# CMPE 212

# Principles of Digital Design

## *Lecture 8*

## Analyzing Switching Circuits

February 17, 2016

www.csee.umbc.edu/~younis/CMPE212/CMPE212.htm

# Lecture’s Overview

## *Previous Lecture:*

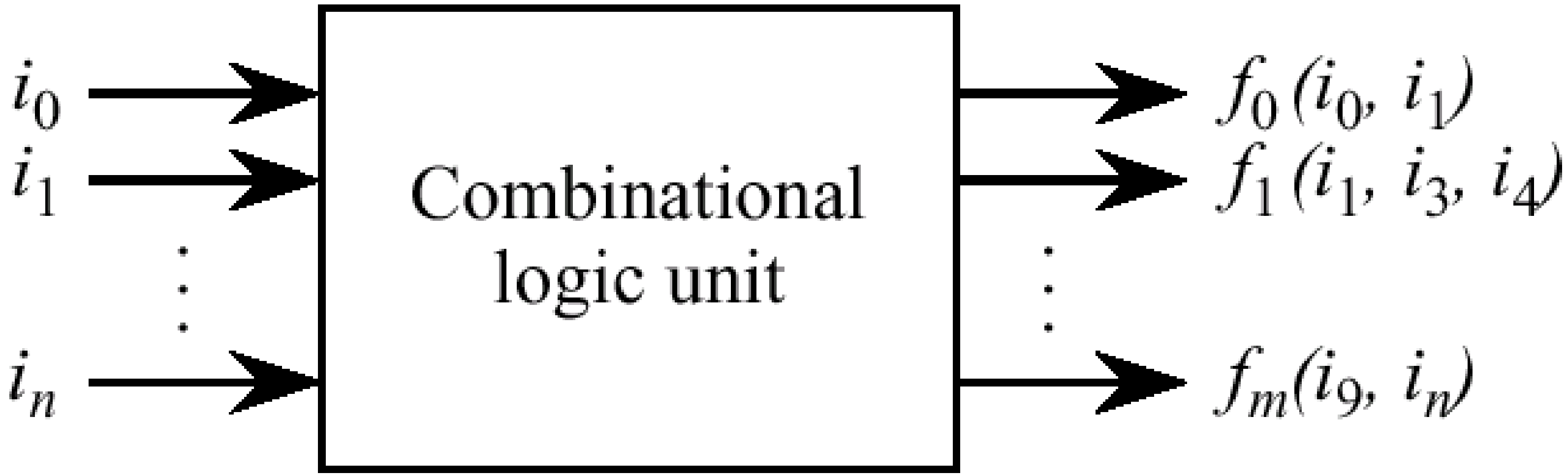
- ➔ Introduction to combinational circuits
  (Truth table and Derivation of logic function)
- ➔ Minterms and Maxterms
- ➔ Sum of products and product of sums
- ➔ Canonical form of switching functions
  (conversion from simplified to canonical form)

## *This Lecture:*

- ➔ Analyzing switching circuits using algebraic methods
- ➔ Analysis of timing diagram

# Combinational Logic

- Translates a set of inputs into a set of outputs according to one or more mapping functions.
- Inputs and outputs for a combination logic unit normally have two distinct (binary) values: high and low, 1 and 0, or 5 volt and 0 volt.
- The outputs of a combinational logic unit (CLU) are strictly functions of the inputs, and the outputs are updated immediately after the inputs change. A set of inputs $i_0 - i_n$ are presented to the CLU, which produces a set of outputs according to mapping functions $f_0 - f_m$




* Slide is courtesy of M. Murdocca and V. Heuring

# Minterm

- A product term in which each variable is present either in true or in complement form
- For *n* variables, there are $2^n$ unique minterms.

| | Minterm | Product |
|---|---|---|
| 000 | $m_0$ | $\bar{A}\ \bar{B}\ \bar{C}$ |
| 001 | $m_1$ | $\bar{A}\ \bar{B}\ C$ |
| 010 | $m_2$ | $\bar{A}\ B\ \bar{C}$ |
| 011 | $m_3$ | $\bar{A}\ B\ C$ |
| 100 | $m_4$ | $A\ \bar{B}\ \bar{C}$ |
| 101 | $m_5$ | $A\ \bar{B}\ C$ |
| 110 | $m_6$ | $A\ B\ \bar{C}$ |
| 111 | $m_7$ | $A\ B\ C$ |

