Market Entry and Plant Location in Multi-Product Firms

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Broad Motivation

- Recent quantitative literature on interdependencies in plant location choices.
  - Helpman et al. (2004), Tintelnot (2017), Oberfield et al. (2023), Antràs et al. (2023), Arkolakis et al. (2023).
- Large literature on product market introduction for multi-product firms
  - Eckel and Neary (2010), Dhingra (2013), Hottman et al. (2016), Head and Mayer (2019), Arkolakis et al. (2021).
- These two literatures study both phenomena in isolation
- Multinational firms are multi-product firms, with each product manufactured and sold in several countries
- Decisions of where to produce and sell each product are naturally interdependent
- Nontrivial implications of country-specific demand/supply/trade policies or shocks
- Focus on the global car industry: recent target of large industrial policy measures

This Paper

- Model with firms deciding where to produce and sell each product in a portfolio.
  - Nested CES demand: cannibalization across a firm's products in each destination.
  - Fixed selling costs: binary choice of whether to sell each product in each destination.
  - Export platforms: cannibalization across a firm's origins for each product.
  - Fixed production costs: binary choice of whether to produce each product in each origin.
  - Trade costs: interdependencies between selling and producing location choices.
  - Caveat: no strategic interactions between firms; monopolistically competitive model.
- Provide an algorithm to solve for the firm's problem.
- Use bounds on the firm's solution & moment inequalities to estimate fixed costs.
- Predict industry equilibrium changes in global car industry in reaction to counterfactual changes in consumption subsidies, production subsidies, and tariffs.

This Paper

- Research Question: How do multi-product firms re-organize global production in response to industrial and trade policies? What are the global implications?

- Methodological Contribution: solution method for single-agent multiple discrete choice problems featuring both pairwise complementarities and substitutabilities

- Key Findings:
  1. Demand-side EV subsidies in the US favor production in Europe and are effective at inducing EV product entry in the US $\rightarrow$ EU is the optimal sourcing location to the US
  2. Marginal cost EV subsidies in the US lead to EV production in the US and product entry worldwide $\rightarrow$ EVs now cheaper to produce in the US
  3. Tariffs have limited effects on the structure of global production $\rightarrow$ US market not sufficiently large to justify the opening of expensive plants that only supply locally
  4. Policies implemented in the US have spillover effects on other countries through changes in firms' production location decisions

Literature

1. Multinational firms and plant location:
   - Helpman et al. (2004), Tintelnot (2017), Oberfield et al. (2023), Antràs et al. (2023), Arkolakis et al. (2023).
2. Multi-product firms and cannibalization:
   - Eckel and Neary (2010), Dhingra (2013), Hottman et al. (2016), Head and Mayer (2019), Arkolakis et al. (2021).
3. Theoretical literature on multinational, multi-product firms.
   - Bernard et al. (2018), Helpman and Niswonger (2022).
4. Combinatorial discrete-choice problems:
   - Solution: Jia (2008), Arkolakis et al. (2023), Alfaro-Ureña et al. (2023), Houde et al (2023), Sabal (2024)
   - Estimation: Holmes (2011), Morales et al. (2019), Fan and Yang (2023)
5. Studies of global automobile industry:
   - Goldberg (1995), Cosar et al. (2018), Head and Mayer (2019), Castro-Vincenzi (2023), Allcott et al. (2024), Head et al. (2024)

Quantitative Model

Setting

- Static industry equilibrium model.
- Models indexed by m belong to segments indexed by s within brands indexed by b.
- A brand may operate in many segments, and many brands may operate in a segment.
- Treat each brand-segment pair as an independent firm.
- Firms operate a finite number of models, which are determined exogenously.
- The firm simultenaously decides where to produce and sell each model in its portfolio.
- Index destinations by $n = 1, \ldots, N$ and origins by $o = 1, \ldots, N$.
- Firms face nested CES demand, with upper nest being a brand-segment.
- Firms are monopolistically competitive and internalize cannibalization across models.

Demand

- In destination n, exogenous expenditure E_{sn} in a quantity aggregator C_{sn} in segment s:

$$C_{sn} = \left(\int_{b \in \Omega_{sn}} (\psi_{bsn} C_{bsn})^{\frac{\sigma^B - 1}{\sigma^B}} db \right)^{\frac{\sigma^B}{\sigma^B - 1}},$$

$$C_{bsn} = \left(\sum_{m=1}^{M_{bs}} I_{mn} (\psi_{mn} C_{mn})^{\frac{\sigma^M - 1}{\sigma^M}} \right)^{\frac{\sigma^M}{\sigma^M - 1}},$$

  with $\sigma^M \geq \sigma^B > 1$.

- Demand equation for a model m equals

$$C_{mn} = A_{sn} (\psi_{bsn})^{\sigma^B - 1} (\psi_{mn})^{\sigma^M - 1} (P_{bsn})^{\sigma^M - \sigma^B} (P_{mn})^{-\sigma^M}.$$

  In empirics, ψ_{bsn} and ψ_{mn} depend on brand and model effects, and covariates.

Production

- Marginal production costs:

$$c_{mo} = \phi_b \phi_o \phi_s \phi_{oh(b)} \phi_m.$$

  In empirics, both $\phi_{oh(b)}$ and ϕ_m are a function of observed characteristics.

- Iceberg trade costs:

$$\tau_{on} = \kappa_n \kappa_{on}.$$

  In empirics, κ_{on} is a function of covariates.

- Price conditional on selling model m in destination n from production location o:

$$P_{mn}(D_m) = \frac{\sigma^B}{\sigma^B - 1} \times \min_{o:D_{mo}=1} \{\tau_{on} c_{mo}\},$$

  with $D_m = \{D_{mo}\}_{o=1}^{N}$ and $D_{mo} = 1$ if model m is produced in o (and zero otherwise).

- Price is independent of how many models brand b sells in destination d.

Extensive Margin Choices

- Fixed costs of selling product m in destination d:

$$F^e_{mn} = \gamma_{nh(b)} + \nu^e_{mn}, \qquad \text{with} \qquad \nu^e_{mn} \sim \mathbb{N}(0, \sigma_e).$$

  In empirics, $\gamma_{nh(b)}$ is a function of observed characteristics:

$$\gamma_{nh(b)} = \gamma_0 + \gamma_1 dist_{nh(b)}.$$

- Fixed costs of producing product m in origin o:

$$F^p_{mo} = d^p_{mo}(\gamma_{oh(b),1} + \nu^p_{mo}), \quad \text{with} \quad \nu^p_{mo} \sim \mathbb{N}(0, \sigma_p) \quad \text{and} \quad d^p_{mo} \sim B(1, \gamma_{oh(b),2}).$$

  In empirics, $\gamma_{oh(b)} = (\gamma_{oh(b),1}, \gamma_{oh(b),2})$ are a function of observed characteristics:

$$\gamma_{oh(b),1} = \gamma_2$$
$$\gamma_{oh(b),2} = \gamma_3$$

- Each brand-segment chooses:

  $M_{bs} \times N$ binary **product entry** decisions $\boldsymbol{I}$,

  $M_{bs} \times N$ binary **production location** decisions $\boldsymbol{D}$.

Firm Problem

- Each brand-segment chooses $I = \{I_{mn}\}_{m,n}$ and $D = \{D_{mo}\}_{m,o}$ to maximize profits:

$$\Pi_{bs}(I, D) = \sum_{n=1}^{N} \sum_{m=1}^{M_{bs}} \{I_{mn}(\pi_{mn}(I_n, \widetilde{D}_n) - F^e_{mn}) - D_{mn}F^p_{mn}\}$$

  with $D_{mn} = 1$ if model m is produced in n (and zero otherwise), $I_{mn} = 1$ if m is sold in n (and zero otherwise), $I_n = \{I_{mn}\}_m$, and $\widetilde{D}_n = \{D_{mo}I_{mn}\}_{m,o}$.

- Variable potential gross profits of selling model m in market n are:

$$\pi_{mn}(I_n, \boldsymbol{D}) = A_{bsn}(P_{bsn}(I_n, \widetilde{D}_n))^{\sigma^M - \sigma^B}(P_{mn}(D_m))^{1-\sigma^M},$$

$$P_{bsn}(I_n, \widetilde{D}_n) = \left(\sum_{m=1}^{M_{bs}} I_{mn} \left(\frac{P_{mn}(D_m)}{\psi_{mn}} \right)^{1-\sigma^M} \right)^{\frac{1}{1-\sigma^M}}.$$

Model: Key Takeaways

- Industry equilibrium model of multi-product production and product entry decisions.

- Nested CES structure: segment-brand-product.

- Firms (brand-segments) have a portfolio of M_{bs} products and choose $M_{bs} \times N$ product entry and production location decisions.

- Internalize:
  1. Geography: trade costs and tariffs.
  2. Cannibalization across their products within markets.
  3. Demand, marginal costs, and fixed production and market entry costs.

Solution Algorithm

Marginal Value & Cross-Partial

- The firm's problem is of the form

$$C^* = \text{argmax}_{C \in \mathcal{B}^J} \pi(C)$$

  with $\mathcal{B}^J$ the set of all 2^J J-dimensional binary vectors: the J-dimensional Boolean set.

- For any $i = \{1, \ldots, J\}$, define the marginal value of discrete choice i under C

$$\sigma_i(C) = D_i(\pi(C)) := \pi(C^{i \to 1}) - \pi(C^{i \to 0}),$$

  where $C^{i \to 1}$ is the vector C with the ith coordinate set to 1 (analogously for $C^{i \to 0}$).

- Thus, $D_i \pi(C)$ is the change in $\pi(C)$ when changing the ith coordinate from zero to one holding all other coordinates constant.

- For any two choices $i, j = \{1, \ldots, J\}$, define the cross partial of i and j under C:

$$\sigma_{ij}(C) = D_{ij}\pi(C) := D_j(D_i(\pi(C))).$$

Complementarities and Substitutabilities

- The key assumption $\pi(C)$ must satisfy for our algorithm to bound the solution to the firm's problem is that the sign of $\sigma_{ij}(C)$ is independent of third choices and known.
- We do not require $\pi(C)$ to be either submodular or supermodular, or to exhibit single crossing differences in choices (Jia, 2008; Arkolakis et al., 2023).

Assumption

For all $C \in \mathcal{B}^N$, and any $i, j \in \{1, 2, \ldots, N\}$, the sign of $\sigma_{ij}(C)$ is known and independent of C.

- In our model, the following pairs have weakly negative cross partials:
  1. D_{mo} and $D_{m'o'}$ for (m, m', o, o');
  2. I_{mn} and $I_{m'n}$ for $m \neq m'$;
  3. D_{mo} and $I_{m'n}$ for all (m, m', o, n);

  and the following pairs have weakly positive cross partials:
  1. D_{mo} and I_{mn} for all (o, n).

Tarski's Theorem

Theorem (Tarski 1955)

Let $(L, \leq)$ be a complete lattice. Suppose $F: L \to L$ is monotone increasing; i.e. for all $x, y \in L$, $x \leq y \Rightarrow F(x) \leq F(y)$. The set of fixed points of F is then a complete lattice with respect to $\leq$.

- Lattice: a partially ordered set $(L, \leq)$ such that for any $\{a, b\} \subseteq L$, the infimum (meet) $a \vee b \in L$ and the supremum (join) $a \wedge b \in L$.
- Complete lattice: a lattice $(L, \leq)$ such that any subset $S \subseteq L$ has a supremum and an infimum in L.
- Any finite lattice is complete.
- Implication of Tarski (1955): starting from the supremum of L, applying the mapping F a finite number of iterations, one converges to supremum of the set of fixed points.

Our Approach: Define Lattice

- Denote as $S(\mathcal{B}^J)$ the set of complete sublattices of $\mathcal{B}^J$.
- If $J = 2$, then

$$\mathcal{B}^J = \{(0,0),(1,0),(0,1),(1,1)\}$$

and

$$\begin{aligned} S(\mathcal{B}^J) = \{&\emptyset, \{(0,0)\}, \{(1,0)\}, \{(0,1)\}, \{(1,1)\}, \{(0,0),(1,0)\}, \{(0,0),(0,1)\}\} \\ &\{(0,0),(1,1)\}, \{(1,0),(1,1)\}, \{(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}, \dots, \\ &\{(0,0),(1,0),(0,1),(1,1)\}\}. \end{aligned}$$

- $(S(\mathcal{B}^J), \subseteq)$ is a lattice with supremum $\mathcal{B}^J$ and infimum $\emptyset$. As it is non-empty and finite, it is also a complete lattice.
- Thus, Tarski (1955) applies to any $F\colon S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ that is monotone increasing.

Our Approach: Define Mapping

- For any $\boldsymbol{C} \in S(\mathcal{B}^J)$, define

$$\overline{\Omega}(\boldsymbol{C}) = \{i = 1, \ldots, J \colon D_i(\pi(\sup_i(\boldsymbol{C}))) < 0\},$$
$$\underline{\Omega}(\boldsymbol{C}) = \{i = 1, \ldots, J \colon D_i(\pi(\inf_i(\boldsymbol{C}))) \geq 0\},$$

where, for each $i = 1, \ldots, N$ and $j = 1, \ldots, N$,

$$[\sup_i(\boldsymbol{C})]_j := \mathbb{1}[\sigma_{ij} \geq 0][\sup(\boldsymbol{C})]_j + \mathbb{1}[\sigma_{ij} < 0][\inf(\boldsymbol{C})]_j,$$
$$[\inf_i(\boldsymbol{C})]_j := \mathbb{1}[\sigma_{ij} \geq 0][\inf(\boldsymbol{C})]_j + \mathbb{1}[\sigma_{ij} < 0][\sup(\boldsymbol{C})]_j;$$

and define $F \colon S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ as,

$$F(\boldsymbol{C}) = \{C \in \boldsymbol{C} \colon C_i = 0 \text{ and } C_{i'} = 1 \; \forall i \in \overline{\Omega}(\boldsymbol{C}) \text{ and } \forall i' \in \underline{\Omega}(\boldsymbol{C})\}.$$

Mapping: Properties and Implementation

- Mapping $F\colon S(\mathcal{B}^J) \to S(\mathcal{B}^J)$ is increasing; i.e., $\boldsymbol{C} \subseteq \boldsymbol{C}' \implies F(\boldsymbol{C}) \subseteq F(\boldsymbol{C}')$.
- Intuition:

$$\begin{aligned}
\boldsymbol{C} \subseteq \boldsymbol{C}' &\implies \forall i \;\; D_i(\pi(\sup_i(\boldsymbol{C}))) \leq D_i(\pi(\sup_i(\boldsymbol{C}'))) \implies \overline{\Omega}(\boldsymbol{C}') \subseteq \overline{\Omega}(\boldsymbol{C}) \\
\boldsymbol{C} \subseteq \boldsymbol{C}' &\implies \forall i \;\; D_i(\pi(\inf_i(\boldsymbol{C}))) \geq D_i(\pi(\inf_i(\boldsymbol{C}'))) \implies \underline{\Omega}(\boldsymbol{C}') \subseteq \underline{\Omega}(\boldsymbol{C})
\end{aligned}$$

and

$$\left.\begin{array}{l} \overline{\Omega}(\boldsymbol{C}') \subseteq \overline{\Omega}(\boldsymbol{C}) \\ \underline{\Omega}(\boldsymbol{C}') \subseteq \underline{\Omega}(\boldsymbol{C}) \end{array}\right\} \implies F(\boldsymbol{C}) \subseteq F(\boldsymbol{C}').$$

- Tarski (1955) implies set of fixed points $\boldsymbol{C}^f$ is a complete lattice, and we can converge to its supremum in a finite number of steps applying F iteratively from $\mathcal{B}^J$.

