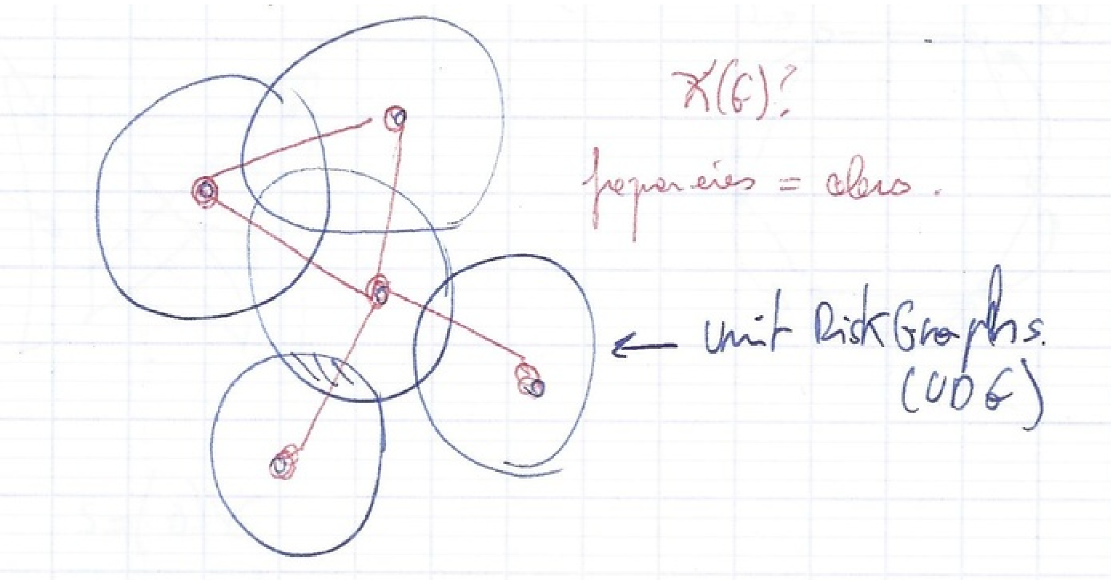
Graph and discrete structure

Louis Esperet (GSCOP) - 4 lectures.
Lecture 3

List coloring

Antenna exemple



Theroem

If G is a UDG, $\chi(G) \leq 3 \cdot \omega(G) - 2$
$\omega(G) \leq \chi(G)$

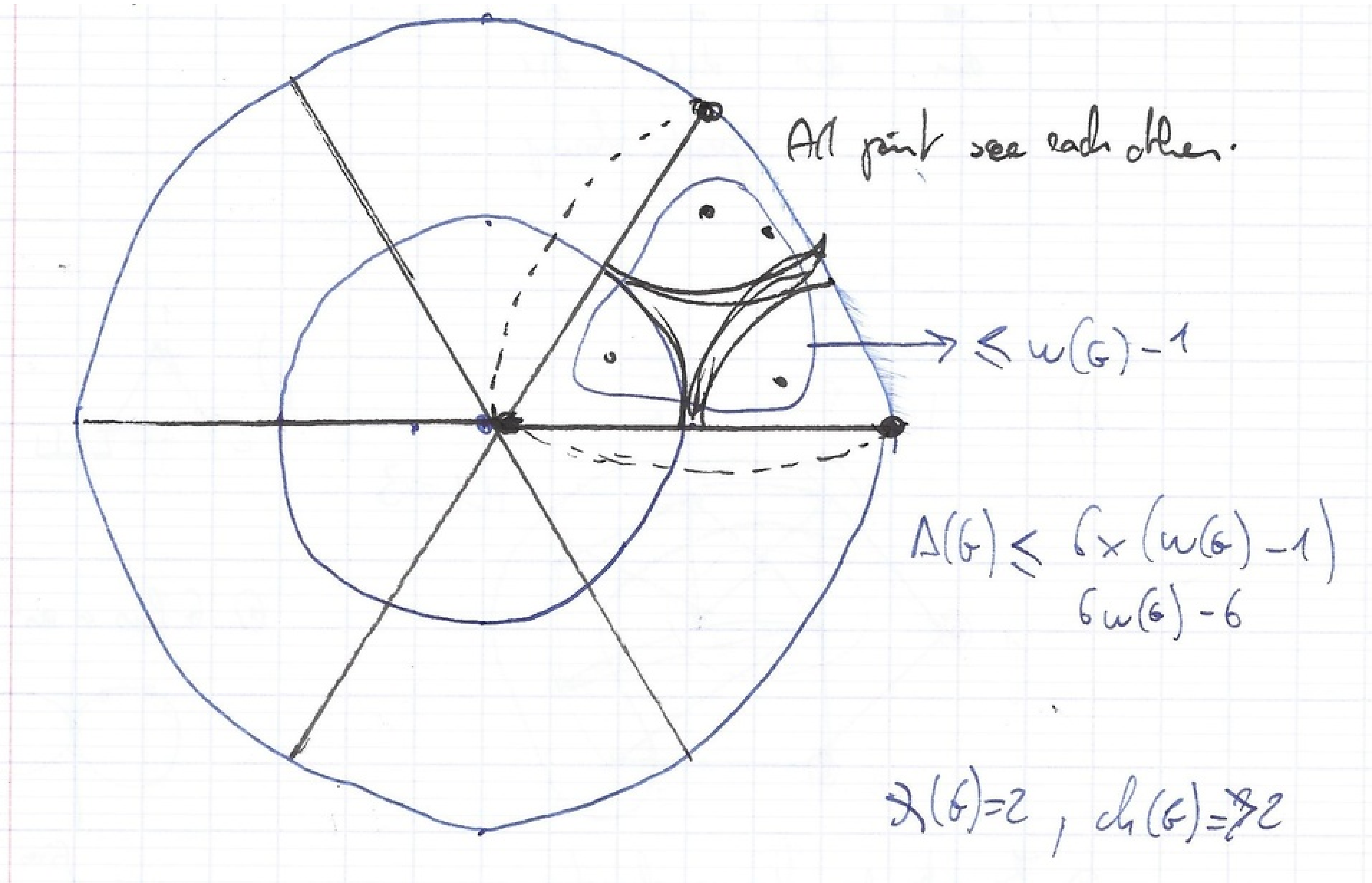
In particular, our proof will give a 3-approximation algorithm.

