CMPE323 Lecture 10 LTI and Block Diagrams

- Exam #1 Wed 10/5/2016
- Content
  - Signals, decomposition, synthesis
  - Time manipulation, delay, advance, compress, expand, linear
  - Even & odd
  - Differentiation, integration
- Systems, Linear, TI, Causal, Static
- Convolution, graphic, analytic, properties

System applications

- **In systems application we're much less worried about the particular and homogeneous solutions than we are about the solution with and without an input**
- **The zero state solution**

$$y_{ZS}(t) = y(t)\Big|_{Y_I=0} = \left(\frac{1}{a-b}\right)\left(e^{-bt} - e^{-at}\right)u(t)$$

- **The zero input solution** $y_{ZI}(t) = y(t)\Big|_{x(t)=0} = y_h(t) = Y_I e^{-at}$
- **And** $y(t) = y_{ZS}(t) + y_{ZI}(t)$

Is the system described linear?

- **The linear combinations have to work for any valid inputs, so choose** $x_1(t) = \alpha e^{-bt}u(t),\ x_2(t) = -x_1(t)$
- **Then, from our solution**

$$y_1(t) = Y_I e^{-at} + \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

$$y_2(t) = Y_I e^{-at} - \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

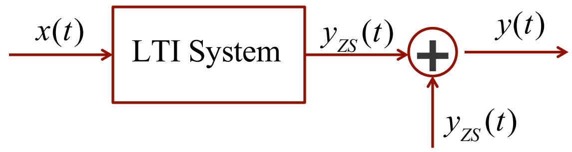
- **If we let** $x(t) = x_1(t) + x_2(t) = 0$ **, a linear system will give**

$$y(t) = Y_I e^{-at} + \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t) + Y_I e^{-at} - \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

$$y(t) = Y_I e^{-at} + Y_I e^{-at} = 2Y_I e^{-at} = 0 \Leftrightarrow Y_I = 0$$

- **So this system is linear iff it is initially at rest!**

In general



- **A system described by a LCCDE is linear if and only if it is initially at rest.**
- **A causal system that is initially at rest is also Time Invariant**

Block Diagrams of LCCDE

- **How would we implement or synthesize a system if we had a LCCDE?**
- **Implementing differentiators is problematic because**
  - **High frequencies in the input are multiplied by a frequency ramp**

$$\frac{d\sin(\omega_o t)}{dt} = \omega_o \cos(\omega_0 t)$$

  - **Impulses can result from differentiation**
- **So we transform the equation into an integral equation**
- **Here are the steps**

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}, \quad \frac{d^0 y(t)}{dx^0} \triangleq y(t)$$

If $M < N$, let $b_k = 0,\ k = M+1, M+2 \ldots N$

If $N < M$, let $a_k = 0,\ k = N+1, N+2 \ldots M$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} b_k \frac{d^k x(t)}{dt^k}, \quad \frac{d^0 y(t)}{dx^0} \triangleq y(t)$$

Now form a set of auxiliary variables or , $y_k(t)$ and $x_k(t)$

$$y_0(t) = y(t), \quad x_0(t) = x(t)$$

$$y_1(t) = \int_{-\infty}^{t} y_o(\tau)\,d\tau, \; x_1(t) = \int_{-\infty}^{t} x_o(\tau)\,d\tau$$

$$y_2(t) = \int_{-\infty}^{t} y_1(\tau)\,d\tau, \; x_2(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau$$

etc...

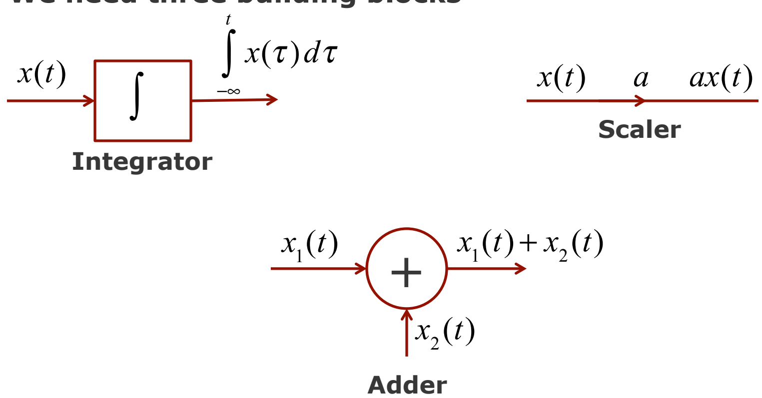
$$y_N(t) = \int_{-\infty}^{t} y_{N-1}(\tau)\,d\tau, \; x_N(t) = \int_{-\infty}^{t} x_{N-1}(\tau)\,d\tau$$

$$\sum_{k=0}^{N} a_k y_{N-k}(t) = \sum_{k=0}^{N} b_k x_{N-k}(t) = \sum_{k=0}^{M} b_k x_{N-k}(t)$$

Where we have integrated both sides N times...
requiring N initial conditions (one for each integral)

