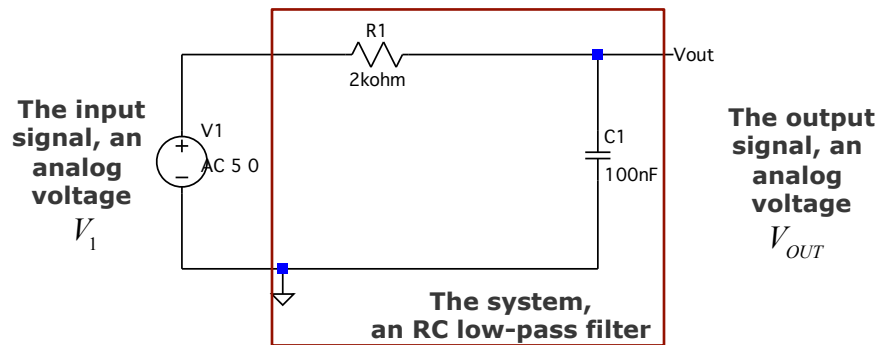


What's it all about?

- A **signal** is a waveform or measurement that represents some physical quantity
- A **system** is a set of operations that transforms *input* signals to *output* signals
- We've seen this kind of thing already in circuits (CMPE306)



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1

Mathematical Description of Signals

- We want to be able to describe signals in three ways
 - Time domain
 - Frequency (or Fourier) Domain
 - Laplace Domain
- Each way emphasizes different qualities of the the signal
- Our method is to decompose complex signals into simpler signals
- In the process, we'll develop a dictionary of standard signals that will prove very useful.

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2

Key Mathematical Characteristic of Signals

- **Periodicity**

- **A signal is periodic if it repeats exactly, that is if**

$$f(\alpha) = f(\alpha \pm nT) \text{ for any } \alpha \text{ and } n$$

T is called the *period*

- **Sinusoids are periodic**

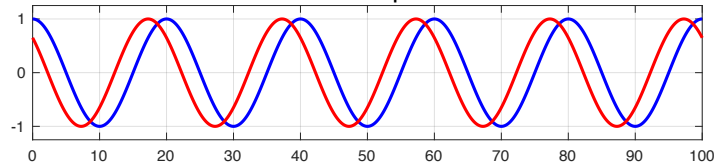
$$x(t) = A \cos(\omega t + \phi), \quad T = \frac{2\pi}{\omega}$$

- **Pulse trains may be periodic**

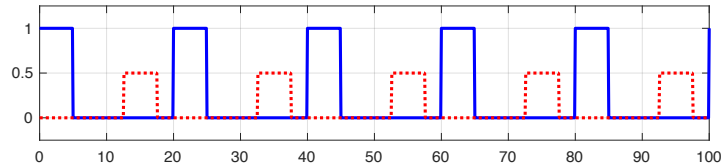
$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT) \text{ where } p(t) \text{ is a specified pulse waveform}$$

