CMPE 330 Spring 2015 Second Midterm Examination—Version 2

(Each problem is worth 10 points)

1. A transmission line with an impedance of $75 + j50\ \Omega$ should be terminated with a resistor and a shorted 50-Ω stub to absorb all power at 100 MHz. What is the resistance and the length of the stub that should be used for the termination? You should find the shortest possible stub length and assume the velocity in the transmission line and the stub are both equal to c.

2. Use the definition of the gradient of a scalar ψ to prove that it is equal to

$$\nabla\psi(x,y,z) = \hat{\mathbf{x}}\frac{\partial\psi(x,y,z)}{\partial x} + \hat{\mathbf{y}}\frac{\partial\psi(x,y,z)}{\partial y} + \hat{\mathbf{z}}\frac{\partial\psi(x,y,z)}{\partial z}$$

   in Cartesian coordinates.

3. Write Maxwell's equations in differential form. Use Gauss's theorem and Stokes theorem to rewrite Maxwell's equations in integral form.

4. Consider a cylindrical capacitor of length L with inner radius a, outer radius b, and with an insulating material that has permittivity ϵ between the conductors. Show that the energy U that is required to charge the capacitor to voltage V is given by

$$U = \frac{\epsilon}{2}\int_v |\mathbf{E}|^2\, dv,$$

   where $\mathbf{E}(r,\theta,z)$ is the electric field inside the capacitor.

5. Suppose you have two current lines separated by 1 m and with 1 A of current flowing in each in the same direction. What is the force per unit length between the wires? Do they attract or repel? Suppose now that you add a third wire with a current of 10 A in the same plane and with the current in the same direction. This third wire is 1 m from one of the original wires and 2 m from the other. What are the directions of the forces acting on the two original wires? Are those two wires pulled apart or pushed together?