CS 234 H.W. #3

1. a.)

From the "CartPole_1000" and "CartPole_~~std~~_1000" plots, it's clear that a score of 200 is achieved well before iteration 100, but after iteration 105 the returns stabilize. Increasing the batch_size hyper-parameter to 2000 caused the agent to achieve the 200 score in 60 iterations and less variance (since the gradient estimates were less noisy), ~~however~~ and the returns ~~fail to~~ stabilize and ~~continue to~~ tend to intermittently oscillate ~~significantly~~ slightly ~~from~~ 200. Decreasing the batch_size to 500 caused the agent to achieve the 200 score in about 20 iterations, much faster than the default size of 1000 and on-par with setting size to 2000. Of course, this came at the expense of higher variance as indicated by the "CartPole_std_500" plot. (noisier gradient estimates). Finally, removing the baseline after re-setting to original configuration settings yielded a significantly less stable agent that only achieves 200 at iteration 120 and variance on par with batch_size = 500 with a baseline.

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b.)

We see from plot "Inverted Pendulum_1000" that for the default configuration settings the agent doesn't reach the required score of 1000 within 100 iterations. However, after doubling batch-size to 2000, the desired target is met ("Inverted Pendulum_2000").

c.)

After adjusting gamma to 0.9, batch size to 50,000, and the policy network to 2-hidden layers of size 32 nodes each, we obtain the required score of 200 within 100 iterations as shown in "Half Cheetah_adj

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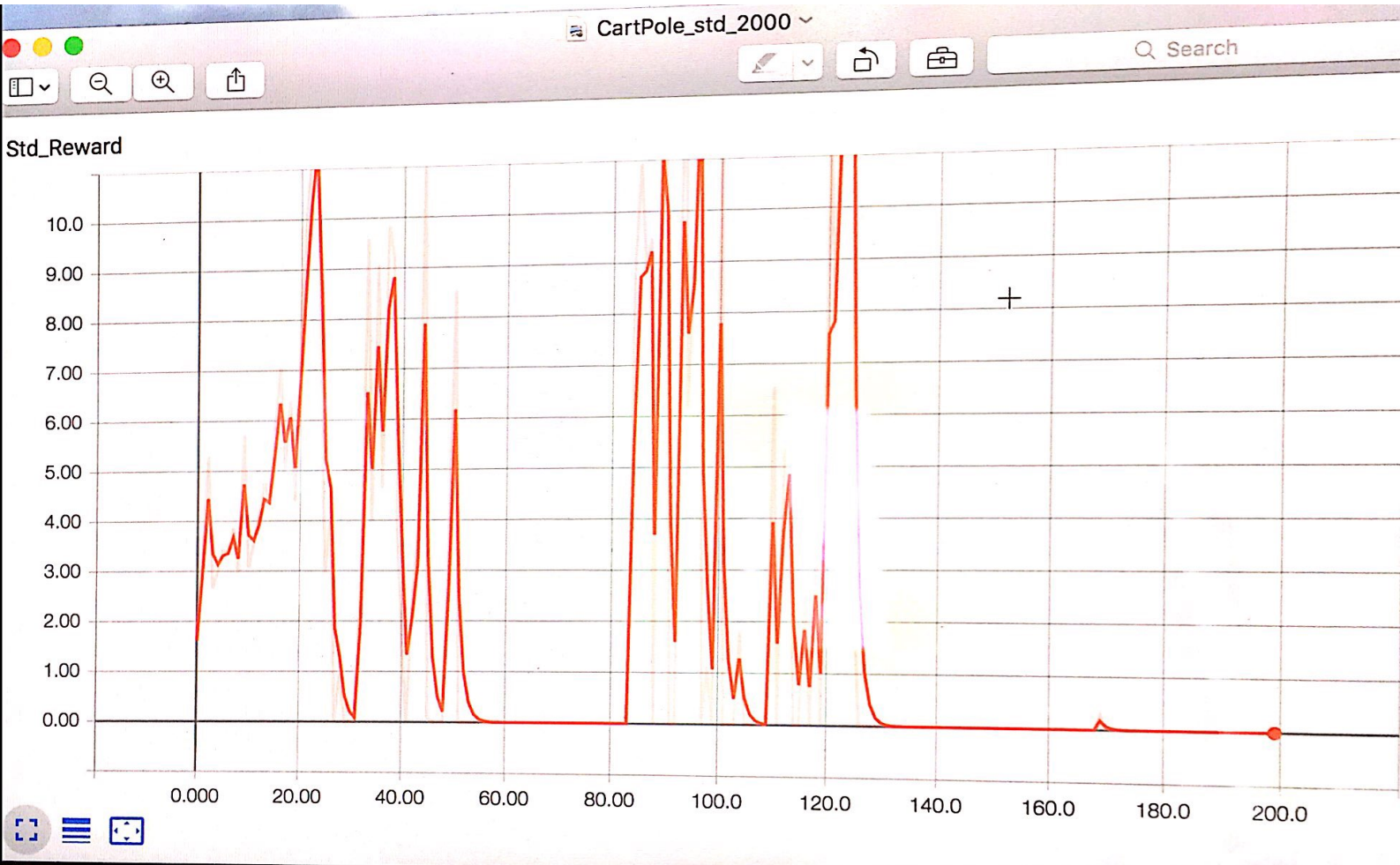

