# Explanation of "On Two Minimax Theorems in Graph"

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## Contents



## 1 Theorems

**THEOREM 1** (Edmonds)**.** *The maximum number of edge-disjoint branchings (rooted at a) equals the minimum number of edges in* $a-$*cuts.*

## 2 Definitions

### Edge-disjoint branchings

Two branching are edge-disjoint if they do not have any internal edge in common.

### Branching :

**Branching One** In a directed graph $D = (V, A)$ a branching is a set of arcs not containing circuits s.t. each node of $D$ is entered at most by one arc in $A'$. (So the arc in $A'$ make up a forest).

**Branching Two** A branching $B$ is an arc set in a digraph $D$ that is a forest such that every node of $D$ is the head of at most one arc of $B$. A branching that is a tree is called an arborescence. A branching that is a spanning tree of $D$ is called a spanning arborescence of $D$. Clearly, in a spanning arborescence B of $D$ every node of $D$ is the head of one arc of $B$ except for one node. This node is called the root of $B$. If $r$ is the root of arborescence $B$ we also say that $B$ is rooted at $r$ or $r-$rooted.

**R-branching** For a subset $R \subset V$, an $R-$branching in $G$ is a spanning forest $B \subset G$ in which all vertices of $V - R$ have outdegree precisely 1.

**r-branching** When $R$ just consists of a single vertex $r$, we refer to $B$ as an $r-$branching.

### k-connected

A graph $G$ (digraph $D$) is called $k-$connected ($k-$diconnected) if every pair s, t of nodes is connected by at least $k$ $[s,t]-$paths ($(s,t)-$dipaths) whose sets of internal nodes are mutually disjoint.

### a-cut

A $a-$cut of $G$ determined by a set $S \subset V(G)$ is the set of edges going from $S$ to $V(G) - S$. It will be denoted by $\Delta_G(S)$. We also set that $\delta_G(S) = |\Delta_G(S)|$.

### r-Cuts and r-Arborescences

Consider a connected digraph $(N, A)$ with $r \in N$ and nonnegative integer arc lengths $l_a$ for $a \in A$. An $r-$arborescence is a minimal arc set that contains an $rv-$dipath for every $v \in N$. It follows that an $r-$arborescence contains $|N| - 1$ arcs forming a spanning tree and each node of $N - r$ is entered by exactly one arc. The minimal transversals of the clutter of r-arborescences are called $r-$cuts.

We define a clutter C to be a family $E(C)$ of subsets of a finite ground set $V(C)$ with the property that $S_1 \nsubseteq S_2$ for all distinct $S_1, S_2 \in E(C)$. $V(C)$ is called the set of vertices and $E(C)$ the set of edges of $C$.

### r-cut 2

An r-cut in a digraph is an edge set $\delta^+(X)$ for some $X \subset V(G)$ with $r \in X$.

### r-arborescence 2

an Arborescence in a digraph $D = (V, A)$ is a set $A'$ of arcs making up a spanning tree such that each node of $D$ is entered by at most one arc in $A'$. It follows that there is exactly one node r that is not entered by any arc of $A'$. This node is called the root of $A'$, and $A'$ is called rooted in $r$, or an $r-$arborescence.

### A note on orientation

A rooted tree is a tree in which one vertex has been designated the root. The edges of a rooted tree can be assigned a natural orientation, either away from or towards the root, in which case the structure becomes a directed rooted tree. When a directed rooted tree has an orientation away from the root, it is called an arborescence, branching, or out-tree; when it has an orientation towards the root, it is called an anti-arborescence or in-tree.

## 3 Examples

#### Simple branching

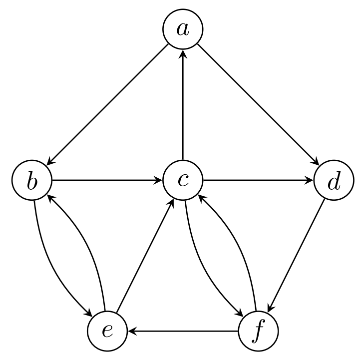


Figure 1: A digraph $G = (A, V)$

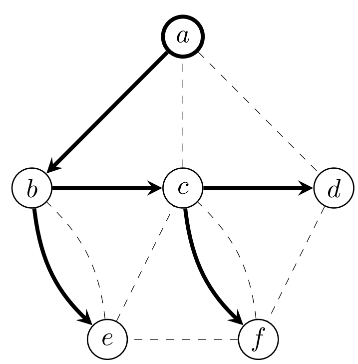


Figure 2: A branching of $G$ routed at $a$.

**Edge disjoint branching**

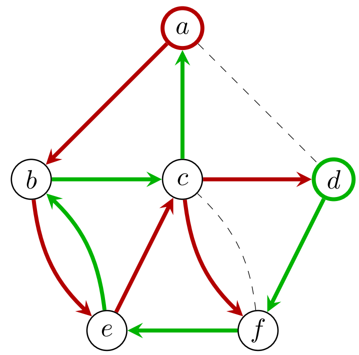


Figure 3: Two edge-disjoint branching routed respectively at $a$ and $d$.

**Edge disjoint a-routed branching**

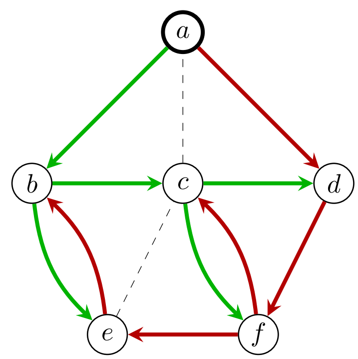


Figure 4: Two edge-disjoint $a$-routed branching.

**a-cut**