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**MATH 407** HW 02

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**A.1** Let A, B, C be subsets of a given set S. Prove the following statements.

**10** $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

**11** $(A \cup B) \times C = (A \times C) \cup (B \times C)$

**A.4** **9** Let $a_1, \ldots, a_n$ be positive real numbers, $G_n$ = $\sqrt[n]{a_1 a_1 \ldots a_n}$, and $A_n$ = $\frac{1}{n}\sum_{i=1}^{n} a_i$. Then $G_n$ is called the gemoteric mean and $A_n$ is called the arithmetic mean. We wish to show that $G_n \leq A_n$.

1. Show that $G_2 \leq A_2$.
2. Show that $G_{2^n} \leq A_{2^n}$ by using induction on $n$.
3. Show that $G_n \leq A_n$.

*Hint:* Let $m$ be such that $2^m \geq n$, and set $a_{n+1}$ = $a_{n+2}$ = $a_{2^m}$ = $A_n$ and apply part (2).

**10** Let $a$ and $b$ be real numbers. Prove the binomial theorem , which states that

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i} \quad \text{where} \quad \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

and $n!$ = $n(n-1)\ldots 2 \cdot 1$ for $n \geq 1$ and $0! = 1$.

*Hint:* $\binom{m+1}{k} = \binom{m}{k} + \binom{m}{k-1}$.

**11** Find a formula for the derivative of the product of $n$ functions, and give a detailed proof by induction (assuming the product rule for the derivative of two functions).

**12** Find a formula for the $n$th derivative of the product of two functions, and give a detailed proof by induction.