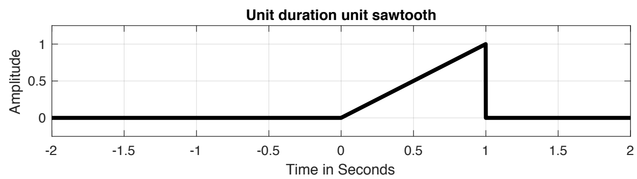
You try

- **Use the unit sawtooth to find the following**

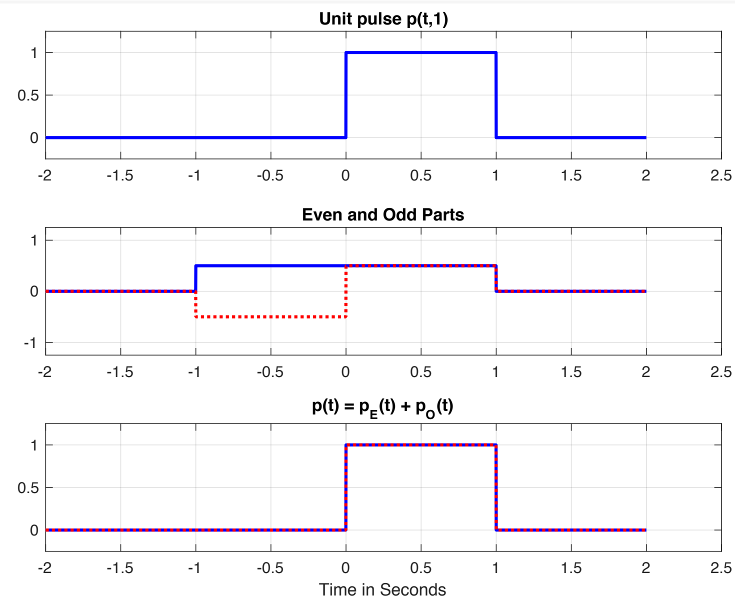
$$f(t) = s(0.5t - 3)$$
$$g(t) = s(2t + 5)$$
$$h(t) = s(-2t + 5)$$



Even and Odd

- **Finally, let's discuss even and odd functions**
- **For an even function** $f(t) = f(-t)$
  - **Examples** $x(t) = t^2,\ x(t) = \cos(\omega t),\ x(t) = A$
- **For an odd function** $f(t) = -f(-t)$
  - **Examples** $x(t) = t^3, x(t) = at,\ x(t) = \sin(\omega t)$
- **But what about a general function, like** $u(t)$ **?**
- **Any function can be decomposed into the sum of an even function and an odd function**

$$f_E(t) = \frac{f(t) + f(-t)}{2}, f_O(t) = \frac{f(t) - f(-t)}{2}$$
$$f(t) = f_E(t) + f_O(t)$$



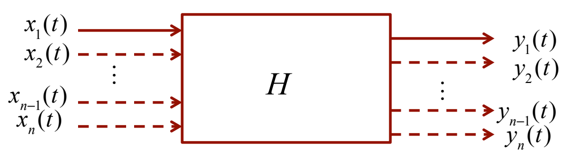
Mathematical Description of Systems

CMPE323

We've been talking about functions of time

- **Functions of time (or space, or temp, or something) are our *signals*…**
- **…and we now know some properties and how to change them.**
- **Now let's turn to the mathematical description of a *system***

- **A *system*, H, takes one or more input signals, $x_n(t)$, and turns it/them into one or more output signals $y_n(t)$**



We might know what's in the box!

