Explanation of "On Two Minimax Theorems in Graph"

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Contents

1 Definitions

1.1 Edge-disjoint paths

Two paths are edge-disjoint if they do not have any internal edge in common.

1.2 Branching :

Branching One In a directed graph $D = (V, A)$ a branching is a set of arcs not containing circuits s.t. each node of D is entered at most by one arc in A'. (So the arc in A' make up a forest).

Branching Two A branching B is an arc set in a digraph D that is a forest such that every node of D is the head of at most one arc of B. A branching that is a tree is called an arborescence. A branching that is a spanning tree of D is called a spanning arborescence of D. Clearly, in a spanning arborescence B of D every node of D is the head of one arc of B except for one node. This node is called the root of B. If r is the root of arborescence B we also say that B is rooted at r or $r-$rooted.

R-branching For a subset $R \subset V$, an $r-$branching in G is a spanning forest $B \subset G$ in which all vertices of $V - R$ have outdegree precisely 1.

r-branching When R just consists of a single vertex r, we refer to B as an $r-$branching.

1.3 k-connected

A graph G (digraph D) is called $k-$connected ($k-$diconnected) if every pair s, t of nodes is connected by at least k $[s, t]-$paths ($(s, t)-$dipaths) whose sets of internal nodes are mutually disjoint.

1.4 a-cut

A $a-$cut of G determined by a set $S \subset V(G)$ is the set of edges going from S to $V(G) - S$. It will be denoted by $\Delta_G(S)$. We also set that $\delta_G(S) = |\Delta_G(S)|$.

1.5 r-Cuts and r-Arborescences

Consider a connected digraph (N, A) with $r \in N$ and nonnegative integer arc lengths l_a for $a \in A$. An $r-$arborescence is a minimal arc set that contains an $rv-$dipath for every $v \in N$. It follows that an $r-$arborescence contains $|N| - 1$ arcs forming a spanning tree and each node of $N - r$ is entered by exactly one arc. The minimal transversals of the clutter of r-arborescences are called $r-$cuts.

1.6 A note on orientation

A rooted tree is a tree in which one vertex has been designated the root. The edges of a rooted tree can be assigned a natural orientation, either away from or towards the root, in which case the structure becomes a directed rooted tree. When a directed rooted tree has an orientation away from the root, it is called an arborescence, branching, or out-tree; when it has an orientation towards the root, it is called an anti-arborescence or in-tree.

2 Definitions examples

2.1 Branchings examples

Taking the next figure as exemple :

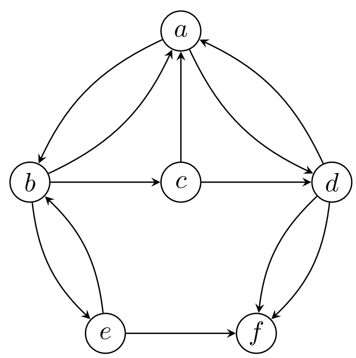

Figure 1: A graph

2.1.1 Simple branching

Not yet.

2.1.2 Edge disjoint branching

Decomposition of the exemple figure in 3 branching routed respectively at a, c, d, e.

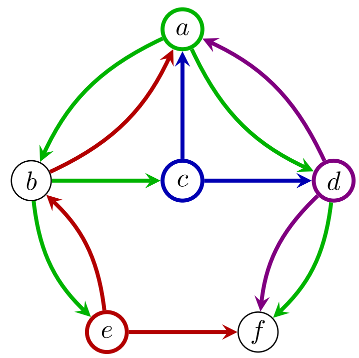

Figure 2: A graph

2.1.3 Edge disjoint $a-$routed branching

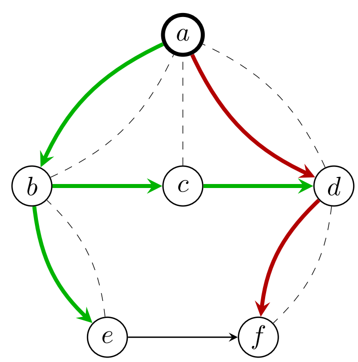

Figure 3: A graph

2.2 Cuts examples

2.2.1 General cut

2.2.2 $a-$cut

3 Theorems

THEOREM 1 (Edmonds)**.** *The maximum number of edge-disjoint branchings (rooted at a) equals the minimum number of edges in $a-$cuts.*