Oblique Incidence: Snell's Laws

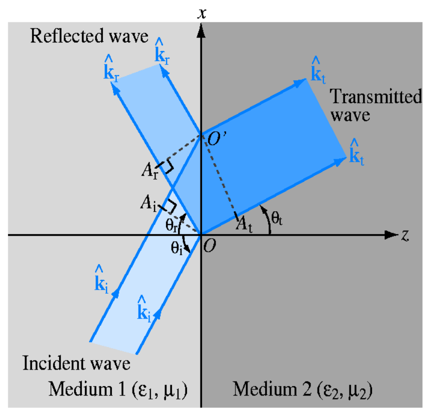
Directions of propagation

The directions of the incident, reflected and transmitted waves are characterized by three unit vectors: $\hat{\mathbf{k}}_i$, $\hat{\mathbf{k}}_r$, and $\hat{\mathbf{k}}_t$.

At a surface with normal $\hat{\mathbf{n}}$, these directions are also characterized by $\cos\theta_i = \hat{\mathbf{k}}_i \cdot \hat{\mathbf{n}}$, $\cos\theta_r = \hat{\mathbf{k}}_r \cdot \hat{\mathbf{n}}$, and $\cos\theta_t = \hat{\mathbf{k}}_t \cdot \hat{\mathbf{n}}$.

In the geometry shown at right:
$\hat{\mathbf{n}} = \hat{\mathbf{z}}, \quad \hat{\mathbf{k}}_i = \hat{\mathbf{x}}\sin\theta_i + \hat{\mathbf{z}}\cos\theta_i$

Goal: Given $\hat{\mathbf{k}}_i$, find $\hat{\mathbf{k}}_r, \hat{\mathbf{k}}_t$.



Ulaby Figure 8-9

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16.1

Oblique Incidence: Snell's Laws

Goal: Given $\hat{\mathbf{k}}_i$, find $\hat{\mathbf{k}}_r, \hat{\mathbf{k}}_t$.

This problem is a ***waves*** problem, not an ***electromagnetics*** problem!

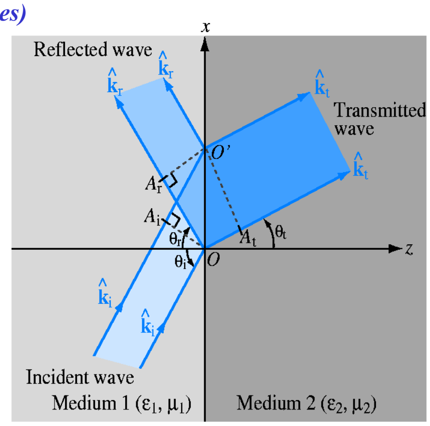
- Maxwell's equations are not needed to solve it
- ***It was solved by Snell (and Descartes) 400 years ago!***
- The solution is the same for *any* wave at an interface

Key idea: The phase of all three waves at every point on the interface must be the same!

$$\omega t - \mathbf{k}_i \cdot \mathbf{R} = \omega t - \mathbf{k}_r \cdot \mathbf{R} = \omega t - \mathbf{k}_t \cdot \mathbf{R}$$

Or $\mathbf{k}_i \cdot \mathbf{R} = \mathbf{k}_r \cdot \mathbf{R} = \mathbf{k}_t \cdot \mathbf{R}$ where

$\mathbf{R} = \overline{OO'}$ is any position vector on the interface



Ulaby Figure 8-9

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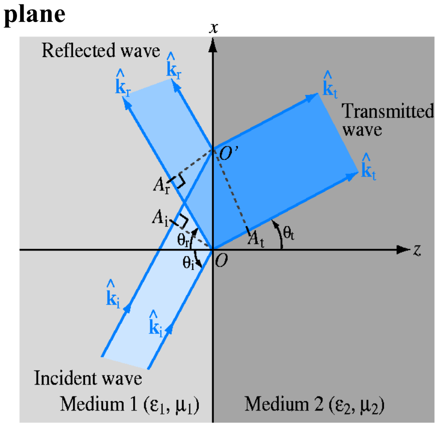
16.2

Oblique Incidence: Snell's Laws

Consequences:

1. **All frequencies are the same:** $\omega_t = \omega_r = \omega_i = \omega$
   [We already used this consequence to equate $\mathbf{k}_i \cdot \mathbf{R} = \mathbf{k}_r \cdot \mathbf{R} = \mathbf{k}_t \cdot \mathbf{R}$]
2. **All wave vectors are in the same plane**
   Let $\mathbf{A} = \mathbf{k}_i \times \hat{\mathbf{n}}$; $\mathbf{A} \perp \hat{\mathbf{n}}$ is on the interface, so
   $\mathbf{k}_r \cdot \mathbf{A} = \mathbf{k}_t \cdot \mathbf{A} = \mathbf{k}_i \cdot \mathbf{A} = 0$
   and all wave vectors are on the plane normal to $\mathbf{A}$.
   [Hence, defining the angles θ_i, θ_r, and θ_t is sufficient.]



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Ulaby Figure 8-9

Oblique Incidence: Snell's Laws

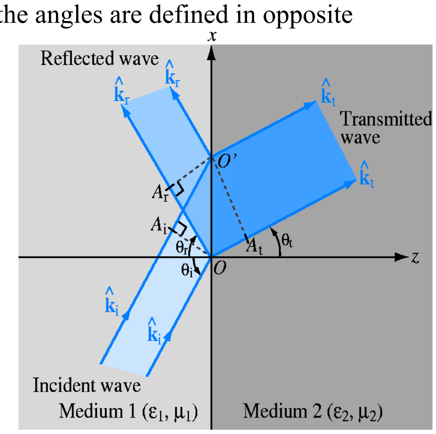
Consequences:

3. **Snell's Law of Reflection:** $\theta_r = \theta_i$
   We have $k_r = k_i = k_1$. In the geometry below, we also have $k_{xr} = k_{xi}$. It follows that $k_{zr} = -k_{zi}$. Since the angles are defined in opposite directions, the equality follows.
4. **Snell's Law of Refraction**

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}}$$

   We have
   $\sin\theta_i = k_{xi} / k_i, \ \sin\theta_t = k_{xt} / k_t$
   We also have
   $k_{xt} = k_{xi}, \ \omega = k_t / u_{p2} = k_i / u_{p1}$

   The result follows after substitution.



