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MATH 407 HW 03

1.1 **4** Use the Euclidean algorithm to find the following greatest common divisors.

a $(6643, 2873)$

Ans $GCD(6643, 2873)$

$$6643 = 2873 \cdot 2 + 197$$
$$2873 = 897 \cdot 3 + 182$$
$$897 = 182 \cdot 4 + 169$$
$$182 = 169 \cdot 1 + 13$$
$$169 = 13 \cdot 13 + 0$$

$\therefore GCD(6643, 2873) = 13$ $\square$

c $(26460, 12600)$

Ans $GCD(26460, 12600)$

$$26460 = 12600 \cdot 2 + 1260$$
$$12600 = 1260 \cdot 10 + 0$$

$\therefore GCD(26460, 12600) = 1260$ $\square$

e $(12091, 8439)$

Ans $GCD(12091, 8439)$

$$12091 = 8439 \cdot 1 + 3652$$
$$8439 = 3652 \cdot 2 + 1135$$
$$3652 = 1135 \cdot 3 + 247$$
$$1135 = 247 \cdot 4 + 147$$
$$247 = 147 \cdot 1 + 100$$
$$147 = 100 \cdot 1 + 47$$
$$100 = 47 \cdot 2 + 6$$
$$47 = 6 \cdot 6 + 5$$
$$6 = 5 \cdot 1 + 1$$
$$5 = 1 \cdot 5 + 0$$

$\therefore GCD(12091, 8439) = 1$ □

6 For each part of Exercise 4, find integers m and n such that (a, b) is expressed in the form $ma + nb$.

a $(6643, 2873)$

Ans □

c $(26460, 12600)$

Ans □

e $(12091, 8439)$

Ans □

7 Let a, b, c be integers. Give a proof for these facts about divisors:

a If $b \mid a$, then $b \mid ac$.

Ans Let $a = mb$, $m \in \mathbb{Z}$.

Multiplying both sides by c:

$a \cdot c = mb \cdot c$

$a \cdot c = mc \cdot b$ (commutative law of multiplication)

Let $n = mb$, $n \in \mathbb{Z}$.

$a \cdot c = n \cdot c$

$\therefore b \mid ac$ if $b \mid a$ □

b If $b \mid a$ and $c \mid b$, then $c \mid a$.

Ans Let $a = mb$ and $b = nc$ for $m, n \in \mathbb{Z}$

$\because b = nc$, $c = b/n$. $\square$

c If $c \mid a$ and $c \mid b$, then $c \mid (ma + nb)$ for any integers m, n.

Ans $\square$

11 Show that if $a > 0$, then $(ab, ac) = a(b, c)$

Ans $\square$

14 For what positive integers n is it true that $(n, n+2) = 2$? Prove your claim.

Ans $\square$

17 Let a, b, n be integers with $n > 1$. Suppose that $a = nq_1 + r_1$ with $0 \le r_1 < n$ and $b = nq_2 + r_2$ with $0 \le r_2 < n$. Prove that $n \mid (a - b)$ if and only if $r_1 = r_2$.

Ans $\square$

19 Let a, b, q, n be integers such that $b \neq 0$ and $a = bq + r$. Prove that $(a, b) = (b, r)$ by showing that (b, r) satisfies the definition of the greatest common divisor of a and b.

Ans $\square$