Gauss's Law

In differential form:

$$\nabla \cdot \mathbf{D} = \rho_V$$

When integrated over a volume, we have

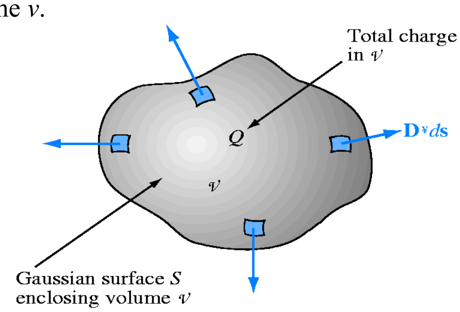
$$\int_v \nabla \cdot \mathbf{D}\, dv = \int_v \rho_V\, dv = Q$$

where Q is the total charge in volume v.
Using Gauss's theorem:

$$\int_v \nabla \cdot \mathbf{D}\, dv = \oint_s \mathbf{D} \cdot d\mathbf{s}$$

We obtain the integral form:

$$\oint_s \mathbf{D} \cdot d\mathbf{s} = Q$$



Ulaby Figure 4-8

UMBC AN HONORS UNIVERSITY IN MARYLAND

Gauss's Law

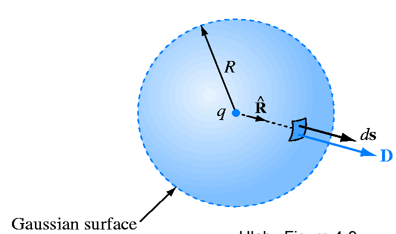
Applications

In some simple cases with high symmetry, solutions may be found directly

- A point charge

$$\mathbf{E}(\mathbf{R}) = \mathbf{D}(\mathbf{R})/\varepsilon = \hat{\mathbf{R}}\frac{q}{4\pi\varepsilon R^2}$$

— This just reproduces Coulomb's law



Ulaby Figure 4-9

- A line charge

$$\mathbf{E}(\mathbf{R}) = \mathbf{E}(r,\phi,z) = \mathbf{D}(\mathbf{R})/\varepsilon = \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon r}$$

- A planar charge

$$\mathbf{E}(\mathbf{R}) = \mathbf{E}(r,\phi,z) = \mathbf{E}(x,y,z) = \mathbf{D}(\mathbf{R})/\varepsilon = \hat{\mathbf{z}}\frac{\rho_S}{2\varepsilon}$$

— We obtained this result earlier by direct integration

UMBC AN HONORS UNIVERSITY IN MARYLAND

Gauss's Law

Applications

The line charge in more detail:

— From symmetry, $\mathbf{D}(\mathbf{R}) = \mathbf{D}(r,\phi,z) = \mathbf{D}(r) = \hat{\mathbf{r}}\, D_r(r)$

Only the r-component of $\mathbf{D}$ is present, and it points in the r-direction

We consider a cylinder of height h surrounding the line charge. It contains a charge $Q = \rho_l h$

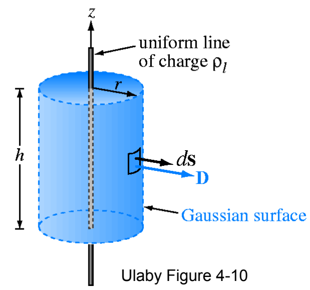
We thus find

$$\int_{z=0}^{h}\int_{\phi=0}^{2\pi} \hat{\mathbf{r}} D_r \cdot \hat{\mathbf{r}}\, r\, d\phi\, dz = \rho_l h$$

which implies $2\pi h D_r r = \rho_l h$

from which we conclude

$$\mathbf{E}(\mathbf{R}) = \hat{\mathbf{r}}\frac{D_r}{\varepsilon} = \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon r}$$



Ulaby Figure 4-10

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.3

Gauss's Law

Applications

An even more important application of the integral formulation in today's world is to serve as the starting point for numerical approaches!

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.4

The Scalar Potential — Voltage

Physical Meaning

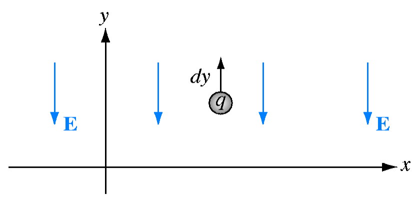
The voltage V between two points indicates the work (energy) required to move a unit charge from the first point to the second

The field exerts a force $\mathbf{F}_{\mathrm{e}} = q\mathbf{E}$ on the charge. An external source of energy — like a battery — is required to move the charges; the external force must counteract the electrical force; $\mathbf{F}_{\mathrm{ext}} = -\mathbf{F}_{\mathrm{e}} = -q\mathbf{E}$.

