MEMO Number CMPE323-Lab03

DATE: September 22, 2016
TO: CMPE323
FROM: EFC LaBerge
SUBJECT: Convolution

1 INTRODUCTION

We have been talking about convolution. In class we developed both the convolution sum

$$y_n = \sum_{k=-\infty}^{\infty} x_k h_{n-k} = \sum_{k=-\infty}^{\infty} x_{n-k} h_k \quad , \qquad 22\backslash* \text{ MERGEFORMAT ()}$$

and the convolution integral

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau) h(\tau)\, d\tau \qquad 33\backslash* \text{ MERGEFORMAT ()}$$

MATLAB, being a program for a digital computer, implements the convolution sum. This lab explores considerations for estimating the convolution integral in the discrete domain, that is, without using MATLAB's Symbolic Algebra toolbox.

2 EQUIPMENT

For this lab, you need a laptop with MATLAB installed.

For the purpose of CMPE323, please use the following naming conventions for all output files:

CMPE323F16_Lab<Lab#>_<Your Campus ID>

For the purpose of CMPE323, please use the following naming conventions for MATLAB scripts or functions that you are required to submit.

<function name>_<Your Campus ID>

Examples will be given in the lab description. Follow the instructions exactly, or you may not get graded!

3 LAB TASKS

You might find it useful to use the MATLAB function `diary` to capture your inputs and outputs.

3.1 Simple Pulses

Use your anonymous function to create two unit amplitude pulses $p(t,1)$ of duration one time unit starting at $t=0$. For this exercise, use a time array `t = [0:.01:5]`. Designate one of the pulses as `x` and one as `h`. Call the output array `yhat`.

Write a MATLAB script to perform the convolution sum given in 2. Although a double loop is permissible, I encourage you to consider how you might do the problem using matrix-vector multiplication. Such an implementation will be much more efficient. Make sure that you include *all* of the convolution response! *Hint: How many points are in the response?*

Can you plot the output, `yhat`, against the time array? Why or why not? If not, what modifications to the time array are necessary to permit plotting. Make these modifications, if any, in a variable called `t2`.

Use pencil and paper (or equivalent) to perform the convolution integral of 3 for these two pulses. Implement your analytic equation in MATLAB using the `t2` time array.

Plot the input, `x`, impulse response, `h`, and convolution sum, `yhat`, and the analytic result, `y` on four separate axes. Use professional practices on your plots. Explain any differences between the convolution sum and the analytic result.

Redo your four plots using the MATLAB built-in function `conv(x,h)`. How should you scale the output of `conv(x,h)` to approximate the convolution integral, and thereby bring the third and fourth plots into approximate agreement? What could you do to improve the approximation to the analytic result given by the convolution integral?

3.2 Unequal Pulses

Replace x with $p(t,2)$ and redo all steps of 3.1. Pay particular attention to any changes that occur in `t2`, and explain any changes in your writeup.

3.3 Offset input

Replace x with $p(t+1,2)$ and redo all steps of 3.1. Pay particular attention to your output as a function of time. Explain why your output makes sense. If your output doesn't make sense, what modifications must you make to `t2` to obtain a meaningful result? How do the modifications depend on the starting point of the input pulse?

3.4 Offset input and offset impulse response

Keep x as $p(t+1,2)$ and replace h with $2p(t-0.5)$ and redo all steps of 3.1. Pay particular attention to your output as a function of time. Explain why your output makes sense. If your output doesn't make sense, what modifications must you make to `t2` to obtain a meaningful result? How do the modifications depend on the starting point of the input pulse and the starting point of the impulse response?

3.5 LTI Characteristic

Using your pulse anonymous function, create input(s) that demonstrate that the convolution is LTI. This should come as not surprise, as we assume the system was LTI to derive the convolution sum and convolution integral.

4 LAB SUBMISSIONS

Submit the following via the Blackboard assignment Lab 3.

Using this lab description document as a template, create a single PDF file named in accordance with the output naming conventions given above. The content must include

a. The outputs and discussions generated in 3.1.

b. The outputs and discussions generated in 3.2.

c. The outputs and discussions generated in 3.3.

d. The outputs and discussions generated in 3.4.

e. The outputs and discussions generated in 3.5.

Professional, high quality writing, math, and graphic (that is plots) presentation is expected, and must be provided for you to earn full credit.

In a separate file, but as part of your submission, include you MATLAB script for 3.1. If you built one big script to handle all elements of the lab, you may include the entire thing. Use good programming practices in your script.