Slide is courtesy of Vishwani D. Agrawal

# Canonical SOP Form

- A Boolean function expressed as a sum of minterms.
- Example: $f(A,B,C) = A\,B + \bar{A}\,C + A\,\bar{C}$

$$= \bar{A}\,\bar{B}C + \bar{A}BC + A\,\bar{B}\,\bar{C} + AB\,\bar{C} + ABC$$

$$= m_1+m_3+m_4+m_6+m_7 = \sum m(1, 3, 4, 6, 7)$$

Truth table with row numbers

| Row No. | A | B | C | f(A,B,C) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 1 |

Slide is courtesy of Vishwani D. Agrawal

# Maxterm

- A summation term in which each variable is present either in true or in complement form.
- For *n* variables, there are $2^n$ unique maxterms.

| | Maxterm | Sum |
|---|---|---|
| 000 | $M_0$ | $A + B + C$ |
| 001 | $M_1$ | $A + B + \bar{C}$ |
| 010 | $M_2$ | $A + \bar{B} + C$ |
| 011 | $M_3$ | $A + \bar{B} + \bar{C}$ |
| 100 | $M_4$ | $\bar{A} + B + C$ |
| 101 | $M_5$ | $\bar{A} + B + \bar{C}$ |
| 110 | $M_6$ | $\bar{A} + \bar{B} + C$ |
| 111 | $M_7$ | $\bar{A} + \bar{B} + \bar{C}$ |

Slide is courtesy of Vishwani D. Agrawal

# Canonical POS Form

- A Boolean function expressed as a product of maxterms.
- Example: $f(A,B,C) = A\,B + \bar{A}\,C + A\,\bar{C}$

$$= (A + B + C)(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

$$= M_0\,M_2\,M_5 = \Pi\,M(0, 2, 5)$$

Truth table with row numbers

| Row No. | A | B | C | f(A,B,C) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 1 |

Slide is courtesy of Vishwani D. Agrawal

# Canonical Forms

- A canonical form completely defines a Boolean function. That is, for every input the canonical form specifies the value of the function.
- Canonical forms of a particular switching function are Unique
- To determine a canonical form:
  - Construct truth table and sum minterms corresponding to 1 outputs, or multiply maxterms corresponding to 0 outputs.
  - Alternatively, use Shannon's expansion theorem:
    - $f(x_1, x_2, \cdots, x_n) = x_1 . f(1, x_2, \cdots, x_n) + \overline{x_2} . f(0, x_2, \cdots, x_n)$
    - $f(x_1, x_2, \cdots, x_n) = [x_1 + f(1, x_2, \cdots, x_n)]\ [\overline{x_2} + f(0, x_2, \cdots, x_n)]$
- Two Boolean functions are identical if and only if their canonical forms are identical.

# Converting to Canonical Form

Example: $f(x) = AB + A\bar{C} + \bar{A}C$

$$\begin{aligned}
&= AB(C + \bar{C}) + A\bar{C}(B + \bar{B}) + \bar{A}C(B + \bar{B}) \\
&= ABC + AB\bar{C} + A\bar{C}B + A\bar{C}\bar{B} + \bar{A}CB + \bar{A}C\bar{B} \\
&= ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C \\
&= m_7 + m_6 + m_6 + m_4 + m_3 + m_1 \\
&= \sum m(1,3,4,6,7)
\end{aligned}$$

Example: $f(A, B, C) = A(A + \bar{C})$

$$\begin{aligned}
A &= (A + \bar{B})(A + B) \\
&= (A + \bar{B} + \bar{C})(A + \bar{B} + C)(A + B + \bar{C})\,(A + B + C) \\
&= M_3 M_2 M_1 M_0
\end{aligned}$$

$$\begin{aligned}
(A + \bar{C}) &= (A + \bar{C} + \bar{B})\,(A + \bar{C} + B) \\
&= (A + \bar{B} + \bar{C})\,(A + B + \bar{C}) = M_3 M_1
\end{aligned}$$

$$f(A, B, C) = (M_3 M_2 M_1 M_0)(M_3 M_1) = \prod M(0,1,2,3)$$

# Analysis of Combinational Circuits

- Digital circuits are designed by transforming a word description of a function into a switching equation and then a circuit

