Chapter 2:

Introduction to the Relational Model

Database System Concepts, 6th Ed.

Class Announcements

- Class discussion: To post questions to TAs and instructor, follow the link under the syllabus tab in the course content menu on brightspace. To ask a question, start a new thread.
- Office hours:
  - Instructor office hours on W 3-4pm in room 856 in 370 Jay Street, as announced already.
  - TA office hours for this week will be announced today. There will be in-person on Tuesday (tomorrow) and online on Friday.
- HW#1 will be available later this week, and will be due around October 5. Submission is via gradescope.

Example of a Relation

attributes
(or columns)

ID	*name*	*dept_name*	*salary*
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

tuples
(or rows)

Attribute Types

- The set of allowed values for each attribute is called the **domain** of the attribute
- Attribute values are (normally) required to be **atomic**; that is, indivisible
- The special value ***null*** is a member of every domain
- The null value causes complications in the definition of many operations

Relation Schema and Instance

- $A_1, A_2, \ldots, A_n$ are *attributes*

- $R = (A_1, A_2, \ldots, A_n)$ is a *relation schema*

  Example:

  $instructor = (ID, name, dept\_name, salary)$

- Formally, given sets $D_1, D_2, \ldots. D_n$ a **relation *r*** is a subset of

  $D_1 \times D_2 \times \ldots \times D_n$

  Thus, a relation is a set of n-tuples $(a_1, a_2, \ldots, a_n)$ where each $a_i \in D_i$

- The current values (**relation instance**) of a relation are specified by a table
- An element ***t*** of ***r*** is a *tuple*, represented by a *row* in a table

Relations are Unordered

- Order of tuples is irrelevant (tuples may be stored in an arbitrary order)
- Example: *instructor* relation with unordered tuples

ID	*name*	*dept_name*	*salary*
22222	Einstein	Physics	95000
12121	Wu	Finance	90000
32343	El Said	History	60000
45565	Katz	Comp. Sci.	75000
98345	Kim	Elec. Eng.	80000
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
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76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
58583	Califieri	History	62000
83821	Brandt	Comp. Sci.	92000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
76543	Singh	Finance	80000

- This is on the logical design level – ordering can be defined on physical level

Database

- A database consists of multiple relations
- Information about an enterprise is broken up into parts

  instructor
  student
  advisor

- Bad design:

  univ (*instructor_ID, name, dept_name, salary, student_ID*, ..)

  results in
  - repetition of information (e.g., two students have the same instructor)
  - the need for null values (e.g., represent a student with no advisor)
- Normalization theory (Chapter 7) deals with how to design "good" relational schemas

Keys

- Let K $\subseteq$ R
- *K* is a **superkey** of *R* if values for *K* are sufficient to identify a unique tuple of ***any possible*** relation *r(R)*
  - Example: {*ID*} and {ID, name} are both superkeys of *instructor.*
  - *However: {name} is not a superkey. (Why?)*
- Superkey *K* is a **candidate key** if *K* is minimal
  Example: {*ID*} is a candidate key for *Instructor*
- One of the candidate keys is selected to be the **primary key**.
  - which one?
- **Foreign key** constraint: Value in one relation must appear in another
  - **Referencing** relation
  - **Referenced** relation

Keys and Foreign Keys: Example

- E-Commerce Database with customers, products, and purchases:

CUSTOMER

CID	CNAME	CADDRESS

PURCHASE

CID	PID	TIMEDATE	PRICE

PRODUCT

PID	PNAME	PDESCRIPTION	PPRICE

Keys and Foreign Keys: Example

- E-Commerce Database with customers, products, and purchases:

