

## Exam Quiz 1 (Given 02/03)

---

Ulaby, et al. Example 1-2

**Question:** A laser beam propagating through the atmosphere is characterized by an electric field intensity given by

$$E(x, t) = 150 \exp(-0.03x) \cos(3 \times 10^{15} t - 10^7 x) \quad (\text{V/m})$$

where  $x$  is the distance from the source in meters. Determine (a) the direction of wave travel, (b) the wave velocity, and (c) the wave amplitude at a distance of 200 m



EQ.P1

## Exam Quiz 1

---

**Solution:** (a) Since the coefficients of  $t$  and  $x$  have the opposite sign, the wave propagates in the  $+x$  direction.

(b) We find that

$$u_p = \frac{\omega}{\beta} = \frac{3 \times 10^{15} \text{ s}^{-1}}{10^7 \text{ m}^{-1}} = 3 \times 10^8 \text{ m/s},$$

which is (of course) the speed of light  $c$  in the vacuum or air.

(c) At  $x = 200$  m, the amplitude of  $E(x, t)$  is

$$E(x, t) = 150 \exp(-0.03 \text{ m}^{-1} \times 200 \text{ m}) = 0.37 \text{ V/m}$$



EQ.S1

## Exam Quiz 2 (Given 02/10)

Paul Quick Review Exercises 6.1 and 6.7 (modified)

**Question:** What are the per unit length capacitance and inductance of a two-wire line whose wires have a radius of 10 mils and a separation of 75 mils? What is the characteristic impedance and the velocity of propagation? (These dimensions are typical for ribbon cables used to interconnect components.)

Parameter	Coaxial	Two wire
$R'$	$\frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_s}{\pi d}$
$L'$	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]$
$G'$	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$
$C'$	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$

$$\mu = 4\pi \times 10^{-7} \text{ H/m}, \quad \epsilon = 1 / \mu c^2$$

EQ.P2



## Exam Quiz 2

**Solution:** 10 mils  $\times 2.54 \times 10^{-5} \text{ m/mil} = 2.54 \times 10^{-4} \text{ m}$ ,  
 75 mils  $= 1.905 \times 10^{-3} \text{ m}$ .  $(d/2a) = 75.0/20.0 = 3.75$ ,  $[(d/2a)^2 - 1]^{1/2} = 3.61$ ,  
 $\ln(3.75 + 3.61) = 1.96$ .  $L' = (\mu_0 / \pi) \times 1.96 = 4 \times 10^{-7} \text{ H/m} \times 1.96 =$   
 $7.85 \times 10^{-7} \text{ H/m}$ .  $C' = (\pi \epsilon_0 / 1.96) = (\pi / 1.96) \times 8.85 \times 10^{-12} \text{ F/m}$   
 $= 1.42 \times 10^{-11} \text{ F/m}$ .  $Z_0 = (L' / C')^{1/2} = 235 \text{ ohms}$ .  $u_p = 1 / (7.85 \times 10^{-7} \text{ H/m} \times$   
 $1.42 \times 10^{-11} \text{ F/m})^{1/2} = 3.00 \times 10^8 \text{ m/s}$ . The velocity equals the speed of light  
 in the vacuum.



EQ.S2

### Exam Quiz 3 (Given 02/10)

#### Review of the Solution to the Telegrapher's Equation

**Question:** The telegrapher's equation may be written

$$-\frac{\partial v(z,t)}{\partial z} = L' \frac{\partial i(z,t)}{\partial t}, \quad -\frac{\partial i(z,t)}{\partial z} = C' \frac{\partial v(z,t)}{\partial t}$$

Show that the solution to this equation may be written

$$v(z,t) = v^+ \left( t - \frac{z}{u_p} \right) + v^- \left( t + \frac{z}{u_p} \right), \quad i(z,t) = \frac{1}{Z_0} v^+ \left( t - \frac{z}{u_p} \right) - \frac{1}{Z_0} v^- \left( t + \frac{z}{u_p} \right)$$

where  $v^+$  and  $v^-$  are arbitrary function whose second derivatives exist and

$$u_p = 1/\sqrt{L'C'}, \quad Z_0 = \sqrt{L'/C'}$$



EQ.P3

### Exam Quiz 3 (Given 02/10)

**Solution:** We may first combine eliminate the current from the telegrapher's equation to obtain the wave equation

$$\frac{\partial^2 v(z,t)}{\partial z^2} = L'C' \frac{\partial^2 v(z,t)}{\partial t^2}$$

