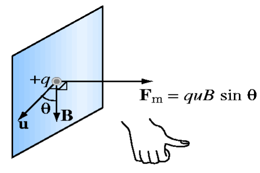
Magnetic Forces

Lorentz Force

The force exerted by the magnetic flux density **B** on a single charge q:

$$\mathbf{F}_{\rm m} = q\mathbf{u}\times\mathbf{B}$$

- The force is at right angles to **u** and **B**
- The magnitude is $quB\sin\theta$



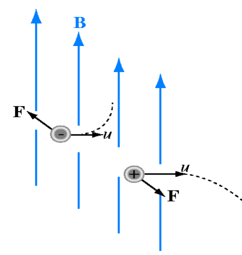
The magnetic force does no work

$$dW = \mathbf{F}_{\rm m}\cdot d\mathbf{l} = (\mathbf{F}_{\rm m}\cdot\mathbf{u})\,dt = 0$$

The total force is given by

$$\mathbf{F} = \mathbf{F}_{\rm e} + \mathbf{F}_{\rm m} = q(\mathbf{E}+\mathbf{u}\times\mathbf{B})$$

This force is called the Lorentz force!



(b) Ulaby Figure 5-1

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Magnetic Forces

Comparison of Electric and Magnetic Forces

Electric Field	Magnetic Field
• Force is in the field direction	• Force is orthogonal to the field
• Field exerts a force that does not depend on charge's speed	• Field exerts a force that is proportional to the charge's speed
• Electric field does work on charges	• Magnetic field does no work on charges

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Magnetic Forces

Example: Uniform Motion in a Magnetic Field

Question: A charge is moving in a uniform magnetic field $\mathbf{B} = \hat{\mathbf{z}}\,B$. Show that the charges move in a spiral

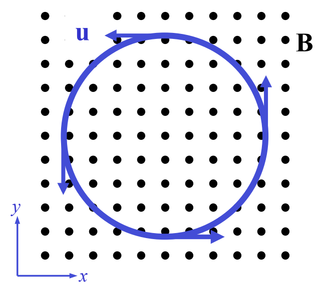
Answer: The equations of motion are

$$F_x = m\frac{du_x}{dt} = qu_yB, \quad F_y = m\frac{du_y}{dt} = -qu_xB, \quad F_z = m\frac{du_z}{dt} = 0$$

$$\Rightarrow \quad u_x = u_{\perp 0}\sin\left(\frac{qB}{m}t + \phi_0\right),$$

$$u_y = u_{\perp 0}\cos\left(\frac{qB}{m}t + \phi_0\right),$$

$$u_z = u_{z0}$$



Magnetic Forces

Example: Uniform Motion in a Magnetic Field

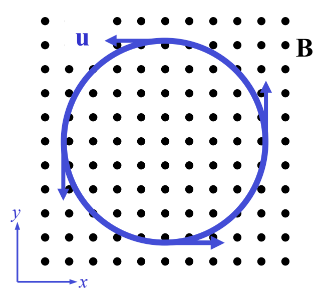
Answer (continued): After further integration, we find

$$x = x_0 - \frac{m}{qB}u_{\perp 0}\cos\left(\frac{qB}{m}t + \phi_0\right), \quad y = y_0 + \frac{m}{qB}u_{\perp 0}\sin\left(\frac{qB}{m}t + \phi_0\right), \quad z = z_0 + u_{z0}t$$

which is indeed a spiral. This motion is called ***cyclotron motion***.

The rotation frequency is independent of the velocity!

By accelerating the charges, using a periodically-varying electric field, it is possible to obtain very high energies.



Magnetic Forces

Current-Carrying Wire

A current I flowing in a wire segment of length $d\mathbf{l}$ experiences a force

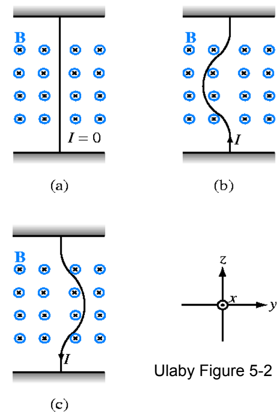
$$d\mathbf{F}_{\mathrm{m}} = dQ\,\mathbf{u}\times\mathbf{B} = (\rho_{\mathrm{V}}\,\mathbf{u}\,A dl)\times\mathbf{B} = (\rho_{\mathrm{V}}\,u\,A d\mathbf{l})\times\mathbf{B} = I\,d\mathbf{l}\times\mathbf{B}$$

- In (a), there is no force
- In (b), the force is to the left
- In (c), the force is to the right

Over a closed loop, we have

$$\mathbf{F}_{\mathrm{m}} = I\left(\oint_C d\mathbf{l}\right)\times\mathbf{B}$$

The force on a closed loop in a constant magnetic field is zero



Ulaby Figure 5-2

Magnetic Forces

Magnetic Torque

A force $\mathbf{F}$ exerted on a moment arm $\mathbf{d}$ exerts a torque $\mathbf{T}$, $\mathbf{T} = \mathbf{d}\times\mathbf{F}$

Electric motors operate using the torques exerted by magnetic fields

Magnetic field in the plane of a current loop:
Arms 1 and 3 of the loop experience forces

$$\mathbf{F}_1 = I(-\hat{\mathbf{y}}b)\times(\hat{\mathbf{x}}B) = \hat{\mathbf{z}}\,I\,bB,$$

