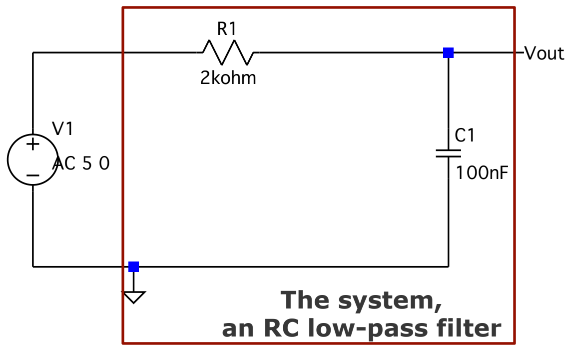
What's it all about?

- A *signal* is a waveform or measurement that represents some physical quantity
- A *system* is a set of operations that transforms *input* signals to *output* signals
- We've seen this kind of thing already in circuits (CMPE306)

The input signal, an analog voltage V_1



The output signal, an analog voltage V_{OUT}

Mathematical Description of Signals

- We want to be able to describe signals in three ways
  - Time domain
  - Frequency (or Fourier) Domain
  - Laplace Domain
- Each way emphasizes different qualities of the the signal
- Our method is to decompose complex signals into simpler signals
- In the process, we'll develop a dictionary of standard signals that will prove very useful.

Key Mathematical Characteristic of Signals

- **Periodicity**
  - **A signal is periodic if it repeats exactly, that is if**

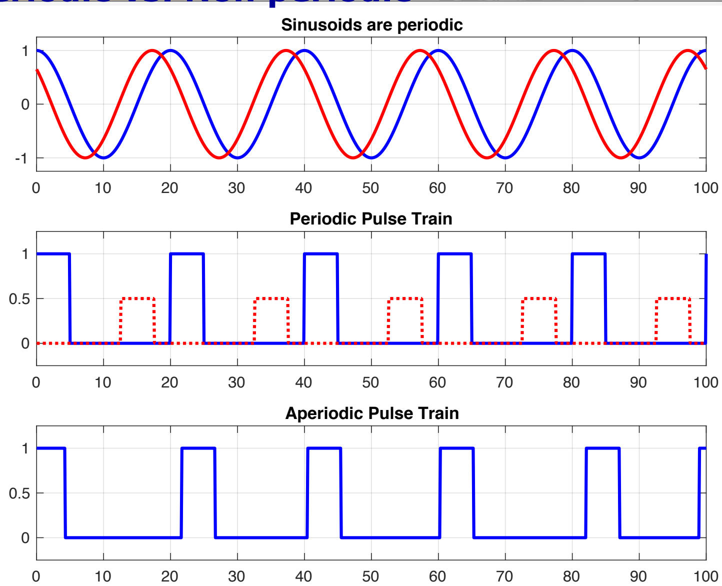
    $f(\alpha) = f(\alpha \pm nT)$ for any α and and n

    T is called the *period*
  - **Sinusoids are periodic**

    $$x(t) = A\cos(\omega t + \phi), \quad T = \frac{2\pi}{\omega}$$
  - **Pulse trains may be periodic**

    $$s(t) = \sum_{-\infty}^{\infty} p(t - kT) \text{ where } p(t) \text{ is a specified pulse waveform}$$

