MEMO Number CMPE323-Lab07
DATE: November 4, 2016
TO: CMPE323
FROM: EFC LaBerge
SUBJECT: Properties of the Fourier Transform Part I

1 INTRODUCTION

This lab explores the time shift, complex modulation, and cosine modulation properties of the Fourier Transform by direct computation. In this lab, we will be using what is known as the Discrete Time Fourier Transform (DTFT), not to be confused with the Discrete Fourier Transform (DFT) or the Fast Fourier Transform (FFT). We'll get to those later. In the DTFT, samples in time are used to compute or estimate the Fourier transform at any convenient set of frequencies. The set of frequencies are not related to the sample times! In the DFT and FFT, the sample times and computation frequencies are closely related!

2 EQUIPMENT

For this lab, you need a laptop with MATLAB installed.

For the purpose of CMPE323, please use the following naming conventions for all output files:

CMPE323F16_Lab<Lab#>_<Your Campus ID>

For the purpose of CMPE323, please use the following naming conventions for MATLAB scripts or functions that you are required to submit.

<function name>_<Your Campus ID>

Examples will be given in the lab description. Follow the instructions exactly, or you may not get graded!

3 LAB TASKS

You might find it useful to use the MATLAB function `diary` to capture your inputs and outputs.

3.1 Computing the Fourier Transform of the basic pulse

Using a time array from [-4.096: 0.001: 4095], use your basic anonymous pulse function to compute the $pulse(t+\frac{\tau}{2},\tau)$ for t = 1. Then shift that pulse by $-t/2$ (to the right) and $+t/2$ (to the left). Plot all three pulses on separate subplots using professional practice.

Develop an algorithm to estimate the Discrete Time Fourier Transform of your three pulses (one at a time!). For the purpose of this lab DTFT is defined

$$\hat{X}(f) = T\sum_{-N}^{+N} x(nT)e^{-j2\pi(nT)f} \tag{1}$$

where $x(t)$ is the time function under consideration (your pulses), T is the sample interval, nT is the time at which the n-th sample of $x(t)$ is computed, and f is the frequency at which the DTFT is performed. For this lab, use $f = [-10:0.1:10]$ Hz. Use your algorithm to perform the Fourier transform on each of your three pulses.

Hint: The fact that the summation in (1) *is a "sum of products" strongly suggests that it would be efficient to use matrix multiplication to compute the values of* $\widehat{X}(f), f = [-10:0.1:10]$. *I suggest putting values of* f *in the different rows and values of* $t = nT$ *in the different columns.*

Using the process performed in class and the time-shift property of the Fourier Transform, compute the analytical result in each case. Program MATLAB to compute the (complex) value of the analytical result for the values of f used in your contribution of $\widehat{X}(f)$.

Plot the magnitude and phase for each of the three Fourier Transforms and, on each of the graphs, the corresponding parameter of the analytical result. In particular, comment on any slope or jaggedness to the shape of the computed (as opposed to analytical) result. Indicate how well your computation matches the theory. Comment on how your plots illustrate the time delay property. Use professional practice on your plots.

3.2 The Complex Modulation Property

Take your three pulse functions ($p_1(t)$, $p_2(t)$, and $p_3(t)$) and multiply each of them by

$$c(t) = e^{j2\pi f_c t}. \tag{2}$$

with $f_c = 5$ Hz. Call the new pulses $x_1(t)$, $x_2(t)$, and $x_3(t)$, and us them to compute the corresponding DTFTs and the corresponding analytical results. Plot all three pulses on separate subplots using professional practice.

On separate graphs, plot the magnitude and phase for each of the three Fourier Transforms and, on each of the graphs, the corresponding parameter of the analytical result. In particular, comment on any slope or jaggedness to the shape of the computed (as opposed to analytical) result. Indicate how well your computation matches the theory. Comment on how your plots illustrate the complex modulation theorem. Use professional practice on your plots.

3.3 The Cosine Modulation Property

A corollary to the Complex Modulation Theorem is the Cosine Modulation Property, which uses the Euler expansion of the cosine and applies the Complex Modulation Property. Take your original pulses (from 3.1, not from 3.2) and multiply them by

$$m(t) = \cos(2\pi f_c t), \tag{3}$$

with $f_c = 5$ Hz. Call the new pulses $w_1(t)$, $w_2(t)$, and $w_3(t)$, and us them to compute the corresponding DTFTs and the corresponding analytical results. Plot all three pulses on separate subplots using professional practice.

On separate graphs, plot the magnitude and phase for each of the three Fourier Transforms and, on each of the graphs, the corresponding parameter of the analytical result. In particular, comment on any slope or jaggedness to the shape of the computed (as opposed to analytical) result. Indicate how well your computation matches the theory. Comment on how your plots illustrate the complex modulation theorem. Use professional practice on your plots synthesizing the waveform

4 LAB SUBMISSIONS

Submit the following via the Blackboard Assignment Lab 7.

Using this lab description document as a template, create a single PDF file named in accordance with the output naming conventions given above. The content must include

a. The outputs and discussions generated in 3.1.

b. The outputs and discussions generated in 3.2.

c. The outputs and discussions generated in 3.3.

Professional, high quality writing, math, and graphic (that is plots) presentation is expected, and must be provided for you to earn full credit.