Units, Magnitude, and Notation

We use SI (Système International) Units:

You must know how to convert mi, in, ft, yd → mm, m, km, etc.!!

Many stupid mistakes have been made by getting this wrong!

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	k
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Substance amount	mole	mol

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.1

Note: International is spelled incorrectly by Ulaby --- systeme is masculine

Stupid mistakes include: Spacecraft missing Mars; Hubble space telescope

In defense contexts: We don't know about them, but they kill our soldiers!

In civilian contexts: They can kill our people. You must get this right!!

Waves

In a lossless medium, one may write the current y as

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$

This is the general form for any kind of wave, including water waves (Ulaby's example)

A = wave amplitude
T = time period
λ = spatial wavelength
ϕ_0 = reference phase

There are 4 basic wave quantities in a lossless medium

We may define the total phase by writing

$$y(x,t) = A\cos\phi(x,t), \quad \text{where} \quad \phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.2

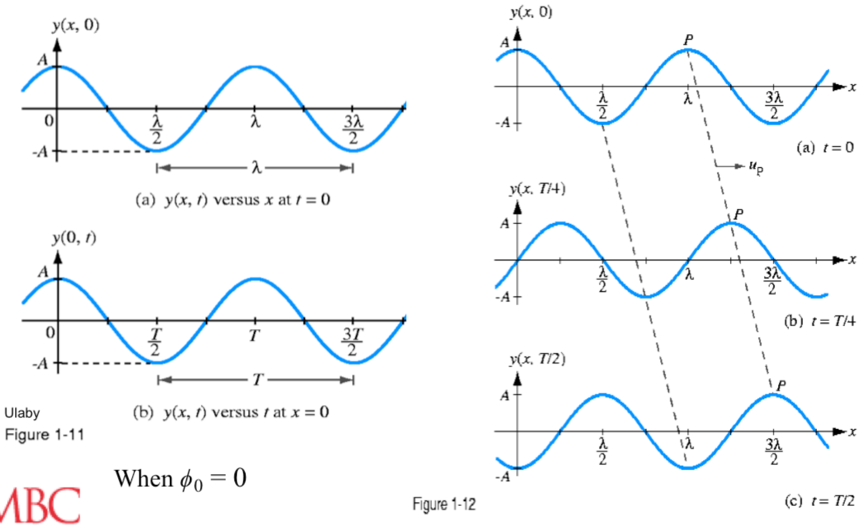
We use $y(x,t)$ instead of $I(x,t)$ to be consistent with Ulaby and because this expression is more general. The key point is that the same wave form applies to ***ANY*** wave of any kind!

Waves in our everyday experience, including water waves, sound waves, surface waves on a drum, a plucked string on a violin, all move in a medium. In the case of electromagnetic waves, there is no medium! Maxwell and others originally thought that there might be (the luminiferous ether), but Einstein and others later showed that they were wrong (or more precisely the hypothesis that an ether exists is unnecessary). As a consequence, electromagnetic waves are an abstraction! No one can show you an electromagnetic wave (except in simulations, which is one of the strengths of simulations). One can, however, determine their consequences.

Why do we worry about cosine (sinusoidally varying) waves instead of square waves or some other shape? It is a fundamental fact of nature that any signal propagating in one dimension can be written as a sum of cosine waves (but they could also be written as a sum of square waves) **AND** (here is the really important point) when the signal power is low enough (low enough depends on the medium that the signal is going through), the cosine waves do not interact with each other!!! Only with the medium! Hence, the importance of Fourier decomposition, which is taught in CMPE 323.

Note that this isn't true just for current or electric fields. It is true for **ANY** signal, sound or pressure waves, water waves, and so on.

Waves



Ulaby
Figure 1-11

When $\phi_0 = 0$

Figure 1-12

UMBC
AN HONORS UNIVERSITY IN MARYLAND

- The phase changes by 2π in one period T or one wavelength λ.
- In Fig. 1-12, the wave moves in the $+x$ direction with a velocity $u_p = \lambda/T$.

2.3

If you keep T fixed and increase x, you will see the same sort of change.