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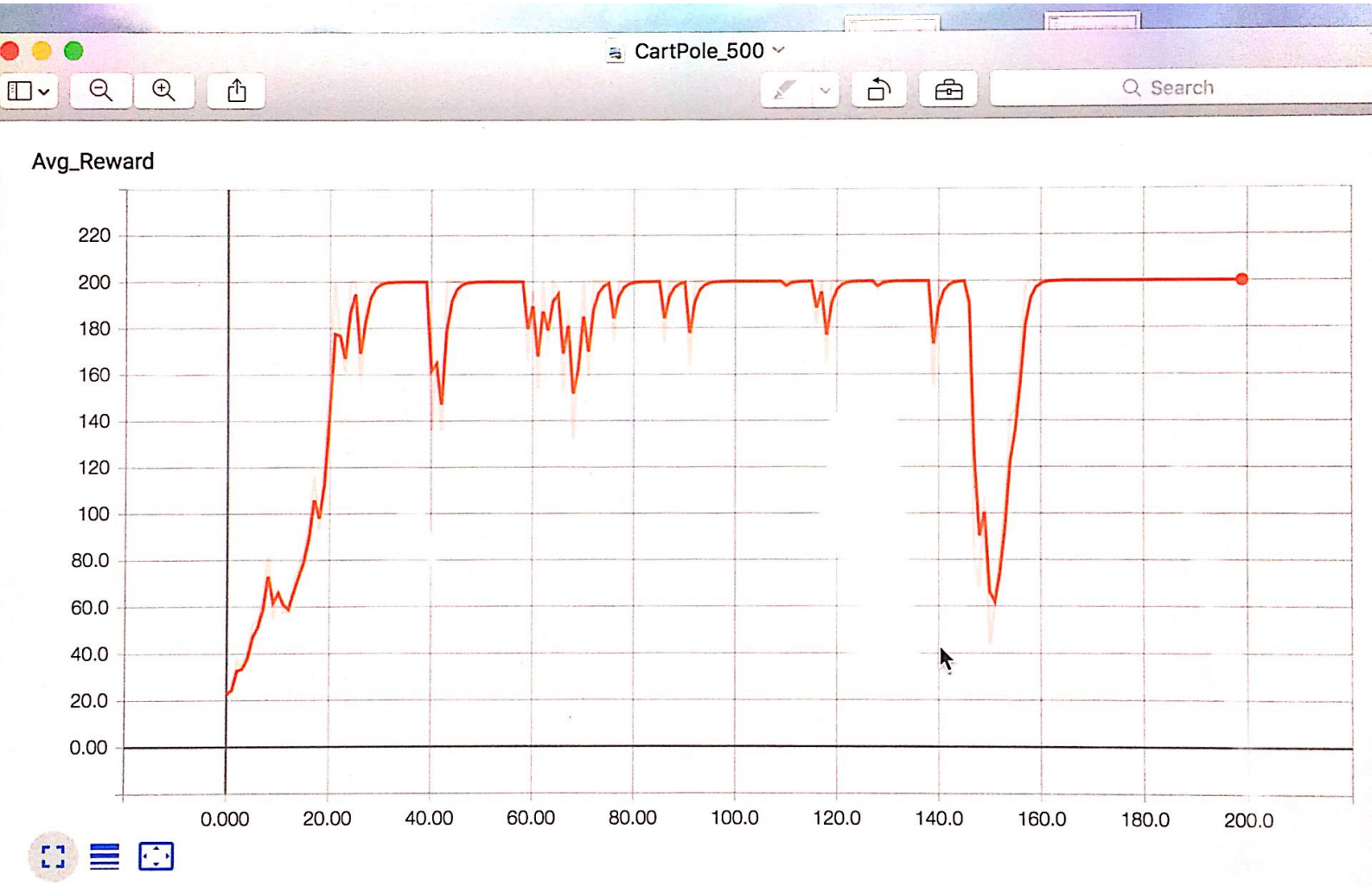