CUSTOMER

CID	CNAME	CADDRESS

PURCHASE

CID	PID	TIMEDATE	PRICE

PRODUCT

PID	PNAME	PDESCRIPTION	PPRICE

Keys and Foreign Keys: Example

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CID	CNAME	CADDRESS

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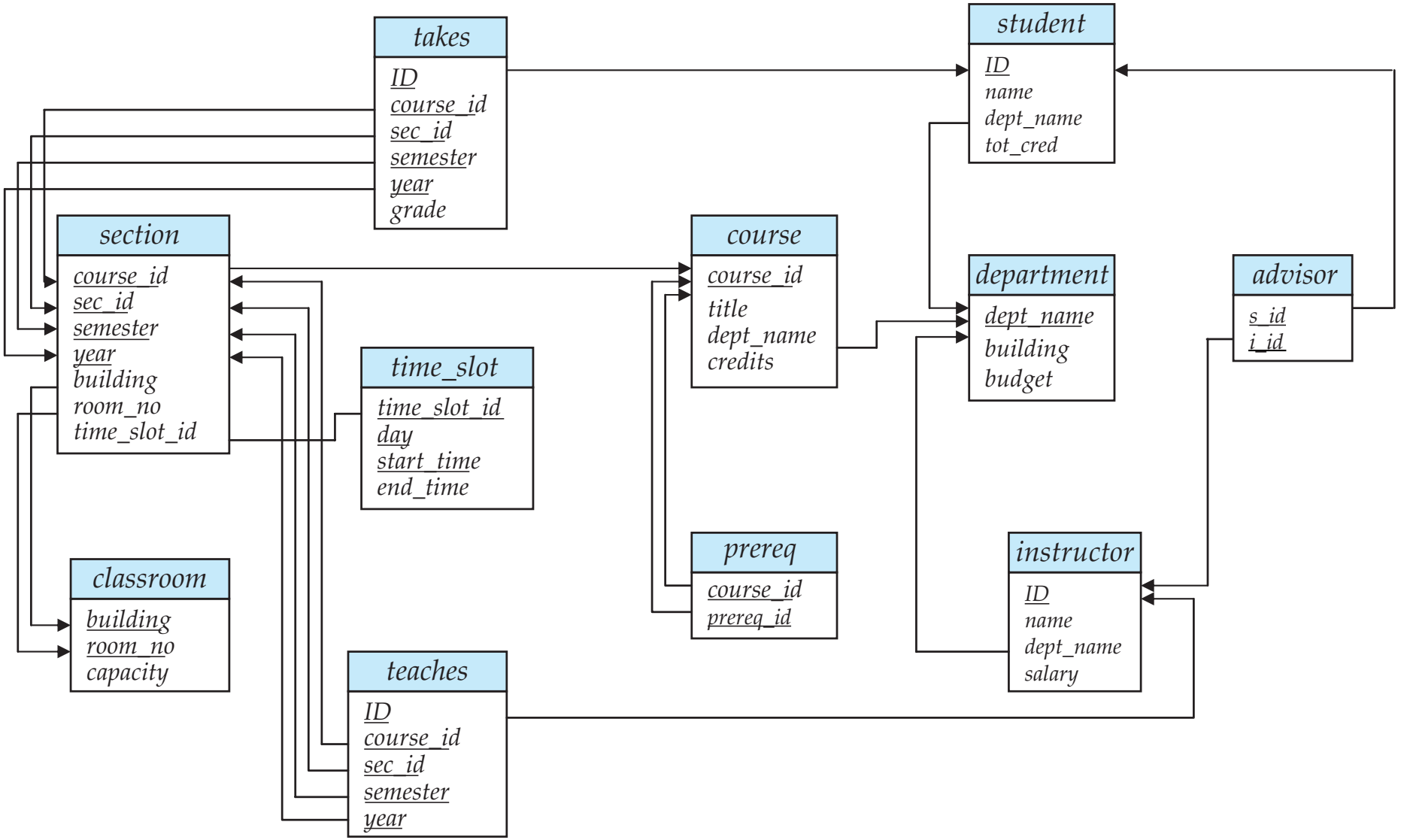
CID	PID	TIMEDATE	PRICE

