Units, Magnitude, and Notation

We use SI (Système International) Units:

You must know how to convert mi, in, ft, yd → mm, m, km, etc.!!

Many stupid mistakes have been made by getting this wrong!

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	k
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Substance amount	mole	mol

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

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2.1

Waves

In a lossless medium, one may write the current y as

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$

This is the general form for any kind of wave, including water waves (Ulaby's example)

A = wave amplitude
T = time period
λ = spatial wavelength
ϕ_0 = reference phase

There are 4 basic wave quantities in a lossless medium

We may define the total phase by writing

$$y(x,t) = A\cos\phi(x,t), \quad \text{where} \quad \phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$$

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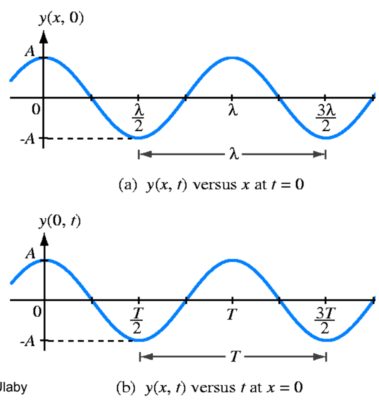
2.2

Waves



Ulaby
Figure 1-11

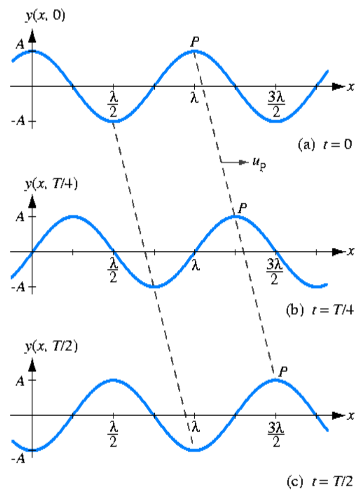


Figure 1-12

When $\phi_0 = 0$

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- The phase changes by 2π in one period T or one wavelength λ.
- In Fig. 1-12, the wave moves in the $+x$ direction with a velocity $u_p = \lambda/T$.

2.3

Waves

Phase velocity u_p:

As time t increases, position x must increase to keep the phase

$$\phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$$

constant. Since ϕ is constant, we have

$$0 = \frac{2\pi}{T} dt - \frac{2\pi}{\lambda} dx.$$

So, a point of constant phase, the phase front, moves with velocity

$$u_p = \frac{dx}{dt} = \frac{\lambda}{T} \quad \text{(m/s)}.$$

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2.4

Waves

Other quantities (derived from the fundamental 4):

$f =$ frequency $= 1/T$ (Hz = s^{-1})
$\omega =$ angular frequency $= 2\pi f$ (rad/s = s^{-1})
$\beta =$ wavenumber $= 2\pi/\lambda$ (rad/m = m^{-1})

In terms of these quantities, we have

$$y(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x + \phi_0\right) = A\cos\left(\omega t - \beta x + \phi_0\right).$$

and

$$u_p = f\lambda = \omega/\beta.$$

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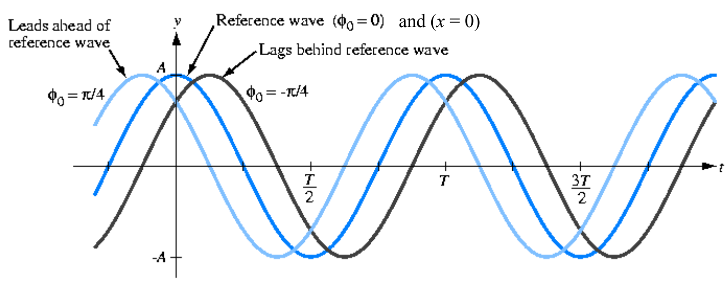
Waves

The phase offset:



Ulaby
Figure 1-13

Another useful representation:

Letting $R = A\cos\phi_0$, $I = A\sin\phi_0$

$$y(x,t) = A\cos\left(\omega t - \beta x + \phi_0\right) = R\cos\left(\omega t - \beta x\right) - I\sin\left(\omega t - \beta x\right).$$

