CMPE323 Lecture 10 LTI and Block Diagrams

1

- **Exam #1 Wed 10/5/2016**
- **Content**
  - **Signals, decomposition, synthesis**
  - **Time manipulation, delay, advance, compress, expand, linear**
  - **Even & odd**
  - **Differentiation, integration**
- **Systems, Linear, TI, Causal, Static**
- **Convolution, graphic, analytic, properties**

2

System applications

- **In systems application we're much less worried about the particular and homogeneous solutions than we are about the solution with and without an input**

- **The zero state solution**

$$y_{ZS}(t) = y(t)\big|_{Y_I=0} = \left(\frac{1}{a-b}\right)\left(e^{-bt} - e^{-at}\right)u(t)$$

- **The zero input solution** $y_{ZI}(t) = y(t)\big|_{x(t)=0} = y_h(t) = Y_I e^{-at}$

- **And** $y(t) = y_{ZS}(t) + y_{ZI}(t)$

Is the system described linear?

- **The linear combinations have to work for any valid inputs, so choose** $x_1(t) = \alpha e^{-bt}u(t),\ x_2(t) = -x_1(t)$

- **Then, from our solution**

$$y_1(t) = Y_I e^{-at} + \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

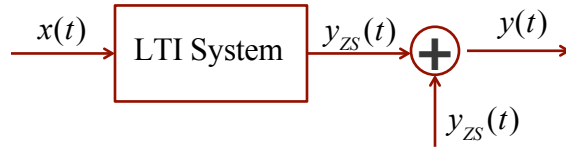$$y_2(t) = Y_I e^{-at} - \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

- **If we let** $x(t) = x_1(t) + x_2(t) = 0$ **, a linear system will give**

$$y(t) = Y_I e^{-at} + \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t) + Y_I e^{-at} - \frac{1}{\alpha}\left(\frac{1}{a-b}\right)(e^{-bt} - e^{-at})u(t)$$

$$y(t) = Y_I e^{-at} + Y_I e^{-at} = 2Y_I e^{-at} = 0 \Leftrightarrow Y_I = 0$$

- **So this system is linear iff it is initially at rest!**

In general

$$x(t) \longrightarrow \boxed{\text{LTI System}} \xrightarrow{y_{ZS}(t)} \; + \; \xrightarrow{y(t)}$$

$$y_{ZS}(t)$$

- A system described by a LCCDE is linear if and only if it is initially at rest.

- A causal system that is initially at rest is also Time Invariant

Block Diagrams of LCCDE

- How would we implement or synthesize a system if we had a LCCDE?

- Implementing differentiators is problematic because

  - High frequencies in the input are multiplied by a frequency ramp

$$\frac{d\sin(\omega_o t)}{dt} = \omega_o \cos(\omega_0 t)$$

  - Impulses can result from differentiation

- So we transform the equation into an integral equation

- Here are the steps

3

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}, \quad \frac{d^0 y(t)}{dx^0} \triangleq y(t)$$

If $M < N$, let $b_k = 0, \ k = M+1, M+2...N$

If $N < M$, let $a_k = 0, \ k = N+1, N+2...M$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} b_k \frac{d^k x(t)}{dt^k}, \quad \frac{d^0 y(t)}{dx^0} \triangleq y(t)$$

Now form a set of auxiliary variables or , $y_k(t)$ and $x_k(t)$

$$y_0(t) = y(t), \ x_0(t) = x(t)$$

$$y_1(t) = \int_{-\infty}^{t} y_o(\tau)d\tau, \ x_1(t) = \int_{-\infty}^{t} x_o(\tau)d\tau$$

$$y_2(t) = \int_{-\infty}^{t} y_1(\tau)d\tau, \ x_2(t) = \int_{-\infty}^{t} x_1(\tau)d\tau$$

etc...

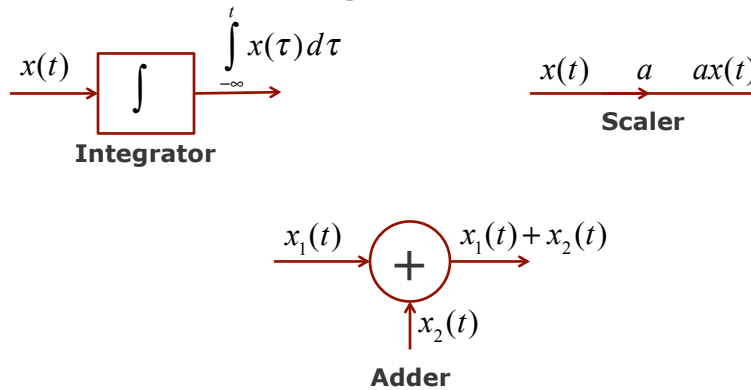$$y_N(t) = \int_{-\infty}^{t} y_{N-1}(\tau)d\tau, \ x_N(t) = \int_{-\infty}^{t} x_{N-1}(\tau)d\tau$$

$$\sum_{k=0}^{N} a_k y_{N-k}(t) = \sum_{k=0}^{N} b_k x_{N-k}(t) = \sum_{k=0}^{M} b_k x_{N-k}(t)$$

