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**CMPE 320** HW 03

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1. With 3 $n$-sided rolls, there are $n^3$ possibilities.
   The probability that either of the pair of persons roll the same face of the die is therefore $n/n^3 = 1/n^2$.
   Therefore

$$\begin{aligned} P(A_{12}) = P(A_{13}) = P(A_{23}) &= \frac{n}{n^3} \\ &= \frac{1}{n^2} \end{aligned}$$

   But if both the events $A_{12}$ and $A_{13}$ takes place, that is both persons $1$ and $2$ and persons $1$ and $3$ roll the same face, then that yields $A_{23}$.
   That is, if both persons $1$ and $2$ and persons $1$ and $3$ roll the same face, then that implies persons $1$ and $3$ rolled the same face.
   But the outcome of person $3$'s roll is not dependent on the other persons.
   That is, pairwise $A_{12}$ and $A_{13}$, $A_{12}$ and $A_{23}$, and $A_{13}$ and $A_{23}$ are independent.
   But if considered individually, they are dependent. $\square$

2. Consider the following counter-example with two independent tosses of a fair coin.
   Let events $B = \{HT, HH\}$ and $C = \{HT, TT\}$ represent tosses where they landed heads and tails respectively.
   Let $A = \{HT, TH\}$ be the event that exactly one toss resulted in heads.
   Then,

$$P(A) = P(B) = P(C) = \frac{1}{2}$$

   And,

$$P(A \cup B) = P(A \cup C) = \frac{1}{4}$$

   Therefore, $A$ and $B$ and $A$ and $C$ are both independent events.
   Therefore, $B$ and $C$ are also independent events.
   However,

$$P(A \cap (B \cup C)) = \frac{1}{4} \neq P(A)P(B \cup C) = \frac{1}{2} \cdot \frac{3}{4}$$

   Therefore, $A$ and $B \cup C$ are dependent $\square$

3. $\square$

4. Let $p_5$ denote the longer path of 5 links from $A$ to $B$,
   and $p_3$ denote the shorter path of 3 links.

   Given, the probability of links failing independently is $q$.
   Therefore, the probability of links not failing is $1 - q$.

   For a successful transmission, all of the links have to not fail.
   Therefore, for path $p_5$ the probability is $P(p_5) = (1-q)^5$
   and for path $p_3$ the probability is $P(p_3) = (1-q)^3$.
   Since the paths are independent of each other,

$$\begin{aligned} P(p_5 \cap p_3) &= P(p_5) \cdot P(p_3) \\ &= (1-q)^5 \cdot (1-q)^3 \\ &= (1-q)^8 \end{aligned}$$

   That is, the probability of both the paths not failing is $(1-q)^8$.
   Therefore, the probability of either the paths not failing for a successful transmission from terminal $A$ to $B$ is:

$$\begin{aligned} P(p_5 \cup p_3) &= P(p_5) + P(p_3) - P(p_5 \cap p_3) \\ &= (1-q)^5 + (1-q)^3 - (1-q)^8 \end{aligned}$$ $\square$

5. $\square$

6. $\square$

7. Given:
   $P(\text{qualified}) = q$,
   $P(\text{not qualified}) = 1 - q$,
   $P(\text{correct answer} \mid \text{qualified}) = p$
   $P(\text{incorrect answer} \mid \text{not qualified}) = p$

Therefore,

$$
\begin{aligned}
P(>15\text{ correct} \mid \text{qualified}) &= \frac{P(>15\text{ correct} \cap \text{qualified})}{P(\text{qualified})} \\
&= \frac{q\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5}{q\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5 + (1-q)\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5} \\
&= \frac{q\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5}{\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5} \\
&= q
\end{aligned}
$$

$\square$

8. Let $C$ represent the 20 random distinct cars chosen for a test drive.

   (a) To find $P(K = 0 \mid C)$, without replacement:

$$
\begin{aligned}
P(K = 0 \mid C) &= \frac{P(K=0)P(C \mid K=0)}{\sum_{i=0}^{9} P(K=i)P(C \mid K=i)} \\
&= \frac{\binom{100}{20}}{\sum_{i=0}^{9}\binom{100-i}{20}} \\
&\approx 0.227
\end{aligned}
$$

$\square$

   (b) To find $P(K = 0 \mid C)$, with replacement:

$$
\begin{aligned}
P(K = 0 \mid C) &= \frac{P(K=0)P(C \mid K=0)}{\sum_{i=0}^{9} P(K=i)P(C \mid K=i)} \\
&= \frac{100^{20}}{\sum_{i=0}^{9}(100-i)^{20}} \\
&\approx 0.213
\end{aligned}
$$

$\square$

9. $\square$

10. The permutations of a word is given by:

$$
\frac{\text{(length of word)!}}{\text{(repetitions of A)!(repetitions of B)!}\ldots\text{(repetitions of Z)!}}
$$

(a) Since there are no repeating characters, the permutation is simply:

$$\begin{aligned}\text{permutations} &= length(\text{children})! \\ &= 8! \\ &= 40320 \qquad \square\end{aligned}$$

(b) Since the characters $o$ repeats 2 times, $k$ repeats 2 times, and $e$ repeats 3 times:

$$\begin{aligned}\text{permutations} &= \frac{length(\text{bookkeeper})!}{(\text{repetitions of o})!(\text{repetitions of k})!(\text{repetitions of e})!} \\ &= \frac{10!}{2!2!3!} \\ &= 151200 \qquad \square\end{aligned}$$