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Ulaby Figure 8-9

Oblique Incidence: Snell's Laws

Consequences in Electromagnetics:

Snell's laws in the following form apply to any waves:

$$\theta_r = \theta_i; \quad \sin\theta_t / \sin\theta_i = u_{p2} / u_{p1}$$

Other forms of Snell's Law of Refraction are useful in electromagnetism

Defining the index of refraction,

$$n = \frac{c}{u_p} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r} \ ^*$$

Snell's Law of Refraction becomes

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \sqrt{\frac{\mu_{r1}\varepsilon_{r1}}{\mu_{r2}\varepsilon_{r2}}} \xrightarrow{\text{for non-magnetic media}} \frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

* Note that the subscript r here refers to "relative," not "reflected."

16.5

Oblique Incidence: Snell's Laws

Critical Angle:

When $n_1 < n_2$, we have $\theta_t < \theta_i$

When $n_1 > n_2$, we have $\theta_t > \theta_i$

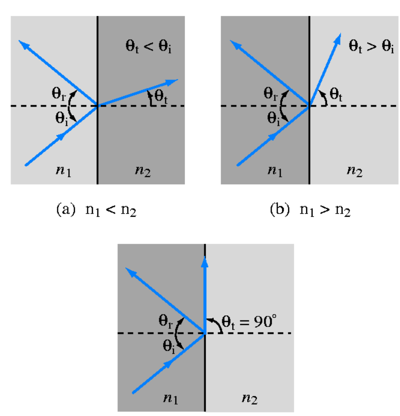
— ***as long as*** $(n_1/n_2)\sin\theta_i < 1$, ***which implies***

$$\theta_i < \theta_c = \sin^{-1}(n_2/n_1)$$

The angle θ_c is referred to as the *critical angle*.

When $\theta_i > \theta_c$, there is no transmitted wave.

This effect is called total internal reflection



(c) $n_1 > n_2$ and $\theta_i = \theta_c$

Ulaby Figure 8-10

16.6

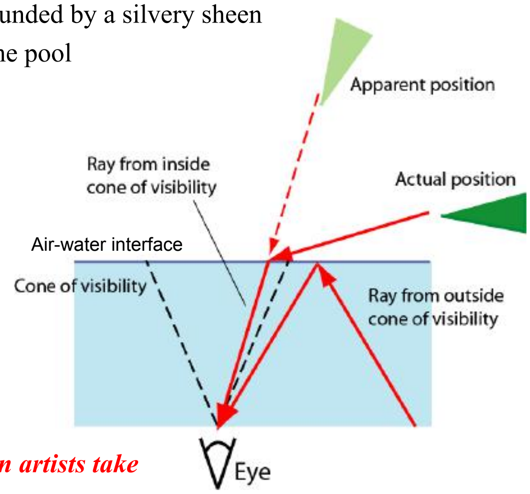
Waves at Boundaries

What you see from the bottom of a pool:

- A circle of visibility, surrounded by a silvery sheen
- Objects near the edge of the pool appear suspended in air

Many similar phenomena exist

- A stick placed in water appears bent
- Coins and other solid objects can also bend



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Magicians and con artists take advantage of this!

Waves at Boundaries

Example: Cone of Visibility and Apparent Height in Water

Question: The index of refraction of water is approximately $n = 1.3$ for visible light. You are looking up from the bottom of a pool. What is the angle that the cone of visibility makes with the surface normal? Suppose that the edge of the pool is 20 feet away and at the edge there is a pennant that is 5 feet above the ground. Ignoring distortion, how high does the pennant appear to be?

Answer: The angle that the cone of visibility makes with the surface is just the critical angle. We have

$$\theta_c = \sin^{-1}(n_2 / n_1) = \sin^{-1}(1/1.3) = 0.88 \text{ rad} = 50°$$

The angle that the light from the pennant makes with respect to the surface normal is given by $\tan\theta_2 = 20/5 = 4.0$, so that $\sin\theta_2 = 0.970$. It follows that $\sin\theta_1 = (1/1.3)\sin\theta_2 = 0.746$, which corresponds to 48.3°.

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Waves at Boundaries

Example: Cone of Visibility and Apparent Height in Water

Question: The index of refraction of water is approximately $n = 1.3$ for visible light. You are looking up from the bottom of a pool. What is the angle that the cone of visibility makes with the surface normal? Suppose that the edge of the pool is 20 feet away and at the edge there is a pennant that is 5 feet above the ground. Ignoring distortion, how high does the pennant appear to be?

Answer (continued): Neglecting distortion, we may assume that the distance from the water appears unchanged, and that it is given by $(20^2 + 5^2)^{1/2} = 20.6$ ft. So, the apparent height is given by $20.6\cos(48.3^{\text{o}}) =$ 14 ft. This angle is close to the critical angle, and there will be considerable distortion!

Fiber Optics

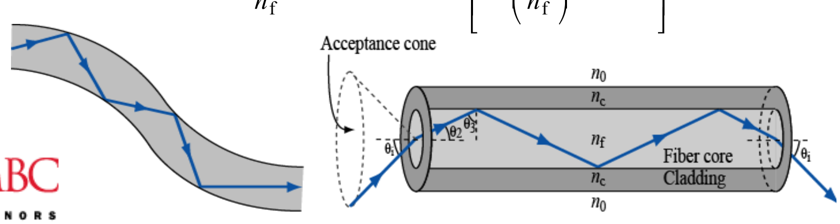
Waves are guided by total internal reflection!

An optical fiber consists of a cylinder of glass, called the *fiber core*, surrounded by a glass with a lower index of refraction, called the *cladding*. We write n_{c} = cladding index and n_{f} = fiber core index.

We must have $\cos\theta_2 = \sin\theta_3 \geq \sin\theta_{\text{c}} = n_{\text{c}} / n_{\text{f}}$

Relative to the incidence angle, we have

$$\sin\theta_2 = \frac{n_0}{n_{\text{f}}}\sin\theta_{\text{i}}; \quad \cos\theta_2 = \left[1 - \left(\frac{n_0}{n_{\text{f}}}\right)^2 \sin^2\theta_{\text{i}}\right]^{1/2}$$



(a) Optical fiber

(b) Successive internal reflections

Ulaby Figure 8.12

Fiber Optics

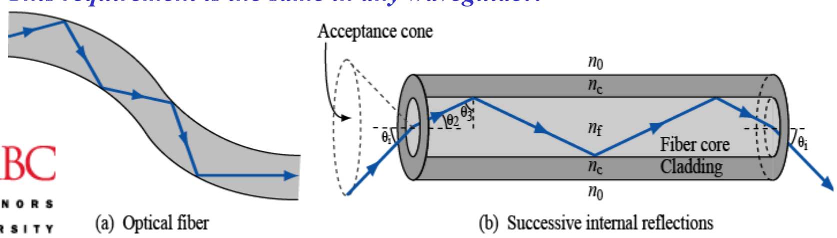
Waves are guided by total internal reflection!