If we let  $\theta = t - z/u_p$ , we find

$$\frac{\partial^2}{\partial z^2} v^+ \left( t - \frac{z}{u_p} \right) = \frac{1}{u_p^2} \frac{d^2 v^+(\theta)}{d\theta^2} \quad \text{and} \quad \frac{\partial^2}{\partial t^2} v^+ \left( t - \frac{z}{u_p} \right) = \frac{d^2 v^+(\theta)}{d\theta^2}$$

Substituting these results into the wave equations, and identity results. Hence any  $v^+$  with two derivatives with satisfy the wave equation. The proof for  $v^-$  is analogous. To obtain the result for the propagation equations we write

$$i(z,t) = i^+ \left( t - \frac{z}{u_p} \right) + i^- \left( t + \frac{z}{u_p} \right),$$

and we substitute into either one of the telegraphers' equation. The result follows from equating terms

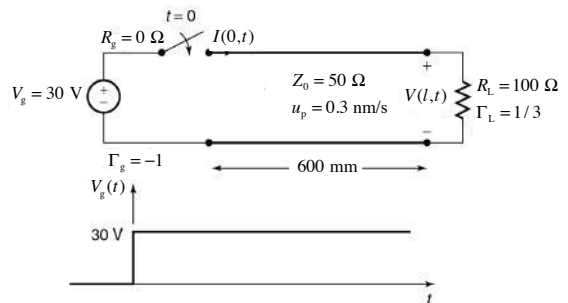


EQ.S3

### Exam Quiz 4 (Given 02/24)

Example (Paul 6.1-modified):

**Question:** For the transmission line shown below, calculate  $I(0,t)$  and  $I(l,t)$  for times up to  $4T$ . In Ulaby's notation, the generator voltage  $V_g = 0$  V for  $t < 0$  and  $V_g = 30$  V for  $t > 0$ . We have  $R_g = 0$ ,  $R_L = 100 \Omega$ , and  $Z_0 = 50 \Omega$ . The line is 600 mm long, and the propagation velocity is  $3.0 \times 10^8$  m/s



EQ.P4

Paul Figure 6-12(a) — Modified

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

### Exam Quiz 4 (Given 02/24)

**Solution:** The line parameters are given by

$$\Gamma_g = \frac{0 - 50}{0 + 50} = -1, \quad \Gamma_L = \frac{100 - 50}{100 + 50} = \frac{1}{3}, \quad T = \frac{l}{u_p} = 2 \text{ ns}$$

For  $t = (0 \text{ ns}, 2 \text{ ns})$ ,  $V(l, t) = 0$ . For  $t = (2 \text{ ns}, 6 \text{ ns})$ ,  $V(l, t) = (\text{incoming voltage}) + (\text{reflected voltage}) = 30 \text{ V} + (1/3) \times 30 \text{ V} = 40 \text{ V}$ . The corresponding current is  $I(l, t) = V(l, t) / R_L = 0.4 \text{ A}$ .

Since the reflected voltage is 10 V, we find that for  $t = (4 \text{ ns}, 8 \text{ ns})$ ,  $V^+(t) = 30 \text{ V} - 10 \text{ V} = 20 \text{ V}$ , and, for  $t = (6 \text{ ns}, 10 \text{ ns})$ ,  $V(l, t) = (4/3) \times 20 \text{ V} = 26.67 \text{ V}$  and  $I(l, t) = 0.2667 \text{ A}$ .

For  $t = (0 \text{ ns}, 4 \text{ ns})$ ,  $I(0, t) = 30 \text{ V} / 50 \Omega = 0.6 \text{ A}$ .

For  $t = (4 \text{ ns}, 8 \text{ ns})$ ,  $I(0, t) = (20 \text{ V} - 10 \text{ V}) / 50 \Omega = 0.2 \text{ A}$ .

$V(0, t) = 30 \text{ V}$  for  $t > 0$ .

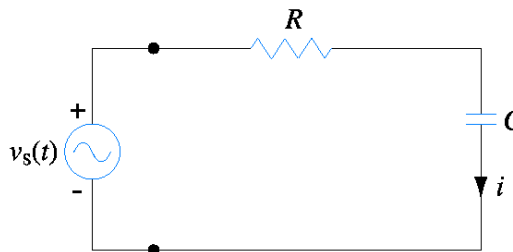
UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.S4

### Exam Quiz 5 (Given 03/03)

**Question:** For the RC circuit shown below, show directly (not using phasors) that if  $v_s(t) = V_0 \sin(\omega t + \phi_0)$  then

$$i(t) = \frac{V_0 \omega C}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t + \phi_0 + \phi_1) \quad \text{with} \quad \phi_1 = \tan^{-1}(\omega RC)$$



UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.P5

### Exam Quiz 5 (Given 03/03)

**Solution:** We begin by assuming a solution in the form  
We substitute  $i(t) = A \cos(\omega t + \phi_0 + \phi_1)$  into the equation

$$Ri(t) + \frac{1}{C} \int i(t) dt = v_s(t)$$

and we obtain

$$RA \cos(\omega t + \phi_0 + \phi_1) + \frac{1}{\omega C} \sin(\omega t + \phi_0 + \phi_1) = V_0 \sin(\omega t + \phi_0) \quad (1)$$

Writing now

$$A \cos(\omega t + \phi_0 + \phi_1) = A \cos \phi_1 \cos(\omega t + \phi_0) + A \sin \phi_1 \sin(\omega t + \phi_0)$$

$$A \sin(\omega t + \phi_0 + \phi_1) = A \cos \phi_1 \sin(\omega t + \phi_0) - A \sin \phi_1 \cos(\omega t + \phi_0) \quad (2)$$

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.S5

### Exam Quiz 5 (Given 03/03)

#### Solution (continued):

Substituting (2) into (1), we obtain

$$\left[ RA \cos \phi_1 - \frac{A}{\omega C} \sin \phi_1 \right] \cos(\omega t + \phi_0) + \left[ RA \cos \phi_1 + \frac{A}{\omega C} \sin \phi_1 \right] \sin(\omega t + \phi_0) = V_0 \sin(\omega t + \phi_0)$$

Separately equating the coefficients of  $\cos(\omega t + \phi_0)$  and  $\sin(\omega t + \phi_0)$ , we obtain

$$RA \cos \phi_1 - \frac{A}{\omega C} \sin \phi_1 = 0, \quad RA \cos \phi_1 + \frac{A}{\omega C} \sin \phi_1 = V_0$$



EQ.S5

### Exam Quiz 5 (Given 03/03)

#### Solution (continued):

Solving for  $A \cos \phi_1$  and  $A \sin \phi_1$ , we obtain

$$A \cos \phi_1 = \frac{\omega C V_0}{1 + \omega^2 R^2 C^2}, \quad A \sin \phi_1 = \frac{\omega R C^2 V_0}{1 + \omega^2 R^2 C^2}$$

From these equations, we obtain

$$\tan \phi_1 = \omega R C \quad \text{or} \quad \phi_1 = \tan^{-1}(\omega R C)$$

and

$$A^2 = \frac{\omega^2 C^2 V_0^2}{1 + \omega^2 R^2 C^2} \quad \text{or} \quad A = \frac{\omega C V_0}{\sqrt{1 + \omega^2 R^2 C^2}}$$



EQ.S5

### Exam Quiz 6 (Given 03/03)

Ulaby Exercise 2.4 (Slide 4.6)

**Question:** A two-wire air line has the following parameters:  $R' = 0.404 \text{ m}\Omega/\text{m}$ ,  $L' = 2.00 \text{ mH/m}$ ,  $G' = 0$ ,  $C' = 5.56 \text{ pF/m}$ . For operation at 5 kHz, determine (a) the attenuation coefficient  $\alpha$ , (b) the wavenumber  $\beta$ , (c) the phase velocity  $u_p$ , and the characteristic impedance  $Z_0$ .



EQ.P6

### Exam Quiz 6 (Given 03/03)

**Solution:**  $\omega = 2 \times 3.14159 \times (5 \times 10^3 \text{ s}^{-1}) = 3.14159 \times 10^4 \text{ s}^{-1}$ .  
 $R' + j\omega L' = (4.04000 \times 10^{-4} + j \times 3.14159 \times 10^4 \times 2.00000 \times 10^{-6}) \Omega/\text{m} = (4.04000 \times 10^{-4} + j \times 6.28318 \times 10^{-2}) \Omega/\text{m} = 6.28319 \times 10^{-2} \times \exp(j \times 1.56436) \Omega/\text{m}$ .  
 $G' + j\omega C' = j \times 1.74673 \times 10^{-7} \Omega^{-1}/\text{m} = 1.74673 \times 10^{-7} \times \exp(j \times 1.57080) \Omega^{-1}/\text{m}$ . Note the small difference in phases! Six digits of accuracy are needed to keep three digits in the attenuation coefficient.  $\gamma^2 = 1.09750 \times 10^{-8} \times \exp(j \times 3.13156) \text{ m}^{-2}$ , so that  $\gamma = 1.04762 \times 10^{-4} \times \exp(j \times 1.56758) \text{ m}^{-1} = 3.37 \times 10^{-7} + j \times 1.04761 \times 10^{-4}$ , so that  $\alpha = 3.37 \times 10^{-7} \text{ Np/m}$  and  $\beta = 1.05 \times 10^{-4} \text{ rad/m}$ .  
We have  $u_p = \omega / \beta = (3.142 / 1.048) \times 10^8 = 3.00 \times 10^8 \text{ m/s}$   
and  $Z_0 = \{(6.283 \times 10^{-2} / 1.747 \times 10^{-7}) \exp[j \times (1.56436 - 1.57080)]\}^{1/2} = (600 - j \times 1.93) \Omega$