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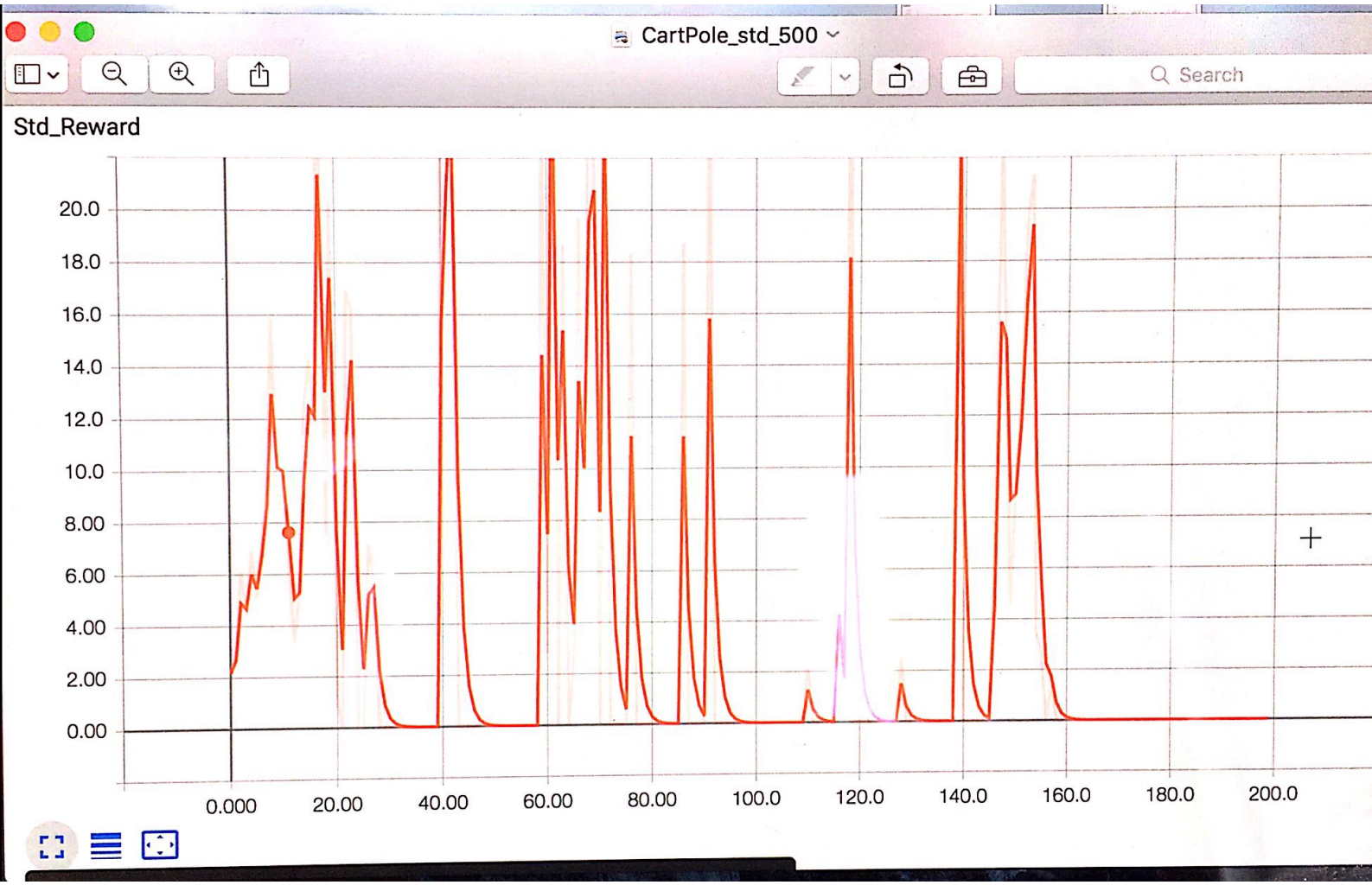