We have

$$dV = \frac{dW}{q} = -\mathbf{E}\cdot d\mathbf{l}, \quad \text{which integrates to}$$

$$V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l}$$



Ulaby Figure 4-11

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.5

The Scalar Potential — Voltage

Physical Meaning

The voltage V between two points is independent of the path that is used to go between those two points

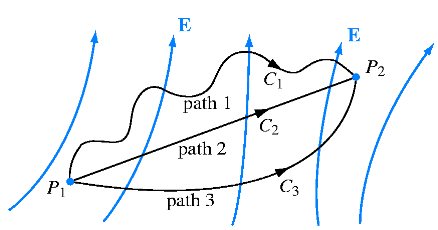
This is the source of: ***Kirchhoff's Voltage Law***

An important consequence: $\oint_C \mathbf{E}\cdot d\mathbf{l} = 0$

In an axiomatic approach, we may derive this result from the second equation of electrostatics: $\nabla\times\mathbf{E} = 0$.

Using Stokes theorem, we have

$$\int_s (\nabla\times\mathbf{E})\cdot d\mathbf{s} = \oint_C \mathbf{E}\cdot d\mathbf{l} = 0$$



Ulaby Figure 4-12

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.6

The Scalar Potential — Voltage

Physical Meaning

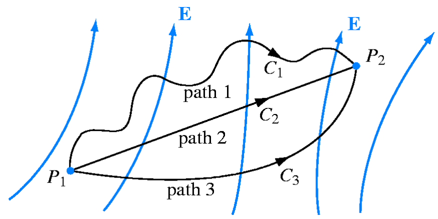
More generally, $\nabla \times \mathbf{E} \neq 0$ ***due to a changing magnetic field and Kirchoff's law breaks down!***

The Concept of Ground

Voltage is defined as *the potential difference* between two points.

To set an absolute voltage reference, we assume that the voltage is zero in the ground, which corresponds to bringing a particle from ∞.

$$V = -\int_{\infty}^{P} \mathbf{E} \cdot d\,\mathbf{l}$$



Ulaby Figure 4-12

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.7

The Scalar Potential — Voltage

Voltage due to Point Charges

From a single charge, using $\mathbf{E} = \hat{\mathbf{R}}\left(q / 4\pi\varepsilon R^2\right)$

$$V = -\int_{\infty}^{P(\mathbf{R})} \left(\hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \right) \cdot \hat{\mathbf{R}}\, dR = \frac{q}{4\pi\varepsilon R}$$

From a charge q that is at $\mathbf{R}_1 \neq 0$, we have

$$V(\mathbf{R}) = \frac{q}{4\pi\varepsilon\, |\mathbf{R} - \mathbf{R}_1|}$$

For multiple charges $q_i, i = 1, 2, \ldots N$ located at $\mathbf{R}_i, i = 1, 2, \ldots N,$ we have

$$V(\mathbf{R}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon\, |\mathbf{R} - \mathbf{R}_i|}$$

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.8

The Scalar Potential — Voltage

Voltage due to Continuous Distributions

By analogy with our earlier definition of the fields due to charge distributions, we have

$$V(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho_{\mathrm{V}}\, dv'}{R'} \quad \text{(volume distribution)}$$

$$V(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{s'} \frac{\rho_{\mathrm{S}}\, ds'}{R'} \quad \text{(surface distribution)}$$

$$V(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{l'} \frac{\rho_{l}\, dl'}{R'} \quad \text{(line distribution)}$$

Voltages are typically much easier to calculate than fields because there is just one scalar quantity to calculate instead of three vector components!

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.9

The Scalar Potential — Voltage

Calculating the Electric Field from the Voltage Field

From the equation $dV = -\mathbf{E}\cdot d\mathbf{l}$, we infer

$$\mathbf{E} = -\nabla V$$

The preferred approach to calculating the field is to first determine the voltage field and then use the equation $\mathbf{E} = -\nabla V$ to determine the electric field

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.10

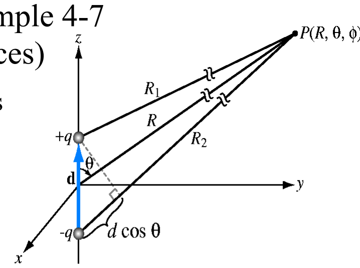
The Scalar Potential — Voltage

Electric Dipole: Ulaby and Ravaioli Example 4-7
(extended to arbitrary distances)

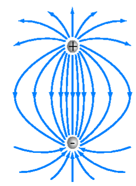
Question: An electric dipole has two charges with same the same magnitude and the opposite sign separated by a distance d. Determine the voltage and the field

Answer: In contrast to Ulaby's example, we calculate the field at any distance. We start with cylindrical coordinates, which are more convenient when the distances are arbitrary. We have

$$V(r,z) = \frac{q}{4\pi\varepsilon\left[r^2+(z-d/2)^2\right]^{1/2}} - \frac{q}{4\pi\varepsilon\left[r^2+(z+d/2)^2\right]^{1/2}}$$



