Worksheet 3

1. For A event of having a Visa card,B having a MasterCard with $p(A) = 0.5$, $p(B) = 0.4$ $P(A \cup B) = 0.25$: Find :

   a) $p(B|A)$:
   $$\frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

   b) $p(B'|A)$:
   $$1 - P(B|A) = 0.5$$

   c) $P(A|B), p(A'|B)$:
   $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.4} = 0.625$$
   $$P(A'|B) = 1 - P(A|B) = 0.38$$

   d) Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?
   $$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = 0.78$$

1. For A event that project 1 is successful,B that project 2 is successful with $p(A) = 0.4$, $p(B) = 0.7$, knowing that A is independent of B, Find :

   a) $p(A|B) = P(A) = 0.4$.

   b) $p(B'|A) = P(B') = 1 - 0.7 = 0.3$.

   c) Given that the at least one project is successful, what is the probability that **only** project A is successful:
   $$P(AB'|A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB')}{P(A \cup B)} = \frac{P(A)P(B')}{P(A \cup B)} = 0.146$$

2. If A and B are independent events show that A' is independent of B and A' is independent from B':
   $$P(B) = P(A' \cap B) + P(A \cap B) \rightarrow P(A' \cap B) = P(B) - P(A)P(B) = P(B)P(A')$$

   Now to prove that A' is independent of B':
   $$P(A \cap B') = P(B') - P(A' \cap B') \rightarrow P(B') - P(A)P(B') = P(A' \cap B')$$
   $$P(A')P(B') = P(A' \cap B')$$