Waves

Phase velocity u_p:

As time t increases, position x must increase to keep the phase

$$\phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$$

constant. Since ϕ is constant, we have

$$0 = \frac{2\pi}{T} dt - \frac{2\pi}{\lambda} dx.$$

So, a point of constant phase, the phase front, moves with velocity

$$u_p = \frac{dx}{dt} = \frac{\lambda}{T} \quad \text{(m/s)}.$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.4

Ulaby in his discussion takes phi_0 = 0. It doesn't matter.

Waves

Other quantities (derived from the fundamental 4):

f = frequency $= 1/T$ (Hz = s^{-1})
ω = angular frequency $= 2\pi f$ (rad/s = s^{-1})
β = wavenumber $= 2\pi/\lambda$ (rad/m = m^{-1})

In terms of these quantities, we have

$$y(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x + \phi_0\right) = A\cos\left(\omega t - \beta x + \phi_0\right).$$

and

$$u_p = f\lambda = \omega/\beta.$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

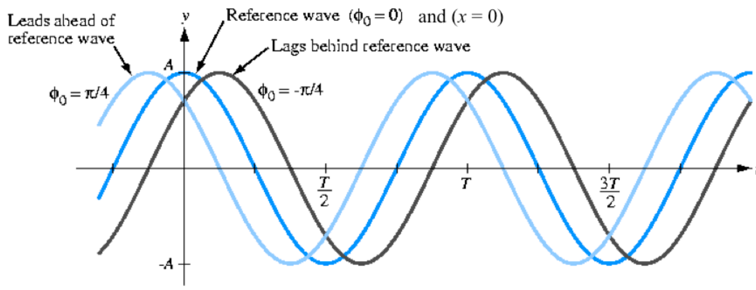
2.5

What are the fundamental 4? A, T, lambda, phi_0

Notes that rad, which is short for radians is unitless in terms of SI units. You cannot use dimensional analysis to keep straight whether you are using angular frequency or regular frequency. There are 2*pi radians in a cycle and omega is 2*pi larger than f for the same wave. This is an easy source of errors. Usually, when frequencies are quoted, they are the ordinary frequency (Hz), but when we do calculations we usually use omega to avoid having factors of 2*pi running around.

Waves

The phase offset:



Ulaby
Figure 1-13

Another useful representation:

Letting $R = A\cos\phi_0$, $I = A\sin\phi_0$

$$y(x,t) = A\cos\left(\omega t - \beta x + \phi_0\right) = R\cos\left(\omega t - \beta x\right) - I\sin\left(\omega t - \beta x\right).$$

This representation will be useful when we discuss phasors

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.6

ULABY 2010

Module 1.1, Module 1.3

Module 1.1: do 1/2 the amplitude; double frequency (how many peaks?), halve frequency (how many peaks);

+90^o (forward or backwards); sine? (-90^o); frequency = 1; what is amplitude at 0.125 (pi/2 for f =2) [0.707 = sqrt(2)/2]

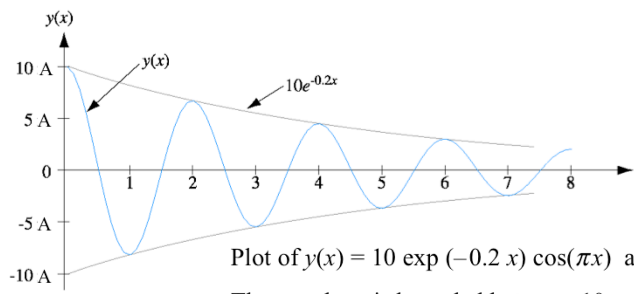
With the frequency = 0.667=2/3; what is the amplitude at 0.125 [0.5]; what about 0.25 [0.832 = sqrt(3)/2]

Module 1.3: lead by pi/4 (green); lag by pi/2 (blue) [BUT THIS IS NOT QUITE RIGHT. WHY? (Zero-crossing should match peak)]; what is the purple curve doing? (leading by pi/2)

Waves

With Loss: $y(x,t) = A\exp(-\alpha x)\cos\left(\omega t - \beta x + \phi_0\right)$

α = attenuation coefficient



Plot of $y(x)$ = 10 exp (–0.2 x) cos(πx) amperes

The envelope is bounded between 10 exp (–0.2 x) and its mirror image –10 exp (–0.2 x)

