Exam Quiz 1 (Given 02/03)

Ulaby, et al. Example 1-2

Question: A laser beam propagating through the atmosphere is characterized by an electric field intensity given by

$$E(x,t) = 150\exp(-0.03x)\cos\left(3\times 10^{15}t - 10^{7}x\right) \quad \text{(V/m)}$$

where x is the distance from the source in meters. Determine (a) the direction of wave travel, (b) the wave velocity, and (c) the wave amplitude at a distance of 200 m

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EQ.P1

Exam Quiz 1

Solution: (a) Since the coefficients of t and x have the opposite sign, the wave propagates in the $+x$ direction.

(b) We find that

$$u_{\mathrm{p}} = \frac{\omega}{\beta} = \frac{3\times 10^{15}\,\mathrm{s}^{-1}}{10^{7}\,\mathrm{m}^{-1}} = 3\times 10^{8}\,\mathrm{m/s},$$

which is (of course) the speed of light c in the vacuum or air.

(c) At x = 200 m, the amplitude of $E(x,t)$ is

$$E(x,t) = 150\exp(-0.03\,\mathrm{m}^{-1}\times 200\,\mathrm{m}) = 0.37\ \text{V/m}$$

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EQ.S1

Exam Quiz 2 (Given 02/10)

Paul Quick Review Exercises 6.1 and 6.7 (modified)

Question: What are the per unit length capacitance and inductance of a two-wire line whose wires have a radius of 10 mils and a separation of 75 mils? What is the characteristic impedance and the velocity of propagation? (These dimensions are typical for ribbon cables used to interconnect components.)

Parameter	Coaxial	Two wire
R'	$\frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_S}{\pi d}$
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$

$$\mu = 4\pi\times10^{-7}\ \text{H/m},\quad \varepsilon = 1/\mu c^2$$

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EQ.P2

Exam Quiz 2

Solution: 10 mils × 2.54×10^{-5} m/mil = 2.54×10^{-4} m, 75 mils = 1.905×10^{-3} m. $(d/2a) = 75.0/20.0 = 3.75$, $[(d/2a)^2 - 1]^{1/2} = 3.61$, $\ln(3.75 + 3.61) = 1.96$. $L' = (\mu_0/\pi)\times1.96 = 4\times10^{-7}$ H/m × 1.96 = 7.85×10^{-7} H/m. $C' = (\pi\varepsilon_0\ /\ 1.96) = (\pi/1.96)\times8.85\times10^{-12}$ F/m $= 1.42\times10^{-11}$ F/m. $Z_0 = (L'/C')^{1/2} = 235$ ohms. $u_p = 1\ /\ (7.85\times10^{-7}$ H/m × 1.42×10^{-11} F/m$)^{1/2} = 3.00\times10^{8}$ m/s. The velocity equals the speed of light in the vacuum.

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EQ.S2

Exam Quiz 3 (Given 02/10)

Review of the Solution to the Telegrapher's Equation

Question: The telegrapher's equation may be written

$$-\frac{\partial v(z,t)}{\partial z} = L'\frac{\partial i(z,t)}{\partial t}, \quad -\frac{\partial i(z,t)}{\partial z} = C'\frac{\partial v(z,t)}{\partial t}$$

Show that the solution to this equation may be written

$$v(z,t) = v^+\left(t - \frac{z}{u_\mathrm{p}}\right) + v^-\left(t + \frac{z}{u_\mathrm{p}}\right), \quad i(z,t) = \frac{1}{Z_0}v^+\left(t - \frac{z}{u_\mathrm{p}}\right) - \frac{1}{Z_0}v^-\left(t + \frac{z}{u_\mathrm{p}}\right)$$

where v^+ and v^- are arbitrary function whose second derivatives exist and

$$u_\mathrm{p} = 1/\sqrt{L'C'}, \quad Z_0 = \sqrt{L'/C'}$$

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EQ.P3

Exam Quiz 3 (Given 02/10)

Solution: We may first combine eliminate the current from the telegrapher's equation to obtain the wave equation

$$\frac{\partial^2 v(z,t)}{\partial z^2} = L'C'\frac{\partial^2 v(z,t)}{\partial t^2}$$

