Sabbir Ahmed

DATE: March 22, 2018

CMPE 320: HW 05

---

1. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period. And assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-2 \le x \le 4 \qquad\qquad -1 \le y - x \le 1.$$

(a) Find the marginal PMFs and the means of X and Y.

□

(b) Find the mean of the trader's profit.

□

2. The MIT football team wins any one game with probability p. and loses it with probability $1-p$. Its performance in each game is independent of its performance in other games. Let L_1 be the number of losses before its first win, and let L_2 be the number of losses after its first win and before its second win. Find the joint PMF of L_1 and L_2.

□

3. A class of n students takes a test in which each student gets an A with probability p, a B with probability q, and a grade below B with probability $1-p-q$, independently of any other student. If X and Y are the numbers of students that get an A and a B, respectively. calculate the joint PMF $p_{x,y}$.

4. Your probability class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let X be the number of students that get an A in your class.

(a) Calculate $E[X]$ by first finding the PMF of X

□

(b) Calculate $E[X]$ by viewing X as a sum of random variables, whose mean is easily calculated.

$\square$

5. A scalper is considering buying tickets for a particular game. The price of the tickets is \$75, and the scalper will sell them at \$150. However, if she can't sell them at \$150, she won't sell them at all. Given that the demand for tickets is a binomial random variable with parameters $n = 10$ and $p = 1/2$, how many tickets should she buy in order to maximize her expected profit?

6. Suppose that X and Y are independent discrete random variables with the same geometric PMF:

$$p_X(k) = p_Y(k) = p(1-p)^{k-1} \qquad\qquad k = 1, 2, \ldots,$$

where p is a scalar with $0, p < 1$. Show that for any integer $n \geq 2$, the conditional PMF

$$P(X = k \mid X + Y = n)$$

is uniform.

7. Consider four independent rolls of a 6-sides die. Let X be the number of 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?

8. Alvin shops for probability books for K hours, where K is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N|K}(n \mid k) = \frac{1}{k}, \qquad\qquad \text{for } n = 1, \ldots, k$$

(a) Find the joint PMF of K and N

$\square$

(b) Find the marginal PMF of N

$\square$

(c) Find the conditional PMF of K given that $N = 2$

□

(d) Find the conditional mean and variance of K, given that he bought at least 2 but no more than 3 books.

□

(e) The cost of each book is a random variable with mean \$30. What is the expected value of his total expenditure? *Hint:* Condition on the events $\{N = 1\}, \ldots, \{N = 4\}$, and use the total expectation theorem.

□