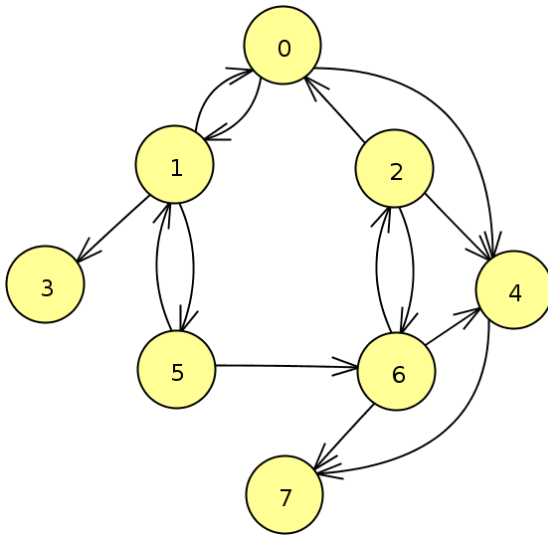


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HW 6 – Version A  
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## Graphs

- Complete a Breadth First Search on the graph:

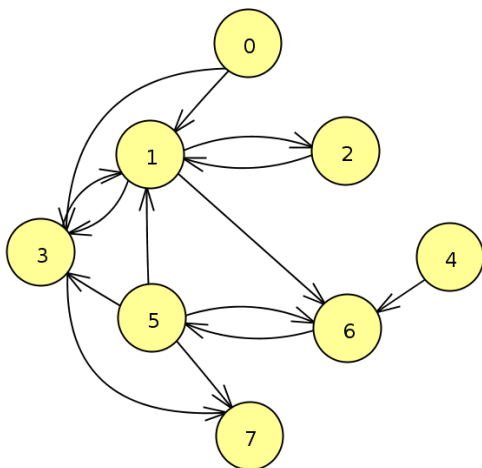


Index	Parent	Visited	BFS Queue
0	2	T	2
1	0	T	0
2	-1	T	4
3	1	T	6
4	2	T	1
5	1	T	7
6	2	T	3
7	4	T	5

<b>Initial:</b>	→ set all values of Visited to false → set Parent as an empty vector → set BFS Queue as an empty queue
<b>Start at 2:</b>	→ enqueue 2 to BFS Queue → assign the 2nd values of Visited to true and Parent to -1 → check neighbors being visited by 2 → find 0, 4, 6 → enqueue the lowest of the neighbors to BFS Queue – 0 → assign the 0th values of Visited to true and Parent to 2 → enqueue the next lowest of the neighbors to BFS Queue – 4 → assign the 4th values of Visited to true and Parent to 2 → enqueue the next lowest of the neighbors to BFS Queue – 6 → assign the 6th values of Visited to true and Parent to 2
<b>Start at 0 (next in BFS Queue):</b>	→ check neighbors being visited by 0 → find 1, 4 → enqueue the lowest of the neighbors to BFS Queue – 1 → assign the 1st values of Visited to true and Parent to 0 → since 4 is already in the BFS Queue and has been visited, ignore it
<b>Start at 4 (next in BFS Queue):</b>	→ check neighbors being visited by 4 → find 7 → enqueue the neighbor to BFS Queue – 7 → assign the 7th values of Visited to true and Parent to 4

<b>Start at 6 (next in BFS Queue):</b>	→ check neighbors being visited by 6 → find 2, 4 → since both 2 and 4 are in the BFS Queue and have been visited, ignore them
<b>Start at 1 (next in BFS Queue):</b>	→ check neighbors being visited by 1 → find 0, 3, 5 → since 4 is already in the BFS Queue and has been visited, ignore it → enqueue the next lowest neighbor to BFS Queue – 3 → assign the 3rd values of Visited to true and Parent to 1 → enqueue the next lowest neighbor to BFS Queue – 5 → assign the 5th values of Visited to true and Parent to 1
<b>Start at 7 (next in BFS Queue):</b>	→ check neighbors being visited by 7 → find 6 → since 4 is already in the BFS Queue and has been visited, ignore it
<b>Start at 3 (next in BFS Queue):</b>	→ check neighbors being visited by 3 → none found, move on
<b>Start at 5 (next in BFS Queue):</b>	→ check neighbors being visited by 5 → find 1, 6 → since both 1 and 6 are in the BFS Queue and have been visited, ignore them

