

## CMPE323 What did we learn

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Week 1 1-1

## Linear, Time Invariant Systems

- **LTI is not everything!**
- **There are many non-linear, non time invariant systems.**
- **But LTI is so common that it is the largest class of systems**
- **Even non linear systems (think GPS) often act like linear systems in the neighborhood of an operating point...**
- **...through use of a first-order Taylor Series expansion**

$$\hat{y}(t;c) \approx y(t;c_0) + \frac{\partial y(t;c)}{\partial c} \Delta c + \mathcal{O}(\Delta c^2)$$

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Week 1 1-2

## Convolution

- Convolution is the sliding & integrating process

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

- ...where we frequently have to evaluate the integral differently in different regions of  $t$
- Remember that the integration is with respect to  $\tau$ , and  $t$  is a parameter when doing the integration!
- There should be no dependence on  $\tau$  after the integration...
- ...although there will obviously be a dependence on  $t$
- Convolution is a *filtering operation*

## Laplace Domain

- The Laplace domain (s-domain) is frequently used for characterizing the performance of linear systems
- ...but not the characteristics of linear signals
- $\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$
- The transfer function  $H(s)$  is the Laplace Transform of the impulse response  $h(t)$  of that system
- The Laplace transform includes specification of the Region of Convergence.
- Many Laplace transforms are rational polynomials in

$$X(s) = \frac{\prod_{k=1}^M (s - z_k)}{\prod_{k=1}^B (s - p_k)}$$

## Inverse Laplace Transforms

- We generally invert Laplace Transforms by means of evaluating the Residues, also known as Partial Fraction Expansion

$$X(s) = \frac{\prod_{k=1}^M (s - z_k)}{\prod_{k=1}^N (s - p_k)} = \sum_{m=1}^N \frac{R_m}{(s - p_m)}$$

- ... with explicit process for dealing with roots of higher order
- Poles and residuals must occur as either real or in complex conjugate pairs

## Right-sided and left-sided signals

- A right sided signal exists for  $t > t_0$
- A left sided signal exists for  $t < t_0$
- A two sided signal exists for  $t_1 < t < t_2$
- For right sided signals, RoC is  $\text{Re}[s] > a$
- For left sided signals, RoC is  $\text{Re}[s] < a$
- For two sided signals, RoC is  $a < \text{Re}[s] < b$
- There are no poles in the Region of Convergence
- For causal signals, all the poles must be in the left half plane
- For anti causal signals, all the poles must be in the right half plane

## Fourier Transforms

- The Fourier (  $\omega$ ,  $f$ , or  $j\omega$  ) domain is used to analyze signals, generally not systems.

- A periodic (power) signal has a Fourier Series

$$c_k = \frac{1}{T} \int_a^{a+T} x(t) e^{-j\frac{2\pi k}{T}t} dt, \quad x(t) = \sum_{n=-\infty}^{\infty} c_k e^{j\frac{2\pi n}{T}t}$$

- A non-periodic (energy) signal has a Fourier Transform


$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = X(f)$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

## The convolution theorem

- The convolution theorem ties the time and Laplace/Fourier domains together


$$\mathcal{L}(x(t) * h(t)) = X(s)H(s)$$



- **Sampling in one domain indicates that the signal is periodic in the other domain**
- **Periodicity in one domain indicates that the signal is sampled (discrete) in the other domain.**
- **If a signal is periodic and sampled in one domain, it is sampled and periodic in the other domain.**

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Week 1 1-9



- **The DFT is a numerical computation that approximates a scaled version of the Fourier Transform**

$$X_{DFT}[n] = \sum_{k=0}^{N-1} x[k] e^{-j \frac{2\pi kn}{N}}$$

- **The DTFT is the Fourier transform of the impulse-sampled piecewise continuous time signal and is continuous in frequency**

$$X_{DTFT}(\omega) = \sum_{k=-\infty}^{\infty} x(t) e^{-j\omega k \Delta t}$$

- **The FFT is an efficient algorithm for computing the DFT. It is most efficient if  $N = 2^M$ . It implicitly assumes both the time and frequency domain signals are periodic, but has no explicit identification with time of frequency**

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## Analog Filters

- A Butterworth filter has a transfer function

$$H(j\omega) = H(s)_{s=j\omega}$$

- ... that is monotonically decreasing away from the center of the passband.
- A Chebyshev Type 1 filter has a transfer function that has ripples in the pass band, but is monotonically decreasing outside of the band
- A Chebyshev Type 2 filter has a transfer function that has ripples in the stop band, but is monotonically decreasing inside the pass band.