Where we have integrated both sides N times...
    requiring N initial conditions (one for each integral)

4

Basic Block Diagram Elements

- **We'll now use this "integrated" form to *synthesize* an implementation of the system described by the LCCDE**

- **We need three building blocks**

$$x(t) \quad \boxed{\int} \quad \overset{t}{\underset{-\infty}{\int}} x(\tau)\,d\tau$$

Integrator

$$x(t) \quad a \quad ax(t)$$

Scaler

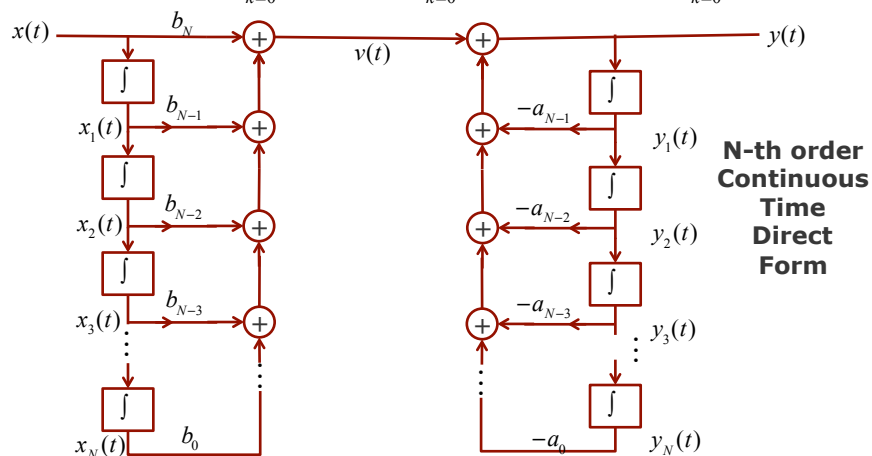$$x_1(t) \quad \bigoplus \quad x_1(t) + x_2(t)$$
$$x_2(t)$$

Adder

9

Synthesis

- **Assume $a_N = 1$ (or divide through to make it so)**

- **Rewrite**

$$y(t) = y_0(t) = \sum_{k=0}^{N} b_k x_{N-k}(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t) = v(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t)$$

