Problem Set #8 Solutions

1. For the most part, we are just applying the basic wave relationships to electromagnetic waves.

   a. We have $\lambda = 2\pi/k = 6.28/0.25 = 25.1 \to 25$ m.

   b. We have $f = u_\mathrm{p}/\lambda = (2.00 \times 10^8)/(25.1) = 7.96 \times 10^6 \to 8.0$ MHz. For future reference, we note that the radial frequency is given by $\omega = 2\pi f = k u_\mathrm{p} = 5.0 \times 10^7$ s^{-1}.

   c. Using the relationship $\epsilon_\mathrm{r} = c^2/\mu_\mathrm{r} u_\mathrm{p}^2$, we have $\epsilon_\mathrm{r} = (8.99 \times 10^{16})/[(2.70) \times (4.00 \times 10^{16})] = 0.833 \to 0.83$.

   d. Here, we will use the relation $\widetilde{\mathbf{H}} = \hat{\mathbf{k}} \times (\widetilde{\mathbf{E}}/\eta)$. We have (to three significant figures for this intermediate calculation), $\eta = 377\sqrt{\mu_\mathrm{r}/\epsilon_\mathrm{r}} = 377 \times 1.800 = 678\ \Omega$. We also note that the wave is propagating in the $-z$-direction, so that $\hat{\mathbf{k}} = -\hat{\mathbf{z}}$. We thus have $\widetilde{\mathbf{H}} = -(\hat{\mathbf{z}} \times \hat{\mathbf{y}})(\widetilde{\mathbf{E}}/\eta) = \hat{\mathbf{x}}(15.0/678)\exp(j0.25z) = \hat{\mathbf{x}}\,0.0221\exp(j0.25z)$ A/m. We conclude $\mathbf{H}(z,t) = \hat{\mathbf{x}}\,0.022\cos(5.0 \times 10^7\,t + 0.2\,z)$ A/m.

2. a. We have $f = (8\pi \times 10^9)/2\pi = 4.00 \times 10^9 \to 4.0$ GHz; $u_\mathrm{p} = c/\sqrt{\epsilon_\mathrm{r}} = (3.00 \times 10^8)/1.66 = 1.81 \times 10^8 \to 1.8 \times 10^8$ m/s; $\lambda = u_\mathrm{p}/f = (1.81 \times 10^8)/(4.00 \times 10^9) = 4.52 \times 10^{-2} \to 4.5$ cm; $k = 2\pi/\lambda = 139 \to 140\ \mathrm{m}^{-1}$; $\eta = 377/\sqrt{\epsilon_\mathrm{r}} = 377/1.66 = 227 \to 230\ \Omega$.

   b. Since the propagation is in the $+z$-direction, the **H**-field must be directed in the $-x$-direction, and we have that its amplitude is given by $H_0 = E_0/\eta = 25/227 = 0.1100$ A/m, so the full expression for the magnetic field becomes

   $$\mathbf{H}(z,t) = -\hat{\mathbf{x}}\,110\cos(8\pi \times 10^9\,t - 139z)\ \mathrm{mA/m}$$

3. The first two polarization states are linear and the last two are elliptical. The polarization angles are explicitly as follows:

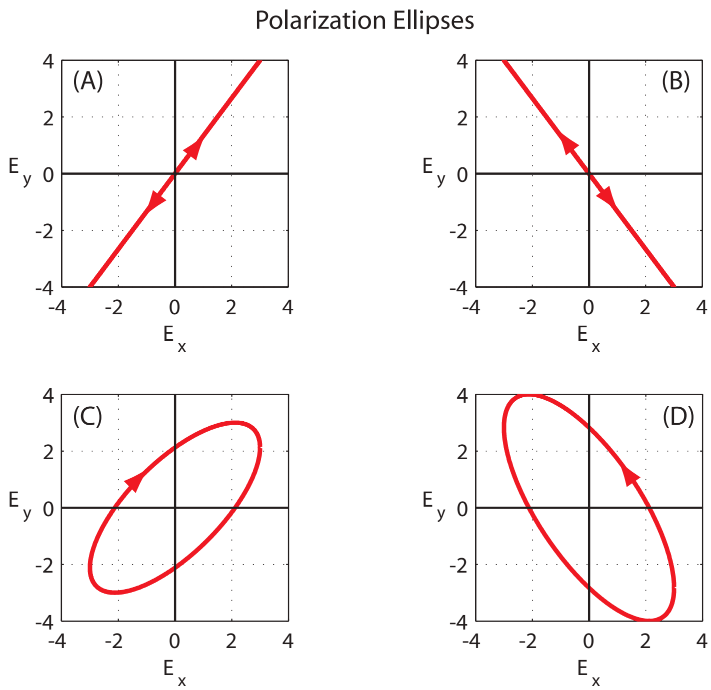
   a. $\psi_0 = \tan^{-1}(4/3) = 0.9273 \simeq 53°$, $\gamma = \psi_0 = 0.9273 \simeq 53°$, $\chi = 0$