EQ.S6

### Exam Quiz 7 (Given 03/03)

---

Ulaby Exercise 2.11 (Slide 4.17)

**Question:** A  $140\ \Omega$  lossless line is terminated in a load impedance  $Z_L = (280 + j182)\ \Omega$ . If  $\lambda = 72\text{ cm}$ , find (a) the reflection coefficient  $\Gamma$ , (b) the VSWR  $S$ , (c) the locations of the voltage maxima and minima.



EQ.P7

### Exam Quiz 7 (Given 03/03)

---

**Solution:** (a)  $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (280 + j182 - 140) / (280 + j182 + 140) = 230 \exp(j0.915) / 458 \exp(j0.409) = 0.50 \exp(j0.51)$ .  
NOTE:  $0.51\text{ rads} = 29^\circ$ . (b)  $S = (1 + |\Gamma|) / (1 - |\Gamma|) = (1 + 0.502) / (1 - 0.502) = 3.0$ . (c)  $l_{\max} = (0.506 \times 72 / 4\pi + n \times 72 / 2)\text{ cm} = 2.9\text{ cm}, 39\text{ cm}, 75\text{ cm}, \dots$ ;  $l_{\min} = (0.506 \times 72 / 4\pi + 18 + n \times 72 / 2)\text{ cm} = 21\text{ cm}, 57\text{ cm}, 93\text{ cm}, \dots$



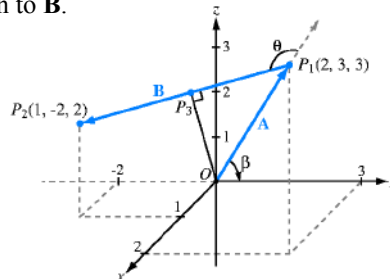
EQ.S7



## Exam Quiz 8 (Given 3/10/2015)

Vectors and angles: Ulaby et al. Example 3-1 (Slide 6.10)

**Question:** In Cartesian coordinates, vector **A** is directed from the origin to the point  $P_1(2, 3, 3)$ , and vector **B** is directed from  $P_1$  to  $P_2(1, -2, 2)$ . Find (a) the vector **A**, its magnitude  $A$ , and its unit vector  $\hat{\mathbf{a}}$ , (b) the angle that **A** makes with the  $y$ -axis, (c) vector **B**, (d) the angle between **A** and **B**, and (e) the perpendicular distance from the origin to **B**.



Ulaby et al., Figure 3-7

**UMBC**  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.P8

## Exam Quiz 8 (Given 3/10/2015)

**Solution:** (a)  $\mathbf{A} = \hat{\mathbf{x}} 2 + \hat{\mathbf{y}} 3 + \hat{\mathbf{z}} 3$

$$A = \sqrt{4 + 9 + 9} = \sqrt{22}$$

$$\hat{\mathbf{a}} = \mathbf{A} / A = (\hat{\mathbf{x}} 2 + \hat{\mathbf{y}} 3 + \hat{\mathbf{z}} 3) / \sqrt{22}$$

(b) The angle  $b$  between **A** and the  $y$ -axis may be found from the expression

$$\beta = \cos^{-1} \left( \frac{\mathbf{A} \cdot \hat{\mathbf{y}}}{A} \right) = \cos^{-1} \left( \frac{3}{\sqrt{22}} \right) = 0.879 \text{ rads} = 50.2^\circ$$

(c)  $\mathbf{B} = \hat{\mathbf{x}} (1 - 2) + \hat{\mathbf{y}} (-2 - 3) + \hat{\mathbf{z}} (2 - 3) = -\hat{\mathbf{x}} - \hat{\mathbf{y}} 5 - \hat{\mathbf{z}}$

$$(d) \quad \theta = \cos^{-1} \left( \frac{\mathbf{A} \cdot \mathbf{B}}{AB} \right) = \cos^{-1} \left( \frac{-20}{\sqrt{22}\sqrt{27}} \right) = 2.533 \text{ rads} = 145.1^\circ$$

(e) The points  $OP_1P_3$  form a right triangle.