Ulaby (modified)
Figure 1-14

UMBC
AN HONORS UNIVERSITY IN MARYLAND

With loss, there are 5 fundamental quantities

2.7

ULABY 2010

Module 1.2: Double the frequency (faster or slower); double frequency; halve the wavelength (faster or slower); Consider (alpha = 1; why does the peak move

to the left of the peaks in the unattenuated case? Will the movement be less or more with larger/smaller alpha). At x =1 and and x =2; about how

much smaller is the value of the attenuated case as opposed to the unattenuated case (1/e = 1/2.72 = 0.37 ~ 1/3 = 0.33)

Waves

With Loss: Ulaby Example 1-2

Question: A laser beam propagating through the atmosphere is characterized by an electric field intensity given by

$$E(x,t) = 150\exp(-0.03x)\cos\left(3\times10^{15}t - 10^{7}x\right) \quad \text{(V/m)}$$

where x is the distance from the source in meters. Determine (a) the direction of wave travel, (b) the wave velocity, and (c) the wave amplitude at a distance of 200 m

Solution: (a) Since the coefficients of t and x have the opposite sign, the wave propagates in the $+x$ direction.

(b) We find that

$$u_\mathrm{p} = \frac{\omega}{\beta} = \frac{3\times10^{15}\,\mathrm{s}^{-1}}{10^{7}\,\mathrm{m}^{-1}} = 3\times10^{8}\,\mathrm{m/s},$$

which is (of course) the speed of light c in the vacuum or air.

(c) At x = 200 m, the amplitude of $E(x,t)$ is

$$E(x,t) = 150\exp(-0.03\,\mathrm{m}^{-1}\times 200\,\mathrm{m}) = 0.37\ \mathrm{V/m}$$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

2.8

Ulaby in his example does not include the units in the problem statement and intermediate steps. But you should always make sure that they match properly.

The speed of light in air is actually slightly less than in the vacuum --- but only very slightly!

The frequency is 3 PHz (petahertz). Above terahertz, it is unusual to refer to frequencies using the notation in slide 1, but it is likely to become more common. At optical frequencies, one usually refers to the vacuum wavelength corresponding to the frequency.

THIS EXAMPLE WILL BE THE FIRST EXAM QUIZ

Dispersion Relations

Dispersion relations: $\beta(\omega)$ and $\alpha(\omega)$ are functions of $f = \omega/2\pi$

Calculating the dispersion relations is an important part of EM theory!

In a homogeneous, isotropic medium, this is straightforward

- homogeneous = the same at all points in space
- isotropic = the same in all orientations (no strains; no crystal structure)

We calculate $\beta(\omega)$ and $\alpha(\omega)$ from $\varepsilon(\omega)$ and $\mu(\omega)$

- **We will do this when we discuss plane waves**

In an inhomogeneous, isotropic medium, we must account for geometry

$\varepsilon(\omega) \rightarrow \varepsilon(\omega,\mathbf{r})$ and $\mu(\omega) \rightarrow \mu(\omega,\mathbf{r})$

- The dispersion relations are determined by geometry as well as frequency
- There can be multiple solutions at one frequency
- **We will do this for simple geometries**

UMBC
AN HONORS UNIVERSITY IN MARYLAND

As a consequence:
The 5 fundamental quantities → 3 independent quantities

2.9

When we study transmission lines, we will calculate dispersion relations.

In waveguides or transmission lines, the number of solutions at any frequency can be arbitrarily large.

NOTE: epsilon and mu are due to microscopic properties of the medium. They relate **E** and **D** (epsilon) and **B** and **H** (mu)

Any wave in any medium is defined in terms of the five fundamental quantities. The reduction from 5 to 3 depends on the type of wave (light, wave, sound,…) and the dispersion relation.

Dispersion Relations

Dispersion relations: $\beta(\omega)$ and $\alpha(\omega)$ are functions of $f = \omega/2\pi$

Calculating the dispersion relations is an important part of EM theory!

