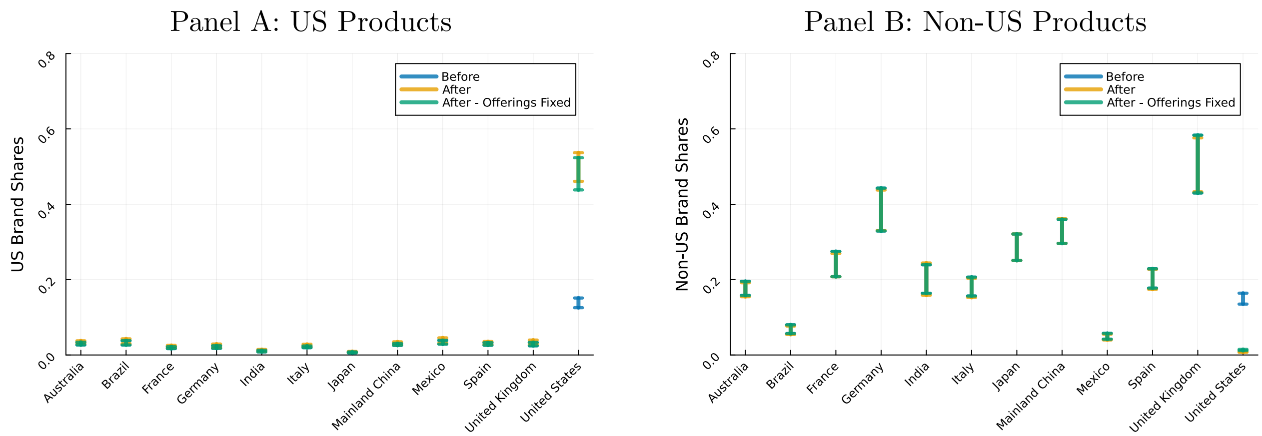
Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 20% marginal cost reduction for US brands. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented. The intervals in green only reflect the intensive margin response.

Figure 50: Number of Products Offered



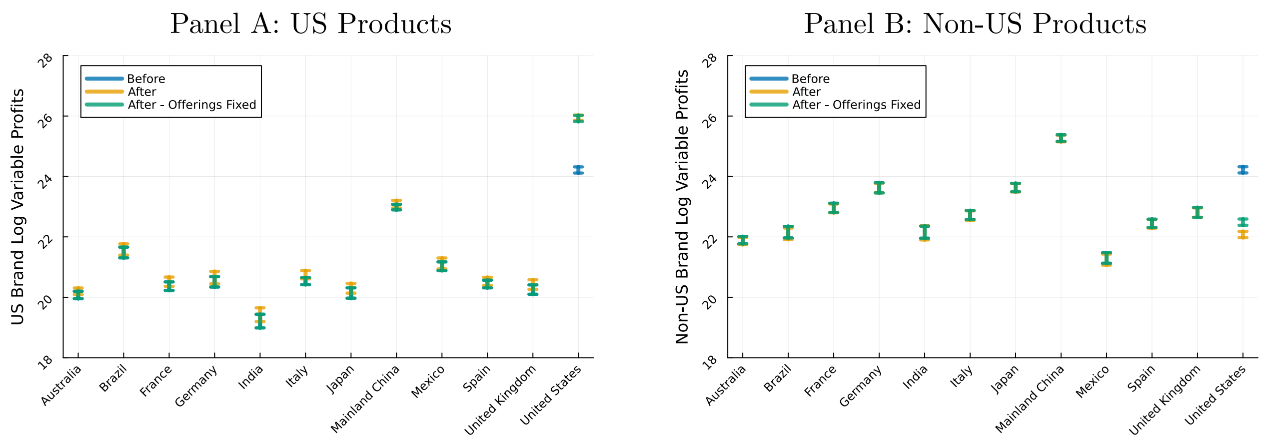
Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

Figure 51: Market Shares



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays similar bounds on the expected total market share of non-US brands before and after the policy is implemented.

