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1 Question 1

1. **Prove that:**

$$-1 < \overset{*}{V}_1 < 1$$

At first, we should mention the equation of $\overset{*}{V}_1$:

$$\overset{*}{V}_1 = \frac{V_1}{h} = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \cos 2\theta \, dz \tag{1}$$

With consideration of symmetry of composite, the equation reduces to:

$$\overset{*}{V}_1 = \frac{V_1}{h} = \frac{2}{h}\int_{0}^{\frac{h}{2}} \cos 2\theta \, dz \tag{2}$$

The integral can be replaced to summation since composite plies are discontinous:

$$\overset{*}{V}_1 = = \frac{2}{h}\sum_{i=1}^{m/2} \cos 2\theta_i h_i \tag{3}$$

m is defined as number of plies and if we take v as volume fraction of plies with specific angle $v_i = \frac{2h_i}{h}$:

$$\overset{*}{V}_1 = = \sum_{i=1}^{m/2} \cos 2\theta_i v_i \tag{4}$$

We know that summation of the fractions should be equal to 1, since it makes the whole composite:

$$v_1 + v_2 + v_3 + ... = 1 \tag{5}$$

We know that maximum of $cos2\theta_i$ is 1 and minimum of it is equal to -1. So maximum of $\cos 2\theta_i v_i$ is equal to 1 and minimum of it is equal to -1. Therefore:

$$-1 < \overset{*}{V}_1 < 1$$

2 Question 2

2. **Prove that:**

$$A_{11} + A_{22} + 2A_{12} = 2(U_1 + U_4)h$$

$$A_{12} = (U_1 + U_4)h - \tfrac{1}{2}(A_{11} + A_{22})$$

$$A_{66} = A_{12} + (U_5 - U_4)h$$

By integeration of Q_{ij} following matrix can be obtained:

$$\begin{array}{c|c} & \begin{array}{ccc} h & U_2 & U_3 \end{array} \\ \hline \begin{array}{c} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \\ A_{16} \\ A_{26} \end{array} & \left[\begin{array}{ccc} U_1 & V_1 & V_2 \\ U_1 & -V_1 & V_2 \\ U_4 & & -V_2 \\ U_5 & & -V_2 \\ & \frac{1}{2}V_3 & V_4 \\ & \frac{1}{2}V_3 & -V_4 \end{array}\right] \end{array} \tag{7}$$

So these equations can be proved:

$$A_{11} + A_{22} + 2A_{12} = hU_1 + U_2V_1 + U_3V_2 + hU_1 - U_2V_1 + U_3V_2 + 2hU_4 - 2U_3V_2 \tag{8}$$

This reduces to:

$$A_{11} + A_{22} + 2A_{12} = 2(U_1 + U_4)h \tag{9}$$

Or in another way:

$$A_{12} = (U_1 + U_4)h - \frac{1}{2}(A_{11} + A_{22}) \tag{10}$$

By using the equation 1:

$$A_{66} = hU_5 - U_3V_2 \tag{11}$$

$$A_{12} + (U_5 - U_4)h = hU_4 - U_3V_2 + (U_5 - U_4)h = hU_5 - U_3V_2 \tag{12}$$

We can substantiate the equation below from equations 5 and 6:

$$A_{66} = A_{12} + (U_5 - U_4)h \tag{13}$$