(a) Electric dipole



(b) Electric-field pattern

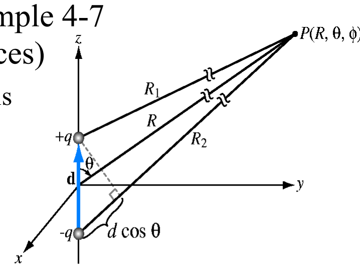
Ulaby Figure 4-13 9.11

The Scalar Potential — Voltage

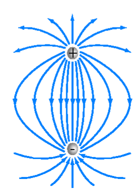
Electric Dipole: Ulaby and Ravaioli Example 4-7
(extended to arbitrary distances)

Answer (continued): We note that the field is independent of ϕ. So, the electric field becomes

$$\mathbf{E}(r,z) = \frac{q\left[\hat{\mathbf{r}}\,r+\hat{\mathbf{z}}(z-d/2)\right]}{4\pi\varepsilon\left[r^2+(z-d/2)^2\right]^{3/2}} - \frac{q\left[\hat{\mathbf{r}}\,r+\hat{\mathbf{z}}(z+d/2)\right]}{4\pi\varepsilon\left[r^2+(z+d/2)^2\right]^{3/2}}$$



(a) Electric dipole



(b) Electric-field pattern

Ulaby Figure 4-13 9.12

The Scalar Potential — Voltage

Electric Dipole: Ulaby and Ravaioli Example 4-7
(extended to arbitrary distances)

Answer (continued): In spherical coordinates, we have

$$\mathbf{E}(R,\theta) = \frac{q\left\{\hat{\mathbf{R}}\left[R-(d/2)\cos\theta\right]+\hat{\boldsymbol{\theta}}(d/2)\sin\theta\right\}}{4\pi\varepsilon\left[R^2 - R\,d\cos\theta + (d/2)^2\right]^{3/2}} - \frac{q\left\{\hat{\mathbf{R}}\left[R+(d/2)\cos\theta\right]-\hat{\boldsymbol{\theta}}(d/2)\sin\theta\right\}}{4\pi\varepsilon\left[R^2 + R\,d\cos\theta + (d/2)^2\right]^{3/2}}$$

In the limit $R \gg d$

$$\frac{1}{\left[R^2 \mp R\,d\cos\theta + (d/2)^2\right]^{3/2}} \simeq \frac{1}{R^3}\left(1 \pm \frac{3d}{2R}\cos\theta\right)$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

The Scalar Potential — Voltage

Electric Dipole: Ulaby and Ravaioli Example 4-7
(extended to arbitrary distances)

Answer (continued): We conclude

$$\mathbf{E}(R,\theta) = \frac{qd}{4\pi\varepsilon R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

Note the cubic falloff with distance and the proportionality to the charge separation! This behavior is characteristic of dipoles.

UMBC AN HONORS UNIVERSITY IN MARYLAND

Electrical Properties of Materials

Conductors and Dielectrics

In electromagnetic theory, we treat materials as conductors or dielectrics

- In conductors, the charges and currents appear in ρ and $\mathbf{J}$.
- In dielectrics, the charges and currents appear in ε and μ.

What about semiconductors?

The answer is complicated. It depends on

- The material properties
- Electric field properties; particularly the frequency

It can be useful to treat different currents and charges in the same material differently; there can be both bound and free charges and currents in the same medium.

UMBC

AN HONORS UNIVERSITY IN MARYLAND

9.15

Conductance and Resistance

Ohm's Law:

The simplest model that relates the current to the electric field is Ohm's law,

$\mathbf{J} = \sigma\, \mathbf{E}$

Two important limits:

- A perfect (ideal) conductor; $\sigma = \infty$, $\mathbf{E} = 0$.
- A perfect (ideal) dielectric; $\sigma = 0$, $\mathbf{J} = 0$.

Real materials can be more complicated; other effects can include

- A tensor response or a nonlinear response.
- A portion of the current that is due to the magnetic field.

UMBC

AN HONORS UNIVERSITY IN MARYLAND

9.16

Conductance and Resistance

Conductivity and Resistance:

We may relate the conductivity and the resistance in a wire of length l and area A:

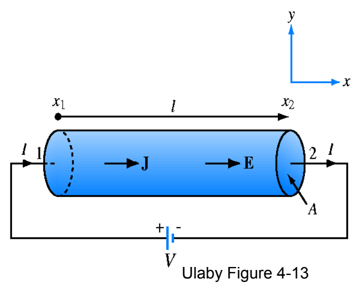
$$V = V_1 - V_2 = -\int_{x_1}^{x_2} \mathbf{E}\cdot d\mathbf{l} = E_x l$$

$$I = \int_A \mathbf{J}\cdot d\mathbf{s} = \int_A \sigma \mathbf{E}\cdot d\mathbf{s} = \sigma E_x A$$