In an anisotropic medium, this becomes complex

$$\mathbf{D}(\mathbf{r},\omega) = \varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) \rightarrow \mathbf{D}(\mathbf{r},\omega) = \mathsf{E}(\mathbf{r},\omega)\bullet\mathbf{E}(\mathbf{r},\omega)\,;$$

$$\mathbf{B}(\mathbf{r},\omega) = \mu(\mathbf{r},\omega)\mathbf{H}(\mathbf{r},\omega) \rightarrow \mathbf{B}(\mathbf{r},\omega) = \mathsf{M}(\mathbf{r},\omega)\bullet\mathbf{H}(\mathbf{r},\omega)$$

where $\mathsf{E}(\mathbf{r},\omega)$ and $\mathsf{M}(\mathbf{r},\omega)$ are 3×3 matrices*

- **We will not discuss anisotropic media in this course**

— This discussion assumes that the medium is *linear*
(Waves at different frequencies do not interact)

— All media become linear when the wave amplitudes A are small enough

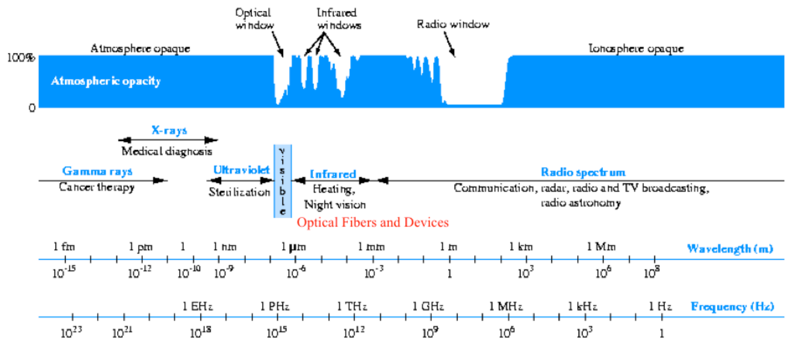
UMBC
AN HONORS UNIVERSITY IN MARYLAND

*Strictly speaking, these are second-order tensors

2.10

Again, it is a remarkable and important fact of nature that in ANY medium whatsoever, the cosine and sine components at different frequencies all propagate without interacting when the signal intensity gets small enough.

Electromagnetic Spectrum



Ulaby (modified)
Figure 1-15

Optical fibers and devices typically operate in the range 0.8 μm to 1.8 μm with 1.5 μm corresponding to minimum loss.

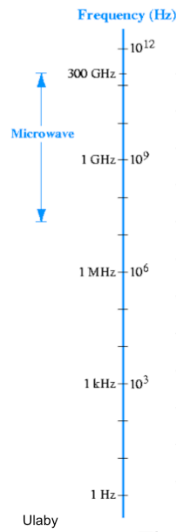
UMBC
AN HONORS UNIVERSITY IN MARYLAND

Note that the minimum loss band in atmospere is at visible light!

Thus, it is sensible that this is the band of electromagnetic radiation that our eyes see!

2.11

Electromagnetic Spectrum



Band	Applications
Extremely High Frequency EHF (30 - 300 GHz)	Radar, advanced communication systems, remote sensing, radio astronomy
Super High Frequency SHF (3 - 30 GHz)	Radar, satellite communication systems, aircraft navigation, radio astronomy, remote sensing
Ultra High Frequency UHF (300 MHz - 3 GHz)	TV broadcasting, radar, radio astronomy, microwave ovens, cellular telephone
Very High Frequency VHF (30 - 300 MHz)	TV and FM broadcasting, mobile radio communication, air traffic control
High Frequency HF (3 - 30 MHz)	Short wave broadcasting
Medium Frequency MF (300 kHz - 3 MHz)	AM broadcasting
Low Frequency LF (30 - 300 kHz)	Radio beacons, weather broadcast stations for air navigation
Very Low Frequency VLF (3 - 30 kHz)	Navigation and position location
Ultra Low Frequency ULF (300 Hz - 3 kHz)	Audio signals on telephone
Super Low Frequency SLF (30 - 300 Hz)	Ionospheric sensing, electric power distribution, submarine communication
Extremely Low Frequency ELF (3 - 30 Hz)	Detection of buried metal objects
$f < 3$ Hz	Magnetotelluric sensing of the earth's structure

Ulaby

Figure 1-16

The wavelength is tied to the frequency through the dispersion relations.

UMBC

AN HONORS UNIVERSITY IN MARYLAND

Waves

Birefringence:

Electromagnetic waves are transverse waves that have two polarizations

That is why polarizing filters work!