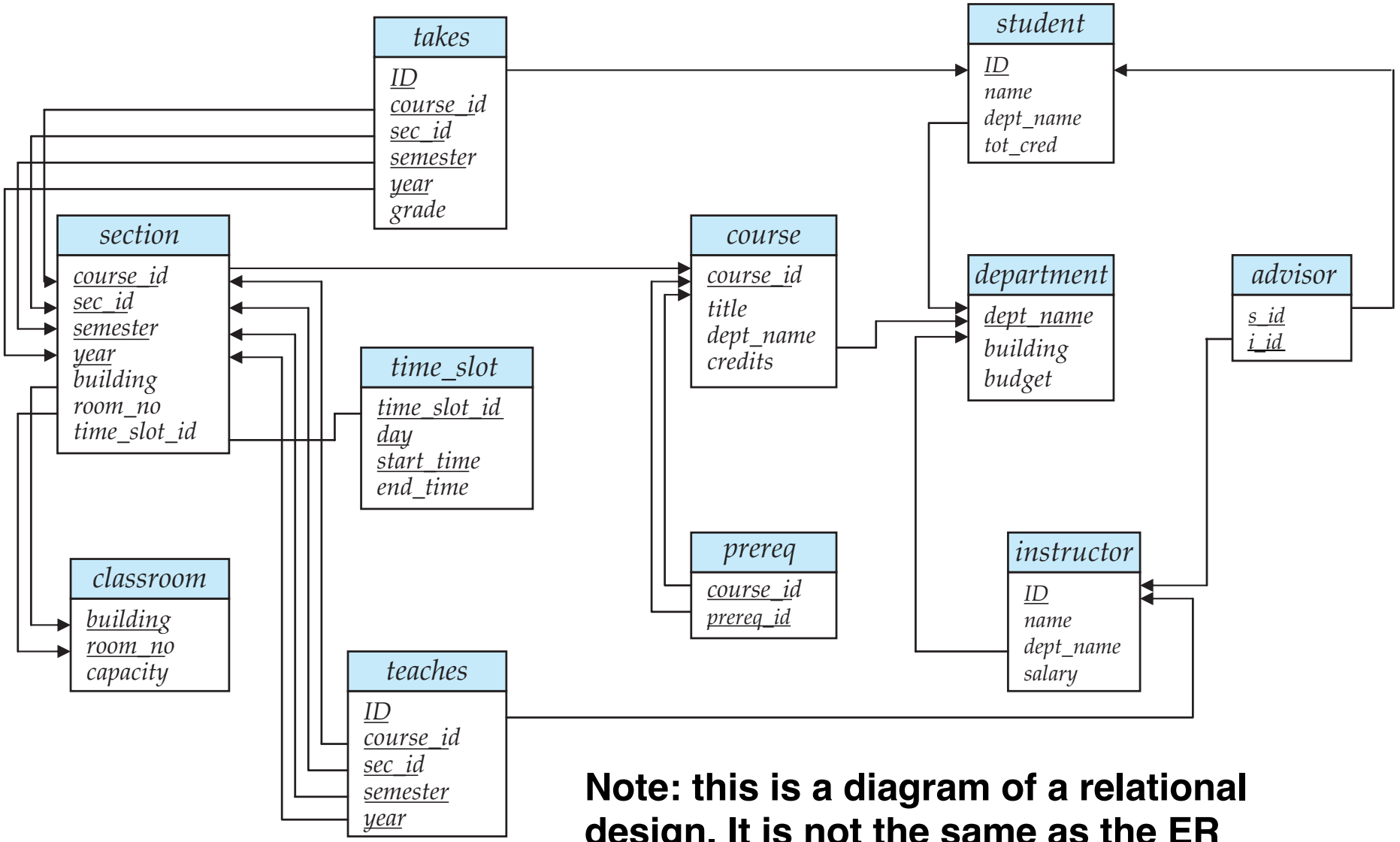
PRODUCT

PID	PNAME	PDESCRIPTION	PPRICE

Schema Diagram for University Database



Schema Diagram for University Database



Note: this is a diagram of a relational design. It is not the same as the ER diagrams we will see in a few weeks.

Relational Query Languages

- Procedural vs.non-procedural, or declarative
- "Pure" languages:
  - Relational algebra (procedural)
  - Relational calculus (non-procedural)
    - Domain relational calculus (DRC)
    - Tuple relational calculus (TRC)
- Non-pure, real languages:
  - SQL
  - Query By Example (QBE)
- But first we introduce relational operators
  - Selection, projection, cross product, ...

Selection of tuples

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- Select tuples with A=B and D > 5

- $\sigma_{A=B \text{ and } D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Selection of Columns (Attributes)

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- Select A and C
  - Projection
  - $\Pi_{A, C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Joining two relations – Cartesian Product

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

r x *s*

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Union of two relations

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s

A	B
α	1
α	2
β	1
β	3

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r - s$

A	B
α	1
β	1

Set Intersection of two relations

- Relation *r, s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r \cap s$

A	B
α	2

Joining two relations – Natural Join

- Let r and s be relations on schemas R and S respectively.
  Then, the "natural join" of relations R and S is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from r and t_s from s.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple t to the result, where
    - t has the same value as t_r on r
    - t has the same value as t_s on s

Natural Join Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

- Natural Join
  - r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Overview of Operators

Symbol (Name)	Example of Use
σ (Selection)	$\sigma_{\text{salary}>=85000}(instructor)$
	Return rows of the input relation that satisfy the predicate.
Π (Projection)	$\Pi_{ID,\ salary}(instructor)$
	Output specified attributes from all rows of the input relation. Remove duplicate tuples from the output.
$\bowtie$ (Natural Join)	$instructor \bowtie department$
	Output pairs of rows from the two input relations that have the same value on all attributes that have the same name.
$\times$ (Cartesian Product)	$instructor \times department$
	Output all pairs of rows from the two input relations (regardless of whether or not they have the same values on common attributes)
$\cup$ (Union)	$\Pi_{name}(instructor) \cup \Pi_{name}(student)$
	Output the union of tuples from the two input relations.

Database Design Example

- Suppose you want to design a database for a bakery selling cakes

- The bakery makes certain cakes: Apple Pie, Chocolate Cake, etc
- Cakes have a price and also need certain ingredients
- Customer can order cakes, and then pick them up on some date
- Bakery also wants to keep track of how much ingredients they have
- Customers may want to search by ingredient
  (e.g., “cakes with chocolate but without peanuts”)

- Let’s design a relational schema for this problem ...

Database Design Example

- Let’s design a relational schema for this problem ...

Database Design Example

- Possible schema:

  CUSTOMER (cid, name, phone, ccn)

  CAKE (cname, price, slices)

  ORDER (oid, cid, cname, pickupdate, orderdate, oprice)

  INGREDIENT (iname, price, amountleft)

  USED_IN (cname, iname, amount)

Database Design Example

- Possible schema:

  CUSTOMER (<u>cid</u>, name, phone, ccn)

  CAKE (<u>cname</u>, price, slices)

  ORDER (<u>oid</u>, cid, cname, pickupdate, orderdate, oprice)

  INGREDIENT (<u>iname</u>, price, amountleft)

  USED_IN (<u>cname, iname</u>, amount)

- Foreign keys:

Database Design Example

- Possible schema:

  CUSTOMER (cid, name, phone, ccn)

  CAKE (cname, price, slices)

  ORDER (oid, cid, cname, pickupdate, orderdate, oprice)

  INGREDIENT (iname, price, amountleft)

  USED_IN (cname, iname, amount)