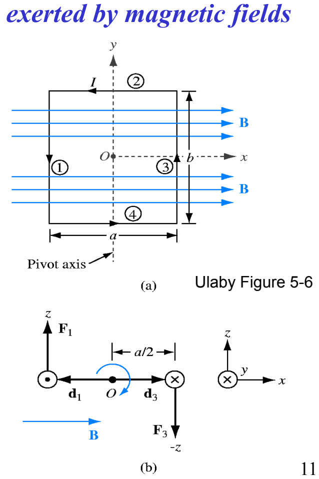
$$\mathbf{F}_3 = I(\hat{\mathbf{y}}b)\times(\hat{\mathbf{x}}B) = -\hat{\mathbf{z}}\,I\,bB$$

The resulting torque is

$$\mathbf{T} = \mathbf{d}_1\times\mathbf{F}_1 + \mathbf{d}_3\times\mathbf{F}_3$$

$$= \left(-\hat{\mathbf{x}}\frac{a}{2}\right)\times\left(\hat{\mathbf{z}}\,I\,bB\right) + \left(\hat{\mathbf{x}}\frac{a}{2}\right)\times\left(-\hat{\mathbf{z}}\,I\,bB\right)$$

$$= \hat{\mathbf{y}}\,I\,ab\,B = \hat{\mathbf{y}}\,I\,A\,B \qquad (A = \text{area of loop})$$



Ulaby Figure 5-6

Magnetic Forces

Magnetic Torque

More generally,

$$\mathbf{T} = \hat{\mathbf{y}}\, I A B \sin\theta = I A\, \hat{\mathbf{n}} \times \mathbf{B}$$

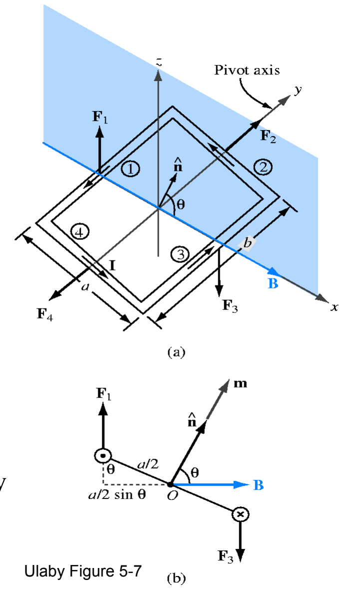
where $\hat{\mathbf{n}}$ is the surface normal
— using the right-hand rule

With N turns, this expression becomes

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad \text{with} \quad \mathbf{m} \equiv \hat{\mathbf{n}}\, N I A$$

$\mathbf{m}$ = ***the magnetic moment***

Since current flows in loops, the magnetic
moment is a fundamental quantity
— analogous to electric dipoles



Ulaby Figure 5-7

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Biot-Savart Law

Magnetic analogue to Coulomb's law

Since magnetic field are generated by currents, it is bit more complicated…

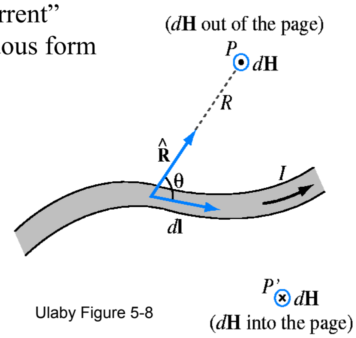
$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad \Rightarrow \quad \mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$

NOTE: There is no such thing as a "point current"
— The Biot-Savart law only exists in continuous form

Surface and Volume Distributions

$$\mathbf{H} = \frac{1}{4\pi} \int_S \frac{\mathbf{J}_S \times \hat{\mathbf{R}}}{R^2}\, ds, \quad \text{(surface current)}$$

$$\mathbf{H} = \frac{1}{4\pi} \int_v \frac{\mathbf{J} \times \hat{\mathbf{R}}}{R^2}\, dv \quad \text{(volume current)}$$



Ulaby Figure 5-8

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Magnetic Fields and Forces

Magnetic Field of a Linear Conductor: Ulaby and Ravaoli, Ex. 5-2

Question: A linear conductor of length *l* and carrying a current *I* is placed along the *z*-axis. What is the magnetic flux density **B** at a point *P* at a distance *r* in the *x*-*y* plane in free space?

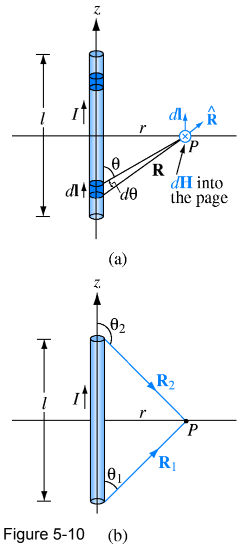
Answer: We have $d\mathbf{l}\times\hat{\mathbf{R}} = dz(\hat{\mathbf{z}}\times\hat{\mathbf{R}}) = \hat{\boldsymbol{\varphi}}\sin\theta\, dz$
and $R = r\csc\theta, \quad z = -r\cot\theta, \quad dz = r\csc^2\theta\, d\theta$
so that

$$\mathbf{H} = \frac{I}{4\pi}\int_{z=-l/2}^{z=l/2}\frac{d\mathbf{l}\times\hat{\mathbf{R}}}{R^2} = \hat{\boldsymbol{\varphi}}\frac{I}{4\pi r}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$= \hat{\boldsymbol{\varphi}}\frac{I}{4\pi r}\left(\cos\theta_1 - \cos\theta_2\right)$$

with

$$\cos\theta_1 = -\cos\theta_2 = \frac{l/2}{\sqrt{r^2 + (l/2)^2}}$$



Ulaby Figure 5-10

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11.9

Magetic Fields and Forces

Magnetic Field of a Linear Conductor: Ulaby and Ravaoli, Ex. 5-2

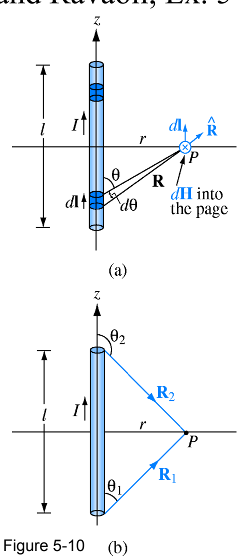
Answer (continued): We conclude

$$\mathbf{B} = \mu_0\mathbf{H} = \hat{\boldsymbol{\varphi}}\frac{\mu_0 I l}{2\pi r\sqrt{4r^2 + l^2}}$$