Periodic vs. Non periodic



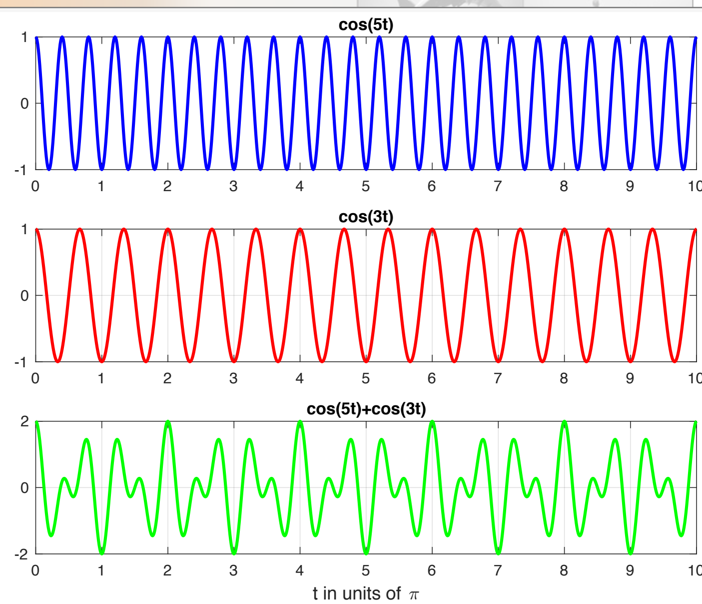
Sums and products of periodic waveforms

- **The sum of two periodic waveforms with periods** T_1 and T_2 **will be periodic iff** $T_1 / T_2 = f_2 / f_1 = n_2 / n_1;\ n_1, n_2 \in \mathbb{Z}$
- **The period of the sum is** $T = n_1 T_1 = n_2 T_2$

  $\cos(5t) + \cos(3t)$ is periodic

  $$T_1 = \frac{2\pi}{5}, T_2 = \frac{2\pi}{3}, \frac{T_1}{T_2} = \frac{3}{5}, \text{ so } n_1 = 5, n_2 = 3$$

  The common period is $T = n_1 T_1 = 2\pi = n_2 T_2$
- **A similar rule holds for the product of periodic waveforms** $(f_1 + f_2)/(f_1 - f_2) = n_2 / n_1,\ n_1, n_2 \in \mathbb{Z}$



Elementary Functions

- **We define several elementary functions (this is also a review from CMPE306!)**
- **A unit step**

$$u(t)=\begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} \text{, we'll leave the definition at } t=0 \text{ for later}$$

- **A unit impulse**

$$\delta(t)=\frac{du}{dt} \Rightarrow \lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon}\delta\left(t\right)dt=1$$

- **The unit impulse isn't a function is the traditional sense…**
- **…but rather a limit of functions…**
- **…and a number of different "shapes" can be used**

The unit impulse is very important

- **The sieving property**

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt=f(t_0)$$

- **Examples**

$$\int_{-\infty}^{\infty} e^{-at}\delta(t-t_0)\,dt=e^{-at_0}\text{, in fact }\int_{t_0-\varepsilon}^{t_0+\varepsilon} e^{-at}\delta(t-t_0)\,dt=e^{-at_0}$$

$$\int_{-\infty}^{\infty} e^{-at}(100\cos\omega t+200\sin\omega t)\delta(t-\tau)\,dt=e^{-a\tau}(100\cos\omega\tau+200\sin\omega\tau)$$

More elementary functions

- **Sinusoids**

  $A\cos\omega t,\ A\sin\left(2\pi ft+\phi\right),\text{etc, periodic with } T=\frac{2\pi}{\omega}=\frac{1}{f}$
- **Exponentials**

  $a_0e^{at}\ \text{(growing)},\ b_0e^{-\alpha t}\ \text{(decaying)}$
- **Complex exponentials**

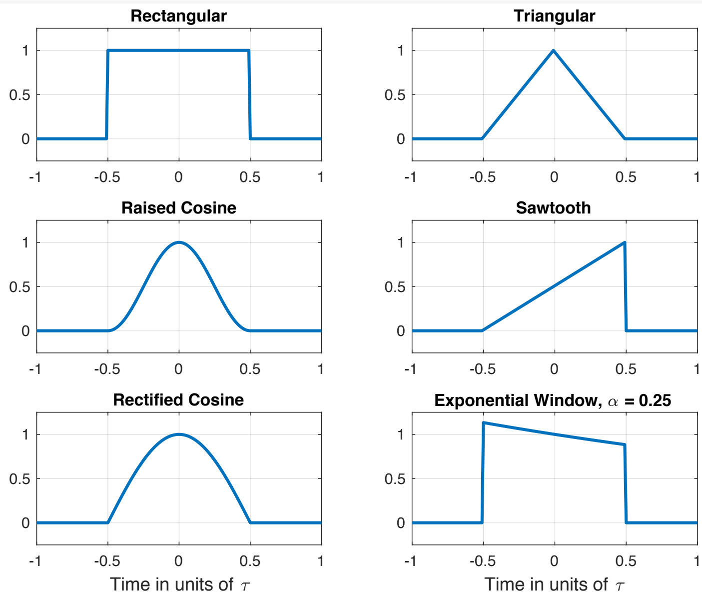
  $Ae^{j\phi}=A\cos\phi+jA\sin\phi$

  $Ae^{j(\omega t+\phi)}=A\cos(\omega t+\phi)+jA\sin(\omega t+\phi)$
- **The sinc function**
  - **Our text:** $\text{sinc}(x)=\frac{\sin x}{x}$
  - **Some other texts and MATLAB:** $\text{sinc}(x)=\frac{\sin \pi x}{\pi x}$

