This task focuses on the analytical and numerical solution of a second-order linear ordinary differential equation (ODE) of the form: ax” + bx’ + cx = d

The ODE is first solved analytically to obtain an exact reference solution. It is then reformulated as a system of first-order equations and solved numerically using three integration methods: Explicit Euler, Implicit Euler, and the fourth-order Runge–Kutta method. The numerical results are compared against the analytical solution to evaluate accuracy.

SOLVING THE DIFFERENTIAL EQUATION

$$ODE:\ \mathrm{ax}'' + \mathrm{bx}' + \mathrm{cx} = \mathrm{d} \Leftrightarrow \mathrm{x}'' + \frac{\mathrm{b}}{\mathrm{a}}\mathrm{x}' + \frac{\mathrm{c}}{\mathrm{a}}\mathrm{x} = \frac{\mathrm{d}}{\mathrm{a}}$$

coefficients: a= 1.94; b= -5.02; c= 3.93; d= -8.12

ODE becomes: x” -2.5876x’ +2.0257x = -4.1855

let a= 1   b= -2.5876   c= 2.0257   d= -4.1855

characteristic equation:

$r^2 - 2.5876r + 2.0257 = 0$

$$\Delta = b^2 - 4ac = (-2.5876)^2 - 4 * 1 * 2.0257 = -1.407 < 0$$

$$r_1 = \delta + \omega i = \frac{2.5876}{2} + \frac{i\sqrt{1.407}}{2} = 1.2938 + 0.5930i$$

$$r_2 = \delta - \omega i = \frac{2.5876}{2} - \frac{i\sqrt{1.407}}{2} = 1.2938 - 0.5930i$$

homogeneous solution:

$$x_h(t) = e^{\delta t}\left(C_1 * \cos(\omega t) + C_2 * \sin(\omega t)\right) = e^{1.2938t}\left(C_1 * \cos(0.593t) + C_2 * \sin(0.593t)\right)$$

particular solution:

$$x_p(t) = K = cst \Rightarrow x_p'(t) = 0\ ;\ x_p''(t) = 0$$

$$\Rightarrow 0 - 2.5876 * 0 + 2.0257K = -4.1855$$

$$K = \frac{-4.1855}{2.0257} = \frac{d}{c} = -2.066$$

general solution:

$$x(t) = x_h(t) + x_p(t) = e^{1.2938t}\left(C_1 * \cos(0.593t) + C_2 * \sin(0.593t)\right) - 2.066$$

initial conditions (t = 0   x(0) = 0.1   x'(0) = 0.0):

$$x(0) = 0.1 \Rightarrow 0.1 = e^{1.2938*0}\left(C_1 * \cos(0.593*0) + C_2 * \sin(0.593*0)\right) - 2.066$$

$$0.1 = C_1 - 2.066 \Rightarrow C_1 = 0.1 + 2.066 = x(0) - x_p = 2.166$$

$$x'(0) = 0.0 \Rightarrow 0.0 = 1.2938 * C_1 + 0.593 * C_2$$

$$\Rightarrow C_2 = -\frac{1.2938}{0.593} \times C_1 = -\frac{\delta}{\omega} \times C_1$$

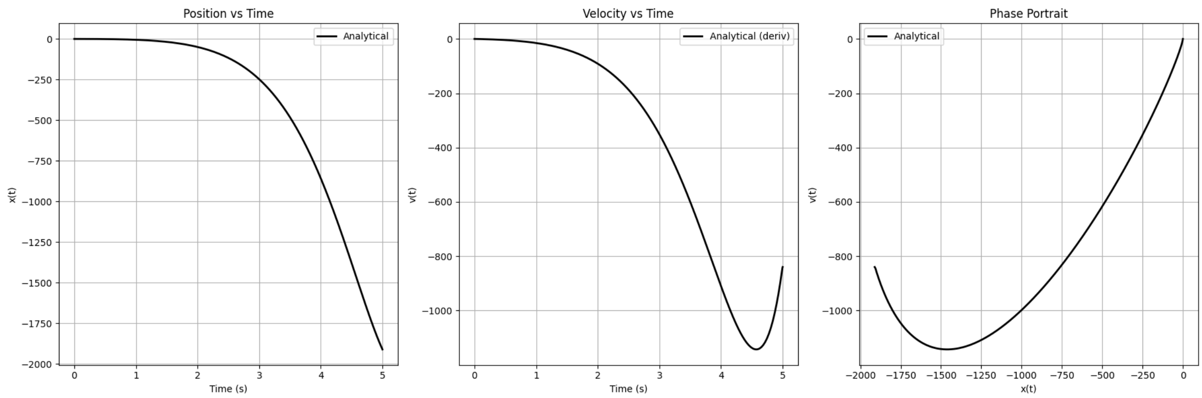
$$C_2 = -\frac{1.2938}{0.593} \times 2.166 = -4.7257$$

