MEMO Number CMPE323-Lab04 rev1

DATE: October 28, 2016
TO: CMPE323
FROM: EFC LaBerge
SUBJECT: Fourier Series and the Gibbs Phenomenon

1 INTRODUCTION

This lab explores the computation of the complex coefficients of a Fourier Series by direction computation and by evaluation of the analytical answer obtained in class.

2 EQUIPMENT

For this lab, you need a laptop with MATLAB installed.

For the purpose of CMPE323, please use the following naming conventions for all output files:

CMPE323F16_Lab<Lab#>_<Your Campus ID>

For the purpose of CMPE323, please use the following naming conventions for MATLAB scripts or functions that you are required to submit.

<function name>_<Your Campus ID>

Examples will be given in the lab description. Follow the instructions exactly, or you may not get graded!

3 LAB TASKS

You might find it useful to use the MATLAB function `diary` to capture your inputs and outputs.

3.1 Computing the coefficients

Using your pulse anonymous function create an "infinite" square wave

$$x(t)=\sum_{n=-N}^{N} p(t-nT)$$ 22* MERGEFORMAT ()

where $p(t)=pulse(t+0.5,1)$, that is $\tau=1$ is the duration or "ON" time of the pulse. Let the period of the waveform by $T=4$. Plot at least seven periods of your periodic waveform, that is, $N\geq 3$. Use a sample rate of at least 10,000 sps. Is this waveform even or odd? Plot the waveform using professional practices.

Analytically compute the Fourier coefficients $c_k$given by the analysis equation

$$c_k=\frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)e^{-j\omega_0 kt},\ \omega_0=2\pi f_0=\frac{2\pi}{T}$$ 33* MERGEFORMAT ()

The integral can extend over any complete period of the square wave. Choose a specific interval and indicate why it is appropriate. Write the closed form, simplified expression for the coefficients as a function of T, τ, and k.

Using the simulated $x(t)$ developed above, numerically compute the coefficients for $x(t)$ implementing 3. Compute the range $k = [-800:800]$ and hold onto it for future use. Your answer will be complex. Examine the imaginary part $\left(\mathrm{Im}[c_k]\right)$ and determine if it contributes to the answer. Based on your conclusion, plot the computed values and the theoretical values and compare them.

3.2 Synthesizing the waveform

Using the c_k computed in 3.1, synthesize an estimate of your periodic waveform over the full time interval (not just one period) you computed in 3.1. The synthesis equation is

$$\hat{x}(t) = \sum_{k=-K}^{K} c_k e^{j\omega_0 kt} ,$$ 44* MERGEFORMAT ()

where K controls the number of terms in the estimate.

Compute estimates for ***`K=[10 50 100 200 400 800].`***

For each estimate, plot the real part of your estimate (the imaginary part should be negligible), and your original waveform, $x(t)$. Comment on any differences.

For each estimate (one estimate for each K), compute the *mean squared error (MSE)* defined by

$$MSE = \frac{1}{T}\int_{\alpha}^{\alpha+T} \left(x(t) - \hat{x}(t)\right)^2 dt$$ 55* MERGEFORMAT ()

Plot the estimates as a function of the value of *K*.

3.3 The Gibbs Phenomenon

The finite sum estimate, $\hat{x}(t)$, of the square wave $x(t)$ exhibits a characteristic shape known as the Gibbs Phenomenon. This characteristic occurs whenever a limited number of Fourier coefficients (or a bandlimited Fourier Transform) is used to estimate a piecewise continuous time waveform. On a single plot, show the Gibbs Phenomenon for all of your *K*-limited estimates of the square wave by plotting only the region $t = [0.45\ 0.55]$. Comment on the shape and extent of the Gibbs phenomenon as the number of terms in the sum increases.

3.4 A different waveform.

Create a new waveform, $z(t) = \sum_{n=-N}^{N} r(t - nT),$ where $r(t) = (t + 0.5)u(t + 0.5)$. This produces a periodic sawtooth wave. Repeat the process used in 3.1 and 3.2 to synthesize $\hat{z}(t)$ from *K*-sums of the Fourier coefficients. Comment on any Gibbs phenomenon effects on your estimates.

4 LAB SUBMISSIONS

Submit the following via the Blackboard assignment Lab 5.

Using this lab description document as a template, create a single PDF file named in accordance with the output naming conventions given above. The content must include

a. The outputs and discussions generated in 3.1.

b. The outputs and discussions generated in 3.2.

c. The outputs and discussions generated in 3.3.

d. The outputs and discussions generated in 3.4.

Professional, high quality writing, math, and graphic (that is plots) presentation is expected, and must be provided for you to earn full credit.