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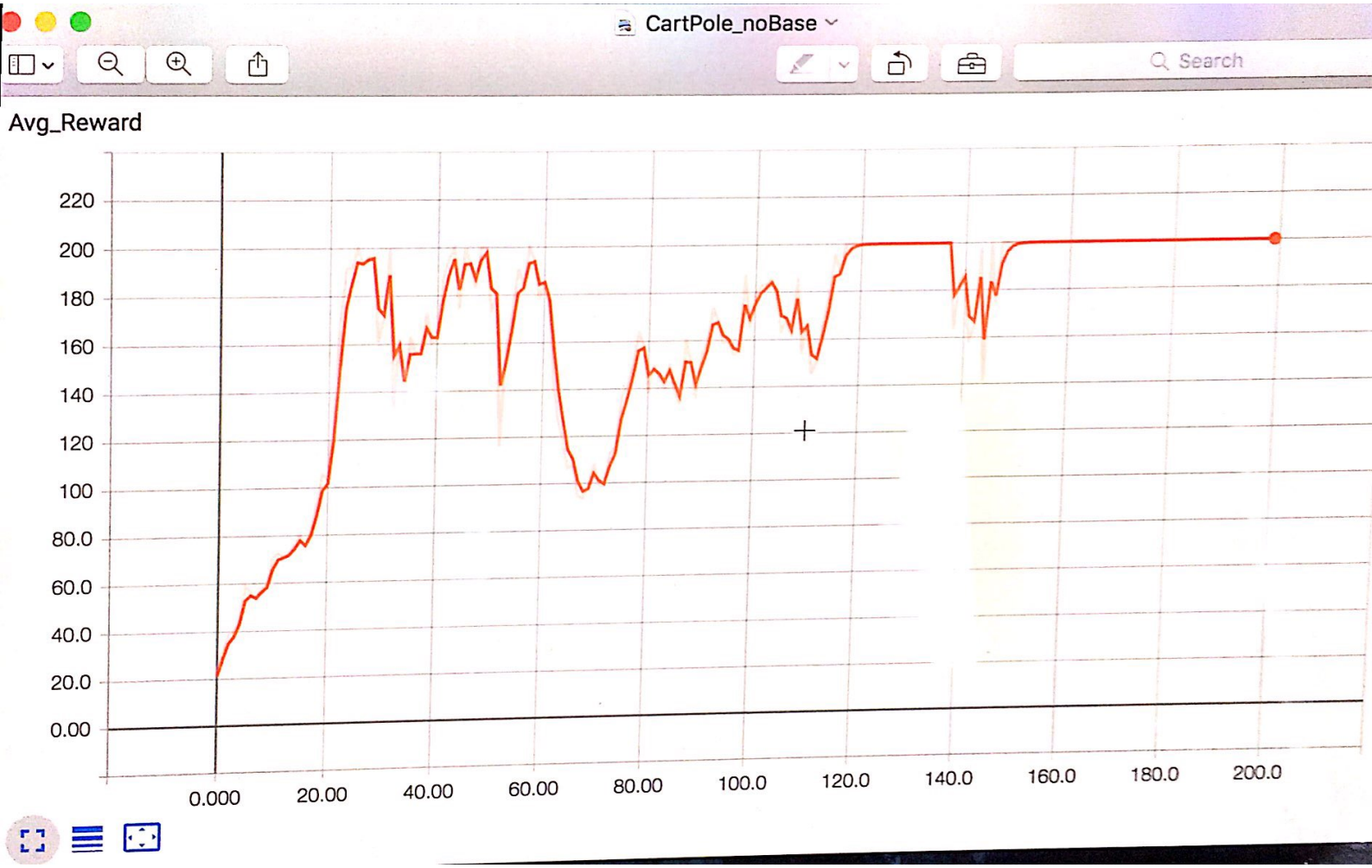

