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MATH 407: HW 12

4.1 **1** Let $f(x)$, $g(x)$, $g(x) \in F[x]$. Show that the following properties hold.

c If $g(x) \mid f(x)$, then $g(x) \cdot h(x) \mid f(x) \cdot h(x)$.

Pf. $\square$

d If $g(x) \mid f(x)$ and $f(x) \mid g(x)$, then $f(x) = kg(x)$ for some $k \in F$.

Pf. $\square$

5 Over the given field $\mathbb{F}$, write $f(x) = q(x)(x - c) + f(c)$ for

b $f(x) = 2x^3 + x^2 - 4x + 3$; $c = 1$; $\mathbb{F} = \mathbb{Q}$;

Pf. $\square$

d $f(x) = x^3 + 2x + 3$; $c = 2$; $\mathbb{F} = \mathbb{Z}_5$;

Pf. $\square$

6 Let p be a prime number. Find all roots of $x^{p-1} - 1$ in $\mathbb{Z}_p$.

Pf. $\square$

7 Show that if c is any element of the field $\mathbb{F}$ and $k > 2$ is an odd integer, then $x + c$ is a factor of $x^k + c^k$.

Pf. $\square$

11 Show that the set $\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$ is closed under addition, subtraction, multiplication, and division.

Pf. $\square$

13 Show that the set of matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $a, b \in \mathbb{R}$, is a field under the operations of matrix addition and multiplication.

Pf. $\square$

17 Let $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ be points in the Euclidean plane $\mathbb{R}^2$ such that x_0, x_1, x_2 are distinct. Show the formula

$$f(x) = \frac{y_0(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + \frac{y_1(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + \frac{y_2(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

defines a polynomial $f(x)$ such that $f(x_0) = y_0$, $f(x_1) = y_1$, and $f(x_2) = y_2$.

Pf. $\square$

18 Use Lagrange's interpolation formula to find a polynomial $f(x)$ such that $f(1) = 0$, $f(2) = 1$, and $f(3) = 4$.

Pf. $\square$