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DATE: April 14, 2018

CMPE 320: HW 07

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1. A radar tends to overestimate the distance of an aircraft, and the error is a normal random variable with a mean of 50 meters and a standard deviation 100 meters. What is the probability that the measured distance will be smaller than the true distance?

□

2. Let X be normal with mean 1 and variance 4. Let $Y = 2X + 3$.

(a) Calculate the PDF of Y.

□

(b) Find $P(Y > 0)$.

□

3. A signal of amplitude $s = 2$ is transmitted from a satellite but is corrupted by noise, and the received signal is $X = s + W$, where W is noise. When the weather is good, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 4. In the absence of any weather information:

(a) Calculate the PDF of X.

□

(b) Calculate the probability that X is between 1 and 3.

□

4. Oscar uses his high-speed modem to connect to the internet. The modem transmits zeros and ones by sending signals -1 and $+1$, respectively. We assume that any given bit has probability p of being a zero. The network cable introduces additive zero-mean Gaussian noise with variance σ^2 (so, the receiver at the other end receives a signal which is the sum of the transmitted signal and the channel noise). The value of the noise is assumed to be independent of the encoded signal value.

(a) Let a be a constant between -1 and 1. The receiver at the other end decides that the signal -1 (respectively, $+1$) was transmitted if the value it receives is less (respectively, more) than a. Find a formula for the probability of making an error.

□

(b) Find a numerical answer for the question of part (a) assuming that $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.

□

5. An old modem can take anywhere from 0 to 30 seconds to establish a connection, with all times between 0 and 30 being equally likely.

(a) What is the probability that if you use this modem you will have to wait more than 15 seconds to connect?

□

(b) Given that you have already waited 10 seconds, what is the probaility of having to wait at least 10 more seconds?

□

6. Consider a random variable X with PDF

$$f_X(x) = \begin{cases} 2x/3, & \text{if } 1 < x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$$

and let A be the event $\{X \geq 1.5\}$. Calculate $E[X]$, $P(A)$ and $E[X \mid A]$.

□

7. Dine, the cook, has good days and bad days with equal frequency. On a good day, the time (in hours) it takes Dino to cook a souffle is described by the PDF

$$f_G(g) = \begin{cases} 2, & \text{if } 1/2 < g \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

but on a bad day, the time it takes is described by the PDF

$$f_B(b) = \begin{cases} 1, & \text{if } 1/2 < b \leq 3/2, \\ 0, & \text{otherwise,} \end{cases}$$

Find the conditional probability that today was a bad day, given that it took Dine less than three quarters of an hour to cook a souffle.

□

8. One of the two wheels of fortune, A and B, is selected by the toss of a fair coin, and the wheel chosen is spun once to determine the value of a random variable X. If wheel A is selected, the PDF of X is

$$f_{X|A}(x \mid A) = \begin{cases} 1, & \text{if } 0 < b \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

If wheel B is selected, the PDF of X is

$$f_{X|B}(x \mid B) = \begin{cases} 3, & \text{if } 0 < b \leq 1/3, \\ 0, & \text{otherwise,} \end{cases}$$

If we are told that the value of X was less than 1/4, what is the conditional probability that wheel A was the one selected.

□