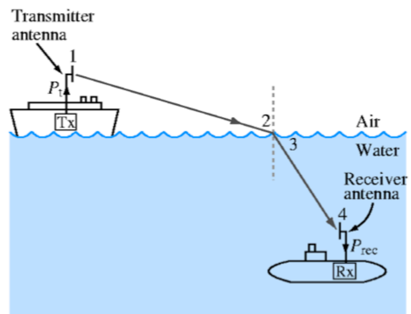
Waves at Boundaries

Flow from the transmitter (Tx) to the receiver (Rx)

- A signal is created electrically and flows through a transmission line
- The signal goes to an antenna, where it is radiated into the air
- When the signal reaches the air-water interface, it is refracted
- At the receiving antenna, the signal is converted to electrical impulses
- The signal flows through a transmission line to a computer
- The data is stored

At every step, Maxwell's equations govern the behavior!

We will now discuss how to calculate the flow of electromagnetic signals from one medium to another



Ulaby Figure 8-1

UMBC AN HONORS UNIVERSITY IN MARYLAND

Waves at Boundaries

Reflection and transmission

When a wave encounters a boundary between two media, part is transmitted and part is reflected

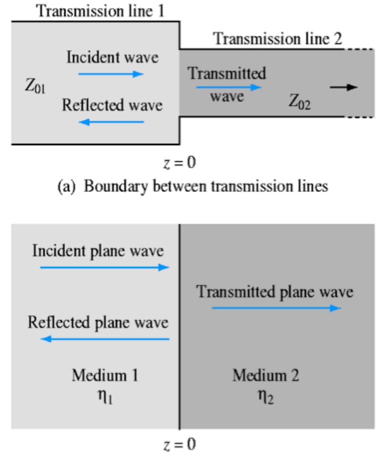
The media are characterized by different values for η_1 and η_2.

This behavior is analogous to what is observed at the boundary of two transmission lines with two different impedances

We will use ***rays*** to represent the flow of electromagnetic waves. Rays are arrows that point in the direction of the **k**-vectors and are orthogonal to the ***wavefronts***

Wavefront = points where the field has constant phase

Ulaby Figure 8-2



(a) Boundary between transmission lines

(b) Boundary between different media

UMBC AN HONORS UNIVERSITY IN MARYLAND

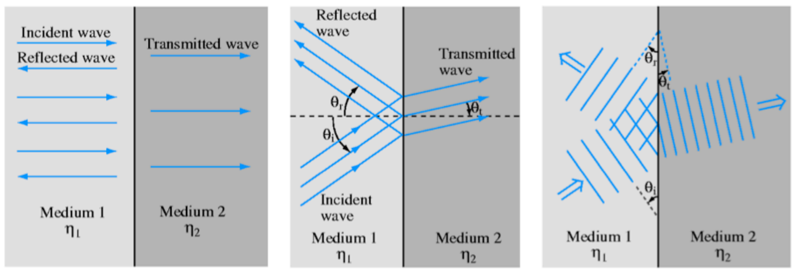
Waves at Boundaries

Normal and oblique incidence

A signal can strike a boundary surface at any angle

- Normal incidence = the **k**-vector of the signal is orthogonal to the surface
- Oblique incidence = the **k**-vector of the signal is not orthogonal to the surface

Normal incidence is simpler to describe and very important;
so we treat it first



(a) Normal incidence (b) Ray representation of oblique incidence (c) Wavefront representation of oblique incidence

Ulaby Figure 8-3

UMBC
AN HONORS UNIVERSITY IN MARYLAND

15.3

As noted earlier (slide 13.2), it is often possible to treat guided waves as transversely modulated plane waves. Light in optical fibers is an example. In that case, the incidence on a boundary is almost always normal.