If we let $\theta = t - z/u_\mathrm{p}$, we find

$$\frac{\partial^2}{\partial z^2}v^+\left(t - \frac{z}{u_\mathrm{p}}\right) = \frac{1}{u_\mathrm{p}^2}\frac{d^2 v^+(\theta)}{d\theta^2} \quad \text{and} \quad \frac{\partial^2}{\partial t^2}v^+\left(t - \frac{z}{u_\mathrm{p}}\right) = \frac{d^2 v^+(\theta)}{d\theta^2}$$

Substituting these results into the wave equations, and identity results. Hence any v^+ with two derivatives with satisfy the wave equation. The proof for and v^- is analogous. To obtain the result for the propagation equations we write

$$i(z,t) = i^+\left(t - \frac{z}{u_\mathrm{p}}\right) + i^-\left(t + \frac{z}{u_\mathrm{p}}\right),$$

and we substitute into either one of the telegraphers' equation. The result follows from equating terms

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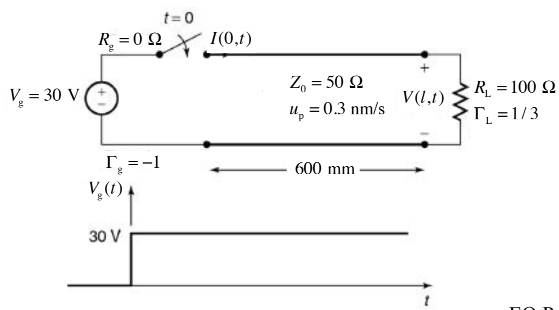
EQ.S3

Exam Quiz 4 (Given 02/24)

Example (Paul 6.1-modified):

Question: For the transmission line shown below, calculate $I(0,t)$ and $I(l,t)$ for times up to $4T$. In Ulaby's notation, the generator voltage $V_g = 0$ V for $t < 0$ and $V_g = 30$ V for $t > 0$. We have $R_g = 0$, $R_L = 100\ \Omega$, and $Z_0 = 50\ \Omega$. The line is 600 mm long, and the propagation velocity is 3.0×10^8 m/s



Paul Figure 6-12(a) — Modified

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Exam Quiz 4 (Given 02/24)

Solution: The line parameters are given by

$$\Gamma_g = \frac{0-50}{0+50} = -1, \quad \Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}, \quad T = \frac{l}{u_p} = 2\,\text{ns}$$

For t = (0 ns, 2 ns), $V(l, t) = 0$. For t = (2 ns, 6 ns), $V(l, t)$ = (incoming voltage) + (reflected voltage) = 30 V + (1/3)×30 V = 40 V. The corresponding current is $I(l, t) = V(l, t) / R_L = 0.4$ A.

Since the reflected voltage is 10 V, we find that for t = (4 ns, 8 ns), $V^+(t)$ = 30 V – 10 V = 20 V, and, for t = (6 ns, 10 ns), $V(l, t)$ = (4/3)×20 V = 26.67 V and $I(l, t)$ = 0.2667 A.

For t = (0 ns, 4 ns), $I(0, t)$ = 30 V / 50 Ω = 0.6 A.

For t = (4 ns, 8 ns), $I(0, t)$ = (20 V – 10 V) / 50 Ω = 0.2 A.

$V(0, t)$ = 30 V for $t > 0$.

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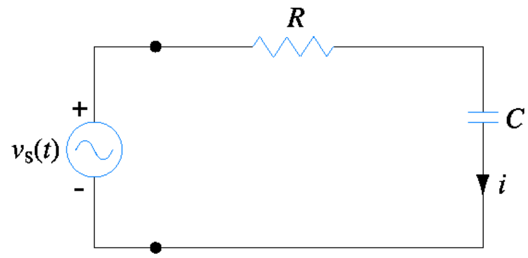
Exam Quiz 5 (Given 03/03)

Question: For the RC circuit shown below, show directly (not using phasors) that if $v_S(t) = V_0 \sin(\omega t + \phi_0)$ then

$$i(t) = \frac{V_0 \omega C}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t + \phi_0 + \phi_1) \quad \text{with} \quad \phi_1 = \tan^{-1}(\omega RC)$$



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EQ.P5

Exam Quiz 5 (Given 03/03)

Solution: We begin by assuming a solution in the form
We substitute $i(t) = A\cos(\omega t + \phi_0 + \phi_1)$ into the equation

$$Ri(t) + \frac{1}{C}\int i(t)\, dt = v_S(t)$$

and we obtain

$$RA\cos(\omega t + \phi_0 + \phi_1) + \frac{1}{\omega C}\sin(\omega t + \phi_0 + \phi_1) = V_0 \sin(\omega t + \phi_0) \quad (1)$$