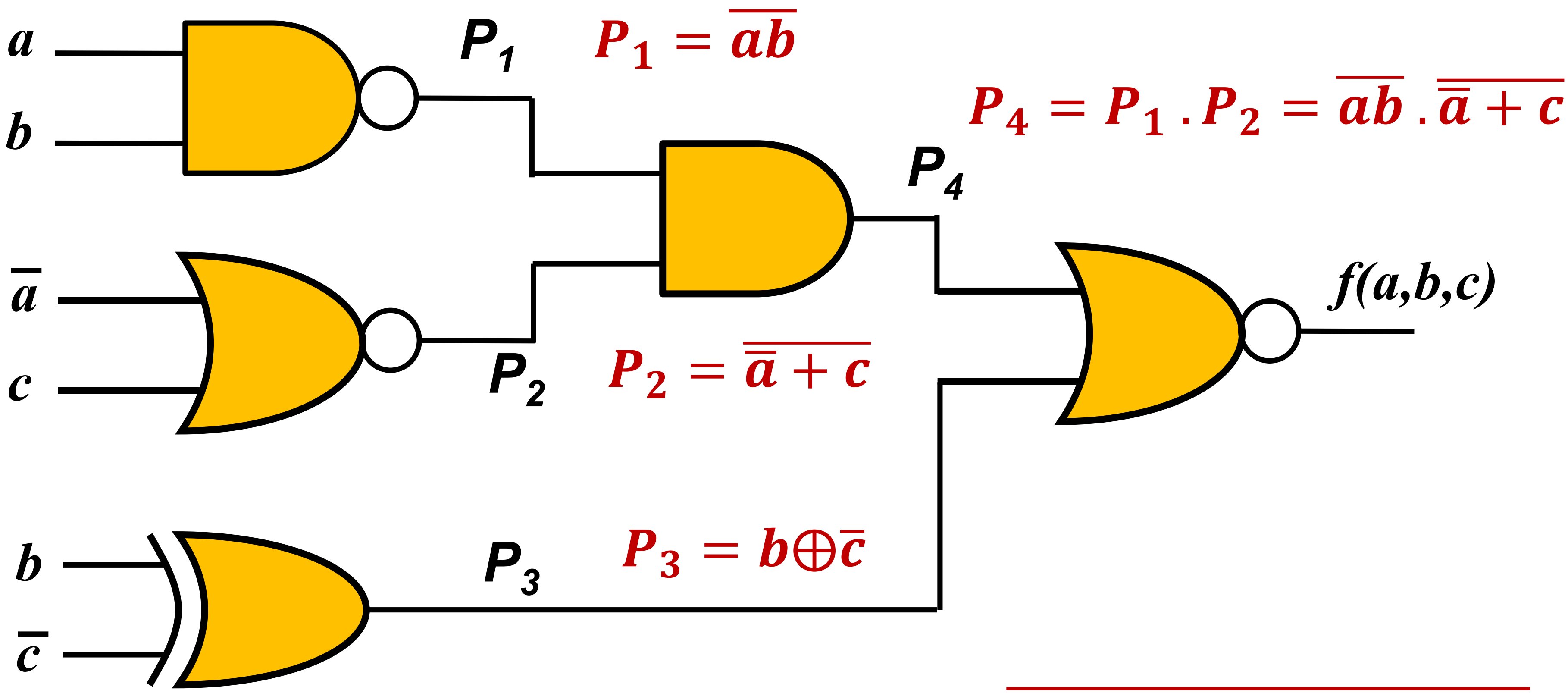
  ➔ Digital circuit analysis is the opposite process

- A digital circuit can be described by:
  1) Switching function (algebraic method)
  2) Hardware design language module
  3) Truth tables
  4) Timing diagram

- Analysis of a logic circuit is used to:
  - determine that its behavior matches specifications
  - transform the circuit to a different format to optimize the implementation

# Algebraic Method

- Derive the Boolean expression and then apply axioms and theorems to simplify
- Example: Simplify the following circuit




$$f(a,b,c) = \overline{P_3 + P_4} = \overline{(b \oplus \overline{c}) + \overline{ab} . \overline{\overline{a} + c}}$$

# Algebraic Simplification

$$f(a,b,c) = \overline{(b \oplus \bar{c}) + \overline{ab}\,.\,\overline{\bar{a} + c}}$$

$$\bar{f}(a,b,c) = (b \oplus \bar{c}) + \overline{ab}\,.\,\overline{\bar{a} + c} \quad = b\,c + \bar{b}\,\bar{c} + \overline{ab}\,.\,\overline{\bar{a} + c}$$