From the relation $V = IR$, we conclude $R = l / \sigma A$

$$R = \frac{V}{I} = \frac{-\int_{x_1}^{x_2} \mathbf{E}\cdot d\mathbf{l}}{\int_A \mathbf{J}\cdot d\mathbf{s}} = \frac{-\int_{x_1}^{x_2} \mathbf{E}\cdot d\mathbf{l}}{\int_A \sigma\mathbf{E}\cdot d\mathbf{s}}$$

NOTE: Conductance is defined as $G = 1/R$



Ulaby Figure 4-13

UMBC AN HONORS UNIVERSITY IN MARYLAND

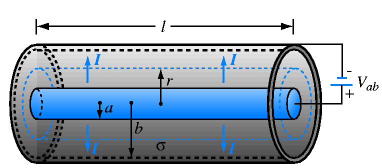
Conductance and Resistance

Conductance of a coaxial cable: Ulaby and Ravaioli Example 4-9

Question: We are now ready to derive the transmission line parameter G' for the coaxial cable geometry. This is the first transmission line parameter that we will derive! A coaxial cable of length l has inner and outer conductors of radius a and b and an insulating layer with a conductivity σ. What is G' ?

Answer: Let I be the current that flows from the inner conductor to the outer conductor. At any distance r, the area through which the current flows is $A = 2\pi r l$. We now have,

$$\mathbf{J} = \hat{\mathbf{r}}\frac{I}{A} = \hat{\mathbf{r}}\frac{I}{2\pi r l} \quad \text{and} \quad \mathbf{E} = \hat{\mathbf{r}}\frac{I}{2\pi\sigma r l}$$



Ulaby Figure 4-15

UMBC AN HONORS UNIVERSITY IN MARYLAND

Conductance and Resistance

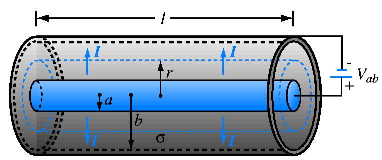
Conductance of a coaxial cable: Ulaby and Ravaioli Example 4-9

Answer (continued): We now have,

$$V_{ab} = -\int_b^a \mathbf{E}\cdot d\mathbf{l} = -\int_b^a \hat{\mathbf{r}}\frac{I}{2\pi\sigma r l}\cdot\hat{\mathbf{r}}\,dr = \frac{I}{2\pi\sigma l}\ln\left(\frac{b}{a}\right)$$

from which we conclude

$$G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab}l} = \frac{2\pi\sigma}{\ln(b/a)}$$



UMBC AN HONORS UNIVERSITY IN MARYLAND

Ulaby Figure 4-15 9.19

Conductance and Resistance

Joule's Law: Power dissipation in a resistor

From basic mechanical theory, we have $\Delta P = \mathbf{F}_\mathrm{V}\cdot\mathbf{u}_\mathrm{V} = \mathbf{E}\cdot\mathbf{J}\,\Delta v$

where

- $\mathbf{F}_\mathrm{V}$ = the average force acting on a small volume of charges
- $\mathbf{u}_\mathrm{V}$ = the average drift velocity of a small volume of charges

and we use $\mathbf{F}_\mathrm{V} = \rho_\mathrm{V}\mathbf{E}\,\Delta v$ and $\mathbf{J} = \rho_\mathrm{V}\mathbf{u}_\mathrm{V}$

from which we may integrate to obtain

$$P = \int_v \mathbf{E}\cdot\mathbf{J}\,dv$$

This relationship is general. When Ohm's law holds

$$P = \int_v \sigma|\mathbf{E}|^2\,dv$$

In the one-dimensional geometry for an electrical wire,

$$P = \int_A \sigma E_x\,ds\int_l E_x\,dl = (\sigma E_x A)(E_x l) = IV = I^2 R$$

UMBC AN HONORS UNIVERSITY IN MARYLAND

This explains the standard circuit formulae for power dissipation! 9.20

Tech Brief 7: Resistive Sensors

Electrical Sensors

- Respond to applied stimulus by generating an electrical signal
- Electrical signal changes depending on intensity of stimulus
  - Voltage, current, or other attribute
- Stimuli include physical, chemical, biological quantities
  - Temperature, pressure, position, distance, motion, velocity, aceleration, concentration (gas or liquid), blood flow, etc.
- Types of sensors
  - **Resistive**
  - Capacitive, inductive, emf sensors (covered later)

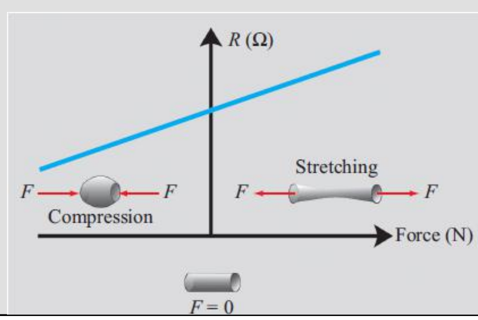
9.21

Tech Brief 7: Resistive Sensors

Piezoresistivity $R = \dfrac{l}{\sigma A}$, Eq. (4.70)

