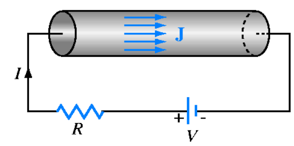
Current Flow in Conductors

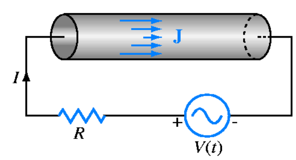
Flow in a wire

- in the DC case, it is nearly constant
- in the AC case, current flow is larger on the conductor surface

This behavior is best understood by first studying a slab geometry



(a) d-c case



(b) a-c case

Ulaby Figure 7-14

UMBC AN HONORS UNIVERSITY IN MARYLAND

Current Flow in Conductors

Flow in a slab

Above the metal slab, we have

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\, E_0, \quad \tilde{\mathbf{H}} = \hat{\mathbf{y}}\, H_0$$

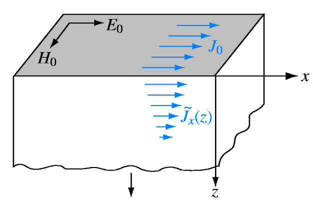
Since the tangential electric field is continuous, we have inside the slab

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\, E_0 \exp(-\alpha z - j\beta z),$$

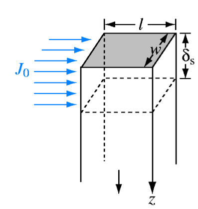
$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\, \frac{E_0}{\eta_c} \exp(-\alpha z - j\beta z)$$

From Ohm's law, we now find the current

$$\begin{aligned}\tilde{\mathbf{J}}(z) &= \hat{\mathbf{x}}\, \tilde{J}_x(z) = \sigma \tilde{\mathbf{E}}(z) \\ &= \hat{\mathbf{x}}\, \sigma E_0 \exp(-\alpha z - j\beta z) \\ &= \hat{\mathbf{x}}\, J_0 \exp(-\alpha z - j\beta z)\end{aligned}$$



(a) Exponentially decaying $\tilde{J}_x(z)$



(b) Equivalent J_0 over skin depth δ_s

Ulaby Figure 7-15

UMBC AN HONORS UNIVERSITY IN MARYLAND

Current Flow in Conductors

Flow in a slab

Integrating to obtain the total current, we find

$$\tilde{I} = w\int_0^\infty J_x(z)\,dz$$

$$= w\int_0^\infty J_0 \exp\left[-(1+j)z/\delta_s\right]dz = \frac{J_0 w\delta_s}{(1+j)}$$

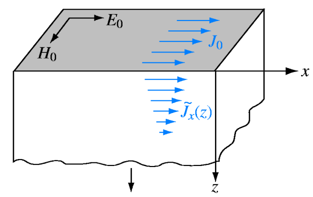
In a slab of width w and length l, we have for the voltage and the impedance

$$\tilde{V} = E_0 l = \frac{J_0}{\sigma} l, \quad Z = \frac{\tilde{V}}{\tilde{I}} = \frac{(1+j)}{\sigma\delta_s}\frac{l}{w} \equiv Z_s \frac{l}{w}$$

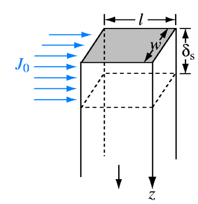
where Z_s = surface impedance

Writing $Z_s = R_s + j\omega L_s$

$$R_s = \text{surface resistance} = \frac{1}{\sigma\delta_s} = \sqrt{\frac{\pi f \mu}{\sigma}}$$



(a) Exponentially decaying $\tilde{J}_x(z)$



(b) Equivalent J_0 over skin depth δ_s

Ulaby Figure 7-15

UMBC AN HONORS UNIVERSITY IN MARYLAND

Current Flow in Conductors

Flow in a coaxial cable

Consider flow in a copper wire at 1 MHz with radius a:

$$\sigma = 5.8\times10^7 \text{ S/m} \quad \Rightarrow \quad \delta_s = 0.066 \text{ mm}$$

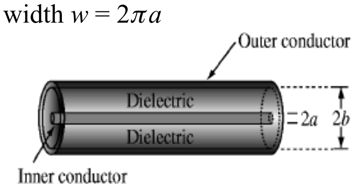
When $a > 5\delta_s$, the wire can be considered "semi-infinite" and the current flows through a surface with width $w = 2\pi a$

On the surface of the inner conductor (surface 1), we have for the resistance per unit length

$$R_1' = \frac{R}{l} = \frac{R_s}{2\pi a}$$

On the outer conductor, we have

$$R_2' = \frac{R_s}{2\pi b}$$



(a) Coaxial cable

(b) Equivalent inner conductor

Ulaby Figure 7-16

UMBC AN HONORS UNIVERSITY IN MARYLAND

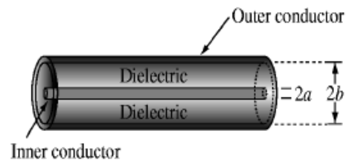
Current Flow in Conductors

Flow in a coaxial cable

Adding the effect of both conductors, we find

$$R' = R_1' + R_2' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

We have now calculated all four transmission line parameters for coaxial cables!