Pulses and Windows

- **Often, we want only a limited segment (in time) of one of the elementary functions.**
- **Such a segment is called a window**
- **Windows will be important later!**
- **We often create a window by multiplying two functions together…**
- **…e.g. Rectified Cosine Window**

$$w_{RC}(t;\tau)=\cos\left(\frac{\pi t}{\tau}\right)w_{RECT}(t-0.5\tau;\tau)$$



Time Averages of Time Waveforms

- **We are usually interested in the energy contained in a waveform or signal…**
- **…as energy is the key to overcoming noise in the system.**

$$\mathcal{E} = \int_{-\infty}^{\infty} x(t)x^*(t)\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt \underbrace{=}_{\substack{\text{real}\\\text{valued}\\\text{signal}}} \int_{-\infty}^{\infty} x^2(t)\,dt$$

- **If our signal has units of volts, then $\mathcal{E}$ has units of volt^2-sec, which is not joules!**
- **When computing the mathematical energy (as above) we assume there is a 1Ω resistor as a scale factor, giving volt^2-sec/ohm = watt-sec = joules!**
- **Using this definition, the energy in a periodic signal is infinite!! (Why?)**

- **For periodic signals, we compute the "average energy per period"**

$$\frac{\mathcal{E}}{T}=\frac{1}{T}\int_{\alpha}^{T+\alpha} x(t)x^*(t)\,dt=\frac{1}{T}\int_{\alpha}^{T+\alpha}|x(t)|^2\,dt\underbrace{=}_{\substack{\text{real}\\\text{valued}\\\text{signal}}}\frac{1}{T}\int_0^T x^2(t)\,dt$$

- **Again using a 1Ω resistor as a scale factor, we wind up with volt^2-sec/ohm-sec=watt-sec/sec=watt…**
- **…so this is power!**
- **A signal with finite energy is called an energy signal**
- **A signal with meaningful power is called a power signal.**
- **Some signals are neither power or energy signals, but these are rare.**