Writing now

$$A\cos(\omega t + \phi_0 + \phi_1) = A\cos\phi_1 \cos(\omega t + \phi_0) + A\sin\phi_1 \sin(\omega t + \phi_0)$$

$$A\sin(\omega t + \phi_0 + \phi_1) = A\cos\phi_1 \sin(\omega t + \phi_0) - A\sin\phi_1 \cos(\omega t + \phi_0) \quad (2)$$

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EQ.S5

Exam Quiz 5 (Given 03/03)

Solution (continued):

Substituting (2) into (1), we obtain

$$\left[RA\cos\phi_1 - \frac{A}{\omega C}\sin\phi_1\right]\cos(\omega t + \phi_0)$$

$$+\left[RA\cos\phi_1 + \frac{A}{\omega C}\sin\phi_1\right]\sin(\omega t + \phi_0) = V_0\sin(\omega t + \phi_0)$$

Separately equating the coefficients of $\cos(\omega t + \phi_0)$ and $\sin(\omega t + \phi_0)$, we obtain

$$RA\cos\phi_1 - \frac{A}{\omega C}\sin\phi_1 = 0, \quad RA\cos\phi_1 + \frac{A}{\omega C}\sin\phi_1 = V_0$$

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EQ.S5

Exam Quiz 5 (Given 03/03)

Solution (continued):

Solving for $A\cos\phi_1$ and $A\sin\phi_1$, we obtain

$$A\cos\phi_1 = \frac{\omega C V_0}{1+\omega^2 R^2 C^2}, \quad A\sin\phi_1 = \frac{\omega R C^2 V_0}{1+\omega^2 R^2 C^2}$$

From these equations, we obtain

$$\tan\phi_1 = \omega RC \quad \text{or} \quad \phi_1 = \tan^{-1}(\omega RC)$$

and

$$A^2 = \frac{\omega^2 C^2 V_0^2}{1+\omega^2 R^2 C^2} \quad \text{or} \quad A = \frac{\omega C V_0}{\sqrt{1+\omega^2 R^2 C^2}}$$

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EQ.S5

Exam Quiz 6 (Given 03/03)

Ulaby Exercise 2.4 (Slide 4.6)

Question: A two-wire air line has the following parameters: $R' = 0.404$ mΩ/m, $L' = 2.00$ mH/m, $G' = 0$, $C' = 5.56$ pF/m. For operation at 5 kHz, determine (a) the attenuation coefficient α, (b) the wavenumber β, (c) the phase velocity u_p, and the characteristic impedance Z_0.

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EQ.P6

Exam Quiz 6 (Given 03/03)

Solution: $\omega = 2 \times 3.14159 \times (5{\times}10^3\ \mathrm{s}^{-1}) = 3.14159{\times}10^4\ \mathrm{s}^{-1}$.
$R' + j\omega L' = (4.04000{\times}10^{-4} + j{\times}3.14159{\times}10^4 \times 2.00000{\times}10^{-6})\ \Omega/\mathrm{m} = (4.04000{\times}10^{-4} + j{\times}6.28318{\times}10^{-2})\ \Omega/\mathrm{m} = 6.28319{\times}10^{-2} \times \exp(j{\times}1.56436)\ \Omega/\mathrm{m}$. $G' + j\omega C' = j{\times}1.74673{\times}10^{-7}\ \Omega^{-1}/\mathrm{m} = 1.74673{\times}10^{-7} \times \exp(j{\times}1.57080)\ \Omega^{-1}/\mathrm{m}$. Note the small difference in phases! Six digits of accuracy are needed to keep three digits in the attenuation coefficient. $\gamma^2 = 1.09750{\times}10^{-8} \times \exp(j{\times}3.13156)\ \mathrm{m}^{-2}$, so that $\gamma = 1.04762{\times}10^{-4} \times \exp(j{\times}1.56758)\ \mathrm{m}^{-1} = 3.37{\times}10^{-7} + j{\times}1.04761{\times}10^{-4}$, so that $\alpha = 3.37{\times}10^{-7}$ Np/m and $\beta = 1.05{\times}10^{-4}$ rad/m.
We have $u_p = \omega / \beta = (3.142/1.048) \times 10^8 = 3.00 \times 10^8$ m/s
and $Z_0 = \{(6.283{\times}10^{-2} / 1.747{\times}10^{-7}) \exp[j{\times}(1.56436 - 1.57080)]\}^{1/2}$
$= (600 - j{\times}1.93)\ \Omega$