Mapping: Properties and Implementation

- Given the definition of C^* as

$$C^* = \text{argmax}_{C \in \mathcal{B}^J} \pi(C)$$

we know that, for all $i = 1, \dots, J$, it holds

$$\begin{aligned}
C_i^* = 0 &\implies D_i(\pi(C^*)) < 0 \implies i \in \overline{\Omega}(\boldsymbol{C}) \implies [F(C^*)]_i = 0; \\
C_i^* = 1 &\implies D_i(\pi(C^*)) \geq 0 \implies i \in \underline{\Omega}(\boldsymbol{C}) \implies [F(C^*)]_i = 1.
\end{aligned}$$

- Therefore, $F(C^*) = C^*$ and, thus, C^* is a fixed point of F; i.e., $C^* \in \boldsymbol{C}^f$.
- Implementation:
    1. Start with $\boldsymbol{C}_0 = \mathcal{B}^J$.
    2. Apply F to obtain $\boldsymbol{C}_1 = F(\boldsymbol{C}_0)$; note that $\boldsymbol{C}_1 \in S(\mathcal{B}^N)$.
    3. Obtain $\boldsymbol{C}_k = F(\boldsymbol{C}_{k-1})$ for $k = 1, 2, \dots,$ until convergence.

Algorithm: Example

- Consider a setting with three choices such that $\sigma_{12} > 0$, $\sigma_{13} < 0$, and $\sigma_{23} < 0$. Then,

$$\boldsymbol{C}_0 = \{(0,0,0),(1,0,0),(0,1,0),(0,0,1),(1,1,0),(1,0,1),(0,1,1),(1,1,1)\}.$$

- As $\sigma_{12} > 0$, $\sigma_{13} < 0$, $\sup(\boldsymbol{C}_0) = (1,1,1)$, and $\inf(\boldsymbol{C}_0) = (0,0,0)$, it holds

$$\sup_1(\boldsymbol{C}_0) = [\cdot, [\sup(\boldsymbol{C}_0)]_2, [\inf(\boldsymbol{C}_0)]_3] = [\cdot, 1, 0].$$

- Similarly:

$$\sup_2(\boldsymbol{C}_0) = [1, \cdot, 0] \qquad \sup_3(\boldsymbol{C}_0) = [0, 0, \cdot]$$

and

$$\inf_1(\boldsymbol{C}_0) = [\cdot, 0, 1] \qquad \inf_2(\boldsymbol{C}_0) = [0, \cdot, 1] \qquad \inf_3(\boldsymbol{C}_0) = [1, 1, \cdot].$$

Algorithm: Example (Cont.)

- Suppose

$$\{D_1(\pi(\sup_1(\boldsymbol{C}_0))) < 0, D_2(\pi(\sup_2(\boldsymbol{C}_0))) \geq 0, D_3(\pi(\sup_3(\boldsymbol{C}_0))) \geq 0\}$$
$$\{D_1(\pi(\inf_1(\boldsymbol{C}_0))) < 0, D_2(\pi(\inf_2(\boldsymbol{C}_0))) < 0, D_3(\pi(\inf_3(\boldsymbol{C}_0))) \geq 0\}$$

  then

$$\overline{\Omega}(\boldsymbol{C}_0) = 1 \qquad \text{and} \qquad \underline{\Omega}(\boldsymbol{C}_0) = 3.$$

- Applying mapping F, we obtain:

$$F(\boldsymbol{C}_0) = \{C \in \boldsymbol{C}_0 : C_1 = 1, C_3 = 0\} = \{(0,1,1),(0,0,1)\} = \boldsymbol{C}_1$$

- And the next step would apply F to $\boldsymbol{C}_1$.

Solving the Model: Mapping 1

- The firm's problem can be expressed as:

$$\max_{I,D} \Pi_{b,s}(I, D)$$

- We apply the algorithm using knowledge of all cross-partials concerning product entry and production location decisions.
- Recall the following cross-partials for $\Pi_{b,s}(I, D)$:
  - In our model, the following pairs have weakly negative cross partials:
    1. D_{mo} and $D_{m'o'}$ for (m, m', o, o');
    2. I_{mn} and $I_{m'n}$ for $m \neq m'$;
    3. D_{mo} and $I_{m'n}$ for all (m, m', o, n);

    and the following pairs have weakly positive cross-partials:

    1. D_{mo} and I_{mn} for all (o, n).

Solving the Model: Mapping 2

Single-Product Firm Case

Suppose that firm (b, s) is a single-product firm.

Insight: we can write down the firm's problem as

$$\max_{D} V_{b,s}(D)$$

where

$$V_{b,s}(D) = \max_{I} \Pi_{b,s}(I, D).$$

We prove that $V_{b,s}$ exhibits decreasing differences across all pairs:

1. D_o and $D_{o'}$ have weakly negative cross-partials.

$\Longrightarrow$ can apply the set-increasing mapping to $V_{b,s}$ to solve optimal production locations.

Solving the Model: Approach 2

Multi-Product Firms

Suppose that we know the solution to all of the firm's discrete choices except those concerning model m: (I^*_{-m}, D^*_{-m}) is known.

- **Proposition 1**: The global arg max D^*_m is the arg max of:

$$V_{b,s,m}(D_m; I^*_{-m}, D^*_{-m}) := \max_{I_m} \sum_n I_{mn}[\Delta\pi^m_n(D_m; I^*_{-m,n}, D^*_{-m,n}) - F^e_{mn}] - \sum_n D_{mn}F^o_{mn}$$

  - $\Delta\pi^m_n$: additional profits in market n when m is offered (conditional on other models' offerings and production locations).
  - **Intuition**: Conditional on the non-m decisions being optimal, the best the firm can do is to maximize the additional profits that it can earn from model m.

- **Proposition 2**: $V_{b,s,m}(D_m; I_{-m}, D_{-m})$ exhibits decreasing differences: D_{mo} and $D_{mo'}$ have weakly negative cross-partials **for any** I_{-m}, D_{-m}.

- **Proposition 3**: $V_{b,s,m}(D_m; I_{-m}, D_{-m})$ exhibits decreasing differences wrt I_{-m} and D_{-m}.

Solving the Model: Mapping 2

Sufficient Conditions

- Propositions 1-3 mean that if

$$\Delta \underline{V}^{o}_{b,s,m}(\overline{D}_m; \overline{I}_{-m}, \overline{D}_{-m}) := V^{m}_{b,s}(\overline{D}^{o\to 1}_m; \overline{I}_{-m}, \overline{D}_{-m}) - V^{m}_{b,s}(\overline{D}^{o\to 0}_m; \overline{I}_{-m}, \overline{D}_{-m}) \geq 0$$

then $D^*_{mo} = 1$ and if

$$\Delta \overline{V}^{o}_{b,s,m}(\underline{D}_m; \underline{I}_{-m}, \underline{D}_{-m}) := V^{m}_{b,s}(\underline{D}^{o\to 1}_m; \underline{I}_{-m}, \underline{D}_{-m}) - V^{m}_{b,s}(\underline{D}^{o\to 0}_m; \underline{I}_{-m}, \underline{D}_{-m}) < 0,$$

then $D^*_{mo} = 0$.

- $\overline{D}$, $\overline{I}$ denote upper bounds and $\underline{D}$, $\underline{I}$ denote lower bounds on the solution.

Solving the Model: Mapping 2

Algorithm

Iterate over such sufficient conditions across models:

1. Start the algorithm at known bounds of the solution: $(\underline{I}, \underline{D})$ and $(\bar{I}, \overline{D})$. At first iteration, initialize with: $(\underline{I}, \underline{D}) = (\mathbf{0}, \mathbf{0})$ and $(\bar{I}, \overline{D}) = (\mathbf{1}, \mathbf{1})$.
2. Set $m = 1$.
3. For each o, if $\Delta \underline{V}^{o}_{b,s,m}(\overline{D}_m; \bar{I}_{-m}, \overline{D}_{-m}) > 0$, then $D^*_{mo} = 1$ so update the lower bound $\underline{D}_{mo} = 1$. If $\Delta \overline{V}^{o}_{m}(\underline{D}_m; \underline{I}_{-m}, \underline{D}_{-m}) < 0$, then $D^*_{mo} = 0$ so update the upper bound $\overline{D}_{mo} = 0$.
4. Iterate until convergence for model m given bounds on other models.
5. Apply Step 3 on model $m = 2$ given updated bounds on model $m = 1$.
6. ... Apply Step 3 model $m = M$ given bounds updated bounds on models $m < M$.
7. Repeat steps 1-6 until global convergence on D is achieved.

Combining the Mappings: First-Step Algorithm

- Mapping 2 for model m: set-increasing mapping used in Mapping 1 applied on $V_{b,s,m}$.
- Mapping 2: composition of set-increasing mappings applied to $\{V_{b,s,m}\}_{m=1}^{M_{b,s}}$.
- **Key**: a composition of set-increasing mappings is set-increasing $\implies$ can combine Mappings 1 and 2 in any order provided we start at known bounds of the solution.
- Combine with Mapping 1 to solve for offerings I given bounds on D.
- In practice, we compose both Mapping 1 and Mapping 2 by applying Mapping 1 between Steps 3 and Steps 4 on the pertinent model m.
  - Mapping 1 is computationally cheaper because there is no max operator (optimization).
  - Solving for sales of model m can speed up the convergence of Mapping 2 when applying Step 3 on other models, in light of Proposition 2.
- We call this combination of set-increasing mappings the "**First-Step Algorithm**".

Algorithm: Illustrating Approach 1

Iteration 1		Locations 1	2	3	4	5
Model 1	Sales			0		1
	Prod.					1
Model 2	Sales					1
	Prod.					1

Iteration 2		Locations 1	2	3	4	5
Model 1	Sales			0		1
	Prod.	0	0		0	1
Model 2	Sales					1
	Prod.					1

Iteration 3		Locations 1	2	3	4	5
Model 1	Sales			0	0	1
	Prod.	0	0		0	1
Model 2	Sales					1
	Prod.					1

Iteration 4		Locations 1	2	3	4	5
Model 1	Sales			0	0	1
	Prod.	0	0		0	1
Model 2	Sales				1	1
	Prod.					1

Refinement: Branching

- First-Step Algorithm: bounds not guaranteed to be informative.

- Show that we can implement the branching algorithm in Arkolakis et al. (2024).

- **Branching Algorithm**:
  1. Choose any decision I_{mn} or D_{mo} such that:

$$[\underline{I}_{mn} = 0 \text{ and } \overline{I}_{mn} = 1] \text{ or } [\underline{D}_{mo} = 0 \text{ and } \overline{D}_{mo} = 1]$$

  2. Apply First-Step Algorithm *conditional* on 1 and conditional on 0.
  3. If obtain full solution both conditional on 1 and conditional on 0, choose the one that yields the highest payoff.
  4. Otherwise, in each branch, choose another decision to set to both 1 and 0 and solve conditional on both cases.
  5. Repeat until convergence.

- **Proposition:** Branching algorithm yields the global solution when composed with FSA.

Branching: Caveats

- Caveat of branching: becomes as computationally infeasible as a brute-force solution in the limit in which the first-step algorithm provides no information across branches.
- Ultimately there are 2^K possible branches and thus payoff evaluations.
- In practice: we put a limit on the number on payoff evaluations. If that limit is surpassed, we return the first-step algorithm solution.

Solution Method: Key Takeaways

- Provided the sign of the interdependence between any pair of choices is known, our method bounds the global optimum.

- Interdependencies in our model
  1. Trade costs $\implies$ value of selling model is higher if production location is closer.
  2. Cannibalization $\implies$ selling distinct models (within a market) and producing distinct models are substitute decisions.
  3. Plant-model increasing returns $\implies$ complementarity between producing and selling the same product in different markets.

- Apply mapping to value function rather than payoff function: yields more informative bounds on production location decisions.

Multinational Firms in Global Car Industry: Summary Statistics

Data

- Data on car production, prices and model characteristics:
  - Source: IHS Markit (Cosar et al. 2018, Head and Mayer 2019, Alcott et al., 2024)
  - Year: 2019.
  - New car registrations: by model (1245) on brand, segment (375 brand-segments), data on production (assembly) country (53) and sales country (77).
  - Model price, quantity sold, and characteristics for Australia, Brazil, China, Spain, France, Germany, UK, India, Italy, Japan, Mexico, and the US.
- Other sources of data:
  - CEPII: geographical distance between countries
  - MacMap: car tariffs
  - World Bank: Income per capita and population per country

Summary Statistics

MNEs are multi-product, with each product produced & sold in several countries.

