Important Special Cases

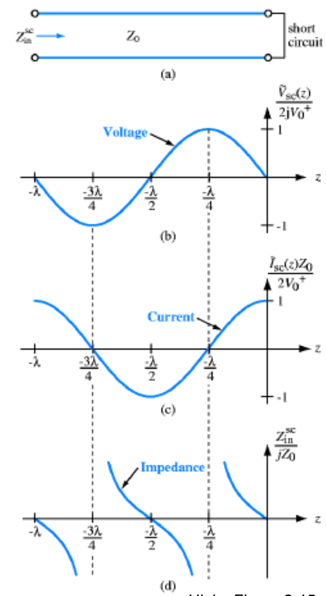
Short-Circuited Line: $\Gamma = -1$

$$\tilde{V}_{sc} = V_0^+[\exp(-j\beta z) - \exp(j\beta z)]$$
$$= -2jV_0^+ \sin\beta z;$$
$$\tilde{I}_{sc} = \frac{V_0^+}{Z_0}[\exp(-j\beta z) + \exp(j\beta z)]$$
$$= \frac{2V_0^+}{Z_0}\cos\beta z$$

Input impedance:

$$Z_{in}^{sc} = [\tilde{V}_{sc}(-l) / \tilde{I}_{sc}(-l)] = jZ_0 \tan\beta l$$

- By varying l over $\lambda / 2$, we can obtain any desired reactance (capacitance or inductance).
- This method is often the most practical!



Ulaby Figure 2-15

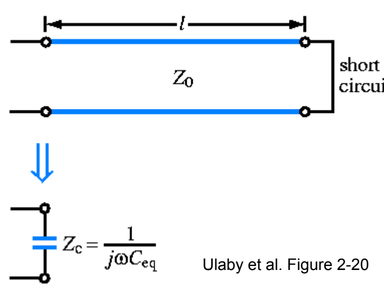
5.1

Important Special Cases

Short-Circuited Line: Ulaby et al. Example 2-8

Question: Choose the length of a shorted 50 Ω lossless transmission line such that its input impedance at 2.25 GHz is equivalent to a capacitor with $C_{eq} = 4$ pF. The wave velocity is $0.75c$.

Answer: We have $\beta = (2\pi\, f / 0.75\, c) = (2\pi \times 2.25{\times}10^9 / 0.75 \times 3.00{\times}10^8)$ = 62.8 rads/m. We require $\tan\beta l = -1 / (Z_0\omega C_{eq}) = -1 / (50 \times 2\pi \times 2.25{\times}10^9 \times 4{\times}10^{-12}) = -0.354$. The arctangent yields $\beta l = (2.80 + n\pi)$ rads, $(n = 0, 1, 2,\ldots) = 2.80, 5.94, 9.08,\ldots$
We conclude l = 4.5 cm, 9.4 cm, 14.5 cm…



Ulaby et al. Figure 2-20

5.2

Important Special Cases

Open-Circuited Line: $\Gamma = +1$

$$\tilde{V}_{\mathrm{oc}} = V_0^+[\exp(-j\beta z) + \exp(j\beta z)] = 2V_0^+ \cos\beta z;$$

$$\tilde{I}_{\mathrm{oc}} = \frac{V_0^+}{Z_0}[\exp(-j\beta z) - \exp(j\beta z)] = \frac{-2jV_0^+}{Z_0}\sin\beta z$$

$$Z_{\mathrm{in}}^{\mathrm{oc}} = [\tilde{V}_{\mathrm{oc}}(-l)/\tilde{I}_{\mathrm{oc}}(-l)] = -jZ_0 \cot\beta l$$

Using a *network analyzer* to measure $Z_{\mathrm{in}}^{\mathrm{sc}}$ and $Z_{\mathrm{in}}^{\mathrm{oc}}$ and the formulae

$$Z_0 = \sqrt{Z_{\mathrm{in}}^{\mathrm{sc}} Z_{\mathrm{in}}^{\mathrm{oc}}}, \quad \tan\beta l = \sqrt{-Z_{\mathrm{in}}^{\mathrm{sc}} / Z_{\mathrm{in}}^{\mathrm{oc}}}$$

we can determine Z_0 and β for a transmission line from two measurements

Important Special Cases

Quarter-Wave Transformer

When $l = \lambda/4 + n\lambda/2$, we have $Z_{\mathrm{in}} = Z_0^2 / Z_{\mathrm{L}}$, allowing us to match a line with one impedance to a load with another.