The magnitude of the line segment

$$OP_3 \text{ is given by } A \sin(\pi - \theta) = \sqrt{22} \sin(0.609) = 2.68$$

**UMBC**  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.S8

### Exam Quiz 9 (Given 3/10/2015)

---

Charge in a sphere: Ulaby et al. Example 3-6 (Slide 6.26)

**Question:** A sphere of radius 2 cm contains a charge of density  $\rho_V$  given by

$$\rho_V = 4 \cos^2 \theta$$

What is the total charge?



EQ.P9

### Exam Quiz 9 (Given 3/10/2015)

---

**Solution:** After converting from cm to m,

$$\begin{aligned} Q &= \int_V \rho_V dv \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{R=0}^{2 \times 10^{-2}} (4 \cos^2 \theta) R^2 \sin \theta dR d\theta d\phi \\ &= 4 \int_0^{2\pi} \int_0^{\pi} \left( \frac{R^3}{3} \right) \Big|_0^{2 \times 10^{-2}} \sin \theta \cos^2 \theta d\theta d\phi \\ &= \frac{64}{9} \times 10^{-6} \int_0^{2\pi} d\phi = \frac{128\pi}{9} \times 10^{-6} = 44.68 \mu\text{C} \end{aligned}$$



EQ.S9

## Exam Quiz 10 (Given 3/24/2015)

Slides 6.17, 6.22, and 6.25

**Question:** Obtain the differential relations for the lengths, surface areas, and volumes in Cartesian, cylindrical, and spherical coordinates using pictures that illustrate the geometry.



EQ.P10

## Exam Quiz 10 (Given 3/24/2015)

**Solution: Cartesian coordinates**

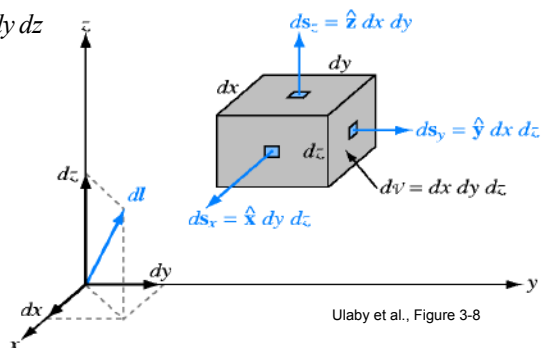
Length:  $d\mathbf{l} = \hat{\mathbf{x}} dl_x + \hat{\mathbf{y}} dl_y + \hat{\mathbf{z}} dl_z = \hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$

Surface area:  $ds_x = \hat{\mathbf{x}} dl_y dl_z = \hat{\mathbf{x}} dy dz$  (y-z plane)

$ds_y = \hat{\mathbf{y}} dz dx = \hat{\mathbf{y}} dx dz$  (x-z plane)

$ds_z = \hat{\mathbf{z}} dx dy$  (x-y plane)

Volume:  $dv = dx dy dz$



Ulaby et al., Figure 3-8

EQ.S10

## Exam Quiz 10 (Given 3/24/2015)

### Solution: Cylindrical coordinates

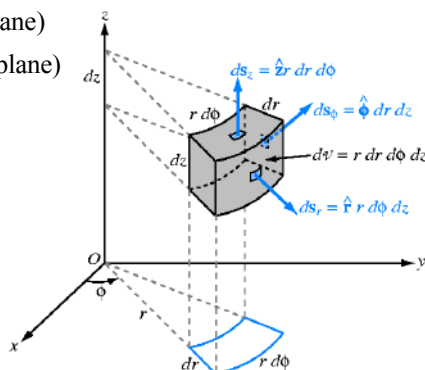
Length:  $d\mathbf{l} = \hat{\mathbf{r}} dl_r + \hat{\boldsymbol{\phi}} dl_\phi + \hat{\mathbf{z}} dl_z = \hat{\mathbf{r}} dr + \hat{\boldsymbol{\phi}} r d\phi + \hat{\mathbf{z}} dz$

Surface area:  $d\mathbf{s}_r = \hat{\mathbf{r}} r d\phi dz$  ( $\phi$ - $z$  plane)

$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}} dr dz$  ( $r$ - $z$  plane)

$d\mathbf{s}_z = \hat{\mathbf{z}} r dr d\phi$  ( $r$ - $\phi$  plane)