Number of:	Mean (unweigh.)	Mean (weight.)	p25	p50	p75	p90	Max
Models (per brand-segment)	3.5	10.2	1	2	5	8	23
Sales countries (per model)	12.1	30.7	1	2	17	43	75
Production countries (per model)	1.5	3.1	1	1	1	3	12

Other Statistics

Number of:	Mean (unweigh.)	Mean (weight.)	p25	p50	p75	p90	Max
Models (per market)	41.4	137.9	11	31	65	102	387
Sales countries (per brand-segment)	18.9	49.8	1	5	40	62	77
Production countries (per brand-segment)	2.4	8.1	1	1	2	5	19
Share exported (by model-prod. loc.)	38.7%	12.9%	0%	6.4%	94%	100%	100%
Share produced in HQs (by brand-segment)	80.2%	44.1%	63.2%	100%	100%	100%	100%
Share sold in HQs (by brand-segment)	61.2%	30.9%	19.9%	78.7%	100%	100%	100%

Estimation

Estimation Strategy

- Parameters entering trade costs and marginal production costs.
  - **Price equation**
- Demand function parameters.
  - **Equations for revenue shares**: model-specific and brand-segment-specific
- Fixed cost parameters.
  - **Moment inequality** approach.

Trade Costs and Variable Production Costs

- If good m sold in n is produced in o, the model predicts:

$$P_{mn} = \frac{\sigma^B}{\sigma^B - 1} \underbrace{\kappa_n \kappa_{on}}_{\tau_{on}} \underbrace{\phi_b \phi_o \phi_s \phi_{oh(b)} \phi_m}_{c_{mo}} .$$

- Estimating equation:

$$\log P_{mn} = \alpha_n + \underbrace{\beta_1 \log \text{tariff}_{on} + \beta_2 \log \text{dist}_{on}}_{\kappa_{on}}$$
$$+ \alpha_b + \alpha_o + \alpha_s + \underbrace{\beta_3 \log dist_{oh(b)}}_{\phi_{oh(b)}} + \underbrace{\beta_4 \log HP_m + \beta_5 \log Weight_m}_{\phi_m} + \varepsilon_{mn}.$$

- Estimates:

$$\hat{\beta}_1 = 0.833^a \quad \hat{\beta}_2 = 0.044^a \quad \hat{\beta}_3 = 0.588^a \quad \hat{\beta}_4 = 0.858^a \quad \hat{\beta}_5 = 0.028^a.$$

Demand Function Parameters

- Model-implied market share in n of a model m within its brand-segment bs is:

$$\log(S_{mn}) = (1-\sigma^M)\log(P_{mn}) + (\sigma^M - 1)\log(P_{bsn}) + (\sigma^M - 1)\log(\psi_{mn}).$$

- Estimating equation:

$$\begin{aligned}\log(S_{mn}) = {} & (1-\sigma^M)\log(P_{mn}) + \alpha_{bsn} \\ & + \underbrace{\alpha_m + \beta_1 \log HP_m \log GDPpc_n + \beta_2 \log Weight_m \log GDPpc_n + \nu_{mn}}_{(\sigma^M - 1)\log(\psi_{mn})}.\end{aligned}$$

- Instrument $\log(P_{mn})$ using $\log \text{tariff}_{on}$ and $\log \text{dist}_{on}$.
- Estimate of within-brand cross-model elasticity:

$$\hat{\sigma}^M = 6.65.$$

Demand Function Parameters (Cont.)

- Model-implied market share in n of a brand b within a segment s is:

$$\log(S_{bsn}) = (1-\sigma^B)\log(P_{bsn}) + (\sigma^B - 1)\log(P_{sn}) + (\sigma^M - 1)\log(\psi_{bsn}).$$

- Estimating equation:

$$\log(S_{bsn}) = (1-\sigma^B)(\hat{\alpha}_{bsn}/(\sigma^M - 1)) + \alpha_{sn} + \underbrace{\alpha_b + \beta_1 \mathbb{1}\{h(b) = n\} + \nu_{mn}}_{(\sigma^M - 1)\log(\psi_{bsn})}.$$

- Estimate of within-segment cross-brand elasticity:

$$\hat{\sigma}^B = 3.19.$$

Fixed Cost Parameters

- Group fixed cost parameters into parameter vector θ.
- Consider moment functions $g(I, D; \theta)$ increasing in $I = \{I_{mn}\}_{m,n}$ and $D = \{D_{mo}\}_{m,o}$.
- Use solution to squeezing algorithm to build moment inequalities:

$$
\begin{aligned}
\frac{1}{M}\sum_{m=1}^{M} \mathbb{E}_\nu[g_m(\underline{I}_{b.s}(\nu), \underline{D}_{b,s}(\nu); \theta)] \leq \\
\frac{1}{M}\sum_{m=1}^{M} g_m(I^{obs}_{b,s}, D^{obs}_{b,s}; \theta) \\
\leq \frac{1}{M}\sum_{m=1}^{M} \mathbb{E}_\nu[g_m(\overline{I}(\nu), \overline{D}(\nu); \theta)].
\end{aligned}
$$

- Use Andrews and Soares (2010) to compute confidence set for θ.
- Rely on asymptotics as $M \to \infty$.
- Clustered variance-covariance matrix at the brand-segment level.

Monotonic Moment Functions

- **Share of sales destinations entered**

$$g_m^1(I_{b,s}, D_{b,s}) = \frac{1}{N-1} \sum_{n=1}^{N} I_{mn}^{b,s}$$

- **Sold in destination in profit bin k:**

$$g_m^{2,k}(I_{b,s}, D_{b,s}) = \mathbb{1}\left\{ \sum_{n=1}^{N} I_{mn}^{b,s} \mathbb{1}\{n \in B^k\} > 0 \right\}$$

- **Percentiles $p \in \{25, 50, 75\}$ of sales destinations interacted with distance to HQ:**

$$g_m^{3,p}(I_{b,s}, D_{b,s}) = Q_p\left(\{I_{mn}^{b,s} dist_{nh(b)} \mathbb{1}\{h(b) \neq n\}\}_{n=1}^{N}\right)$$

Monotonic Moment Functions (continued)

- **Share of production locations entered (excluding HQ country)**:

$$g_m^4(I_{b,s}, D_{b,s}) = \frac{1}{L}\sum_{o=1}^{L} D_{mo}^{b,s}\mathbb{1}\{h(b) \neq o\}$$

- **Produced in location in profit bin k (exc. HQ) and produced in at least 2 locations**:

$$g_m^{5,k}(I_{b,s}, D_{b,s}) = \mathbb{1}\left\{\sum_{o=1}^{L} D_{mo}^{b,s}\mathbb{1}\{o \in W^k\}\mathbb{1}\{h(b) \neq o\} > 0\right\}\mathbb{1}\left\{\sum_{o=1}^{L} D_{mo}^{b,s} > 1\right\}$$

- **Produced in the HQ country**:

$$g_m^6(I_{b,s}, D_{b,s}) = \mathbb{1}\{D_{mh(b)} = 1\}$$

Profit Bin Construction

- **Sales Bins**: Construct k quantile categories across all model-destination pairs (m, n) for

$$A_{sn}\psi_{bsn}^{\sigma^B-1}(\psi_{mn}/\overline{\phi}_m)^{\sigma^M-1}$$

  where $\overline{\phi}_m$ is the mean marginal cost of model m across all origins.
- **Production Bins**: Construct k quantile categories across all model-origin pairs (m, o) for

$$\phi_o^{1-\sigma^M}\sum_{n=1}^{N}\tau_{on}^{1-\sigma^M}A_{sn}\psi_{bsn}^{\sigma^B-1}(\psi_{mn})^{\sigma^M-1}$$

  - Mimic market access measure in Redding and Venables (2004)

Fixed Cost Estimates

	95% Confidence Set Limits
Market Entry Fixed Cost	
γ_0 (Constant)	[-3.0, 1.0]
γ_1 (Distance to HQ)	[1.5, 3.0]
σ_e (Std Shock)	[4.0, 6.0]
Production Plant Fixed Costs	
γ_2 (Constant)	[1100.0, 2600.0]
γ_3 (Free HQ plant probability)	[3.8, 4.3]
σ_p (Std Shock)	[400.0, 1000.0]
Observations	1130

Notes: Confidence sets computed using Andrews and Soares (2010) and a grid search. Units are in millions of USD. We cluster the variance covariance matrix at the brand-segment level.

Estimation: Key Takeaways

1. Cannibalization forces matter: $\hat{\sigma}^M > \hat{\sigma}^B$.
   - Within-brand-segment varieties are more substitutable than across.

2. Trade costs: increase in distance to the production plant (elasticity of 0.04) and with tariffs (elasticity of 0.83).

3. Moment Inequalities: built moment inequalities that use our solution method.
   - Moments required to be monotone in extensive margin decisions.
   - Production plant fixed costs much greater than product entry fixed costs.
   - Estimates imply firms have an incentive to build export platforms with high market access.

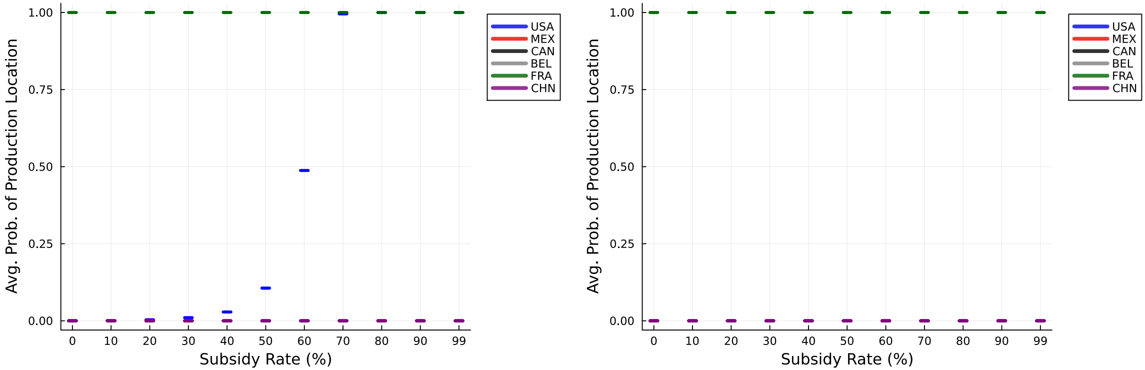
Quantification: 2-Product Firm Comparative Statics

Quantification: 2-Product Firm

- One brand-segment: Peugeot-wagon.
- Two models: Peugeot 308 (non-electric) and Peugeot 508 (electric).
- Initial firm choices:
  - Both models produced only in France.
  - Neither model sold in the US.
- Match these initial firm production choices and explore impact of counterfactual changes in the US depending on initial sales choices.
- Counterfactual changes:
  - Production subsidy (to marginal costs) for electric model in the US.
  - Consumption subsidy for electric model in the US.