This representation will be useful when we discuss phasors

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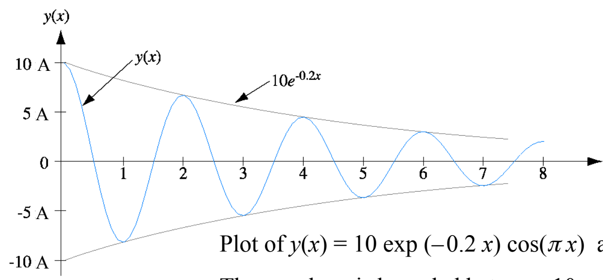
Waves

With Loss: $y(x,t) = A\exp(-\alpha x)\cos\left(\omega t - \beta x + \phi_0\right)$

α = attenuation coefficient



Plot of $y(x)$ = 10 exp (–0.2 x) cos(πx) amperes

The envelope is bounded between 10 exp (–0.2 x) and its mirror image –10 exp (–0.2 x)

Ulaby (modified)
Figure 1-14

With loss, there are 5 fundamental quantities

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2.7

Waves

With Loss: Ulaby Example 1-2

Question: A laser beam propagating through the atmosphere is characterized by an electric field intensity given by

$$E(x,t) = 150\exp(-0.03x)\cos\left(3\times10^{15}t - 10^{7}x\right) \quad \text{(V/m)}$$

where x is the distance from the source in meters. Determine (a) the direction of wave travel, (b) the wave velocity, and (c) the wave amplitude at a distance of 200 m

Solution: (a) Since the coefficients of t and x have the opposite sign, the wave propagates in the $+x$ direction.

(b) We find that

$$u_\text{p} = \frac{\omega}{\beta} = \frac{3\times10^{15}\,\text{s}^{-1}}{10^{7}\,\text{m}^{-1}} = 3\times10^{8}\,\text{m/s},$$

which is (of course) the speed of light c in the vacuum or air.

(c) At x = 200 m, the amplitude of $E(x,t)$ is

$$E(x,t) = 150\exp(-0.03\,\text{m}^{-1}\times 200\,\text{m}) = 0.37\ \text{V/m}$$

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2.8

Dispersion Relations

Dispersion relations: $\beta(\omega)$ and $\alpha(\omega)$ are functions of $f = \omega/2\pi$

Calculating the dispersion relations is an important part of EM theory!

In a homogeneous, isotropic medium, this is straightforward

- homogeneous = the same at all points in space
- isotropic = the same in all orientations (no strains; no crystal structure)

We calculate $\beta(\omega)$ and $\alpha(\omega)$ from $\varepsilon(\omega)$ and $\mu(\omega)$

- **We will do this when we discuss plane waves**

In an inhomogeneous, isotropic medium, we must account for geometry

$\varepsilon(\omega) \to \varepsilon(\omega,\mathbf{r})$ and $\mu(\omega) \to \mu(\omega,\mathbf{r})$

- The dispersion relations are determined by geometry as well as frequency
- There can be multiple solutions at one frequency
- **We will do this for simple geometries**

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As a consequence:

The 5 fundamental quantities → 3 independent quantities

2.9

Dispersion Relations

Dispersion relations: $\beta(\omega)$ and $\alpha(\omega)$ are functions of $f = \omega/2\pi$

Calculating the dispersion relations is an important part of EM theory!

In an anisotropic medium, this becomes complex

$\mathbf{D}(\mathbf{r},\omega) = \varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) \to \mathbf{D}(\mathbf{r},\omega) = \mathsf{E}(\mathbf{r},\omega)\bullet\mathbf{E}(\mathbf{r},\omega)$;

$\mathbf{B}(\mathbf{r},\omega) = \mu(\mathbf{r},\omega)\mathbf{H}(\mathbf{r},\omega) \to \mathbf{B}(\mathbf{r},\omega) = \mathsf{M}(\mathbf{r},\omega)\bullet\mathbf{H}(\mathbf{r},\omega)$

where $\mathsf{E}(\mathbf{r},\omega)$ and $\mathsf{M}(\mathbf{r},\omega)$ are 3×3 matrices*