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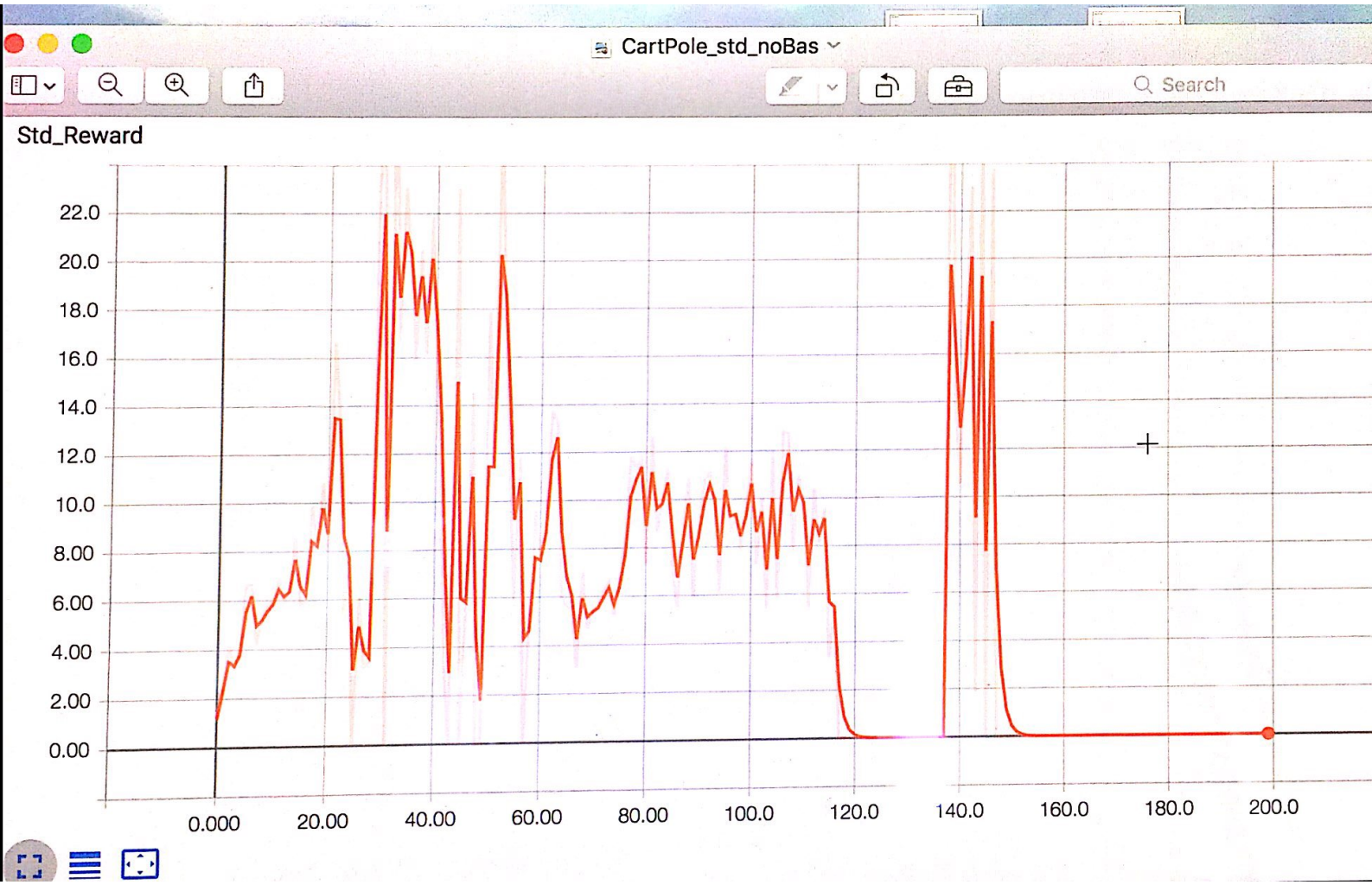