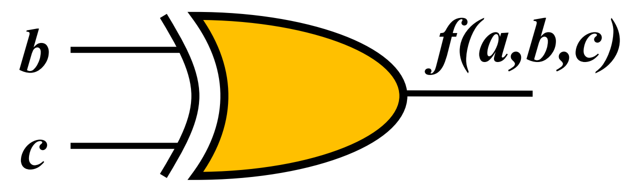
$$= b\,c + \bar{b}\,\bar{c} + (\bar{a} + \bar{b}).(a\,\bar{c})$$

$$= b\,c + \bar{b}\,\bar{c} + \bar{a}\,a\,\bar{c} + \bar{b}\,a\,\bar{c}$$

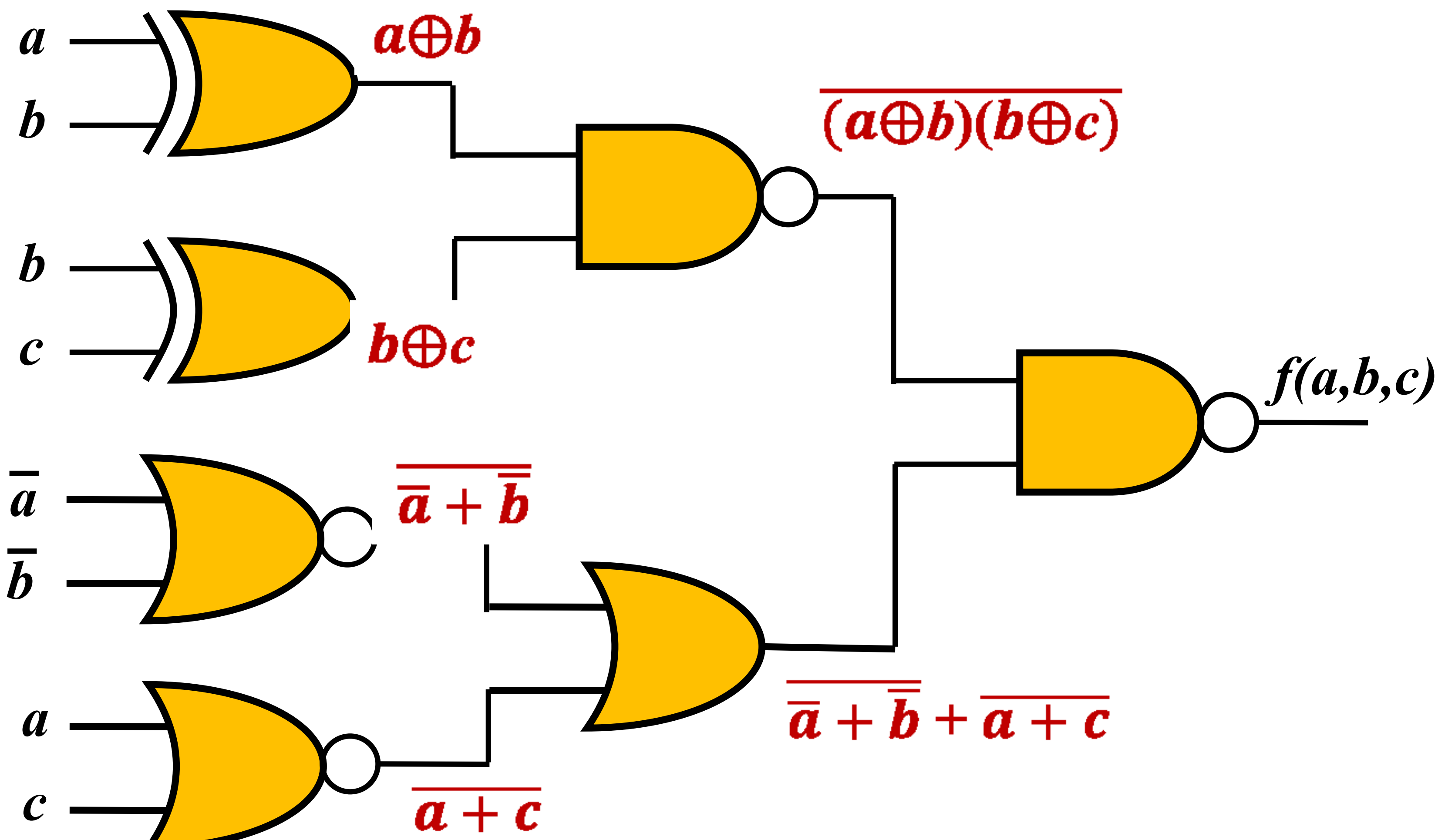
$$= b\,c + \bar{b}\,\bar{c} + 0 + a\,\bar{b}\,\bar{c} \quad = b\,c + \bar{b}\,\bar{c}\,(1 + a)$$

$$= b\,c + \bar{b}\,\bar{c} = (b \odot c)$$

$$f(a,b,c) = \overline{(b \odot c)} = (b \oplus c)$$



# Example: Algebraic Method




$$f(a,b,c) = \overline{\overline{(a\oplus b)(b\oplus c)}.\ (\overline{\overline{a}+\overline{b}} + \overline{a+c})}$$