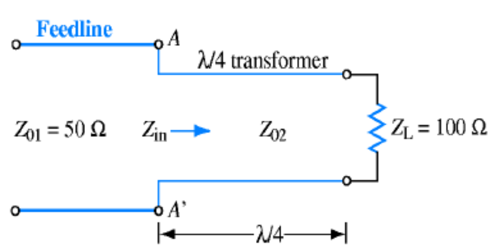
Ulaby et al. Example 2-10

Question: A Z_{01} = 50 Ω transmission line is to be matched to a resistive load with Z_{L} = 100 Ω. What should be the characteristic impedance Z_{02} of a quarter-wave transformer?

Answer: We want Z_{in} = 50 Ω, to avoid reflections, entering the quarter-wave transformer, so that

$$Z_{02} = \sqrt{Z_{\mathrm{in}} Z_{\mathrm{L}}}$$

$$= \sqrt{50 \times 100} = 70.7\ \Omega$$



Ulaby Figure 2-22

Power Flow

Instantaneous Power

We find for the incident power on the load

$$P^{\mathrm{i}}(t) = v^{\mathrm{i}}(t)\cdot i^{\mathrm{i}}(t) = \mathrm{Re}\left[|V_0^+|\exp j\phi^+ \exp(j\omega t)\right]\cdot \mathrm{Re}\left[\frac{|V_0^+|}{Z_0}\exp j\phi^+ \exp(j\omega t)\right]$$

$$= \frac{|V_0^+|^2}{Z_0}\cos^2(\omega t + \phi^+)$$

Similarly for the reflected power,

$$P^{\mathrm{r}}(t) = v^{\mathrm{r}}(t)\cdot i^{\mathrm{r}}(t) = -|\Gamma|^2 \frac{|V_0^+|^2}{Z_0}\cos^2(\omega t + \phi^+ + \theta_{\mathrm{r}})$$

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5.5

Power Flow

Average Power (Transferred to the Load)

Of more importance is the power flow averaged over a period.
— We calculate this value in both the time domain and the phasor domain

Time Domain:

$$P_{\mathrm{av}}^{\mathrm{i}} = \frac{1}{T}\int_0^T P^{\mathrm{i}}(t)\,dt = \frac{|V_0^+|^2}{Z_0}\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\cos^2(\omega t + \phi^+)\,dt = \frac{|V_0^+|^2}{2Z_0};$$

$$P_{\mathrm{av}}^{\mathrm{r}} = \frac{1}{T}\int_0^T P^{\mathrm{r}}(t)\,dt = -|\Gamma|^2\frac{|V_0^+|^2}{2Z_0}; \quad P_{\mathrm{av}} = P_{\mathrm{av}}^{\mathrm{i}} + P_{\mathrm{av}}^{\mathrm{r}} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma|^2\right)$$

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5.6

Power Flow

Average Power (Transferred to the Load)

Of more importance is the power flow averaged over a period.
— We calculate this value in both the time domain and the phasor domain

Phasor Domain: Our starting point is the key equation:

$$P_{\text{av}} = (1/2)\,\text{Re}\left(\tilde{V} \cdot \tilde{I}^*\right)$$

This expression and similar equations appear in many contexts!

— Substitution yields:

$$P^{\text{i}}_{\text{av}} = \frac{1}{2}\text{Re}\left(V_0^+ \cdot \frac{V_0^{+*}}{Z_0}\right) = \frac{|V_0^+|^2}{2Z_0}; \quad P^{\text{r}}_{\text{av}} = \frac{1}{2}\text{Re}\left[\Gamma V_0^+ \cdot \left(-\frac{\Gamma^* V_0^{+*}}{Z_0}\right)\right] = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}$$

which is the same as what was obtained before in the time domain

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The Smith Chart

Points on the Complex Γ-Plane:

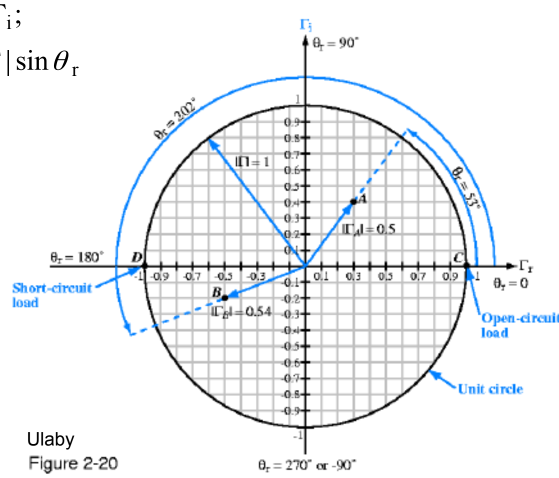
$$\Gamma = |\Gamma| \exp j\theta_{\text{r}} = \Gamma_{\text{r}} + j\Gamma_{\text{i}};$$

$$\Gamma_{\text{r}} = |\Gamma|\cos\theta_{\text{r}}; \quad \Gamma_{\text{i}} = |\Gamma|\sin\theta_{\text{r}}$$

Examples:

- $\Gamma_A = 0.3 + j0.4$,
- $\Gamma_B = -0.5 - j0.2$

NOTE: All points have $|\Gamma| \leq 1$ and lie on or within the unit circle



Ulaby
Figure 2-20

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The Smith Chart

Normalized Load Impedances: $z_L = Z_L / Z_0$

We have

$$\Gamma = \frac{z_L - 1}{z_L + 1}; \quad z_L = \frac{1+\Gamma}{1-\Gamma}$$

Writing the normalized load impedance the sum of a normalized resistance r_L and a normalized reactance x_L, so that $z_L = r_L + j x_L$, we obtain

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}, \quad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2},$$

Keeping r_L fixed and allowing x_L to vary, we get circles on the Γ-plane

Keeping x_L fixed and allowing r_L to vary, we get circles on the Γ-plane

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The Smith Chart

Circles of constant r_L and x_L

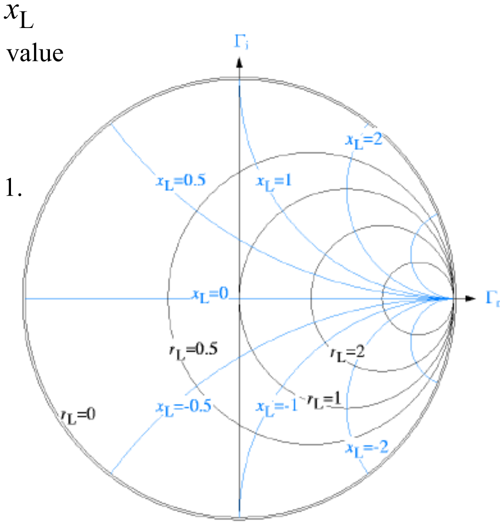
(1) Only $r_L \geq 0$ is allowed; any value of x_L is possible

(2) Circles of constant r_L are contained within $|\Gamma| = 1$; $r_L = 0$ corresponds to $|\Gamma| = 1$.

(3) No circles of constant x_L are contained within $|\Gamma| = 1$

(4) All circles touch $\Gamma = 1$

(5) Circles of constant r_L and circles of constant x_L intersect at 90° angles.



Ulaby Figure 2-21

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The Smith Chart

Examples of use

(1) *Finding* Γ: Given r_L and x_L, the x- and y-coordinates give Γ_i and Γ_r. The magnitude and phase give $|\Gamma|$ and θ_r.

(2) *Finding* Z_{in}: Using the expression

$$z_{in} = \frac{Z_{in}}{Z_0} = \frac{1+|\Gamma|\exp\left[j(\theta_r - 2\beta l)\right]}{1-|\Gamma|\exp\left[j(\theta_r - 2\beta l)\right]}$$

we conclude that z_{in} is related to z_L by a rotation by an angle $-2\beta l$ on the Smith chart.

(3) *Finding* SWR *and voltage minima and maxima*: On the circle of constant $|\Gamma|$, SWR = r_L when $\Gamma_i = 0$, corresponding to P_{max}. The angular rotation gives the minima and maxima, normalized to wavelength.