From the condition $\cos\theta_2 \geq n_c / n_f$, it follows that

$$\sin\theta_a = \frac{1}{n_0}(n_f^2 - n_c^2)^{1/2}$$

where θ_a is the acceptance angle, inside of which the light is confined.

Not every angle can propagate! The allowed angles are quantized by the requirement of phase coherence. (Waves that arrive at the same point by different paths must have the same phase.)
This requirement is the same in any waveguide!!



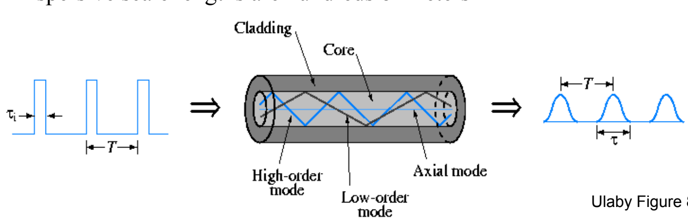
(a) Optical fiber (b) Successive internal reflections

Ulaby Figure 8.12 16.11

Fiber Optics

Two kinds of optical fibers [λ = 1.0 – 1.3 μm]

- **Single-mode fibers:** [diameter < 10 μm] Only one mode (actually two polarization modes) propagate. Used for long-distance communications
  - — Dispersion is intra-modal (due to frequency spread; birefringence)
  - — Dispersive scale lengths are hundreds of kilometers
- **Multi-mode fibers:** [diameter ~ 100 μm] Many modes can propagate. Used for short-haul applications, e.g., 1- and 10-Gbs ethernet
  Now making a "come-back" for long-haul communications!
  - — Dispersion is inter-modal (due to different velocities of different modes)
  - — Dispersive scale lengths are hundreds of meters



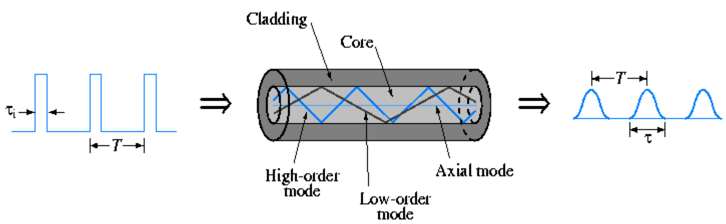
Ulaby Figure 8.13 16.12

Fiber Optics

Modal Dispersion in Multi-mode fibers

Allowed spreading criterion: $T \geq 2\tau$

Spreading occurs because all rays travel at a velocity $u_p = c/n_f$, and rays at the edge of the acceptance cone must travel a longer distance to reach the same z-point along the fiber than do rays that go straight down the center. Hence, these rays arrive at a later time. [Other rays in the acceptance cone arrive in between these two extremes.]



Ulaby Figure 8.13

Fiber Optics

Modal Dispersion in Multi-mode fibers

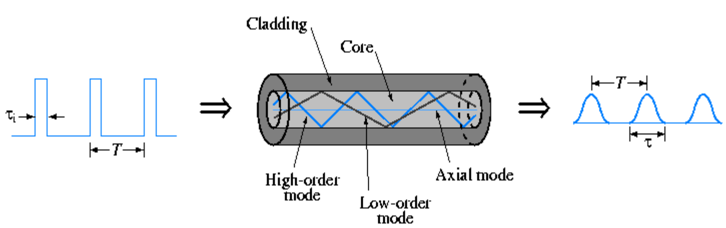
We have

$$l_{\max} = \frac{l}{\cos\theta_2} = l\frac{n_f}{n_c},\ l_{\min} = l \quad \Rightarrow \quad t_{\max} = \frac{l_{\max}}{u_p} = \frac{l n_f^2}{c n_c},\ t_{\min} = \frac{l_{\min}}{u_p} = \frac{l}{c} n_f$$

We thus find

$$\tau = t_{\max} - t_{\min} = \frac{l n_f}{c}\left(\frac{n_f}{n_c} - 1\right), \text{ so that } f_p = \frac{1}{T} = \frac{1}{2\tau} = \frac{c n_c}{2 l n_f (n_f - n_c)}$$

where f_p = data rate



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Ulaby Figure 8.13

Fiber Optics

Data Rate in Optical Fibers: Ulaby and Ravaioli Example 8-5

Question: A 1 km long optical fiber (in air) is made of a fiber core with an index of refraction of 1.52 and a cladding with an index of refraction of 1.49. Determine (a) the acceptance angle θ_a, and (b) the maximum data rate f_p.

Answer: For the acceptance angle, we have the equation

$$\sin\theta_a = \frac{1}{n_0}(n_f^2 - n_c^2)^{1/2} = [(1.52)^2 - (1.49)^2]^{1/2} = 0.300$$

which corresponds to θ_a = 0.305 rads = 17.5°. For the maximum data rate, we have

$$f_p = \frac{c n_c}{2 l n_f (n_f - n_c)} = \frac{(3\times10^8)\times1.49}{2\times10^3\times1.52\times(1.52-1.49)} = 5 \text{ Mb/s}^*$$

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* We lose two places of accuracy in the data rate because of the subtraction 1.52 – 1.49.

16.15

Tech Brief 15: Lasers

Light Amplification by Stimulated Emission of Radiation

Source of monochromatic, coherent, narrow beam light

First laser built by Maiman in 1960.

Basic Principles

Atom can be modeled by a nucleus surrounded by a cloud of electrons at discrete energy levels. Adding energy can raise an electron to a higher energy level.

Spontaneous emission happens when an electron moves to a lower state without external stimulus and emits a photon.

Stimulated emission occurs when incident light (photons) induces an electron to decay and emit a photon of the same energy, wavelength and phase.

Applications – too numerous to mention, but includes CD/DVD players, bar-code readers, eye surgery, etc.