- Stretching a conductor by an external force decreases A and increases l
- Greek work *piezein* means to press
- Resistance relationship approximately modeled by a linear equation, where a_0 is the piezoresistive coefficient

$$R = R_0\left(1 + \frac{\alpha F}{A_0}\right)$$



UMBC AN HONORS UNIVERSITY IN MARYLAND

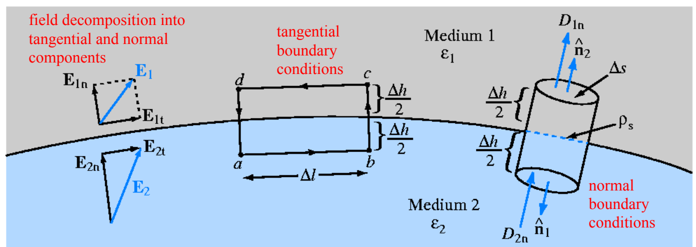
9.22

Boundary Conditions

Field Decomposition:

We first decompose the fields in media 1 and 2, indicated $\mathbf{E}_1$ and $\mathbf{E}_2$ into tangential and normal components

$$\mathbf{E}_1 = \mathbf{E}_{1\,\text{t}} + \mathbf{E}_{1\,\text{n}} \quad \text{and} \quad \mathbf{E}_2 = \mathbf{E}_{2\,\text{t}} + \mathbf{E}_{2\,\text{n}}$$



UMBC AN HONORS UNIVERSITY IN MARYLAND

Ulaby Figure 4-18

9.23

Boundary Conditions

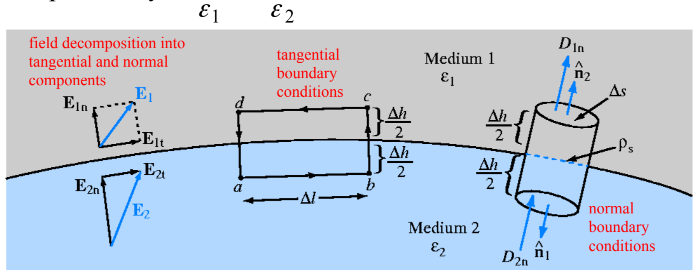
Tangential conditions:

We follow the path *abcda* shown in the figure, we use $\oint_C \mathbf{E}\cdot d\mathbf{l} = 0$, and we let $\Delta h \to 0$. We also note that in this limit, $|b-a| = |d-c| = \Delta l$. We have

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = \int_a^b \mathbf{E}_2 \cdot d\mathbf{l} + \int_c^d \mathbf{E}_1 \cdot d\mathbf{l} = (E_{2\text{t}} - E_{1\text{t}})\Delta l = 0,$$

We conclude:

$$E_{1\text{t}} = E_{2\text{t}} \quad \text{or equivalently} \quad \frac{D_{1\text{t}}}{\varepsilon_1} = \frac{D_{2\text{t}}}{\varepsilon_2}$$



UMBC AN HONORS UNIVERSITY IN MARYLAND

Ulaby Figure 4-18

9.24

Boundary Conditions

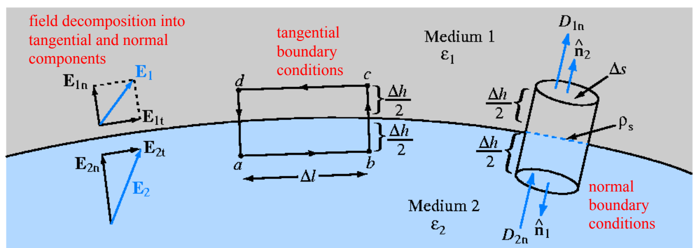
Normal conditions:

We use Gauss's law on the bottom and top of the pill box in the figure,

$$\oint_s \mathbf{D}\cdot d\mathbf{s} = \int_{\text{top}} \mathbf{D}_1\cdot\hat{\mathbf{n}}_2\, ds + \int_{\text{bottom}} \mathbf{D}_2\cdot\hat{\mathbf{n}}_1\, ds = \rho_S\,\Delta s$$

Along with the relation $\hat{\mathbf{n}}_1 = -\hat{\mathbf{n}}_2$, to obtain

$$\hat{\mathbf{n}}_2\cdot\left(\mathbf{D}_1 - \mathbf{D}_2\right) = D_{1\text{n}} - D_{2\text{n}} = \rho_S \quad \text{or equivalently} \quad \varepsilon_1 E_{1\text{n}} - \varepsilon_2 E_{2\text{n}} = \rho_S$$



Ulaby Figure 4-18

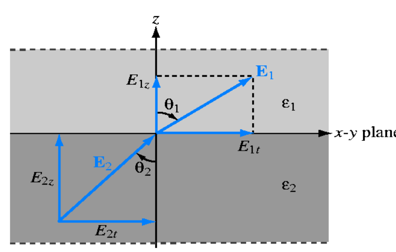
Boundary Conditions

Application of Boundary Conditions: Ulaby and Ravaioli, Ex. 4-10

Question: The x-y plane at $z = 0$ is a charge-free boundary separating two dielectric media with permittivities ε_1 and ε_2. If the electric field in medium 1 is $\mathbf{E}_1 = \hat{\mathbf{x}}\,E_{1x} + \hat{\mathbf{y}}\,E_{1y} + \hat{\mathbf{z}}\,E_{1z}$, find the electric field $\mathbf{E}_2$ in medium 2 and the angles θ_1 and θ_2.

