CENTRAL LIMIT THEOREM ASSIGNMENT

1) Let x be a random variable with $\mu = 10$ and $\sigma = 4$. A sample of size 100 is taken from this population. Find the probability that the sample mean of these 100 observation is less than 9.

$$\text{Standard Error} = \frac{\sigma}{\sqrt{N}} = \frac{4}{\sqrt{100}} = 0.4$$

$P(x<9)$

$$z = \frac{x-\mu}{SE} = \frac{9-10}{0.4} = -2.5$$

$$P(x<9) = 0.0062$$

2) The Data Science classes are being conducted at 8th floor of a building and students must use lift to reach there floor. An elevator can transport a maximum of 550 kgs.

$\mu = 50\text{kg}$ $\sigma = 15\text{kg}$ $N = 10$

$$\text{Standard Error} = \frac{15}{\sqrt{10}} = 4.74$$

$$\bar{x} = \frac{550}{10} = 55$$

$$P(\text{all 10 students}) \bar{x} = ?\ z = \frac{55-50}{4.74} = 1.05$$

$$P(\text{all 10 students}) = 0.85$$

3) From past experience, it is known that the number of tickets purchased by a passenger . . .

$M = 2.4 \quad \sigma = 2.0 \quad N = \cancel{2}00$

$\text{Standard Error} = \frac{\sigma}{\sqrt{N}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$

$x = \frac{250}{100} = 2.5$

$z = \frac{2.5 - 2.4}{0.2} = \frac{0.1}{0.2} = 0.5$

Probability $= 0.69$ (or) 69%.

4) An office in the army needs 35 men for a mission average IQ scor 35 men must be greater than 98 pt.

$M = 96 \quad \sigma = 16 \quad N = 35$

$\text{Standard Error} = \frac{\sigma}{\sqrt{N}} = \frac{16}{\sqrt{35}} = 2.7$

$z = \frac{x - M}{\sigma/\sqrt{N}} = \frac{98 - 96}{2.7} = 0.74$

$P(x > 98) = 1 - P(x < 98)$

$P(x < 98) = 0.77$

$P(x > 98) = 1 - 0.77 = 0.23$

$\boxed{P(x > 98) = 23\%}$

CENTRAL LIMIT THEOREM

5) Engineers must consider the breadths of male heads when designing motorcycle helmet. Men have head breadths that are normally distributed with a mean of 6.0 inch and a standard deviation of 1.0 inch.

a) if a male is selected, find prob that his head breadth is less than 6.2 inch.

$$z = \frac{x-\mu}{\sigma} = \frac{6.2-6}{1} = 0.2$$

$$P = 0.5793$$

b) find the probability that 100 randomly selected men have a mean breadth that is less than 6.2 inch

$$\text{Sample Error} = \frac{\sigma}{\sqrt{N}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = 0.1$$

$$z = \frac{6.2-6}{0.1} = 2$$

$$P(x < 6.2) = 0.97 \text{ (or) } 97\%.$$

6) A production Manager for Safeguard Helmet company plans an initial run of 100 Helmets. Seeing the results part(b) the manager reasons that all helmets should be made for men with head breadths less than 6.2 inch, because they would fit all but a few men. What is wrong with that reasoning)

As per central limit theorem, the sampling distribution of the sample means approaches a normal distribution when the sample sizes over 30. But here in part(B) we have taken only one samples. So we can't decide based on the one sample.

7) The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days, if 25 women are randomly selected, find the prob that lengths of pregnancy mean less than 260 days?

$$\mu = 268 \quad \sigma = 15 \quad N = 25$$

$$z = \frac{x - \mu}{\sigma/\sqrt{N}} = \frac{260 - 268}{15/\sqrt{25}} = \frac{-8}{3} = -2.67$$

$$\boxed{P(x < 260) = 0.0038}$$

CENTRAL LIMIT THEOREM

8) if 25 women are put on a special diet just before they become pregnant and they end up having a mean length of pregnancy of less than 260 days. Does it appear that the diet has an effect on the length of pregnancy?

Yes, it appears that diet has an effect on the length of pregnancy. Since in this experiment, population is considered instead of sample.