Periodic vs. Non periodic

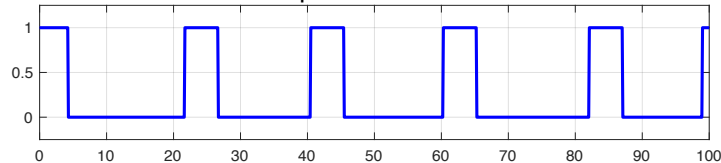
Sinusoids are periodic



Periodic Pulse Train



Aperiodic Pulse Train



Sums and products of periodic waveforms

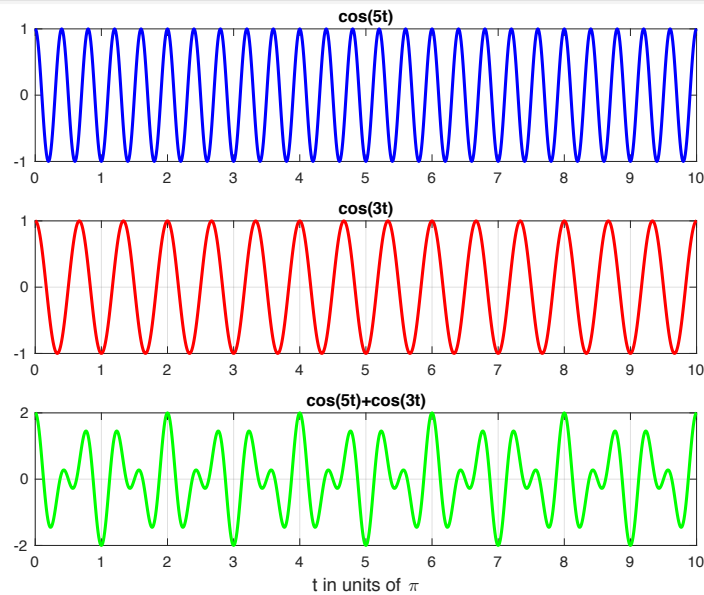
- The sum of two periodic waveforms with periods T_1 and T_2 will be periodic iff $T_1 / T_2 = f_2 / f_1 = n_2 / n_1$; $n_1, n_2 \in \mathbb{Z}$
- The period of the sum is $T = n_1 T_1 = n_2 T_2$

$\cos(5t) + \cos(3t)$ is periodic

$$T_1 = \frac{2\pi}{5}, T_2 = \frac{2\pi}{3}, \frac{T_1}{T_2} = \frac{3}{5}, \text{ so } n_1 = 5, n_2 = 3$$

The common period is $T = n_1 T_1 = 2\pi = n_2 T_2$

- A similar rule holds for the product of periodic waveforms $(f_1 + f_2) / (f_1 - f_2) = n_2 / n_1$, $n_1, n_2 \in \mathbb{Z}$



Elementary Functions

- We define several elementary functions (this is also a review from CMPE306!)

- A unit step

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}, \text{ we'll leave the definition at } t = 0 \text{ for later}$$

- A unit impulse

$$\delta(t) = \frac{du}{dt} \Rightarrow \lim_{\varepsilon \rightarrow 0} \int_{-\varepsilon}^{+\varepsilon} \delta(t) dt = 1$$

- The unit impulse isn't a function in the traditional sense...
- ...but rather a limit of functions...
- ...and a number of different "shapes" can be used

The unit impulse is **very** important

- The sieving property

$$\int_{-\infty}^{\infty} f(t) \delta(t - t_0) dt = f(t_0)$$

- Examples

$$\int_{-\infty}^{\infty} e^{-at} \delta(t - t_0) dt = e^{-at_0}, \text{ in fact } \int_{t_0 - \varepsilon}^{t_0 + \varepsilon} e^{-at} \delta(t - t_0) dt = e^{-at_0}$$

$$\int_{-\infty}^{\infty} e^{-at} (100 \cos \omega t + 200 \sin \omega t) \delta(t - \tau) dt = e^{-a\tau} (100 \cos \omega \tau + 200 \sin \omega \tau)$$

More elementary functions

- **Sinusoids**

$A\cos\omega t$, $A\sin(2\pi ft + \phi)$, etc, periodic with $T = \frac{2\pi}{\omega} = \frac{1}{f}$

- **Exponentials**

$a_0 e^{at}$ (growing), $b_0 e^{-\alpha t}$ (decaying)

- **Complex exponentials**

$$Ae^{j\phi} = A\cos\phi + jA\sin\phi$$

$$Ae^{j(\omega t + \phi)} = A\cos(\omega t + \phi) + jA\sin(\omega t + \phi)$$

- **The sinc function**

- **Our text:** $\text{sinc}(x) = \frac{\sin x}{x}$

- **Some other texts and MATLAB:** $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$

Pulses and Windows

- **Often, we want only a limited segment (in time) of one of the elementary functions.**