Answer: Let $\mathbf{E}_2 = \hat{\mathbf{x}}\,E_{2x} + \hat{\mathbf{y}}\,E_{2y} + \hat{\mathbf{z}}\,E_{2z}$. From the boundary conditions, we have $E_{2x} = E_{1x}$, $E_{2y} = E_{1y}$, and $E_{2z} = (\varepsilon_1/\varepsilon_2)E_{1z}$. We conclude

$$\mathbf{E}_2 = \hat{\mathbf{x}}\,E_{1x} + \hat{\mathbf{y}}\,E_{1y} + \hat{\mathbf{z}}\,\frac{\varepsilon_1}{\varepsilon_2}E_{1z}.$$



Ulaby Figure 4-19

Boundary Conditions

Application of Boundary Conditions: Ulaby and Ravaioli Ex. 4-10

Answer (continued): The tangential and normal components of $\mathbf{E}_1$ and $\mathbf{E}_2$ are given by

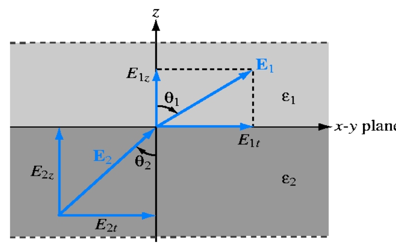
$$E_{1t} = \left(E_{1x}^2 + E_{1y}^2\right)^{1/2}, \quad E_{2t} = \left(E_{2x}^2 + E_{2y}^2\right)^{1/2}, \quad E_{1n} = E_{1z}, \quad E_{2n} = E_{2z}$$

from which we infer

$$\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\left(E_{1x}^2 + E_{1y}^2\right)^{1/2}}{E_{1z}}, \quad \tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\left(E_{2x}^2 + E_{2y}^2\right)^{1/2}}{E_{2z}} = \frac{\left(E_{1x}^2 + E_{1y}^2\right)^{1/2}}{(\varepsilon_1/\varepsilon_2)E_{1z}}$$

The angles are related by

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_2}{\varepsilon_1}$$



Ulaby Figure 4-19

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.27

Boundary Conditions

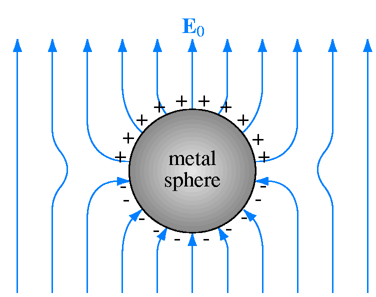
Dielectric-Conductor Interface:

If medium 2 is the conductor, then we have $\mathbf{E}_2 = 0$ and $\mathbf{D}_2 = 0$ everywhere — including the interface with medium 1

It follows that $E_{1t} = D_{1t} = 0$ and $\varepsilon_1 E_{1n} = D_{1n} = \rho_S$

which can be combined to yield $\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = \hat{\mathbf{n}}\,\rho_S$ at the conductor surface

Field lines are always normal to the surface of a conductor!



Ulaby Figure 4-21

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.28

Capacitance

We are now ready to derive the capacitances that we used in the section on transmission lines

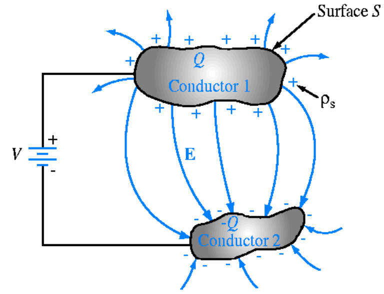
Basic theory:

When a voltage is applied between two conductors, the conductors accumulate an equal and opposite charge that distributes itself so that the surface is at a single potential and there is no electric field to move charges on the surface.