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EQ.S6

Exam Quiz 7 (Given 03/03)

Ulaby Exercise 2.11 (Slide 4.17)

Question: A 140 Ω lossless line is terminated in a load impedance $Z_L = (280 + j\,182)$ Ω. If $\lambda = 72$ cm, find (a) the reflection coefficient Γ, (b) the VSWR S, (c) the locations of the voltage maxima and minima.

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EQ.P7

Exam Quiz 7 (Given 03/03)

Solution: (a) $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (280 + j\,182 - 140) / (280 + j\,182 + 140) = 230 \exp(j\,0.915) / 458 \exp(j\,0.409) = 0.50 \exp(j\,0.51)$. NOTE: 0.51 rads = 29°. (b) $S = (1 + |\Gamma|) / (1 - |\Gamma|) = (1 + 0.502) / (1 - 0.502) = 3.0$. (c) $l_{max} = (0.506 \times 72 / 4\pi + n \times 72 / 2)$ cm = 2.9 cm, 39 cm, 75 cm,…; $l_{min} = (0.506 \times 72 / 4\pi + 18 + n \times 72 / 2)$ cm = 21 cm, 57 cm, 93 cm,…

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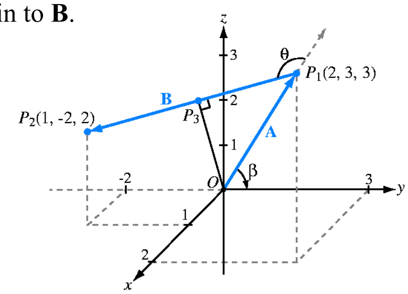
EQ.S7

Exam Quiz 8 (Given 3/10/2015)

Vectors and angles: Ulaby et al. Example 3-1 (Slide 6.10)

Question: In Cartesian coordinates, vector **A** is directed from the origin to the point $P_1(2, 3, 3)$, and vector **B** is directed from P_1 to $P_2(1, -2, 2)$. Find (a) the vector **A**, its magnitude A, and its unit vector $\hat{\mathbf{a}}$, (b) the angle that **A** makes with the y-axis, (c) vector **B**, (d) the angle between **A** and **B**, and (e) the perpendicular distance from the origin to **B**.



Ulaby et al., Figure 3-7

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EQ.P8

Exam Quiz 8 (Given 3/10/2015)

Solution: (a) $\mathbf{A} = \hat{\mathbf{x}}\,2 + \hat{\mathbf{y}}3 + \hat{\mathbf{z}}\,3$

$$A = \sqrt{4+9+9} = \sqrt{22}$$

$$\hat{\mathbf{a}} = \mathbf{A}/A = \left(\hat{\mathbf{x}}\,2 + \hat{\mathbf{y}}3 + \hat{\mathbf{z}}\,3\right)\Big/\sqrt{22}$$

(b) The angle b between **A** and the y-axis may be found from the expression

$$\beta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\hat{\mathbf{y}}}{A}\right) = \cos^{-1}\left(\frac{3}{\sqrt{22}}\right) = 0.879 \text{ rads} = 50.2°$$

(c) $\mathbf{B} = \hat{\mathbf{x}}\,(1-2) + \hat{\mathbf{y}}(-2-3) + \hat{\mathbf{z}}\,(2-3) = -\hat{\mathbf{x}} - \hat{\mathbf{y}}5 - \hat{\mathbf{z}}$

(d) $$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\frac{-20}{\sqrt{22}\sqrt{27}}\right) = 2.533 \text{ rads} = 145.1°$$

(e) The points OP_1P_3 form a right triangle. The magnitude of the line segment OP_3 is given by $A\sin(\pi-\theta) = \sqrt{22}\sin(0.609) = 2.68$

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EQ.S8

Exam Quiz 9 (Given 3/10/2015)

Charge in a sphere: Ulaby et al. Example 3-6 (Slide 6.26)

Question: A sphere of radius 2 cm contains a charge of density ρ_V given by

$$\rho_V = 4\cos^2\theta$$

What is the total charge?