Normal Incidence

Lossless Media

We consider two media that are lossless with the boundary at $z = 0$

— the media are characterized by ε_1, μ_1 and ε_2, μ_2

An x-polarized plane wave is normally incident

We then have

Incident wave:

$$\tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{i}} \exp(-jk_1 z), \quad \tilde{\mathbf{H}}^{\mathrm{i}}(z) = \frac{1}{\eta_1}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{y}} \frac{E_0^{\mathrm{i}}}{\eta_1} \exp(-jk_1 z)$$

Reflected wave:

$$\tilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{r}} \exp(jk_1 z), \quad \tilde{\mathbf{H}}^{\mathrm{r}}(z) = -\frac{1}{\eta_1}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{r}}(z) = -\hat{\mathbf{y}} \frac{E_0^{\mathrm{r}}}{\eta_1} \exp(jk_1 z)$$

Transmitted wave:

$$\tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{t}} \exp(-jk_2 z), \quad \tilde{\mathbf{H}}^{\mathrm{t}}(z) = \frac{1}{\eta_2}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{y}} \frac{E_0^{\mathrm{t}}}{\eta_2} \exp(-jk_2 z)$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

15.4

Normal Incidence

Lossless Media

Incident wave:

$$\tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{i}} \exp(-jk_1 z), \quad \tilde{\mathbf{H}}^{\mathrm{i}}(z) = \frac{1}{\eta_1}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{y}}\frac{E_0^{\mathrm{i}}}{\eta_1}\exp(-jk_1 z)$$

Reflected wave:

$$\tilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{r}} \exp(jk_1 z), \quad \tilde{\mathbf{H}}^{\mathrm{r}}(z) = -\frac{1}{\eta_1}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{r}}(z) = -\hat{\mathbf{y}}\frac{E_0^{\mathrm{r}}}{\eta_1}\exp(jk_1 z)$$

Transmitted wave:

$$\tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{t}} \exp(-jk_2 z), \quad \tilde{\mathbf{H}}^{\mathrm{t}}(z) = \frac{1}{\eta_2}\hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{y}}\frac{E_0^{\mathrm{t}}}{\eta_2}\exp(-jk_2 z)$$

The incident and transmitted wave propagate in the +z-direction

The reflected wave propagates in the –z-direction

Mathematical Consequences: $jkz \rightarrow -jkz$ and $\hat{\mathbf{y}} \rightarrow -\hat{\mathbf{y}}$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

Normal Incidence

Lossless Media

There are no free charges or currents

→ The **E** and **H** fields are all continuous across the boundary

Fields in Medium 1 ($z < 0$)**:**

$$\tilde{\mathbf{E}}_1(z) = \tilde{\mathbf{E}}^{\mathrm{i}}(z) + \tilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}}\,[E_0^{\mathrm{i}} \exp(-jk_1 z) + E_0^{\mathrm{r}} \exp(jk_1 z)],$$

$$\tilde{\mathbf{H}}_1(z) = \tilde{\mathbf{H}}^{\mathrm{i}}(z) + \tilde{\mathbf{H}}^{\mathrm{r}}(z) = \hat{\mathbf{y}}\frac{1}{\eta_1}[E_0^{\mathrm{i}} \exp(-jk_1 z) - E_0^{\mathrm{r}} \exp(jk_1 z)]$$

Fields in Medium 2 ($z > 0$)**:**

$$\tilde{\mathbf{E}}_2(z) = \tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}}\,E_0^{\mathrm{t}} \exp(-jk_2 z), \quad \tilde{\mathbf{H}}_2(z) = \tilde{\mathbf{H}}^{\mathrm{t}}(z) = \hat{\mathbf{y}}\frac{E_0^{\mathrm{t}}}{\eta_2} \exp(-jk_2 z)$$

Matching the fields at $z = 0$:

$$\tilde{\mathbf{E}}_1(0) = \tilde{\mathbf{E}}_2(0) \quad \rightarrow \quad E_0^{\mathrm{i}} + E_0^{\mathrm{r}} = E_0^{\mathrm{t}},$$

$$\tilde{\mathbf{H}}_1(0) = \tilde{\mathbf{H}}_2(0) \quad \rightarrow \quad \frac{E_0^{\mathrm{i}}}{\eta_1} - \frac{E_0^{\mathrm{r}}}{\eta_1} = \frac{E_0^{\mathrm{t}}}{\eta_2}$$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

Normal Incidence

Lossless Media

Reflected and transmitted amplitudes

$$E_0^{\mathrm{r}} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} \equiv \Gamma E_0^{\mathrm{i}}, \quad E_0^{\mathrm{t}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} \equiv \tau E_0^{\mathrm{i}}$$

- Γ = reflection coefficient
- τ = transmission coefficient; $\tau = 1 + \Gamma$

Another expression: Using $\eta_1 = \eta_0 / \sqrt{\varepsilon_{\mathrm{r1}}}, \quad \eta_2 = \eta_0 / \sqrt{\varepsilon_{\mathrm{r2}}}$
we have

$$\Gamma = \frac{\sqrt{\varepsilon_{\mathrm{r2}}} - \sqrt{\varepsilon_{\mathrm{r1}}}}{\sqrt{\varepsilon_{\mathrm{r2}}} + \sqrt{\varepsilon_{\mathrm{r1}}}}$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

Ulaby 2001

15.7

Note existence of standing waves by analogy with transmission lines. See next slide first.