solution with initial conditions:

$$x(t) = e^{1.2938t}(2.166 * \cos(0.593t) - 4.7257 * \sin(0.593t)) - 2.066$$

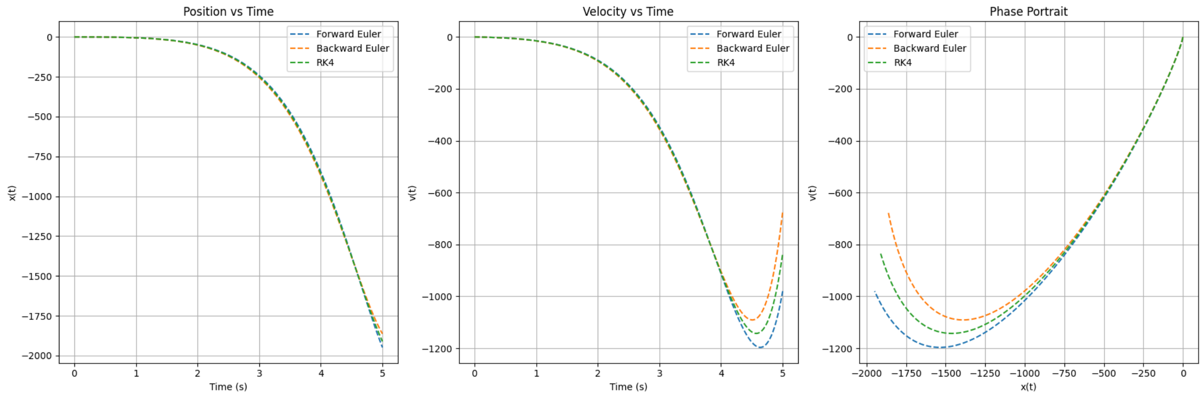
analytical solution:

https://github.com/saban0-lab/simrobs_group_2025/blob/main/SRS/practice_1/submissions/509109_Zongo_Saban_Task1/Integrator_Analy.ipynb



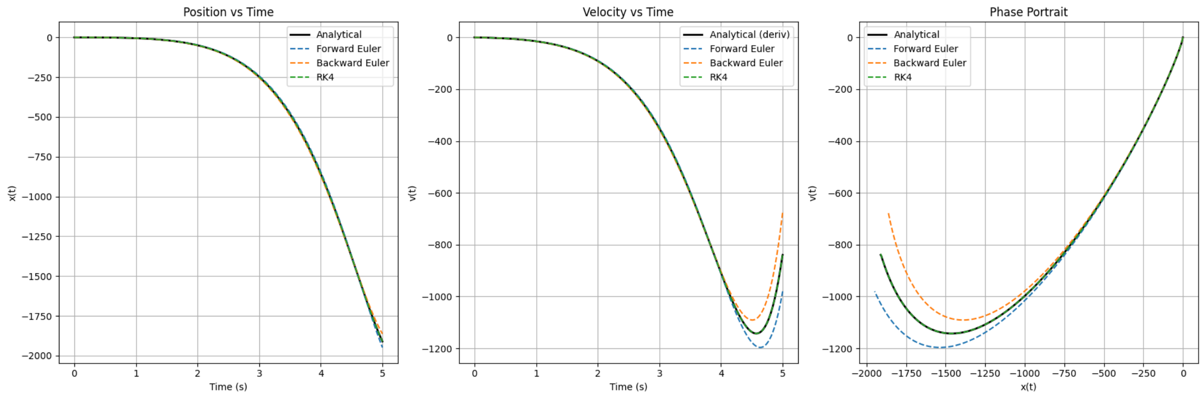
numerical solution:

https://github.com/saban0-lab/simrobs_group_2025/blob/main/SRS/practice_1/submissions/509109_Zongo_Saban_Task1/Integrator_Num.ipynb



Comparison and Discussion:

https://github.com/saban0-lab/simrobs_group_2025/blob/main/SRS/practice_1/submissions/509109_Zongo_Saban_Task1/Integrators.ipynb



The analytical solution provides the exact reference for position, velocity, and phase behavior.

Forward Euler shows significant deviation over time, with growing oscillations or instability evident in the phase portrait spiraling outward indicating poor energy conservation.

Backward Euler is overly dissipative; it damps the solution artificially, causing trajectories to spiral inward in the phase portrait, even when the true system conserves energy.

RK4 closely matches the analytical solution in all three plots, demonstrating high accuracy and good qualitative behavior.

conclusion: RK4 is clearly superior for this problem, while Forward and Backward Euler suffer from instability and excessive numerical damping, respectively.