Key

1 airline A   32 36 46 33 38

airline B   33 31 28 39 38

$\bar{x}_A = \frac{32+\dots+38}{5} = 36.8 \qquad s_A^2 = \frac{\sum_{i=1}^{5}(x_i-\bar{x}_A)^2}{4} = 26.7$

$\bar{x}_B = \frac{33+\dots+38}{5} = 33.8 \quad , \quad s_B^2 = 21.7$

a) 90% C.I for μ_A = as we are assuming normality and we don't have σ^2, with a small sample size

$\mu_A \in \bar{x}_A \mp t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \quad , \alpha = .1$

$\mu_A \in \left[36.8 \mp 2.132 \sqrt{\frac{26.7}{5}} \right] = [31.87 , 41.72]$

b) $n = \left\lceil \frac{2 t_{\alpha/2, n-1}\, s}{w} \right\rceil^2 = \left\lceil \frac{2(2.132(26.7))}{1} \right\rceil^2$

$= 485.4 \simeq \underline{486}$

c) a prediction interval = $\bar{x}_B \mp t_{\alpha/2, n_B-1}\, s_B \sqrt{1+\frac{1}{n_B}}$

$= 33.8 \mp 2.132 \sqrt{21.7} \sqrt{1+\frac{1}{5}} = [22.92 , 44.68]$

d) 90% lower C.I for σ_B^2 : $\sigma_B^2 \geq \frac{(n_B-1)\, s_B^2}{\chi^2_{\alpha, n-1}}$

$\sigma_B^2 \geq \frac{4(21.7)}{7.779} \Rightarrow \sigma_B^2 \geq 11.158$

$\boxed{e}$ $H_0: M_A = 35$ vs $H_1: M_A > 35$

For $\alpha = 0.05$ and at $M_A = 38$ we want $1-\beta = 0.9 \Rightarrow \beta = 0.1$

we are assuming to know $\sigma_A = 2$, $n_A = ?$

$$n = \left[\frac{\sigma(z_\alpha + z_\beta)}{M_0 - M}\right]^2 = \left[\frac{2(1.645 + 1.28)}{35 - 38}\right]^2$$

$$= 3.8025 \simeq \underline{\underline{4}}$$

$\boxed{f}$ $H_0: \sigma_A^2 = \sigma_B^2$ vs $H_1: \sigma_A^2 \neq \sigma_B^2$

$$F = \frac{s_A^2}{s_B^2} = \frac{26.7}{21.7} = 1.23$$

$$F_{\alpha/2, 4, 4} = F_{0.1/2, 4, 4} = f_{0.05, 4, 4} = 15.98$$

$F_{0.05, 4, 4} > F \Rightarrow$ we fail to reject H_0 at $\underline{\underline{\alpha = 0.1}}$

$\boxed{g}$ $H_0: M_A = M_B$ vs $H_1: M_A > M_B \equiv H_1: M_A - M_B > 0$

For $\alpha = 0.1$

We have two indep samples with normality assumed also we dont have σ^2, and small sample size $\Rightarrow$ we will use two-sample-t-test :

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} = 0.964$$

We have an upper tail $H_1 \Rightarrow$ if $t \geq t_{\alpha, v}$ Then we reject H_0

$$v = \left(\frac{s_A^2}{n_A} \quad \frac{s_B^2}{n_B}\right)^2 \Bigg/ \left[\frac{(s_A^2/n_A)^2}{n_A-1} + \frac{(s_B^2/n_B)^2}{n_B-1} \right] \simeq 7.9$$

$v \simeq 7$

now $t_{0.1,7} = 1.415 \Rightarrow t_{\alpha,v} > t \Rightarrow$ we fail to reject H_0 at $\alpha = 0.1$

$\boxed{h}$ $\mu_A - \mu_B \in \left[\bar{x}_A - \bar{x}_B \mp t_{\alpha/2,v} \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}} \right]$

$\left[(36.8 - 33.8) \mp 2.365 \sqrt{\frac{26.7}{5} + \frac{21.2}{5}} \right]$

$\boxed{i}$ $H_0 : P_A = 0.2$ vs $H_1 : P_A < 0.2$

$\hat{P} = \frac{12}{64} = 0.1875$

$z = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.1872 - 0.2}{\sqrt{\frac{0.1872(1-.1872)}{64}}} = -0.2625$

for $\alpha = 0.05$ we have a lower tail : $-z_\alpha = -1.645$

$\Rightarrow z > -z_\alpha \Rightarrow$ we fail to reject H_0 at $\alpha = 0.05$

$\boxed{2}$ indoor = 0.65 0.62 0.43

outdoor = 0.72 0.61 0.55

indoor - outdoor = -0.07 0.01 -0.12

$H_0 : \mu_d = 0$ vs $H_1 : \mu_d < 0$ " outdoor is higher "

$t_d = \frac{\bar{x}_d}{s_D/\sqrt{n_d}} = -1.58$ $c - t_{0.1,2} = -1.886 \Rightarrow t > -t_{\alpha,df} \Rightarrow$ we fail to reject H_0 at $\alpha = 0.1$

$\boxed{3}$ $X \sim \text{poisson}(\mu)$ $n = 36$ $\bar{x} = 5.2$ $s = 2.2$

$H_0: \mu = 4.5$ vs $H_1: \mu > 4.5$

for $\alpha = 0.05$, we have a large sample

$$z = \frac{5.2 - 4.5}{2.2/\sqrt{36}} = 1.909$$

for an upper tail H_1: $z_\alpha = 1.645$

we reject H_0 at $\alpha = 0.05$ as $z > z_\alpha$

$\boxed{4}$

Group	1	2	3	4	5	6	sum
n	4	3	9	2	5	4	27
$\bar{X}$	0.36	0.33	0.31	0.4	0.33	0.38	
S	0.1	0.13	0.21	0.2	0.09	0.17	
$X_{\cdot}$	1.44	0.99	2.79	.8	1.65	1.52	$9.19 = X_{\cdot\cdot}$

$H_0: \mu_1 = \mu_2 = \dots = \mu_6$ vs H_1: at least one μ is diff

$$SSE = \frac{s_1^2 + \dots + s_6^2}{6} = 0.02467$$ (we have unequal sample size)

$$SS_{tr} = \sum_{i=1}^{6} \frac{1}{n_i} x_{i\cdot}^2 - \frac{1}{n} x_{\cdot\cdot}^2 \quad , \quad n = \sum_{i=1}^{6} n_i = 27 \quad , \quad x_{1\cdot} = \bar{x}_1 n = 1.44$$

$$SS_{tr} = \left[\left[\frac{(1.44)^2}{4} + \frac{(.99)^2}{3} + \dots + \frac{(1.52)^2}{4} \right] - \frac{1}{27} (9.19)^2 \right] = 3.1521 - 3.128 \doteq 0.024096$$

$$SST = SSTr + SSE = 0.048766$$

ANOVA table

	df	SS	MS	F
treatment	5	0.024096	0.00482	4.10145
Error	21	0.02467	0.001175	
total	26	0.048766		

we have $F_{0.05,5,21} = 2.68$

As $F > F_{\alpha,5,21}$ we reject H_0 at $\alpha = 0.05$