When $l \gg r$, we have

$$\mathbf{B} = \hat{\boldsymbol{\varphi}}\frac{\mu_0 I}{2\pi r}$$

NOTE: From the result for segments of length l, we can build up arbitrary curves.



Ulaby Figure 5-10

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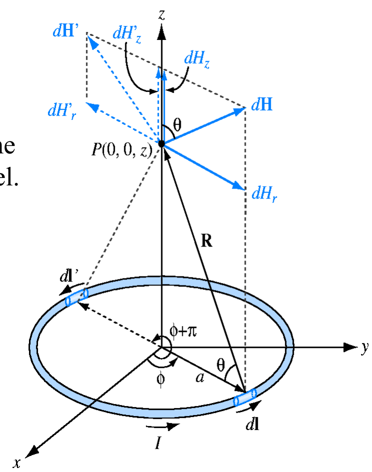
11.10

Magnetic Fields and Forces

Magnetic Field of a Circular Loop: Ulaby and Ravaioli, Ex. 5-4

Question: A circular loop of radius a on the x-y plane is carrying a current I. Determine the magnetic field $\mathbf{H}$ along the z-axis.



Ulaby Figure 5-12

Answer: A current segment $d\mathbf{l}$ creates a magnetic field in both the z- and r-directions. The z-components add, while the r-components cancel. We have

$$dH_z = dH\cos\theta = \frac{I\cos\theta}{4\pi(a^2+z^2)}dl$$

$$= \frac{I}{4\pi(a^2+z^2)}\frac{a}{(a^2+z^2)^{1/2}}dl$$

$$= \frac{Ia}{4\pi(a^2+z^2)^{3/2}}dl$$

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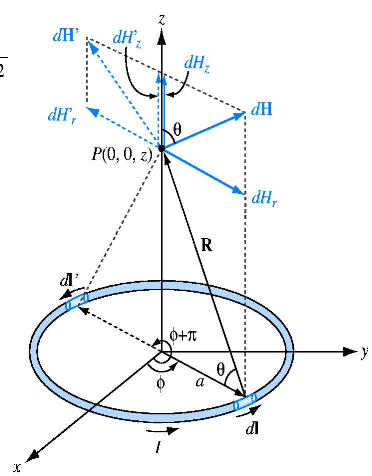
11.11

Magnetic Fields and Forces

Magnetic Field of a Circular Loop: Ulaby and Ravaioli Ex. 5-4

Answer (continued): Integrating over the loop, we obtain

$$\mathbf{H} = \hat{\mathbf{z}}\frac{Ia}{4\pi(a^2+z^2)^{3/2}}\oint dl = \hat{\mathbf{z}}\frac{Ia^2}{2(a^2+z^2)^{3/2}}$$



Ulaby Figure 5-12

When z is large, this becomes:

$$\mathbf{H} = \hat{\mathbf{z}}\frac{Ia^2}{2|z|^3} = \hat{\mathbf{z}}\frac{m}{2\pi|z|^3}$$

where m is the magnetic moment

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11.12

Magnetic Fields and Forces

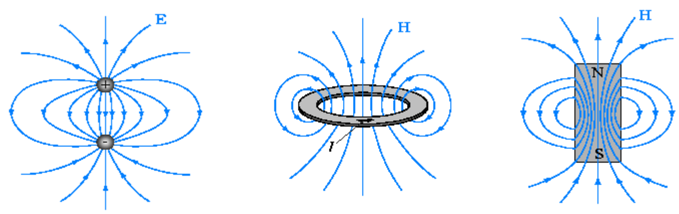
Magnetic Dipole Field:

More generally, we have far away from a current loop at $P(R, \theta, \phi)$

$$\mathbf{H} = \frac{m}{4\pi R^3}\left(\hat{\mathbf{R}}2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

This expression is analogous to what we found for the electric dipole.

Near to the dipole, the fields do not look the same.



(a) Electric dipole (b) Magnetic dipole (c) Bar magnet

Ulaby Figure 5-13

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11.13

Magnetic Fields and Forces

Force between two conductors

Since conductors create fields, one conductor will exert a force on another

— The field $\mathbf{B}_1$ due to the current I_1 at the location of current I_2 is given by

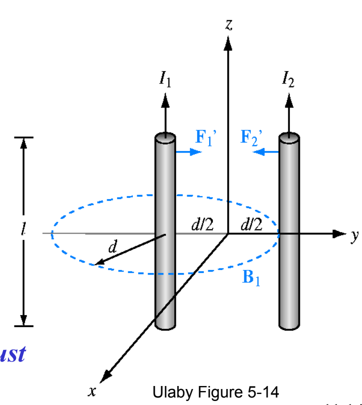
$$\mathbf{B}_1 = -\hat{\mathbf{x}}\frac{\mu_0 I}{2\pi d}$$

so that the force on a wire of length l is

$$\mathbf{F}_2 = I_2 l\,\hat{\mathbf{z}} \times \mathbf{B}_1 = -\hat{\mathbf{y}}\frac{\mu_0 I_1 I_2 l}{2\pi d}$$

and the force per unit length is

$$\mathbf{F}_2' = \frac{\mathbf{F}_2}{l} = -\hat{\mathbf{y}}\frac{\mu_0 I_1 I_2}{2\pi d}$$



Ulaby Figure 5-14

From Newton's second law, the force exerted by wire 2 on wire 1 must be equal and opposite!