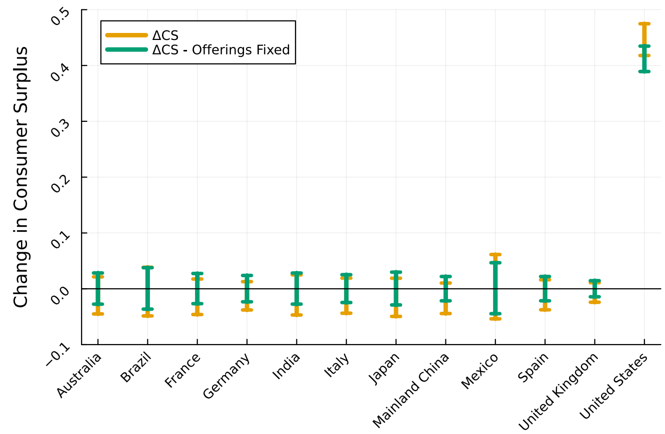
Figure 52: Variable Profits



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.

50% Consumption Subsidy Under Point HL

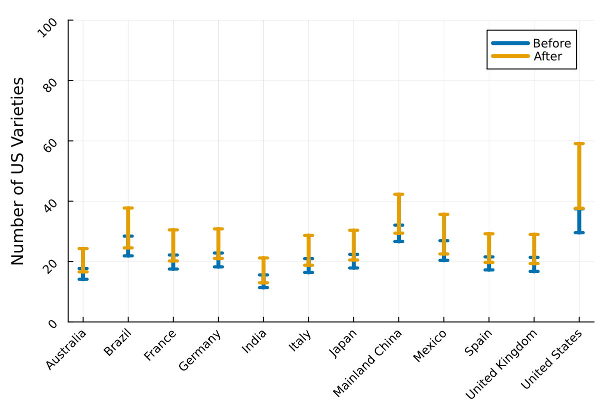
Figure 53: Change in Consumer Surplus



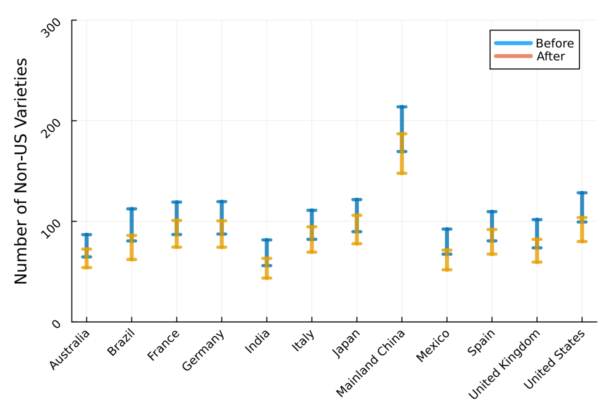
Notes: This figure plots, for each country, a lower and an upper bound on the expected change in consumer surplus following a 20% marginal cost reduction for US brands. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in orange show the change in expected consumer surplus accounting for the change in the (bounds of) the equilibrium distribution of product offerings following the policy. The intervals in green show the change in expected consumer surplus using the bounds on the distribution of product offerings before the policy is implemented. The intervals in green only reflect the intensive margin response.