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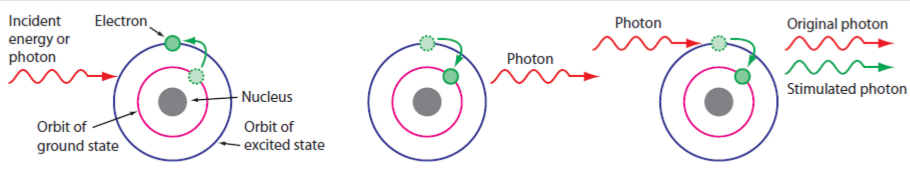
16.16

Tech Brief 15: Lasers



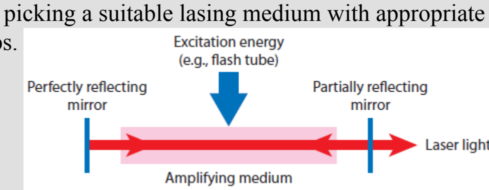
(a) Pumping electron to excited state (b) Spontaneous emission (c) Stimulated emission

Lasing medium – solid, liquid, or gas.

Electrons are pumped (electronically, optically, or otherwise) to higher states. Some atoms will spontaneously emit, which induces other atoms to emit photons at the same wavelength.

Light then travels back and forth in the cavity, with some escaping and the rest stimulating more emission.

Wavelength may be chosen by picking a suitable lasing medium with appropriate energy gaps.



Oblique Incidence: Reflection and Transmittance

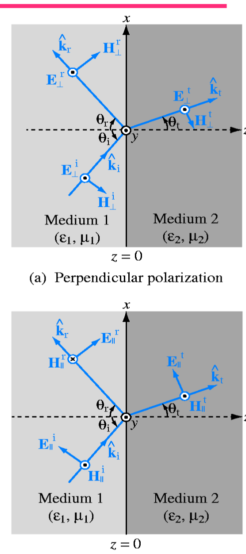
Two special polarizations: TE and TM

With normal incidence:

- Geometry is cylindrically symmetric
- The polarizations are ***degenerate***
  - → any choice of the *x*- and *y*-axes is allowed
  - → the polarization state is unchanged by reflection and refraction

With oblique incidence:

- There are two special cases:
  - (1) **E** || surface; (2) **H** || surface
- The surface interaction ***breaks the degeneracy***
  - → these two cases have different coefficients of reflection and refraction
  - → the polarization state of these two special cases is unchanged by reflection and refraction
  - → the polarization state of any other case *is changed by reflection and refraction*



(a) Perpendicular polarization

(b) Parallel polarization

Ulaby Figure 8-14

Example: Circular polarization becomes elliptical

Oblique Incidence: Reflection and Transmittance

Two special polarizations: TE and TM

This is a serious complication!

Definitions

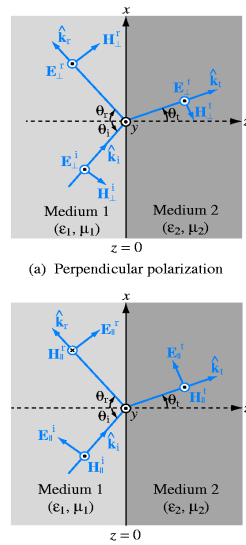
Plane of incidence = plane defined by $\hat{\mathbf{k}}$ and $\hat{\mathbf{n}}$

We note:

- Case (1): **E** || surface
  - → **E** is perpendicular to the plane of incidence
- Case (2): **H** || surface
  - → **E** is parallel to the plane of incidence

More definitions:

- Case (1): *perpendicular polarization*
  or *transverse electric polarization* (TE)
- Case (2): *parallel polarization*
  or *transverse magnetic polarization* (TM)



(a) Perpendicular polarization

(b) Parallel polarization

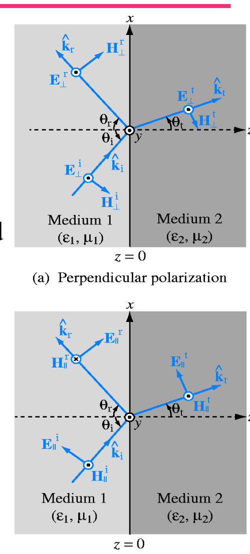
Ulaby Figure 8-14

Oblique Incidence: Reflection and Transmittance

Method of analysis for arbitrary input

1. Decompose the input polarization into a sum of perpendicular and parallel polarizations
2. Separately determine the reflected and transmitted amplitudes for the two special polarizations
3. Sum the results to find the reflected and transmitted waves

Finding the reflection and transmission coefficients for the special polarizations is required to analyze the reflection and transmission for any polarization!



(a) Perpendicular polarization

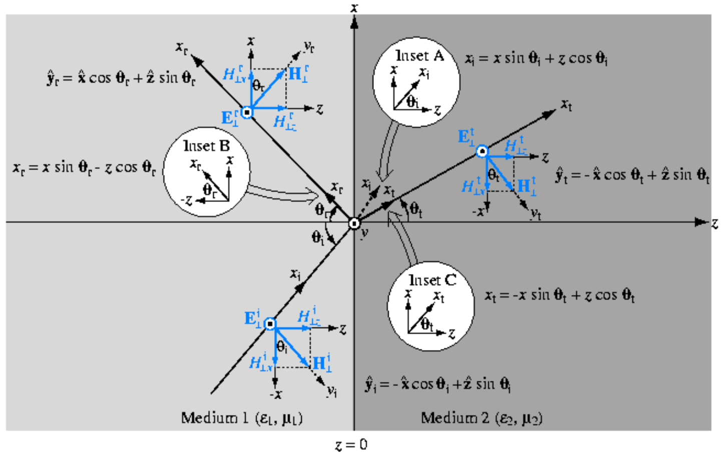
(b) Parallel polarization

Ulaby Figure 8-14

Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization

We take the *x*-*z* plane as our plane of incidence: $\hat{\mathbf{k}}_{\mathrm{i}} = \hat{\mathbf{x}}\sin\theta_{\mathrm{i}} + \hat{\mathbf{z}}\cos\theta_{\mathrm{i}}$



Ulaby Figure 8.15

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Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization

Incident Wave:

$$\tilde{\mathbf{E}}_{\perp}^{\mathrm{i}} = \hat{\mathbf{y}}\, E_{\perp 0}^{\mathrm{i}} \exp\left[-jk_1(x\sin\theta_{\mathrm{i}} + z\cos\theta_{\mathrm{i}})\right],$$

$$\tilde{\mathbf{H}}_{\perp}^{\mathrm{i}} = \frac{1}{\eta_1}\hat{\mathbf{k}}_{\perp}^{\mathrm{i}} \times \tilde{\mathbf{E}}_{\perp}^{\mathrm{i}} = (-\hat{\mathbf{x}}\cos\theta_{\mathrm{i}} + \hat{\mathbf{z}}\sin\theta_{\mathrm{i}})\frac{E_{\perp 0}^{\mathrm{i}}}{\eta_1}\exp\left[-jk_1(x\sin\theta_{\mathrm{i}} + z\cos\theta_{\mathrm{i}})\right]$$

with $k_1 = \omega\sqrt{\mu_1\varepsilon_1},\ \eta_1 = \sqrt{\mu_1/\varepsilon_1}$