Proof of $\leq 6 \cdot \omega(G) - 5$

First we proove that $\chi(G) \leq 6 \cdot \omega(G) - 5$.
Indeed, we proove that $\Delta(G) \leq 6 \cdot \omega(G) - 6$.

Counter exemple

Proof by exemple



Proof of $\leq 3 \cdot \omega(G) - L$

Exercice for next time : finish the proof.
Idea of the proof for $\chi(G) \leq 3 \cdot \omega(G) - L$

Definition

Definition : list coloring

A k-list-assignement for G is an assignement of lists $L(v)$ for each vertex v. Such that, $\forall v, |L(v)| = k$.

An L-coloring of G is a proper coloring c such that $\forall v, c(v) \in L(v)$

Definition : k-list-colorable

G is k-list-colorable if

Theorem on even cycle

Even cycle are 2-colorable.

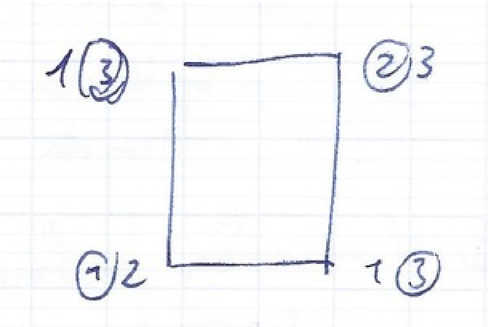
Proof that even cycle are two colorable

for any k-list-assignement G is L-colorable.

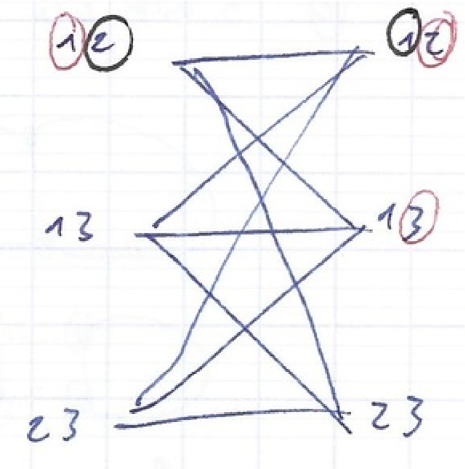
Defition : Choice number

The *choice number* of G is the smallest k such that G is k-list-colorable $ch(G)$.

Exemple



G is k-list-colorable $6RightarrowG$ is L-colorable with $L(v) = \{1, 2, \cdots, 3\}, \forall v$
$\Leftrightarrow \chi(G) \leq k, G$ is k-colorable.