- Possible queries:
  - IDs of customers with name “J. Smith”
  - Names of cakes containing more than 12oz of chocolate
  - Names of customers who bought “apple pie”
  - IDs of customers who have never bought a cake with peanuts

Chapter 6: Formal Relational Query Languages

Relational Algebra

Domain Relational Calculus

(Tuple Relational Calculus)

Query By Example

Relational Algebra

- Procedural language
- Based on relational operations introduced in chapter 2
- Three sets of RA operations
  - Basic RA operations:
    - select, project, union, set diff., cartesian product, rename
  - Additional RA operations:
    - Can be expressed using basic ones
    - But make it easier to write queries
    - intersection, assignment, joins, division
  - Extended RA:
    - Increases power of RA
    - Cannot be expressed with basic RA
    - Aggregation and group_by

Relational Algebra

- Six basic operators
  - select: σ
  - project: Π
  - union: $\cup$
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- All operators take one or two relations as inputs and produce a new relation as a result
- Additional operators defined on top of these 6 later

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Select Operation

- Notation: $\sigma_p(r)$
- p is called the **selection predicate**
- Defined as:

$$\sigma_p(\boldsymbol{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
Each **term** is one of:

<attribute> *op* <attribute> or <constant>

where *op* is one of: =, $\neq$, >, $\geq$. <. $\leq$

- Example of selection:

$$\sigma_{\textit{dept\_name}=\text{“Physics”}}(\textit{instructor})$$

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Project Operation

- Notation: $\Pi_{A_1, A_2, \ldots, A_k}(r)$

  where A_1, A_2 are attribute names and r is a relation name.
- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the *dept_name* attribute of *instructor*

$$\Pi_{ID,\ name,\ salary}(instructor)$$

Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

A	B
α	1
α	2
β	1
β	3

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- *r* – *s*:

A	B
α	1
β	1

Cartesian-Product Operation – Example

- Relations *r, s*:

A	*B*
α	1
β	2

r

C	*D*	*E*
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	*B*	*C*	*D*	*E*
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Cartesian-Product Operation

- Notation $r \times s$
- Defined as:

$$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \varnothing$).
- If attributes of *r(R)* and *s(S)* are not disjoint, then renaming must be used.

Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$

- $r \times s$

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \times s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

Rename Operation

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.
- Allows us to refer to a relation by more than one name.
- Example: $\rho_x(E)$

  returns the expression E under the name X
- If a relational-algebra expression E has arity n, then

$$\rho_{x(A_1, A_2, \ldots, A_n)}(E)$$

  returns the result of expression E under the name X, and with the attributes renamed to $A_1, A_2, \ldots, A_n$.
- E.g.. Given relation CAKE(cname, price, slices)

$$\rho_{\textit{cheapcake(cakename, price, slices)}} \left(\sigma_{\textit{price} < 10}(\text{CAKE}) \right)$$

Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let E_1 and E_2 be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1 \times E_2$
  - $\sigma_p(E_1)$, P is a predicate on attributes in E_1
  - $\Pi_S(E_1)$, S is a list consisting of some of the attributes in E_1
  - $\rho_x(E_1)$, x is the new name for the result of E_1

Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Assignment
- Outer join
- Division

Set-Intersection Operation – Example

- Relation *r, s*:

A	*B*
α	1
α	2
β	1

r

A	*B*
α	2
β	3

s

- $r \cap s$

A	*B*
α	2

Natural-Join Operation

- Notation: r ⋈ s
- Let *r* and *s* be relations on schemas *R* and *S* respectively.
  Then, r ⋈ s is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from *s*.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on *s*
- Example:

  $R = (A, B, C, D)$

  $S = (E, B, D)$
  - Result schema = (A, B, C, D, E)
  - $r \bowtie s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B = s.B \wedge r.D = s.D}\left(r \times s\right)\right)$$

Natural Join Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

- r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Natural Join and Theta Join

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Comp. Sci.”}}(instructor \bowtie teaches \bowtie course))$
- Natural join is associative
  - (*instructor* $\bowtie$ *teaches*) $\bowtie$ *course* is equivalent to *instructor* $\bowtie$ (*teaches* $\bowtie$ *course*)
- Natural join is commutative
  - *instruct* $\bowtie$ *teaches* is equivalent to *teaches* $\bowtie$ *instructor*
- The **theta join** operation $r \bowtie_\theta s$ is defined as
  - $r \bowtie_\theta s = \sigma_\theta (r \times s)$

  where θ (theta) is an arbitrary condition

Natural Join and Theta Join

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Comp. Sci.”}}(instructor \bowtie teaches \bowtie course))$
- Natural join is associative
  - (*instructor* $\bowtie$ *teaches*) $\bowtie$ *course* is equivalent to
    instructor $\bowtie$ (*teaches* $\bowtie$ *course*)
- Natural join is commutative
  - *instruct* $\bowtie$ *teaches* is equivalent to
    teaches $\bowtie$ *instructor*
- The **theta join** operation $r \bowtie_\theta s$ is defined as
  - $r \bowtie_\theta s = \sigma_\theta(r \times s)$

Careful: this assumes instructors only teach in “their” department!