2. Complete a Depth First Search on this graph:

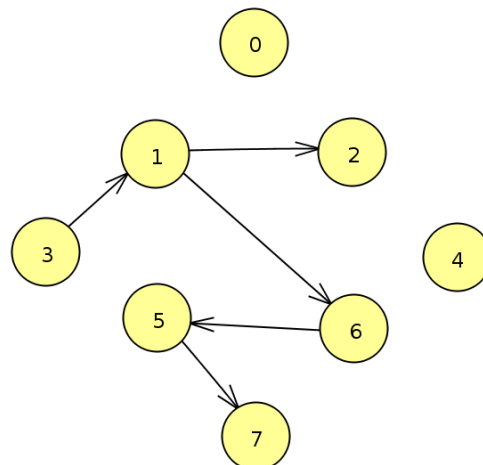


Index	Parent	Visited	DFS Stack
0		F	DFS(3)
1	3	T	DFS(1)
2	1	T	DFS(2)
3	-1	T	DFS(6)
4		F	DFS(5)
5	6	T	DFS(7)
6	1	T	
7	5	T	

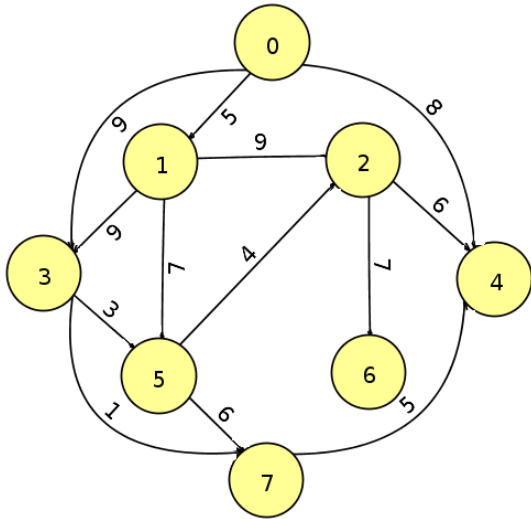
<b>Initial:</b>	→ set all values of Visited to false → set Parent as an empty vector → set DFS Stack as an empty stack
<b>Start at 3:</b>	→ add 3 to the DFS Stack → assign the 3 <sup>rd</sup> values of Visited to true and Parent to -1 → check neighbors being visited by 3 → find 1, 7 → traverse to the lower of the neighbors - 1
<b>Start at 1:</b>	→ add 1 to the DFS Stack with an indent → assign the 1 <sup>st</sup> values of Visited to true and Parent to 3

	→ check neighbors being visited by 1 → find 2, 3, 6 → traverse to the lower of the neighbors - 2
<b>Start at 2:</b>	→ add 2 to the DFS Stack with two indents → assign the 2 <sup>nd</sup> values of Visited to true and Parent to 1 → check neighbors being visited by 2 → found 1 → since 1 has already been visited, search through 2 dies and recurses back to its parent at 1
<b>Start at 1:</b>	→ find the other neighbors 3, 6 → since 3 has already been visited, skip it → traverse to the next neighbor – 6
<b>Start at 6:</b>	→ add 6 to the DFS Stack with two indents → assign the 6 <sup>th</sup> values of Visited to true and Parent to 1 → check neighbors being visited by 6 → found 5 → traverse to 5
<b>Start at 5:</b>	→ add 5 to the DFS Stack with three indents → assign the 5 <sup>th</sup> values of Visited to true and Parent to 6 → check neighbors being visited by 5 → found 1, 3, 7 → since 1 and 3 are visited, skip and traverse to 7
<b>Start at 7:</b>	→ add 7 to the DFS Stack with four indents → assign the 7 <sup>th</sup> values of Visited to true and Parent to 5 → check neighbors being visited by 7 → none found → search through 7 dies, recurse back to its parent at 5
<b>Start at 5:</b>	→ since all its neighbors have been visited, recurse to its parent at 6
<b>Start at 6:</b>	→ since all its neighbors have been visited, recurse to its parent at 1
<b>Start at 1:</b>	→ since all its neighbors have been visited, recurse to its parent at 3
<b>Start at 3</b>	→ depth first search on the graph terminates

