دانشکده مهندسی مکانیک

درس مکانیک کامپوزیت پیشرفته

تمرین:تمرین چهارم

نام و نام خانوادگی: صبا عباس‌زاده منتظری

شماره دانشجویی: ۴۰۳۷۴۳۳۷۶

نام استاد: دکتر محمود مهرداد شکریه

تاریخ: ۱۳ اسفند ۱۴۰۳

Contents

List of Figures

1 Question 1

1. Derive the transformation of stiffness of a unidirectional composite from one off-axis orientation to another.

Following formula is used for the calculations:

$$\{\sigma_i\}_{1',2',6'} = \left[T_\sigma^-\right] \left[Q_{ij}\right]_{1,2,6} \left[T_\varepsilon^+\right] \{\varepsilon_j\}_{1',2',6'} \tag{1}$$

The code below implements the equation above. It calculates the transformation matrix from one off-axis coordinate to another:

```
%%%%%%%%%%%%%%%%%%%
syms Q11 Q12 Q16 Q21 Q22 Q26 Q61 Q62 Q66 m n
Q=[Q11,Q12,Q16;Q12,Q22,Q26;Q16,Q26,Q66];
T_stress_neg=[m^2,n^2,-2*m*n;n^2,m^2,2*m*n;m*n,-m*n,(m^2-n^2)];
T_strain_pos=[m^2,n^2,m*n;n^2,m^2,-m*n;-2*m*n,2*m*n,(m^2-n^2)];
Q_transformed=simplify(T_stress_neg*Q*T_strain_pos)
```

The answer that matlab gave us can be simplified to:

	Q'_{11}	Q'_{22}	Q'_{12}	Q'_{66}	Q'_{16}	Q'_{26}
Q_{11}	m^4	n^4	$2m^2n^2$	$4m^2n^2$	$-4m^3n$	$-4mn^3$
Q_{22}	n^4	m^4	$2m^2n^2$	$4m^2n^2$	$4mn^3$	$4m^3n$
Q_{12}	m^2n^2	m^2n^2	m^4+n^4	$-4m^2n^2$	$2\left(m^3n-mn^3\right)$	$2\left(mn^3-m^3n\right)$
Q_{66}	m^2n^2	m^2n^2	$-2m^2n^2$	$\left(m^2-n^2\right)^2$	$2\left(m^3n-mn^3\right)$	$2\left(mn^3-m^3n\right)$
Q_{16}	m^3n	$-mn^3$	mn^3-m^3n	$2\left(mn^3-m^3n\right)$	$m^4-3m^2n^2$	$3m^2n^2-n^4$
Q_{26}	mn^3	$-m^3n$	m^3n-mn^3	$2\left(m^3n-mn^3\right)$	$3m^2n^2-n^4$	$m^4-3m^2n^2$

2 Question 2

1. Repeat the process from question 1 for the compliance.

$$\{\sigma_i\}_{1',2',6'} = \left[T_\varepsilon^-\right] \left[Q_{ij}\right]_{1,2,6} \left[T_\sigma^+\right] \{\varepsilon_j\}_{1',2',6'} \tag{2}$$

The code below implements the equation above. It calculates the transformation matrix from one off-axis coordinate to another for compliance matrix:

```
%%%%%%%%%%%%%%%%%
```

```
syms S11 S12 S16 S21 S22 S26 S66 m n
Q=[Q11,Q12,Q16;Q12,Q22,Q26;Q16,Q26,Q66];
T_stress_pos=[m^2,n^2,2*m*n;n^2,m^2,-2*m*n;-m*n,m*n,(m^2-n^2)];
T_strain_neg=[m^2,n^2,-m*n;n^2,m^2,m*n;2*m*n,-2*m*n,(m^2-n^2)];
S_transformed=simplify(T_strain_neg*Q*T_stress_pos)
```

The answer of code can be represented as the matrix below:

	S'_{11}	S'_{22}	S'_{12}	S'_{66}	S'_{16}	S'_{26}
S_{11}	m^4	n^4	$2m^2n^2$	m^2n^2	$-2m^3n$	$-2mn^3$
S_{22}	n^4	m^4	$2m^2n^2$	m^2n^2	$2mn^3$	$2m^3n$
S_{12}	m^2n^2	m^2n^2	m^4+n^4	$-m^2n^2$	m^3n-mn^3	mn^3-m^3n
S_{66}	$4m^2n^2$	$4m^2n^2$	$-8m^2n^2$	$(m^2-n^2)^2$	$4(m^3n-mn^3)$	$4(mn^3-m^3n)$
S_{16}	$2m^3n$	$-2mn^3$	$2(mn^3-m^3n)$	mn^3-m^3n	$m^4-3m^2n^2$	$3m^2n^2-n^4$
S_{26}	$2mn^3$	$-2m^3n$	$2(m^3n-mn^3)$	m^3n-mn^3	$3m^2n^2-n^4$	$m^4-3m^2n^2$

3 Question 3

3. Are there invariants associated with the transformed engineering constants? An average Young's modulus can be defined from the area under the transformed Young's modulus; e.g., in Figure 3.17. What is the relation between this and that derived from the transformed compliance (1/U1 in Figure 3.12)?