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11.14

Integral Form of Magnetostatic Equations

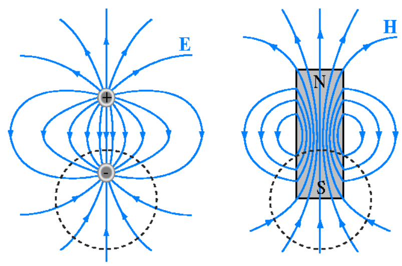
Gauss's Law for Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \Leftrightarrow \quad \oint_S \mathbf{B} \cdot ds = 0, \text{ where } S \text{ is any closed surface}$$

- There is no magnetic charge
- Magnetic flux lines always form continuous closed loops
- You cannot have an isolated north pole or south pole in a magnet

— The flux entering a region surrounding a charge in an electric dipole is non-zerio

— The flux entering a region surrounding a pole in a magnetic dipole (bar magnet) is zero.



(a) Electric dipole (b) Bar magnet

Ulaby Figure 5-15

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11.15

Integral Form of Magnetostatic Equations

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \Leftrightarrow \quad \oint_C \mathbf{H} \cdot dl = I, \text{ where}$$

- C is any closed contour
- I is the current flowing through *any* surface that is attached to C

Why emphasize *any?*

In three dimensions:

— The volume surrounded by a closed surface S is unambiguous

— ***The surface S attached to a closed contour C is ambiguous***

(There are many of them)

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11.16

Integral Form of Magnetostatic Equations

Ampere's Law

$\nabla \times \mathbf{H} = \mathbf{J} \quad \Leftrightarrow \quad \oint_C \mathbf{H} \cdot dl = I$, where

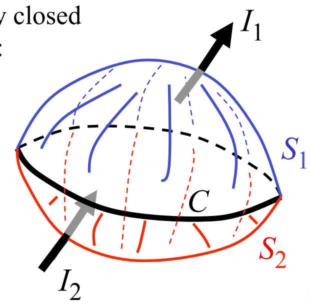
- C is any closed contour
- I is the current flowing through *any* surface that is attached to C

Some consequences:

- Current cannot be created or destroyed ($I_1 = I_2$)
- The sum of the currents flowing into any closed surface is zero … which in turn implies:

Kirchhoff's Current Law: The sum of the currents flowing into any node is zero.

— To prove the theorem, you surround node by an arbitrary closed surface.



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Ampere's Law

In situations with high symmetry, one can calculate the field

- current line
- current sheet
- torus

This is analogous to using Gauss's law in electrostatics, but there is no magnetic charge

Again, as in the case of electrostatics:

An even more important application of the integral formulation in today's world is to serve as the starting point for numerical approaches!

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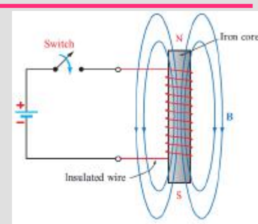
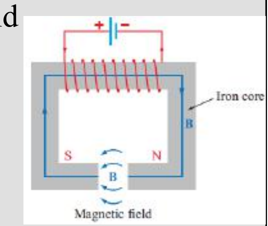
Tech Brief 10: Electromagnets

First developed by William Sturgeon in 1820s



Basic Principle

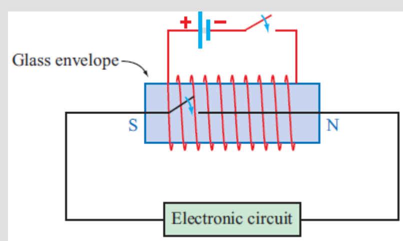
- Can be constructed in many shapes
  - linear solenoid
  - horseshoe
- Electric current flows through insulated wire coiled around a core
- Produces magnetic field with bar magnet-like field
- Field strength proportional to
  - Current
  - Number of turns
  - Permeability of core
    - Field strength increased by ferromagnetic core



Tech Brief 10: Electromagnets

Applications

- Motors
- Relays
- Loud speakers
- Magnetic levitation
- Many others

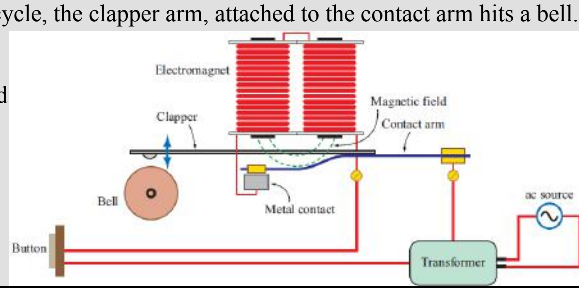


Doorbell

Electromagnet pulls on a metal bar anchored on one side (contact arm). Pulling on the bar severs current flow, so the arms snaps back, restoring current. This happens many times per second, and on each cycle, the clapper arm, attached to the contact arm hits a bell.

