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MATH 407: HW 08

3.4 **4** Show that $\mathbb{Z}_5^\times$ is not isomorphic to $\mathbb{Z}_8^\times$ by showing that the first group has an element of order 4 but the second group does not

The elements in each of the groups

$$\{[1],[2],[3],[4]\} \in \mathbb{Z}_5^\times$$
$$\{[1],[3],[5],[7]\} \in \mathbb{Z}_8^\times$$

For $\mathbb{Z}_5^\times$

$$[5]^2 = [1]$$

Therefore, $o()$

□

7 Let G be a group. Show that the group $(G, *)$ defined in Exercise 3 of Section 3. 1 is isomorphic to G.

□

11 Let G be the set of all matrices in $GL_2(\mathbb{Z}_3)$ of the form $\begin{bmatrix} \text{m} & \text{b} \\ 0 & 1 \end{bmatrix}$. That is, $m, b \in \mathbb{Z}_3$ and $m \neq [0]_3$. Show that G is a subgroup of $GL_2(\mathbb{Z}_3)$ that is isomorphic to S_3.

□

17 Let $\phi : G_1 \to G_2$ be a group isomorphism. Prove that if H is asubgroup of G_1, then $\phi(H) = \{y \in G_2 \mid y = \phi(h) \text{ for some } h \in H\}$ is a subgroup of G_2.

Since $\phi : G_1 \to G_2$ is a group isomorphism, $\phi(e_1) = e_2$
Since H is a subgroup,

$$e_1 \in H$$
$$\Rightarrow e_2 \in \phi(H)$$

A non-empty set G is a subgroup if $xy^{-1} \in G$, $\forall\, x, y \in G$

Let $x, y \in \phi(H)$

Then, there exists $h_1, h_2 \in H$, such that

$$\phi(h_1) = x$$
$$\phi(h_2) = y$$

Also, since ϕ is homomorphic,

$$\begin{aligned}
\phi(h_2^{-1}) &= (\phi(h_2))^{-1} \\
&= y^{-1} \\
\phi(h_1 h_2^{-1}) &= \phi(h_1)\phi(h_2^{-1}) \\
&= xy^{-1}
\end{aligned}$$

Since H is a subgroup, $h_1 h_2^{-1} \in H$, $\forall\, h_1, h_2 \in H$

Therefore,

$$\begin{aligned}
\phi(h_1 h_2^{-1}) &= xy^{-1} \\
&\in \phi(H)
\end{aligned}$$

That is, $\phi(h_1 h_2^{-1}) \in \phi(H)$, $\forall\, x, y \in \phi(H)$ $\square$