Volume:  $dV = r dr d\phi dz$



Ulaby et al., Figure 3-10

EQ.S10

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

## Exam Quiz 10 (Given 3/24/2015)

### Solution: Spherical coordinates

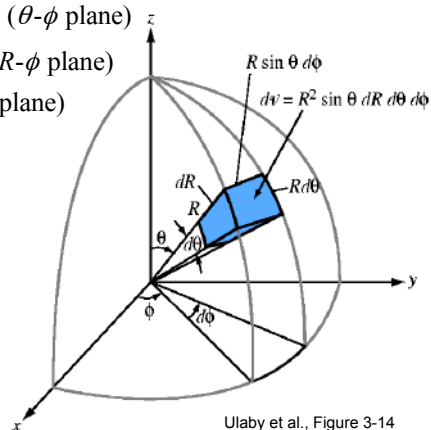
Length:  $d\mathbf{l} = \hat{\mathbf{R}} dl_R + \hat{\boldsymbol{\theta}} dl_\theta + \hat{\boldsymbol{\phi}} dl_\phi = \hat{\mathbf{R}} dR + \hat{\boldsymbol{\theta}} R d\theta + \hat{\boldsymbol{\phi}} R \sin \theta d\phi$

Surface area:  $d\mathbf{s}_R = \hat{\mathbf{R}} R^2 \sin \theta d\theta d\phi$  ( $\theta$ - $\phi$  plane)

$d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}} R \sin \theta dR d\phi$  ( $R$ - $\phi$  plane)

$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}} R dR d\theta$  ( $R$ - $\theta$  plane)

Volume:  $dV = R^2 \sin \theta dR d\theta d\phi$



Ulaby et al., Figure 3-14

EQ.S10

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

### Exam Quiz 11 (Given 3/24/2015)

Slides 7.26, 7.27, 7.28, and 7.29

**Question:** Show using an appropriate picture and symmetry that the general expression for the curl in an orthogonal coordinate system may be written

$$\nabla \times \mathbf{A} = \hat{\mathbf{x}}_1 \left[ \frac{1}{h_2 h_3} \left( \frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right) \right] + \hat{\mathbf{x}}_2 \left[ \frac{1}{h_1 h_3} \left( \frac{\partial}{\partial x_3} h_1 A_1 - \frac{\partial}{\partial x_1} h_3 A_3 \right) \right] \\ + \hat{\mathbf{x}}_3 \left[ \frac{1}{h_1 h_2} \left( \frac{\partial}{\partial x_1} h_2 A_2 - \frac{\partial}{\partial x_2} h_1 A_1 \right) \right]$$

where  $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3$  are unit vectors in the 1- 2- and 3-directions and  $h_1, h_2, h_3$  are the corresponding differential lengths. Use this result to obtain the expression in cylindrical coordinates.

**UMBC**  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.P11

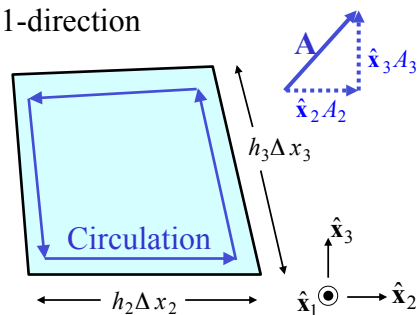
### Exam Quiz 11 (Given 3/24/2015)

**Solution:** The circulation in the 1-direction may be written

$$\Delta C_1 = \Delta_2 [A_3(h_3 \Delta x_3)] - \Delta_3 [A_2(h_2 \Delta x_2)] \\ \approx \Delta x_2 \Delta x_3 \left( \frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right)$$

so that

$$\lim_{\Delta s_1 \rightarrow 0} \frac{\Delta C_1}{\Delta s_1} = \frac{1}{h_2 h_3} \left( \frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right)$$



**UMBC**  
AN HONORS  
UNIVERSITY  
IN MARYLAND

The other directions may be obtained by using the  $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$  symmetry.

EQ.S11

### Exam Quiz 11 (Given 3/24/2015)

---

In cylindrical coordinates:  $h_1 = 1$ ,  $h_2 = r$ ,  $h_3 = 1$

After substitution into the general expression, we find

$$\nabla \times \mathbf{A} = \hat{\mathbf{r}} \left[ \frac{1}{r} \left( \frac{\partial}{\partial \phi} A_z - \frac{\partial}{\partial z} r A_\phi \right) \right] + \hat{\boldsymbol{\phi}} \left( \frac{\partial}{\partial z} A_r - \frac{\partial}{\partial r} A_z \right) \\ + \hat{\mathbf{z}} \left[ \frac{1}{r} \left( \frac{\partial}{\partial r} r A_\phi - \frac{\partial}{\partial \phi} A_r \right) \right]$$