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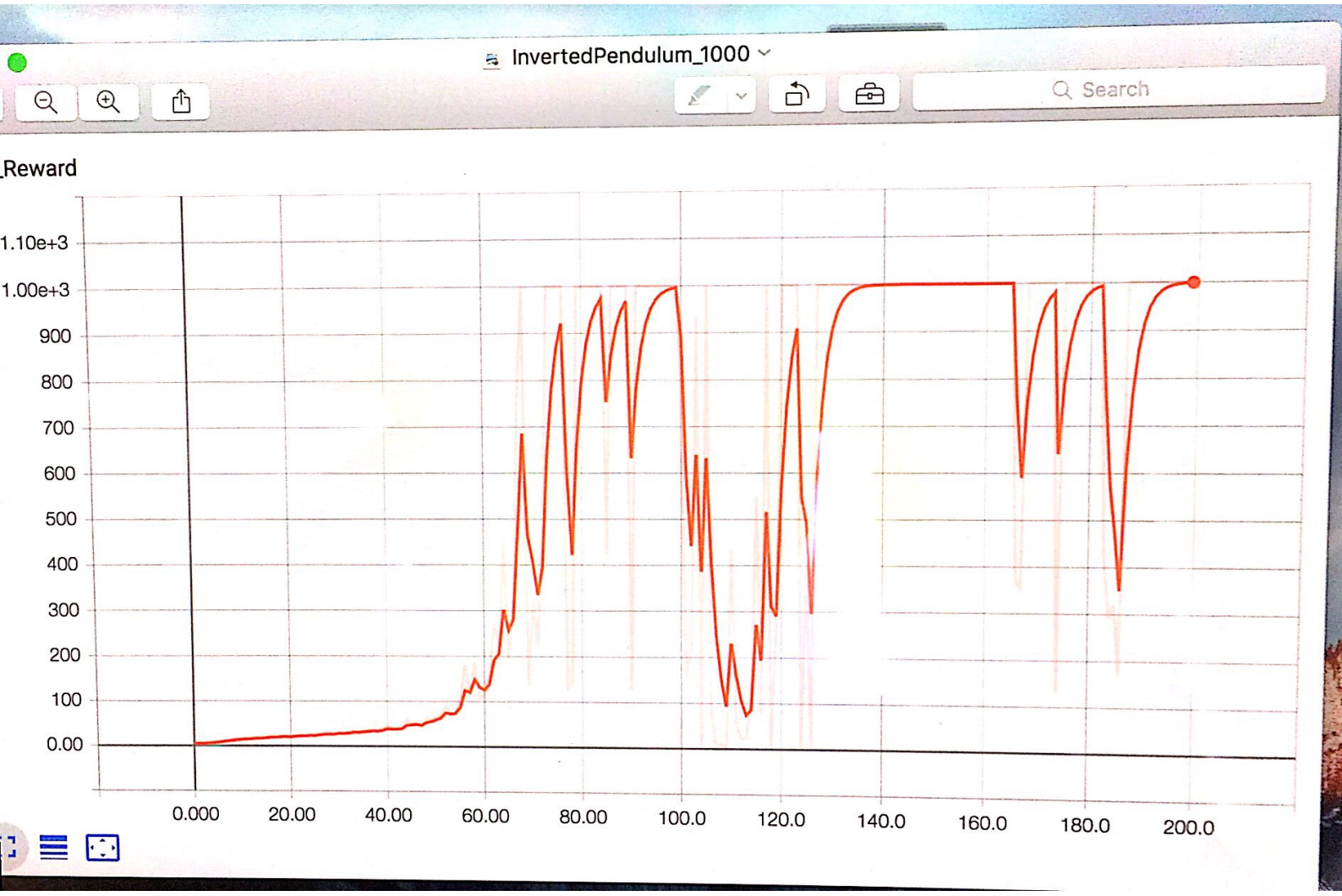