# Algebraic Simplification

$$f(a,b,c) = \overline{\overline{(a\oplus b)(b\oplus c)}.\ \left(\overline{\bar{a}+\bar{b}}+\overline{a+c}\right)}$$

$$= \overline{\overline{(a\oplus b)(b\oplus c)}} + \overline{\left(\overline{\bar{a}+\bar{b}}+\overline{a+c}\right)}$$

$$= (a\oplus b)(b\oplus c) + (\bar{a}+\bar{b})(a+c)$$

$$= (a\bar{b}+\bar{a}\,b)(b\bar{c}+\bar{b}\,c) + \bar{a}a + \bar{a}c + \bar{b}a + \bar{b}c$$

$$= a\bar{b}b\bar{c} + a\bar{b}\bar{b}c + \bar{a}bb\bar{c} + \bar{a}b\bar{b}c + 0 + \bar{a}c + \bar{b}a + \bar{b}c$$

$$= 0 + a\bar{b}c + \bar{a}b\bar{c} + \bar{a}c + \bar{b}a + \bar{b}c$$

$$= \bar{a}b\bar{c} + \bar{a}c + \bar{b}a + \bar{b}c$$

Consensus

$$= \bar{a}b\bar{c} + \bar{a}c + \bar{b}a \qquad = \bar{a}(b\bar{c}+c) + a\bar{b}$$

$$= \bar{a}(b+c) + a\bar{b} = \bar{a}b + \bar{a}c + a\bar{b} = (a\oplus b) + \bar{a}c$$

# Truth Table Method

- Build the truth table for the circuit and then derive a simplified switching function using SOP or POS
- Example:

to simplify the previous circuit

$$\prod M(0,1,7)$$

| $a$ | $b$ | $c$ | $\overline{\overline{(a\oplus b)(b\oplus c)}.\ (\overline{\bar{a}+\bar{b}}+\overline{a+c})}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$$f(a,b,c) = (a+b+c)(\bar{a}+\bar{b}+c)(\bar{a}+\bar{b}+\bar{c}) = (a+b+c)(\bar{a}+\bar{b})$$

$$= a\bar{a}+a\bar{b}+b\bar{a}+b\bar{b}+\bar{a}c+\bar{b}c \quad = a\bar{b}+b\bar{a}+\bar{a}c+\bar{b}c$$

$$= a\bar{b}+b\bar{a}+\bar{a}c+\bar{b}c \quad = a\bar{b}+b\bar{a}+\bar{a}c \quad = (a\oplus b)+\bar{a}c$$