### Exam Quiz 12 (Given 3/31/2015)

---

Electric Field due to Two Point Charges: Ulaby et al., Ex. 4-3, Slides 8.16, 8.17

**Question:** Two point charges with  $q_1 = 2 \times 10^{-5}$  C and  $q_2 = -4 \times 10^{-5}$  C are located at (1, 3, -1) and (-3, 1, -2), respectively, in a Cartesian coordinate system. Find (a) the electric field  $\mathbf{E}$  at (3, 1, -2) and (b) the force on a charge  $q_3 = 8 \times 10^{-5}$  C located at that point

### Exam Quiz 12 (Given 3/31/2015)

**Solution:** a) Since  $\epsilon = \epsilon_0$  and there are two charges, we have

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1(\mathbf{R} - \mathbf{R}_1)}{|\mathbf{R} - \mathbf{R}_1|^3} + \frac{q_2(\mathbf{R} - \mathbf{R}_2)}{|\mathbf{R} - \mathbf{R}_2|^3} \right]$$

with

$$\mathbf{R}_1 = \hat{x} + \hat{y}3 - \hat{z}$$

$$\mathbf{R}_2 = -\hat{x}3 + \hat{y} - \hat{z}2$$

$$\mathbf{R} = \hat{x}3 + \hat{y} - \hat{z}2$$

After substitution, we find

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \left[ \frac{2(\hat{x}2 - \hat{y}2 - \hat{z})}{27} - \frac{4(\hat{x}6)}{216} \right] \times 10^{-5} = \frac{\hat{x} - \hat{y}4 - \hat{z}2}{108\pi\epsilon_0} \times 10^{-5} \text{ V/m}$$



(b) Using the force equation, we have

$$\mathbf{F} = q_3\mathbf{E} = 8 \times 10^{-5} \times \frac{\hat{x} - \hat{y}4 - \hat{z}2}{108\pi\epsilon_0} \times 10^{-5} = \frac{\hat{x}2 - \hat{y}8 - \hat{z}4}{27\pi\epsilon_0} \times 10^{-10} \text{ N}$$

EQ.S12

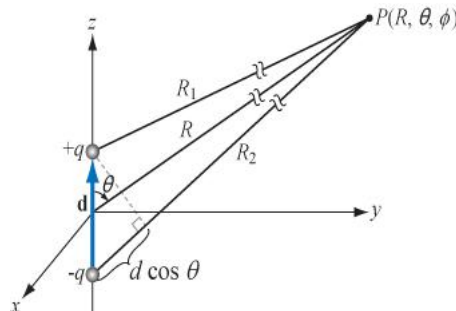
### Exam Quiz 13 (Given 4/07/2013)

Electric Dipole: Ulaby et al., Ex. 4-7, Slides 9.12–9.14

**Question:** Suppose you have two charges of magnitude  $q$  and opposite sign, as shown below. Show that when  $R \gg d$

$$V = \frac{qd \cos \theta}{4\pi\epsilon R^2}$$

Find the electric field.



EQ.P13

### Exam Quiz 13 (Given 4/07/2015)

**Solution:** The voltage may be written as

$$V = \frac{1}{4\pi\epsilon} \left( \frac{q}{R_1} + \frac{-q}{R_2} \right)$$

From the figure, we find

$$\begin{aligned} \frac{1}{R_1} &= \left[ \left( R - \frac{d}{2} \cos \theta \right)^2 + \left( \frac{d}{2} \sin \theta \right)^2 \right]^{-1/2} \\ &= \frac{1}{R} \left( 1 - \frac{d}{R} \cos \theta + \frac{d^2}{2R^2} \right)^{-1/2} ; \quad \frac{1}{R} \left( 1 + \frac{d}{2R} \cos \theta \right) \end{aligned}$$

and similarly

$$\frac{1}{R_2} ; \quad \frac{1}{R} \left( 1 - \frac{d}{2R} \cos \theta \right)$$

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.S13

### Exam Quiz 13 (Given 4/07/2015)

**Solution (continued):** So, after substitution in the expression for  $V$ ,

$$V = \frac{qd \cos \theta}{4\pi\epsilon R^2}$$

To obtain  $E$ , we use the expression for the gradient in spherical coordinates, when the  $\phi$ -variation can be ignored

$$\begin{aligned} \mathbf{E} &= -\nabla V = -\hat{\mathbf{R}} \frac{\partial V}{\partial R} - \hat{\boldsymbol{\theta}} \frac{1}{R} \frac{\partial V}{\partial \theta} \\ &= \frac{qd}{4\pi\epsilon R^3} \left( \hat{\mathbf{R}} 2 \cos \theta + \hat{\boldsymbol{\theta}} \sin \theta \right) \end{aligned}$$