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EQ.P9

Exam Quiz 9 (Given 3/10/2015)

Solution: After converting from cm to m,

$$Q = \int_v \rho_V \, dv$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{R=0}^{2\times 10^{-2}} \left(4\cos^2\theta\right) R^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$= 4\int_0^{2\pi}\int_0^{\pi} \left(\frac{R^3}{3}\right)\Bigg|_0^{2\times 10^{-2}} \sin\theta\cos^2\theta \, d\theta \, d\phi$$

$$= \frac{64}{9}\times 10^{-6}\int_0^{2\pi} d\phi = \frac{128\pi}{9}\times 10^{-6} = 44.68 \ \mu\text{C}$$

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EQ.S9

Exam Quiz 10 (Given 3/24/2015)

Slides 6.17, 6.22, and 6.25

Question: Obtain the differential relations for the lengths, surface areas, and volumes in Cartesian, cylindrical, and spherical coordinates using pictures that illustrate the geometry.

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EQ.P10

Exam Quiz 10 (Given 3/24/2015)

Solution: Cartesian coordinates

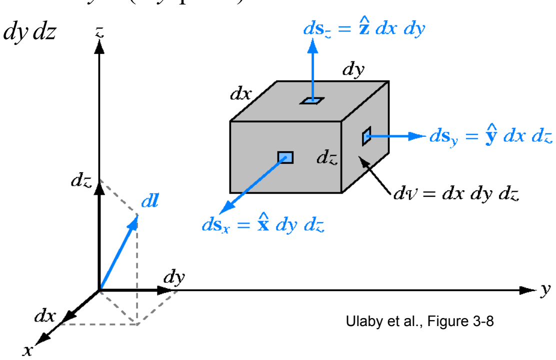
Length: $d\mathbf{l} = \hat{\mathbf{x}}\, dl_x + \hat{\mathbf{y}}\, dl_y + \hat{\mathbf{z}}\, dl_z = \hat{\mathbf{x}}\, dx + \hat{\mathbf{y}}\, dy + \hat{\mathbf{z}}\, dz$

Surface area: $d\mathbf{s}_x = \hat{\mathbf{x}}\, dl_y\, dl_z = \hat{\mathbf{x}}\, dy\, dz$ (y-z plane)

$d\mathbf{s}_y = \hat{\mathbf{y}}\, dz\, dx = \hat{\mathbf{y}}\, dx\, dz$ (x-z plane)

$d\mathbf{s}_z = \hat{\mathbf{z}}\, dx\, dy$ (x-y plane)

Volume: $dv = dx\, dy\, dz$



Ulaby et al., Figure 3-8

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EQ.S10

Exam Quiz 10 (Given 3/24/2015)

Solution: Cylindrical coordinates

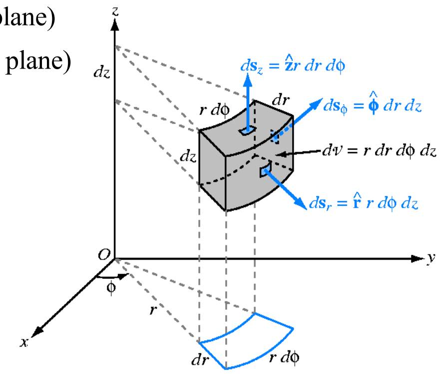
Length: $d\mathbf{l} = \hat{\mathbf{r}}\,dl_r + \hat{\boldsymbol{\phi}}\,dl_\phi + \hat{\mathbf{z}}\,dl_z = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r d\phi + \hat{\mathbf{z}}\,dz$

Surface area: $d\mathbf{s}_r = \hat{\mathbf{r}}\,r d\phi\, dz$ (ϕ-z plane)

$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ (r-z plane)

$d\mathbf{s}_z = \hat{\mathbf{z}}\,r dr\, d\phi$ (r-ϕ plane)

Volume: $dv = r dr\, d\phi\, dz$



Ulaby et al., Figure 3-10

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EQ.S10

Exam Quiz 10 (Given 3/24/2015)