The key point: The variation of the input impedance as a function of position in the transmission line can be traced along a circle on the Smith chart!

Applications — Paul

High-Speed Digital Interconnects

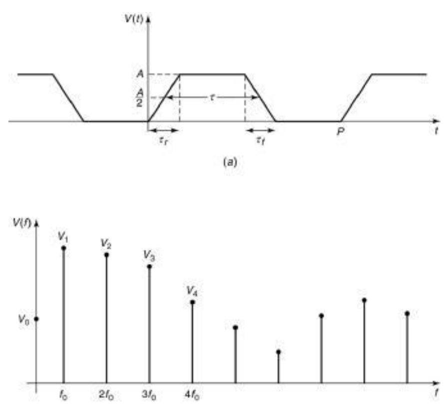
Consider a clock signal with rise and fall times τ_r and τ_f, a pulse duration τ and a repetition period $P = 1 / f_0$. The spectrum will show harmonics at multiples of f_0.

We define:

duty cycle = τ / P

50% duty cycles are typical

We will consider a 1 MHz clock signal with 20 ns rise and fall times and a 50% duty cycle in the next figure



Paul Figure 6.36

Applications — Paul

High-Speed Digital Interconnects

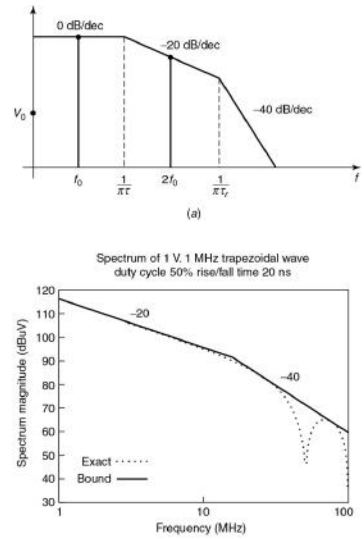
Three spectral components are visible

(1) A falloff of 0 dB/decade up to $f = 1/\pi\tau$

(2) A falloff of 20 dB/decade up to $f = 1/\pi\tau_r$

(3) A falloff of 40 dB/decade beyond $f = 1/\pi\tau_r$

Frequencies up to $3/\pi\tau_r$ are important. For example, that means that the spectrum of a signal with 1 ns rise and fall times is contained within 1 GHz.



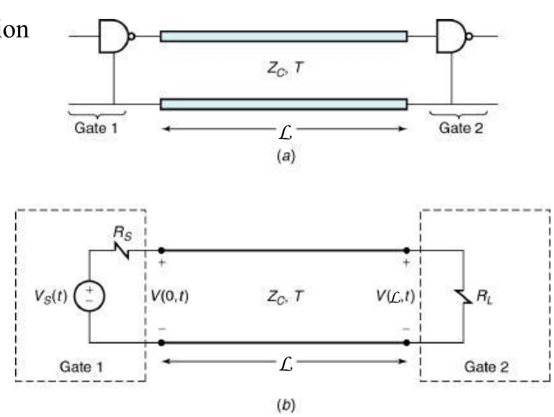
Paul Figure 6.37 (b)

Applications — Paul

High-Speed Digital Interconnects

We model two gates connected through a land as a transmission line.

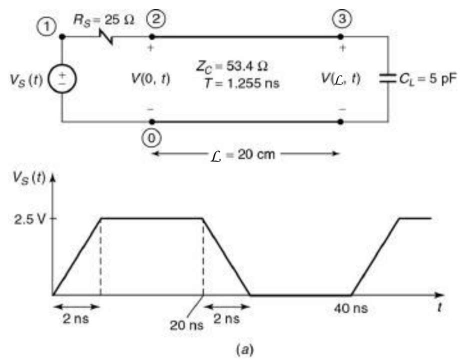
These lands produce "ringing" in combination with high-frequency signal components