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

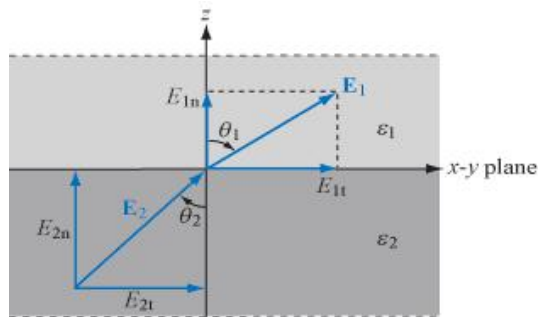
EQ.S13



### Exam Quiz 14 (Given 4/07/2015)

Boundary conditions and application: Ulaby et al., Ex. 4-10, Slides 9.23–9.26

**Question:** Use the two laws of electrostatics to derive the boundary conditions at the interface of two dielectric media with dielectric constants  $\epsilon_1$  and  $\epsilon_2$  and no surface charge. For the geometry shown below, find the relationship between  $\theta_1$  and  $\theta_2$ .



UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.P14

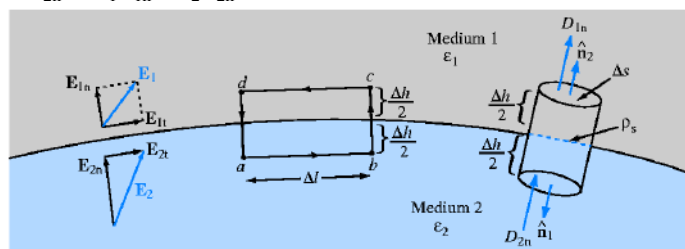
### Exam Quiz 14 (Given 4/07/2015)

**Solution:** To derive the tangential boundary condition, we use the relation  $\nabla \times \mathbf{E} = 0$  or  $\oint \mathbf{E} \cdot d\mathbf{l} = 0$  around a closed path

Using the geometry in the middle of the figure below and letting  $\Delta h \rightarrow 0$ , we infer  $E_{1t} - E_{2t} = 0$  or  $E_{1t} = E_{2t}$ . To derive the normal boundary condition, we use the relation

$$\nabla \cdot \mathbf{D} = 0 \quad \text{or} \quad \int_S \mathbf{D} \cdot \hat{\mathbf{n}} ds = 0 \quad \text{over a closed surface}$$

Using the pillbox shape shown to the right and letting  $\Delta h \rightarrow 0$ , we infer  $D_{1n} = D_{2n}$  or  $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$ .



UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND

EQ.S14

### Exam Quiz 14 (Given 4/07/2015)

---

**Solution (continued):** Since  $E_{1t} = E_{2t}$  and  $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$ , we conclude

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{E_{1t}}{E_{1n}} \cdot \frac{E_{2n}}{E_{2t}} = \frac{E_{1t} / E_{2t}}{E_{1n} / E_{2n}} = \frac{\epsilon_1}{\epsilon_2}$$

### Exam Quiz 15 (Given 4/14/2015)

---

Coaxial cable transmission line parameters: Ulaby et al., Ex. 4-9, Slides 9.18–9.19, 9.31, and 11.29

**Question:** Find the conductance per unit length and capacitance per unit length for a coaxial cable

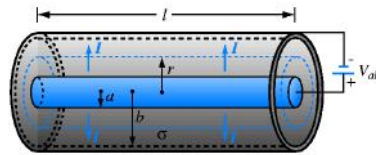
### Exam Quiz 15 (Given 4/14/2015)

**Solution:** We first determine the conductance per unit length. Let  $I$  be the current that flows from the inner conductor to the outer conductor. At any distance  $r$ , the area through which the current flows is  $A = 2\pi r l$ . We now have,

$$\mathbf{J} = \hat{\mathbf{r}} \frac{I}{A} = \hat{\mathbf{r}} \frac{I}{2\pi r l} \quad \text{and} \quad \mathbf{E} = \hat{\mathbf{r}} \frac{I}{2\pi \sigma r l}$$

from which we conclude

$$G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab} l} = \frac{2\pi \sigma}{\ln(b/a)}$$



To find the capacitance and inductance per unit length, we use the relations

$$C' = \frac{\epsilon}{\sigma} G' = \frac{2\pi \epsilon}{\ln(b/a)}, \quad L' = \frac{\epsilon \mu}{C'} = \frac{\mu}{2\pi} \ln(b/a)$$