- **…but again, we might not**
- **We can know (by test) some properties**
  - **Linearity**
  - **Time Invariance**
  - **Causality**
  - **"Left/Right sidedness"**
  - **Stabilty**
  - **Discrete time or continuous time**
  - **Discrete amplitude (quantized) or analog**
  - **Static/Dynamic**
  - **Feedback/No Feedback**

Linearity

- Many, even most, systems we deal with are linear…
- …or at least linear over some range of input signals
- A system is *linear* if it has two properties (same as in CMPE306!)
  - Superposition: the output of the sum is the sum of the outputs
  - Scaleability: the output of a constant times the input is the same constant times the output
- What this means is that a system H is linear iff

$$ax_1(t)+bx_2(t) \longrightarrow \boxed{H} \longrightarrow ay_1(t)+by_2(t)$$

- Where $H(x_1)=y_1$ and $H(x_2)=y_2$

Example

- A perfect summation
- Inputs are discrete in time and discrete in amplitude, so we can treat them like integers

$$\ldots x_{-1},x_0,x_1,x_2,x_3\ldots \longrightarrow \boxed{H} \longrightarrow y_k=\sum_{-\infty}^{k} x_n$$

- Is it linear?
- Test for the properties. Let there be two input sequences: $\ldots x_{-1},x_0,x_1,x_2,x_3\ldots,$ and $\ldots w_{-1},w_0,w_1,w_2,w_3\ldots$
- Use the $\{x_k\}$ sequence and observe $y_k^{(x)}=\sum_{-\infty}^{k} x_n$
- Use the $\{w_k\}$ sequence and observe $y_k^{(w)}=\sum_{-\infty}^{k} w_n$
- Use $v_k=ax_k+bw_k$: $y_k^{(v)}=\sum_{-\infty}^{k} v_k=\sum_{-\infty}^{k}\left(ax_k+bw_k\right)=a\sum_{-\infty}^{k} x_k+b\sum_{-\infty}^{k} w_k$

$$=ay_k^{(x)}+by_k^{(w)} \Rightarrow \text{This system is linear!}$$

Non-linear example

$$y(t) = x^2(t)$$

Time Invariance (TI)

- **A system is *time invariant* (TI) iff a delay in the input causes a corresponding delay in the output, without otherwise changing the output signal**

$$y(t) = H\big(x(t)\big) = x^2(t)$$

Let $w(t) = x(t-T)$ (a simple delay)

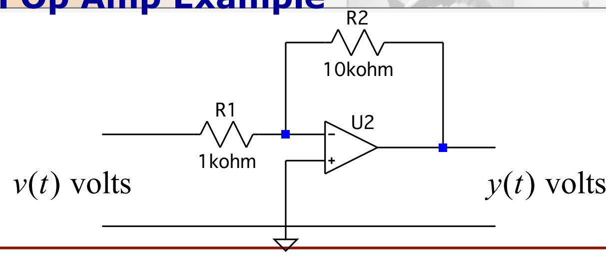
$$y^{(w)}(t) = w^2(t) = x^2(t-T) = y^{(x)}(t-T)$$

Not linear, but time invariant

- **Example** $y(t) = kx(t)$ with k a known constant

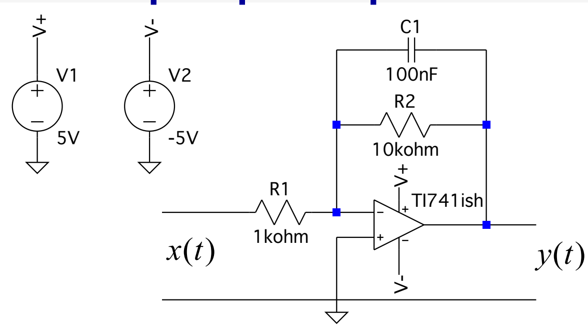
  Is it TI?

Ideal Op Amp Example



Linear? TI? Static?

Non-ideal op amp example: L? TI? Static?



Linear? TI? Static?

Impulse response

- One of the most useful characterizations of a system is its "impulse" response

$$\delta(t) \longrightarrow \boxed{\quad H \quad} \longrightarrow h(t)$$

- Consider a discrete time, discrete amplitude, "N-point average"

$$y_k = H(x_k) = \frac{1}{N} \sum_{n=k-N+1}^{k} x_n$$

- Linear? TI? Static? Causal?
- What's the impulse response?