Magnetic Relays

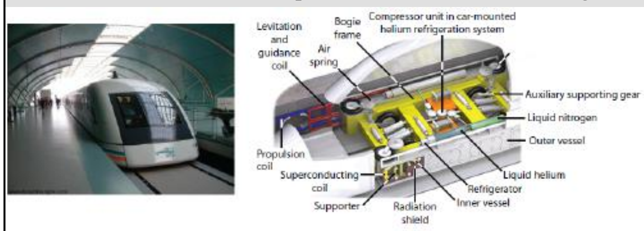
A circuit breaker switched electromagnetically



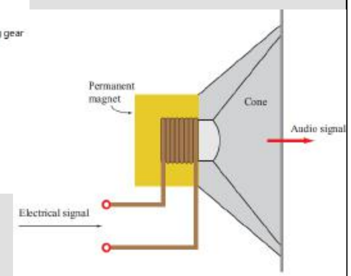
Tech Brief 10: Electromagnets

Loudspeaker

Uses a combination of stationary permanent magnet and a movable electromagnet. Cone attached to electromagnet moves in response to electrical signal. Cone movement creates sound waves with same spectral content as electrical signal



(a) Maglev train (b) Internal workings of the Maglev train



Magnetic Levitation

Maglev trains achieve speeds as high as 500 km/hr because there is no friction between the train and the track. The train carries superconducting electromagnets that induce currents in coils built into the guide rails alongside the train. The magnetic interaction both levitates and propels the train along the track.

11.21

Vector Potential

In electrostatics, we have

$$\mathbf{E} = -\nabla V, \quad V = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho_V}{R'}\, dv'$$

In magnetostatics, we have

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{A} = \frac{\mu}{4\pi}\int_{v'} \frac{\mathbf{J}}{R'}\, dv'$$

The constraint $\nabla \cdot \mathbf{B} = 0$ is automatically satisfied since $\nabla \cdot (\nabla \times \mathbf{C}) = 0$ for any vector **C**.

The vector potential — because it is a vector — is less useful than the scalar potential for doing calculations.

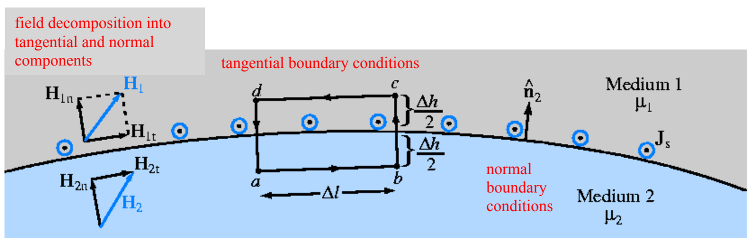
11.22

Boundary Conditions

Normal boundary conditions:

In electrostatics: $\oint_S \mathbf{D}\cdot d\mathbf{s} = Q \quad \Rightarrow \quad D_{1\mathrm{n}} - D_{2\mathrm{n}} = \rho_{\mathrm{S}}$

In magnetostatics: $\oint_S \mathbf{B}\cdot d\mathbf{s} = 0 \quad \Rightarrow \quad B_{1\mathrm{n}} - B_{2\mathrm{n}} = 0, \quad \mu_1 H_{1\mathrm{n}} - \mu_2 H_{2\mathrm{n}} = 0$



Ulaby Figure 5-24

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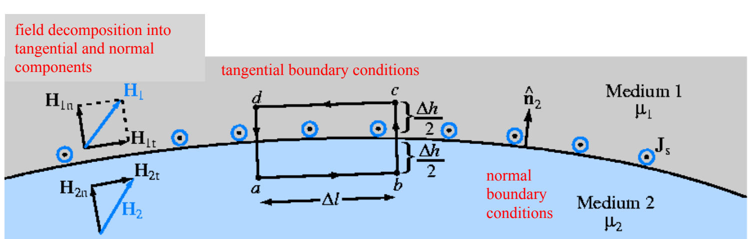
11.23

Boundary Conditions

Tangential boundary conditions:

In electrostatics: $\oint_C \mathbf{E}\cdot d\mathbf{l} = 0 \quad \Rightarrow \quad E_{1\mathrm{t}} - E_{2\mathrm{t}} = 0$

In magnetostatics: $\oint_C \mathbf{H}\cdot d\mathbf{l} = I \quad \Rightarrow \quad H_{1\mathrm{t}} - H_{2\mathrm{t}} = J_{\mathrm{S}}, \quad \frac{B_{1\mathrm{t}}}{\mu_1} - \frac{B_{2\mathrm{t}}}{\mu_2} = J_{\mathrm{S}}$



Ulaby Figure 5-24

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11.24

Boundary Conditions

Summary of electric and magnetic boundary conditions

	electric boundary conditions	magnetic boundary conditions
normal conditions	$D_{1n} - D_{2n} = \rho_S$	$B_{1n} - B_{2n} = 0$
tangential conditions	$E_{1t} - E_{2t} = 0$	$H_{1t} - H_{2t} = J_S$

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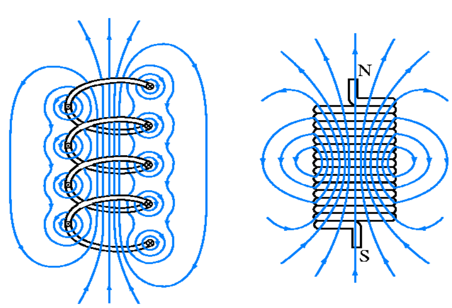
Inductance

Solenoids

Inductors typically use *solenoids* that have many circular loops; using many loops increases the inductance. When tightly wound — the usual case — the field resembles the field of a bar magnet

Solenoid parameters:

N = total number of turns
l = total length
n = turns per unit length = N / l
a = solenoid radius
I = current



(a) Loosely wound solenoid (b) Tightly wound solenoid

Ulaby Figure 5-25

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Inductance

Solenoids

We recall that the field on the z-axis of a loop is $\mathbf{H} = \hat{\mathbf{z}} \dfrac{I' a^2}{2(a^2 + z^2)^{3/2}}$