Assignment Operation

- The assignment operation (←) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
  - Motivation from algebra: suppose you want to compute

    y = (a+b+c)^2 + (a+b+c)^3 + (a+b+c)^4

    Shorter: x = a+b+c

    y = x^2 + x^3 + x^4

Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist
  - All comparisons involving *null* are (roughly speaking) **false** by definition.
    - We shall study precise meaning of comparisons with nulls later

Outer Join – Example

- Relation *instructor1*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches1*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

Outer Join – Example

- Join

  instructor ⋈ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

- Left Outer Join

  instructor ⟕ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*

Outer Join – Example

- Right Outer Join *instructor* ⟖ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
76766	null	null	BIO-101

- Full Outer Join *instructor* ⟗ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*
76766	null	null	BIO-101

- Outer join can be expressed using basic operations
  - e.g. r ⟗ s can be written as

$$(r \bowtie s) \cup (r - \Pi_R(r \bowtie s)) \times \{(null, \ldots, null)\}$$

Theta vs. Natural vs. Outer Join

- CUSTOMER (cid, cname, joindate)
  PURCHASE (cid, pid, buydate)
- Theta Join: more general than Natural Join
- Natural Join: special case, join on common attributes
- Works most of the time, but what if we use “date” in both?
- Inner versus outer join is an independent choice

Null Values

- It is possible for tuples to have a null value, denoted by *null*, for some of their attributes
- *null* signifies an unknown value or that a value does not exist.
- The result of any arithmetic expression involving *null* is *null.*
- Aggregate functions simply ignore null values (as in SQL)
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same (as in SQL)

Null Values

- Comparisons with null values return the special truth value: *unknown*
  - If *false* was used instead of *unknown*, then *not (A < 5)* would not be equivalent to *A >= 5*
- Three-valued logic using the truth value *unknown*:
  - OR: (*unknown* **or** *true*) = *true*,
    (*unknown* **or** *false*) = *unknown*
    (*unknown* **or** *unknown*) = *unknown*
  - AND: (*true* **and** *unknown*) = *unknown,*
    (*false* **and** *unknown*) = *false,*
    (*unknown* **and** *unknown*) = *unknown*
  - NOT*:* (**not** *unknown*) = *unknown*
  - In SQL "*P* **is unknown**" evaluates to true if predicate *P* evaluates to *unknown*
- Result of select predicate is treated as *false* if it evaluates to *unknown*

Division Operator

- Given relations r(R) and s(S), such that $S \subset R$, $r \div s$ is the largest relation t(R-S) such that

  $t \times s \subseteq r$

- E.g. let $r(ID, course\_id) = \Pi_{ID,\ course\_id}(takes)$ and

  $s(course\_id) = \Pi_{course\_id}(\sigma_{dept\_name=\text{“Biology”}}(course))$

  then $r \div s$ gives us students who have taken all courses offered by the Biology department

- Can write $r \div s$ as

$$temp1 \leftarrow \Pi_{R-S}(r)$$
$$temp2 \leftarrow \Pi_{R-S}((temp1 \times s) - \Pi_{R-S,S}(r))$$
$$result = temp1 - temp2$$

  - See use of assignment: the result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.

Division Operation – Example

- Relations *r, s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
ϵ	6
ϵ	1
β	2

r

B
1
2

s

- *r* ÷ *s*:

A
α
β

Another Division Example

- Relations *r, s*:

A	*B*	*C*	*D*	*E*
α	a	α	a	*1*
α	a	γ	a	*1*
α	a	γ	b	*1*
β	a	γ	a	*1*
β	a	γ	b	*3*
γ	a	γ	a	*1*
γ	a	γ	b	*1*
γ	a	β	b	*1*

r

D	*E*
a	*1*
b	*1*

s

- *r* ÷ *s*:

A	*B*	*C*
α	a	γ
γ	a	γ

Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions

Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\prod_{F_1, F_2, \ldots, F_n}(E)$$

- E is any relational-algebra expression
- Each of F_1, F_2, …, F_n are are arithmetic expressions involving constants and attributes in the schema of E.
- Given relation *instructor(ID, name, dept_name,* salary) where salary is annual salary, get the same information but with monthly salary

$$\prod_{ID,\ name,\ dept\_name,\ salary/12}(instructor)$$

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Aggregate operation** in relational algebra

$$_{G_1,G_2,\ldots,G_n}\mathcal{G}_{F_1(A_1),F_2(A_2,\ldots,F_n(A_n)}(E)$$

  E is any relational-algebra expression

  - $G_1, G_2 \ldots, G_n$ is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

- Note: Some books/articles use γ instead of $\mathcal{G}$ (Calligraphic G)

Aggregate Operation – Example

- Relation *r*:

A	*B*	*C*
α	α	7
α	β	7
β	β	3
β	β	10

- $\mathcal{G}_{\mathbf{sum(c)}}(\mathrm{r})$

sum(*c*)
27

Aggregate Operation – Example

- Find the average salary in each department

  $dept\_name\ \mathcal{G}\ \mathbf{avg}(salary)\ (instructor)$

ID	*name*	*dept_name*	*salary*
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

dept_name	*avg_salary*
Biology	72000
Comp. Sci.	77333
Elec. Eng.	80000
Finance	85000
History	61000
Music	40000
Physics	91000

Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

$$dept\_name\ \mathcal{G}\ \textbf{max}(salary)\ \textbf{as}\ max\_sal\ (instructor)$$

- Suppose we want to find instructor who made highest salary
- Do not nest inside selection operator as follows:

$$A \leftarrow \mathcal{G}\ \textbf{max}(salary)\ \textbf{as}\ max\_sal\ (instructor)$$

$$\sigma_{salary = A}\ (instructor)$$

- A is a table with one row and one column
- No nested RA expressions (i.e., tables) inside selection condition

Aggregate Functions (Cont.)