Production Subsidy for Electric Model in the US

Production Location

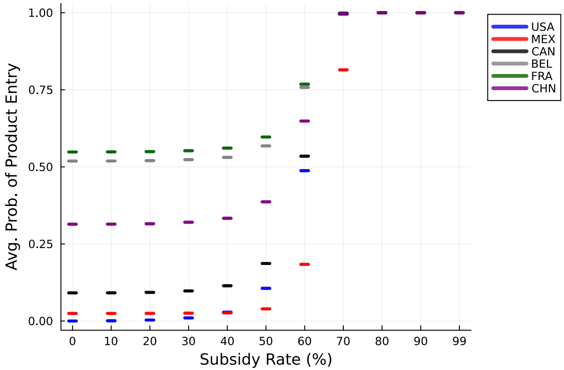


(a) Electric

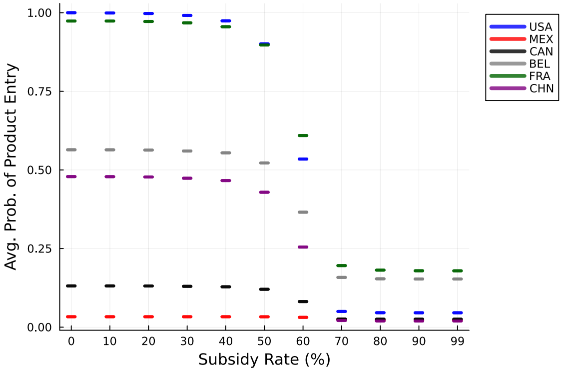
(b) Non-Electric

Production Subsidy for Electric Model in the US

Product Entry



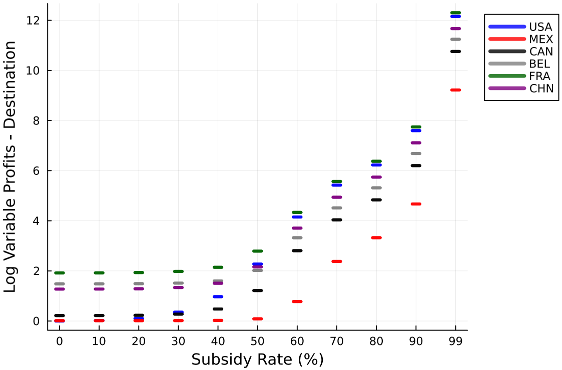
(a) Electric



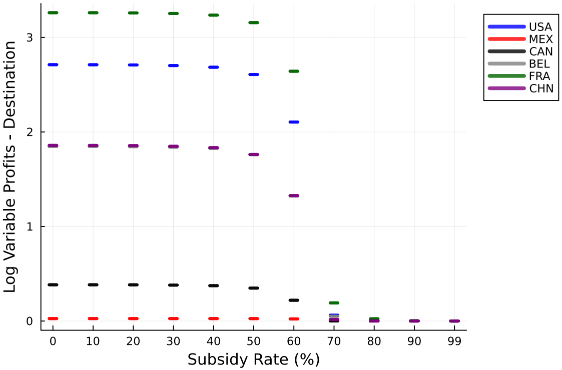
(b) Non-Electric

Production Subsidy for Electric Model in the US

Variable Profits by Destination



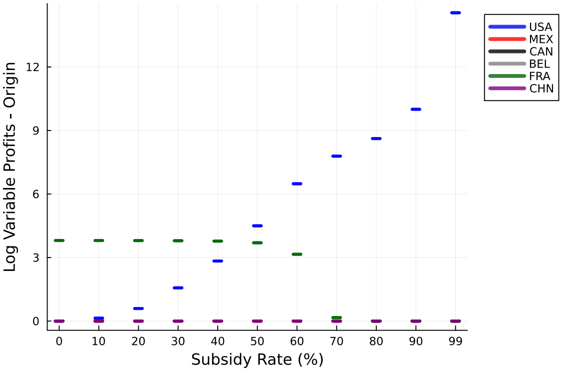
(a) Electric



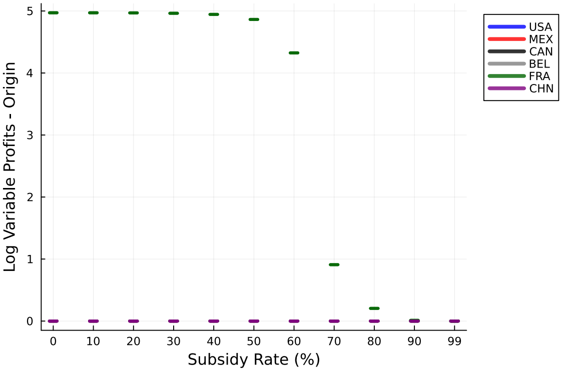
(b) Non-Electric

Production Subsidy for Electric Model in the US

Sales by Origin



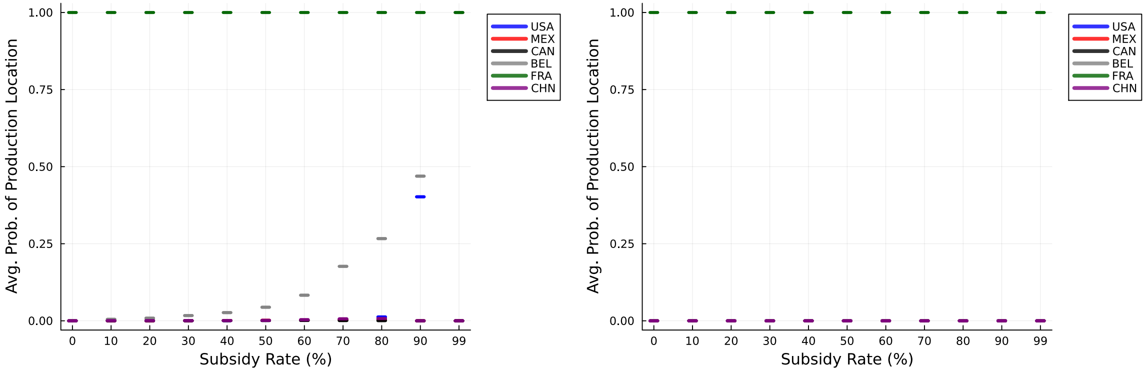
(a) Electric



(b) Non-Electric

Consumption Subsidy for Electric Model in the US

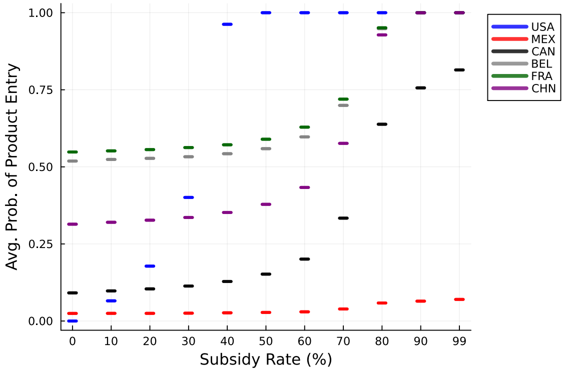
Production Location



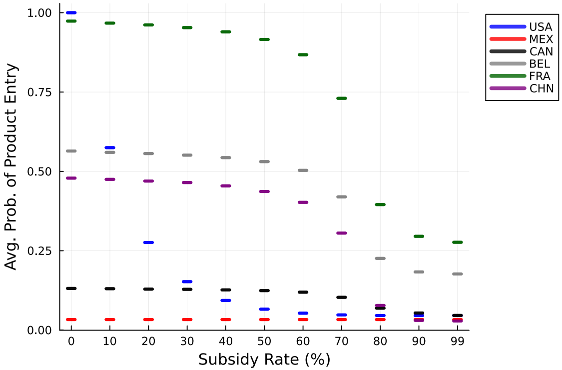
(a) Electric (b) Non-Electric

Production Subsidy for Electric Model in the US

Product Entry



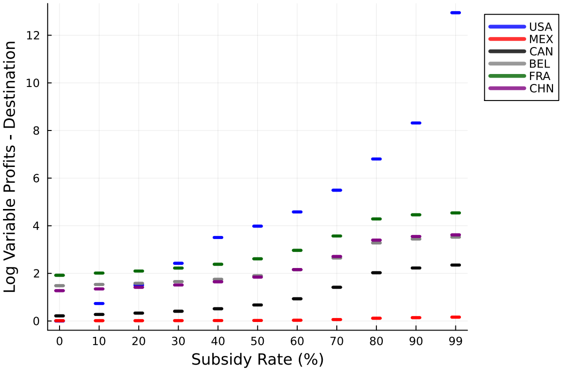
(a) Electric



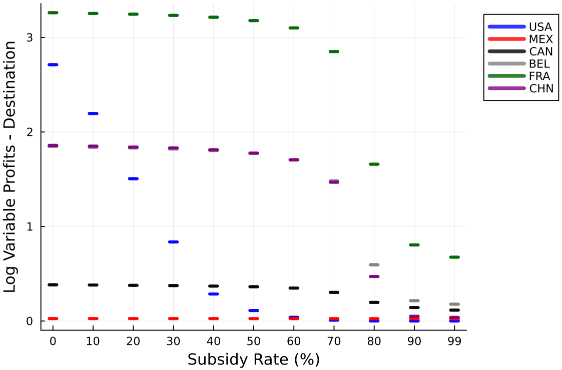
(b) Non-Electric

Production Subsidy for Electric Model in the US

Variable Profits by Destination



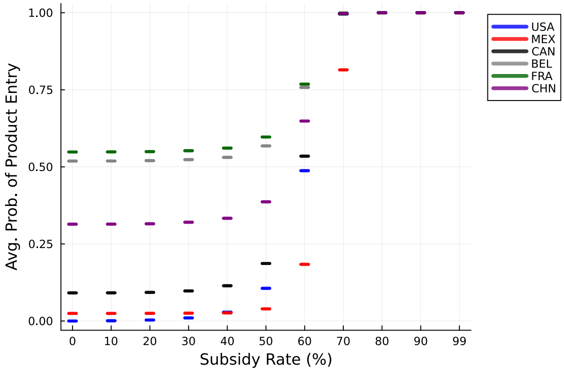
(a) Electric



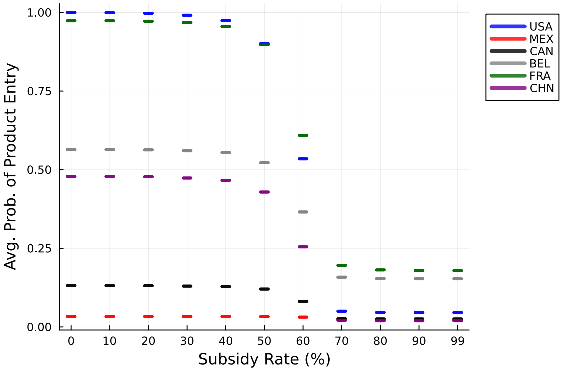
(b) Non-Electric

Production Subsidy for Electric Model in the US

Product Entry



(a) Electric



(b) Non-Electric

Quantification: Aggregate Policy Effects

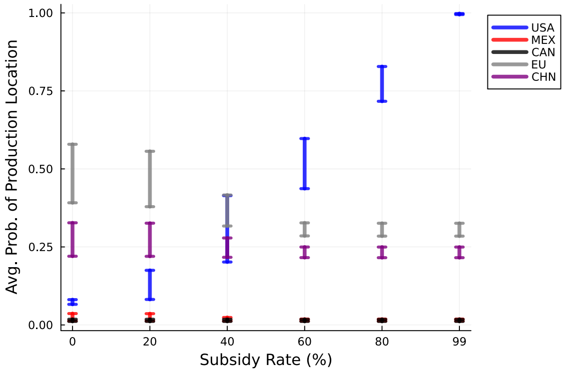
Aggregate Counterfactual Policy Experiments

- 128 draws from the distributions of fixed costs and free plant lotteries.
- Solve model for all brand-segments and under each of the draws under various policies and fixed cost parameters in the confidence set:
  - **Production and Consumption Subsidies in the US favoring Electric Vehicles** at subsidy rates $s \in \{0, 20, 40, 60, 80, 99\}$
  - **Tariffs to World and to the EU on Electric Vehicles** (in addition to observed tariffs): percentage point increases of $t \in \{0, 20, 40, 60, 80, 5000\}$
- Report bounds on levels and changes by averaging over fixed cost and lottery draws
- Bounds reflect:
  1. Parameter uncertainty
  2. Solution uncertainty
- Report counterfactual outcomes in levels and in changes.
  - Can compute changes at given parameter values and only then take the max or min across parameter values.

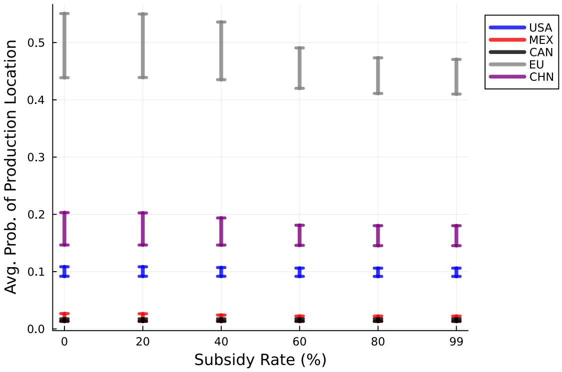
Production Subsidy on Electric Products in the US

Bounds on Levels

Production Location



(a) Electric

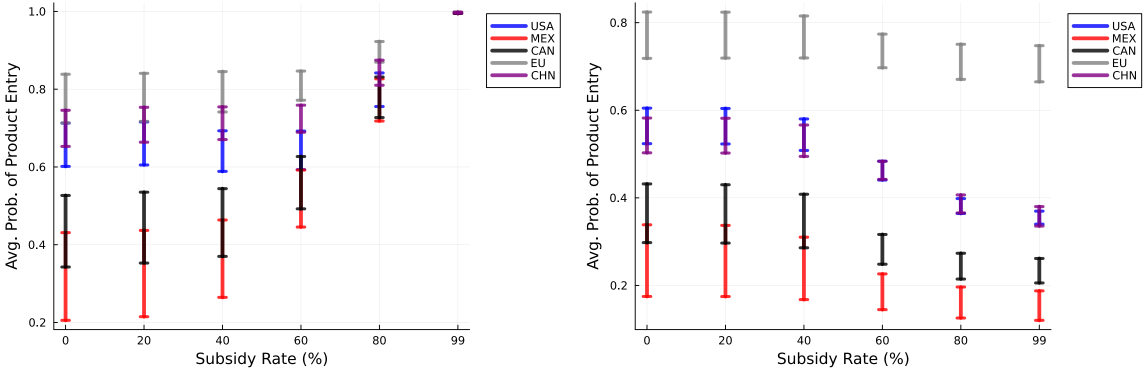


(b) Non-Electric

Production Subsidy on Electric Products in the US

Bounds on Levels

Product Entry

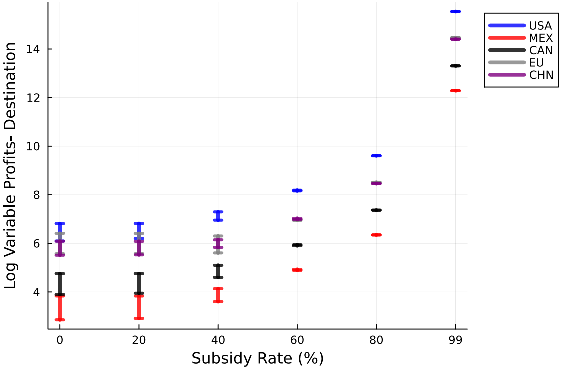


(a) Electric (b) Non-Electric

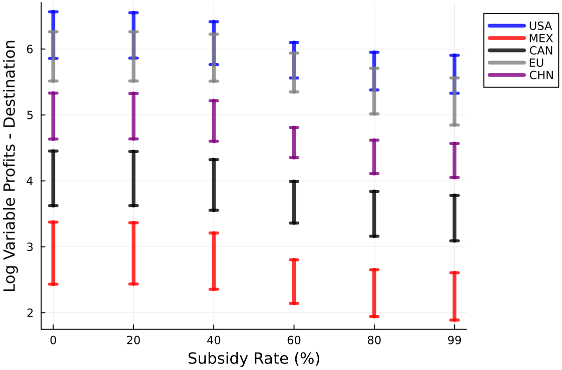
Production Subsidy on Electric Products in the US

Bounds on Levels

Variable Profits - Destination



(a) Electric

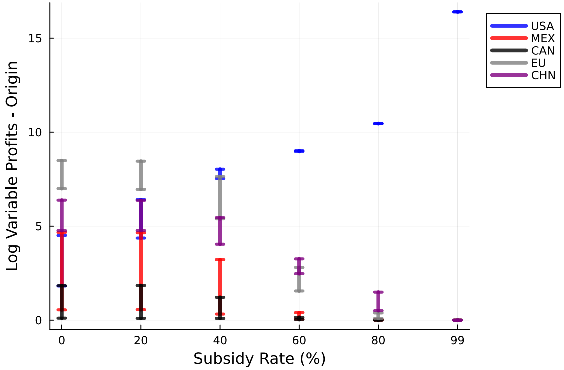


(b) Non-Electric

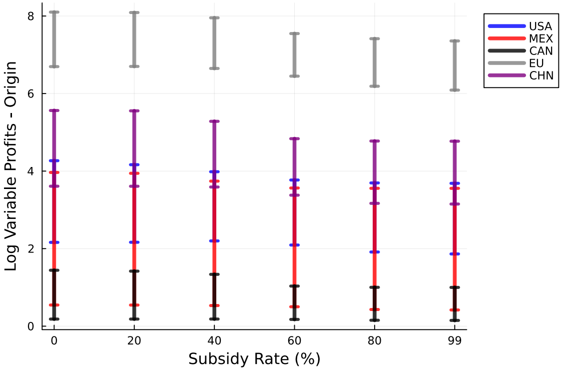
Production Subsidy on Electric Products in the US