In many crystals, glasses, solids—including optical fibers (used in communication systems)—the two polarizations have slightly different dispersion relations and move with different velocities.

The result is a 2π phase shift over long distances!

Example: The ionosphere is birefringent at radio frequencies of 1 MHz. The radio waves propagate at close to the speed of light in a vacuum, but the wavenumber of the two polarization (left- and right-circularly polarized differ by 0.1%. Over what distance do the relative phases shift by 2π?

Answer: We have

UMBC

AN HONORS UNIVERSITY IN MARYLAND

$$\frac{u_{p2} - u_{p1}}{u_{p1}} = \frac{\beta_1}{\omega}\left(\frac{\omega}{\beta_2} - \frac{\omega}{\beta_1}\right) = -\left(\frac{\beta_2 - \beta_1}{\beta_2}\right)$$

2.13

Formerly described in Ulaby modules.

Waves

Example (continued): The ionosphere is birefringent at radio frequencies of 1 MHz. The radio waves propagate at close to the speed of light in a vacuum, but the wavenumber of the two polarization (left- and right-circularly polarized differ by 0.1%. Over what distance do the relative phases shift by 2π?

Answer: We have

$$\frac{u_{\text{p2}} - u_{\text{p1}}}{u_{\text{p1}}} = \frac{\beta_1}{\omega}\left(\frac{\omega}{\beta_2} - \frac{\omega}{\beta_1}\right) = -\left(\frac{\beta_2 - \beta_1}{\beta_2}\right)$$

Since the difference between the two velocities $\Delta u_{\text{p}} = 10^{-3} u_{\text{p}}$ in magnitude, where the difference between u_{p1} and u_{p2} can be neglected. We may similarly write $\Delta\beta = 10^{-3}\beta$. It follows that $\Delta\phi = (\Delta\beta)z = 2\pi$ when

$$z = \frac{2\pi}{\Delta\beta} = \frac{\lambda}{10^{-3}} = 1000\frac{c}{f} = \frac{(10^3)\times(3\times10^8)}{10^6} = 3\times10^5\ \text{m} = 300\ \text{km}$$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

2.14

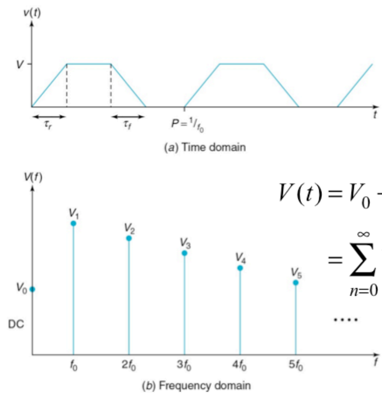
Formerly described in Ulaby modules. Note that this is mathematically equivalent to replacing the complete Taylor expansion of Delta-beta and-Delta u_p with the first term. How big is the error in this estimate? (Under 1%, but only two significant figures are reliable.)

Spectral Analysis

Which frequencies are present?

Following Paul, we consider a digital signal with a rise and fall time that is short compared to the base signal and generates high frequency components. The highest frequency is given approximately by $1/t_{\max}$.

Paul
Fig. 1-3 a/b



With a periodically repeating signal, the only frequencies that appear are multiples of the baseband $f_0 = 1/T$, where T is the period.

$$V(t) = V_0 + V_1\cos(\omega_0 t + \phi_1) + V_2\cos(2\omega_0 t + \phi_2) + \cdots$$

$$= \sum_{n=0}^{\infty} V_n \cos(\omega_n t + \phi_n) \quad \text{with } V_n = V(\omega_n)$$

....

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.15

The time evolution and the frequency spectrum are related to each other by the FOURIER TRANSFORM,
which you learn about in CMPE 323. Because different frequencies evolve independently in linear systems, understanding the relationship between the time and frequency domains is a key tool in both electrical and computer engineering.

With periodic functions, you have the discrete Fourier transform, which produces components at the multiple of the baseband frequency.

With non-periodic functions that go to zero as t --> +/- infinity, you have the regular Fourier transform that has a continuum of components.

