دانشکده مهندسی مکانیک

درس مکانیک کامپوزیت پیشرفته

تمرین:تمرین اول

نام و نام خانوادگی: صبا عباس‌زاده منتظری

شماره دانشجویی: ۴۰۳۷۴۳۳۷۶

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تاریخ: ۵ اسفند ۱۴۰۳

1 Question 1

1.Prove C and S matrices are symmetric.
We should start by definition of W (strain energy density function) which is defined in elasticity problems.

$$\sigma_{ij}{=}\frac{\partial W}{\partial \varepsilon_{ij}} \tag{1}$$

The stiffness matrix can be defined like the equation below according to the constitutive law. We also substitute the equation above to the stress equation.

$$C_{ijkl}{=}\frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}}{=}\frac{\partial^2 W}{\partial \varepsilon_{ij}\partial \varepsilon_{kl}} \tag{2}$$

Since W is scaler and is twice differentiable and is sufficiently smooth, the mixed partial derivatives are equal:

$$\frac{\partial^2 W}{\partial \varepsilon_{ij}\partial \varepsilon_{kl}}{=}\frac{\partial^2 W}{\partial \varepsilon_{kl}\partial \varepsilon_{ij}} \tag{3}$$

So according to the equation above, the stiffness matrix is symmetrical.

$$C_{ijkl} = C_{klij} \tag{4}$$

2 Question 2

2. Prove which components of the C matrix are zero for monoclinic, orthotropic, transversely isotropic, and isotropic materials.

Monoclinic: Consider plane of symmetry is normal to direction 3:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

consider strain and stress under rotation:

$$\sigma\prime = R\sigma R^T \tag{5}$$

$$\varepsilon\prime = R\varepsilon R^T \tag{6}$$

If we express the constitutive law in voigt notation:

$$\begin{bmatrix} \sigma_{1'} \\ \sigma_{2'} \\ \sigma_{3'} \\ \tau_{2'3'} \\ \tau_{3'\mathrm{r}} \\ \tau_{1'} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1'} \\ \varepsilon_{2'} \\ \varepsilon_{3'} \\ \gamma_{23'} \\ \gamma_{3'} \\ \gamma_{12'} \end{bmatrix}$$

Applying equations 5 and 6, the formula below can be calculated:

$$\begin{gathered}\sigma_1{=}\sigma_{1'}, \sigma_2{=}\sigma_{2'}, \sigma_3{=}\sigma_{3'}\\ \tau_{23}{=}-\tau_{2'3'}, \tau_{31}{=}-\tau_{3'1'}, \tau_{12}{=}\tau_{1'2'}\\ \varepsilon_1{=}\varepsilon_{1'}, \varepsilon_2{=}\varepsilon_{2'}, \varepsilon_3{=}\varepsilon_{3'}\\ \gamma_{23}{=}-\gamma_{2'3'}, \gamma_{31}{=}-\gamma_{3'1'}, \gamma_{12}{=}\gamma_{1'2'}\end{gathered} \tag{7}$$

We Know that stresses can be calculated in transfomed and untransformed coordinate by using their constitutive law:

$$\begin{gathered}\sigma_1{=}C_{11}\varepsilon_1{+}C_{12}\varepsilon_2{+}C_{13}\varepsilon_3{+}C_{14}\gamma_{23}{+}C_{15}\gamma_{31}{+}C_{16}\gamma_{12}\\ \sigma_{1'}{=}C_{11}\varepsilon_{1'}{+}C_{12}\varepsilon_{2'}{+}C_{13}\varepsilon_{3'}{+}C_{14}\gamma_{23'}{+}C_{15}\gamma_{3'1'}{+}C_{16}\gamma_{1'2'}\end{gathered} \tag{8}$$

By applying the conditions of 5 and 6 in the equations above, the formula below can be calculated:

$$\sigma_1{=}C_{11}\varepsilon_1{+}C_{12}\varepsilon_2{+}C_{13}\varepsilon_3{-}C_{14}\gamma_{23}{-}C_{15}\gamma_{31}{+}C_{16}\gamma_{12} \tag{9}$$