(a) Coaxial cable

(b) Equivalent inner conductor

Ulaby Figure 7-16

UMBC AN HONORS UNIVERSITY IN MARYLAND

Electromagnetic Power

Poynting Flux:

To determine the power flow due to electromagnetic waves, we define the ***Poynting flux vector:*** $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, which has units of (V/m)×(A/m) = W/m^2

$\mathbf{S}$ = Power density carried by a wave

The power that is carried through an aperture is given by

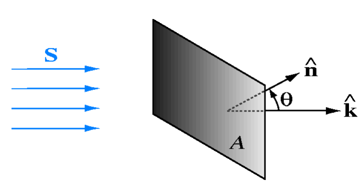
$$P = \int_A \mathbf{S} \cdot \hat{\mathbf{n}}\, dA$$

which depends on both the orientation and size of the aperture.

For a constant flow through a planar aperture: $P = SA\cos\theta,\ S \equiv |\mathbf{S}|$

With $\mathbf{E} \rightarrow V$ and $\mathbf{H} \rightarrow I$, this is analogous to the expression for power flow in a transmission line

$$P(z,t) = v(z,t)i(z,t)$$



Ulaby Figure 7-17

UMBC AN HONORS UNIVERSITY IN MARYLAND

Electromagnetic Power

Average Power Density:

The Poynting flux varies with time.
Of greater interest than the time-varying power is the average power $\mathbf{S}_{\text{av}}$

In transmission lines, we had

$$P_{\text{av}}(z) = \tfrac{1}{2}\text{Re}\left[\tilde{V}(z)\tilde{I}^*(z)\right]$$

For electromagnetic waves, we have

$$\mathbf{S}_{\text{av}} = \tfrac{1}{2}\text{Re}\left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right]$$

The proof is analogous

UMBC

AN HONORS UNIVERSITY IN MARYLAND

14.7

Electromagnetic Power

Average Power Density: Plane Waves in a Lossless Medium

Consider propagation in the $+z$-direction

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\, E_x(z) + \hat{\mathbf{y}}\, E_y(z) = \left(\hat{\mathbf{x}}\, E_{x0} + \hat{\mathbf{y}}\, E_{y0}\right)\exp(-jkz)$$

where E_{x0} and E_{y0} are arbitrary complex constants that give the magnitude and phase of the two (independent) polarizations

We note that:

$$\tilde{\mathbf{H}}(z) = \frac{1}{\eta}\hat{\mathbf{z}} \times \tilde{\mathbf{E}} = \frac{1}{\eta}\left(-\hat{\mathbf{x}}\, E_{y0} + \hat{\mathbf{y}}\, E_{x0}\right)\exp(-jkz)$$

so that

$$\mathbf{S}_{\text{av}} = \tfrac{1}{2}\text{Re}\left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right] = \hat{\mathbf{z}}\frac{1}{2\eta}\left(E_{x0}E_{x0}^* + E_{y0}E_{y0}^*\right) = \hat{\mathbf{z}}\frac{1}{2\eta}\tilde{\mathbf{E}} \cdot \tilde{\mathbf{E}}^* = \hat{\mathbf{z}}\frac{|\tilde{\mathbf{E}}|^2}{2\eta}$$

The powers of the two independent polarizations add!

UMBC

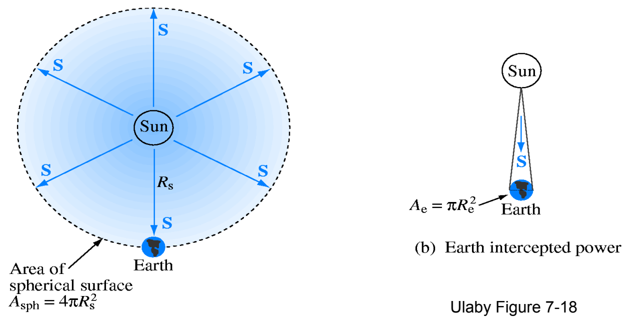
AN HONORS UNIVERSITY IN MARYLAND

14.8

Electromagnetic Power

Solar Power: Ulaby and Ravaioli Example 7-5

Question: If solar illumination has a power density of 1 kW/m^2 at the Earth's surface, find (a) the power radiated by the sun, (b) the total power intercepted by the Earth, and (c) the corresponding electric field, assuming that incident power is at a single frequency. The radius of the Earth's orbit, R_S , is 1.5 × 10^8 km, and the Earth's radius, R_E, is 6,380 km.