Definition of capacitance: $C = Q/V$

The electric field at the surface of a conductor is given by

$$E = E_n = \hat{\mathbf{n}} \cdot \mathbf{E} = \rho_S / \varepsilon$$



Ulaby Figure 4-23

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.29

Capacitance

Basic theory:

On the $+V$ surface, we have

$$Q = \int_s \rho_S \, ds = \int_s \varepsilon \, \hat{\mathbf{n}} \cdot \mathbf{E} \, ds = \int_s \varepsilon \, \mathbf{E} \cdot d\mathbf{s}$$

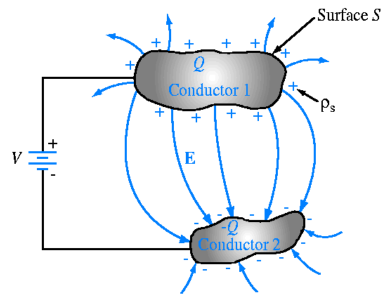
The voltage V is related to $\mathbf{E}$ by

$$V = V_{12} = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l}$$

where P_1 is on the $+V$ conductor and P_2 is on the $-V$ conductor

We conclude:

$$C = \frac{\int_s \varepsilon \, \mathbf{E} \cdot d\mathbf{s}}{-\int_l \mathbf{E} \cdot d\mathbf{l}}$$



Ulaby Figure 4-23

UMBC
AN HONORS UNIVERSITY IN MARYLAND

9.30

Capacitance

Relation of Resistance and Capacitance:

When the material between the conductors has conductivity σ, we found earlier (slide 9.17)

$$R = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_s \sigma \mathbf{E} \cdot d\mathbf{s}}$$

When σ and ε are uniform, it follows: $RC = \varepsilon / \sigma$.

Hence, if we know one, we can find the other!

Example: A coaxial cable.

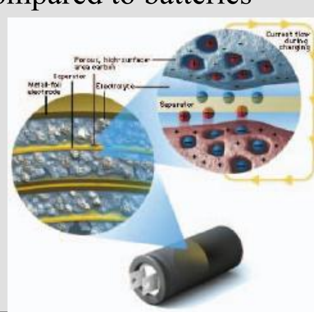
We showed earlier (slide 9.19): $R = \ln(b/a)/2\pi\sigma l$. It follows that $C = 2\pi\varepsilon l / \ln(b/a)$.

9.31

Tech Brief 8: Supercapacitors as Batteries

<u>Electrochemical double-layer capacitor (EDLC)</u>

- Energy storage process a hybrid of capacitor and electrochemical voltaic battery
- Batteries store more energy that capacitors, but capacitors charge and discharge more rapidly.
- Energy density (measured in watt-hours per kg) lower for capacitors and supercapacitors compared to batteries
- Power density is opposite case



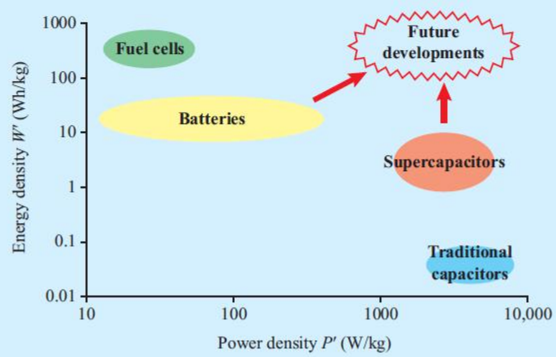
9.32

Tech Brief 8: Supercapacitors as Batteries

Figure TF8-2: Examples of systems that use supercapacitors. (Courtesy of Railway Gazette International; BMW; NASA; Applied Innovative Technologies.)

Energy Storage Devices

Feature	Traditional Capacitor	Supercapacitor	Battery
Energy density W' (Wh/kg)	$\sim 10^{-2}$	1 to 10	5 to 150
Power density P' (W/kg)	1,000 to 10,000	1,000 to 5,000	10 to 500
Charge and discharge rate T	10^{-3} sec	$\sim$ 1 sec to 1 min	$\sim$ 1 to 5 hrs
Cycle life N_c	∞	$\sim 10^6$	$\sim 10^3$



UMBC AN HONORS UNIVERSITY IN MARYLAND

Boundary Conditions

Parallel Plate Capacitor: Ulaby and Ravaioli Example 4-11

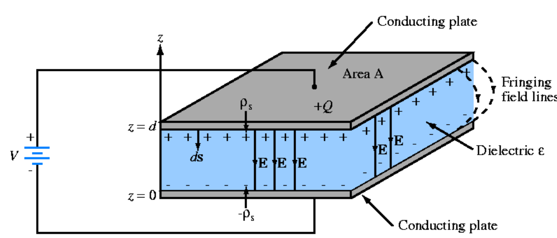
Question: Find the capacitance of a parallel plate capacitor in which each plate has surface area A and they are separated by a distance d. Ignore the fringing fields that appear in any real capacitor.

