Sabbir Ahmed

**DATE:** March 24, 2018

**CMPE 320:** HW 05

---

1. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let $X$ and $Y$ be the price changes of A and B, respectively, over a certain time period. And assume that the joint PMF of $X$ and $Y$ is uniform over the set of integers $x$ and $y$ satisfying

$$-2 \leq x \leq 4 \qquad\qquad -1 \leq y - x \leq 1.$$

(a) Find the marginal PMFs and the means of $X$ and $Y$.

Given:

$$X \in \{x : -2 \leq x \leq 4\}$$

$$Y \in \{y : x - 1 \leq y \leq x + 1,\ x \in X\}$$

Therefore, the pairs $(x, y)$ consist of:

$$\begin{aligned}(x, y) \in \{&(-2,-3),(-2,-2),(-2,-1),\\ &(-1,-2),(-1,-1),(-1,0),\dots,\\ &(4,3),(4,4),(4,5)\}\end{aligned}$$

Totalling in $7 \times 3 = 21$ pairs.

Therefore, the joint PMF is

$$p_{X,Y}(x, y) = \begin{cases} 1/21, & \text{if } -2 \leq x \leq 4, -1 \leq y - x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The marginal PMF are

$$\begin{aligned} p_X(x) &= \sum_y p_{X,Y}(x, y) \\ &= \begin{cases} 3/21, & \text{if } -2 \leq x \leq 4, \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

$$p_Y(y) = \sum_x p_{X,Y}(x,y)$$

$$= \begin{cases} 1/21, & \text{if } y = -3, 5, \\ 2/21, & \text{if } y = -2, 4, \\ 3/21, & \text{if } -1 \leq x \leq 3, \\ 0, & \text{otherwise} \end{cases}$$

The means,

$$E[X] = \sum_x x \cdot p_X(x)$$

$$= \frac{3}{21}((-2) + (-1) + 0 + 1 + 2 + 3 + 4)$$

$$= \frac{3}{21}(7)$$

$$= 1$$

$$E[Y] = \sum_y y \cdot p_Y(y)$$

$$= \frac{1}{21}((-3) + 5) + \frac{2}{21}((-2) + 4) + \frac{3}{21}((-1) + 0 + 1 + 2 + 3)$$

$$= 1$$ □

(b) Find the mean of the trader's profit.

$$100E[X] + 200E[Y] = 100(1) + 200(1)$$

$$= 300$$ □

**2**. The MIT football team wins any one game with probability $p$, and loses it with probability

$1 - p$. Its performance in each game is independent of its performance in other games. Let $L_1$ be the number of losses before its first win, and let $L_2$ be the number of losses after its first win and before its second win. Find the joint PMF of $L_1$ and $L_2$.

For $L_1 = 0$, $L_2 = 0$,

$$\begin{aligned} P(L_1 = 0, L_2 = 0) &= p \cdot p \\ &= p^2 \end{aligned}$$

For $L_1 = 0$, $L_2 = 1$,

$$\begin{aligned} P(L_1 = 0, L_2 = 1) &= p \cdot ((1-p) \cdot p) \\ &= p^2(1-p) \end{aligned}$$

Similarly, for $L_1 = 1$, $L_2 = 0$,

$$\begin{aligned} P(L_1 = 1, L_2 = 0) &= ((1-p) \cdot p) \cdot p \\ &= p^2(1-p) \end{aligned}$$

For $L_1 = 0$, $L_2 = 2$,

$$\begin{aligned} P(L_1 = 0, L_2 = 2) &= p \cdot ((1-p) \cdot (1-p) \cdot p) \\ &= p^2(1-p)^2 \end{aligned}$$

For $L_1 = 0$, $L_2 = 3$,

$$\begin{aligned} P(L_1 = 0, L_2 = 3) &= p \cdot ((1-p) \cdot (1-p) \cdot (1-p) \cdot p) \\ &= p^2(1-p)^3 \end{aligned}$$

And so on. Therefore, the general expression is:

$$p^2(1-p)^{L_1+L_2}$$

with the PMF:

$$p_{L_1,L_2}(L_1, L_2) = p^2(1-p)^{L_1+L_2} \qquad \square$$

**3**. A class of $n$ students take a test in which each student gets an A with probability $p$, a B with probability $q$, and a grade below B with probability $1-p-q$, independently of any other student. If $X$ and $Y$ are the numbers of students that get an A and a B, respectively. calculate the joint PMF $p_{x,y}$.

$\square$

**4**. Your probability class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let $X$ be the number of students that get an A in your class.

(a) Calculate $E[X]$ by first finding the PMF of $X$

Let $Y$ and $Z$ represent the number of undergraduate and graduate students who receive an A, respectively.

$$\begin{aligned} p_Y(y) &= \binom{250}{y}\left(\frac{1}{3}\right)^y\left(1-\frac{1}{3}\right)^{250-y} \\ &= \binom{250}{y}\left(\frac{1}{3}\right)^y\left(\frac{2}{3}\right)^{250-y} \\ p_Z(z) &= \binom{50}{z}\left(\frac{1}{2}\right)^z\left(1-\frac{1}{2}\right)^{50-z} \\ &= \binom{50}{z}\left(\frac{1}{2}\right)^{50} \end{aligned}$$

$\square$

(b) Calculate $E[X]$ by viewing $X$ as a sum of random variables, whose mean is easily calculated.

$\square$

**5**. A scalper is considering buying tickets for a particular game. The price of the tickets is \$75, and the scalper will sell them at \$150. However, if she can't sell them at \$150, she won't sell them at all. Given that the demand for tickets is a binomial random variable with parameters $n = 10$ and $p = 1/2$, how many tickets should she buy in order to maximize her expected profit?

$$\begin{aligned} i &= (n+1)p \\ &= (10+1)(0.5) \\ &= 5.5 \approx 6 \end{aligned}$$

Therefore, she should buy 6 tickets in order to maximize her expected profit □

**6**. Suppose that $X$ and $Y$ are independent discrete random variables with the same geometric PMF:

$$p_X(k) = p_Y(k) = p(1-p)^{k-1} \qquad\qquad k = 1, 2, \ldots,$$

where $p$ is a scalar with $0, p < 1$. Show that for any integer $n \geq 2$, the conditional PMF

$$P(X = k \mid X + Y = n)$$

is uniform.

□

**7**. Consider four independent rolls of a 6-sides die. Let $X$ be the number of 1s and let $Y$ be the number of 2s obtained. What is the joint PMF of $X$ and $Y$?

□

**8**. Alvin shops for probability books for $K$ hours, where $K$ is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books $N$ that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N|K}(n \mid k) = \frac{1}{k}, \qquad\qquad \text{for } n = 1, \ldots, k$$

(a) Find the joint PMF of $K$ and $N$

□

(b) Find the marginal PMF of $N$

□

(c) Find the conditional PMF of $K$ given that $N = 2$

□

(d) Find the conditional mean and variance of $K$, given that he bought at least 2 but no more than 3 books.

□

(e) The cost of each book is a random variable with mean $30. What is the expected value of his total expenditure? *Hint:* Condition on the events $\{N = 1\}, \ldots, \{N = 4\}$, and use the total expectation theorem.

□