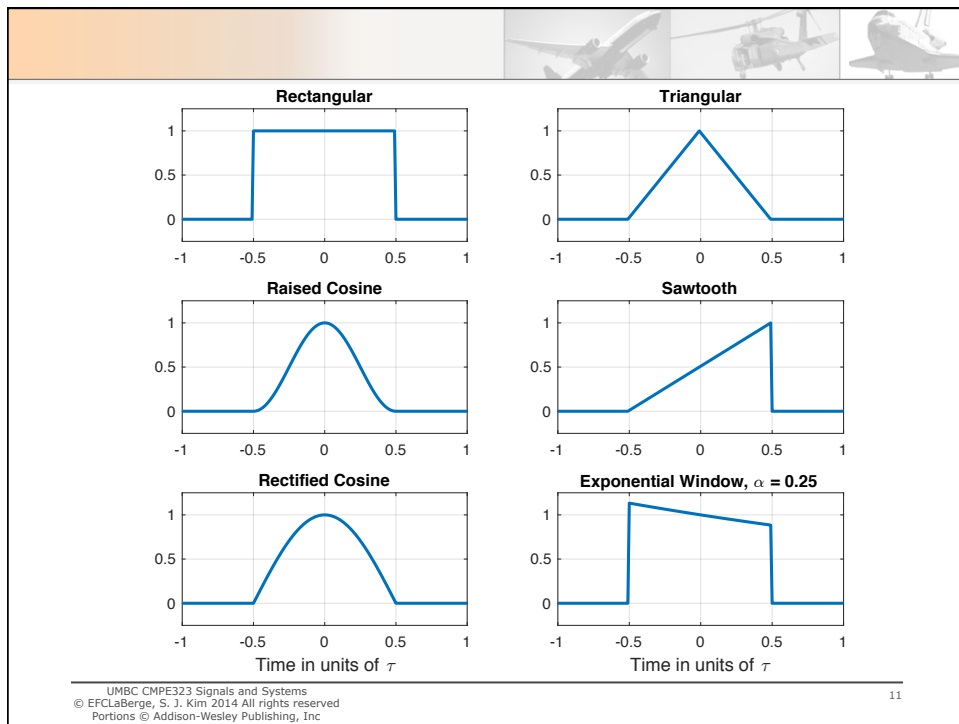
- **Such a segment is called a window**

- **Windows will be important later!**

- **We often create a window by multiplying two functions together...**

- **...e.g. Rectified Cosine Window**

$$w_{RC}(t; \tau) = \cos\left(\frac{\pi t}{\tau}\right) w_{RECT}(t - 0.5\tau; \tau)$$



11

Time Averages of Time Waveforms

- We are usually interested in the energy contained in a waveform or signal...
- ...as energy is the key to overcoming noise in the system.

$$\mathcal{E} = \int_{-\infty}^{\infty} x(t)x^*(t)dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \underset{\substack{\text{real} \\ \text{valued} \\ \text{signal}}}{=} \int_{-\infty}^{\infty} x^2(t)dt$$

- If our signal has units of volts, then \mathcal{E} has units of volt²-sec, which is not joules!
- When computing the mathematical energy (as above) we assume there is a 1 Ω resistor as a scale factor, giving volt²-sec/ohm = watt-sec = joules!
- Using this definition, the energy in a periodic signal is infinite!! (Why?)

12

- For periodic signals, we compute the “average energy per period”

$$\frac{\mathcal{E}}{T} = \frac{1}{T} \int_{\alpha}^{T+\alpha} x(t)x^*(t) dt = \frac{1}{T} \int_{\alpha}^{T+\alpha} |x(t)|^2 dt \stackrel{\substack{\text{real} \\ \text{valued} \\ \text{signal}}}{=} \frac{1}{T} \int_0^T x^2(t) dt$$

- Again using a 1Ω resistor as a scale factor, we wind up with volt²-sec/ohm-sec=watt-sec/sec=watt...
- ...so this is power!
- A signal with finite energy is called an energy signal
- A signal with meaningful power is called a power signal.
- Some signals are neither power or energy signals, but these are rare.