9) The weight of adult males are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds. Based on this info, solve the following?

a) What is the probability that one randomly selected male will weigh more than 190 pounds?

$$z = \frac{190 - 172}{29} = \frac{18}{29} = 0.62$$

$P(x < 190) = 0.7324$

$P(x > 190) = 1 - 0.7324$

$$\boxed{P(x > 190) = 0.27}$$

b) What is the probability that 25 randomly selected adult males will have a mean weight of more than 190 pounds?

$x = 190 \quad N = 25 \quad \mu = 172 \quad \sigma = 29$

$$z = \frac{190 - 172}{29/\sqrt{25}} = \frac{18 \times 5}{29} = 3.1$$

$$\boxed{P(x \geq 190) = 1 - 0.99 = 0.01}$$

c) An elevator at a men's fitness center has a top maximum allowable weight is 4750 pounds. If 25 randomly selected men cram into the elevator, what is probability it will be over maximum weight?

* Probability will be 1%. Same as above part (b))

CLT

10) The amount of impurity in a batch of a chemical product is a random variable with a mean value 4.0 g and a standard deviation of 1.5 g. If 50 batched are independently, what is the (approx) probability that the average amount of impurity in these 50 batches is between 3.5 g and 3.8 g?

$\mu = 4 \quad \sigma = 1.5 \quad N = 50$

$x = 3.5, \quad Z_1 = \dfrac{3.5 - 4}{1.5/\sqrt{50}} = \dfrac{-0.5}{0.212} = -2.36$

$P_1 = 0.0091$

$x = 3.8, \quad Z_2 = \dfrac{3.8 - 4}{0.212} = -0.94$

$P_2 = 0.1736$

Between 3.5g & 3.8g = $P_2 + P_1 = 0.1736 + 0.0091$

$= 0.1827$

$= 18\%$

11) Suppose the Age a Student graduates from Salem State is normally distributed, if the mean age is 23.1 years and standard deviation is 3.1 years. What is the prob that 6 randomly selected student had a mean age at graduation was greater than 27

$$\mu = 23.1 \quad \sigma = 3.1 \quad N = 6$$

$$z = \frac{27 - 23.1}{3.1/\sqrt{6}} = \frac{3.9 \times 2.45}{3.1} = 3.08$$

$$P(x>27) = 1 - P(x<27) = 1 - 0.99$$

$$\boxed{P(x>27) = 0.01 \text{ (or) } 1\%}$$

CLT

12) While checking receipts at Reds, it was determined that the average spend on food per table was \$21.50 with a $\sigma = 2.222$. If we can assume that the amount of money spent was normally distributed, what is the prob that the average of 8 checks between \$20 & \$23

$\mu = 21.50 \quad \sigma = 2.22 \quad N = 8$

$$z_1 = \frac{23 - 21.5}{2.22/\sqrt{8}} = \cancel{1.5} \frac{4.24}{2.22} = 1.9$$

$P_1 = 0.9719$

$$z_2 = \frac{20 - 21.5}{2.22/\sqrt{8}} = \frac{-1.5 \times 2.82}{2.22} = -1.9$$

$P_2 = 0.0281$

$$\boxed{P(20 > 23) = P_1 + P_2 = 100\%}$$

13) Suppose the grades in a finite mathematics class are normally distributed with a mean of 75 & Stand dev. 5

a) What is the probability that a randomly selected student had a grade of atleast 83?

$\mu = 75 \quad \sigma = 5$

$$z = \frac{83-75}{5} = \frac{8}{5} = 1.6$$

$P(x < 83) = 1 - 0.9452$

$$\boxed{P(x < 83) = 0.06 \text{ (or) } 6\%}$$

b) What is the probability that the average grade of 5 randomly selected students was atleast 83

$\mu = 75 \quad \sigma = 5$

$$z = \frac{83-75}{5/\sqrt{5}} = \frac{8 \times \sqrt{5}}{5} = 3.5$$

$$\boxed{P(x < 83) = 0\%}$$

14) The average age of major ~~release~~ league baseball players is 28.3 years and std.dev of 2.3 years. If we can assume that ages are normally distributed What is prob that the average age of 10 randomly Red Sox players is less than 27 years

$M = 28.3 \quad \sigma = 2.3 \quad N = 10 \quad x = 27$

$$Z = \frac{x - M}{\sigma / \sqrt{N}} = \frac{27 - 28.3}{2.3 / \sqrt{10}} = \frac{-1.3 \times \sqrt{10}}{2.3}$$

$Z = 1.79$

$P(x < 27) = 0.9633$ (or) 96%