[See slide 11.11]

An incremental length dz has $n\,dz$ turns and a current $I' = In\,dz$

Its contribution to the flux density is

$$d\mathbf{B} = \mu\, d\mathbf{H} = \hat{\mathbf{z}} \frac{\mu n I a^2}{2(a^2 + z^2)^{3/2}} dz$$

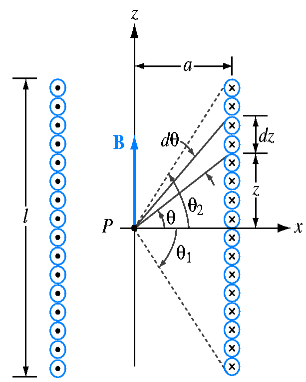
Integrating, we obtain

$$\mathbf{B} = \hat{\mathbf{z}} \frac{\mu n I}{2}\left(\sin\theta_2 - \sin\theta_1\right)$$

which for a long solenoid becomes

$$\mathbf{B} = \hat{\mathbf{z}}\mu n I = \hat{\mathbf{z}}\mu \frac{N}{l} I$$

In this context, "long" means $l / a \gg 1$



Ulaby Figure 5-26

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11.27

Inductance

Self-Inductance (= The usual inductance)

We define the magnetic flux "linking" (passing through) a surface as

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

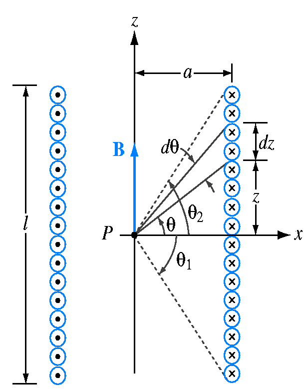
In a solenoid, which has an approximately constant magnetic field,

$$\Phi = \int_S \hat{\mathbf{z}}\left(\mu \frac{N}{l} I\right) \cdot \hat{\mathbf{z}}\, ds = \mu \frac{N}{l} IS$$

The magnetic flux linkage Λ in an N-loop structure is defined as

$$\Lambda = N\Phi \text{ (in general)}$$

$$= \mu \frac{N^2}{l} IS \text{ (for a solenoid)}$$



Ulaby Figure 5-26

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Inductance

The inductance is defined as

$$L = \Lambda / I \text{ (in general)} = \mu \frac{N^2}{l} S \text{ (for a solenoid)}$$

$$= \Phi / I \text{ (for two-conductor structures in transmission lines)}$$

Coaxial cable

$$\mathbf{B} = \hat{\boldsymbol{\varphi}} \frac{\mu I}{2\pi r}$$

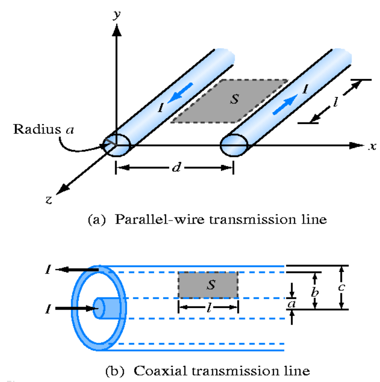
Hence, the flux through S is

$$\Phi = l \int_a^b B\, dr = l \int_a^b \frac{\mu I}{2\pi r} dr = \frac{\mu I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

We conclude

$$L' = \frac{L}{l} = \frac{\Phi}{Il} = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$$

Note: $L'C' = \mu\varepsilon$. This result is general!



Ulaby Figure 5-27

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Magnetic Energy

From circuit theory

The voltage across an inductor: $v = L\, dI/dt$

The power expended: $p = IV$

The energy stored is thus: $W_\text{m} = \int p\, dt = \int I'V'\, dt = L \int_0^I I'dI' = \frac{1}{2} L I^2$

Applying this expression to a solenoid

$$W_\text{m} = \frac{1}{2} L I^2 = \frac{1}{2} \left(\mu \frac{N^2}{l} S\right) \left(\frac{Bl}{\mu N}\right)^2 = \frac{1}{2} \frac{B^2}{\mu} (lS) = \frac{1}{2} \mu H^2 v$$

volume

More generally, $W_\text{m} = \frac{1}{2} \int_V \mu H^2 dv \equiv \int_V w_\text{m}\, dv$

where $w_\text{m} = \frac{1}{2} \mu H^2$ is the energy density

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Applications (Paul)

High-Voltage Transmission Lines: Health Issues

High-voltage transmission lines carry three-phase voltages. The voltage between lines is typically V_{LL} = 765 kV. What is the voltage at a man's head, standing below?

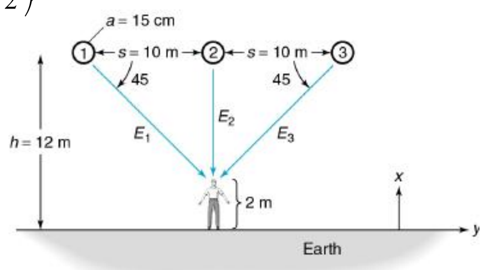
We have $V = 765\text{ kV} = (\rho_l / 2\pi\varepsilon_0)\ln(10/0.15)$, so that $\rho_l = 10.1\ \mu\text{C/m}$

so that

$$E_1 = \frac{\rho_l}{2\pi\varepsilon_0}\frac{1}{\sqrt{(10)^2+(10)^2}}\left(-\frac{\hat{\mathbf{x}}}{\sqrt{2}}+\frac{\hat{\mathbf{y}}}{\sqrt{2}}\right) = -\hat{\mathbf{x}}\,9.1\text{ kV/m} + \hat{\mathbf{y}}\,9.1\text{ kV/m}$$

$$E_2 = -\hat{\mathbf{x}}\,18.2\text{ kV/m}$$

$$E_3 = -\hat{\mathbf{x}}\,9.1\text{ kV/m} - \hat{\mathbf{y}}\,9.1\text{ kV/m}$$



Paul Figure 3-49

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Applications (Paul)