- **We will not discuss anisotropic media in this course**

— This discussion assumes that the medium is *linear*
(Waves at different frequencies do not interact)

— All media become linear when the wave amplitudes A are small enough

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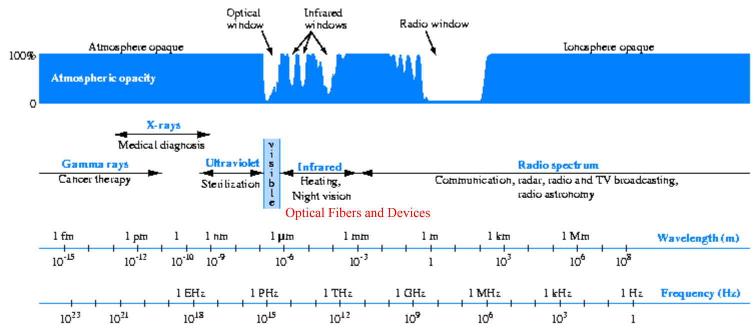
*Strictly speaking, these are second-order tensors

2.10

Electromagnetic Spectrum



Ulaby (modified)
Figure 1-15

Optical fibers and devices typically operate in the range 0.8 μm to 1.8 μm with 1.5 μm corresponding to minimum loss.

Note that the minimum loss band in atmospere is at visible light!

Thus, it is sensible that this is the band of electromagnetic radiation that our eyes see!

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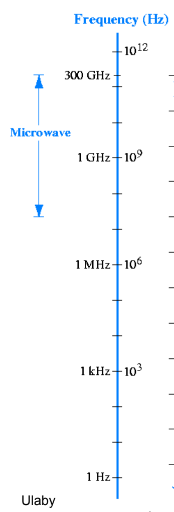
Electromagnetic Spectrum



Band	Applications
Extremely High Frequency EHF (30 - 300 GHz)	Radar, advanced communication systems, remote sensing, radio astronomy
Super High Frequency SHF (3 - 30 GHz)	Radar, satellite communication systems, aircraft navigation, radio astronomy, remote sensing
Ultra High Frequency UHF (300 MHz - 3 GHz)	TV broadcasting, radar, radio astronomy, microwave ovens, cellular telephone
Very High Frequency VHF (30 - 300 MHz)	TV and FM broadcasting, mobile radio communication, air traffic control
High Frequency HF (3 - 30 MHz)	Short wave broadcasting
Medium Frequency MF (300 kHz - 3 MHz)	AM broadcasting
Low Frequency LF (30 - 300 kHz)	Radio beacons, weather broadcast stations for air navigation
Very Low Frequency VLF (3 - 30 kHz)	Navigation and position location
Ultra Low Frequency ULF (300 Hz - 3 kHz)	Audio signals on telephone
Super Low Frequency SLF (30 - 300 Hz)	Ionospheric sensing, electric power distribution, submarine communication
Extremely Low Frequency ELF (3 - 30 Hz)	Detection of buried metal objects
$f < 3$ Hz	Magnetotelluric sensing of the earth's structure

Ulaby
Figure 1-16

The wavelength is tied to the frequency through the dispersion relations.

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Waves

Birefringence:

Electromagnetic waves are transverse waves that have two polarizations

That is why polarizing filters work!

In many crystals, glasses, solids—including optical fibers (used in communication systems)—the two polarizations have slightly different dispersion relations and move with different velocities.

The result is a 2π phase shift over long distances!

Example: The ionosphere is birefringent at radio frequencies of 1 MHz. The radio waves propagate at close to the speed of light in a vacuum, but the wavenumber of the two polarization (left- and right-circularly polarized differ by 0.1%. Over what distance do the relative phases shift by 2π?

Answer: We have

$$\frac{u_{p2}-u_{p1}}{u_{p1}} = \frac{\beta_1}{\omega}\left(\frac{\omega}{\beta_2}-\frac{\omega}{\beta_1}\right) = -\left(\frac{\beta_2-\beta_1}{\beta_2}\right)$$

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2.13

Waves

Example (continued): The ionosphere is birefringent at radio frequencies of 1 MHz. The radio waves propagate at close to the speed of light in a vacuum, but the wavenumber of the two polarization (left- and right-circularly polarized differ by 0.1%. Over what distance do the relative phases shift by 2π?

