Name: ____

1. (5 points) Consider the rod-cutting problem for a rod of length six with the profit for each length of rod given in Table 1.

i	1	2	3	4	5	6
p_i	2	5	8	12	12	15

Table 1: Profit p_i for a rod of length i

(a) Complete the following table of values for $r[i]$ and $s[i]$. Show all work.

Solution:

i	1	2	3	4	5	6
r_i	2	5	8	**12**	**14**	**17**
s_i	1	2	3	**4**	**1**	**2**

Note: For s_4, s_5, and s_6, the values 4, 4, and 4 are also acceptable. If this solution is given for part (a), then the answer to part (b) is (4, 2).

To compute r_4:

$$r_4 = \max(p_1 + r_3, p_2 + r_2, p_3 + r_1, p_4 + r_0) = \max(10, 10, 10, 12) = 12$$

which occurs with an initial cut of four. To compute r_5:

$$r_5 = \max(p_1 + r_4, p_2 + r_3, p_3 + r_2, p_4 + r_1, p_5 + r_0) = \max(14, 13, 13, 14, 12) = 14$$

which occurs with an initial cut of one *or* four. The computation of r_6 is similar.

(b) Use the s table to determine the optimal cuts for a rod of length six. Explain your answer.

Solution: The value of s_6 is two, so we should make an initial cut of length two, leaving a rod of length four. s_4 is four, so we leave this piece uncut. Thus the optimal cuts for a rod of length six are (2, 4).

(continued on other side)

2. (5 points) I own and operate a small delivery truck; every morning I go to a warehouse, load the truck with items awaiting delivery, and deliver them. My truck can carry W pounds of goods. When I arrive at the warehouse, there are n items $x_1, x_2, \ldots, x_n$ waiting to be delivered, each with weight w_i and value v_i, $i = 1, 2, \ldots n$. There are always more items waiting than I can carry in my truck. I want to maximize the total value of my load.

Let $A = \langle x_{i_1}, x_{i_2}, \ldots, x_{i_m} \rangle$ be a solution for a truck with capacity W. That is, the sum of the weights of the items is at most W,

$$w_{i_1} + w_{i_2} + \cdots + w_{i_m} \leq W,$$

and the total value of the load, $v_{i_1} + v_{i_2} + \cdots + v_{i_m}$, is as large as possible. Define $A' = \langle x_{i_1}, x_{i_2}, \ldots, x_{i_{m-1}} \rangle$; that is, A' is A with x_{i_m} removed.

(a) Prove that the packages in A' will fit in a truck with capacity $W' = W - w_{i_m}$.

Solution: We are given that $w_{i_1} + w_{i_2} + \cdots + w_{i_m} \leq W$. Subtracting w_{i_m} from both sides gives

$$w_{i_1} + w_{i_2} + \cdots + w_{i_{(m-1)}} \leq W - w_{i_m},$$

Since the left-hand side is the weight of A', we see that the load will fit in a truck with capacity $W - w_{i_m}$.

(b) Use proof by contradiction to show that A' is a solution for a truck with capacity W'.

Solution: Suppose A' were not a solution for a truck with capacity W'. Then there would be a load of weight at most $W - w_{i_m}$ that is more valuable. Adding item x_{i_m} to this load yields a solution to the original problem with greater total value, contradicting the supposition A was optimal.