High-Voltage Transmission Lines: Health Issues

The fields are all 120$^{\text{o}}$ out of phase, so that

$$E = \left|E_1 + E_2\exp(-j2\pi/3) + E_3\exp(j2\pi/3)\right| = 18.2\text{ kV/m}$$

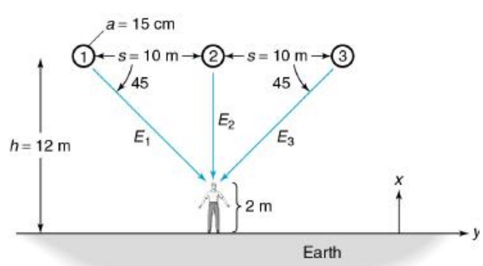
This is strong enough to make a fluorescent light bulb glow!

A similar calculation shows that B = 21.4 μ T = half earth's field at surface

What are the health risks? Is there a resonance in the brain at 60 Hz?

Studies are ambiguous…

You should avoid exposure to large fields!



Paul Figure 3-49

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Applications (Paul)

Electrostatic Discharge

Voltages of about 3 kV can lead to electrostatic discharge (air breakdown)
— This can be a serious problem with computer equipment

Poor conductors (good dielectrics) accumulate charge
— including dry human skin; hence we ground our hands when working sensitive equipment

Some materials give up electrons easily: air, human skin
Some materials accept electrons easily: teflon, silicon

A rough estimate: The capacitance of sphere of radius a is $C = 4\pi\varepsilon_0 a$. Taking a for the human body as 1 m, we find C = 100 pF. With only 1 μC on the body, we find V = 10 kV. That is enough to cause discharges!

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Applications (Paul)

Interference

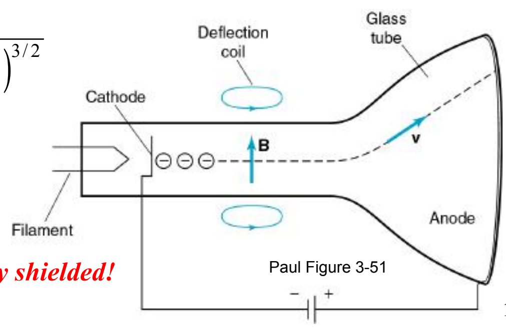
Video displays and power wires are a major source of interference

Traditional video displays work by boiling electrons off a cathode, accelerating them in an electric field, deflecting them in a magnetic field, and having them hit a phosphorescent material on the anode.

Magnetic fields can be quite large relative to the Earth's field (50 μT)
Using:

$$B = \frac{\mu_0 I N}{2} \frac{a^2}{\left(a^2 + d^2\right)^{3/2}}$$

with a = 5 cm, d = 10 cm,
I = 0.6 A, and N = 1000,
we find B = 700 μT



Paul Figure 3-51

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This must be properly shielded!

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Applications (Paul)

Parasitic Effects

Since charges and currents are linked, you cannot have capacitance without inductance — and vice versa!

The inductance and capacitance of two well-separated wires of length l are

$$C = \frac{\pi \varepsilon_0 l}{\ln(s/a)}, \quad L = \frac{\mu_0 l}{\pi} \ln(s/a)$$

where s = wire separation, a = wire radii

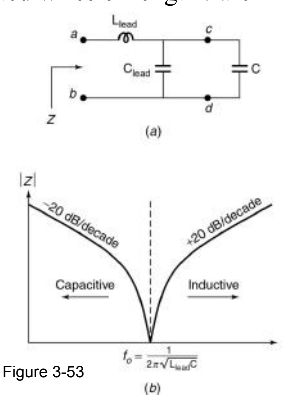
As leads to another capacitor, they create a parasitic inductance and the equivalent circuit shown

Example: $l = 0.5$ in, $s = 0.25$ in, $a = 16$ mils, $C = 1000$ pF. We find $L_{\text{lead}} = 14$ nH and

$C_{\text{lead}} = 0.1$ pF (negligible)

We have $f_0 = 40$ MHz

Above this frequency, the capacitor becomes an inductor



Paul Figure 3-53

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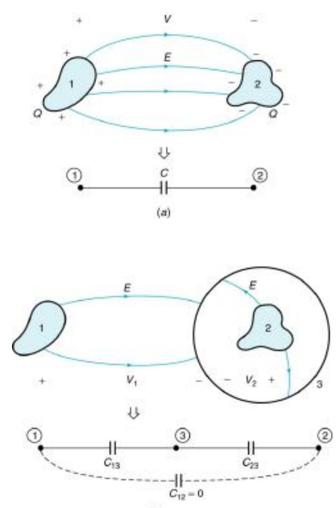
Applications (Paul)

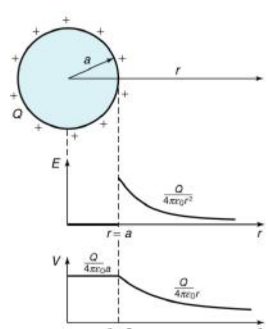
Electrostatic Shielding

Inside a conducting surface (Faraday cage), the voltage is fixed and the field is zero.

The bodies of airplanes and cars make decent Faraday cages.