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$$A \leftarrow \mathcal{G}\ \textbf{max}(salary)\ \textbf{as}\ max\_sal\ (instructor)$$

$\sigma_{salary = A}\ (instructor)$ -- this is wrong!

- A is a table with one row and one column
- No nested RA expressions (i.e., tables) inside selection condition

Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

$$dept\_name\ \mathcal{G}\ \textbf{avg}(salary)\ \textbf{as}\ avg\_sal\ (instructor)$$

- Correct:

$$A \leftarrow\ \mathcal{G}\ \textbf{max}(salary)\ \textbf{as}\ max\_sal\ (instructor)$$

$$\sigma_{\text{salary = A.max\_sal}}\ (instructor\ X\ A)$$

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations can be expressed using the assignment operator
- Compute new state of table, then assign it to table
- But this does not give much insight so we skip it ...

Multiset Relational Algebra

- Pure relational algebra removes all duplicates
  - e.g. after projection
- Multiset relational algebra retains duplicates, to match SQL semantics
  - SQL duplicate retention was initially for efficiency, but is now a feature
- Multiset relational algebra defined as follows
  - selection: has as many duplicates of a tuple as in the input, if the tuple satisfies the selection
  - projection: one tuple per input tuple, even if it is a duplicate
  - cross product: If there are m copies of $t1$ in r, and n copies of $t2$ in s, there are m x n copies of $t1.t2$ in r x s
  - Other operators similarly defined
    - E.g. union: $m + n$ *copies,* intersection: min(m, n) copies difference: min(0, $m - n$) copies

SQL and Relational Algebra

- **select** *A1, A2, .. An*
  from *r1, r2, …, rm*
  where P

  is equivalent to the following expression in multiset relational algebra

$$\Pi_{A1, .., An}(\sigma_{P}(r1 \times r2 \times .. \times rm))$$

- **select** *A1, A2,* **sum***(A3)*
  from *r1, r2, …, rm*
  where P
  group by *A1, A2*

  is equivalent to the following expression in multiset relational algebra

$$A1, A2\ \mathcal{G}_{\textbf{sum}(A3)}(\sigma_{P}(r1 \times r2 \times .. \times rm)))$$

Example Database

- E-Commerce Database with customers, products, and purchases:

CUSTOMER

CID	CNAME	CADDRESS

PURCHASE

CID	PID	TIMEDATE	PRICE

PRODUCT

PID	PNAME	PDESCRIPTION	PPRICE

Example Database

CUSTOMER

CID	CNAME	CADDRESS

PURCHASE

CID	PID	TIMEDATE	PRICE

PRODUCT

PID	PNAME	PDESCRIPTION	PPRICE

- CIDs of Customers named “John Smith”
- Names of customers who have ordered something costing >$10
- Names customers who have ordered an item called “IPhone 9”
- CID of customers named “John Smith” who have ordered an “IPhone9”
- For each customer, how much money they have spent.
- For each product, how many items were bought.

Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- It is the set of all tuples t such that predicate P is true for t
- t is a *tuple variable*, $t[A]$ denotes the value of tuple t on attribute A
- $t \in r$ denotes that tuple t is in relation r
- P is a *formula* similar to that of the predicate calculus

Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g., $<$, $\leq$, $=$, $\neq$, $>$, $\geq$)
3. Set of connectives: and ($\wedge$), or (v), not ($\neg$)
4. Implication ($\Rightarrow$): x $\Rightarrow$ y, if x if true, then y is true

$$x \Rightarrow y \equiv \neg x \text{ v } y$$

5. Set of quantifiers:
   - $\exists\, t \in r\,(Q\,(t\,)) \equiv$ "there exists" a tuple in t in relation r such that predicate $Q\,(t\,)$ is true
   - $\forall\, t \in r\,(Q\,(t\,)) \equiv Q$ is true "for all" tuples t in relation r

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than $80,000

$$\{t \mid t \in instructor \wedge t[salary] > 80000\}$$

- As in the previous query, but output only the *ID* attribute value

$$\{t \mid \exists\ s \in \text{instructor}\ (t[ID] = s[ID] \wedge s[salary] > 80000)\}$$

Notice that a relation on schema (*ID*) is implicitly defined by the query

Example Queries

- Find the names of all instructors whose department is in the Watson building

$$\{t \mid \exists s \in \textit{instructor}\ (t[\textit{name}] = s[\textit{name}] \wedge \exists u \in \textit{department}\ (u[\textit{dept\_name}] = s[\textit{dept\_name}] \wedge u[\textit{building}] = \text{“Watson”}))\}$$