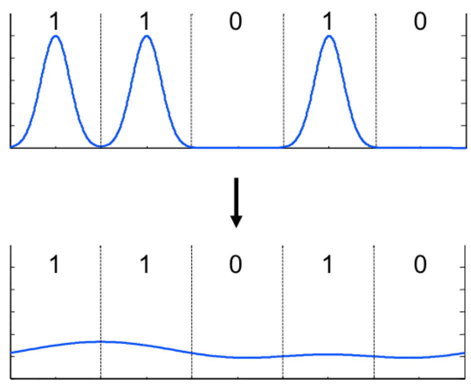
With non-periodic functions that do not go to zero, like a random string of pulses, you have an average Fourier transform with both continuous and discrete components.

Spectral Analysis

Why are high harmonics / large frequency spreads bad?

(1) The high harmonics lead to undesired electronic coupling in digital systems.

(2) The high harmonics lead to pulse spreading in communications systems because different frequencies have different values of u_p.



Chromatic Dispersion:

After spreading a factor of 5 in this return-to-zero example, the digital 1's cannot be distinguished from the digital 0's

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.16

Strictly speaking, it is the variation of u_g = d(beta)/d(omega) that leads to dispersion. Of course, the two are related

u_g = u_p + omega*[d(u_p)/d(omega)].

In many systems, u_g ~ u_p (like glass fibers).

Complex Numbers and Phasors

A ***complex number*** z is written: $z = x + jy$, where $j = \sqrt{-1}$

We also write $x = \mathrm{Re}(z)$, $y = \mathrm{Im}(z)$

In ***polar form***, we have: $z = |z|\exp(j\theta) = |z|e^{j\theta} = |z|\angle\theta$,

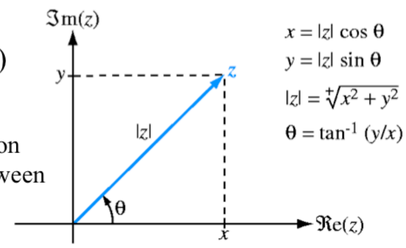
where $|z|$ is the magnitude and θ is the phase.

From ***Euler's identity***, $\exp(j\theta) = \cos\theta + j\sin\theta$, we find

$x = |z|\cos\theta$, $y = |z|\sin\theta$,

$|z| = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$

Graphical representation of the relationship between rectangular and polar coordinates



Ulaby
Figure 1-17

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.17

I prefer exp notation. It is like cos or sin, indicating a function. The notation e^. is common. I rarely use the angle notation.

Euler's identity can be proved by using the Taylor expansions for cos, sin, and exp.

Complex Numbers and Phasors

The ***complex conjugate*** z^* is defined:

$$z^* = x - jy = |z|\exp(-j\theta), \quad \text{so that} \quad |z| = \sqrt{zz^*}$$

Mathematical operations:

- Equality: $z_1 = z_2 \quad \leftrightarrow \quad x_1 = x_2$ and $y_1 = y_2$
- Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
- Multiplication: $z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$
  $= |z_1||z_2|\exp\left[j(\theta_1 + \theta_2)\right]$
- Division: $\dfrac{z_1}{z_2} = \dfrac{(x_1 x_2 + y_1 y_2) + j(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2} = \dfrac{|z_1|}{|z_2|}\exp\left[j(\theta_1 - \theta_2)\right]$
- Powers: $z^r = |z|^r \exp(jr\theta)$, where r is any real number

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.18

Can verify magnitude relationship and others by substitution (and should!)

Complex Numbers and Phasors

The ***complex conjugate*** z^* is defined:

$$z^* = x - jy = |z| \exp(-j\theta), \quad \text{so that} \quad |z| = \sqrt{z z^*}$$

Mathematical operations:

- Logarithm: $\log z = \log |z| + j(\theta + 2\pi n)$, where n is any integer

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.19

Note that there are an infinity of values for each z. We have seen this sort of thing in other functions with the square root (2 values), cube root (3 values), and so on.