Examples

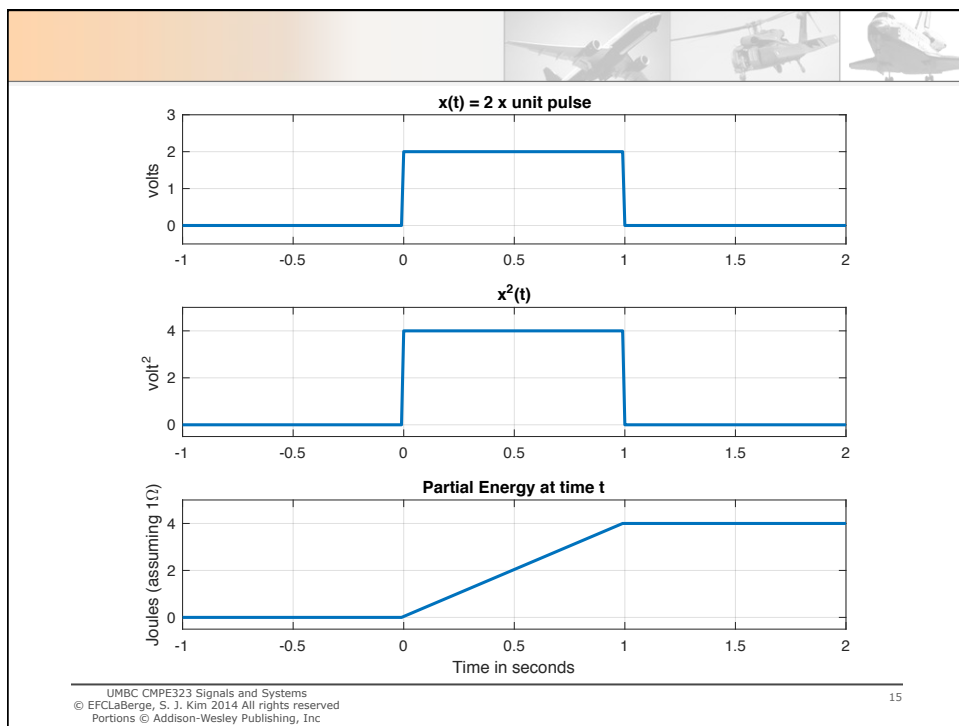
- Unit pulse

$$p(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = 2p(t)$$

$$x^2(t) = \begin{cases} 4 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{E} = \int_{-\infty}^{\infty} x^2(t) dt = \int_0^1 4 dt = 4 \text{ joules}$$



