

NOTE: Solutions are available for many of the problems. You must show complete work for full credit.

When nothing else is stated, carry out calculations to four significant figures and report three.

1. Determine the wavelength at the following frequencies in SI and in English units [based on Paul 1.2.1, p. 23]. Report answers to three significant figures. Note: The answers in Paul's book, which are sometimes given to many significant figures, are not reliable past the third significant figure, because he uses $c = 3.000 \times 10^8$ m/s, rather than the true value to four significant figures, which is $c = 2.998 \times 10^8$ m/s. In the fifth significant digit, the speed of light in air is lower than in vacuum by a factor of about 1.0003, and we would have to take that into account if we were reporting four or five digits. In general, you should keep at least one more digit in your calculations than you report. In parts (c) and (i), you must keep two more digits. Why? In general, *do not report more significant digits than you have!*
 - a. LORAN C long-range navigation 100 Hz [3000 km, 1860 mi]
 - b. Submarine communication 1.5 kHz
 - c. Automatic detection finder in aircraft 330 kHz [908 m, 0.564 mi]
 - d. AM radio transmission 1.0 MHz
 - e. Amateur radio 30 MHz [9.99 m, 32.8 ft]
 - f. FM radio transmission 125 MHz
 - g. Instrument landing system 310 MHz [96.7 cm, 3.17 ft]
 - h. Satellite 6.2 GHz
 - i. Remote sensing 43 GHz [6.97 mm, 274 mils]
2. Determine the following physical dimensions in wavelength, which is their electrical dimension [modified from Paul 1.2.2, p. 23]. Report answers to three significant digits, except in part (d), where should only report two. Why?
 - a. A 100 km length of a 50 Hz power transmission line (European power system) [0.0167]
 - b. A 500 ft AM broadcast antenna operating at 400 kHz
 - c. A 5 ft FM broadcast antenna operating at 110 Mhz [0.559]
 - d. A 2 in land on a printed circuit board at 3 GHz, assuming a velocity of propagation in the land of 1.5×10^8 m/s.
3. A sinusoidal current wave is described. Determine the velocity of propagation and the wavelength. From the distance d that the wave travels, determine the time delay and

the phase shift. [modified from Paul 1.2.3, p. 23]. Report results to two significant figures.

- a. $i(t, z) = I_0 \cos(2\pi \times 10^6 t - 2.2 \times 10^{-2} z)$, $d = 5 \text{ km}$ [$u_p = 2.9 \times 10^8 \text{ m/s}$, $\lambda = 290 \text{ m}$, $T = 18 \text{ } \mu\text{s}$, $\phi = 6300^\circ$]
 - b. $i(t, z) = I_0 \cos(6\pi \times 10^9 t - 75.4z)$, $d = 3 \text{ in}$
 - c. $i(t, z) = I_0 \cos(30\pi \times 10^7 t - 3.15z)$, $d = 15 \text{ ft}$ [$u_p = 3.0 \times 10^8 \text{ m/s}$, $\lambda = 2.0 \text{ m}$, $T = 15 \text{ ns}$, $\phi = 830^\circ$]
 - d. $i(t, z) = I_0 \cos(6\pi \times 10^3 t - 0.126 \times 10^{-3} z)$, $d = 40 \text{ mi}$
4. The voltage of an electromagnetic wave travelling on a transmission line is given by [modified from Ulaby, et al. 1.13, p. 44]

$$v(z, t) = 4 \exp(-\alpha z) \sin(\pi \times 10^9 t - 5\pi z),$$

where z is the distance in meters from the generator.

- a. Find the frequency, wavelength, and phase velocity of the wave
 - b. At $z = 2 \text{ m}$, the amplitude of the wave was measured to be 1 V . Find α
5. A signal propagates in a birefringent optical fiber. A birefringent optical fiber has two different light polarizations that propagate at slightly different velocities. The vacuum wavelength of the light is $1.5 \text{ } \mu\text{m}$. The bit rate is 10 Gbits/s . The index of refraction, $n = \beta c/\omega = 1.5$ and the index difference between the two polarizations is given by $\Delta n/n = \Delta\beta/\beta = 10^{-5}$.
- a. What is the frequency of the light? [$2.00 \times 10^{14} \text{ Hz}$]
 - b. What is the speed of light in the optical fiber? [$2.00 \times 10^8 \text{ m/s}$]
 - c. What is the time separation between bits (bit period)? [100 ps]
 - d. What is the velocity difference between the two polarizations? [$2.00 \times 10^3 \text{ m/s}$]
 - e. How far can the signal propagate before the signals in the two polarizations separate by a bit period? [2.0 km]
6. Evaluate each of the following complex numbers and express the result in rectangular form [modified from Ulaby, et al. 1.16, p. 45]: (a) $z_1 = 4 \exp(j\pi/3)$, (b) $z_2 = \sqrt{3} \exp(j3\pi/4)$, (c) $z_3 = 2 \exp(j\pi/2)$, (d) $z_4 = j^3$, (e) $z_5 = j^{-2}$, (f) $z_6 = (1 - j)^5$, (g) $z_7 = (1 - j)^{1/2}$
7. Derive Ulaby, et al.'s Eq. (1.68) directly without using phasors. [Hint: You will have to begin by assuming that $i(t)$ has the form $i(t) = A \cos(\omega t + \phi_0 - \phi_1)$. The task is to determine A and ϕ_1 . After substitution into the time domain equation, Eq. 1.56, you will have to use the difference formulae for the cosine and sine functions, $\cos(A - B) = \cos A \cos B + \sin A \sin B$ and $\sin(A - B) = \sin A \cos B - \cos A \sin B$, to find the coefficients of $\cos(\omega t + \phi_0)$ and $\sin(\omega t + \phi_0)$. That will give you two equations

for two unknowns, $A \cos \phi_1$ and $A \sin \phi_1$. After solving those equations, you can solve the problem.]

8. A series RLC circuit is connected to a generator with a voltage $v_s(t) = V_0 \cos(\omega t + \pi/3)$ [Ulaby, et al. 1.28, p. 46]
 - a. Write the loop voltage equation in terms of the current $i(t)$, R , L , C , and $v_s(t)$
 - b. Obtain the corresponding phasor domain equation
 - c. Solve the equation to find an expression for the phasor current \tilde{I}