Examples

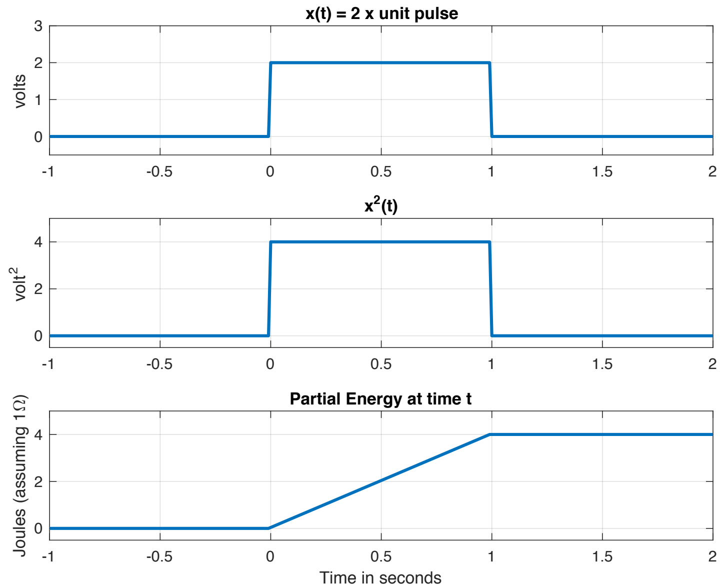
- **Unit pulse**

$$p(t)=\begin{cases}1 & 0\le t<1\\ 0 & \text{otherwise}\end{cases}$$

$$x(t)=2p(t)$$

$$x^2(t)=\begin{cases}4 & 0\le t<1\\ 0 & \text{otherwise}\end{cases}$$

$$\mathcal{E}=\int_{-\infty}^{\infty}x^2(t)\,dt=\int_0^1 4dt=4\text{ joules}$$



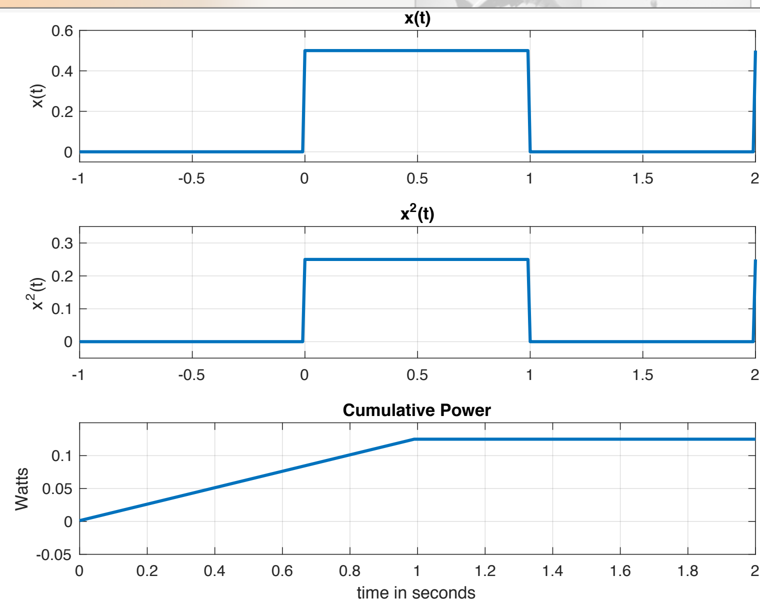
- **Pulse train**

$$p(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{-\infty}^{\infty} 0.5\, p(t - 2n)$$

$$x^2(t) = \sum_{-\infty}^{\infty} 0.25\, p(t - 2n)$$

$$P = \frac{1}{2} \int_0^2 0.25\, dt = 0.125 \text{ watts}$$



You try

- **Compute the power or energy, as appropriate**

$$a(t) = \begin{cases} e^{-0.5t} & 0 \le t < 2 \\ 0 & \text{o/w} \end{cases} \quad \text{(exponential pulse)}$$

$$b(t) = \sum_{k=-\infty}^{\infty} a(t-4k)$$

$$c(t) = 120\cos(120\pi t + \pi/3)$$

$$d(t) = \begin{cases} 120\cos(120\pi t) & -\frac{1}{240} \le t < \frac{1}{240} \\ 0 & \text{o/w} \end{cases}$$

$$f(t) = \begin{cases} 0.25t & 0 \le t < 4 \\ 0 & \text{o/w} \end{cases}$$

$$g(t) = \sum_{k=-\infty}^{\infty} f(t-16k)$$

$a(t)$ is an energy signal

$$\mathcal{E} = \int_0^2 (e^{-0.5t})^2\, dt = \int_0^2 (e^{-1t})\, dt = 1 - e^{-2}$$

$b(t)$ is periodic with period 4, therefore it is a power signal

$$P = \frac{1}{4}\int_0^4 (b(t))^2\, dt = \frac{1}{4}\left(\int_0^2 (e^{-0.5t})^2\, dt + \int_2^4 (0)^2\, dt\right) = \frac{1-e^{-2}}{4}$$

$c(t)$ is periodic with period $\frac{2\pi}{120\pi} = \frac{1}{60}$. The phase does not affect the period, $c(t)$ is a power signal

Let $120\pi t_0 = \frac{\pi}{3} \Rightarrow t_0 = \frac{1}{360}$, $c(t) = 120\cos(120\pi(t + t_0))$

$$P = \left(\frac{1}{60}\right)120^2 \int_{-1/360}^{-1/360+1/60} \cos^2(120\pi(t+t_0)\, dt = \left(\frac{1}{60}\right)120^2 \int_0^{1/60} \cos^2 120\pi\tau\, d\tau$$

$$= \left(\frac{1}{60}\right)120^2 \int_0^{1/60} \left(\frac{1}{2} + \frac{1}{2}\cos 240\pi\tau\right) d\tau$$

$$= 120^2\, \frac{\left((1/60) - 0\right)}{2\times 60} + 120^2\, \frac{\left(\sin\frac{240\pi}{60} - \sin 0\right)}{2\times 60\times 240\pi} = \frac{120^2}{2} = 7200 \text{ watts}$$