Bounds on Levels

Variable Profits - Origin



(a) Electric

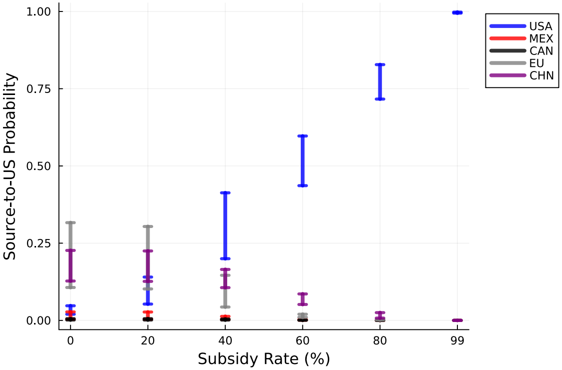


(b) Non-Electric

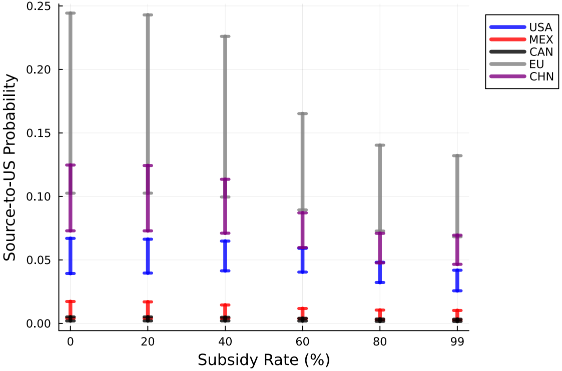
Production Subsidy on Electric Products in the US

Bounds on Levels

Probability of Sourcing to the US



(a) Electric

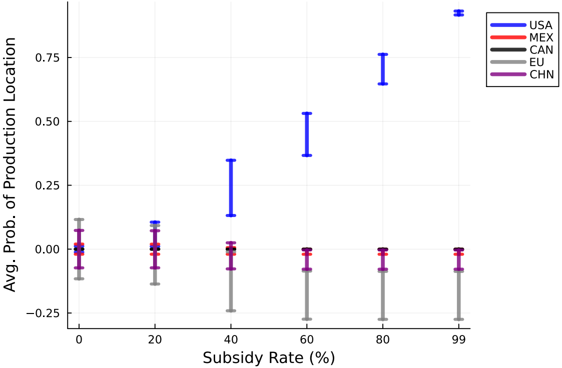


(b) Non-Electric

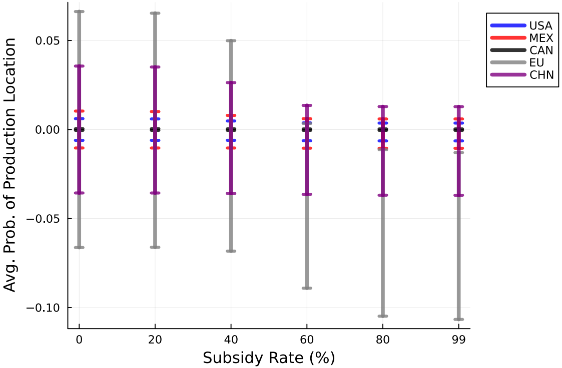
Production Subsidy on Electric Products in the US

Bounds on Changes

Production Location



(a) Electric

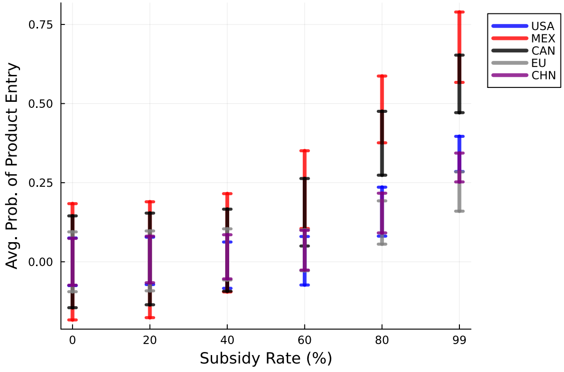


(b) Non-Electric

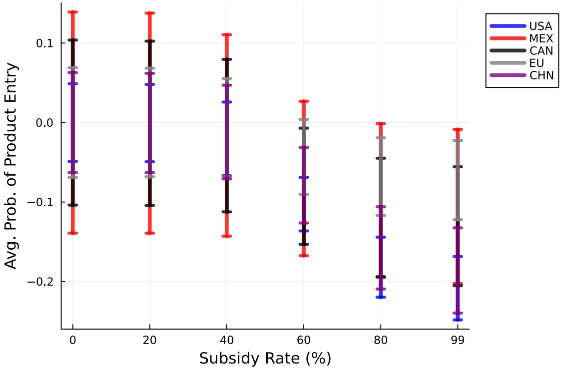
Production Subsidy on Electric Products in the US

Bounds on Changes

Product Entry



(a) Electric

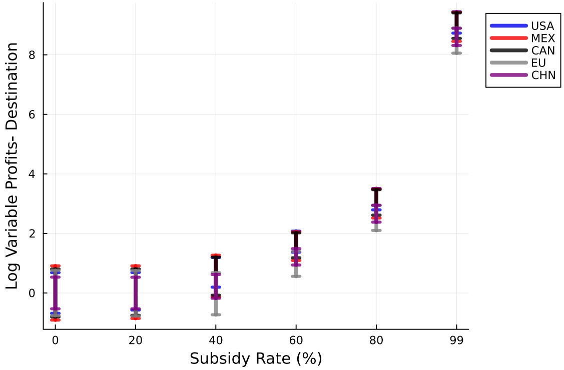


(b) Non-Electric

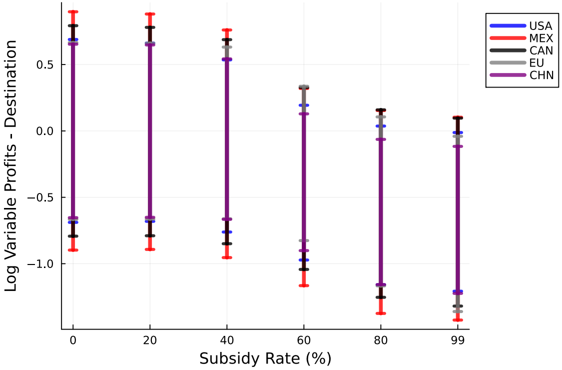
Production Subsidy on Electric Products in the US

Bounds on Changes

Variable Profits - Destination



(a) Electric

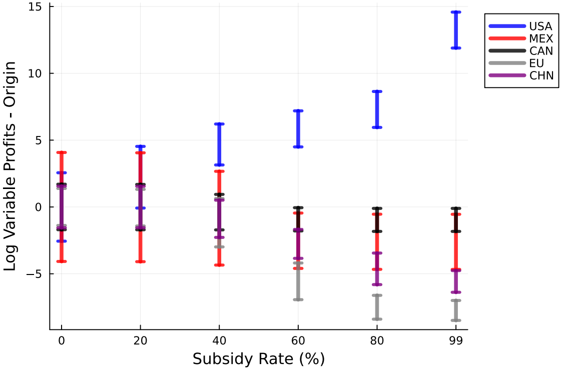


(b) Non-Electric

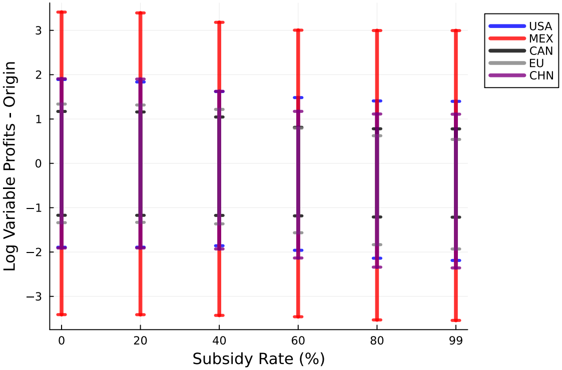
Production Subsidy on Electric Products in the US

Bounds on Changes

Variable Profits - Origin



(a) Electric

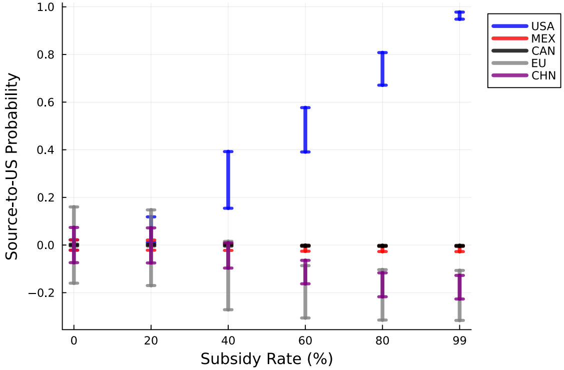


(b) Non-Electric

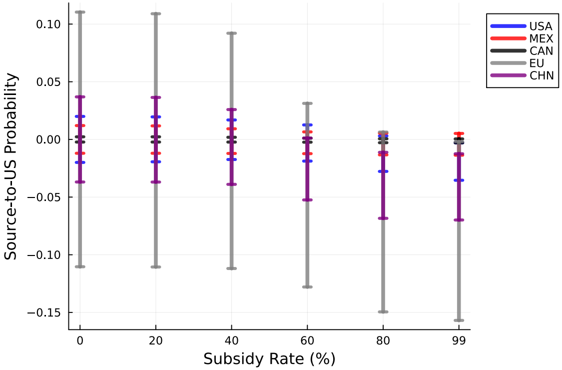
Production Subsidy on Electric Products in the US

Bounds on Changes

Probability of Sourcing to the US



(a) Electric

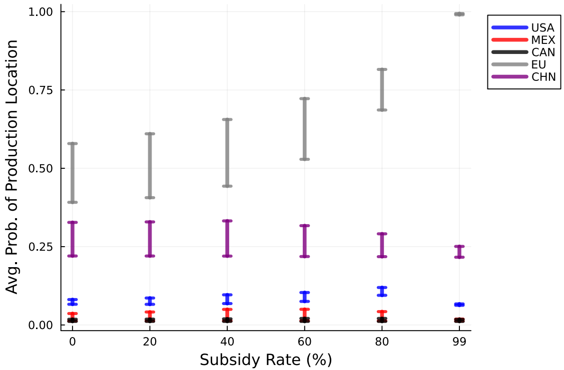


(b) Non-Electric

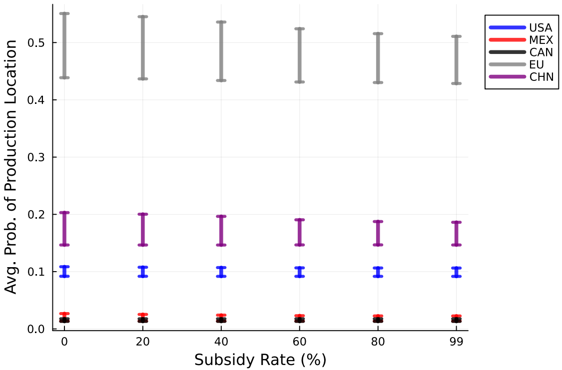
Consumption Subsidy on Electric Products in the US

Bounds on Levels

Production Location



(a) Electric

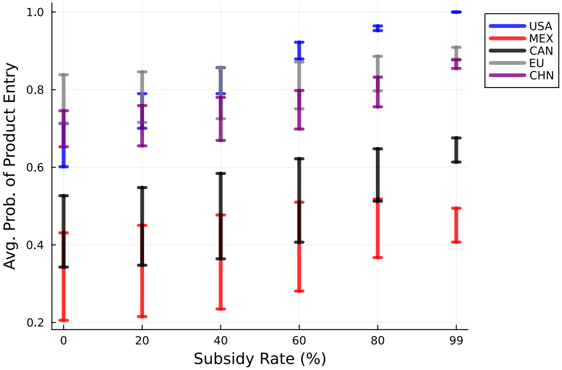


(b) Non-Electric

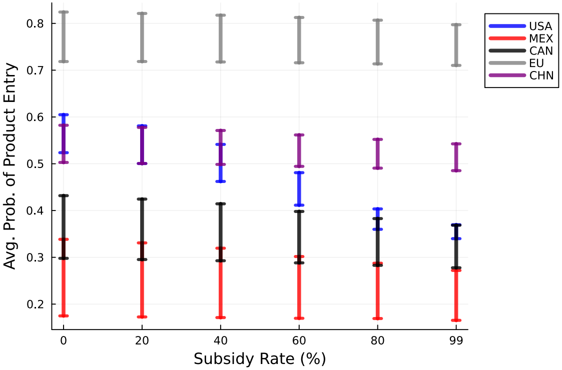
Consumption Subsidy on Electric Products in the US

