CMPE 330 **Spring 2015** **Problem Set #5**

NOTE: You must show complete work for full credit. Report numerical solutions to two significant figures unless otherwise specified.

1. An electron beam shaped like a circular cylinder of radius r_0 carries a charge density given by

$$\rho_{\mathrm{V}} = -\frac{\rho_0 a^2}{a^2 + r^2}, \qquad (\mathrm{C/m}^3)$$

   where a and ρ_0 are positive constants, and the beam's axis is coincident with the z-axis. [modified from Ulaby et al. 4.8, p. 227]

   a. Determine the total charge contained in a length L of the beam.

   b. If the electrons are moving in the $+z$-direction with a uniform speed u, determine the magnitude and direction of the current crossing the $z = 0$ plane.

   Note: Ulaby et al.'s statement of the problem does not treat the units of r consistently with the rest of his text. In his statement of the problem, r must be unitless, since otherwise it makes no sense to add 1 and r^2. Elsewhere in his text, r has units of length. To avoid this issue, I restated the problem. Please use my statement of the problem.

2. Electric charge is distributed along an arc in free space located in the x-y plane and defined by $r = 2$ cm and $0 \le \phi \le \pi/4$. If $\rho_l = 5$ μC/m, find $\mathbf{E}$ at $(0, 0, z)$ and then evaluate it at: (a) the origin, (b) $z = 5$ cm, and (c) $z = -5$ cm. [modified from Ulaby et al. 4.15, p. 227]

   Note: In Ulaby's statement of the problem, he did not specify that the arc is in free space, but that is the only way to get agreement with his solutions. In general, if the dielectric constant is not specified, you should assume that $\epsilon = \epsilon_0$.

3. Multiple charges at different locations are said to be in equilibrium if the force acting on any one of them is identical in magnitude and direction to the force acting on any of the others. Suppose we have two negative charges, one located at the origin and carrying charge $-9e$, and the other located on the positive x-axis at a distance d from the first one and carrying charge $-36e$. Determine the location, polarity, and magnitude of a third charge whose polarization would bring the entire system into equilibrium [Ulaby et al. 4.18, pp. 227–228]

4. Given the electric flux density, $\mathbf{D} = \hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x - 2y)$ C/m^3, determine [modified from Ulaby 4.22, p. 228]

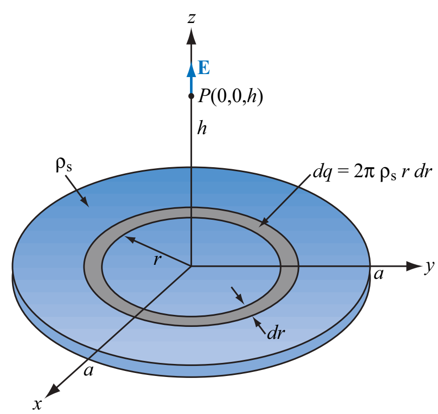
   a. ρ_{V} by applying Gauss's law, $\nabla \cdot \mathbf{D} = \rho_{\mathrm{V}}$.

   b. The total charge Q enclosed in a cube 2 m on side, located in the first octant with three of its sides coincident with the x-, y-, and z-axes and one of its corners at the origin, obtained by integrating over the charge density.

c. The total charge Q in the cube, obtained by applying the integral form of Gauss's law, $\int_S \mathbf{D} \cdot d\mathbf{s} = Q$.

Note: In Ulaby's statement of the problem, he did not specify how to find Q in part (b). The idea of the problem is to find it in two different ways and show that they are equal!

5. A circular disk of radius a, shown in the accompanying figure [Ulaby et al. Fig. 4-7], is located on the x-y plane with its center at the origin. It has a uniform surface charge ρ_S. [Ulaby 4.31, p. 229]



   a. Obtain an expression for the electrostatic potential V at a point $P(0, 0, z)$ on the z-axis.