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{t \mid \exists s \in \textit{section}\ (t[\textit{course\_id}] = s[\textit{course\_id}] \wedge s[\textit{semester}] = \text{“Fall”} \wedge s[\text{year}] = 2009) \vee \exists u \in \textit{section}\ (t[\textit{course\_id}] = u[\textit{course\_id}] \wedge u[\textit{semester}] = \text{“Spring”} \wedge u[\text{year}] = 2010)\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{t \mid \exists s \in section\ (t[course\_id] = s[course\_id] \wedge s[semester] = \text{“Fall”} \wedge s[year] = 2009) \wedge \exists u \in section\ (t[course\_id] = u[course\_id] \wedge u[semester] = \text{“Spring”} \wedge u[year] = 2010)\}$$

- Find the set of all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\{t \mid \exists s \in section\ (t[course\_id] = s[course\_id] \wedge s[semester] = \text{“Fall”} \wedge s[year] = 2009) \wedge \neg \exists u \in section\ (t[course\_id] = u[course\_id] \wedge u[semester] = \text{“Spring”} \wedge u[year] = 2010)\}$$

Safety of Expressions

- It is possible to write tuple calculus expressions that generate infinite relations.
- For example, $\{ t \mid \neg\, t \in r \}$ results in an infinite relation if the domain of any attribute of relation r is infinite
- To guard against the problem, we restrict the set of allowable expressions to safe expressions.
- An expression $\{t \mid P(t)\}$ in the tuple relational calculus is *safe* if every component of t appears in one of the relations, tuples, or constants that appear in P
  - NOTE: this is more than just a syntax condition.
    - E.g. $\{ t \mid t[A] = 5 \vee \textbf{true} \}$ is not safe **---** it defines an infinite set with attribute values that do not appear in any relation or tuples or constants in P.

Universal Quantification

- Find all students who have taken all courses offered in the Biology department
  - $\{t \mid \exists\, r \in \mathit{student}\,(t\,[\mathit{ID}] = r\,[\mathit{ID}]) \wedge$
    $(\forall\, u \in \mathit{course}\,(u\,[\mathit{dept\_name}] = \text{“Biology”} \Rightarrow$
    $\exists\, s \in \mathit{takes}\,(t\,[\mathit{ID}] = s\,[\mathit{ID}\,] \wedge$
    $s\,[\mathit{course\_id}] = u\,[\mathit{course\_id}]))\}$
  - Note that without the existential quantification on student, the above query would be unsafe if the Biology department has not offered any courses.

Domain Relational Calculus

- A nonprocedural query language equivalent in power to the tuple relational calculus
- Each query is an expression of the form:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n) \}$$

  - $x_1, x_2, \ldots, x_n$ represent domain variables
  - P represents a formula similar to those in predicate calculus

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than $80,000
  - $\{< i, n, d, s > \mid < i, n, d, s > \in instructor \wedge s > 80000\}$
- As in the previous query, but output only the *ID* attribute value
  - $\{< i > \mid \exists\, n, d, s\ (< i, n, d, s > \in instructor \wedge s > 80000)\}$
- Find the names of all instructors whose department is in the Watson building

$$\{< n > \mid \exists\, i, d, s\ (< i, n, d, s > \in instructor \wedge \exists\, b, a\ (< d, b, a > \in department \wedge b = \text{“Watson”}\ ))\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{<c> \mid \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, r, t> \in section \wedge s = \text{“Fall”} \wedge y = 2009) \vee \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, r, t> \in section] \wedge s = \text{“Spring”} \wedge y = 2010)\}$$

This case can also be written as

$$\{<c> \mid \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, r, t> \in section \wedge ((s = \text{“Fall”} \wedge y = 2009) \vee (s = \text{“Spring”} \wedge y = 2010))\}$$

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{<c> \mid \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, r, t> \in section \wedge s = \text{“Fall”} \wedge y = 2009) \wedge \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, r, t> \in section] \wedge s = \text{“Spring”} \wedge y = 2010)\}$$

Relative Expressive Power

- Save TRC & DRC are as powerful as basic relational algebra
- Extended relational algebra is
  - More powerful than TRC/DRC
  - useful for defining SQL semantics
  - useful for designing SQL queries
  - useful for understanding SQL execution
- Relational calculus is
  - useful for designing SQL queries (sometimes)
  - The basis for QBE
- Different viewpoints: relational calculus vs. relational algebra
- Limitation for all three languages: transitive closure

Query By Example

- A graphical query language which is based (roughly) on domain relational calculus
- Two dimensional syntax – system creates templates of relations that are requested by users
- Queries are expressed "by example" on skeleton tables (forms)

branch	branch-name	branch-city	assets

customer	customer-name	customer-street	customer-city

loan	loan-number	branch-name	amount

Queries on One Relation

- Find all loan numbers at the Perryridge branch.

loan	*loan_number*	*branch_name*	*amount*
	P._x	Perryridge	

  - _x is a variable (optional; can be omitted in above query since it is not used elsewhere in the query)
  - P. means print (display)
  - duplicates are removed by default
  - To retain duplicates use P.ALL

loan	*loan_number*	*branch_name*	*amount*
	P.ALL.	Perryridge	

Queries on One Relation (Cont.)