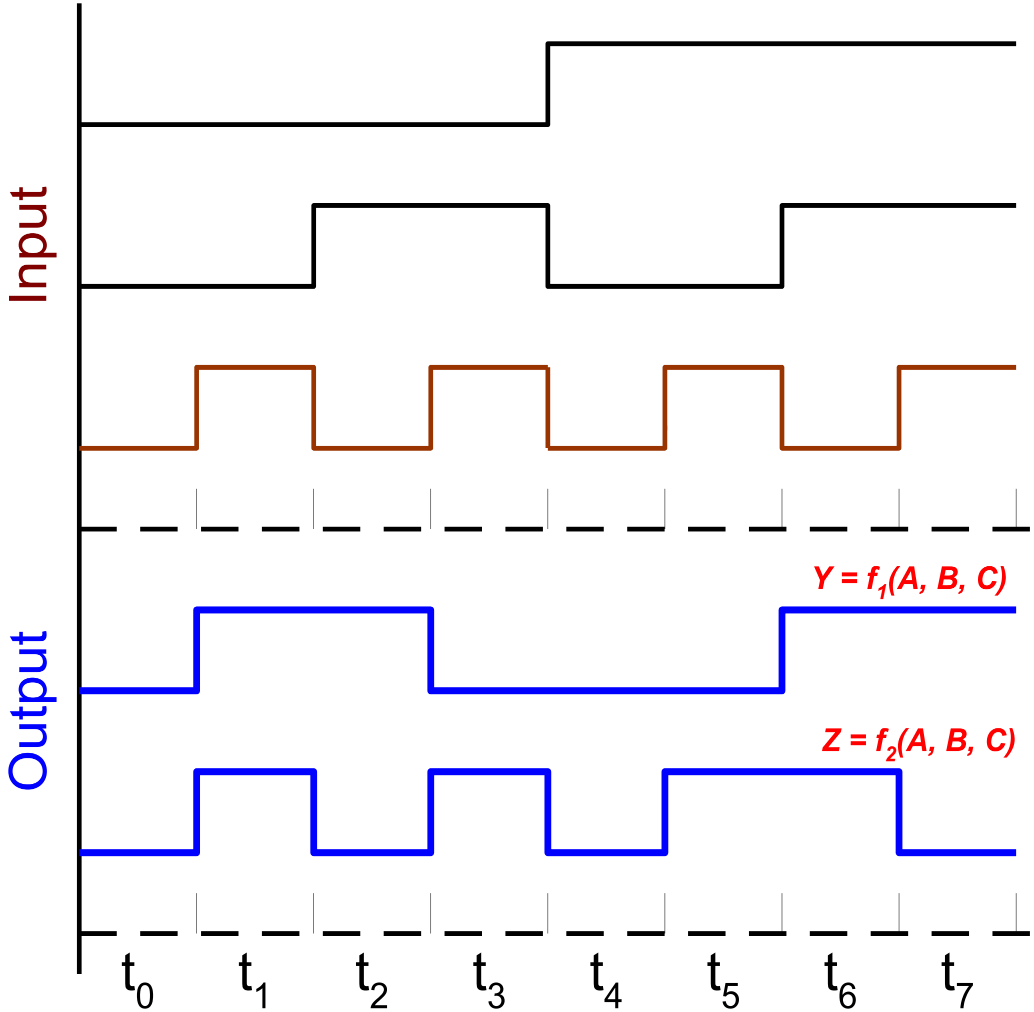
# Timing Diagram

- Apply a sequence of input and observe corresponding output
- Useful for analyzing the propagation delay:

| Time | Input (A, B, C) | Output | |
|---|---|---|---|
| | | $f_1(A, B, C)$ | $f_2(A, B, C)$ |
| $t_0$ | 000 | 0 | 0 |
| $t_1$ | 001 | 1 | 1 |
| $t_2$ | 010 | 1 | 0 |
| $t_3$ | 011 | 0 | 1 |
| $t_4$ | 100 | 0 | 0 |
| $t_5$ | 101 | 0 | 1 |
| $t_6$ | 110 | 1 | 1 |
| $t_7$ | 111 | 1 | 0 |