Reflected Wave:

$$\tilde{\mathbf{E}}_{\perp}^{\mathrm{r}} = \hat{\mathbf{y}}\, E_{\perp 0}^{\mathrm{r}} \exp\left[-jk_1(x\sin\theta_{\mathrm{r}} - z\cos\theta_{\mathrm{r}})\right],$$

$$\tilde{\mathbf{H}}_{\perp}^{\mathrm{r}} = (\hat{\mathbf{x}}\cos\theta_{\mathrm{r}} + \hat{\mathbf{z}}\sin\theta_{\mathrm{r}})\frac{E_{\perp 0}^{\mathrm{r}}}{\eta_1}\exp\left[-jk_1(x\sin\theta_{\mathrm{r}} - z\cos\theta_{\mathrm{r}})\right]$$

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Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization

Transmitted Wave:

$$\tilde{\mathbf{E}}_{\perp}^{\mathrm{t}} = \hat{\mathbf{y}}\, E_{\perp 0}^{\mathrm{t}} \exp\left[-jk_2(x\sin\theta_{\mathrm{t}} + z\cos\theta_{\mathrm{t}})\right],$$

$$\tilde{\mathbf{H}}_{\perp}^{\mathrm{t}} = (-\hat{\mathbf{x}}\cos\theta_{\mathrm{t}} + \hat{\mathbf{z}}\sin\theta_{\mathrm{t}})\frac{E_{\perp 0}^{\mathrm{t}}}{\eta_2}\exp\left[-jk_2(x\sin\theta_{\mathrm{t}} + z\cos\theta_{\mathrm{t}})\right]$$

with $k_2 = \omega\sqrt{\mu_2\varepsilon_2},\ \eta_2 = \sqrt{\mu_2/\varepsilon_2}$

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Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization

Boundary conditions:

— The tangential components of **E** and **H** must be continuous

$$(\tilde{E}_{\perp y}^{\mathrm{i}} + \tilde{E}_{\perp y}^{\mathrm{r}})|_{z=0} = \tilde{E}_{\perp y}^{\mathrm{t}}|_{z=0}, \quad (\tilde{H}_{\perp x}^{\mathrm{i}} + \tilde{H}_{\perp x}^{\mathrm{r}})|_{z=0} = \tilde{H}_{\perp x}^{\mathrm{t}}|_{z=0}$$

These conditions become

$$E_{\perp 0}^{\mathrm{i}} \exp(-jk_1 x\sin\theta_{\mathrm{i}}) + E_{\perp 0}^{\mathrm{r}} \exp(-jk_1 x\sin\theta_{\mathrm{r}}) = E_{\perp 0}^{\mathrm{t}} \exp(-jk_2 x\sin\theta_{\mathrm{t}})$$

$$-\frac{E_{\perp 0}^{\mathrm{i}}}{\eta_1}\cos\theta_{\mathrm{i}} \exp(-jk_1 x\sin\theta_{\mathrm{i}}) + \frac{E_{\perp 0}^{\mathrm{r}}}{\eta_1}\cos\theta_{\mathrm{r}} \exp(-jk_1 x\sin\theta_{\mathrm{r}}) = \frac{E_{\perp 0}^{\mathrm{t}}}{\eta_2}\cos\theta_{\mathrm{t}} \exp(-jk_2 x\sin\theta_{\mathrm{t}})$$

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Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization: Boundary Conditions

From the phase-matching condition, we have:

$$k_1 \sin\theta_{\mathrm{i}} = k_1 \sin\theta_{\mathrm{r}} = k_2 \sin\theta_{\mathrm{t}}$$

These conditions imply Snell's laws!

Our boundary conditions now become

$$E^{\mathrm{i}}_{\perp 0} + E^{\mathrm{r}}_{\perp 0} = E^{\mathrm{t}}_{\perp 0}, \quad \frac{\cos\theta_{\mathrm{i}}}{\eta_1}(-E^{\mathrm{i}}_{\perp 0} + E^{\mathrm{r}}_{\perp 0}) = \frac{\cos\theta_{\mathrm{t}}}{\eta_2} E^{\mathrm{t}}_{\perp 0}$$

where we made use of $\theta_{\mathrm{r}} = \theta_{\mathrm{i}}$

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Oblique Incidence: Reflection and Transmittance

Perpendicular Polarization: Fresnel Coefficients

We conclude

$$\Gamma_\perp = \frac{E^{\mathrm{r}}_{\perp 0}}{E^{\mathrm{i}}_{\perp 0}} = \frac{\eta_2\cos\theta_{\mathrm{i}} - \eta_1\cos\theta_{\mathrm{t}}}{\eta_2\cos\theta_{\mathrm{i}} + \eta_1\cos\theta_{\mathrm{t}}}, \quad \tau_\perp = \frac{E^{\mathrm{t}}_{\perp 0}}{E^{\mathrm{i}}_{\perp 0}} = \frac{2\eta_2\cos\theta_{\mathrm{i}}}{\eta_2\cos\theta_{\mathrm{i}} + \eta_1\cos\theta_{\mathrm{t}}} = 1 + \Gamma_\perp$$

We also have in a non-magnetic medium ($\mu_1 = \mu_2 = \mu_0$)

$$\Gamma_\perp = \frac{\cos\theta_{\mathrm{i}} - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}}{\cos\theta_{\mathrm{i}} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}}$$

These are called the Fresnel coefficients for perpendicular polarization

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Oblique Incidence: Reflection and Transmittance