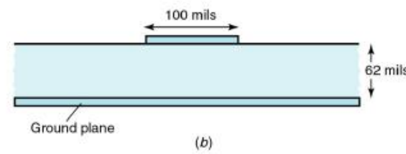


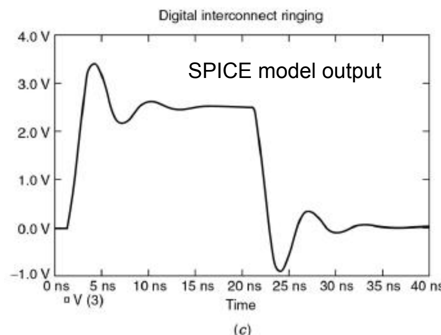
Paul Figure 6.38

Applications—Paul

Example: Two CMOS inverters with land interconnects







- Inverter is modeled as a capacitor
- Ringing is evident

Paul Figure 6.40

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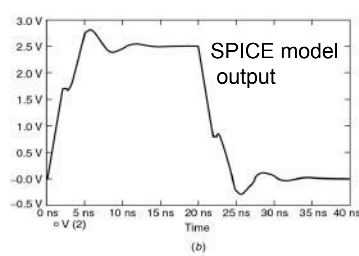
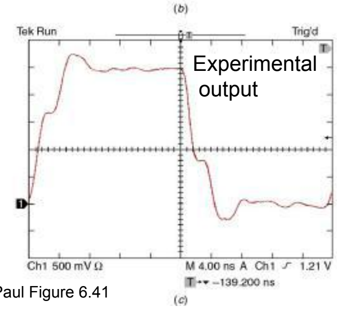
Applications—Paul

Example: Two CMOS inverters with land interconnects

```
Paul Example on p. 319:
* Ringing in Lands Between Two CMOS Gates
VS    1  0  PWL(0 0 2N 2.5 20N 2.5 22N 0 40N 0)
RS    1  2  25
T     2  0  3  0  Z0=53.4  TD=1.255N
CL    3  0  5P
LL    5  0  0.318U
.TRAN 0.04N  40N  0  0.04N
.PROBE
.END
```

A matching scheme must be implemented to eliminate the ringing





Paul Figure 6.41

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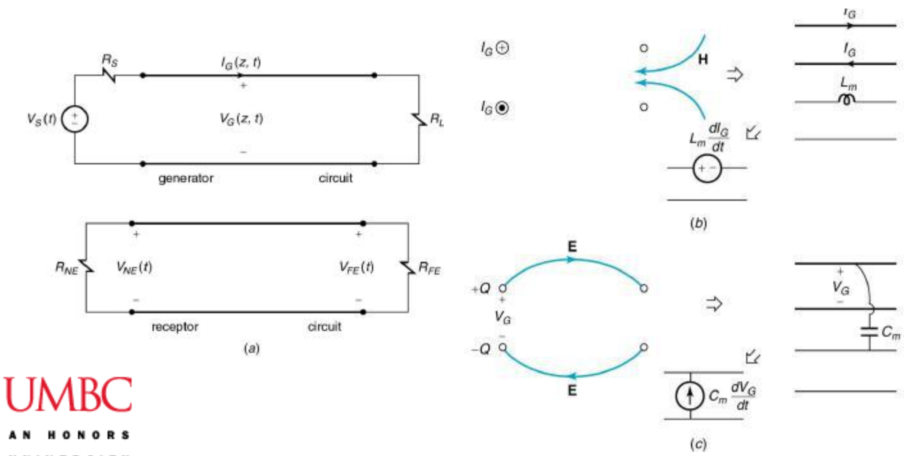
Applications — Paul

Crosstalk Between Transmission Lines

There is a “hidden schematic” connecting two nearby transmission lines



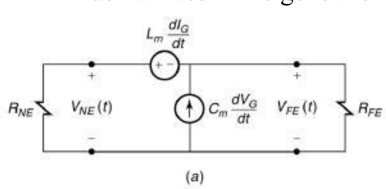
Paul Figure 6.49

5.17

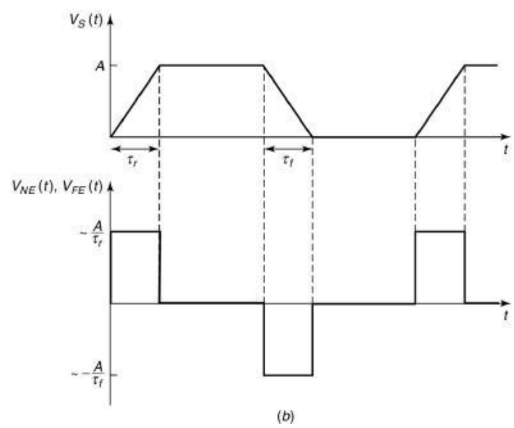
Applications — Paul

Crosstalk Between Transmission Lines

Capacitive and Inductive coupling appear proportional to the time derivatives in the generator circuit