Basic Block Diagram Elements

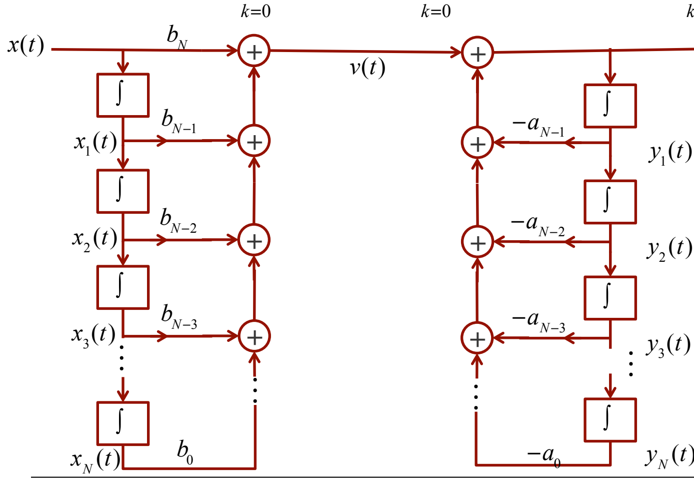
- **We'll now use this "integrated" form to *synthesize* an implementation of the system described by the LCCDE**
- **We need three building blocks**



Synthesis

- **Assume** $a_N = 1$ **(or divide through to make it so)**
- **Rewrite**

$$y(t) = y_0(t) = \sum_{k=0}^{N} b_k x_{N-k}(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t) = v(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t)$$



N-th order Continuous Time Direct Form

Draw the Direct Form Implementation

- **Example** $\dfrac{dy(t)}{dt} + \alpha y(t) = \beta \dfrac{dx(t)}{dt} + \gamma x(t)$

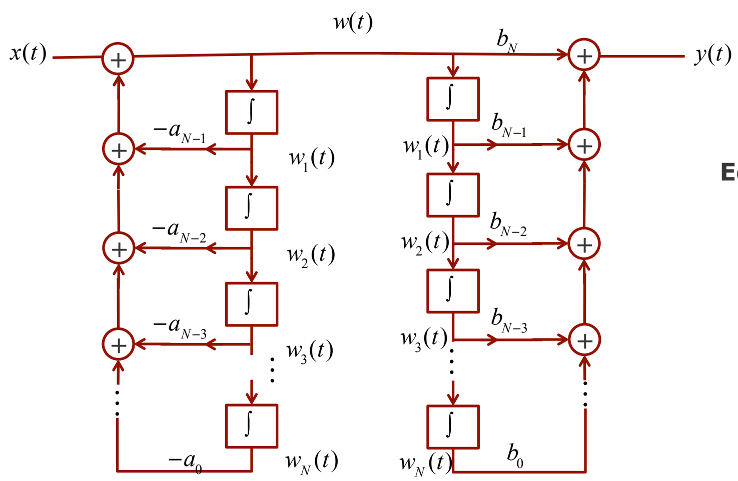
Concatenation

- **Our direct form is the concatenation of two systems:**

$$\boxed{v(t) = \sum_{k=0}^{N} b_k x_{N-k}(t)} \longrightarrow \boxed{y(t) = v(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t)}$$

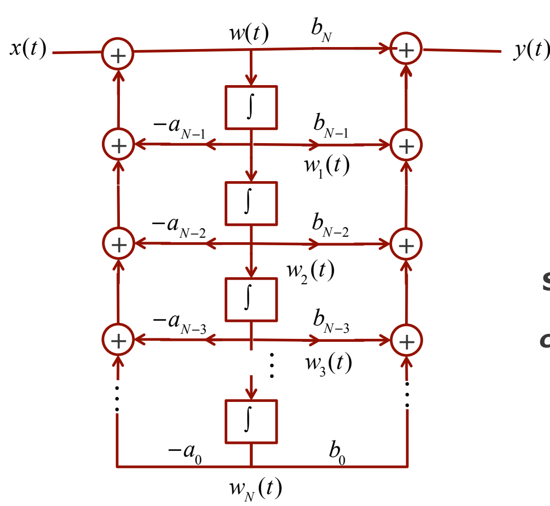
- **Convolution is associative and commutative, so we can swap the order (defining a new intermediate fnc)**

$$\boxed{w(t) = x(t) - \sum_{k=0}^{N-1} a_k x_{N-k}(t)} \longrightarrow \boxed{y(t) = \sum_{k=0}^{N} b_k w_{N-k}(t)}$$



Equivalent System

Direct Form II Structure



Direct Form II Structure is *canonical*

- **Consider**

$$2\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\frac{dx(t)}{dt}+3x(t)$$

- **Draw the Direct Form II implementation**

- **Define the centroid of a signal** $x(t)$ **to be**

$$C_x=\int_{-\infty}^{\infty}\tau x(\tau)\,d\tau\,/\,A_x,\ \text{where } A_x=\int_{-\infty}^{\infty}x(\tau)\,d\tau$$

- **Define the k-th central moment of a signal to be**

$$\sigma_x^k=\int_{-\infty}^{\infty}(\tau-C_x)^k x(\tau)\,d\tau\,/\,A_x$$

- **Define our usual unit pulse** $p(t)=\begin{cases}1 & 0\le t<1\\ 0 & otherwise\end{cases}$
- **Do the following in MATLAB**
  - **1) Compute** C_p,σ_p^2 for the shifted pulse $p(t+0.5)$
  - **2) Convolve** $p(t+0.5)$ **with itself** $p_2(t)=p(t+0.5)*p(t+0.5)$ **and compute** $C_{p_2},\sigma_{p_2}^2$
  - **3) Convolve** $p_2(t)$ with itself $p_4(t)=p_2(t)*p_2(t)$ **and compute** $C_{p_4},\sigma_{p_4}^2$
  - **4) Repeat to compute** $p_8(t)=p_4(t)*p_4(t),\ C_{p_8},\sigma_{p_8}^2$
  - **5) Provide plots. Superimpose the plot of** $p_8(t)$ **with** $Ke^{-(t-C_{p_8})^2/(2\sigma_{p_8}^2)}$**, choosing *K* to make the peaks match**