Do Module 8.1. lambda-1 = 3.0 m, lambda-2 = 1.5 m, eta-1 = 377 ohms, eta-2 = 189 ohms, Gamma = -0.33, tau = 0.67, E_1,max = 13.3, E_2,max = 6.7, l_max = 0.75 m, l_min = 0m.

Normal Incidence

Standing Wave Ratio

Reflected and transmitted amplitudes

$$E_0^{\mathrm{r}} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} \equiv \Gamma E_0^{\mathrm{i}}, \quad E_0^{\mathrm{t}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} \equiv \tau E_0^{\mathrm{i}}$$

- G = reflection coefficient
- *t* = transmission coefficient; *t* = 1 + G

This result generalizes to the case where h₁ and h₂ are complex.

When h_1 is real, we have

$$\left|\tilde{\mathbf{E}}_1(z)\right|^2 = \left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2k_1 z + \theta_{\mathrm{r}})\right] |E_0^{\mathrm{i}}|^2 \quad \text{with} \quad \Gamma = |\Gamma| \exp(j\theta_{\mathrm{r}})$$

Just as in the case of transmission lines,
we can define a standing-wave ratio and determine points
where the amplitude oscillations are maxima and minima

UMBC
AN HONORS
UNIVERSITY
IN MARYLAND

Ulaby 2001

15.8

When eta_1 is complex we get exponential decay in opposite directions, which complicates the description of the z-variation.

See Ulaby for details on the definitions of SWR and the minima and maxima locations. Do module 8.1, noting the analogy with transmission lines, so that E_1-max = (1+|Gamma|)*10 volts = 13.3 volts.

Answers: lambda_1 = 3, lambda_2 = 1.5, eta_1 = 377, eta_2 = 188, Gamma = -0.33, tau = 0.67, E_1,max = 13.3, E_2,max = 6.7, l_max = 0.75, l_min = 0

Normal Incidence

Transmission Line Analogies

Plane Wave	Transmission Line
$\tilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{i}}[\exp(-jk_1 z) + \Gamma \exp(jk_1 z)]$	$\tilde{V}_1(z) = V_0^{+}[\exp(-j\beta_1 z) + \Gamma \exp(j\beta_1 z)]$
$\tilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}}\dfrac{E_0^{\mathrm{i}}}{\eta_1}[\exp(-jk_1 z) - \Gamma \exp(jk_1 z)]$	$\tilde{I}_1(z) = \dfrac{V_0^{+}}{Z_{01}}[\exp(-j\beta_1 z) - \Gamma \exp(j\beta_1 z)]$
$\tilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}}\, \tau E_0^{\mathrm{i}} \exp(-jk_2 z)$	$\tilde{V}_2(z) = V_0^{+} \tau \exp(-j\beta_2 z)$
$\tilde{\mathbf{H}}_2(z) = \hat{\mathbf{y}}\, \tau \dfrac{E_0^{\mathrm{i}}}{\eta_2} \exp(-jk_2 z)$	$\tilde{I}_2(z) = \tau \dfrac{V_0^{+}}{Z_{02}} \exp(-j\beta_2 z)$
$\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$	$\Gamma = (Z_{02} - Z_{01})/(Z_{02} + Z_{01})$
$\tau = 2\eta_2/(\eta_2 + \eta_1)$	$\tau = 2Z_{02}/(Z_{02} + Z_{01})$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