E1 can be calculated by integerating from 0 to 90 degrees..

$$E1 = Q_{11} - v_{21}Q_{12} + v_{61}Q_{16} \tag{3}$$

By substitution of Q_{ij} in the form of invariants (equation 4) in equation 1, we can rewrite and integral the mean modulus as:

	1	R_1	R_2
Q_{11}	U_1	$\cos 2\theta$	$\cos 4\theta$
Q_{22}	U_1	$-\cos 2\theta$	$\cos 4\theta$
Q_{12}	U_4	0	$-\cos 4\theta$
Q_{66}	U_5	0	$-\cos 4\theta$
Q_{16}	0	$\frac{1}{2}\sin 2\theta$	$\sin 4\theta$
Q_{26}	0	$\frac{1}{2}\sin 2\theta$	$-\sin 4\theta$

(4)

Therefore:

$$\bar{E}_1\frac{\pi}{2} = \int_0^{\pi/2} (U_1 + U_2\cos(2\theta) + U_3\cos(4\theta))\,d\theta$$
$$- \bar{v}_{21}\int_0^{\pi/2} (U_4 - U_3\cos(4\theta))\,d\theta \tag{5}$$
$$+ \bar{v}_{61}\int_0^{\pi/2} (0.5U_2\sin(2\theta) + U_3\sin(4\theta))\,d\theta$$

$$\bar{E}_1 = U_1 - \bar{v}_{21}U_4 + \frac{2}{\pi}\bar{v}_{61}U_2 \tag{6}$$

The same way E2 is calculated:

$$E2 = Q_{22} - v_{12}Q_{21} + v_{62}Q_{26} \tag{7}$$

$$\bar{E}_2\frac{\pi}{2} = \int_0^{\pi/2} (U_1 - U_2\cos(2\theta) + U_3\cos(4\theta))\,d\theta$$
$$- \bar{v}_{12}\int_0^{\pi/2} (U_4 - U_3\cos(4\theta))\,d\theta \tag{8}$$
$$+ \bar{v}_{62}\int_0^{\pi/2} (0.5U_2\sin(2\theta) - U_3\sin(4\theta))\,d\theta$$

E2 can be simplified to:

$$\bar{E}_2 = U_1 - \bar{v}_{12}U_4 + \frac{2}{\pi}\bar{v}_{62}U_2 \tag{9}$$

For mean of E6:

$$E6 = Q_{66} + v_{16}Q_{61} + v_{26}Q_{62} \tag{10}$$

By integration:

$$\bar{E}_6 = U_5 + \frac{2}{\pi}(\bar{v}_{16} + \bar{v}_{26})U_2 \tag{11}$$

For comparing this to the moduli extracted from compliance, we should perform the steps below. We invariants of compliance by V_i to avoid confusion from the invariants of stiffness matrix.

$$\varepsilon_1 = \sigma_1 S_{11} = \frac{\sigma_1}{E_1} \tag{12}$$

We substitute invariants from the matrix below. In this question we use V instead of U to avoid confusion.

$$\begin{array}{c|ccc} & 1 & R_1 & R_2 \\ \hline S_{11} & V_1 & -\cos 2\theta & -\cos 4\theta \\ S_{22} & V_1 & \cos 2\theta & -\cos 4\theta \\ S_{12} & V_4 & & \cos 4\theta \\ S_{66} & V_5 & & 4\cos 4\theta \\ S_{16} & & -\sin 2\theta & -2\sin 4\theta \\ S_{26} & & -\sin 2\theta & 2\sin 4\theta \end{array} \tag{13}$$

From integration of S_{11} in invariant form:

$$\bar{E}_1 = \frac{1}{V_1} \tag{14}$$

And the same way:

$$\bar{E}_2 = \frac{1}{V_1} \tag{15}$$

$$\bar{E}_6 = \frac{1}{V_5} \tag{16}$$

In Which v_{ij}can be calculated in the form of compliance invariants:

Coupling coefficients:

$$v_{21} = -\frac{S_{21}}{S_{11}} \tag{17}$$

$$\bar{v}_{21} = \frac{V_4}{V_1} \tag{18}$$

$$v_{61} = \frac{S_{61}}{S_{11}} \tag{19}$$

$$\bar{v}_{61} = -\frac{V_2}{V_1} \tag{20}$$

And the same way:

$$\bar{v}_{16} = -\frac{V_2}{V_5} \tag{21}$$

$$\bar{v}_{12} = \frac{V_4}{V_1} \tag{22}$$

$$\bar{v}_{26} = \frac{V_2}{V_5} \tag{23}$$

$$\bar{v}_{62} = \frac{V_2}{V_1} \tag{24}$$

By using these equations we can derive the relation between the invariants. e.g:

$$U_1 - \bar{v}_{21}U_4 + \frac{2}{\pi}\bar{v}_{61}U_2 = \frac{1}{V_1} \tag{25}$$

4 Question 4

4. What difficulties are involved for testing of an off-axis unidirectional composite? What is the difference in response between the tubular and flat specimens with off-axis ply orientation? Examine the cases of uniaxial extension, pure shear, and hydrostatic pressure. What kind of stresses are induced in the load introduction points (the ends)? What load and displacement controls are desired for these tests?

Difficulties:

The reason lies behind the coupling effects. Due to the complexities of dealing with shear and normal stress interactions, off-axis testing often requires more advanced setups (strain gauges) and interpretations.

Differences in resonse:

First: Tubular component can't be considered 2D and the stiffness matrix becomes more complicated and needs more tests.

Second: Deriving stiffness or compliance for tabular specimen is possible in cylindrical coordinate and converting that to cartesian for flat usages can be hard.

Uniaxial: With Uniaxial extension we can derieve E1 and two coupling coefficients with rosette strain gauge (if we consider tubular specimen 2D).

shear and torsion: With Uniaxial extension we can derieve E6 and two coupling coefficients with rosette strain gauge (if we consider tubular specimen 2D).

Hydrostatic: In tubular we derive E_r which is out of plane. In flat specimen it gives us E2 and coupling coefficients.

Result:Tests are not enough for tubular specimen but it is enough for flat one.

At the ends comperessive stresses are concentrated which we try to reduce by special fixtures and end-tabs.

Displacement control or load control:

In these tests, shear and extension, load control is desired since we derive compliance and compliance is easier for deriving off-axis constants.

But being load control and displacement control also depends on the type of composite.

5 Question 5

2.Please interpret the following behaviors.

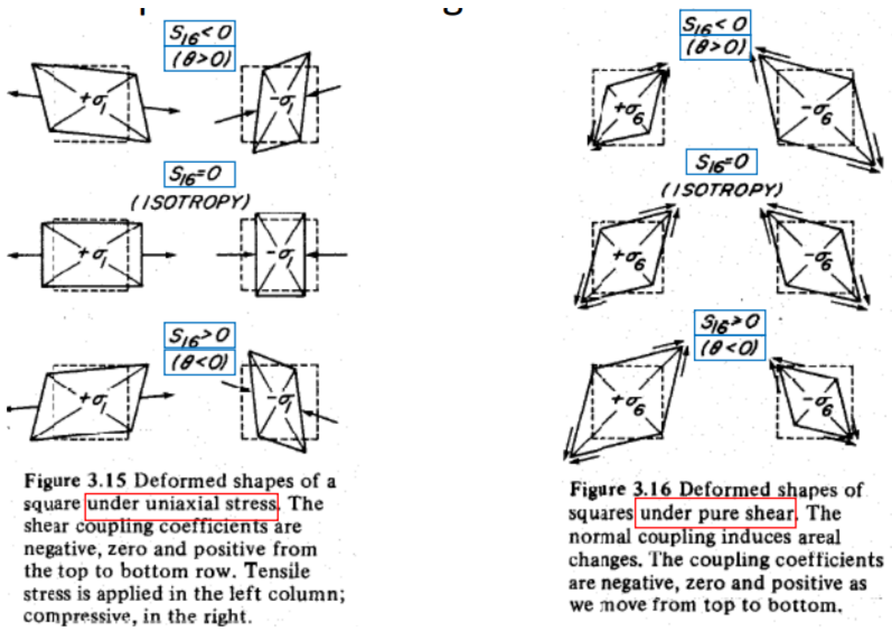

Figure 1: Schematic of problem

Left Part:

$$
\begin{array}{c|ccc}
 & \sigma_1 & \sigma_2 & \sigma_6 \\
\hline
\epsilon_1 & \frac{1}{E_1} & -\frac{\nu_{21}}{E_1} & \frac{\nu_{61}}{E_1} \\
\epsilon_2 & -\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & \frac{\nu_{62}}{E_2} \\
\epsilon_6 & \frac{\nu_{16}}{E_6} & \frac{\nu_{26}}{E_6} & \frac{1}{E_6}
\end{array}
\tag{26}
$$

when $S_{16} = S_{61} < 0$ with $\sigma_1 > 0$, the shearing strain $\varepsilon_6 < 0$, therefore the left side goes up and right side goes down and expands in direction-1.