Parallel Polarization

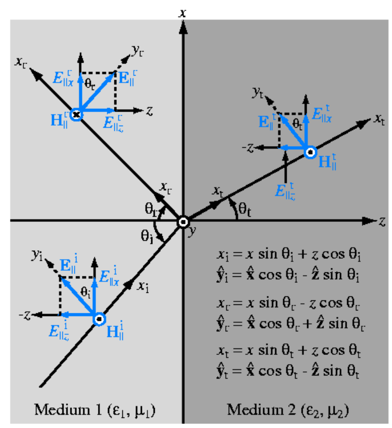
We proceed by analogy with the perpendicular case and find:

$$\Gamma_{\parallel} = \frac{E^{\mathrm{r}}_{\parallel 0}}{E^{\mathrm{i}}_{\parallel 0}} = \frac{\eta_2 \cos\theta_{\mathrm{t}} - \eta_1 \cos\theta_{\mathrm{i}}}{\eta_2 \cos\theta_{\mathrm{t}} + \eta_1 \cos\theta_{\mathrm{i}}},$$

$$\tau_{\parallel} = \frac{E^{\mathrm{t}}_{\parallel 0}}{E^{\mathrm{i}}_{\parallel 0}} = \frac{2\eta_2 \cos\theta_{\mathrm{i}}}{\eta_2 \cos\theta_{\mathrm{t}} + \eta_1 \cos\theta_{\mathrm{i}}}$$

$$= (1+\Gamma_{\parallel})\frac{\cos\theta_{\mathrm{i}}}{\cos\theta_{\mathrm{t}}}$$

These are the Fresnel coefficients for parallel polarization



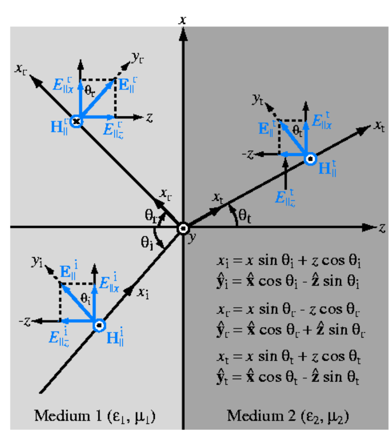
Ulaby Figure 8.16

Oblique Incidence: Reflection and Transmittance

Parallel Polarization

For non-magnetic materials, we have:

$$\Gamma_{\parallel} = \frac{-(\varepsilon_2/\varepsilon_1)\cos\theta_{\mathrm{i}} - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}}{(\varepsilon_2/\varepsilon_1)\cos\theta_{\mathrm{i}} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}}$$



Ulaby Figure 8.16

Oblique Incidence: Reflection and Transmittance

Brewster Angle, θ_B

At this incidence angle, $\Gamma = 0$, and all the energy is transmitted.

It is also called the polarizing angle, because it is not the same for the two polarizations

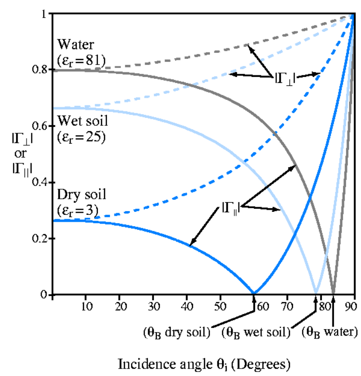
Perpendicular polarization:

Condition: $\eta_2 \cos\theta_i - \eta_1 \cos\theta_t = 0$

We find

$$\sin\theta_{B\perp} = \left[\frac{1-(\mu_1\varepsilon_2/\mu_2\varepsilon_1)}{1-(\mu_1/\mu_2)^2}\right]^{1/2}$$

For non-magnetic materials, there is no perpendicular Brewster angle!



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Ulaby Figure 8.17 16.29

Oblique Incidence: Reflection and Transmittance

Brewster Angle, θ_B

Parallel polarization:

Condition: $\eta_2 \cos\theta_t - \eta_1 \cos\theta_i = 0$

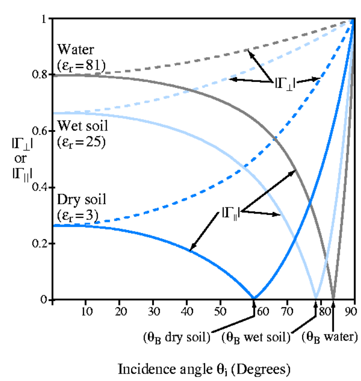
We find

$$\sin\theta_{BP} = \left[\frac{1-(\varepsilon_1\mu_2/\varepsilon_2\mu_1)}{1-(\varepsilon_1/\varepsilon_2)^2}\right]^{1/2}$$

which in the case of non-magnetic materials implies

$$\theta_{BP} = \sin^{-1}\left[\frac{1}{1+(\varepsilon_1/\varepsilon_2)}\right]^{1/2}$$

$$= \tan^{-1}\left(\sqrt{\varepsilon_2/\varepsilon_1}\right)$$



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Ulaby Figure 8.17 16.30

Oblique Waves at Boundaries

Example: Oblique Wave on a Soil Surface
[Modified from Ulaby and Ravaioli Example 8-6]

Question: A right-circularly polarized wave radiated by a distant antenna is incident in air upon a soil surface at $z = 0$. The electric field is given by

$$\mathbf{E}^{\mathrm{i}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}}\frac{\sqrt{3}}{2} - \hat{\mathbf{z}}\frac{1}{2}\right)100\cos\left[\omega t - 2\pi\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)\right]$$

$$+\frac{1}{\sqrt{2}}\hat{\mathbf{y}}100\sin\left[\omega t - 2\pi\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)\right] \text{ V/m},$$

$$= (\hat{\mathbf{x}}61.2 - \hat{\mathbf{z}}35.4)\cos(\omega t - \pi x - 1.73\pi z) + \hat{\mathbf{y}}70.7\sin(\omega t - \pi x - 1.73\pi z) \text{ V/m}$$

and the soil can be assumed to be a lossless dielectric with a relative permittivity of 4.

(a) Verify that $\mathbf{E}^{\mathrm{i}}$ corresponds to the field of a right-circularly polarized wave and determine k_1, k_2, and θ_{i}.

(b) Find expressions for the electric field phasors in air and soil. What is the ellipticity of the transmitted wave?

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Oblique Waves at Boundaries

Example: Oblique Wave on a Soil Surface
[Modified from Ulaby et al. Example 8-6]

Question: A right-circularly polarized wave radiated by a distant antenna is incident in air upon a soil surface at z = 0. The electric field is given by

$$\mathbf{E}^{\mathrm{i}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}}\frac{\sqrt{3}}{2} - \hat{\mathbf{z}}\frac{1}{2}\right)100\cos\left[\omega t - 2\pi\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)\right]$$

$$+\frac{1}{\sqrt{2}}\hat{\mathbf{y}}100\sin\left[\omega t - 2\pi\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)\right] \text{ V/m},$$

$$= (\hat{\mathbf{x}}61.2 - \hat{\mathbf{z}}35.4)\cos(\omega t - \pi x - 1.73\pi z) + \hat{\mathbf{y}}70.7\sin(\omega t - \pi x - 1.73\pi z) \text{ V/m}$$

and the soil can be assumed to be a lossless dielectric with a relative permittivity of 4.