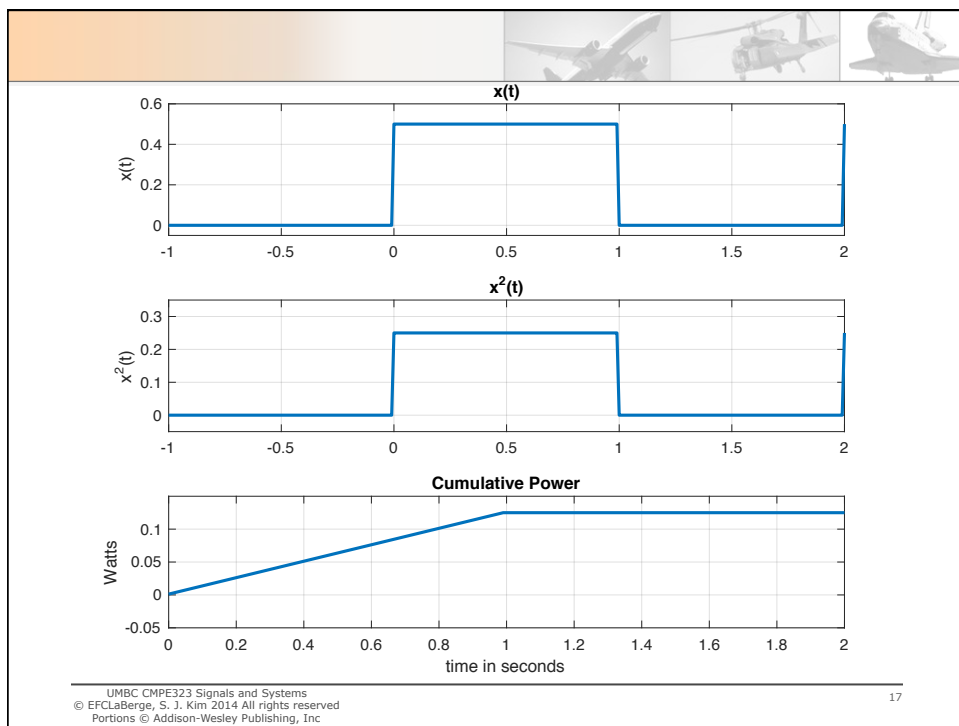
■ Pulse train

$$p(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{-\infty}^{\infty} 0.5p(t - 2n)$$

$$x^2(t) = \sum_{-\infty}^{\infty} 0.25p(t - 2n)$$

$$P = \frac{1}{2} \int_0^2 0.25 dt = 0.125 \text{ watts}$$



You try

- Compute the power or energy, as appropriate

$$a(t) = \begin{cases} e^{-0.5t} & 0 \leq t < 2 \\ 0 & \text{o/w} \end{cases} \quad (\text{exponential pulse})$$

$$b(t) = \sum_{k=-\infty}^{\infty} a(t - 4k)$$

$$c(t) = 120 \cos(120\pi t + \pi/3)$$

$$d(t) = \begin{cases} 120 \cos(120\pi t) & -\frac{1}{240} \leq t < \frac{1}{240} \\ 0 & \text{o/w} \end{cases}$$

$$f(t) = \begin{cases} 0.25t & 0 \leq t < 4 \\ 0 & \text{o/w} \end{cases}$$

$$g(t) = \sum_{k=-\infty}^{\infty} f(t - 16k)$$

$a(t)$ is an energy signal

$$\mathcal{E} = \int_0^2 (e^{-0.5t})^2 dt = \int_0^2 (e^{-1t}) dt = 1 - e^{-2}$$

$b(t)$ is periodic with period 4, therefore it is a power signal

$$P = \frac{1}{4} \int_0^4 (b(t))^2 dt = \frac{1}{4} \left(\int_0^2 (e^{-0.5t})^2 dt + \int_2^4 (0)^2 dt \right) = \frac{1 - e^{-2}}{4}$$

$c(t)$ is periodic with period $\frac{2\pi}{120\pi} = \frac{1}{60}$. The phase does not affect

the period, $c(t)$ is a power signal

$$\text{Let } 120\pi t_0 = \frac{\pi}{3} \Rightarrow t_0 = \frac{1}{360}, \quad c(t) = 120 \cos(120\pi(t + t_0))$$

$$P = \left(\frac{1}{60} \right) 120^2 \int_{-1/360}^{-1/360+1/60} \cos^2(120\pi(t + t_0)) dt = \left(\frac{1}{60} \right) 120^2 \int_0^{1/60} \cos^2 120\pi\tau d\tau$$

$$= \left(\frac{1}{60} \right) 120^2 \int_0^{1/60} \left(\frac{1}{2} + \frac{1}{2} \cos 240\pi\tau \right) d\tau$$

$$= 120^2 \frac{((1/60) - 0)}{2 \times 60} + 120^2 \frac{\left(\sin \frac{240\pi}{60} - \sin 0 \right)}{2 \times 60 \times 240\pi} = \frac{120^2}{2} = 7200 \text{ watts}$$