Normal Incidence

Power Flow in Lossless Media

We have

$$\mathbf{S}_{\mathrm{av1}} = \tfrac{1}{2}\mathrm{Re}\left[\tilde{\mathbf{E}}_1(z)\times\tilde{\mathbf{H}}_1^*(z)\right]$$

$$= \tfrac{1}{2}\mathrm{Re}\left\{\hat{\mathbf{x}}\,E_0^{\mathrm{i}}[\exp(-jk_1 z)+\Gamma\exp(jk_1 z)]\times\hat{\mathbf{y}}\,\frac{E_0^{\mathrm{i}*}}{\eta_1}[\exp(jk_1 z)-\Gamma\exp(-jk_1 z)]\right\}$$

$$= \hat{\mathbf{z}}\frac{|E_0^{\mathrm{i}}|^2}{2\eta_1}(1-\Gamma^2)$$

Note that cross-terms cancel! As a consequence:

$$\mathbf{S}_{\mathrm{av1}} = \mathbf{S}_{\mathrm{av}}^{\mathrm{i}} + \mathbf{S}_{\mathrm{av}}^{\mathrm{r}}$$

with $\mathbf{S}_{\mathrm{av}}^{\mathrm{i}} = \hat{\mathbf{z}}\dfrac{|E_0^{\mathrm{i}}|^2}{2\eta_1}$ and $\mathbf{S}_{\mathrm{av}}^{\mathrm{r}} = -\hat{\mathbf{z}}\Gamma^2\dfrac{|E_0^{\mathrm{i}}|^2}{2\eta_1} = -\Gamma^2\mathbf{S}_{\mathrm{av}}^{\mathrm{i}}$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

15.10

If we replace Gamma^2 with |Gamma|^2, this expression is valid when medium 2 is lossy; that is what Ulaby does

Normal Incidence

Power Flow in Lossless Media

We also have

$$\mathbf{S}_{\text{av2}} = \tfrac{1}{2}\text{Re}\left[\tilde{\mathbf{E}}_2(z)\times\tilde{\mathbf{H}}_2^*(z)\right]$$

$$= \tfrac{1}{2}\text{Re}\left[\hat{\mathbf{x}}\,\tau E_0^{\text{i}}\exp(-jk_2 z)\times\hat{\mathbf{y}}\,\tau\frac{E_0^{\text{i}*}}{\eta_2}\exp(jk_2 z)\right] = \hat{\mathbf{z}}\tau^2\frac{|E_0^{\text{i}}|^2}{2\eta_2}$$

Using the relation

$$\frac{\tau^2}{\eta_2} = \frac{2}{\eta_2+\eta_1} = \frac{1-\Gamma^2}{\eta_1}$$

we conclude

$$\mathbf{S}_{\text{av1}} = \mathbf{S}_{\text{av2}}$$

And energy is conserved! As it should be

UMBC AN HONORS UNIVERSITY IN MARYLAND

15.11

If we let tau^2 become |tau|^2, then the relation is correct when medium 1 is lossy. Ulaby et al. do that.

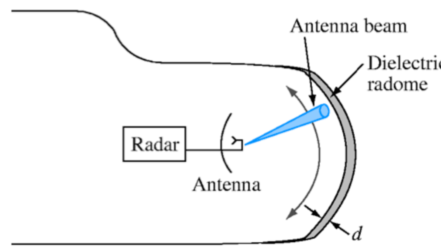
Again, the point is not to prove that energy is conserved. The point is to prove that Maxwell’s equations are consistent with this law of nature.

Normal Incidence

Radar Radome Design: Ulaby et al. Example 8-1

Question: A 10 GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal. Over the narrow extent of the antenna beam, we can assume that the radome shape is planar. If the radome material is a lossless dielectric with $\mu_r = 1$ and $\varepsilon_r = 9$, choose the thickness d such that the radome appears transparent to the radar beam. Mechanical integrity requires $d > 2.3$ cm.

Answer: This is an impedance-matching problem — analogous to impedance-matching problems that we saw in the study of transmission lines.



Ulaby Figure 8-5

UMBC
AN HONORS UNIVERSITY IN MARYLAND

15.12

At least, the planar approximation is a good place to start. For exact answers, you want to use numerical methods. This is the right approximation procedure. You start with a simple theoretical estimate. You can then use that to check the more sophisticated calculations.

Even if the radome is transparent, there can be transients that lead to reflections that burn out the transmitter. Thus, it is important to calculate them! In some years, I give this issue as an exercise in Problem Set 8.