Paul Figure 6.50

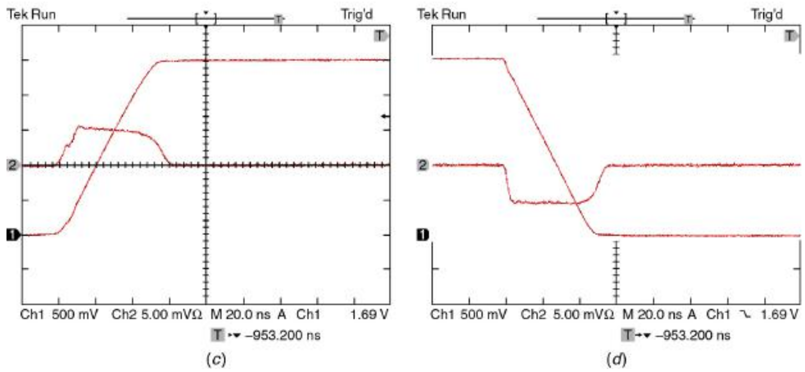


5.18

Applications — Paul

Crosstalk Between Transmission Lines

Experimentally observed near-end crosstalk



Paul Figure 6.52

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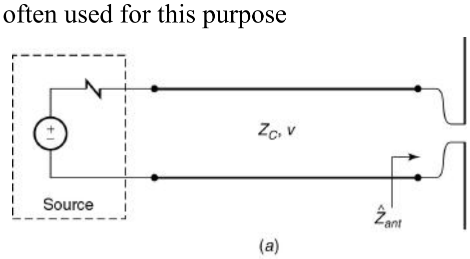
5.19

Applications — Paul

Antenna Matching

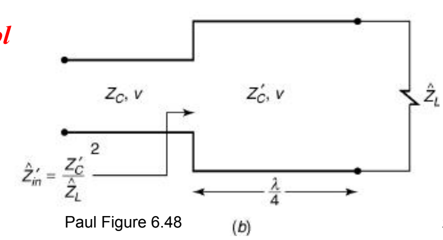
A quarter wave transformer is often used for this purpose

Every antenna has an effective impedance — often close to the vacuum impedance of 377 Ω.



Quarter wave matching allows one to match the antenna to a different impedance.

However: one must carefully control the frequency and length!



Paul Figure 6.48

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5.20

Applications Using The Smith Chart

Impedance Matching Techniques

1. *Quarter-wavelength transformer*

   Ulaby and Ravaioli 2015 Module 2.7
2. *Discrete Element Matching*: a capacitor or inductor can be inserted at an appropriate location along the transmission line to cancel match the line to the load

   Ulaby and Ravaioli 2015 Module 2.8
3. *Single-Stub Tuning*: Instead of a lumped element, a shorted stub of appropriate length can be placed on the line to match the impedance.

   Ulaby and Ravaioli 2015 Module 2.9

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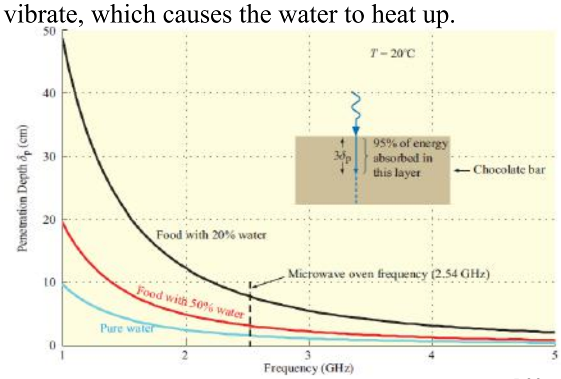
5.21

Technology Brief 3: Microwave Ovens

Invented by accident in 1940s at Ratheon by Percy Spencer while working on magnetrons for radar.

When water molecules are subjected to an electric field, they rotate to orient their dipole with the field. Since the microwave field direction changes rapidly, the molecules vibrate, which causes the water to heat up.