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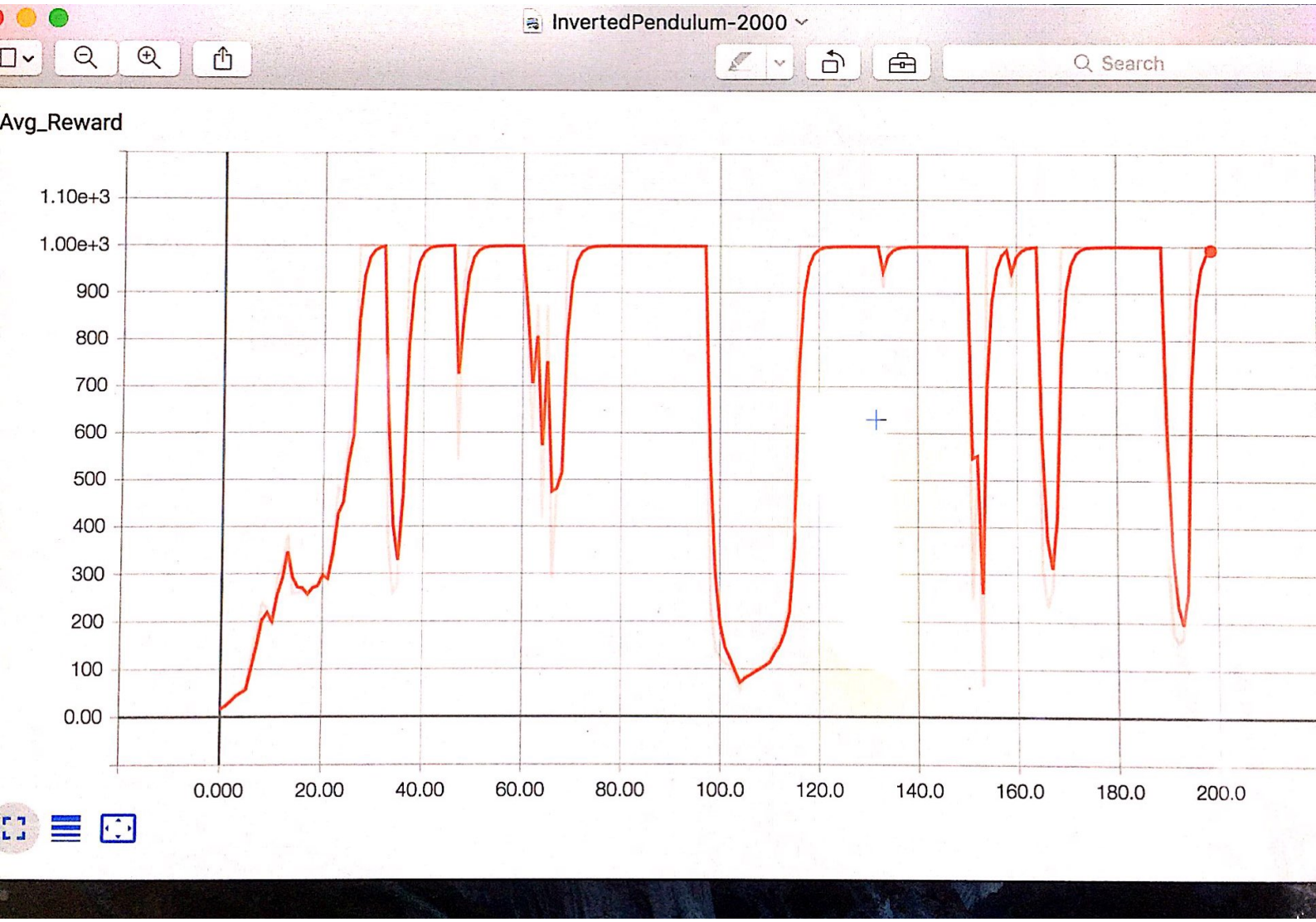

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2. a.) By definition,

$$V_\pi(s) = E_\pi\left[\sum_{t=1}^{\infty} \gamma^{t-1} r_t \mid s_1 = s\right]$$

$$= \sum_{t=1}^{\infty} \gamma^{t-1} E_\pi[r_t \mid s_1 = s]$$

$$\therefore \left| V_\pi(s) - \sum_{t=1}^{T'} \gamma^{t-1} E_\pi[r_t \mid s_1 = s] \right|$$

$$= \left| \sum_{t=1}^{\infty} \gamma^{t-1} E_\pi[r_t \mid s_1 = s] - \sum_{t=1}^{T'} \gamma^{t-1} E_\pi[r_t \mid s_1 = s] \right|$$