Bounds on Levels

Product Entry



(a) Electric

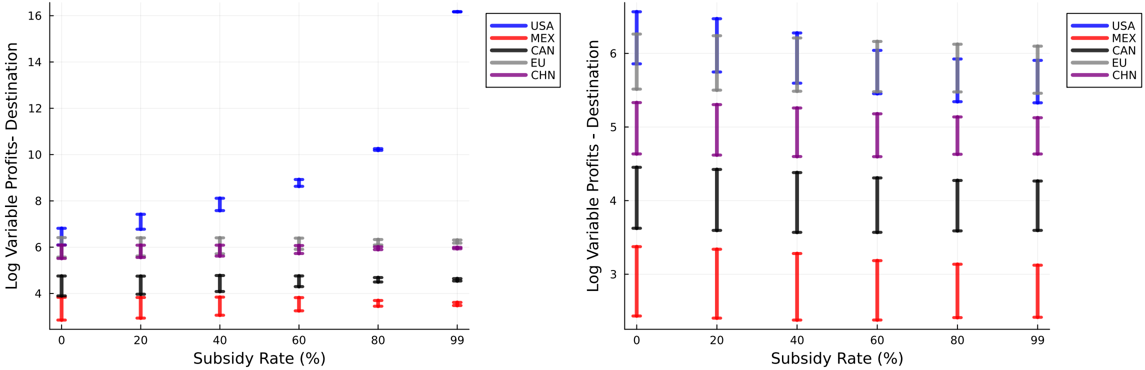


(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Bounds on Levels

Variable Profits - Destination

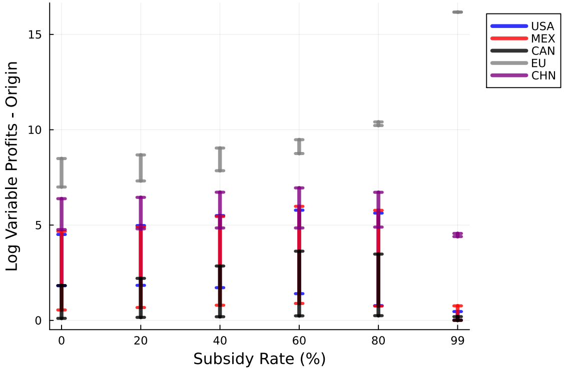


(a) Electric (b) Non-Electric

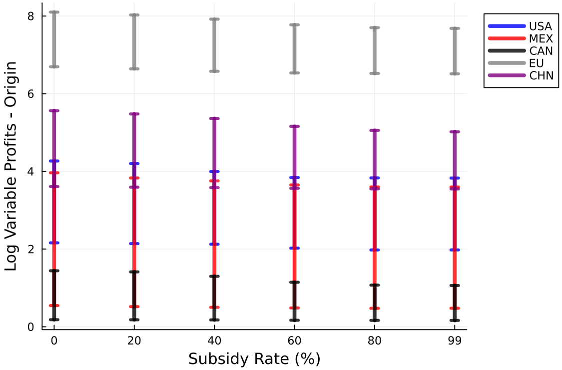
Consumption Subsidy on Electric Products in the US

Bounds on Levels

Variable Profits - Origin



(a) Electric

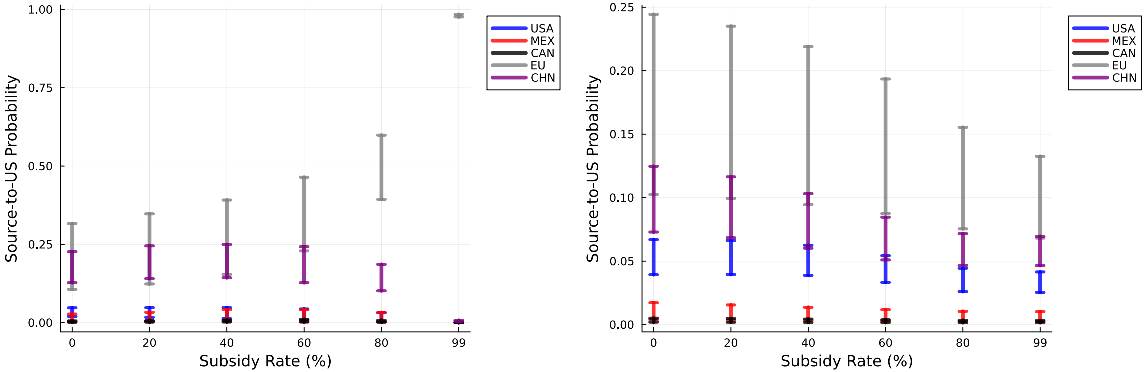


(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Bounds on Levels

Probability of Sourcing to the US

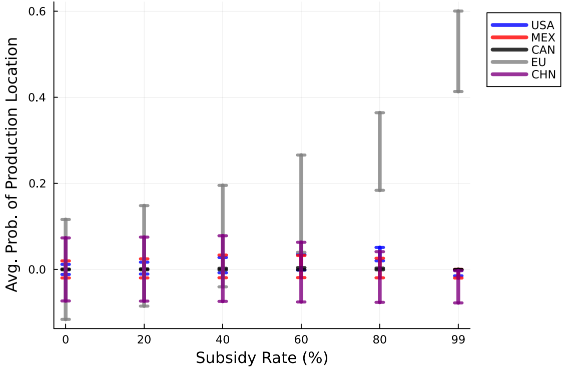


(a) Electric (b) Non-Electric

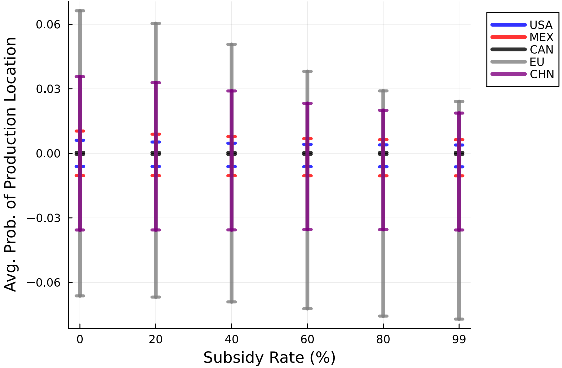
Consumption Subsidy on Electric Products in the US

Bounds on Changes

Production Location



(a) Electric

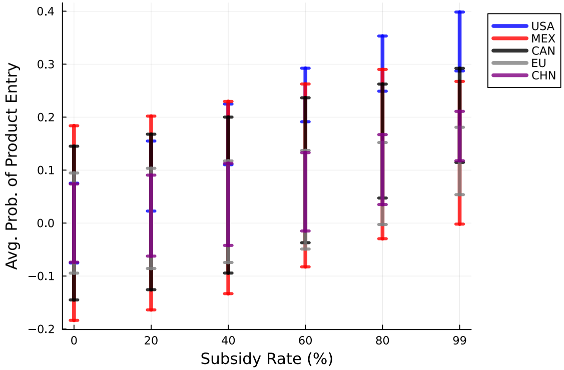


(b) Non-Electric

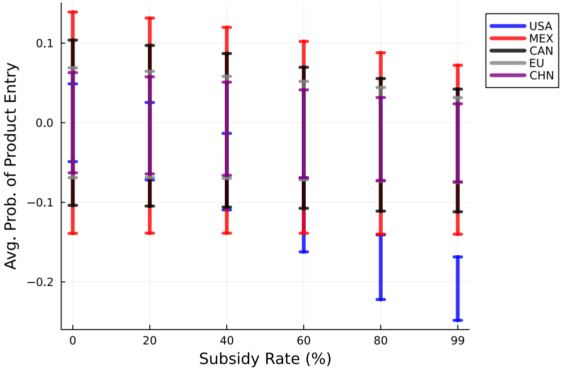
Consumption Subsidy on Electric Products in the US

Bounds on Changes

Product Entry



(a) Electric

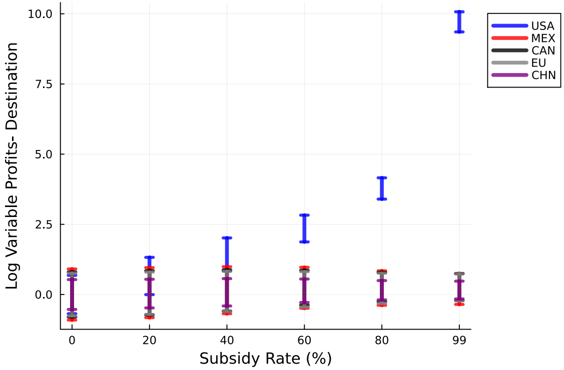


(b) Non-Electric

Consumption Subsidy on Electric Products in the US

Bounds on Changes

Variable Profits - Destination



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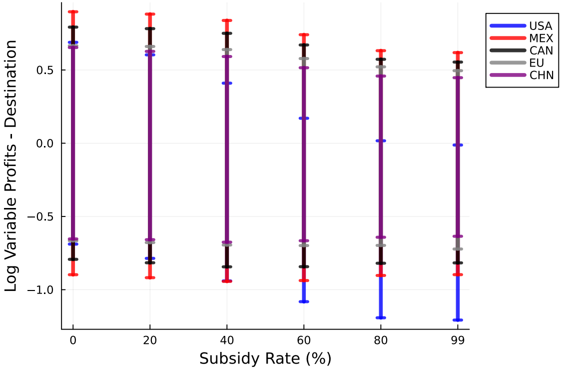
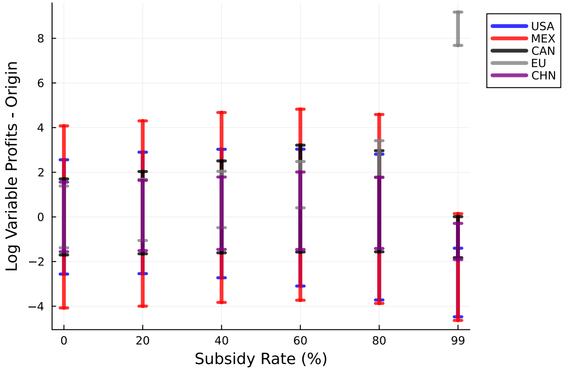


(b) Non-Electric

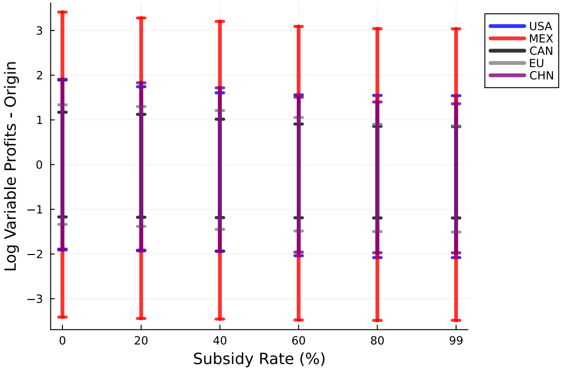
Consumption Subsidy on Electric Products in the US

Bounds on Changes

Variable Profits - Origin



(a) Electric

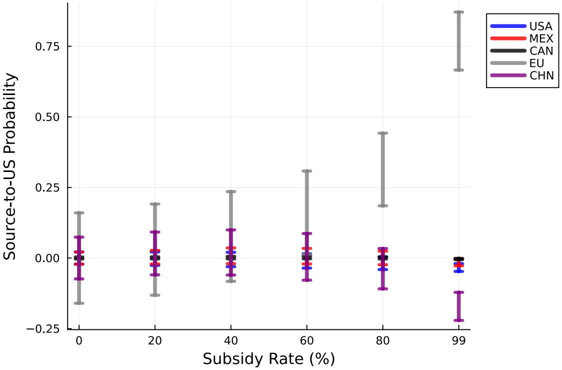


(b) Non-Electric

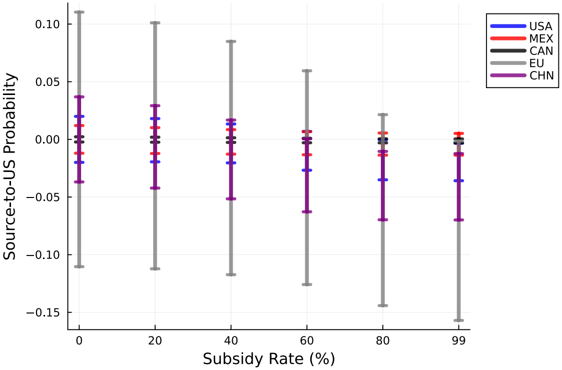
Consumption Subsidy on Electric Products in the US

Bounds on Changes

Probability of Sourcing to the US



(a) Electric

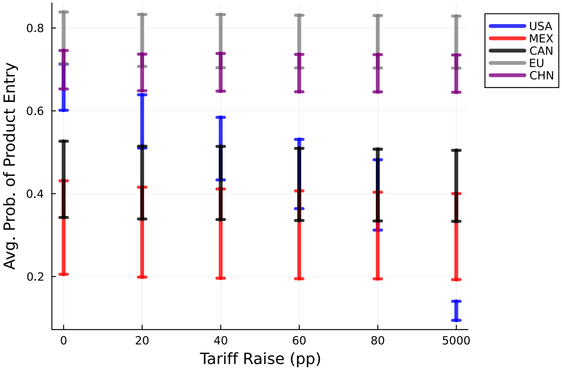


(b) Non-Electric

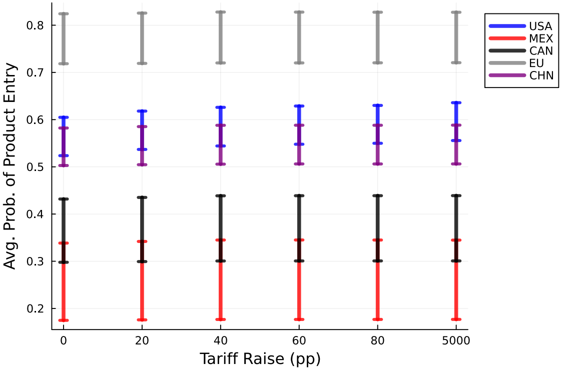
Tariffs on Electric Products Not Produced in the US

Bounds on Levels

Product Entry



(a) Electric

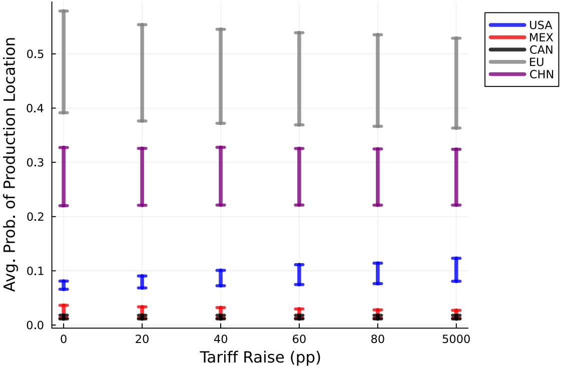


(b) Non-Electric

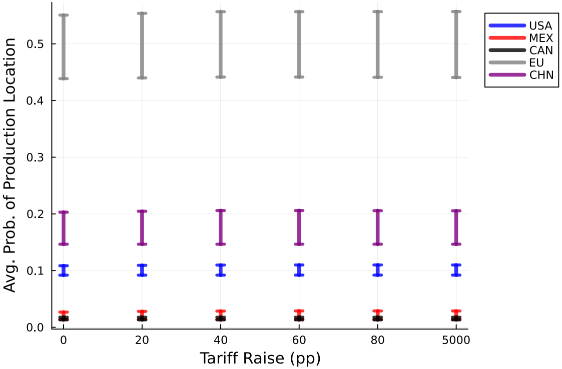
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Production Location