**Final connected graph after a depth first search:**



3. Complete a Dijkstra's Shortest Path algorithm on this graph:



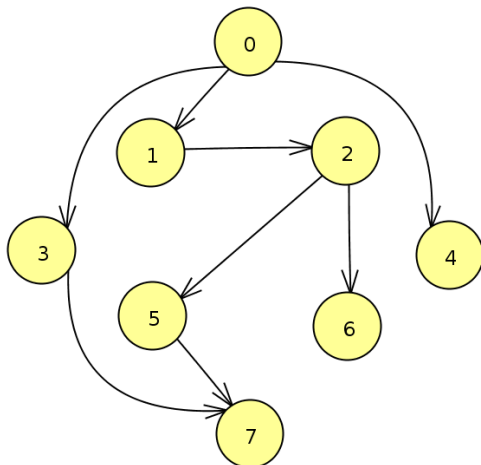
Vertex	Known	Cost	Parent	Path
0	T	5	1	1 0
1	T	0	-1	1
2	T	9	1	1 2
3	T	9	1	1 3
4	T	13	0	1 0 4
5	T	7	1	1 5
6	T	16	2	1 2 6
7	T	10	3	1 3 7

<b>Initial:</b>	<ul style="list-style-type: none"> <li>→ set all values of Known to false and Path to -1</li> <li>→ set Cost as an empty vector</li> </ul>
<b>Start at 1:</b>	<ul style="list-style-type: none"> <li>→ assign the 1<sup>st</sup> values of Known to true and Cost to 0</li> <li>→ check neighbors being visited by 1</li> <li>→ find 0, 2, 3, 5</li> <li>→ traverse to the lower of the neighbors - 0</li> </ul>
<b>Start at 0:</b>	<ul style="list-style-type: none"> <li>→ assign the 0<sup>th</sup> values of Cost to 5 and Path to 1</li> <li>→ traverse to the next lower of the neighbors of 1 - 2</li> </ul>
<b>Start at 2:</b>	<ul style="list-style-type: none"> <li>→ assign the 2<sup>nd</sup> values of Cost to 9 and Path to 1</li> <li>→ traverse to the next lower of the neighbors of 1 - 3</li> </ul>
<b>Start at 3:</b>	<ul style="list-style-type: none"> <li>→ assign the 3<sup>rd</sup> values of Cost to 9 and Path to 1</li> <li>→ traverse to the last of the neighbors of 1 - 6</li> </ul>
<b>Start at 5:</b>	<ul style="list-style-type: none"> <li>→ assign the 5<sup>th</sup> values of Cost to 7 and Path to 1</li> <li>→ traversal of the neighbors of 1 is done</li> </ul>
<b>Start at 1:</b>	<ul style="list-style-type: none"> <li>→ find the least cost of the neighbors</li> <li>→ find 0 with a cost of 5</li> <li>→ traverse back to 0</li> </ul>
<b>Start at 0:</b>	<ul style="list-style-type: none"> <li>→ assign the 0<sup>th</sup> value of Known to true</li> <li>→ check neighbors being visited by 0</li> <li>→ find 1, 3, 4</li> <li>→ since 1 has already been visited and its cost 0 is smaller than the new cost of 5, don't update and move on</li> <li>→ since 3 has a cost of 9 which is smaller than the new cost of 5 + 9, don't update and move on</li> <li>→ traverse to the last of its neighbors - 4</li> </ul>
<b>Start at 4:</b>	<ul style="list-style-type: none"> <li>→ assign the 4<sup>th</sup> values of Cost to 8 + 5 and Path to 0</li> <li>→ traversal of the neighbors of 0 is done</li> </ul>
<b>Start at 1:</b>	<ul style="list-style-type: none"> <li>→ find the least cost of the neighbors</li> <li>→ find 5 with a cost of 7</li> </ul>