   b. $\psi_0 = \tan^{-1}(4/3) = 0.9273 \simeq 53°$, $\gamma = -\psi_0 = 0.9273 \simeq -53°$, $\chi = 0$

   c. $\psi_0 = \tan^{-1}(1) = 0.7854 = 45°$,
   $\gamma = (1/2)\tan^{-1}[\tan(1.5708)\cos(0.7854)] = 0.7854 = 45°$,
   $\chi = (1/2)\sin^{-1}[\sin(1.5708)\sin(0.7854)] = 0.3927 = 22.5°$

   d. $\psi_0 = \tan^{-1}(4/3) = 0.9273 \simeq 53°$,
   $\gamma = (1/2)\tan^{-1}[\tan(1.8546)\cos(-2.3562)] = 0.5898 \simeq 34°$,
   $\chi = (1/2)\sin^{-1}[\sin(1.8546)\sin(-2.3562)] = -0.3731 \simeq -21°$

   We show next the loci for the four cases that we are considering. The arrows indicate the direction in which the tip of the field vector moves when it is being observed at a fixed location. The direction is ambiguous in the case of linear polarization since the field vector moves in both directions along the locus at different times.



The MATLAB listing follows:

```
% Ellipse
%
% Plots the polarization ellipse for four different choices
% of amplitudes and phase differences

% Input parameters
Ax = [3.0 3.0 3.0 3.0];      % Field x-amplitudes
Ay = [4.0 4.0 3.0 4.0];      % Field y-amplitudes
delta = [0.0 180.0 45.0 -135.0];     % Phase offsets in degrees

angle = (pi/180)*delta       % Phase offsets in radians

% Calculate and plot loci
theta = 0:0.001:1;  theta = theta*2*pi;      %set up angle array
for iplot = 1:4
    Ex = Ax(iplot)*cos(theta);  Ey=Ay(iplot)*cos(theta+angle(iplot));
        %determine the electric and magnetic fields
    subplot(2,2,iplot), plot(Ex,Ey,'r','linewidth',2)
    hold on
    subplot(2,2,iplot), plot([-4 4], [0 0],'k','linewidth',1)
    subplot(2,2,iplot), plot([0 0], [-4 4],'k','linewidth',1)
    axis square
    axis([-4.0 4.0 -4.0 4.0])
    grid
    xlabel('E_x');  ylabel('E_y');
    hold off
end
```

4. Since the real and imaginary parts of η_c are equal, we are in the limit of a good conductor. We first note $\alpha = 1/\delta_s = 0.500\ \mathrm{m}^{-1}$. We also note that we may usefully rewrite $\eta_c = (1+j)(35.0/\sqrt{2}) = (1+j)24.7 \to (1+j)25\ \Omega$.

   a. We have $\sigma = (1+j)\alpha/\eta_c = 0.500/24.7 \to 20$ mS/m.

b. We have $\lambda = 2\pi/\beta = 2\pi/\alpha = 2\pi\delta_{\mathrm{s}} \to 13$ m.

c. We have $u_{\mathrm{p}} = \lambda f = 12.56 \times (1.00 \times 10^{6}) \to 1.3 \times 10^{7}$ m/s.

5. For copper, we have $\epsilon_{\mathrm{r}} = 1.0$, $\mu_{\mathrm{r}} = 1.0$, and $\sigma = 5.8 \times 10^{7}$ S/m. The skin depth in copper at 1 kHz is given by $\delta_{\mathrm{c}} = (\pi f \mu\sigma)^{-1/2} = 0.209$ cm, which is considerably smaller than the block height of 20 cm. The a-c resistance for flow in the x-direction is given by $R_{\mathrm{a-c}} = l/w\delta_{\mathrm{s}}\sigma$, where l is the length of the slab in the x-direction and w is the width in the y-direction. We have that the usual d-c resistance is given by $R_{\mathrm{d-c}} = l/wd\sigma$, where d is the depth of the slab. Hence, we find that $R_{\mathrm{a-c}}/R_{\mathrm{d-c}} = d/\delta_{\mathrm{s}} = 20.0/0.209 \to 96$.

6. The corresponding magnetic field is given by $H(R) = (3000/377R) = 7.96/R$ A/m. The Poynting flux is thus $S = 0.5 \times 3000 \times 7.96/R^{2} = 1.19 \times 10^{4}/R^{2}$ W/m^{2}. Converting units, this value becomes $S = 1.19 \times 10^{3}/R^{2}$ mW/cm^{2}. Hence, we must have $R^{2} > 1.19 \times 10^{3}$ or $R > 35$ m. We would probably want $R > 40$ m to be really safe.