

Data Structure and Algorithms

Lab6

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Homework Question:

In this exercise, we will explore different types of randomized algorithms. We say that a randomized algorithm is a **Las Vegas algorithm** if it is always correct (that is, it returns the right answer with probability 1), but the running time is a random variable. We say that a randomized algorithm is a **Monte Carlo algorithm** if there is some probability that it is incorrect. For example, QuickSort (with a random pivot) is a Las Vegas algorithm, since it always returns a sorted array, but it might be slow if we get very unlucky.

We will revisit the Majority Element problem to get more insight on randomized algorithms.

Answer:

Algorithm	Monte Carlo or Las Vegas?	Expected Running Time	Worst-case running time	Probability of returning a majority element
Algorithm 1	Las Vegas	$O(n)$	infinite	1
Algorithm 2	Monte Carlo	$O(n)$	$O(n)$	$1 - \frac{1}{2^{100}}$
Algorithm 3	Las Vegas	$O(n)$	$O(n^2)$	1

Algorithm 1 : It always gives correct result, so it is Las Vegas Algorithm.

Expected Running Time = Choosing a random p takes $O(1)$ and $\text{isMajority}(P,p)$ takes $O(n)$. So, expected running time is $O(n)$.

Worst-case running time = Infinite (for every choice of $p \in P$, p is not the majority element).

Probability of returning a majority element = $P[\text{failure}] = 0$. That's why Probability of returning a majority element is 1.

Algorithm 2 : It does not always gives you correct result, so it is Monte Carlo Algorithm.

Expected Running Time = Choosing a random p takes $O(1)$ and $\text{isMajority}(P,p)$ takes $O(n)$. So, expected running time is $O(n)$.

Worst-case running time = $O(n)$ whether it is wrong or right.

Probability of returning a majority element:

Let P be the probability to find majority element

$$P[\text{failure}] = \frac{1}{2^{100}}$$

$$\text{Probability of returning a majority element} = 1 - \frac{1}{2^{100}}$$

Algorithm 3 : It always gives correct result, so it is Las Vegas Algorithm.

Expected Running Time = $O(n)$.

Worst-case running time = $O(n) + O(n^2) = O(n^2)$

Probability of returning a majority element = $P[\text{failure}] = 0$. That's why Probability of returning a majority element is 1.
