Math-Phys Quiz 1 Questions: Given 02/07/2017

1. Determine the wavelength that corresponds to the frequency $f = 75$ Hz. Determine the frequency that corresponds to the wavelength $\lambda = 2.0$ μm.
2. Write θ in radians, $\sin\theta$ and $\cos\theta$ for following angles $\theta = 30°,\ 45°,\ 60°,\ 90°,\ 120°,\ 135°,\ 150°,$ and $180°$. Example: $30°$: $\theta = \pi/6 = 0.502$, $\sin\theta = 0.5$, $\cos\theta = \sqrt{3}/2 = 0.866$.

Exam Quiz 1 Question: Given 02/07/2017

1. **Ulaby, et al., Example 1-2 (modified):**

   A laser beam propagating through the atmosphere is characterized by an electric field intensity given by

   $$E(x,t) = 200\exp(-0.02x)\cos\left(6\times10^{15}t - 2\times10^{7}x\right) \quad \text{(V/m)}$$

   where x is the distance from the source in meters. Determine (a) the direction of wave travel, (b) the wave velocity, and (c) the wave amplitude at a distance of 100 m

Math-Phys Quiz 1 Solutions:

1. $\lambda = c/f = 3.0 \times 10^8/75 = 4.0 \times 10^6$ m $= 4000$ km; $f = c/\lambda = 3.0 \times 10^8/2.0 \times 10^{-6} = 1.5 \times 10^{14}$ Hz $= 150$ THz.

2. $\theta = 30°$: $\theta = \pi/6 = 0.52$, $\sin\theta = 0.5$, $\cos\theta = \sqrt{3}/2 = 0.87$;
   $\theta = 45°$: $\theta = \pi/4 = 0.79$, $\sin\theta = \cos\theta = \sqrt{2}/2 = 0.71$;
   $\theta = 60°$: $\theta = \pi/3 = 1.05$, $\sin\theta = \sqrt{3}/2 = 0.87$, $\cos\theta = 0.5$;
   $\theta = 90°$: $\theta = \pi/2 = 1.6$, $\sin\theta = 0$, $\cos\theta = 1$;
   $\theta = 120°$: $\theta = 2\pi/3 = 2.1$, $\sin\theta = \sqrt{3}/2 = 0.87$, $\cos\theta = 0.5$;
   $\theta = 135°$: $\theta = \pi/4 = 0.79$, $\sin\theta = -\cos\theta = \sqrt{2}/2 = 0.71$;
   $\theta = 150°$: $\theta = 3\pi/4 = 2.36$, $\sin\theta = 0.5$, $\cos\theta = -0.87$;
   $\theta = 180°$: $\theta = \pi = 3.14$, $\sin\theta = 0$, $\cos\theta = -1$

Exam Quiz 1 Solutions:

1. (a) The wave propagates in the positive x-direction. (b) The wave velocity is given by $u_\mathrm{p} = 6 \times 10^{15}/2 \times 10^7 = 3 \times 10^8$ m/s. (c) At a distance of 100 m, the amplitude is given by $A = 200\exp(-0.02 \times 100) = 200\exp(-2) = 200 \times 0.135 = 27.1$ V/m.