	→ traverse back to 5
<b>Start at 5:</b>	→ assign the 5 <sup>th</sup> value of Known to true → check neighbors being visited by 5 → find 1, 2, 3, 7 → since 1 has already been visited and its cost 0 is smaller than the new cost of 7, don't update and move on → since 2 has a cost of 9 which is smaller than the new cost of 9 + 7, don't update and move on → since 3 has a cost of 9 which is smaller than the new cost of 9 + 3, don't update and move on → traverse to the last of its neighbors - 7
<b>Start at 7</b>	→ assign the 7 <sup>th</sup> values of Cost to 7 + 6 and Path to 5 → traversal of the neighbors of 5 is done
<b>Start at 1:</b>	→ find the least cost of the neighbors → find 2 and 3 with costs of 9 → traverse back to 2 because of the value of its key being lower
<b>Start at 2:</b>	→ assign the 2 <sup>nd</sup> value of Known to true → check neighbors being visited by 2 → find 1, 4, 5, 6 → since 1 has already been visited and its cost 0 is smaller than the new cost of 9, don't update and move on → since 4 has a cost of 13 which is smaller than the new cost of 9 + 6, don't update and move on → since 5 has a cost of 7 which is smaller than the new cost of 9 + 4, don't update and move on → traverse to the last of its neighbors – 6
<b>Start at 6</b>	→ assign the 6 <sup>th</sup> values of Cost to 9 + 7 and Path to 2 → traversal of the neighbors of 2 is done
<b>Start at 3:</b>	→ assign the 3 <sup>rd</sup> value of Known to true → check neighbors being visited by 3 → find 0, 1, 5, 7 → since 0 has already been visited and its cost 5 is smaller than the new cost of 9 + 5, don't update and move on → since 1 has already been visited and its cost 0 is smaller than the new cost of 9, don't update and move on → since 5 has a cost of 7 which is smaller than the new cost of 9 + 3, don't update and move on → since 7 has a cost of 13 which is greater than the new cost of 9 + 1, update its cost to 9 + 1 and Parent to 3 → traversal of the neighbors of 3 is done
<b>Start at 7</b>	→ since the next lowest cost is assigned to 7 with a 10 → assign the 7 <sup>th</sup> value of Known to true → check neighbors being visited by 7 → find 3, 4, 5 → since 3 has already been visited and its cost 9 is smaller than the new cost of 1 + 9, don't update and move on

	<ul style="list-style-type: none"> <li>→ since 4 has a cost of 13 is smaller than the new cost of <math>10 + 13</math>, don't update and move on</li> <li>→ since 5 has already been visited and its cost 7 is smaller than the new cost of <math>10 + 7</math>, don't update and move on</li> <li>→ traversal of the neighbors of 4 is done</li> </ul>
<b>Start at 4:</b>	<ul style="list-style-type: none"> <li>→ since the next lowest cost is assigned to 4 with a 13</li> <li>→ assign the 4<sup>th</sup> value of Known to true</li> <li>→ check neighbors being visited by 4</li> <li>→ find 0, 2, 7</li> <li>→ since 0 has already been visited and its cost 5 is smaller than the new cost of <math>8 + 5</math>, don't update and move on</li> <li>→ since 2 has already been visited and its cost 9 is smaller than the new cost of <math>6 + 9</math>, don't update and move on</li> <li>→ since 7 has a cost of 10 which is smaller than the new cost of <math>5 + 13</math>, don't update and move on</li> <li>→ traversal of the neighbors of 4 is done</li> </ul>
<b>Start at 6:</b>	<ul style="list-style-type: none"> <li>→ since the last of the nodes</li> <li>→ assign the 6<sup>th</sup> value of Known to true</li> <li>→ since the only neighbor of 6 is 2 which has been visited, its cost cannot be updated</li> <li>→ Dijkstra's Shortest Path algorithm terminates</li> </ul>

4. Complete a Topological sort on this graph:

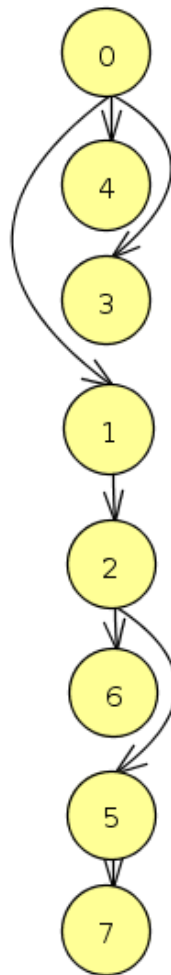


Vertex	Path	Topological order
0	DFS(0)	7
1	DFS(1)	5
2	DFS(2)	6
3	DFS(5)	2
4	DFS(7)	1
5	DFS(6)	3
6	DFS(3)	4
7	DFS(4)	0