Figure 54: Number of Products Offered

Panel A: US Products

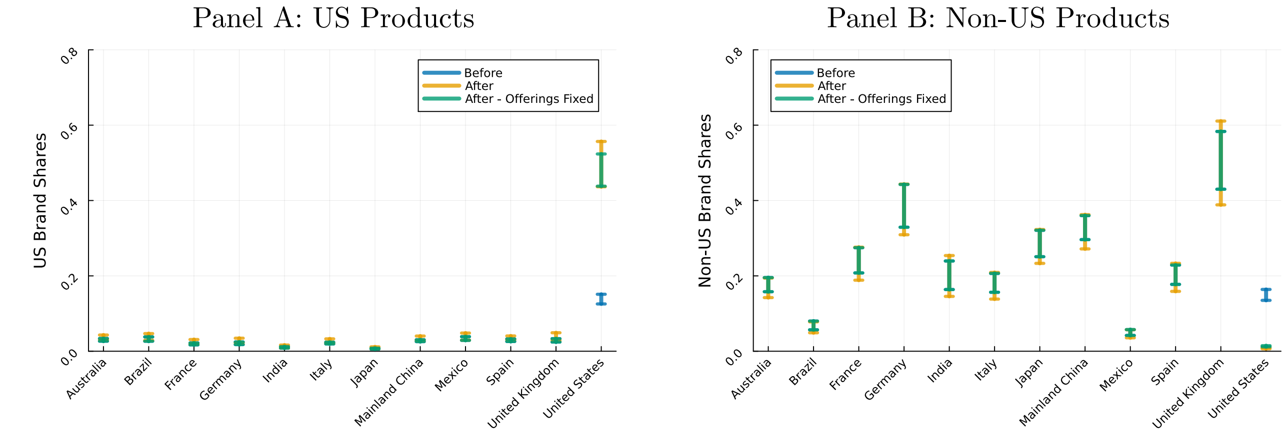


Panel B: Non-US Products



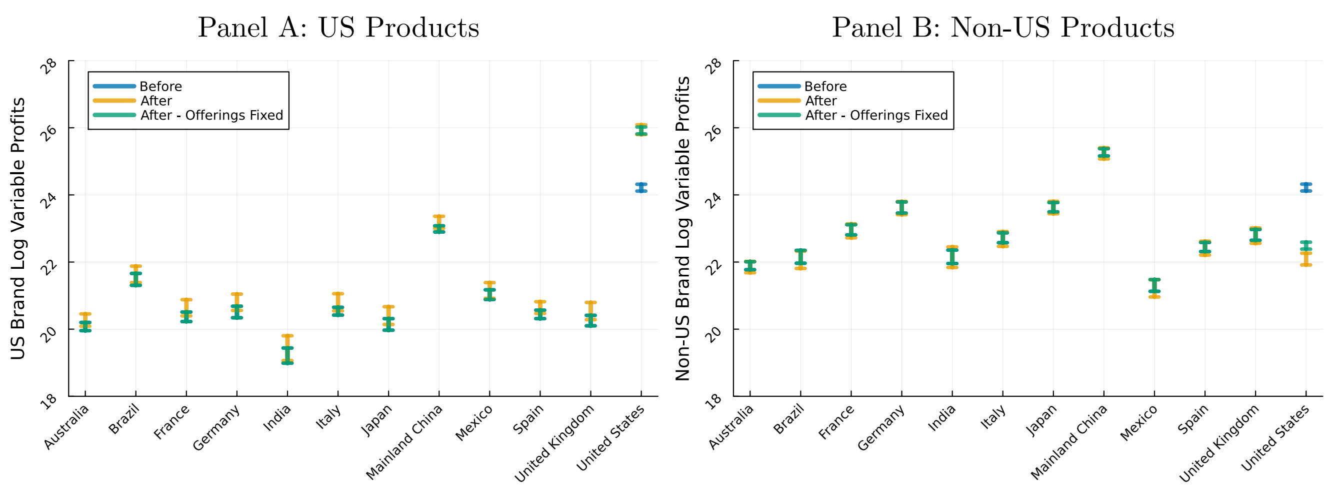
Notes: Panel A displays bounds on the expected number of US-branded products offered across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. Panel B displays the corresponding bounds on the expected number of non-US-branded products before and after the policy is implemented.

Figure 55: Market Shares



Notes: Panel A displays bounds on the expected total market share of US brands across countries before (blue) and after (orange) a 50% consumer subsidy on US-branded products in the United States. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand market shares after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays similar bounds on the expected total market share of non-US brands before and after the policy is implemented.

Figure 56: Variable Profits



Notes: Panel A displays bounds on the expected (log) total variable profits of US brands across countries before (blue) and after (orange) a 20% reduction in US brands' marginal costs. The expectation is over bounds on the probability distribution of firms' offerings and demand and marginal cost shocks. The intervals in green are the bounds on US-brand (log) total variable profits after the policy is implemented, computed using the bounds on the distribution of product offerings in each market before the policy is implemented. The intervals in green only reflect the intensive margin response. Panel B displays the corresponding bounds on the expected (log) total variable profits of non-US brands before and after the policy is implemented.