

1. Given,

Probability of not losing the first game: $p_1 = 0.4$

Probability of losing the first game: $p_1^c = 1 - 0.4 = 0.6$

Probability of not losing the second game: $p_2 = 0.7$

Probability of losing the second game: $p_2^c = 1 - 0.7 = 0.3$

Therefore, the $pmf(X)$ where $X = 0, 1, 2, 4$ represents the number of points earned over the weekend:

$$\begin{aligned}P(X = 0) &= p_1^c \cdot p_2^c \\&= 0.6 \cdot 0.3 \\&= 0.18\end{aligned}$$

$$\begin{aligned}P(X = 1) &= \frac{p_1^c \cdot p_2}{2} + \frac{p_1 \cdot p_2^c}{2} \\&= \frac{0.6 \cdot 0.7}{2} + \frac{0.4 \cdot 0.3}{2} \\&= 0.27\end{aligned}$$

□

2. Given $p = 1/649640$. Therefore,

$$\begin{aligned}P(X \geq 1) &= 1 - P(X = 0) \\&= 1 - \left(\frac{649640 - 1}{649640} \right)^{649640} \\&= 1 - \left(1 - \frac{1}{649640} \right)^{649640}\end{aligned}$$

If $n = 649640$

$$\begin{aligned}
 P(X \geq 1) &= 1 - \left(1 - \frac{1}{n}\right)^n \\
 &= 1 - \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n \\
 &= 1 - \frac{1}{e}
 \end{aligned}
 \quad \square$$

3. A claim is first filed with the probability

$$pq = (0.05)(1 - 0.05)^{n-1} = (0.05)(0.095)^{n-1} \quad \square$$

4. (a) $Y = X \pmod{3}$

$$\begin{aligned}
 P(Y = 0) &= P(X = \{0, 3, 6, 9\}) \\
 &= \frac{4}{10} \\
 &= 0.4
 \end{aligned}$$

$$\begin{aligned}
 P(Y = 0) &= P(X = \{0, 3, 6, 9\}) \\
 &= \frac{4}{10} \\
 &= 0.4
 \end{aligned}$$

$$\begin{aligned}
 P(Y = 1) &= P(X = \{1, 4, 7\}) \\
 &= \frac{3}{10} \\
 &= 0.3
 \end{aligned}$$

$$\begin{aligned}
 P(Y = 2) &= P(X = \{2, 5, 8\}) \\
 &= \frac{3}{10} \\
 &= 0.3
 \end{aligned}$$

(b) $Y = 5 \pmod{X + 1}$

$$\begin{aligned} P(Y = 0) &= P(X = \{0, 4\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y = 1) &= P(X = \{1, 5\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y = 2) &= P(X = \{2\}) \\ &= \frac{1}{10} \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} P(Y = 5) &= P(X = \{5, 6, 7, 8, 9\}) \\ &= \frac{5}{10} \\ &= 0.5 \end{aligned}$$

☐

5.

☐

6.

☐

7.

☐

8.

☐

9.

☐

10.

☐