(a) Electric



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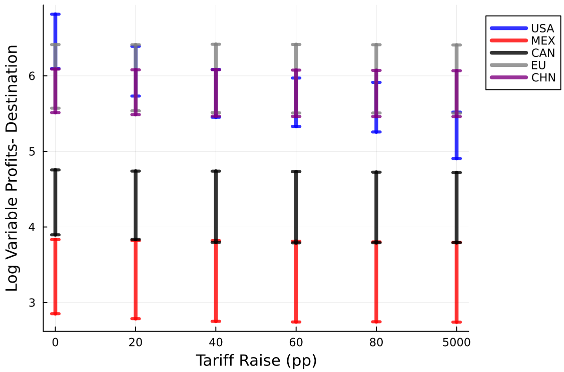
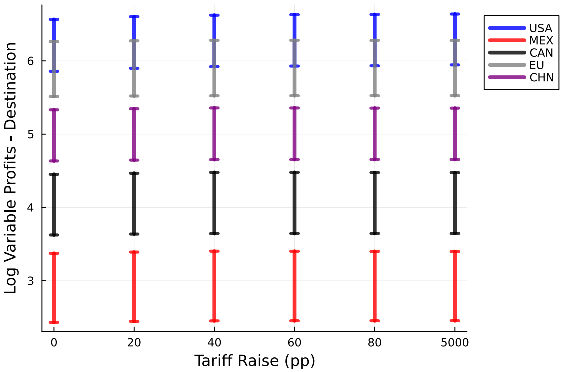
Tariffs on Electric Products Not Produced in the US

Bounds on Levels

Variable Profits - Destination



(a) Electric



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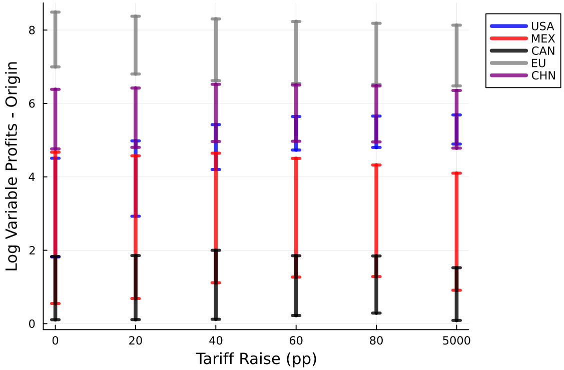
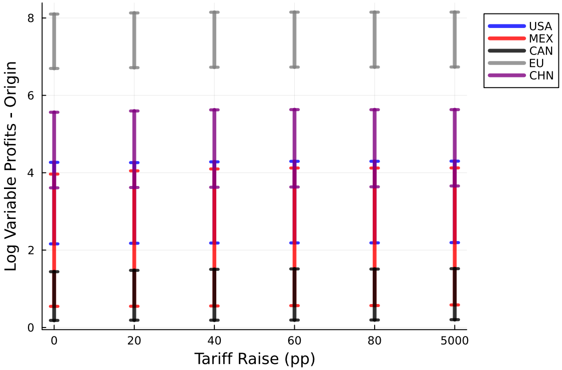
Tariffs on Electric Products Not Produced in the US

Bounds on Levels

Variable Profits - Origin



(a) Electric



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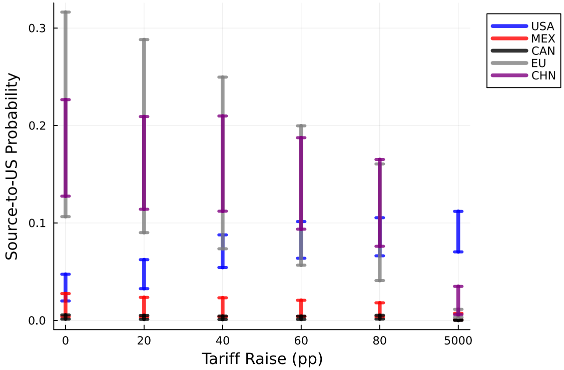
Tariffs on Electric Products Not Produced in the US

Bounds on Levels

Probability of Sourcing to the US



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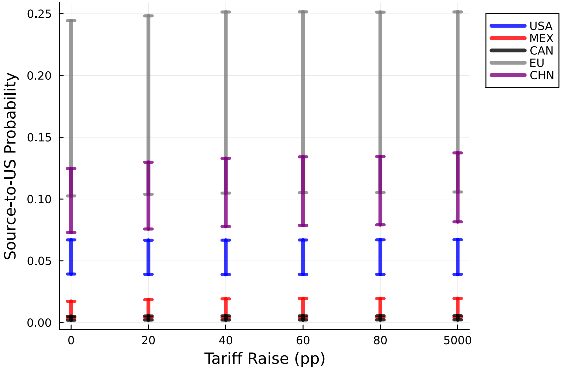
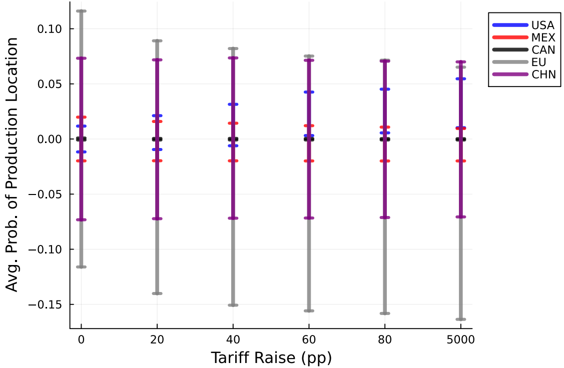


(b) Non-Electric

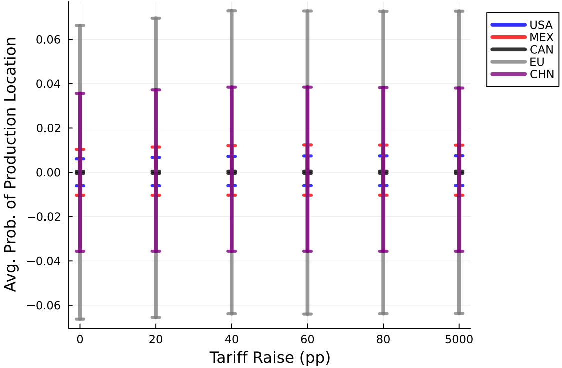
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Production Location



(a) Electric

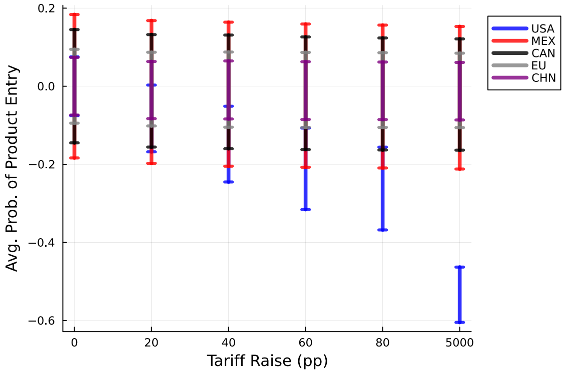


(b) Non-Electric

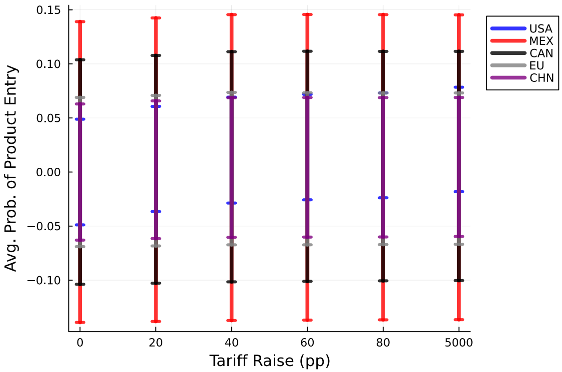
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Product Entry



(a) Electric



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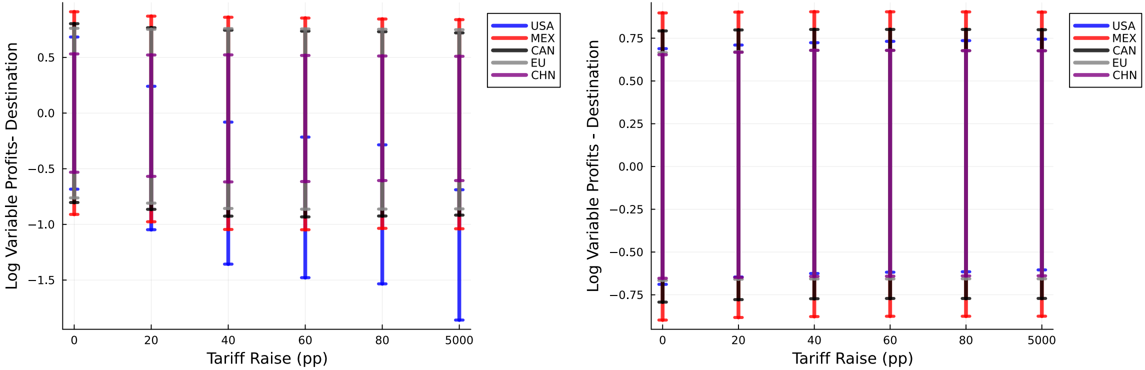
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Variable Profits - Destination

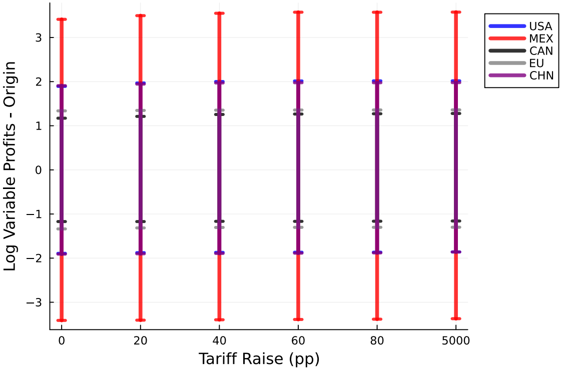


(a) Electric (b) Non-Electric

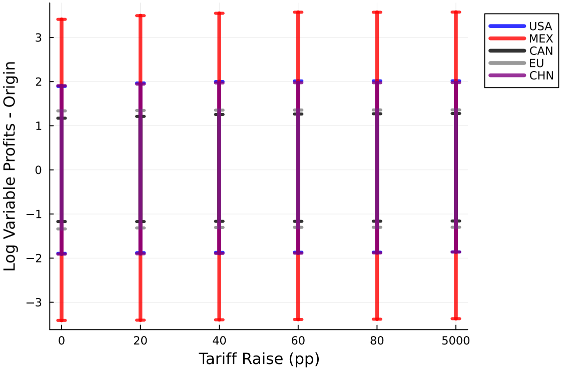
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Variable Profits - Origin



(a) Electric

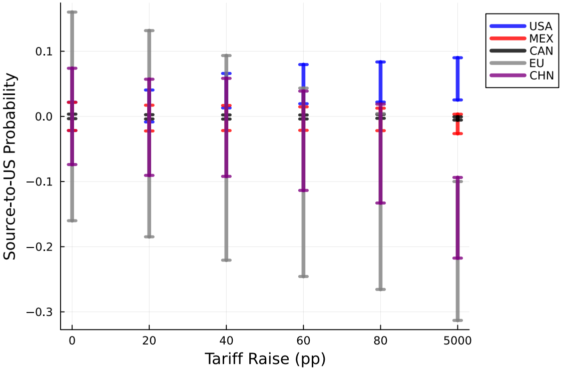


(b) Non-Electric

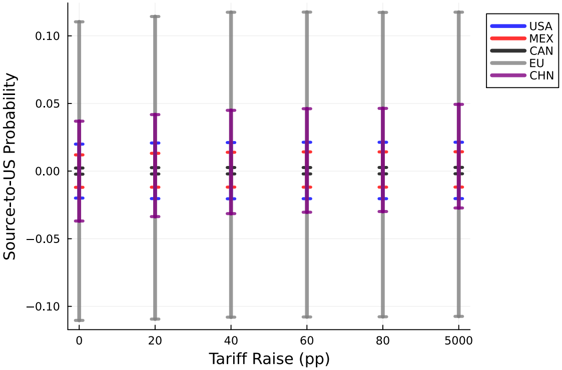
Tariffs on Electric Products Not Produced in the US

Bounds on Changes

Probability of Sourcing to the US



(a) Electric

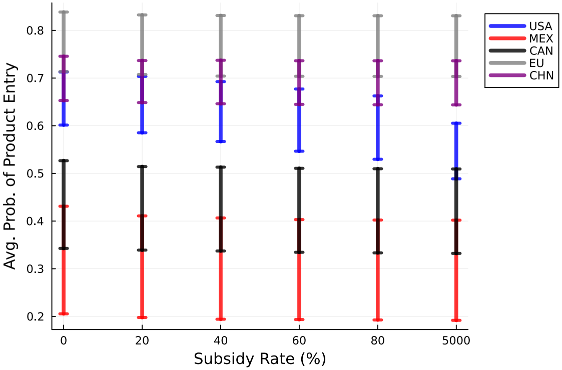


(b) Non-Electric

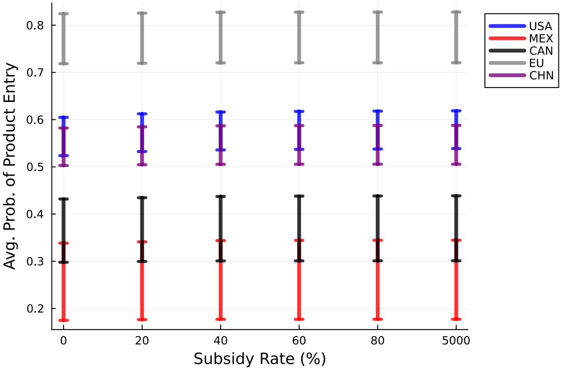
Tariffs on Electric Products Produced in the EU