$$= \left| \sum_{t=T'+1}^{\infty} \gamma^{t-1} E_\pi[r_t \mid s_1 = s] \right|$$

as $r_t \in [0, 1], \Rightarrow E_\pi[r_t \mid s_1 = s] \leq 1$

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This expression becomes

$$\leq \sum_{t=T'+1}^{\infty} \gamma^{t-1} \quad \left(\text{Infinite geo. series}\right)$$

$$= \frac{\gamma^{T'}}{1-\gamma}$$ Now, As $T' \geq \log_{\frac{1}{\gamma}} \frac{H}{\varepsilon}$,

$$\Rightarrow \left(\frac{1}{\gamma}\right)^{T'} \geq \frac{H}{\varepsilon} \Rightarrow \gamma^{T'} \leq \frac{\varepsilon}{H}$$

$$\therefore \left| V_\pi(s) - \sum_{t=1}^{T'} \gamma^{t-1} E_\pi(r_t \mid s_1 = s) \right| \leq \frac{\varepsilon}{H(1-\gamma)}$$

$\therefore$ as $H = \frac{1}{1-\gamma}$,

$$\boxed{\left| V_\pi(s) - \sum_{t=1}^{T'} \gamma^{t-1} E_\pi(r_t \mid s_1 = s) \right| \leq \varepsilon}$$

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b.)

First things first: we define

$$d(t) = f(T+t) - f(t)$$
$$= \sqrt{T+t} - \sqrt{t}$$

Now, $d'(t) = \dfrac{1}{2\sqrt{T+t}} - \dfrac{1}{2\sqrt{t}} = \dfrac{\sqrt{t} - \sqrt{T+t}}{2\sqrt{t^2+Tt}}$

$< 0 \quad \forall\, t, T > 0$

$\therefore$ $d(t)$ is a decreasing function.

So

$$\left| E_* [V_*(S_t)] - E_A [V_A(S_t)] \right| \leq \varepsilon$$

$$\Rightarrow f(T'+t-1) - f(T'+t-1) + 2\varepsilon' \leq \varepsilon$$

$\Rightarrow$ For any $t^* \geq t$, say $\left(\frac{1+\varepsilon}{\varepsilon}\right)(t-1) = t^* - 1$,

$$f(T'+t^*-1) - f(T'+t^*-1) + 2\varepsilon' \leq \varepsilon$$

Upon substitution for T' and t^*,

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$$\leq \sqrt{\log_{\frac{1}{4}} \frac{4}{\varepsilon'} + \left(\frac{1+\varepsilon}{\varepsilon}\right)^2 \left(\log_{\frac{1}{4}} \frac{4}{\varepsilon}\right)^2 - \left(\frac{1+\varepsilon}{\varepsilon}\right) \log_{\frac{1}{4}} \frac{4}{\varepsilon} + 2\varepsilon'}$$

choosing $\varepsilon' = \frac{\varepsilon}{4}$, we obtain

(let $x = \log_{\frac{1}{4}} \frac{4}{\varepsilon}$)

$$\leq \sqrt{x + \left(\frac{1+\varepsilon}{\varepsilon}\right)^2 x^2 - \left(\frac{1+\varepsilon}{\varepsilon}\right) x + \frac{\varepsilon}{2}}$$

$$\leq \sqrt{\left(\frac{1+\varepsilon}{\varepsilon} x + \frac{\varepsilon}{2(1+\varepsilon)}\right)^2 - \frac{\varepsilon^2}{4(1+\varepsilon^2)} - \left(\frac{1+\varepsilon}{\varepsilon}\right) x + \frac{\varepsilon}{2}}$$

(∵ completing the square)

$$\leq \sqrt{\left(\left(\frac{1+\varepsilon}{\varepsilon}\right) x + \frac{\varepsilon}{2(1+\varepsilon)}\right)^2} - \left(\frac{1+\varepsilon}{\varepsilon}\right) x + \frac{\varepsilon}{2}$$

$$\leq \frac{\varepsilon}{2(1+\varepsilon)} + \frac{\varepsilon}{2}$$

$$\leq \frac{\varepsilon}{2}\left(\frac{2+\varepsilon}{1+\varepsilon}\right) \quad \varepsilon > 0 \Rightarrow \frac{2+\varepsilon}{1+\varepsilon} < 2$$

$$\leq \varepsilon$$

∴ Result is proven

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