Answer: We have

$$\frac{u_{p2}-u_{p1}}{u_{p1}} = \frac{\beta_1}{\omega}\left(\frac{\omega}{\beta_2}-\frac{\omega}{\beta_1}\right) = -\left(\frac{\beta_2-\beta_1}{\beta_2}\right)$$

Since the difference between the two velocities $\Delta u_p = 10^{-3} u_p$ in magnitude, where the difference between u_{p1} and u_{p2} can be neglected. We may similarly write $\Delta\beta = 10^{-3}\beta$. It follows that $\Delta\phi = (\Delta\beta)z = 2\pi$ when

$$z = \frac{2\pi}{\Delta\beta} = \frac{\lambda}{10^{-3}} = 1000\frac{c}{f} = \frac{(10^3)\times(3\times10^8)}{10^6} = 3\times10^5 \text{ m} = 300 \text{ km}$$

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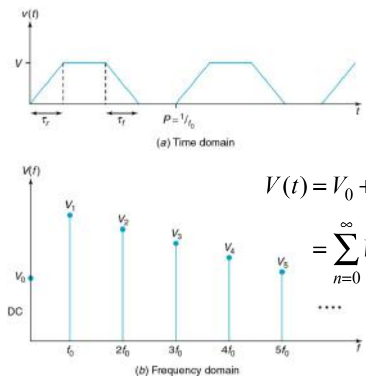
2.14

Spectral Analysis

Which frequencies are present?

Following Paul, we consider a digital signal with a rise and fall time that is short compared to the base signal and generates high frequency components. The highest frequency is given approximately by $1/t_{max}$.

Paul
Fig. 1-3 a/b



With a periodically repeating signal, the only frequencies that appear are multiples of the baseband $f_0 = 1/T$, where T is the period.

$$V(t) = V_0 + V_1\cos(\omega_0 t + \phi_1) + V_2\cos(2\omega_0 t + \phi_2) + \cdots$$

$$= \sum_{n=0}^{\infty} V_n \cos(\omega_n t + \phi_n) \quad \text{with } V_n = V(\omega_n)$$

....

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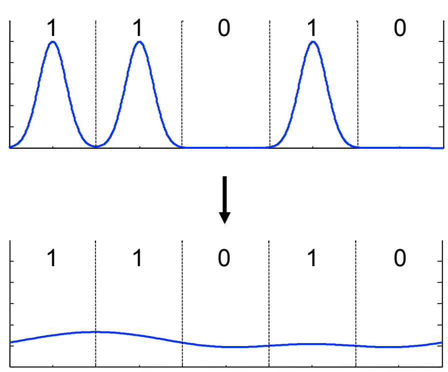
2.15

Spectral Analysis

Why are high harmonics / large frequency spreads bad?

(1) The high harmonics lead to undesired electronic coupling in digital systems.

(2) The high harmonics lead to pulse spreading in communications systems because different frequencies have different values of u_p.



Chromatic Dispersion:

After spreading a factor of 5 in this return-to-zero example, the digital 1's cannot be distinguished from the digital 0's

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2.16

Complex Numbers and Phasors

A ***complex number*** z is written: $z = x + jy$, where $j = \sqrt{-1}$

We also write $x = \text{Re}(z)$, $y = \text{Im}(z)$

In ***polar form***, we have: $z = |z|\exp(j\theta) = |z|e^{j\theta} = |z|\angle\theta$,

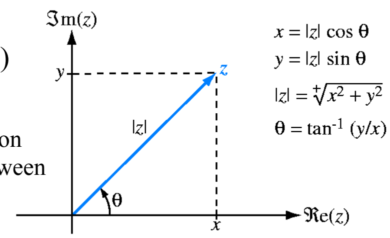
where $|z|$ is the magnitude and θ is the phase.