Normal Incidence

Radar Radome Design: Ulaby Example 8-1

Answer (continued): There will be no reflection if the input impedance matches the air impedance η_0. For a single polarization, we may define the input impedance as $\eta_{\rm in}(z) = |\tilde{\mathbf{E}}_2(z)|/|\tilde{\mathbf{H}}_2(z)|$ since medium 2 corresponds to the transmission line. It follows that

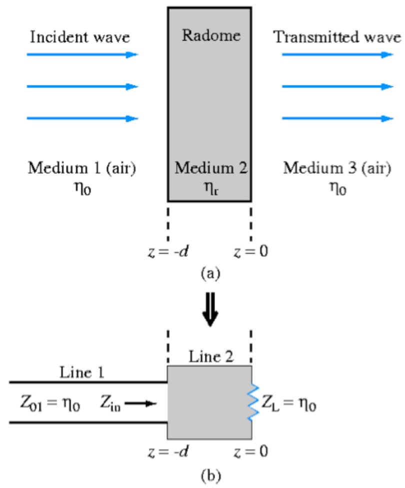
$$\eta_{\rm in}(z) = \eta_{\rm r}\left(\frac{\exp(-jk_{\rm r}z) + \Gamma\exp(jk_{\rm r}z)}{\exp(-jk_{\rm r}z) - \Gamma\exp(jk_{\rm r}z)}\right)$$

with $\Gamma = (\eta_0 - \eta_{\rm r})/(\eta_0 + \eta_{\rm r})$. So, we have

$$\eta_{\rm in}(z) = \eta_{\rm r}\left(\frac{\eta_0 - j\eta_{\rm r}\tan(k_{\rm r}z)}{\eta_{\rm r} - j\eta_0\tan(k_{\rm r}z)}\right)$$

$$= \eta_0 \quad \text{when} \quad k_{\rm r}z = -n\pi$$

(where n is an integer)



Ulaby Figure 8-6

UMBC
AN HONORS UNIVERSITY IN MARYLAND

15.13

We choose the minus sign for convenience in the condition on k_r z.

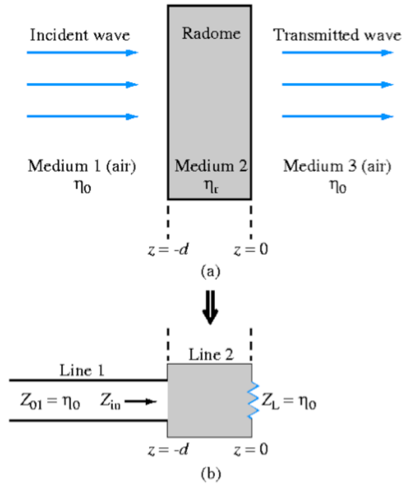
Normal Incidence

Radar Radome Design: Ulaby Example 8-1

Answer (continued): So, we choose

$$d = -z = n\lambda_r / 2 = nc / 2f\sqrt{\varepsilon_r} = n \times 0.5 \text{ cm}$$

The value n = 5 is the minimum that allows us to obey the condition for structural integrity, and we conclude d = 2.5 cm.



Ulaby Figure 8-6

UMBC
AN HONORS UNIVERSITY IN MARYLAND

Normal Incidence

Lossy Media

We may generalize our results to lossy media by using the transformation

$$jk \to \gamma, \quad \eta \to \eta_c$$

We thus obtain in medium 1:

$$\tilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}}\, E_0^i [\exp(-\gamma_1 z) + \Gamma \exp(\gamma_1 z)], \quad \tilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}} \frac{E_0^i}{\eta_{c1}} [\exp(-\gamma_1 z) - \Gamma \exp(\gamma_1 z)]$$

and in medium 2:

$$\tilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}}\, \tau E_0^i \exp(-\gamma_2 z), \quad \tilde{\mathbf{H}}_2(z) = \hat{\mathbf{y}}\, \tau \frac{E_0^i}{\eta_{c2}} \exp(-\gamma_2 z)$$

with $\gamma_1 = \alpha_1 + j\beta_1$, $\gamma_2 = \alpha_2 + j\beta_2$, and

$$\Gamma = \frac{\eta_{c2} - \eta_{c1}}{\eta_{c2} + \eta_{c1}}, \quad \tau = \frac{2\eta_{c2}}{\eta_{c2} + \eta_{c1}}$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

15.15

Normal Incidence

Normal Incidence on a Metal Surface: Ulaby et al. Example 8-3

Question: A 1 GHz x-polarized TEM wave traveling in the $+z$-direction and is incident in air upon a metal surface coincident with the x-y plane at $z = 0$. The incident electric field amplitude is 12 mV/m, and we have for copper $\varepsilon_{\mathrm{r}} = 1$, $\mu_{\mathrm{r}} = 1$, and $\sigma = 5.8\times10^{7}$ S/m. Obtain expressions for the instantaneous fields in the air medium. Assume that the metal surface is more than five times the skin depth in thickness.

