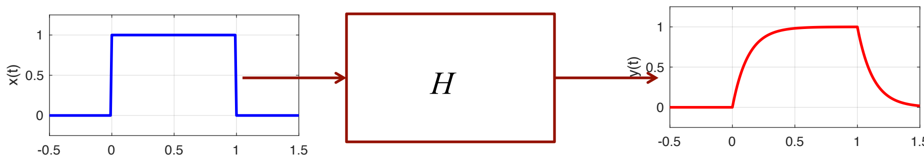
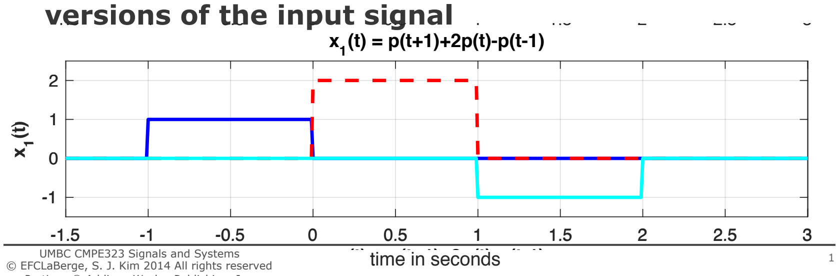
For LTI systems

- If we know the input output relationship for a signal if interest, like a pulse…

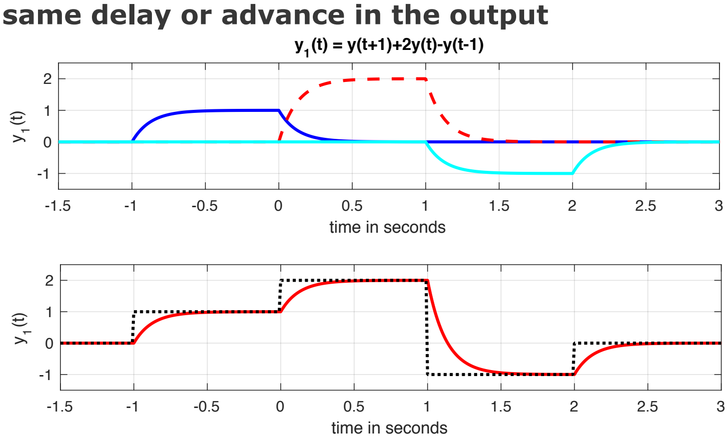


- …we can find the input output relationship for signals that can be decomposed into sums of delayed versions of the input signal



…continued

- By linearity, the output of the sum is the sum of the outputs…
- …by TI, a delay or advance in the input results in the same delay or advance in the output



So you try (solution on the board and posted)

Let H be LTI.

Let $x(t) = p(t,1)$ our unit amplitude, unit duration pulse

Let $h^{(p)}(t) = p(t,1) - p(t-1,1)$ (Our book writes this as $\left\{\underset{\uparrow}{1}, -1\right\}$)

1) Find the response to $x_1(t) = x(t) + 3x(t-1) - 2x(t-2) - 1x(t-3)$

2) Sketch the total response as a function of time.

3) Is this system causal?

4) Write the input in the form of a summation on k for different delays, with a different coefficient a_k for each delay.

5) Write an expression for $y_1(t) = H(x_1)$ as a summation of the outputs corresponding to the delayed inputs.

The convolution sum

- **The answer to the previous problem can be generalized to the *convolution sum***

$$y(t) = \sum_{k=-\infty}^{\infty} a_k h^{(p)}(t - kT)$$

- **If we're just interested in the values at the delayed times (not the intervening times), as in a discrete time system**

$$y_n = \sum_{k=-\infty}^{\infty} a_k h_{(n-k)}$$

- **Carefully note the indices!**
- **Have we seen this before?**
- **…yes, but you didn't recognize it**