From ***Euler's identity***, $\exp(j\theta) = \cos\theta + j\sin\theta$, we find

$x = |z|\cos\theta$, $\quad y = |z|\sin\theta$,

$|z| = \sqrt{x^2 + y^2}$, $\quad \theta = \tan^{-1}(y/x)$

Graphical representation of the relationship between rectangular and polar coordinates



Ulaby
Figure 1-17

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2.17

Complex Numbers and Phasors

The ***complex conjugate*** z^* is defined:

$z^* = x - jy = |z|\exp(-j\theta)$, so that $|z| = \sqrt{z z^*}$

Mathematical operations:

- Equality: $z_1 = z_2 \quad \leftrightarrow \quad x_1 = x_2$ and $y_1 = y_2$
- Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
- Multiplication: $z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$
  $= |z_1||z_2|\exp\left[j(\theta_1 + \theta_2)\right]$
- Division: $\dfrac{z_1}{z_2} = \dfrac{(x_1 x_2 + y_1 y_2) + j(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2} = \dfrac{|z_1|}{|z_2|}\exp\left[j(\theta_1 - \theta_2)\right]$
- Powers: $z^r = |z|^r \exp(jr\theta)$, where r is any real number

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2.18

Complex Numbers and Phasors

The ***complex conjugate*** z^* is defined:

$$z^* = x - jy = |z|\exp(-j\theta), \quad \text{so that} \quad |z| = \sqrt{zz^*}$$

Mathematical operations:

- Logarithm: $\log z = \log|z| + j(\theta + 2\pi n)$, where n is any integer

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Complex Numbers and Phasors

Working with phasors: Ulaby and Ravaioli Example 1-3

Question: Given two complex numbers, $V = 3 - j4$ and $I = -2 - j3$, (a) Express V and I in polar form, and find (b) VI, (c) VI^*, (d) V/I, (e) $I^{1/2}$

Answer: (a) V is in the fourth quadrant and I is in the third quadrant. (See figure.)

$$|V| = \sqrt{3^2 + 4^2} = 5, \quad \theta_V = \tan^{-1}(-4/3) = -0.972, \quad \text{so that} \quad V = 5\exp(-j0.972)$$

$$|I| = \sqrt{2^2 + 3^2} = 3.61, \quad \theta_I = \tan^{-1}(3/2) - \pi = -2.159, \quad \text{so that} \quad I = 3.61\exp(-j2.159)$$

(b) $VI = 5e^{-j0.972} \times 3.61e^{-j2.159} = 18.05e^{-j3.131}$

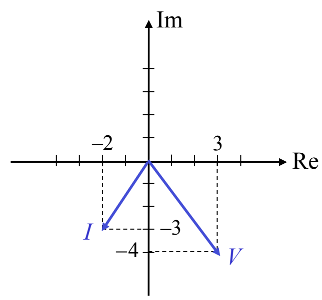
(c) $VI^* = 5e^{-j0.972} \times 3.61e^{j2.159} = 18.05e^{j1.187}$

(d) $V/I = (5/3.61)e^{-j0.972 + j2.159} = 1.39e^{j1.187}$

(e) $\sqrt{I} = \pm(3.61)^{1/2}e^{-j(2.159/2)} = \pm 1.90e^{-j1.080}$

Note: $-1.90e^{-j1.080} = 1.90e^{j2.062}$

Angles are only unique to within 2π



Based on Ulaby Fig. 1-18

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Complex Numbers and Phasors

Why do we work with complex numbers?

The linear integro-differential equations that describe circuits and electromagnetic waves — and in fact any waves — become much easier to solve!

The concept of a **phasor** plays a key role

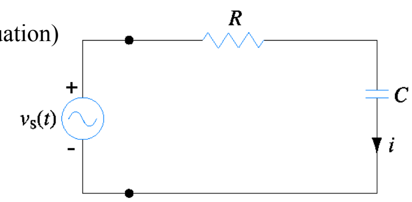
Consider a simple RC circuit with a voltage source $v_s(t) = V_0 \sin(\omega t + \phi_0)$

Kirchoff's voltage law implies

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = v_s(t)$$

(time domain equation)

- Direct time domain solution is a bit messy and involves "guessing" the integral.
- The phasor approach is simpler and deductive