Bounds on Levels

Product Entry



(a) Electric

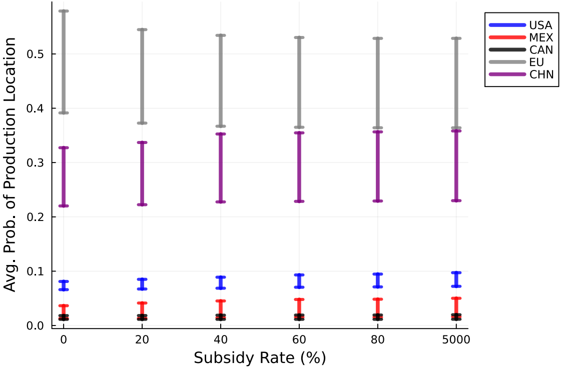


(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Bounds on Changes

Production Location



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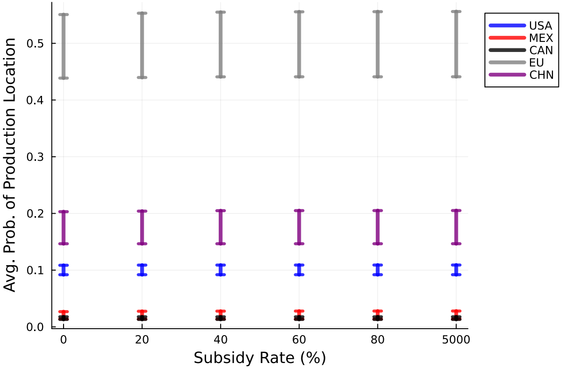
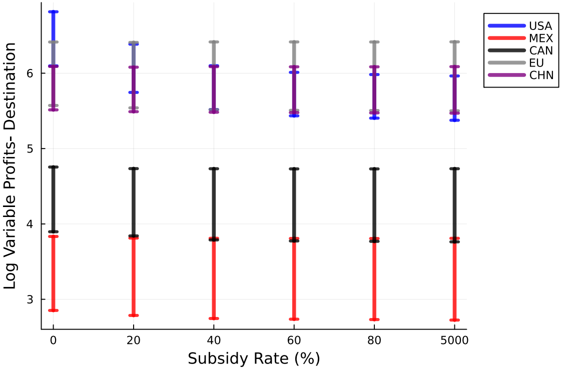


(b) Non-Electric

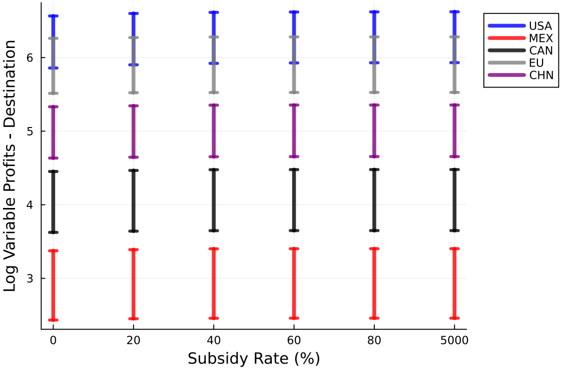
Tariffs on Electric Products Produced in the EU

Bounds on Levels

Variable Profits - Destination



(a) Electric



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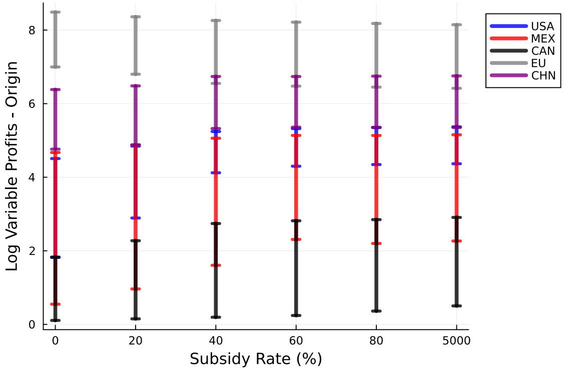
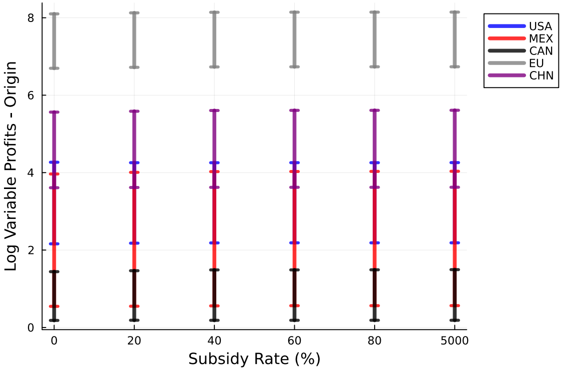
Tariffs on Electric Products Produced in the EU

Bounds on Levels

Variable Profits - Origin



(a) Electric

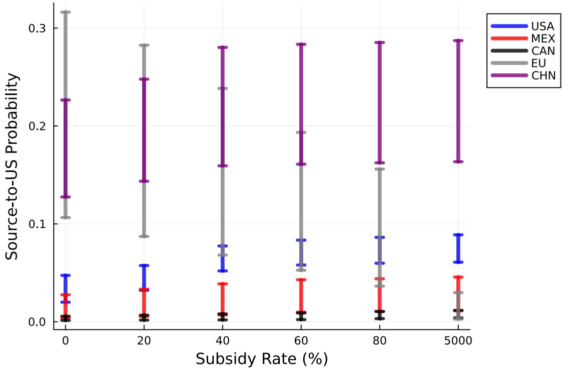


(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Bounds on Levels

Probability of Sourcing to the US



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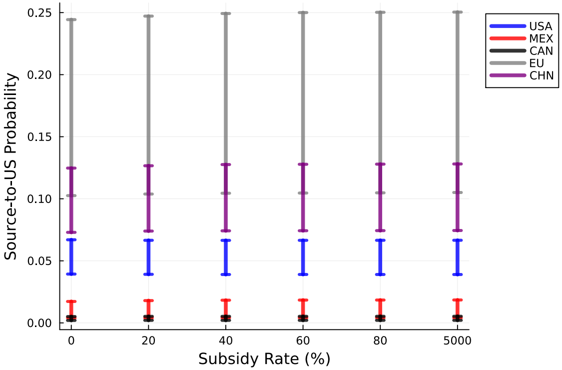
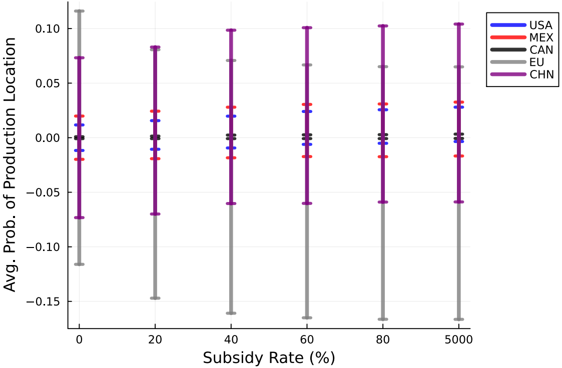


(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Bounds on Changes

Production Location



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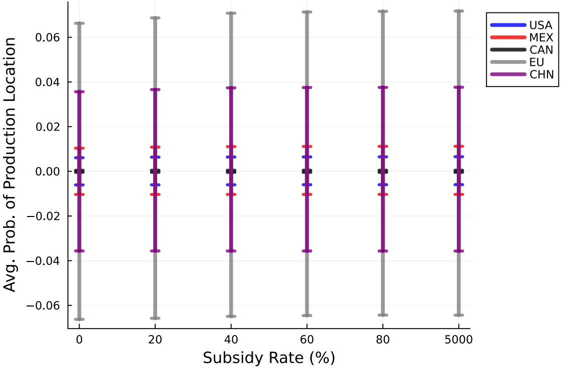
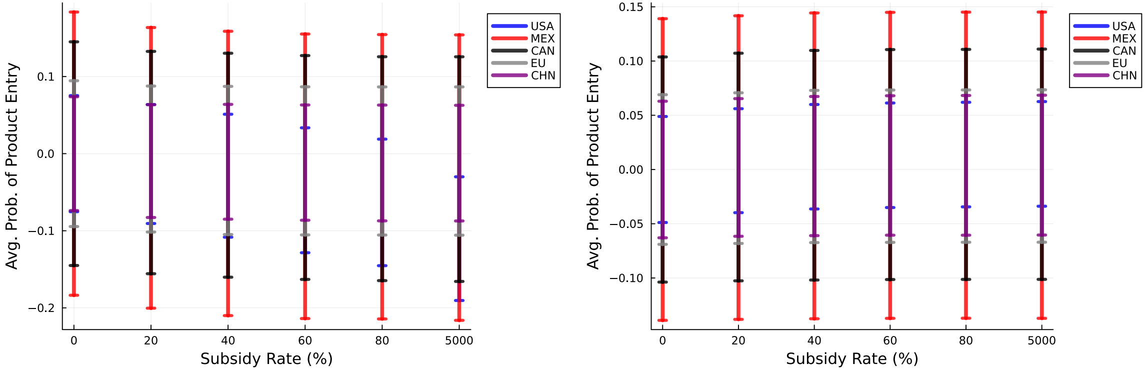


(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Bounds on Changes

Product Entry



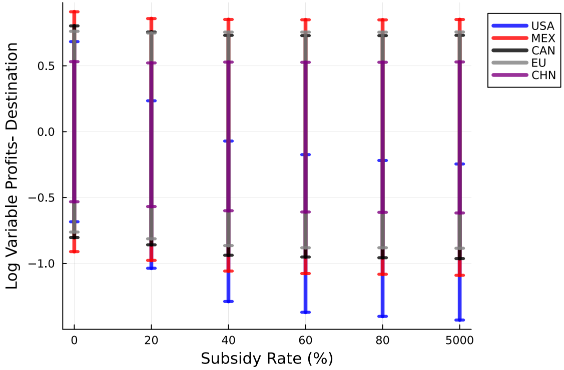
(a) Electric

(b) Non-Electric

Tariffs on Electric Products Produced in the EU

Bounds on Changes

Variable Profits - Destination



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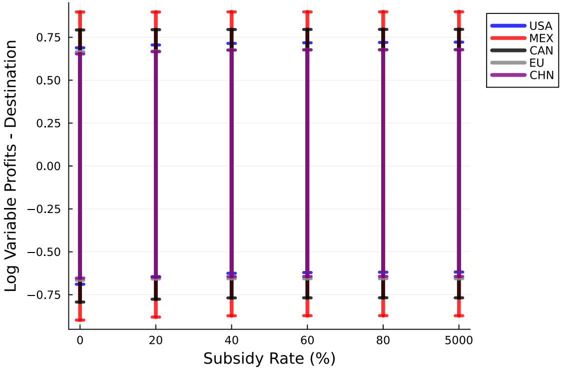

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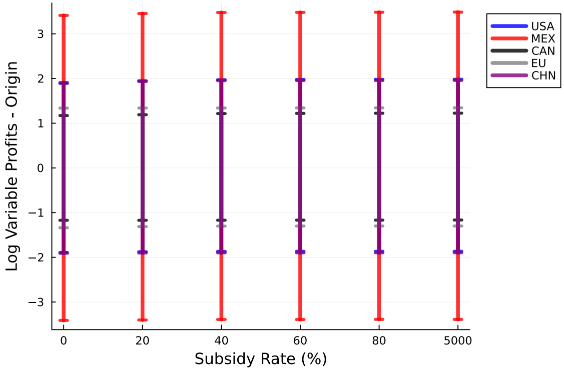
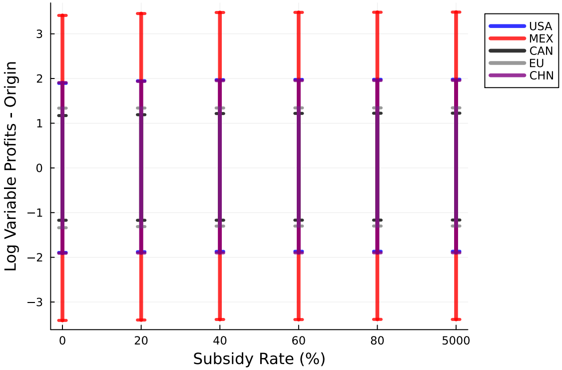
Tariffs on Electric Products Produced in the EU

Bounds on Changes

Variable Profits - Origin



(a) Electric



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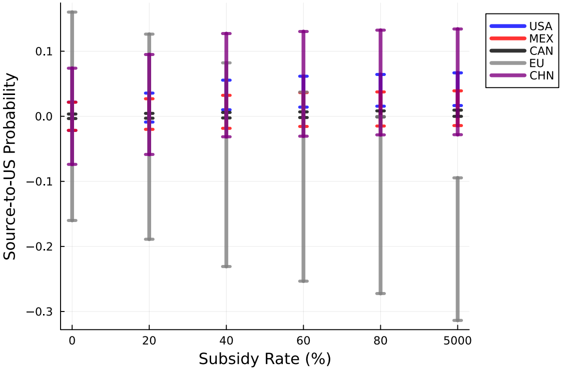
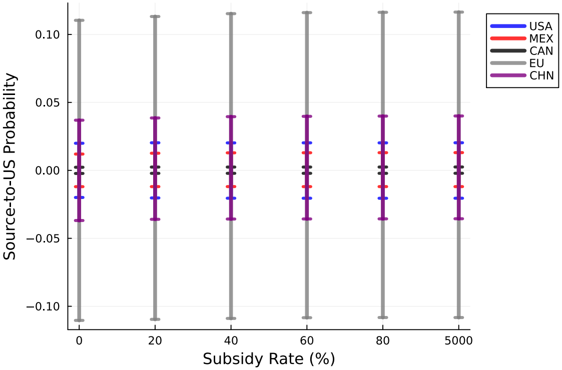
Tariffs on Electric Products Produced in the EU

Bounds on Changes

Probability of Sourcing to the US



(a) Electric



(b) Non-Electric

Conclusion

- Model of multinational and multi-product firms.
- Novel algorithm for combinatorial discrete choice problems with complementarities and substitutabilities.
- Algorithm requires that for any two decisions the sign of the interdependency is known and independent of third choices.
- Showed how to exploit two different types of interdependence to apply the algorithm: interdependencies on the payoff and on the value function.
- Moment inequalities to use algorithm in estimation.
- Evaluate firm-level responses to consumption and production subsidies, and tariffs.