(a) Radiated solar power

(b) Earth intercepted power

Ulaby Figure 7-18

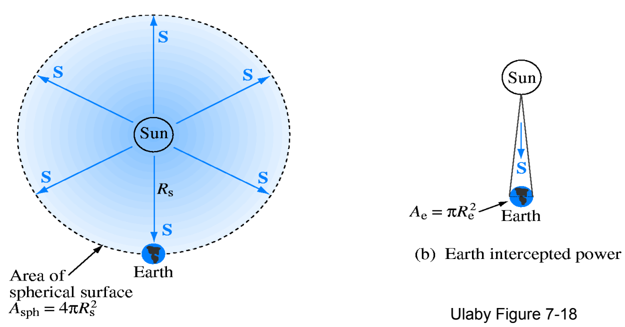
UMBC
AN HONORS UNIVERSITY IN MARYLAND

Electromagnetic Power

Solar Power: Ulaby and Ravaioli Example 7-5

Answer: (a) Assuming that the sun radiates isotropically, the total power equals $S_{av}\,A_{sph}$, where A_{sph} is the area of the sphere at the Earth's radius, so that

$$P_{sun} = S_{av}\left(4\pi R_S^2\right) = (1\times10^3)\times4\pi\times(1.5\times10^{11})^2 = 2.8\times10^{26}\ \text{W}$$



(a) Radiated solar power

(b) Earth intercepted power

Ulaby Figure 7-18

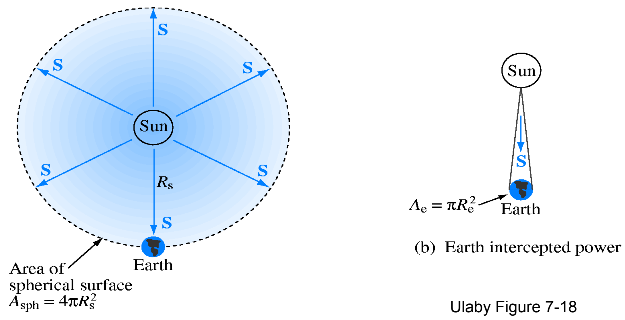
UMBC
AN HONORS UNIVERSITY IN MARYLAND

Electromagnetic Power

Solar Power: Ulaby and Ravaioli Example 7-5

Answer (continued): (b) The power intercepted by the Earth is given by its cross-section $A_{\mathrm{E}} = \pi R_{\mathrm{E}}^2$, so that

$$P_{\text{int}} = S_{\text{av}}\left(\pi R_{\mathrm{E}}^2\right) = (1\times10^3)\times\pi\times(6.4\times10^6)^2 = 1.3\times10^{17}\ \text{W}$$



(a) Radiated solar power

(b) Earth intercepted power

Ulaby Figure 7-18

UMBC AN HONORS UNIVERSITY IN MARYLAND

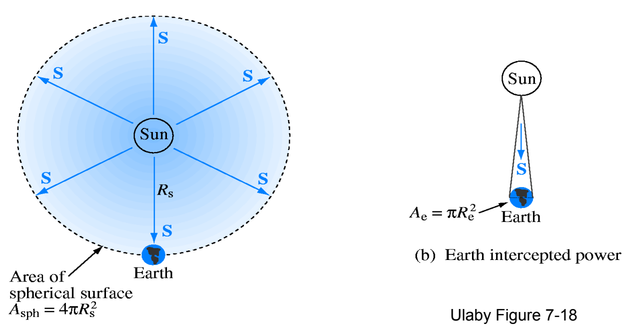
14.11

Electromagnetic Power

Solar Power: Ulaby and Ravaioli Example 7-5

Answer (continued): (c) The power density S_{av} is related to the magnitude of the electric field $|E_0|$ by $S_{\text{av}} = |E_0|^2/2\eta_0$ where $\eta_0 = 377\ \Omega$ for air, so that

$$|E_0| = (2\eta_0 S_{\text{av}})^{1/2} = (2\times377\times10^3)^{1/2} = 870\ \text{V/m}$$