(c) At what incident angle would the reflected polarization be linear?

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Oblique Waves at Boundaries

Example: Oblique Wave on a Soil Surface

Answer: (a) In order for $\mathbf{E}^{\mathrm{i}}$ to correspond to an electromagnetic wave, we must have $\mathbf{k}_{\mathrm{i}} \cdot \mathbf{E}^{\mathrm{i}} = k_1(\hat{\mathbf{k}}_{\mathrm{i}} \cdot \mathbf{E}^{\mathrm{i}}) = 0$. From the argument of the cosine and sine, we have $\mathbf{k}_{\mathrm{i}} = 2\pi\left(\hat{\mathbf{x}}\sin 30° + \hat{\mathbf{z}}\cos 30°\right)$, so that $k_1 = 2\pi \text{ m}^{-1}$, $\theta_{\mathrm{i}} = 30°$, and $\mathbf{k}_{\mathrm{i}} \cdot \mathbf{E}^{\mathrm{i}} = 0$. We also have $k_2 = \varepsilon_{\mathrm{r}}^{1/2} k_1 = 4\pi \text{ m}^{-1}$. While not needed to answer this problem, we note that we also have $\lambda_1 = 1$ m, $f = 300$ MHz, and $\omega = 6\pi \times 10^8 \text{ s}^{-1}$.

To verify that $\mathbf{E}^{\mathrm{i}}$ is right-circularly polarized, it is useful to change coordinate systems. We let $x' = x\cos\theta_{\mathrm{i}} - z\sin\theta_{\mathrm{i}}$, $y' = y$, $z' = x\sin\theta_{\mathrm{i}} + z\cos\theta_{\mathrm{i}}$, which implies $\hat{\mathbf{x}}' = \hat{\mathbf{x}}\cos\theta_{\mathrm{i}} - \hat{\mathbf{z}}\sin\theta_{\mathrm{i}}$, $\hat{\mathbf{y}}' = \hat{\mathbf{y}}$, $\mathbf{z}' = \hat{\mathbf{x}}\sin\theta_{\mathrm{i}} + \hat{\mathbf{z}}\cos\theta_{\mathrm{i}}$. In this new coordinate system, our expression for the incident field becomes $\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{x}}' 70.7\cos(\omega t - 2\pi z') + \hat{\mathbf{y}}' 70.7\sin(\omega t - 2\pi z')$ V/m, which corresponds to right-circular polarization.

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Oblique Waves at Boundaries

Example: Oblique Wave on a Soil Surface

Answer (continued): (b) From Snell's laws, we have $2\sin\theta_{\mathrm{t}} = \sin\theta_{\mathrm{r}} = \sin\theta_{\mathrm{i}}$, which implies $\theta_{\mathrm{r}} = 30°$, along with $\sin\theta_{\mathrm{t}} = 0.250$, $\cos\theta_{\mathrm{t}} = 0.968$, and $\theta_{\mathrm{t}} = 14.5°$. The Fresnel coefficients are

$$\Gamma_\perp = \frac{0.866 - \sqrt{4 - 0.25}}{0.866 + \sqrt{4 - 0.25}} = -0.382, \quad \tau_\perp = (1 - 0.382) = 0.618,$$

$$\Gamma_\parallel = \frac{-4 \times 0.866 + \sqrt{4 - 0.25}}{4 \times 0.866 + \sqrt{4 - 0.25}} = -0.283, \quad \tau_\parallel = (1 - 0.283)\frac{0.866}{0.968} = 0.641$$

The incident electric field phasors are

$$\tilde{\mathbf{E}}_\perp^{\mathrm{i}} = -j\hat{\mathbf{y}} 70.7\exp(-j\pi x - j1.73\pi z), \quad \tilde{\mathbf{E}}_\parallel^{\mathrm{i}} = (\hat{\mathbf{x}} 61.2 - \hat{\mathbf{z}} 35.4)\exp(-j\pi x - j1.73\pi z),$$

which implies

$$\tilde{\mathbf{E}}_\perp^{\mathrm{r}} = j\hat{\mathbf{y}} 27.0\exp(-j\pi x + j1.73\pi z), \quad \tilde{\mathbf{E}}_\parallel^{\mathrm{r}} = (-\hat{\mathbf{x}} 17.3 - \hat{\mathbf{z}} 10.0)\exp(-j\pi x + j1.73\pi z)$$

and

$$\tilde{\mathbf{E}}_\perp^{\mathrm{t}} = -j\hat{\mathbf{y}} 43.7\exp(-j\pi x - j3.87\pi z),$$

$$\tilde{\mathbf{E}}_\parallel^{\mathrm{t}} = (\hat{\mathbf{x}} 43.9 - \hat{\mathbf{z}} 11.3)\exp(-j\pi x - j3.87\pi z) \text{ V/m}$$

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Oblique Waves at Boundaries

Example: Oblique Wave on a Soil Surface

Answer (continued): (b-continued) We have $\tilde{\mathbf{E}}_{\text{air}} = \tilde{\mathbf{E}}^{\text{i}}_{\perp} + \tilde{\mathbf{E}}^{\text{i}}_{\parallel} + \tilde{\mathbf{E}}^{\text{r}}_{\perp} + \tilde{\mathbf{E}}^{\text{r}}_{\parallel}$ and $\tilde{\mathbf{E}}_{\text{soil}} = \tilde{\mathbf{E}}^{\text{t}}_{\perp} + \tilde{\mathbf{E}}^{\text{t}}_{\parallel}$. Explicitly, in the time domain, we have

$$\mathbf{E}_{\text{soil}} = (\hat{\mathbf{x}}\,0.968 - \hat{\mathbf{z}}\,0.250)\,45.3\cos(\omega t - \pi x - 3.87\pi z)$$
$$+\hat{\mathbf{y}}\,43.7\sin(\omega t - \pi x - 3.87\pi z)\ \text{V/m}$$

