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CMPE 212
HW 2

① Convert into canonical SOP form:

$$f(A,B,C,D) = \bar{A}B + BD + AC + \bar{B}\bar{C}$$

$\rightarrow \bar{A}B(C+\bar{C})(D+\bar{D}) + BD(A+\bar{A})(C+\bar{C}) + AC(B+\bar{B})(D+\bar{D}) + \bar{B}\bar{C}(A+\bar{A})(D+\bar{D})$

$\rightarrow (\bar{A}BC + \bar{A}B\bar{C})(D+\bar{D}) + (ABD + \bar{A}BD)(C+\bar{C}) + (ABC + A\bar{B}C)(D+\bar{D}) + (A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C})(D+\bar{D})$

$\rightarrow \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + ABCD + AB\bar{C}D + \bar{A}BCD + \bar{A}B\bar{C}D$
$+ ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

$\rightarrow \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$
$+ A\bar{B}C\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD + A\bar{B}CD$

$\rightarrow 0000 + 0001 + 0100 + 0101 + 0111 + 0110 + 1000 + 1001 + 1011 +$
$1010 + 1101 + 1110 + 1111$

$\rightarrow 0_{10} + 1_{10} + 4_{10} + 5_{10} + 7_{10} + 6_{10} + 8_{10} + 9_{10} + 11_{10} + 10_{10} + 13_{10} + 14_{10} + 15_{10}$

$\rightarrow m_0 + m_1 + m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{13} + m_{14} + m_{15}$

$= \sum m(0,1,4,5,6,7,8,9,10,11,13,14,15)$ □

② Convert into canonical POS form:

$$f(x,y,z) = (\bar{x}+\bar{y}+\bar{z})(y+\bar{z})(x+y)(z+\bar{x})$$

$\rightarrow (\bar{x}+\bar{y}+\bar{z}) \equiv 111 \equiv 7_{10} \rightarrow M_7$

$\rightarrow (y+\bar{z}) \equiv (x+y+\bar{z})(\bar{x}+y+\bar{z}) \equiv (001)(101) \rightarrow M_1 M_5$

$\rightarrow (x+y) \equiv (x+y+z)(x+y+\bar{z}) \equiv (000)(001) \rightarrow M_0 M_1$

$\rightarrow (z+\bar{x}) \equiv (\bar{x}+y+z)(\bar{x}+\bar{y}+z) \equiv (100)(110) \rightarrow M_4 M_6$

$\rightarrow M_7 \cdot M_1 M_5 \cdot M_0 M_1 \cdot M_4 M_6 \rightarrow \prod M(0,1,4,5,6,7)$ □

③ Truth value for:

$$f(a,b,c) = \bar{c}(\bar{b}+a)(a+\bar{c})(b+c)$$

Expressed in its canonical POS form:

$\rightarrow \bar{c} \equiv (a+\bar{c})(\bar{a}+\bar{c}) \equiv (a+b+\bar{c})(a+\bar{b}+\bar{c})(\bar{a}+b+\bar{c})(\bar{a}+\bar{b}+\bar{c})$
$\rightarrow (001)(011)(101)(111) \rightarrow M_1 M_3 M_5 M_7$

$\rightarrow (\bar{b}+a) \equiv (a+\bar{b}+c)(a+\bar{b}+\bar{c}) \rightarrow (010)(011) \rightarrow M_2 M_3$

$\rightarrow (a+\bar{c}) \equiv (a+b+\bar{c})(a+\bar{b}+\bar{c}) \rightarrow (001)(011) \rightarrow M_1 M_3$

$\rightarrow (b+c) \equiv (a+b+c)(\bar{a}+b+c) \rightarrow (000)(100) \rightarrow M_0 M_4$

$\rightarrow M_1 M_3 M_5 M_7 \cdot M_2 M_3 \cdot M_1 M_3 \cdot M_0 M_4 \equiv M_0 M_1 M_2 M_3 M_4 M_5 M_7$

$\rightarrow \prod M(0,1,2,3,4,5,7)$

Row	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

□

4) a) $f_a(a,b,c,d) = \prod M(1,2,6,7,11,12,14,15)$

$\rightarrow M_1 M_2 M_6 M_7 M_{11} M_{12} M_{14} M_{15}$

$\equiv (a+b+c+\bar d)(a+b+\bar c+d)(a+\bar b+\bar c+\bar d)(a+\bar b+\bar c+\bar d)$
$(\bar a+b+\bar c+\bar d)(\bar a+\bar b+c+d)(\bar a+\bar b+\bar c+d)(\bar a+\bar b+\bar c+\bar d)$

$\rightarrow (a+b)(a+\bar b+\bar c)(\bar a)(\bar a+\bar b+\bar c)$
$\rightarrow (a+\bar c)(\bar a+\bar b+\bar c)(\bar a) \rightarrow \bar a(\bar b+\bar c)$
$\rightarrow \bar a(\overline{bc})$ $\square$

b) $f_b(a,b,c,d) = \sum m(0,4,6,7,9,10,12,13,14)$

$\rightarrow m_0 m_4 m_6 m_7 m_9 m_{10} m_{12} m_{13} m_{14}$

$\equiv \bar a\bar b\bar c\bar d + \bar a b\bar c\bar d + \bar a b c\bar d + \bar a bcd + a\bar b\bar c d$
$+ abc\bar d + ab\bar c\bar d + ab\bar c d + abc\bar d$

$\rightarrow \bar a\bar c\bar d + \bar a b\bar c\bar d + \bar a bc\bar d + \bar a bcd + a\bar b\bar c d + a\bar b c\bar d + ab\bar c\bar d + ab\bar c d + abc\bar d$
$\rightarrow \bar a\bar c\bar d + \bar a b\bar d + d + a\bar d + ab$
$\rightarrow \bar a\bar d(b+\bar c) + a(b+\bar d) + d$ $\square$