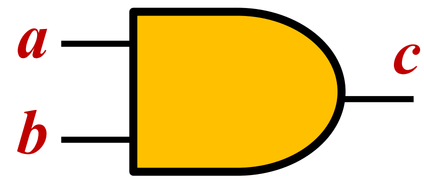
$$f_1(A,B,C) = \sum m(1,2,6,7)$$

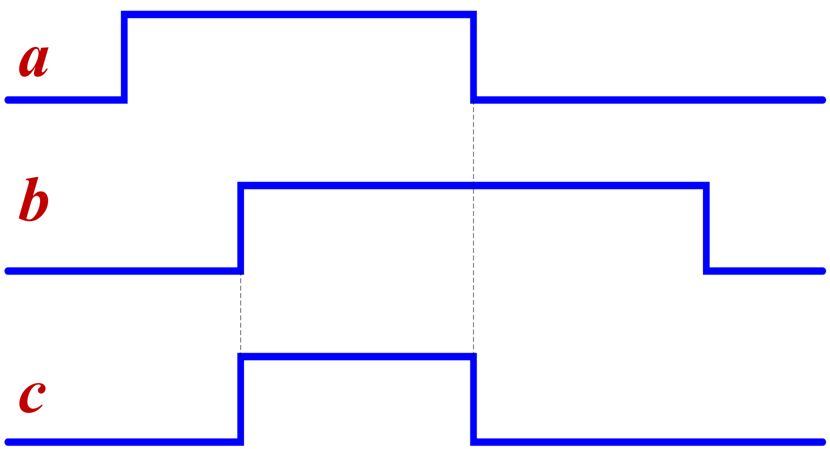
$$f_2(A,B,C) = \sum m(1,3,5,6)$$



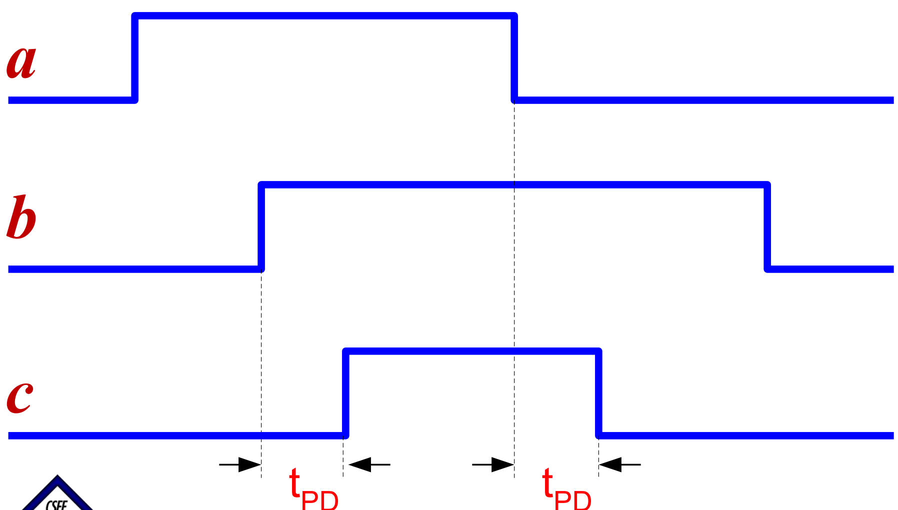
# Propagation Delay

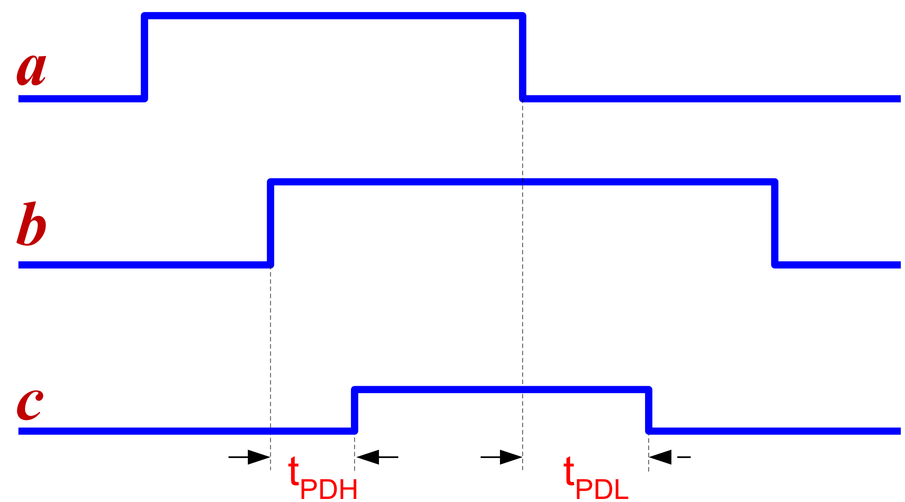
- Propagation delay is the time for the output to become ready after the input gets changed




- Propagation delay depends on the microelectronics technology and size
- Rising and falling time may differ
- $t_{PD} = ½ (t_{PDH} + t_{PDL})$



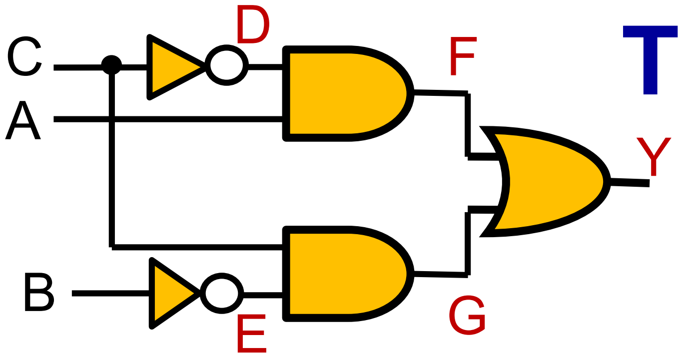



# Important Physical Characteristics

- Physical characteristics vary depending on the microelectronics technology used in the design and fabrication
- There is a trade-off between speed and power dissipation