Complex Numbers and Phasors

Working with phasors: Ulaby Example 1-3

Question: Given two complex numbers, $V = 3 - j4$ and $I = -2 - j3$, (a) Express V and I in polar form, and find (b) VI, (c) VI^*, (d) V/I, (e) $I^{1/2}$

Answer: (a) V is in the fourth quadrant and I is in the third quadrant. (See figure.)

$$|V| = \sqrt{3^2 + 4^2} = 5, \quad \theta_V = \tan^{-1}(-4/3) = -0.972, \quad \text{so that } V = 5\exp(-j0.972)$$

$$|I| = \sqrt{2^2 + 3^2} = 3.61, \quad \theta_I = \tan^{-1}(3/2) - \pi = -2.159, \quad \text{so that } I = 3.61\exp(-j2.159)$$

(b) $VI = 5e^{-j0.972} \times 3.61e^{-j2.159} = 18.05e^{-j3.131}$

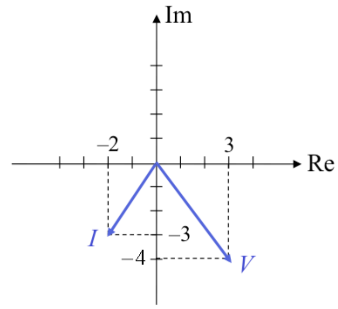
(c) $VI^* = 5e^{-j0.972} \times 3.61e^{j2.159} = 18.05e^{j1.187}$

(d) $V/I = (5/3.61)e^{-j0.972+j2.159} = 1.39e^{j1.187}$

(e) $\sqrt{I} = \pm(3.61)^{1/2}e^{-j(2.159/2)} = \pm 1.90e^{-j1.080}$

Note: $-1.90e^{-j1.080} = 1.90e^{j2.062}$

Angles are only unique to within 2π



Based on Ulaby Fig. 1-18

UMBC AN HONORS UNIVERSITY IN MARYLAND

2.20

Can verify magnitude relationship and others by substitution (and should!)

Ulaby uses degrees in his example. I will use rads. To convert, you write (phase in rads) =(2*pi/360)*(phase in degrees)

In both cases, you are unitless! Dimensional analysis does not help and you have to watch what you are doing.

In this example, Ulaby uses positive degrees for I instead of negative degrees. We can get his result (in degrees) by adding 360 to ours, once ours have been converted from radians to degrees. [–0.972 = –53.1°, –2.159 = –123.7°, –3.131 = –176.8°, 1.187 = 70.6°]

Complex Numbers and Phasors

Why do we work with complex numbers?

The linear integro-differential equations that describe circuits and electromagnetic waves — and in fact any waves — become much easier to solve!

The concept of a **phasor** plays a key

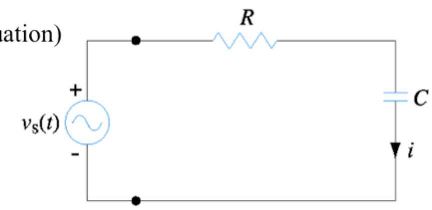
Consider a simple RC circuit with a voltage source $v_s(t) = V_0 \sin(\omega t + \phi_0)$

Kirchoff's voltage law implies

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = v_s(t)$$

(time domain equation)

- Direct time domain solution is a bit messy and involves "guessing" the integral.
- The phasor approach is simpler and deductive



Ulaby Fig. 1-19

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.21

The basic idea that we will be using is that the derivative of an exponential (complex or real) is still an exponential; so is an integral. Look at how we use this concept…

Following Ulaby, we will apply the technique to an example

Complex Numbers and Phasors

Step 1: Adopt a cosine reference

$$v_s(t) = V_0 \sin(\omega t + \phi_0) = V_0 \cos(\omega t + \phi_0 - \pi / 2)$$

Step 2: Express time-dependent variables as phasors

- In general, we write any time-dependent variable $z(t)$ as

$$z(t) = \mathrm{Re}\left[\tilde{Z} \exp(j\omega t)\right]$$

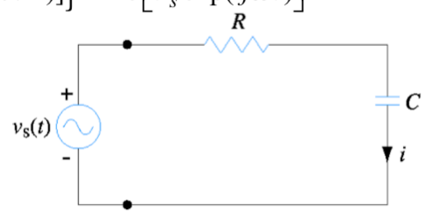
where $\tilde{Z}$ is time-independent and is referred to as a ***phasor***

- In our particular case, we write

$$v_s(t) = \mathrm{Re}\left\{V_0 \exp[j(\omega t + \phi_0 - \pi / 2)]\right\} = \mathrm{Re}\left[\tilde{V}_s \exp(j\omega t)\right]$$

where

$$\tilde{V}_s = V_0 \exp[j(\phi_0 - \pi / 2)]$$



Ulaby Fig. 1-19

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.22

Note that we are using the multiplication of exponentials here.

