CMPE323 What did we learn

Linear, Time Invariant Systems

- **LTI is not everything!**
- **There are many non-linear, non time invariant systems.**
- **But LTI is so common that it is the larges class of systems**
- **Even non linear systems (think GPS) often act like linear systems in the neighborhood of an operating point…**
- **…through use of a first-order Taylor Series expansion**

$$\hat{y}(t;c) \approx y(t;c_0) + \frac{\partial y(t;c)}{\partial c}\Delta c + \mathcal{O}\left(\Delta c^2\right)$$

Convolution

- Convolution is the sliding & integrating process

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$$

- ...where we frequently have to evaluate the integral differently in different regions of t
- Remember that the integration is with respect to τ, and t is a parameter when doing the integration!
- There should be no dependence on τ after the integration...
- ...although there will obviously be a dependence on t
- Convolution is a *filtering operation*

Laplace Domain

- The Laplace domain (s-domain) is frequently used for characterizing the performance of linear systems
- ...but not the characteristics of linear signals
- $\mathcal{L}\big(x(t)\big) = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$
- The transfer function $H(s)$ is the Laplace Transform of the impulse response $h(t)$ of that system
- The Laplace transform includes specification of the Region of Convergence.
- Many Laplace transforms are rational polynomials in

$$X(s) = \frac{\prod_{k=1}^{M}(s-z_k)}{\prod_{k=1}^{B}(s-p_k)}$$

Inverse Laplace Transforms

- We generally invert Laplace Transforms by means of evaluating the Residues, also known as Partial Fraction Expansion

$$X(s) = \frac{\prod_{k=1}^{M}(s - z_k)}{\prod_{k=1}^{N}(s - p_k)} = \sum_{m=1}^{N} \frac{R_m}{(s - p_m)}$$

- … with explict process for dealing with roots of higher order
- Poles and residuals must occur as either real or in complex conjugate pairs

Right-sided and left-sided signals

- A right sided signal exists for $t > t_0$
- A left sided signal exists for $t < t_0$
- A two sided signal exists for $t_1 < t < t_2$

- For right sided signals, RoC is $\mathrm{Re}\left[s\right] > a$
- For left sided signals, RoC is $\mathrm{Re}\left[s\right] < a$
- For two sided signals, RoC is $a < \mathrm{Re}\left[s\right] < b$

- There are no poles in the Region of Convergence
- For causal signals, all the poles must be in the left half plane
- For anti causal signals, all the poles must be in the right half plane

Fourier Transforms

- The Fourier ($\omega, f,$ or $j\omega$) domain is used to analyze signals, generally not systems.
- A periodic (power) signal has a Fourier Series

$$c_k = \frac{1}{T}\int_a^{a+T} x(t)e^{-j\frac{2\pi k}{T}t}\,dt,\; x(t) = \sum_{n=-\infty}^{\infty} c_k e^{j\frac{2\pi n}{T}t}$$

- A non-periodic (energy) signal has a Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt = X(f)$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

The convolution theorem

- The convolution theorem ties the time and Laplace/ Fourier domains together

$$\mathcal{L}\big(x(t) * h(t)\big) = X(s)H(s)$$

- Sampling in one domain indicates that the signal is periodic in the other domain
- Periodicity in one domain indicates that the signal is sampled (discrete) in the other domain.
- If a signal is periodic and sampled in one domain, it is sampled and periodic in the other domain.

- The DFT is a numerical computation that approximates a scaled version of the Fourier Transform

$$X_{DFT}\left[n\right]=\sum_{k=0}^{N-1}x[k]e^{-j\frac{2\pi kn}{N}}$$

- The DTFT is the Fourier transform of the impulse-sampled piecewise continuous time signal and is continuous in frequency

$$X_{DTFT}(\omega)=\sum_{k=-\infty}^{\infty}x(t)e^{-j\omega k\Delta t}$$

- The FFT is an efficient algorithm for computing the DFT. It is most efficient if $N=2^M$ It implicitly assumes both the time and frequency domain signals are periodic, but has no explicit identifiction with time of frequency

Analog Filters

- **A Butterworth filter has a transfer function**

$$H(j\omega) = H(s)_{s=j\omega}$$

- **… that is monotonically decreasing away from the center of the passband.**
- **A Chebyshev Type 1 filter has a transfer function that has ripples in the pass band, but is monotonically decreasing outside of the band**
- **A Chebyshev Type 2 filter has a transfer function that has ripples in the stop band, but is monotonically decreasing inside the pass band.**