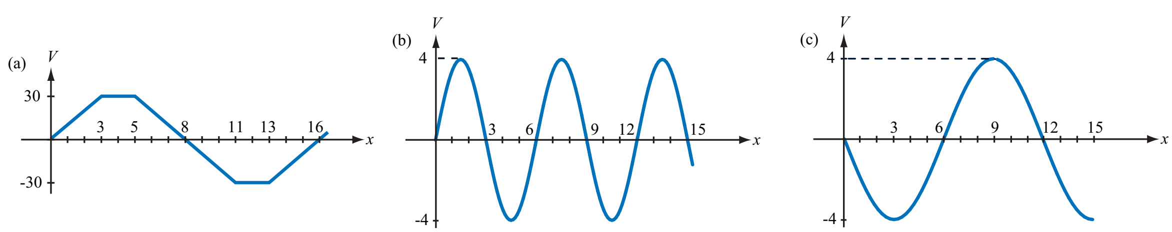
   b. Use this result to find $\mathbf{E}$ and compare it to the result that we obtained directly using Coulomb's law. See slide 8.21 or Ulaby et al. Example 4-5.

6. For each of the distributions of the electric potential in the figure shown below [Ulaby, Fig. P4.36]: [modified from Ulaby et al. 4.36, p. 229]

   a. Find a functional form for the voltage and, from that, derive a functional form for the electric field.

   b. Use a computer to plot the electric field as a function of x. Note that there is only one non-zero component. Show your listing as well as your output.

In all cases, the vertical axis is in volts, the horizontal axis is in meters, and the voltage is independent of y and z.



7. A coaxial resistor of length l consists of two concentric cylinders. The inner cylinder has radius a and is made of a material with conductivity σ_1, and the outer cylinder, extending between $r = a$ and $r = b$, is made of a material with conductivity σ_2. If the two ends of the resistor are capped with conducting plates, show that the resistance between the two ends is $l/[\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2)]$. [Ulaby et al. 4.44, p. 230]

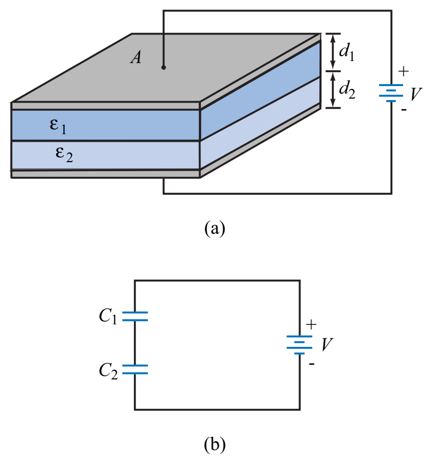
8. A 2 cm conducting sphere is embedded in a charge-free dielectric medium with $\epsilon_{2\mathrm{r}} = 9$. (a) If $\mathbf{E}_2 = -\hat{\mathbf{R}}\, 3\cos\theta$ V/m in the surrounding region, find the charge density on the sphere's surface. (b) In Ulaby's statement of the problem in his 2007 edition, it reads,

"If $\mathbf{E}_2 = \hat{\mathbf{R}}\, 3\cos\theta - \hat{\boldsymbol{\theta}}\, 3\sin\theta$ V/m in the surrounding region, find the charge density on the surface." This formulation must be wrong. Why? [modified from Ulaby 2007, p. 199]

9. The capacitor shown in the accompanying figure [Ulaby et al., Fig. P4.58] consists of parallel dielectric layers. Use energy considerations to show that the overall capacitance of the capacitor, C, is equal to the series combination of the individual layers C_1 and C_2, so that

$$C = \frac{C_1 C_2}{C_1 + C_2}, \tag{4.8.1}$$

where $C_1 = \epsilon_1 A/d_1$ and $C_2 = \epsilon_2 A/d_2$. [Ulaby et al. 4.58, p. 233]



a. Let V_1 and V_2 be the electric potentials across the upper and lower dielectrics respectively. What are the corresponding fields E_1 and E_2? By applying the appropriate boundary condition at the interface of the two dielectrics, obtain explicit expressions for E_1 and E_2 in terms of ϵ_1, ϵ_2, V, and the indicated dimensions of the capacitor.

b. Calculate the energy stored in each of the dielectric layers and then use the sum to obtain an expression for C.

c. Show that C is given by Eq. (4.8.1) above for C.