Paul Figure 3-55



Paul Figure 3-54

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Applications (Paul)

Electric Generator

When wires turn, positive and negative charges experience opposite forces, leading to current flow and an effective EMF = electromotive force

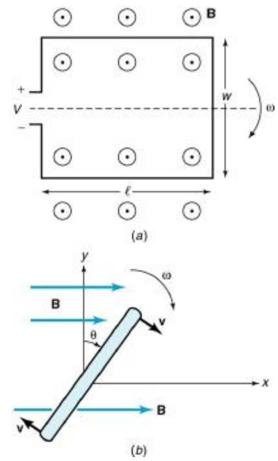
$$V = -\int \mathbf{E}' \cdot d\,\mathbf{l} = \int (\mathbf{u} \times \mathbf{B}) \cdot d\,\mathbf{l}$$

$$= \left(\frac{w\omega}{2}\right) B \sin(\omega t)\left(2l\right) = \omega BA \sin(\omega t)$$

where ω = rotation frequency

$u = w\omega/2$ = rotation velocity

$A = wl$ = area enclosed by current loop



This is how electrical generators work!

Mechanical energy is converted into electrical energy

Paul Figure 3-57

Applications (Paul)

Electric Motor

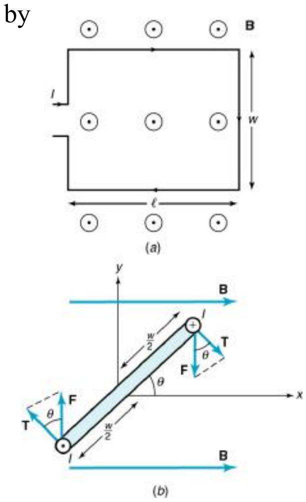
We have already shown that the torque T is given by

$$T = \frac{IBA}{2}\cos\theta$$

Allowing B to vary periodically,

$$B = B_0 \cos(\omega t)$$

we can make the wires turn in one direction



This is how electrical motors work!

Electrical energy is converted into mechanical energy

Paul Figure 3-58

Tech Brief 11: Inductive Sensors

Basic principle: magnetic coupling between different coils

Applications

- Position/displacement measurement
- Proximity detection
- Other related applications

Linear Variable Differential Transformer (LVDT)

- Primary core connected to AC source (1-10 kHz typical)
- Pair of secondary coils
- All coils share a common ferromagnetic core
- Secondary coils are connected in opposition, so when core is in center, there is zero output
- When core is not in center, output voltage is nonzero, indicating position of core
- Core attached to outside by a nonmagnetic rod
- Approximately linear relationship between output voltage amplitude and displacement over a wide range

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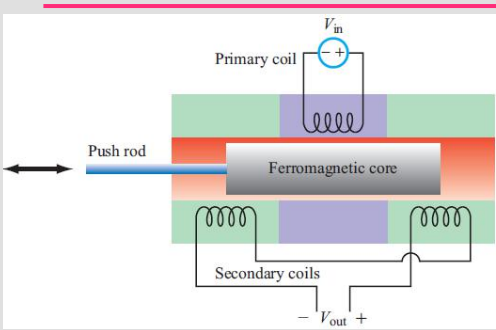
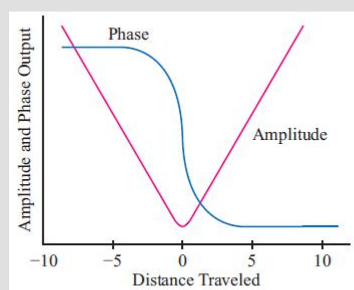
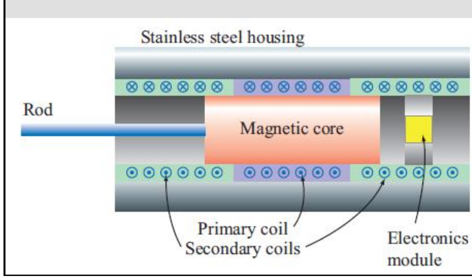
Tech Brief 11: Inductive Sensors







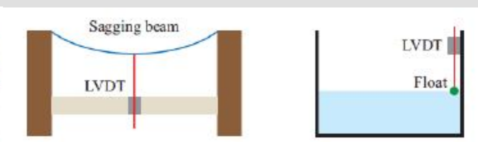


Figure TF11-4: LVDT for measuring beam deflection and as a fluid-level gauge.

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Tech Brief 11: Inductive Sensors

Eddy Current Proximity Sensor

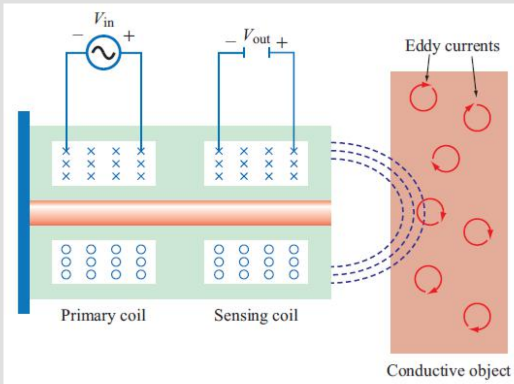
•Sensitive indicator of a conductor

•When an object is in front of the secondary coil, the magnetic field of the coil produces eddy (circular currents in the object.

•Eddy currents generate magnetic fields of their own opposing the field in the coil.

•Reduction in flux in coil decreases output voltage

•Magnitude of voltage change depends on object's conductive properties and distance from sensor



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Assignment

Reading: Ulaby and Ravaioli, Chapter 6

Problem Set 6: Some notes.

- There are 7 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Please watch significant digits.
- Get started early!

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