By comparing the equations 9 and first equation of 8, It can be substatiated:

$$0 = 2C_{14}\gamma_{23}{+}2C_{15}\gamma_{31} \tag{10}$$

So C_{14}, C_{15} are zero.
By doing the same for the next stresses we conclude: C_{24}, C_{25}, C_{34}, C_{35}, C_{46}, $C_{56} = 0$
And By the symmetry of stiffness matrix, the matrix reduces to:

$$\begin{bmatrix}\sigma_{1'}\\ \sigma_{2'}\\ \sigma_{3'}\\ \tau_{2'3'}\\ \tau_{31'}\\ \tau_{12'}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16}\\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26}\\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44} & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{55} & 0\\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{1'}\\ \varepsilon_{2'}\\ \varepsilon_{3'}\\ \gamma_{23'}\\ \gamma_{31'}\\ \gamma_{12'}\end{bmatrix} \tag{11}$$

Orthotropic:
By applying another transformation matrix, since we know that Orthotropic materials are symmetric to two plane too, we apply another Q matrix(1-2,1-3 and 2-3) : If we apply the following Q to the result of monoclinic analysis we can determine zero terms:

$$Q = \begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Due to this matrix:

$$\tau_{5'} = -\tau_6, \tau_{6'} = -\tau_6$$

Therefore:

$$\begin{gathered}\tau_4{=}C_{44}\varepsilon_4 + C_{45}\varepsilon_5\\ \tau_{4'}{=}C_{44}\varepsilon_4 - C_{45}\varepsilon_5\end{gathered} \tag{12}$$

So

$$C_{45} = 0$$

Also:

$$\begin{gathered}\tau_6{=}C_{16}\varepsilon_1 + C_{26}\varepsilon_2 + C_{36}\varepsilon_3\\ \tau_{6'}{=}-C_{16}\varepsilon_1 - C_{26}\varepsilon_2 - C_{36}\varepsilon_3\end{gathered} \tag{13}$$

So

$$C_{16}, C_{26}, C_{36} = 0$$

The stiffness matrix reduces to the equation below.

$$\begin{bmatrix} \sigma_{1'} \\ \sigma_{2'} \\ \sigma_{3'} \\ \tau_{4'} \\ \tau_{5'} \\ \tau_{6'} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1'} \\ \varepsilon_{2'} \\ \varepsilon_{3'} \\ \gamma_{4'} \\ \gamma_{5'} \\ \gamma_{6'} \end{bmatrix} \tag{14}$$

Transversely isotropic:
A transversely isotropic material is a type of material that has one plane of symmetry but behaves isotropically in all directions within that plane. This means the material exhibits the same properties in all directions within the plane. So the zero terms are the same as orthotopic but since it is isotropic in 2-3 plane isotropic symmetry:

$$C_{22} = C_{33}, C_{12} = C_{13}, C_{55} = C_{66}$$

$$\begin{bmatrix} \sigma_{1'} \\ \sigma_{2'} \\ \sigma_{3'} \\ \tau_{4'} \\ \tau_{5'} \\ \tau_{6'} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & (C_{22} - C_{23})/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1'} \\ \varepsilon_{2'} \\ \varepsilon_{3'} \\ \gamma_{4'} \\ \gamma_{66'} \\ \gamma_{6'} \end{bmatrix} \tag{15}$$

Isotropic: In isotropic material zero terms are the same but Cs are related to each other since it shows the same behavior in all axis.

3 Question 3

3 In 2-D plane stress conditions, how many independent constants are exist for monoclinic, orthotropic, transversely isotropic, and isotropic materials?
Monoclinic: In 2D the 13 independents reduce to 6 since it has

$$C_{11}, C_{12}, C_{16}, C_{22}, C_{26}, C_{66}$$

Orthotropic: In 2D the 9 independents reduce to 4 since it has

$$C_{11}, C_{12}, C_{22}, C_{66}$$

Transversely isotropic: In 2D the 5 independents reduce to 3. We know that in transversely isotropic:

$$C_{22} = C_{66}$$

So the 4 independents of orthotopic reduces to 3.
Isotropic: the independents of isotropic material in 3D and 2D are the same.

$$C_{22}, C_{66}$$