<b>Initial:</b>	<ul style="list-style-type: none"> <li>→ set Path and Topological order as empty vectors</li> <li>→ topological sort begins with the vertex with the lowest value</li> </ul>
<b>Start at 0:</b>	<ul style="list-style-type: none"> <li>→ assign the 0<sup>th</sup> value of Path to DFS(0)</li> <li>→ start depth first search from 0</li> <li>→ look for neighbors visited by 0</li> <li>→ find 1, 3, 4</li> <li>→ traverse to the vertex with the lowest value – 1</li> </ul>
<b>Start at 1:</b>	<ul style="list-style-type: none"> <li>→ assign the 1<sup>st</sup> value of Path to DFS(1) with an indent</li> <li>→ look for neighbors visited by 1</li> </ul>

	<ul style="list-style-type: none"> <li>→ find 2</li> <li>→ traverse to 2</li> </ul>
<b>Start at 2:</b>	<ul style="list-style-type: none"> <li>→ assign the 1<sup>st</sup> value of Path to DFS(2) with two indents</li> <li>→ look for neighbors visited by 2</li> <li>→ find 5, 6</li> <li>→ traverse to the vertex with the lowest value – 5</li> </ul>
<b>Start at 5</b>	<ul style="list-style-type: none"> <li>→ assign the 5<sup>th</sup> value of Path to DFS(5) with three indents</li> <li>→ look for neighbors visited by 5</li> <li>→ find 7</li> <li>→ traverse to 7</li> </ul>
<b>Start at 7</b>	<ul style="list-style-type: none"> <li>→ assign the 7<sup>th</sup> value of Path to DFS(7) with four indents</li> <li>→ look for neighbors visited by 7</li> <li>→ none found</li> <li>→ add 7 to Topological order</li> <li>→ recurse back to its parent at 5</li> </ul>
<b>Start at 5</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 5</li> <li>→ none found</li> <li>→ add 5 to Topological order</li> <li>→ recurse back to its parent at 2</li> </ul>
<b>Start at 2</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 5</li> <li>→ found 6</li> <li>→ traverse to 6</li> </ul>
<b>Start at 6</b>	<ul style="list-style-type: none"> <li>→ assign the 6<sup>th</sup> value of Path to DFS(6) with three indents</li> <li>→ look for neighbors visited by 6</li> <li>→ none found</li> <li>→ add 6 to Topological order</li> <li>→ recurse back to its parent at 2</li> </ul>
<b>Start at 2</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 2</li> <li>→ none found</li> <li>→ add 2 to Topological order</li> <li>→ recurse back to its parent at 1</li> </ul>
<b>Start at 1</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 1</li> <li>→ none found</li> <li>→ add 1 to Topological order</li> <li>→ recurse back to its parent at 0</li> </ul>
<b>Start at 0</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 0</li> <li>→ found 3, 4</li> <li>→ traverse to the vertex with the lowest value – 3</li> </ul>
<b>Start at 3</b>	<ul style="list-style-type: none"> <li>→ assign the 3<sup>rd</sup> value of Path to DFS(3) with an indent</li> <li>→ look for neighbors visited by 3</li> <li>→ found 7</li> <li>→ since 7 has already been visited and added, move on</li> <li>→ add 3 to Topological order</li> <li>→ recurse back to its parent at 0</li> </ul>
<b>Start at 0</b>	<ul style="list-style-type: none"> <li>→ look for other neighbors visited by 0</li> <li>→ found 4</li> <li>→ traverse to its last neighbor - 4</li> </ul>

<b>Start at 4</b>	→ assign the 4 <sup>th</sup> value of Path to DFS(4) with an indent → look for neighbors visited by 4 → none found → add 4 to Topological order → recurse back to its parent at 0
<b>Start at 0</b>	→ no other neighbors left → add 0 to Topological order
<b>Topological order vector</b>	→ vertices currently are in order: 7, 5, 6, 2, 1, 3, 4, 0 → flip the vector so the vertices are: 0, 4, 3, 1, 2, 6, 5, 7 → reconnect the vertices to verify topological order of the vertices



Final topological sorted graph