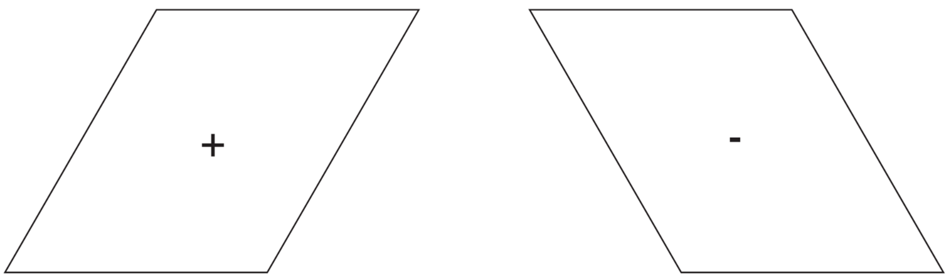

Figure 2: Shear sign

when $S_{16} = S_{61} = 0$ with $\sigma_1 > 0$, the shearing strain $\varepsilon_6 = 0$, the element wont distort and just expands in direction-1. when $S_{16} = S_{61} > 0$ with $\sigma_1 > 0$, the shearing strain $\varepsilon_6 > 0$, the element will distort. left side goes down and right side goes up, and expands in direction-1.

These processes happen vice versa for the stress with the opposite sign.

Right side:

For positive shear loadings in left side:// when $S_{16} = S_{61} < 0$ with $\sigma_6 > 0$, the shearing strain will be $\varepsilon_6 > 0$,and expands in contracts direction-1.

when $S_{16} = S_{61} = 0$ with $\sigma_6 > 0$, the shearing strain will be $\varepsilon_6 > 0$,and it won't change in direction-1.

when $S_{16} = S_{61} > 0$ with $\sigma_6 > 0$, the shearing strain will be $\varepsilon_6 > 0$,and it will expand in direction-1.

These processes happen vice versa for the shearing stress with the opposite sign.

6 Question 6

6. Write a computer code to calculate the engineering constants of an off-axis UD ply made of T300/5208 composites. Run your code for q = 45 degrees?

This code is written by on-axis compliance matrix and transformation matrices for deriving off-axis matrix. At the end off-axis constants are calculated.

```
load('Transformations')
load('compliance')
syms teta
%%properties of the composite
T300_5208=struct('Ex',181,...
    'Ey',10.3,...
    'Vx',0.28,...
    'Es',7.17);
% Compute Vy dynamically and add it to the structure. It is computed by
% supposing symmetricity.
T300_5208.Vy = T300_5208.Vx * (T300_5208.Ey / T300_5208.Ex);
%%Calculation of on-axis stiffness for T300_5208
S_T300_5208=subs(S,fieldnames(T300_5208), struct2cell(T300_5208));
%%Calculation of Positive stress transformation matrix for 45 degree
TP=subs(T_stress_positive,[teta],deg2rad(45));
```

```
%%Calculation of Negative strain transformation matrix for 45 degree
TN=subs(T_strain_negative,[teta],deg2rad(45));
%%Calculation of off-axis compliance for T300_5208
S_off=TN*S_T300_5208*TP,4;
% Create a structure
Constants_off = struct(...
    'E1', double(1/S_off(1,1)), ...
    'v21',double(-S_off(2,1)/S_off(1,1)), ...
    'v61',double(-S_off(3,1)/S_off(1,1)), ...
    'E2', double(1/S_off(2,2)), ...
    'v12',double(-S_off(1,2)/S_off(2,2)), ...
    'v62',double(S_off(3,2)/S_off(2,2)), ...
    'E6', double(1/S_off(3,3)),...
    'v16',double(S_off(1,3)/S_off(3,3)), ...
    'v26',double(S_off(2,3)/S_off(3,3)) ...
)
save('Constants_off.mat','Constants_off')
```

Off-axis constants:

$$\begin{Bmatrix} E_1 \\ v_{21} \\ v_{61} \\ E_2 \\ v_{12} \\ v_{62} \\ E_6 \\ v_{16} \\ v_{26} \end{Bmatrix} = \begin{Bmatrix} 16.737 \\ 0.167 \\ 0.766 \\ 16.737 \\ 0.167 \\ -0.766 \\ 9.460 \\ -0.433 \\ -0.433 \end{Bmatrix} \tag{27}$$

The answers are in total compliance with the table below.