- **Find the loan numbers of all loans made jointly to Smith and Jones.**

borrower	*customer_name*	*loan_number*
	Smith	P._x
	Jones	_x

- **Find all customers who live in the same city as Jones**

customer	*customer_name*	*customer_street*	*customer_city*
	P._x		_y
	Jones		_y

Queries on Several Relations

- Find the names of all customers who have a loan from the Perryridge branch.

loan	*loan-number*	*branch-name*	*amount*
	_x	Perryridge	

borrower	*customer-name*	*loan-number*
	P._y	_x

Negation in QBE

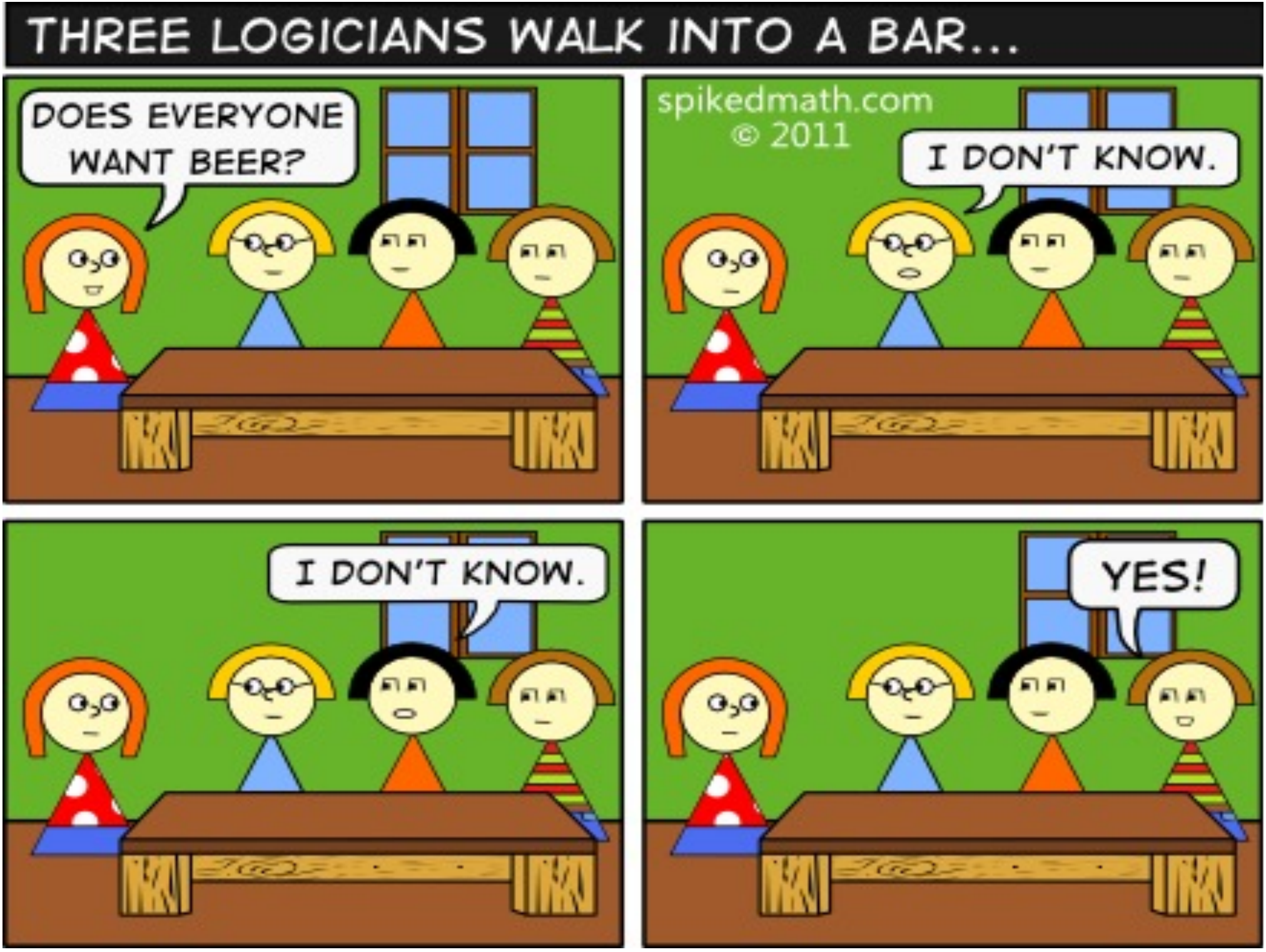
- Find the names of all customers who have an account at the bank, but do not have a loan from the bank.

depositor	*customer-name*	*account-number*
	P._*x*	

borrower	*customer-name*	*loan-number*
¬	_*x*	

¬ means "there does not exist"

Example: Quantization and 3-Valued Logic


THREE LOGICIANS WALK INTO A BAR...
DOES EVERYONE WANT BEER?
spikedmath.com
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I DON'T KNOW.
I DON'T KNOW.
YES!