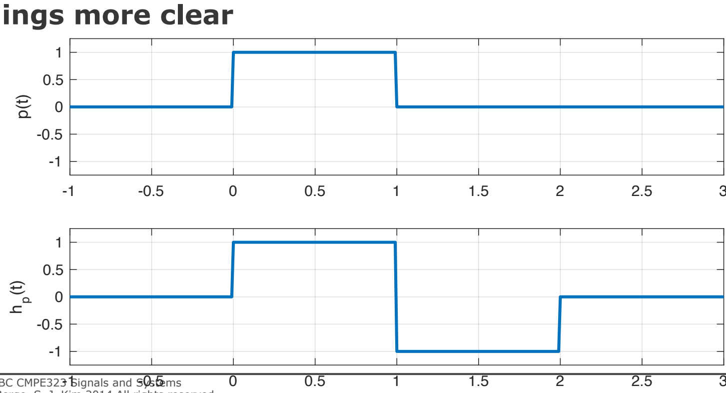
Consider multi-digit multiplication

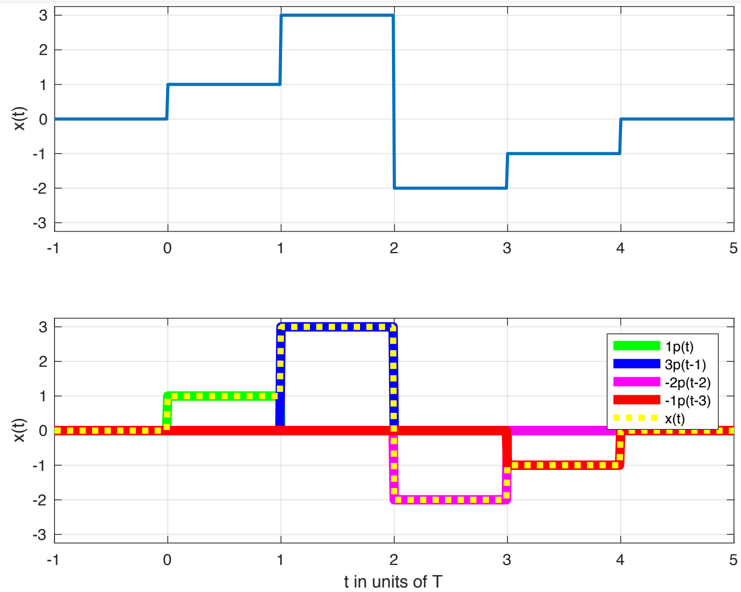
$$\begin{array}{llll} x_3 & x_2 & x_1 & x_0 \quad \Leftarrow \text{subscripts = powers of 10} \\ h_3 & h_2 & h_1 & h_0 \end{array}$$

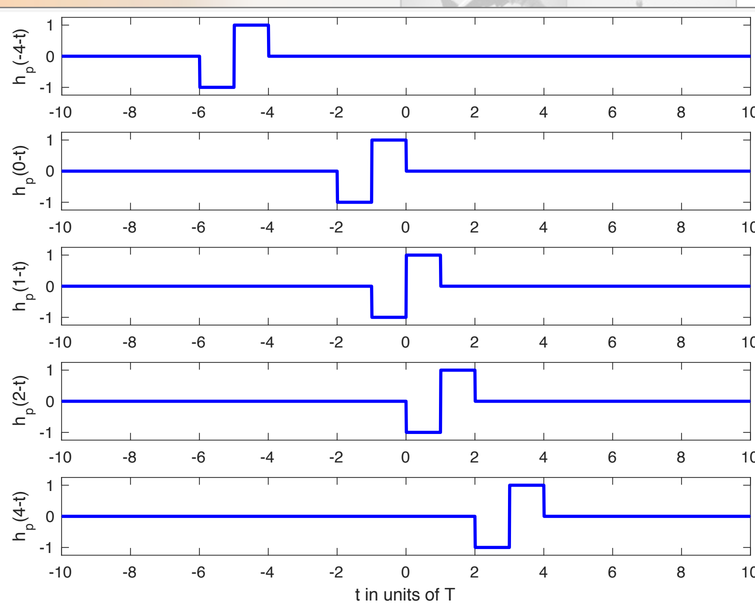
$$\begin{array}{lc} 0 & x_0 h_0 \\ 1 & x_0 h_1 + x_1 h_0 \\ 2 & x_0 h_2 + x_1 h_1 + x_2 h_0 \\ 3 & x_0 h_3 + x_1 h_2 + x_2 h_1 + x_3 h_0 \\ 4 & (\ldots + x_{-1} h_5 + x_0 h_4) + x_1 h_3 + x_2 h_2 + x_3 h_1 + (x_4 h_0 + x_5 h_{-1} \ldots) \\ 5 & (\ldots) + x_2 h_3 + x_3 h_2 + (\ldots) \\ 6 & (\ldots) + x_3 h_3 + (\ldots) \end{array}$$

In general: $y_n = \sum_{k=-\infty}^{\infty} x_n h_{n-k} \overset{!!}{=} \sum_{k=-\infty}^{\infty} x_{n-k} h_n$

Breaking the sum apart

- **What does it mean graphically?**
- **Let**
  $h_0 = 1, h_1 = -1 = \left\{ \underset{\uparrow}{1}, -1 \right\}; x_0 = 1, x_2 = 3, x_3 = -2, x_4 = -1 = \left\{ \underset{\uparrow}{1}, 3, -2, -1 \right\}$
- **What does** h_{n-k} **look like for** $n = -4, 0, 1, 2, 4$?
- **You may use the "unit pulse" sketches to make things more clear**
- 


x(t)
t in units of T
1p(t)
3p(t-1)
-2p(t-2)
-1p(t-3)
x(t)


$h_p(-4-t)$
$h_p(0-t)$
$h_p(1-t)$
$h_p(2-t)$
$h_p(4-t)$
t in units of T