5) a) $f(x,y,z) = (y+\bar x)(xy+z) + xy\bar x + \bar x\bar y z + (x+y)(\bar x+z)$

$\rightarrow (xy + yz + \bar x z + 0) + 0 + \bar x\bar y z + (0 + xz + \bar x y + yz)$
$\rightarrow xy + \bar x z + \bar x\bar y z + xz + \bar x y$
$\rightarrow y + z + \bar x\bar y z$ $\square$

b) $f(w,x,Q) = (Q+\bar w)(x+\bar Q)(w+x+Q)(\bar w+\bar x)$

$\rightarrow (xQ + 0 + \bar w x + \bar w\bar Q)(0 + \overset{0}{\cancel{w\bar x}} + \overset{0}{\cancel{\bar w x}} + 0 + \bar w Q + \bar x Q)$
$\rightarrow (xQ + \bar w\bar Q)(\bar w Q + \bar x Q) \rightarrow \bar w x Q + 0 + 0 + \bar w\bar x Q$
$\rightarrow \bar w Q$ $\square$

c) $f(A,B,C) = \overline{\overline{(\bar{A}+\bar{B})}(A+\bar{A}B)(\bar{A}+\bar{B}+\bar{A}\bar{B}C)+\overline{(A+B)(\bar{A}+C)}}$

$\bar{f}(A,B,C) = \overline{(\bar{A}+\bar{B})}(A+\bar{A}B)(\bar{A}+\bar{B}+\bar{A}\bar{B}C)+\overline{(A+B)(\bar{A}+C)}$

$\rightarrow (\overline{\bar{A}\bar{B}})(A+B)(\overline{AB}+\bar{A}\bar{B}C)+(\overline{AC+\bar{A}B+BC})$

$\rightarrow AB(A+B)(\overline{AB}+\bar{A}\bar{B}C)+(\overline{AC+\bar{A}B})$

$\rightarrow AB(\overline{AB}+\bar{A}\bar{B}C)+(\overline{AC+\bar{A}B})$

$\rightarrow 0+0+(\overline{AC+\bar{A}B}) \longrightarrow \overline{AC+\bar{A}B}$

$\therefore \bar{f}(A,B,C) = \overline{AC+\bar{A}B}$, $f(A,B,C) = AC+\bar{A}B = C+B$ □

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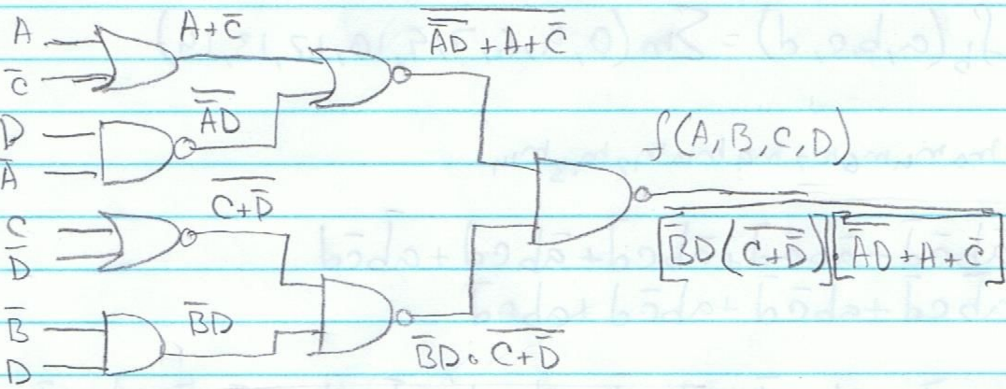


$f(A,B,C,D) = \overline{(\bar{B}D(\overline{C+\bar{D}}))(\overline{\overline{\bar{A}D}+A+C})}$

$\rightarrow \overline{\bar{B}D(\overline{C+\bar{D}})} + \overline{\bar{A}D} + A + C$

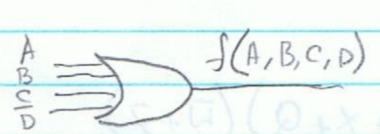
$\rightarrow \overline{\bar{B}D(\bar{C}D)} + A + \bar{D} + A + C$

$\rightarrow \overline{\bar{B}\bar{C}D} + A + C + \bar{D}$

$\rightarrow B + C + \bar{D} + A + C + \bar{D}$

$\rightarrow A + B + C + \bar{D}$

$\therefore$



□

(1)

Time	Input		Output	
	(A,B,C)	m_n	Y	Z
t_0	000	m_0	1	0
t_1	001	m_1	1	1
t_2	010	m_2	0	1
t_3	011	m_3	1	0
t_4	100	m_4	1	0
t_5	101	m_5	1	0
t_6	110	m_6	0	1
t_7	111	m_7	0	1

$\therefore Y = f_1(A,B,C) = \sum m(0,1,3,4,5)$

$Z = f_2(A,B,C) = \sum m(1,2,6,7)$

$\rightarrow Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C$

$\rightarrow \bar{A}\bar{B} + \bar{A}BC + A\bar{B}$

$\rightarrow \bar{A}BC + \bar{B} \rightarrow \bar{A}\bar{B}C$

$\rightarrow Z = \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$

$\rightarrow \bar{A}\bar{B}C + B\bar{C} + ABC$

$\rightarrow B\bar{C} + C(AB + \bar{A}\bar{B})$

$\therefore Y = \bar{A}\bar{B}C,\ Z = B\bar{C} + C(AB + \bar{A}\bar{B})$ $\square$