Answer: In medium 1 (air), $\alpha = 0$, and

$$\beta = k_1 = \frac{\omega}{c} = \frac{2\pi\times10^{9}}{3\times10^{8}} = \frac{20\pi}{3}\ \mathrm{m}^{-1}, \quad \eta_1 = \eta_0 = 377\ \Omega, \quad \lambda = \frac{2\pi}{k_1} = 0.3\ \mathrm{m}.$$

At $f = 1$ GHz, copper is an excellent conductor because

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon_{\mathrm{r}}\varepsilon_0} = \frac{5.8\times10^{7}}{(2\pi\times10^{9})\times(10^{-9}/36\pi)} = 1\times10^{9} \gg 1$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

15.16

Normal Incidence

Normal Incidence on a Metal Surface: Ulaby et al. Example 8-3

Answer (continued): We obtain for the intrinsic impedance

$$\eta_{c2} = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\left[\frac{\pi\times 10^9 \times (4\pi\times 10^{-7})}{5.8\times 10^7}\right]^{1/2} = 8.25\,(1+j)\ \mathrm{m}\Omega$$

This is very small in magnitude compared to η_0, so the copper surface acts like a short circuit, and we have

$$\Gamma = \frac{\eta_{c2}-\eta_0}{\eta_{c2}+\eta_0} \simeq -1$$

so that we find

$$\tilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}}\, E_0^{\mathrm{i}}[\exp(-jk_1 z) - \exp(jk_1 z)] = -\hat{\mathbf{x}}\, j2E_0^{\mathrm{i}}\sin(k_1 z),$$

$$\tilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}}\,\frac{E_0^{\mathrm{i}}}{\eta_1}[\exp(-jk_1 z) + \exp(jk_1 z)] = \hat{\mathbf{y}}\,2\frac{E_0^{\mathrm{i}}}{\eta_1}\cos(k_1 z)$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

Normal Incidence

Normal Incidence on a Metal Surface: Ulaby et al. Example 8-3

Answer (continued): Returning to the time domain

$$\mathbf{E}_1(z,t) = \mathrm{Re}\left[\tilde{\mathbf{E}}_1(z)\exp(j\omega t)\right]$$

$$= \hat{\mathbf{x}}\, 2E_0^{\mathrm{i}} \sin(k_1 z)\sin(\omega t)$$

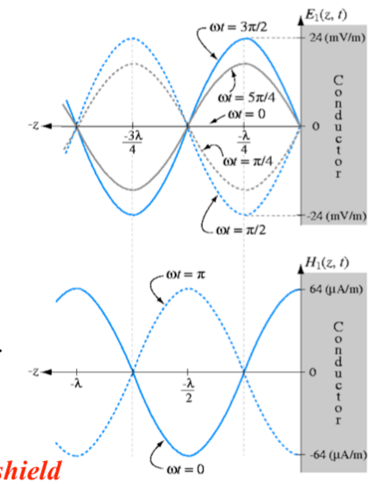
$$= \hat{\mathbf{x}}\, 24 \sin(20\pi z/3)\sin(2\pi\times 10^9 t)\ \text{mV/m},$$

$$\mathbf{H}_1(z,t) = \mathrm{Re}\left[\tilde{\mathbf{H}}_1(z)\exp(j\omega t)\right]$$

$$= \hat{\mathbf{y}}\, 2\frac{E_0^{\mathrm{i}}}{\eta_1}\cos(k_1 z)\cos(\omega t)$$

$$= \hat{\mathbf{y}}\, 64 \cos(20\pi z/3)\cos(2\pi\times 10^9 t)\ \mu\text{A/m}$$

The standing-wave patterns are shown to the left. Note that the **E**-field is shorted out, while the **H**-field remains large.



Ulaby Figure 8-8

UMBC
AN HONORS UNIVERSITY IN MARYLAND

As a consequence, it is harder to shield magnetic fields than electric fields

15.18

The standing wave patterns are the same as in a shorted transmission line.