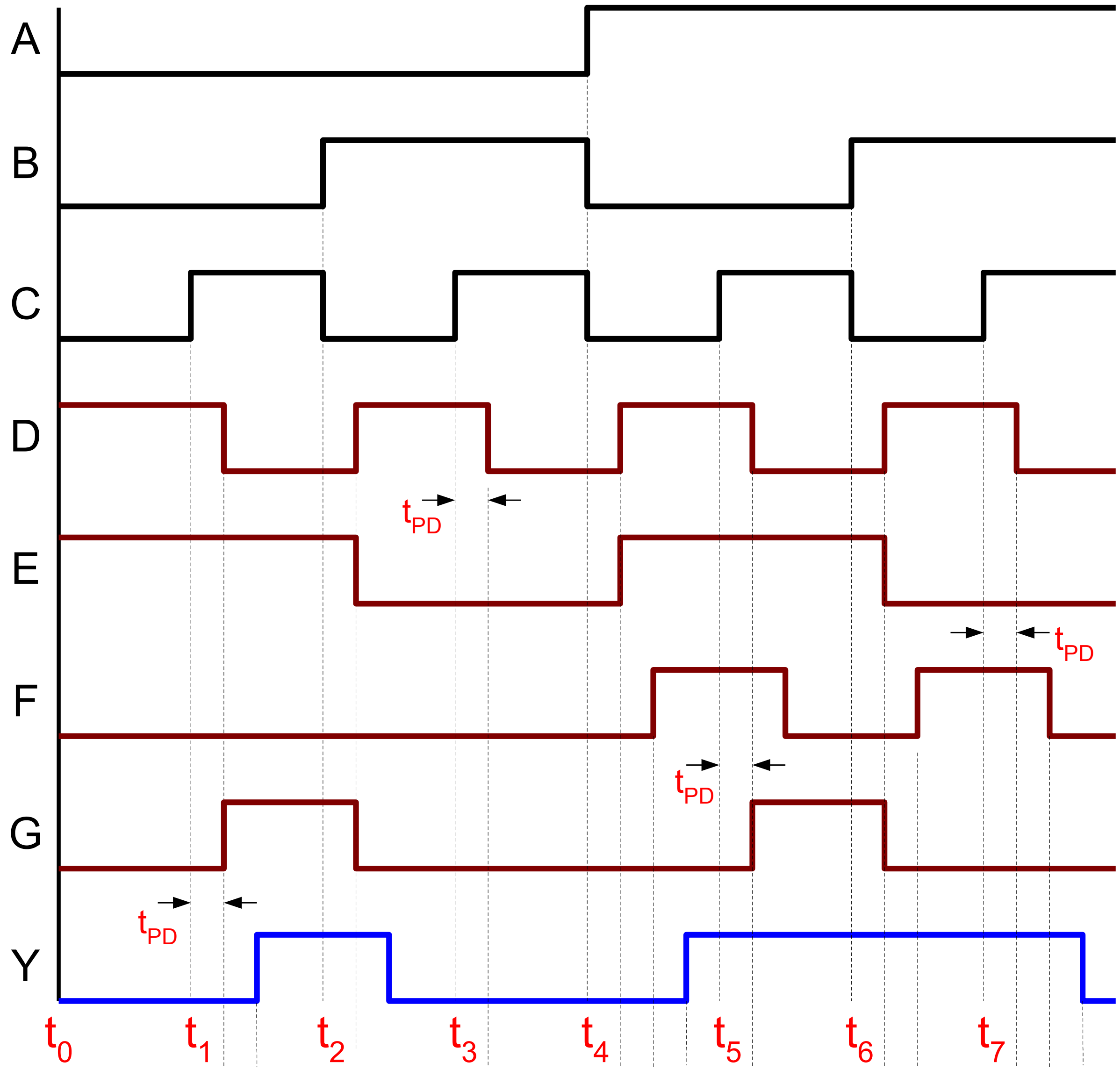
| Logic family | Propagation Delay $t_{PD}$ (ns) | Power Dissipation Per Gate (mW) | Technology |
|---|---|---|---|
| **7400** | 10 | 10 | Standard TTL |
| **74H00** | 6 | 22 | High-speed TTL |
| **74L00** | 33 | 1 | Low-power TTL |
| **74LS00** | 9.5 | 2 | Low-power Schottky TTL |
| **74S00** | 3 | 19 | Schottky TTL |
| **74ALS00** | 3.5 | 1.3 | Advanced low-power Schottky TTL |
| **74AS00** | 3 | 8 | Advanced Schottky TTL |
| **74HC00** | 8 | 0.17 | High-speed CMOS |

# Timing Diagram with Delay




| ABC | Y=f(A,B,C) |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 0 |
| 011 | 0 |
| 100 | 1 |
| 101 | 1 |
| 110 | 1 |
| 111 | 0 |

$$f(A,B,C) = \sum m(1,4,5,6)$$



# Conclusion

- ***Summary***
  - ➔ Canonical form of switching functions
    (conversion from simplified to canonical form)
  - ➔ Analyzing switching circuits using algebraic methods
    (Truth table and Derivation of logic function)
  - ➔ Analysis of timing diagram
  - ➔ Effect of physical characteristics
    (propagation delay and power dissipation)
- ***Next Lecture***
  - ➔ Synthesis of combinational logic circuits

Reading assignment: Section 2.4 in the textbook