Ulaby Fig. 1-19

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2.21

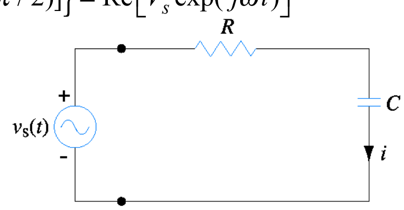
Complex Numbers and Phasors

Step 1: Adopt a cosine reference

$$v_s(t) = V_0 \sin(\omega t + \phi_0) = V_0 \cos(\omega t + \phi_0 - \pi/2)$$

Step 2: Express time-dependent variables as phasors

- In general, we write any time-dependent variable $z(t)$ as
  $z(t) = \mathrm{Re}\left[\tilde{Z}\exp(j\omega t)\right]$ where $\tilde{Z}$ is time-independent and is referred to as a ***phasor***
- In our particular case, we write
  $v_s(t) = \mathrm{Re}\left\{V_0 \exp[j(\omega t + \phi_0 - \pi/2)]\right\} = \mathrm{Re}\left[\tilde{V}_s \exp(j\omega t)\right]$
  where
  $\tilde{V}_s = V_0 \exp[j(\phi_0 - \pi/2)]$



Ulaby Fig. 1-19

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2.22

Complex Numbers and Phasors

Step 2: Express time-dependent variables as phasors (continued)

- We now write $i(t) = \mathrm{Re}\left[\tilde{I}\exp(j\omega t)\right]$, where $i(t)$ and $\tilde{I}$ are unknown

The goal is to solve for $\tilde{I}$, knowing $\tilde{V}_s$, which will allow us to find $i(t)$

- We will make use of two important properties

$$\frac{di(t)}{dt} = \mathrm{Re}\left[j\omega\tilde{I}\exp(j\omega t)\right] \quad \text{and} \quad \int i(t)\,dt = \mathrm{Re}\left[\frac{1}{j\omega}\tilde{I}\exp(j\omega t)\right]$$

Step 3: Recast the equation in phasor form

$$R\,\mathrm{Re}\left[\tilde{I}\exp(j\omega t)\right] + \frac{1}{C}\mathrm{Re}\left[\frac{1}{j\omega}\tilde{I}\exp(j\omega t)\right] = \mathrm{Re}\left[\tilde{V}_s\exp(j\omega t)\right]$$

This equation holds at all points in time if and only if

$$\left(R + \frac{1}{j\omega C}\right)\tilde{I} = \tilde{V}_s \qquad \text{(phasor/frequency/Fourier domain)}$$

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2.23

Complex Numbers and Phasors

Step 4: Solve the phasor equation

$$\tilde{I} = \frac{\tilde{V}_s}{R + 1/(j\omega C)} = V_0\exp[j(\phi_0 - \pi/2)]\left[\frac{j\omega C}{1 + j\omega CR}\right]$$

$$= \frac{V_0\,\omega C}{\left(1 + \omega^2 R^2 C^2\right)^{1/2}}\exp[j(\phi_0 - \phi_1)], \quad \text{where} \quad \phi_1 = \tan^{-1}(\omega RC)$$

Step 5: Solve the time domain equation

$$i(t) = \mathrm{Re}\left[\tilde{I}\exp(j\omega t)\right] = \frac{V_0\,\omega C}{\left(1 + \omega^2 R^2 C^2\right)^{1/2}}\cos(\omega t + \phi_0 - \phi_1)$$

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2.24

Assignment

Reading: Ulaby and Ravaioli, Chapter 2

Problem Set 1: Some notes

- There are 8 problems. Many of the answers to these problems have been provided by either Ulaby or by me. YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- A key issue in numerical calculation is not presenting more significant figures than you have. You cannot have more significant figures than are in your input data. Please watch that; Ulaby is not careful about it.
- Generally, I ask for 3 significant figures, which means that you want to calculate with at least 4. When I want a different number, I tell you. Sometimes, I ask you *why* I want more.

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CAVEAT: MATLAB keeps 14–15 digits. That can seem infinite, but it isn't!

2.25