Statistics 355 Worksheet 2

ANSWER KEY

Problem 1: Beverages

Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

Define the following events:
C = event that a randomly selected adult regularly consumes coffee
S = event that a randomly selected adult regularly consumes carbonated soda

$$P(C) = 0.55 \qquad P(S) = 0.45 \qquad P(C \cup S) = 0.70$$

(a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

$$P(C \cap S) = P(C) + P(S) - P(C \cup S) = 0.55 + 0.45 - 0.70 = \mathbf{0.30}$$

(b) What is the probability that a randomly selected adult does not regularly consume any of these two products?

$$P[(C \cup S)^c] = 1 - P(C \cup S) = 1 - 0.70 = \mathbf{0.30}$$

(c) What is the probability that a randomly selected adult only drinks coffee?

$$P(C \cap S^c) = P(C) - P(C \cap S) = 0.55 - 0.30 = \mathbf{0.25}$$

or, alternatively,
$$P(C \cap S^c) = P(C \cup S) - P(S) = 0.70 - 0.45 = \mathbf{0.25}$$

(d) What is the probability that a randomly selected adult likes **EXACTLY** one beverage?

$$P(C \cap S^c) + P(C^c \cap S) = P(C \cup S) - P(C \cap S) = 0.70 - 0.30 = \mathbf{0.40}$$

or, alternatively,

$$P(C \cap S^c) + P(C^c \cap S) = P(C) - P(C \cap S) + P(S) - P(C \cap S)$$
$$= P(C) + P(S) - 2P(C \cap S) = 0.55 + 0.45 - 2(0.30) = \mathbf{0.40}$$

Problem 2: Expired Juice

A batch of 500 containers for frozen orange juice contains five that are expired. Two are selected, at random, without replacement from the batch.

(a) What is the probability that both are not expired?

$$\frac{\binom{5}{0}\binom{495}{2}}{\binom{500}{2}} = \left(\frac{495}{500}\right)\left(\frac{494}{499}\right) = \mathbf{\color{red}0.9801}$$

(b) What is the probability that both are expired?

$$\frac{\binom{5}{2}\binom{495}{0}}{\binom{500}{2}} = \left(\frac{5}{500}\right)\left(\frac{4}{499}\right) = \mathbf{\color{red}0.0001}$$

(c) What is the probability that the second one selected is expired given that the first one was expired?

$$\frac{4}{499} = \mathbf{\color{red}0.0080}$$

Three containers are selected at random without replacement from the batch.

(a) What is the probability that the third one selected is expired given that the first and second ones selected were not expired?

$$\frac{5}{498} = \mathbf{\color{red}0.0100}$$

(b) What is the probability that all three are expired?

$$\frac{\binom{5}{3}\binom{495}{0}}{\binom{500}{3}} = \left(\frac{5}{500}\right)\left(\frac{4}{499}\right)\left(\frac{3}{498}\right) = \mathbf{\color{red}0.0000005}$$