Stability

- We'll define stability here…
- …and come back to it when we discuss Laplace techniques
- A system (not necessarily LTI, causal, or static) is STABLE if the application of a bounded input introduces a bounded output

$$\text{BIBOS:}\ \ 0 \le x(t) \le K \Rightarrow \left| y(t) \right| \le M$$

$$\text{UBIBOS:}\ \ \left| x(t) \right| \le K \Rightarrow \left| y(t) \right| \le M$$

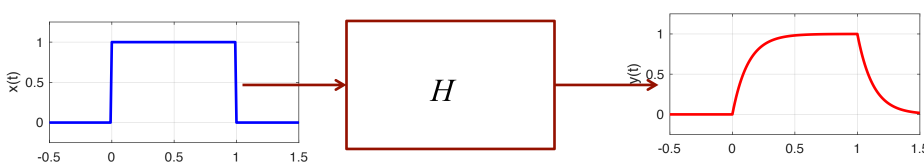
- Is our previous op amp circuit stable?

Using the properties

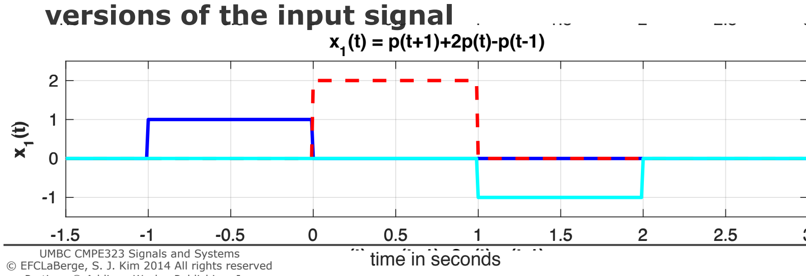
- Why do we care?
- Remember that we're interested in finding the output from a known (but arbitrary) input
- If the system is LTI, and if we can break the input into pieces whose output is known …
- …we can apply the pieces one at a time…
- …and then sum the outputs to get the total output
- Let's see

For LTI systems

- If we know the input output relationship for a signal if interest, like a pulse…

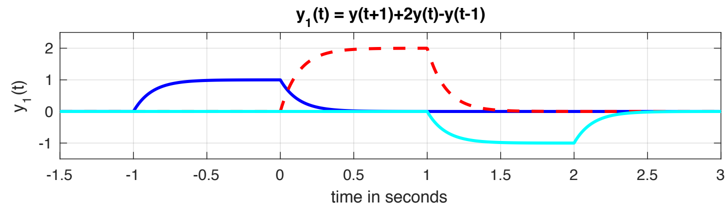


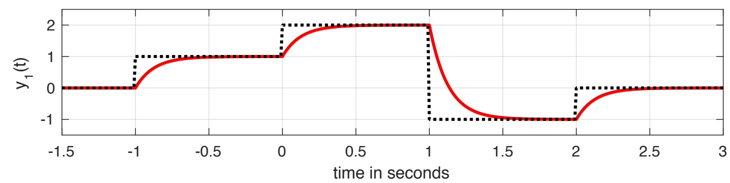
- …we can find the input output relationship for signals that can be decomposed into sums of delayed versions of the input signal



…continued

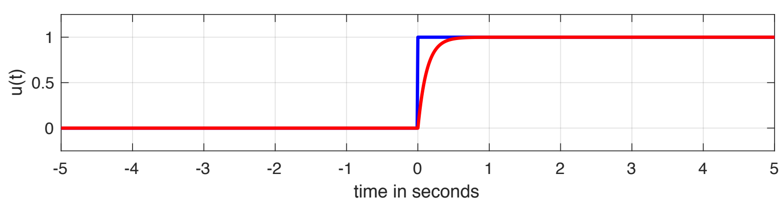
- By linearity, the output of the sum is the sum of the outputs…
- …by TI, a delay or advance in the input results in the same delay or advance in the output





So what is the impulse response?

- We might (should) ask what the impulse response is
- Imagine the input is a very long pulse…



- …which is essential a step, so we have the step response…
- The impulse is the derivative of the step, and the system is LTI, so the output of the derivative is the derivative of the output (check this out for yourselves)

Impulse response continued