

# Machine Learning Portfolio Optimization: Beyond Mean-Variance Analysis

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## 1 Introduction

### 1.1 Background

Modern portfolio theory (MPT), introduced by Harry Markowitz in his 1952 paper (1), revolutionized investment management by providing a mathematical framework for diversification. MPT demonstrates that a portfolio's risk and return should be evaluated not in isolation, but in terms of how individual assets contribute to the overall portfolio performance. By using variance as a measure of risk, MPT allows investors to construct diversified portfolios that maximize expected return for a given level of risk. While MPT's reliance on historical data and assumptions of normally distributed returns has been foundational, its limitations, such as static covariance matrices and linear return assumptions, have led researchers to explore more sophisticated approaches. Recent advances in machine learning, such as those outlined by Gu et al. (2020) (2), offer promising solutions to these challenges by capturing nonlinear patterns, incorporating alternative data, and adapting to evolving market conditions. This project builds on the principles of MPT while leveraging machine learning to enhance portfolio optimization and achieve superior risk-adjusted performance.

### 1.2 Project Overview

This project develops a hybrid portfolio optimization model that combines classical mean-variance optimization with ML-based return and risk prediction. The goal is to improve risk-adjusted returns by leveraging alternative data and nonlinear modeling techniques.

### 1.3 Outline

The remainder of this report is organized as follows: Section 2 derives the hybrid model, Section 2.5 presents backtesting results, and Section 3 concludes with a discussion of strengths, weaknesses, and future work.

## 2 Model Derivation

### 2.1 Assumptions

We assume:

- (i) Investors are rational and risk-averse.
- (ii) Asset returns are not necessarily normally distributed.
- (iii) ML models can capture complex, nonlinear relationships in financial data.

### 2.2 Classical Mean-Variance Optimization

The classical mean-variance optimization problem is formulated as:

$$\begin{aligned} & \min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^T \Sigma \mathbf{w} \\ \text{subject to } & \mathbf{w}^T \boldsymbol{\mu} = p \\ \text{and } & \sum_{i=1}^n w_i = 1, \quad w_i \geq 0 \quad \forall i, \end{aligned} \tag{1}$$

where  $\mathbf{w}$  is the portfolio weight vector,  $\Sigma$  is the covariance matrix,  $\boldsymbol{\mu}$  is the expected return vector, and  $p$  is the target return.

### 2.3 Machine Learning Augmentation

To enhance the classical mean-variance framework while maintaining analytical simplicity, we introduce machine learning only for the estimation of expected returns. The covariance matrix is kept identical to the classical approach, computed from historical returns using a rolling window. This preserves the well-understood risk structure of mean-variance optimization while improving the return forecasts.

**ML-based return estimation.** A Gradient Boosting Machine (GBM) is trained to predict one-step-ahead expected returns for each asset. The model uses features such as technical indicators (e.g., moving averages, momentum), volatility measures, and inflation in the Eurozone. The resulting predictions form the expected return vector

$$\hat{\boldsymbol{\mu}}_{\text{ML}},$$

which replaces the historical mean in the optimization step. GBM is chosen for its ability to capture nonlinear dynamics and interactions commonly present in financial markets.

**Classical covariance estimation.** To maintain stability and interpretability, the covariance matrix is computed exactly as in the standard mean-variance model. Specifically, we use the sample covariance matrix of historical returns over a rolling estimation window:

$$\hat{\Sigma} = \text{Cov}(r_{t-k+1}, \dots, r_t).$$

This approach avoids the complexity and instability of attempting to predict full covariance matrices with ML, while still providing a reliable estimate of risk.

**Integrated optimization.** The optimized portfolio weights are obtained by solving the classical mean-variance problem using the ML-predicted expected returns  $\hat{\mu}_{\text{ML}}$  and the covariance matrix  $\hat{\Sigma}$ .

## 2.4 Hybrid Optimization Problem

The hybrid model integrates machine learning-based return forecasts with the classical mean-variance framework. Let  $\hat{\mu}_{\text{ML}}$  denote the vector of expected returns predicted by the GBM model, and let  $\hat{\Sigma}$  denote the sample covariance matrix computed from historical returns. The optimization problem is then formulated as:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2} \mathbf{w}^T \hat{\Sigma} \mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^T \hat{\mu}_{\text{ML}} = p, \\ & \sum_{i=1}^n w_i = 1, \\ & w_i \geq 0, \quad i = 1, \dots, n, \end{aligned} \tag{2}$$

where  $\mathbf{w}$  is the portfolio weight vector and  $p$  the target return. The objective minimizes variance, subject to return, budget, and long-only constraints. The covariance matrix is classically estimated, while return forecasts use a machine learning model, combining stability with ML-driven predictive power.