Complex Numbers and Phasors

Step 2: Express time-dependent variables as phasors (continued)

- We now write $i(t) = \mathrm{Re}\left[\tilde{I}\exp(j\omega t)\right]$, where $i(t)$ and $\tilde{I}$ are unknown

  The goal is to solve for $\tilde{I}$, knowing $\tilde{V}_s$, which will allow us to find $i(t)$

- We will make use of two important properties

$$\frac{d\,i(t)}{dt} = \mathrm{Re}\left[j\omega\tilde{I}\exp(j\omega t)\right] \quad \text{and} \quad \int i(t)\,dt = \mathrm{Re}\left[\frac{1}{j\omega}\tilde{I}\exp(j\omega t)\right]$$

Step 3: Recast the equation in phasor form

$$R\,\mathrm{Re}\left[\tilde{I}\exp(j\omega t)\right] + \frac{1}{C}\mathrm{Re}\left[\frac{1}{j\omega}\tilde{I}\exp(j\omega t)\right] = \mathrm{Re}\left[\tilde{V}_s\exp(j\omega t)\right]$$

This equation holds at all points in time if and only if

$$\left(R + \frac{1}{j\omega C}\right)\tilde{I} = \tilde{V}_s \qquad \text{(phasor/frequency/Fourier domain)}$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

2.23

Ulaby refers to this equation as the “phasor domain” equation. As he notes, it is essentially the frequency or Fourier domain, and we are in effect determining the Fourier coefficient of the frequency omega for the current from the Fourier coefficient for the voltage. If the driver has multiple frequencies, we solve for each frequency separately and add them together. In linear systems, the different frequencies can all be treated independently.

Clearly, the phasor equation is sufficient to guarantee that the real part holds. Why is it necessary? The reason is that when omega*t = pi/2, exp(j*pi/2) = j multiplies every term, and we see that the imaginary parts also MUST be equal. Adding the imaginary part to the real part and cancelling out the common factor of exp(j*omega*t) leads to the phasor equation.

Complex Numbers and Phasors

Step 4: Solve the phasor equation

$$\tilde{I} = \frac{\tilde{V}_s}{R + 1/(j\omega C)} = V_0 \exp[j(\phi_0 - \pi/2)] \left[\frac{j\omega C}{1 + j\omega CR} \right]$$

$$= \frac{V_0 \omega C}{\left(1 + \omega^2 R^2 C^2\right)^{1/2}} \exp[j(\phi_0 - \phi_1)], \quad \text{where} \;\; \phi_1 = \tan^{-1}(\omega RC)$$

Step 5: Solve the time domain equation

$$i(t) = \mathrm{Re}\left[\tilde{I} \exp(j\omega t) \right] = \frac{V_0 \omega C}{\left(1 + \omega^2 R^2 C^2\right)^{1/2}} \cos(\omega t + \phi_0 - \phi_1)$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.24

Ulaby also has an example of an RL circuit. This approach works with any circuit --- or any wave for that matter.

Ulaby has a complete set of time domain <--> phasor domain equations in Table 1-5.

Assignment

Reading: Ulaby, et al. Chapter 2

Problem Set 1: Some notes

- There are 8 problems. Many of the answers to these problems have been provided by either Ulaby or by me. YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- A key issue in numerical calculation is not presenting more significant figures than you have. You cannot have more significant figures than are in your input data. Please watch that; Ulaby is not careful about it.
- Generally, I ask for 3 significant figures, which means that you want to calculate with at least 4. When I want a different number, I tell you. Sometimes, I ask you *why* I want more.

UMBC
AN HONORS UNIVERSITY IN MARYLAND

2.25

We are beginning the discussion of transmission lines!

Paul (for example) makes a good point about the issues with translating English units. Essentially, the work is an extensive exercise in unit translation. It is important to feel comfortable with that.

At the same time he is sloppy about the number of significant figures. An example is where he uses 3.00 x 10^8 for the speed of light, which is only good to three places, but presents frequencies or wavelengths with five places. Ulaby is no better.