(a) Radiated solar power

(b) Earth intercepted power

Ulaby Figure 7-18

UMBC AN HONORS UNIVERSITY IN MARYLAND

14.12

Electromagnetic Power

Average Power Density: Plane Waves in a Lossy Medium

As in the lossless medium, we consider propagation in the $+z$-direction

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\, E_x(z) + \hat{\mathbf{y}} E_y(z) = \left(\hat{\mathbf{x}}\, E_{x0} + \hat{\mathbf{y}} E_{y0}\right)\exp(-\alpha z)\exp(-jkz)$$

$$\tilde{\mathbf{H}}(z) = \frac{1}{\eta_c}\left(-\hat{\mathbf{x}}\, E_{y0} + \hat{\mathbf{y}} E_{x0}\right)\exp(-\alpha z)\exp(-jkz)$$

so that

$$\mathbf{S}_{av} = \tfrac{1}{2}\mathrm{Re}\left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right] = \hat{\mathbf{z}}\frac{1}{2}\left|E_0\right|^2 \exp(-2\alpha z)\,\mathrm{Re}\left(\frac{1}{\eta_c^*}\right)$$

Writing $\eta_c = |\eta_c|\exp(j\theta_\eta)$, we conclude

$$\mathbf{S}_{av} = \hat{\mathbf{z}}\frac{1}{2}\frac{|E_0|^2}{|\eta_c|}\exp(-2\alpha z)\cos\theta_\eta$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

IMPORTANT NOTE:
The attenuation rate of the power is TWICE the attenuation rate of the amplitude

14.13

Electromagnetic Power

Decibel Scale:

Because it is convenient for dealing with power ratios and exponential attenuation, it is very common to characterize powers using a logarithmic scale. If $G = P_1/P_2$, then $G\ [\mathrm{dB}] \equiv 10\log(P_1/P_2)$

Some examples:

- If $P_1/P_2 = 10$: $G = 10$ dB
- If $P_1/P_2 = 100$: $G = 20$ dB
- If $P_1/P_2 = 2$: $G = 3$ dB (actually 3.01… dB, but the difference is neglected)

When amplitudes are used to characterize powers, the definition changes.

If we measure power by measuring the voltage at a resistor, $P_1 = V_1^2/R$ and $P_2 = V_2^2/R$, then

$$G\ [\mathrm{dB}] \equiv 10\log(P_1/P_2) = 10\log\left(\frac{V_1^2/R}{V_2^2/R}\right)$$

$$= 20\log(V_1/V_2) = 20\log(g) \equiv g\ [\mathrm{dB}]$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

14.14

Electromagnetic Power

Decibel Scale:

While originally defined in terms of *ratios*, one often uses dB to characterize absolute values by using a reference power.

Some examples:

- In communications: P [dBm] = 10 log$(P\,/\,1\text{ mW})$
- In acoustics: I [dB-SPL] = 10 log $(I\,/\,10^{-12}\text{ W/m}^2)$ [I = power density]

There are many different variants in use that are used in different engineering fields!

UMBC AN HONORS UNIVERSITY IN MARYLAND

14.15

Electromagnetic Power

Power Received by a Submarine: Ulaby Example 7-6

Question: A submarine at a depth of 200 m uses a wire antenna to receive signal transmissions at 1 kHz. Determine the power density incident upon the submarine from the electromagnetic wave of Ulaby et al. Example 7-4 [slides 13.28–13.30].

Answer: The system parameters are

$$|E_0| = 4.44\text{ mV/m},\ \alpha = 0.126\text{ Np/m, and } \eta_c = 0.044\exp(j\pi/4)$$

So, we have

$$\begin{aligned}\mathbf{S}_{av} &= \hat{\mathbf{z}}\frac{1}{2}\frac{|E_0|^2}{|\eta_c|}\exp(-2\alpha z)\cos\theta_\eta \\ &= \hat{\mathbf{z}}\frac{(4.44\times10^{-3})^2\times0.707}{2\times0.044}\exp(-0.252\,z) \\ &= \hat{\mathbf{z}}(1.58\times10^{-4})\exp(-0.252\times200) \\ &= \hat{\mathbf{z}}\,2.0\times10^{-26}\text{ W/m}^2\end{aligned}$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

14.16

Assignment

Reading: Ulaby et al., Chapter 8-1 through 8-5

Problem Set 8: Some notes.

- There are 6 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Please watch significant digits

UMBC
AN HONORS UNIVERSITY IN MARYLAND

14.17