Penetration depth $\delta_p = 1/2\alpha$ of microwaves into food depends on frequency, temperature, and water content.



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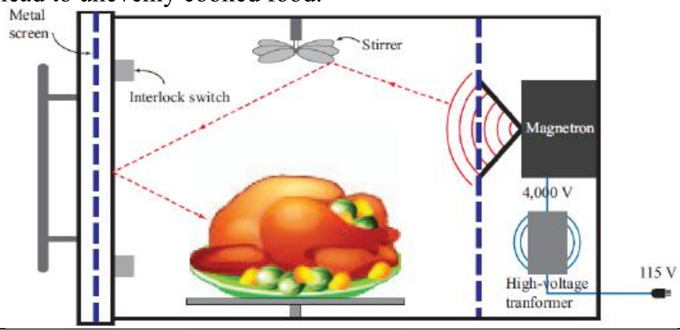
5.22

Technology Brief 3: Microwave Ovens

The magnetron, which generates the microwaves, needs 4 kV to operate.

The oven is made of metal, which contains the microwaves. When the oven door is glass, there is a mesh to prevent the escape of the radiation. Microwaves cannot pass through the metal screen if the mesh width is much smaller than the wavelength of the microwave ($\lambda \approx 12$ cm at 2.5 GHz).

The stirrer mitigates the effect of the standing wave pattern created in the cavity, which would lead to unevenly cooked food.



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5.23

Technology Brief 4: Cancer Zappers

Microwave Ablation

Microwave Ablation uses the same process as a microwave oven to destroy cancerous tissues. The ablation catheter (a transmission line) delivers 60 W of power at 915 MHz. The cancer cells are destroyed by overheating.

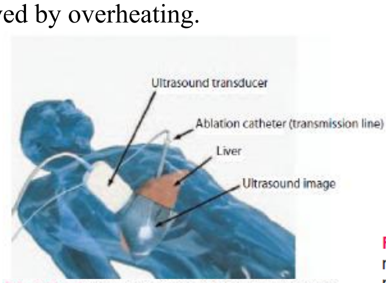

Figure TF4-1: Microwave ablation for liver cancer treatment.

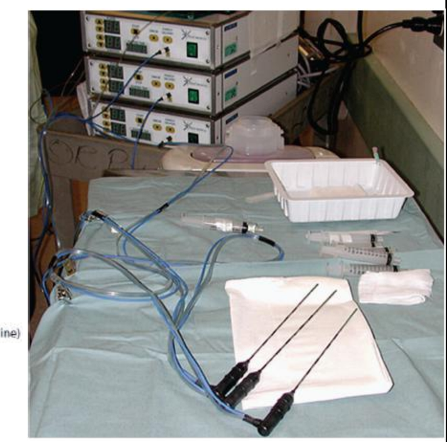

Figure TF4-2: Photograph of the setup for a percutaneous microwave ablation procedure in which three single microwave applicators are connected to three microwave generators. (Courtesy of *RadioGraphics*, October 2005, pp. 569–583.)

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5.24

Technology Brief 4: Cancer Zappers

High-Power Nanosecond Pulses

Research shows that extremely short pulses of extremely high power radiation are effective in destroying malignant tumors. The pulses are about 200 ns long and have a peak power of 180 kW. The energy in each pulse is thus around 4 mJ. Even with the low energy, the high voltage is effective in destroying tumor cells.

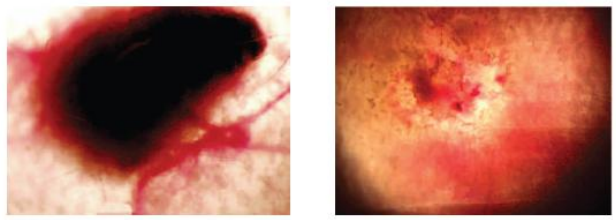

Figure TF4-4: A skin tumor in a mouse before (top) and 16 days after (bottom) treatment with nanoseconds-long pulses of voltage. (Courtesy of *IEEE Spectrum*, August 2006.)

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Assignment

Reading: Ulaby, Chapter 3

Problem Set 3: Some notes.

- There are 7 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Please watch significant digits.
- Get started early!

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