Applications — Paul Chapter 5

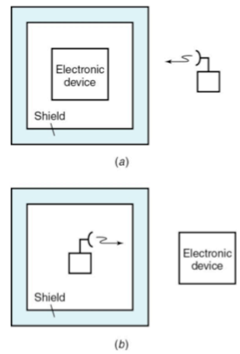
Shielding

Shielded enclosures are used to either (a) prevent a signal from outside the enclosure from interfering with equipment inside or (b) vice versa. Using the same geometry as in the radome problem (and Ulaby et al.'s notation), we find analogously

$$\left|\frac{\tilde{E}_0^{\rm i}}{\tilde{E}_0^{\rm t}}\right| = \left|\frac{(\eta_0+\eta_{\rm c})^2}{4\eta_0\eta_{\rm c}}\right|\left|1-\left(\frac{\eta_0-\eta_{\rm c}}{\eta_0+\eta_{\rm c}}\right)^2\exp\left[-\frac{2d}{\delta}(1+j)\right]\right|\exp\left(\frac{d}{\delta}\right)$$

which in the limit of a good conductor with high reflections and many (> 5) times the width of the skin depth becomes

$$\left|\frac{\tilde{E}_0^{\rm i}}{\tilde{E}_0^{\rm t}}\right| \simeq \left|\frac{\eta_0}{4\eta_{\rm c}}\right|\exp\left(\frac{d}{\delta}\right)$$



Paul Figure 5.19

UMBC
AN HONORS UNIVERSITY IN MARYLAND

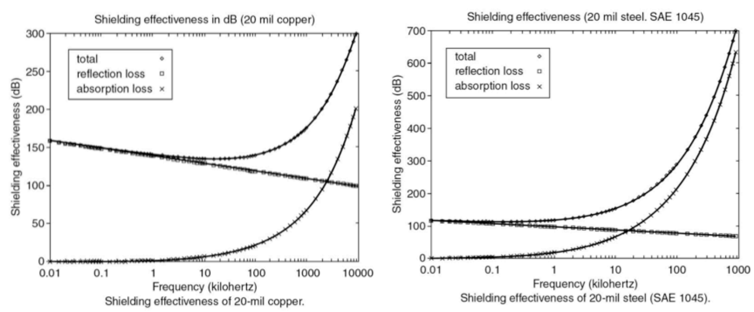
Applications — Paul Chapter 5

Shielding

We define the shielding effectiveness of the enclosure:

$$\mathrm{SE} \equiv 20\log\left|\tilde{E}_0^{\mathrm{i}} / \tilde{E}_0^{\mathrm{t}}\right| = 20\log\left|\eta_0 / 4\eta_{\mathrm{c}}\right| + 20\log\exp\left(d/\delta\right)$$

There is a reflection term and an absorption term. Below are results for 20 mil sheets of copper and steel. Reflection dominates below 2 MHz for copper and below 20 kHz for steel.



Paul Figure 5.21

UMBC AN HONORS UNIVERSITY IN MARYLAND

15.20

Another useful property of dB is that it allows us to express multiplicative factors additively. Note that we are only talking about electrical shielding here. Magnetic fields will experience the absorption --- but not nearly as much reflection.

Applications — Paul Chapter 5

Microwave Health Hazards

Microwave devices work at about 2 GHz. The human body has $\sigma = 1.5$ S/m, $\varepsilon_r = 50$, and $\mu_r = 1$. Regulatory agencies set safe levels at 10 mW/cm^2, corresponding to $|E_0|$ = 275 V/m. Damage comes from skin heating. How much power is absorbed at the "safe" level?

We begin by noting that $\sigma / \omega\varepsilon = 0.27$. The human body is a quasi-conductor at this frequency. We have $\tau = 0.244(0.99 + j0.11)$ and $\gamma_2 = 39.6 + j\,298$, so that $\alpha_2 = 39.6$ and $\delta = 2.5$ cm. We find

$$S_{\text{diss}} = |\tau|^2 \frac{|E_0^{\text{i}}|^2}{2|\eta_2|} \cos\theta_\eta = 42.5 \text{ W/m}^2 = 4.25 \text{ mW/cm}^2.$$

Slightly over half the power is reflected.

UMBC

AN HONORS UNIVERSITY IN MARYLAND