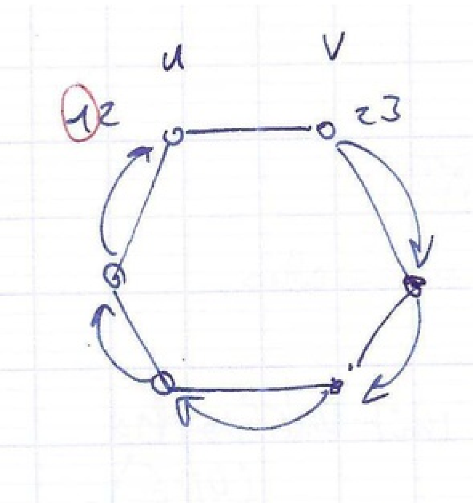


Theorem on even cycle

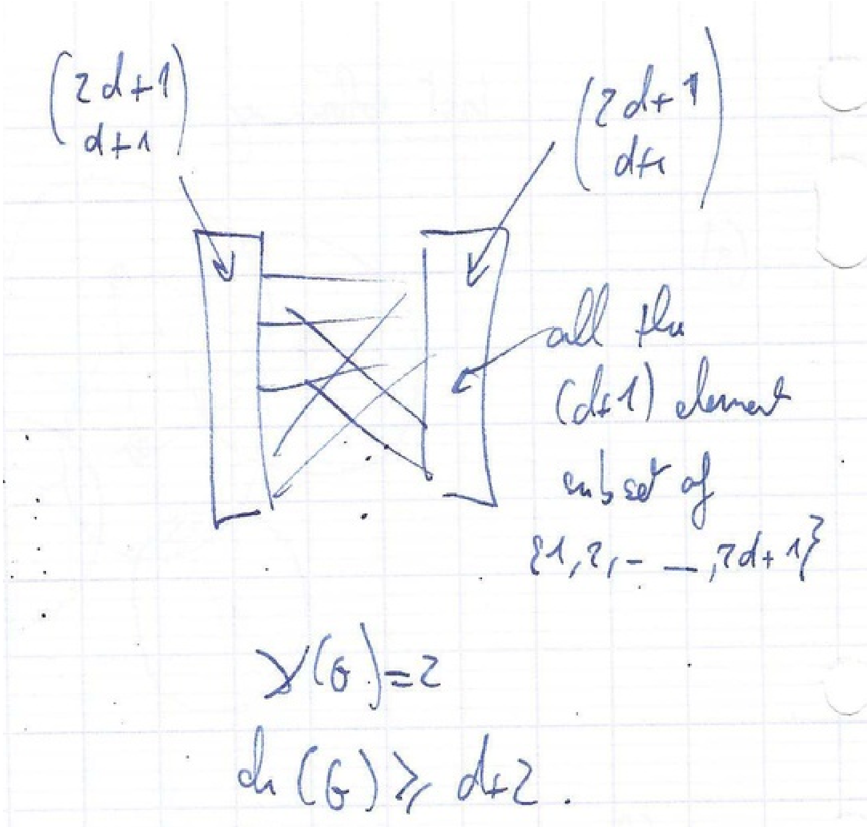
Even cycle are 2-colorable.

Proof that even cycle are two-list-colorable

- all the list are the same(say $\{1, 2\}$) $\rightarrow$ (same as k-colorable) ok since even cycle are two colorable.
- There are at least 2 different lists. $\Rightarrow$ two adjacent vertices u, v have different lists (because cycle are connected).



$L(u)$ has a color that does'nt appear in $L(v)$.

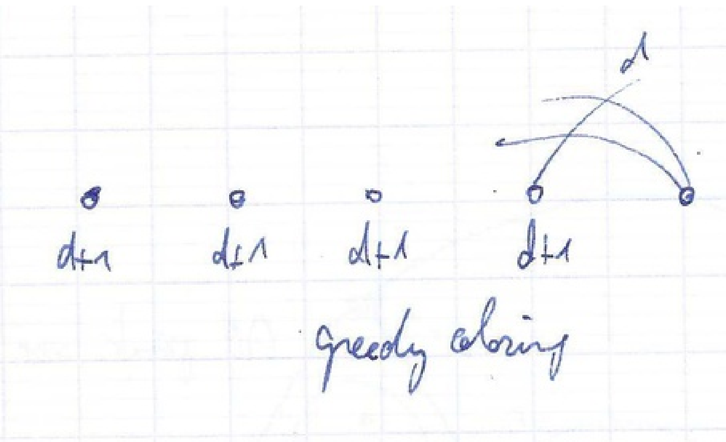


Exercice : proove that $ch(G) \geq d+2$

Therorem on g-degenerate

If G is d-degenerate, then $ch(G) \leq d+1$.