Making the transformation, $x' = x\cos\theta_{\text{t}} - z\sin\theta_{\text{t}}$, $y' = y$, $z' = x\sin\theta_{\text{t}} + z\cos\theta_{\text{t}}$, we have $\mathbf{E}_{\text{soil}} = \hat{\mathbf{x}}'\,45.3\cos(\omega t - 4\pi z') + \hat{\mathbf{y}}'\,43.7\sin(\omega t - 4\pi z')$ V/m, which corresponds to a right-elliptically polarized wave, with

$$\gamma = 0 \quad \text{and} \quad \chi = \tan^{-1}(43.7/45.3) = 44.0°.$$

(c) A linearly polarized wave will be reflected when the incident angle equals the Brewster angle for the parallel polarization. This angle is given by

$$\theta_{\text{i}} = \theta_{\text{BP}} = \tan^{-1}\left(\sqrt{\varepsilon_2/\varepsilon_1}\right) = \tan^{-1} 2 = 63.4°$$

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Power Conservation

Reflectivity and Transmittivity

Consider an illuminated spot. We have for the average power densities

$$S^{\text{i}}_{\perp} = \frac{|E^{\text{i}}_{\perp 0}|^2}{2\eta_1},\quad S^{\text{r}}_{\perp} = \frac{|E^{\text{r}}_{\perp 0}|^2}{2\eta_1},\quad S^{\text{t}}_{\perp} = \frac{|E^{\text{t}}_{\perp 0}|^2}{2\eta_2},$$

with the cross-sectional areas,

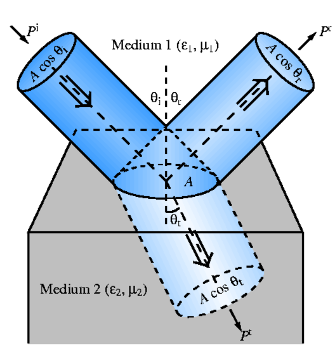
$$A_{\text{i}} = A\cos\theta_{\text{i}},\quad A_{\text{r}} = A\cos\theta_{\text{r}},\quad A_{\text{t}} = A\cos\theta_{\text{t}}.$$

The average powers carried by the beams are

$$P^{\text{i}}_{\perp} = S^{\text{i}}_{\perp}A_{\text{i}} = \frac{|E^{\text{i}}_{\perp 0}|^2}{2\eta_1}A\cos\theta_{\text{i}},$$

$$P^{\text{r}}_{\perp} = S^{\text{r}}_{\perp}A_{\text{r}} = \frac{|E^{\text{r}}_{\perp 0}|^2}{2\eta_1}A\cos\theta_{\text{r}},$$

$$P^{\text{t}}_{\perp} = S^{\text{t}}_{\perp}A_{\text{t}} = \frac{|E^{\text{t}}_{\perp 0}|^2}{2\eta_2}A\cos\theta_{\text{t}}$$



Ulaby Figure 8.18

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Power Conservation

Reflectivity and Transmittivity

We now define a reflectivity

$$R_\perp = \frac{P_\perp^{\mathrm{r}}}{P_\perp^{\mathrm{i}}} = \frac{|E_{\perp 0}^{\mathrm{r}}|^2 \cos\theta_{\mathrm{r}}}{|E_{\perp 0}^{\mathrm{i}}|^2 \cos\theta_{\mathrm{i}}} = \left|\frac{E_{\perp 0}^{\mathrm{r}}}{E_{\perp 0}^{\mathrm{i}}}\right|^2 = |\Gamma_\perp|^2$$

and a transmittivity

$$T_\perp = \frac{P_\perp^{\mathrm{t}}}{P_\perp^{\mathrm{i}}} = \frac{|E_{\perp 0}^{\mathrm{t}}|^2 \eta_1 \cos\theta_{\mathrm{t}}}{|E_{\perp 0}^{\mathrm{i}}|^2 \eta_2 \cos\theta_{\mathrm{i}}} = |\tau_\perp|^2 \left(\frac{\eta_1 \cos\theta_{\mathrm{t}}}{\eta_2 \cos\theta_{\mathrm{i}}}\right)$$

Conservation of power implies: $R_\perp + T_\perp = 1$

Defining analogously, $R_{||} = |\Gamma_{||}|^2$ and $T_{||} = |\tau_{||}|^2 \left(\frac{\eta_1 \cos\theta_{\mathrm{t}}}{\eta_2 \cos\theta_{\mathrm{i}}}\right)$

we have $R_{||} + T_{||} = 1$

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Tech Brief 16: Bar-Code Readers

Bar codes

Consist of parallel bars of certain widths encoding a binary sequence. Laser scanners can read the bar code and transfer the information to a computer.

Basic Operation

Laser focused a mirror rotation 6000 rpm, creating a fan beam. Beam direction changes constantly. One direction should include the bar code to be read. Light reflects off the white parts, but not the black parts, which is detected by the sensor and converted to an electrical signal. To eliminate interference, a glass filter is used to block ambient light.

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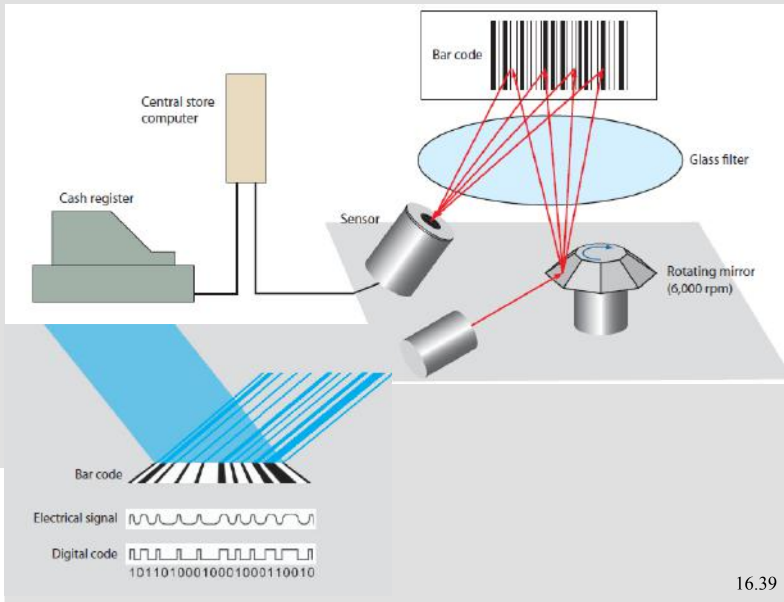
Tech Brief 16: Bar-Code Readers



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Assignment

Reading: NONE! Prepare for final exam.

Problem Set 9: Some notes.

- There are 6 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- The problems are non-trivial; please get started early.
- Please watch significant digits.

16.40