N-th order Continuous Time Direct Form

10

5

Draw the Direct Form Implementation

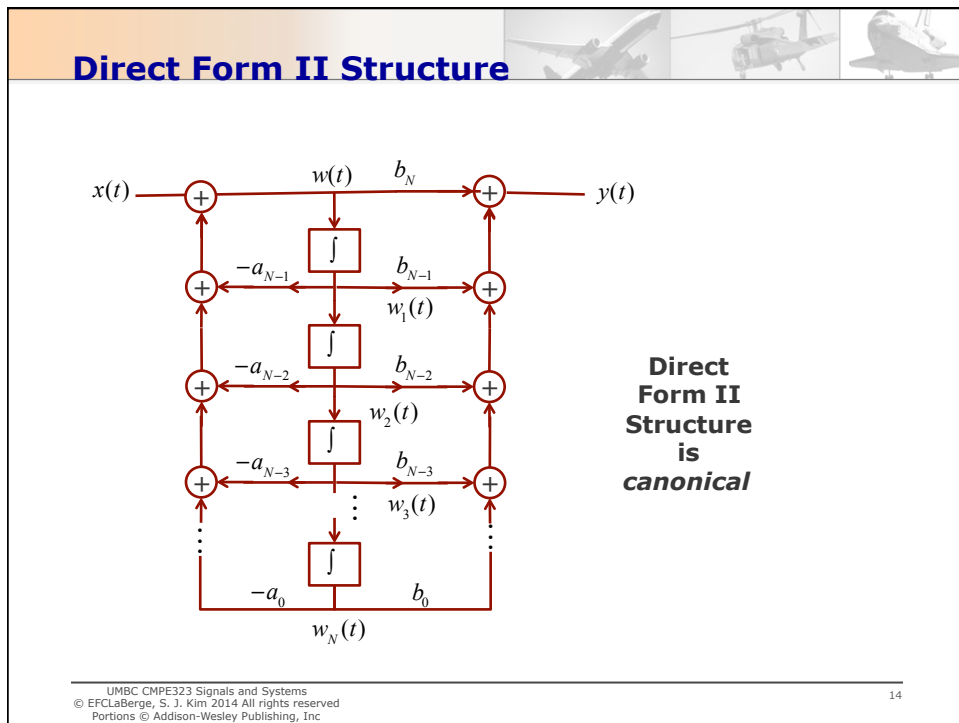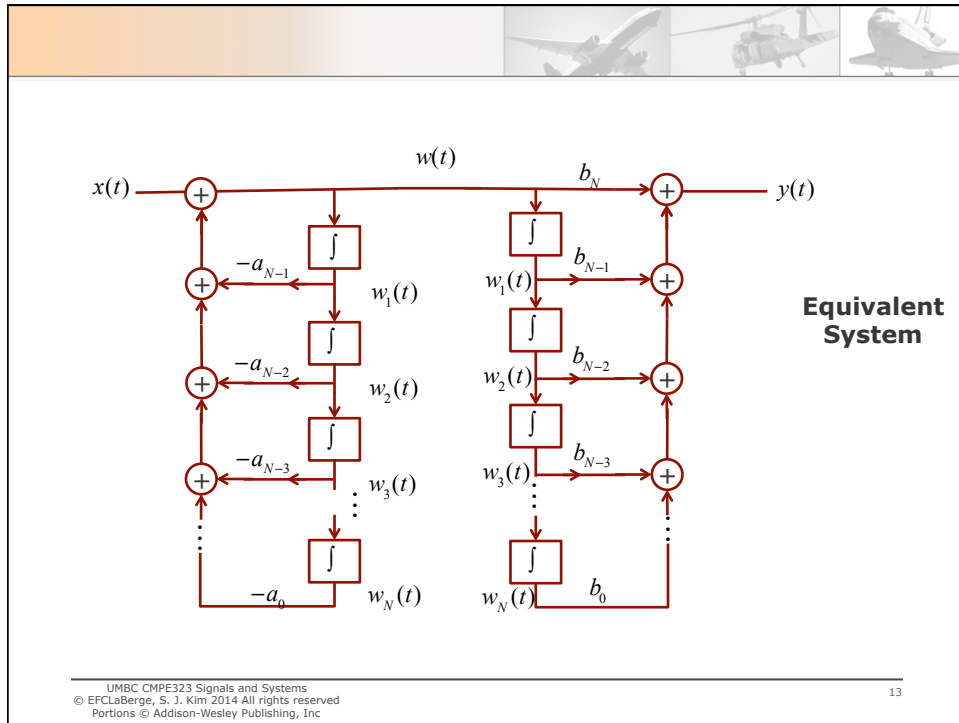- **Example** $\dfrac{dy(t)}{dt} + \alpha y(t) = \beta \dfrac{dx(t)}{dt} + \gamma x(t)$

Concatenation

- **Our direct form is the concatenation of two systems:**

$$v(t) = \sum_{k=0}^{N} b_k x_{N-k}(t)$$ → $$y(t) = v(t) - \sum_{k=0}^{N-1} a_k y_{N-k}(t)$$

- **Convolution is associative and commutative, so we can swap the order (defining a new intermediate fnc)**

$$w(t) = x(t) - \sum_{k=0}^{N-1} a_k x_{N-k}(t)$$ → $$y(t) = \sum_{k=0}^{N} b_k w_{N-k}(t)$$

6

Equivalent System

Direct Form II Structure

Direct Form II Structure is _canonical_

- **Consider**

$$2\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} + 3x(t)$$

- **Draw the Direct Form II implementation**

15

- **Define the centroid of a signal $x(t)$ to be**

$$C_x = \int_{-\infty}^{\infty} \tau x(\tau)\,d\tau / A_x, \text{ where } A_x = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

- **Define the k-th central moment of a signal to be**

$$\sigma_x^k = \int_{-\infty}^{\infty} (\tau - C_x)^k x(\tau)\,d\tau / A_x$$

- **Define our usual unit pulse** $p(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & otherwise \end{cases}$

- **Do the following in MATLAB**
  - **1) Compute** C_p, σ_p^2 for the shifted pulse $p(t+0.5)$
  - **2) Convolve** $p(t+0.5)$ **with itself** $p_2(t) = p(t+0.5)*p(t+0.5)$ **and compute** $C_{p_2}, \sigma_{p_2}^2$
  - **3) Convolve** $p_2(t)$ with itself $p_4(t) = p_2(t)*p_2(t)$ **and compute** $C_{p_4}, \sigma_{p_4}^2$
  - **4) Repeat to compute** $p_8(t) = p_4(t)*p_4(t)$, $C_{p_8}, \sigma_{p_8}^2$
  - **5) Provide plots. Superimpose the plot of** $p_8(t)$ **with** $Ke^{-(t-C_{p_8})^2/(2\sigma_{p_8}^2)}$**, choosing K to make the peaks match**

16