Proof



Corollary

Planar graph are 6-list-colorable.

- 1993 : planar graps are 5-list-colorable.
- 1994 : some planar graph are not 4-list-colorable.

Theorem (Thomassen)

Planar graph are 5-list-colorable.

Instead we will proove the following by induction.

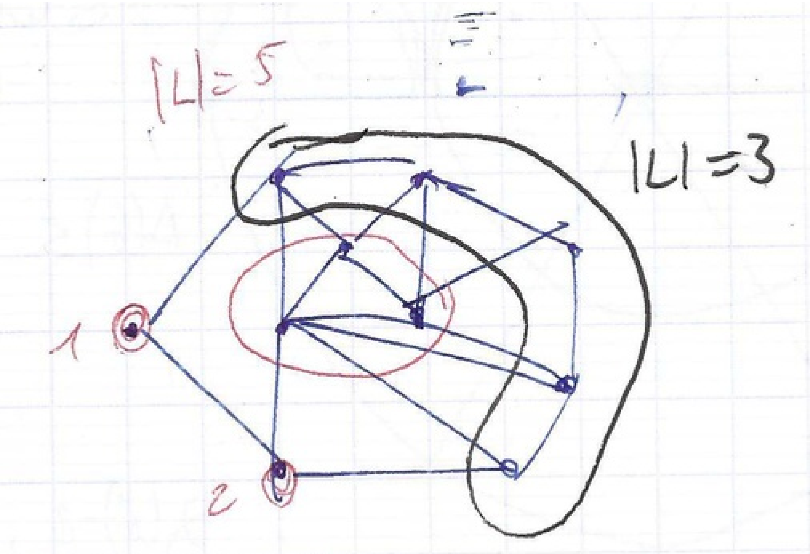
If G is a planar near-triangulation.

- 2 adjacent vetices of the outerface have a fixed different color.
- The other vertices of the outerface have a list of size 3.
- The rest of the vertices have lists of size 5.

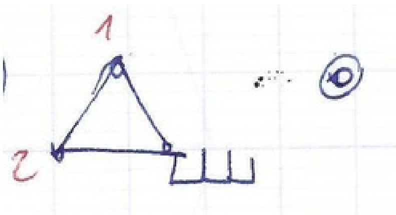
You can color the graph.

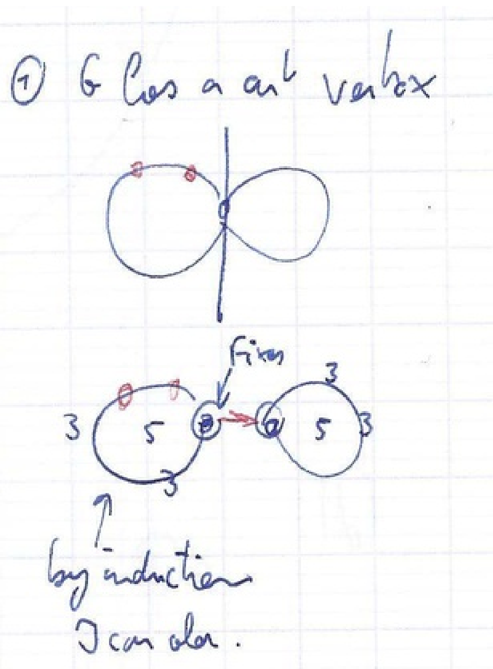
Near triangulation

All the inner fae are triangulated.

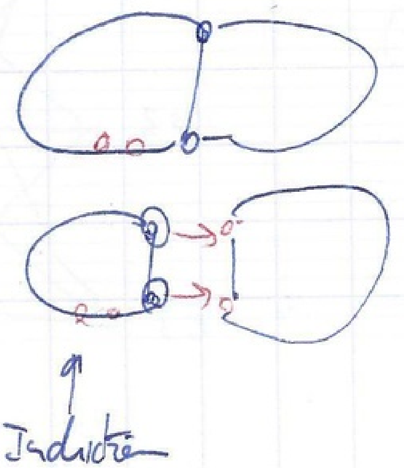


$n = 3$





② The outer face has a chord.



③ No ~~?~~ outer vertices ~~?~~ chord.