Solution: Spherical coordinates

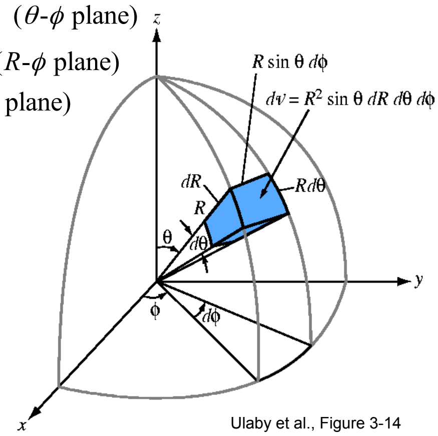
Length: $d\mathbf{l} = \hat{\mathbf{R}}\,dl_R + \hat{\boldsymbol{\theta}}\,dl_\theta + \hat{\boldsymbol{\phi}}\,dl_\phi = \hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}\,R\,d\theta + \hat{\boldsymbol{\phi}}\,R\sin\theta\, d\phi$

Surface area: $d\mathbf{s}_R = \hat{\mathbf{R}}\,R^2 \sin\theta\, d\theta\, d\phi$ (θ-ϕ plane)

$d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}\,R\sin\theta\, dR\, d\phi$ (R-ϕ plane)

$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,R\, dR\, d\theta$ (R-θ plane)

Volume: $dv = R^2 \sin\theta\, dR\, d\theta\, d\phi$



Ulaby et al., Figure 3-14

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EQ.S10

Exam Quiz 11 (Given 3/24/2015)

Slides 7.26, 7.27, 7.28, and 7.29

Question: Show using an appropriate picture and symmetry that the general expression for the curl in an orthogonal coordinate system may be written

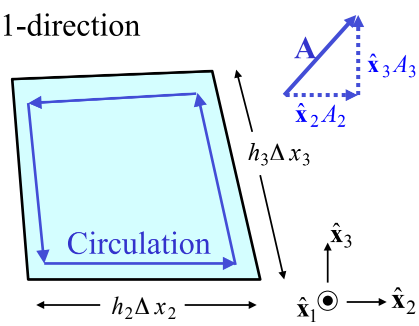
$$\nabla \times \mathbf{A} = \hat{\mathbf{x}}_1 \left[\frac{1}{h_2 h_3} \left(\frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right) \right] + \hat{\mathbf{x}}_2 \left[\frac{1}{h_1 h_3} \left(\frac{\partial}{\partial x_3} h_1 A_1 - \frac{\partial}{\partial x_1} h_3 A_3 \right) \right]$$
$$+ \hat{\mathbf{x}}_3 \left[\frac{1}{h_1 h_2} \left(\frac{\partial}{\partial x_1} h_2 A_2 - \frac{\partial}{\partial x_2} h_1 A_1 \right) \right]$$

where $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3$ are unit vectors in the 1- 2- and 3-directions and h_1, h_2, h_3 are the corresponding differential lengths. Use this result to obtain the expression in cylindrical coordinates.

Exam Quiz 11 (Given 3/24/2015)

Solution: The circulation in the 1-direction may be written

$$\Delta C_1 = \Delta_2 \left[A_3 (h_3 \Delta x_3) \right] - \Delta_3 \left[A_2 (h_2 \Delta x_2) \right]$$
$$\simeq \Delta x_2 \Delta x_3 \left(\frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right)$$



so that

$$\lim_{\Delta s_1 \to 0} \frac{\Delta C_1}{\Delta s_1} = \frac{1}{h_2 h_3} \left(\frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right)$$

The other directions may be obtained by using the $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ symmetry.

Exam Quiz 11 (Given 3/24/2015)

In cylindrical coordinates: $h_1 = 1, \quad h_2 = r, \quad h_3 = 1$

After substitution into the general expression, we find

$$\nabla \times \mathbf{A} = \hat{\mathbf{r}}\left[\frac{1}{r}\left(\frac{\partial}{\partial \phi} A_z - \frac{\partial}{\partial z} r A_\phi\right)\right] + \hat{\boldsymbol{\varphi}}\left(\frac{\partial}{\partial z} A_r - \frac{\partial}{\partial r} A_z\right)$$
$$+ \hat{\mathbf{z}}\left[\frac{1}{r}\left(\frac{\partial}{\partial r} r A_\phi - \frac{\partial}{\partial \phi} A_r\right)\right]$$

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EQ.S11

Exam Quiz 12 (Given 3/31/2015)

Electric Field due to Two Point Charges: Ulaby et al., Ex. 4-3, Slides 8.16, 8.17

