MILP Model and Benders Decomposition Process

MILP Model

Objective Function

Maximize:

$2x_1 + 6x_2 + 4y$

Subject to Constraints

1. $x_1 + x_2 + y \leq 200$

2. $x_1 \leq 100$

3. $x_2 \leq 100$

Variable Types

x_1, x_2 are continuous variables

y is an integer variable

Benders Decomposition - Iterative Process

This document details the iterative process of Benders Decomposition for the given optimization problem. The process involves solving subproblems to generate feasibility and optimality cuts, which are then added to the master problem. The process continues until the lower bound (LB) and upper bound (UB) converge.

Iteration 1

Subproblem Status: Infeasible

Subproblem Objective Value: -1800.0

Shadow Prices: [6.0, 0.0, 0.0]

Cut Added: Feasibility Cut

Lower Bound (LB): -inf

Upper Bound (UB): inf

Initially, the master problem starts with an arbitrary y value of 500.

Master Problem LP Model (y = 500):

Model MasterProblem
LP format - for model browsing. Use MPS format to capture full model detail.

```
Maximize
 z
Subject To
 R0: - 6 y >= -1200
Bounds
Generals
 y
End
```

Updated y_star: 500

New UB: inf

Iteration 2

Subproblem Status: Optimal

Subproblem Objective Value: 800.0

Shadow Prices: [0.0, 2.0, 6.0]

Cut Added: Optimality Cut

Lower Bound (LB): 800.0

Upper Bound (UB): 1e+30

Master Problem LP Model:

```
\ Model MasterProblem
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
 z
Subject To
 R0: - 6 y >= -1200
 R1: - 4 y + z <= 800
Bounds
Generals
 y
End
```

Updated y_star: 200.0

New UB: 1600.0

Iteration 3

Subproblem Status: Optimal

Subproblem Objective Value: -0.0

Shadow Prices: [6.0, 0.0, 0.0]

Cut Added: Optimality Cut

Lower Bound (LB): 800.0

Upper Bound (UB): 1600.0

Master Problem LP Model:

```
\ Model MasterProblem
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
 z
Subject To
 R0: - 6 y >= -1200
 R1: - 4 y + z <= 800
 R2: 2 y + z <= 1200
Bounds
Generals
 y
End
```

Updated y_star: 67.0

New UB: 1066.0

Iteration 4

Subproblem Status: Optimal

Subproblem Objective Value: 666.0

Shadow Prices: [2.0, 0.0, 4.0]

Cut Added: Optimality Cut

Lower Bound (LB): 934.0

Upper Bound (UB): 1066.0

Master Problem LP Model:

```
\ Model MasterProblem
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
 z
Subject To
 R0: - 6 y >= -1200
 R1: - 4 y + z <= 800
 R2: 2 y + z <= 1200
 R3: - 2 y + z <= 800
Bounds
Generals
 y
End
```

Updated y_star: 100.0

New UB: 1000.0

Iteration 5

Subproblem Status: Optimal

Subproblem Objective Value: 600.0

Shadow Prices: [2.0, 0.0, 4.0]

Cut Added: Optimality Cut

Lower Bound (LB): 1000.0

Upper Bound (UB): 1000.0

Master Problem LP Model:

```
\ Model MasterProblem
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
 z
Subject To
 R0: - 6 y >= -1200
 R1: - 4 y + z <= 800
 R2: 2 y + z <= 1200
 R3: - 2 y + z <= 800
 R4: - 2 y + z <= 800
Bounds
Generals
 y
End
```

Updated y_star: 100.0

New UB: 1000.0

Conclusion

The Benders Decomposition process converged after 5 iterations.

Final amount in y: 100.0

Final objective value: 1000.0