$(\mathbb{Z}_6, +)$

+	0	1	2	3	4	5
0	0	1	2			
1	1	2	3			
2	2	3	4			
3	3	4	5			
4	4	5		1		
5	5	·	1			

Commutative group table is symmetric

Subgroups of Groups

If $(G, *)$ is a group and $H \subseteq G$ then H is a subgroup iff $(H, *)$ is a group.

First $*$ must be

Let e's

$$e_H e = e_H$$
$$e_H e_H = e_H \Rightarrow e = e_H$$

$$h_H^{-1} h = e = h^{-1} h$$
$$h_H^{-1} = h^{-1}$$

<u>Th</u> $H \subseteq G$ is a subgroup of G iff

i) (Closure) $*$ is binary operation on H

ii) $e \in H$

iii) $h \in H$ implies $h^{-1} \in H$

<u>Th</u> H is a subgroup iff for any a, b in H $ab^{-1} \in H$ $[H \neq \phi]$

<u>Pf</u> Choose $a \in H$

ii) $e = aa^{-1} \in H \Rightarrow$ ii)

$a^{-1} = ea^{-1} \in H$

i) $ab = a(b^{-1})^{-1} \in H$

Th $H \subseteq (\mathbb{Z},+)$ is a subgroup iff
$H = d\mathbb{Z}$ for some $d \in \mathbb{Z}^+$

Proof $a-b \in H$ for any $a,b \in H$

$d=0$ $d\mathbb{Z} = \{0\}$ Trivial subgroup

(proper, nontrivial) $d=1$ $d\mathbb{Z} = \mathbb{Z}$ Improper subgroup

Subgroups of S_3

$\{(1),(1,2,3),(1,3,2)\} = A_3$

$\{(1),(1,2)\}, \{(1),(1,3)\}, \{(1),(2,3)\}$

$(1,2)\circ(1,3) = (1,3,2)$ $\begin{pmatrix} a & b & c \\ a & c & b \end{pmatrix}$

$a, b\, a$

$(a,b)(a,c)$

A_n = even permutations subgroup of S_n

$\pi_1 = \tau_1 \cdots \tau_{2l}$ $\pi_2 = \tau_1' \cdots \tau_{2m}'$

$\pi_1 \pi_2^{-1} = \underbrace{(\tau_1 \cdots \tau_{2l})(\tau_{2m}' \cdots \tau_1')}_{\text{even \# permutations}}$