## 2.5 Backtesting Results

To evaluate the performance of the proposed hybrid model, we conducted a backtest over the period from 2005 to 2025. The analysis compares the cumulative returns of three portfolios: the hybrid model (integrating machine learning), the classical mean-variance approach, and an equal-weighted benchmark. The results are summarized in Table 1, which highlights the annualized returns and Sharpe ratios for each strategy.

The backtest was performed using a diversified sample of European stocks, as detailed in Table 2. Note that the CAC 40 index for the 2024-2025 interval was used for comparison but excluded from the sample to focus on individual stock performance.

Table 1: Performance Metrics (2005–2025)

| Model         | Annualized Return | Volatility | Sharpe Ratio |
|---------------|-------------------|------------|--------------|
| Hybrid (ML)   | 11.09%            | 18.35%     | 0.60         |
| Mean-Variance | 12.04%            | 18.04%     | 0.67         |
| CAC 40        | 13.39%            | 18.96%     | 0.44         |

Table 2: Stock Sample for Backtesting

|        |         |        |         |         |
|--------|---------|--------|---------|---------|
| AI.PA  | AIR.PA  | ALV.DE | ASML.AS | BAYN.DE |
| DG.PA  | ENEL.MI | ITX.MC | LDO.MI  | LIN.DE  |
| MC.PA  | OR.PA   | RMS.PA | RNO.PA  | SAN.PA  |
| SAP.DE | SU.PA   | TTE.PA | VOE.VI  | VOW3.DE |

## 2.6 Model Assessment

### Strengths:

- **Mean-Variance Optimization:** Provides the best balance of risk and return with the highest Sharpe ratio (0.67) among all strategies. It achieves 12.04% annualized returns with the lowest volatility (18.04%) of all tested approaches, demonstrating superior risk-adjusted performance.
- **Hybrid ML Approach:** Incorporates alternative data sources (technical indicators and macroeconomic data) that could potentially capture more complex market dynamics than classical methods.
- **Robust Backtesting Framework:** The 20-year backtest period (2005-2025) captures multiple market regimes, including the 2008 financial crisis and COVID-19 pandemic, providing comprehensive performance evaluation.

### Weaknesses:

- **ML Underperformance:** The hybrid ML approach currently underperforms both the classical mean-variance strategy and the CAC 40 benchmark in terms of raw returns and risk-adjusted performance.
- **Feature Limitations:** The current feature set (technical indicators and Eurozone inflation) may not provide sufficient predictive power to outperform classical methods.
- **Implementation Complexity:** The ML-enhanced strategy adds computational complexity without currently demonstrating commensurate performance benefits.

## 3 Conclusion

This project developed and evaluated a hybrid portfolio optimization model combining classical mean-variance optimization with machine learning-based return predictions. Our back-testing results over the 2005-2025 period, using a diversified sample of European stocks, reveal several important findings:

The classical mean-variance optimization approach emerges as the superior strategy among those tested. With an annualized return of 12.04% and volatility of 18.04%, it achieves the highest Sharpe ratio of 0.67, indicating optimal risk-adjusted returns. This strategy effectively balances return maximization with risk management, aligning with the core principles of modern portfolio theory.

While our machine learning approach demonstrates potential by incorporating alternative data and nonlinear modeling techniques, it currently underperforms the classical mean-variance strategy. The hybrid model's 11.09% annualized return and 18.35% volatility result in a lower Sharpe ratio of 0.60, suggesting that the additional complexity does not translate to better performance in this implementation.

Comparing both strategies to the CAC 40 benchmark reveals that while the ML approach achieves better risk management (lower volatility than CAC 40's 18.96%), it falls short in absolute returns (11.09% vs CAC 40's 13.39%). The classical mean-variance strategy strikes the best balance, achieving near-benchmark returns with superior risk characteristics.

These results suggest that for our current implementation and dataset, the classical mean-variance optimization remains the preferred approach. However, the framework developed here provides a foundation for future improvements in ML-based portfolio optimization. Potential avenues for enhancement include incorporating more predictive features, refining the machine learning models, and exploring alternative optimization approaches that might better leverage the ML predictions.

## References

- [1] H. Markowitz, “Portfolio Selection,” *The Journal of Finance (New York)*, vol. 7, no. 1, pp. 77–91, 1952, doi: 10.2307/2975974
- [2] S. Gu, B. Kelly, and D. Xiu, “Empirical Asset Pricing via Machine Learning,” *The Review of Financial Studies*, vol. 33, no. 5, pp. 2223–2273, 2020, doi: 10.1093/rfs/hhaa009

## A Project Code

The project code is available on GitHub at <https://github.com/sabanpoulos/Machine-Learning-Portfolio-Optimization>.

## B Data Used

Table 3: Datasets Used in the Project

| Data Type                | Source          | Time Period |
|--------------------------|-----------------|-------------|
| Stock Prices             | FMP API         | 2005–2025   |
| Macroeconomic Indicators | ECB Data Portal | 2005–2025   |