Question: Two point charges with $q_1 = 2 \times 10^{-5}$ C and $q_2 = -4 \times 10^{-5}$ C are located at (1, 3, –1) and (–3, 1, –2), respectively, in a Cartesian coordinate system. Find (a) the electric field $\mathbf{E}$ at (3, 1, –2) and (b) the force on a charge $q_3 = 8 \times 10^{-5}$ C located at that point

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EQ.P12

Exam Quiz 12 (Given 3/31/2015)

Solution: a) Since $\varepsilon = \varepsilon_0$ and there are two charges, we have

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1(\mathbf{R}-\mathbf{R}_1)}{|\mathbf{R}-\mathbf{R}_1|^3} + \frac{q_2(\mathbf{R}-\mathbf{R}_2)}{|\mathbf{R}-\mathbf{R}_2|^3}\right]$$

with

$$\mathbf{R}_1 = \hat{\mathbf{x}} + \hat{\mathbf{y}}3 - \hat{\mathbf{z}}$$
$$\mathbf{R}_2 = -\hat{\mathbf{x}}3 + \hat{\mathbf{y}} - \hat{\mathbf{z}}2$$
$$\mathbf{R} = \hat{\mathbf{x}}3 + \hat{\mathbf{y}} - \hat{\mathbf{z}}2$$

After substitution, we find

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2(\hat{\mathbf{x}}2 - \hat{\mathbf{y}}2 - \hat{\mathbf{z}})}{27} - \frac{4(\hat{\mathbf{x}}6)}{216}\right]\times 10^{-5} = \frac{\hat{\mathbf{x}} - \hat{\mathbf{y}}4 - \hat{\mathbf{z}}2}{108\pi\varepsilon_0}\times 10^{-5}\ \text{V/m}$$

(b) Using the force equation, we have

$$\mathbf{F} = q_3\mathbf{E} = 8\times 10^{-5}\times\frac{\hat{\mathbf{x}} - \hat{\mathbf{y}}4 - \hat{\mathbf{z}}2}{108\pi\varepsilon_0}\times 10^{-5} = \frac{\hat{\mathbf{x}}2 - \hat{\mathbf{y}}8 - \hat{\mathbf{z}}4}{27\pi\varepsilon_0}\times 10^{-10}\ \text{N}$$

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EQ.S12

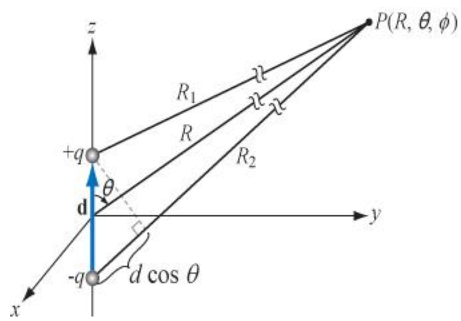
Exam Quiz 13 (Given 4/07/2013)

Electric Dipole: Ulaby et al., Ex. 4-7, Slides 9.12–9.14

Question: Suppose you have two charges of magnitude q and opposite sign, as shown below. Show that when $R \gg d$

$$V = \frac{qd\cos\theta}{4\pi\varepsilon R^2}$$

Find the electric field.



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EQ.P13

Exam Quiz 13 (Given 4/07/2015)

Solution: The voltage may be written as

$$V = \frac{1}{4\pi\varepsilon}\left(\frac{q}{R_1} + \frac{-q}{R_2}\right)$$

From the figure, we find

$$\frac{1}{R_1} = \left[\left(R - \frac{d}{2}\cos\theta\right)^2 + \left(\frac{d}{2}\sin\theta\right)^2\right]^{-1/2}$$

$$= \frac{1}{R}\left(1 - \frac{d}{R}\cos\theta + \frac{d^2}{2R^2}\right)^{-1/2} ; \frac{1}{R}\left(1 + \frac{d}{2R}\cos\theta\right)$$

and similarly

$$\frac{1}{R_2} ; \frac{1}{R}\left(1 - \frac{d}{2R}\cos\theta\right)$$

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EQ.S13

Exam Quiz 13 (Given 4/07/2015)

Solution (continued): So, after substitution in the expression for V,

$$V = \frac{qd\cos\theta}{4\pi\varepsilon R^2}$$

To obtain E, we use the expression for the gradient in spherical coordinates, when the ϕ-variation can be ignored

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{R}}\frac{\partial V}{\partial R} - \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta}$$

$$= \frac{qd}{4\pi\varepsilon R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

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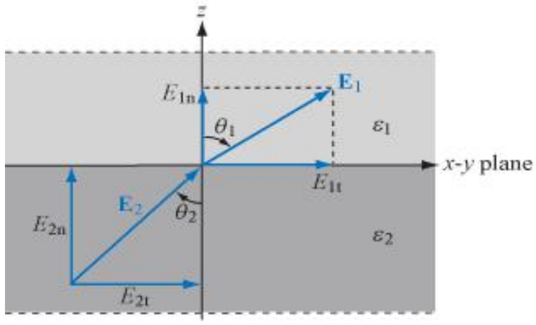
EQ.S13

Exam Quiz 14 (Given 4/07/2015)

Boundary conditions and application: Ulaby et al., Ex. 4-10, Slides 9.23–9.26

Question: Use the two laws of electrostatics to derive the boundary conditions at the interface of two dielectric media with dielectric constants ε_1 and ε_2 and no surface charge. For the geometry shown below, find the relationship between θ_1 and θ_2.



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EQ.P14

Exam Quiz 14 (Given 4/07/2015)

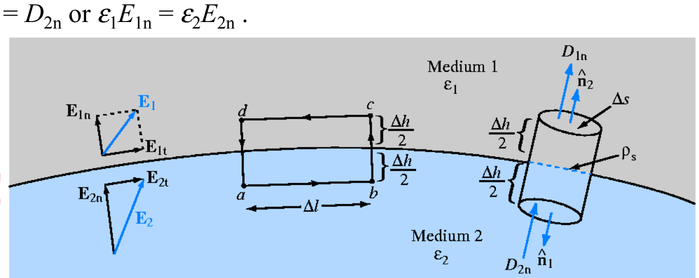
Solution: To derive the tangential boundary condition, we use the relation

$\nabla \times \mathbf{E} = 0$ or $\oint \mathbf{E} \bullet d\mathbf{l} = 0$ around a closed path

Using the geometry in the middle of the figure below and letting $\Delta h \to 0$, we infer $E_{1t} - E_{2t} = 0$ or $E_{1t} = E_{2t}$. To derive the normal boundary condition, we use the relation

$\nabla \bullet \mathbf{D} = 0$ or $\int_S \mathbf{D} \bullet \hat{\mathbf{n}}\, ds = 0$ over a closed surface

Using the pillbox shape shown to the right and letting $\Delta h \to 0$, we infer $D_{1n} = D_{2n}$ or $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$.



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EQ.S14

Exam Quiz 14 (Given 4/07/2015)

Solution (continued): Since $E_{1t} = E_{2t}$ and $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$, we conclude

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{1t}}{E_{1n}} \cdot \frac{E_{2n}}{E_{2t}} = \frac{E_{1t} / E_{2t}}{E_{1n} / E_{2n}} = \frac{\varepsilon_1}{\varepsilon_2}$$

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EQ.S14

Exam Quiz 15 (Given 4/14/2015)

Coaxial cable transmission line parameters: Ulaby et al., Ex. 4-9, Slides 9.18–9.19, 9.31, and 11.29

Question: Find the conductance per unit length and capacitance per unit length for a coaxial cable

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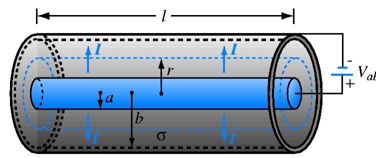
EQ.P15

Exam Quiz 15 (Given 4/14/2015)

Solution: We first determine the conductance per unit length. Let I be the current that flows from the inner conductor to the outer conductor. At any distance r, the area through which the current flows is $A = 2\pi r l$. We now have,

$$\mathbf{J} = \hat{\mathbf{r}}\frac{I}{A} = \hat{\mathbf{r}}\frac{I}{2\pi r l} \quad \text{and} \quad \mathbf{E} = \hat{\mathbf{r}}\frac{I}{2\pi\sigma r l}$$



from which we conclude

$$G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab} l} = \frac{2\pi\sigma}{\ln(b/a)}$$

To find the capacitance and inductance per unit length, we use the relations

$$C' = \frac{\varepsilon}{\sigma}G' = \frac{2\pi\varepsilon}{\ln(b/a)}, \quad L' = \frac{\varepsilon\mu}{C'} = \frac{\mu}{2\pi}\ln(b/a)$$

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EQ.S15