Answer: On the upper plate, we have $\rho_S = Q / A$. It follows that $\mathbf{E} = -\hat{\mathbf{z}} E = -\hat{\mathbf{z}} (Q / \varepsilon A)$. We also have

$$V = -\int_0^d \mathbf{E} \cdot d\mathbf{l} = E d$$

We conclude:

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon A}{d}$$



Ulaby Figure 4-24

UMBC AN HONORS UNIVERSITY IN MARYLAND

Electrostatic Energy

Work performed in charging a capacitor:

The charges already on a capacitor repel new charges that are added. Work must be done to add the new charges. The voltage on the capacitor is related to its charge by the relation: $V = q / C$. The increment of work that is required to add an increment of charge is: $dW_{\mathrm{e}} = V\,dq = (q/C)\,dq$. We conclude that

$$W_{\mathrm{e}} = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \quad (V = Q/C)$$

Field Energy

The energy may be written in terms of the electric field as

$$W_{\mathrm{e}} = \frac{1}{2}\frac{\varepsilon A}{d}(Ed)^2 = \frac{1}{2}\varepsilon E^2(Ad) = \frac{1}{2}\varepsilon E^2 v \quad (v = \text{volume})$$

UMBC

AN HONORS UNIVERSITY IN MARYLAND

9.35

Electrostatic Energy

Field Energy

We now define an electrostatic potential energy density as

$$w_{\mathrm{e}} = \frac{W_{\mathrm{e}}}{v} = \frac{1}{2}\varepsilon E^2$$

The electrostatic potential energy generalizes to

$$W_{\mathrm{e}} = \frac{1}{2}\int_v \varepsilon E^2\,dv$$

in any geometry. We can use this energy to do work on other charges.

Where is the energy?

Is it in the fields or in the charges?

UMBC

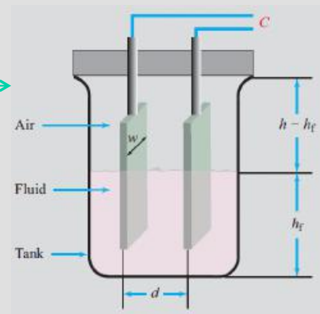
AN HONORS UNIVERSITY IN MARYLAND

9.36

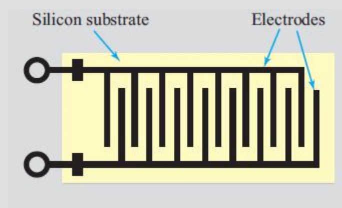
Tech Brief 9: Capacitive Sensors

Capacitive sensors respond to changes in geometry (space between conductors) or dielectric effects (changes in the insulator between conductors)

- Field gauge
  - Sensitive to permittivity changes



- Humidity Sensor
  - Patterned to enhance *A/d* ratio
  - Relative humidity affects air permittivity



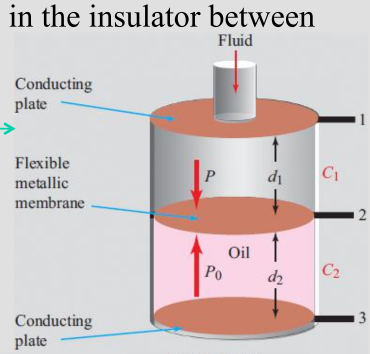
UMBC AN HONORS UNIVERSITY IN MARYLAND

9.37

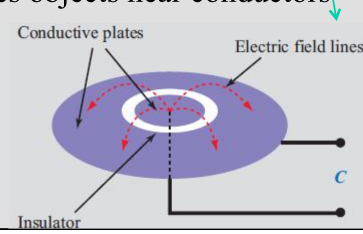
Tech Brief 9: Capacitive Sensors

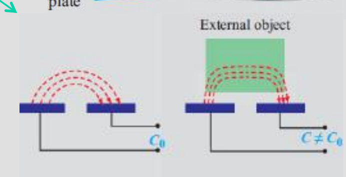
Capacitive sensors respond to changes in geometry (space between conductors) or dielectric effects (changes in the insulator between conductors)

- Pressure Sensor
  - Two capacitors in series



- Noncontact Sensor
  - Senses objects near conductors





(a) Adjacent-plates capacitor (b) Perturbation field

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.38

Tech Brief 9: Capacitive Sensors

Capacitive sensors respond to changes in geometry (space between conductors) or dielectric effects (changes in the insulator between conductors)

- Fingerprint Imager
  - Two dimensional array of capacitive sensors
  - Records electrical representation of fingerprint

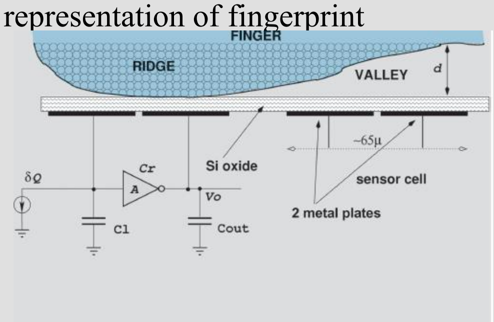


Figure TF9-7: Fingerprint representation. (Courtesy of Dr. M. Tartagni, University of Bologna, Italy.)

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.39

Assignment

Reading: Ulaby and Ravaioli, Chapter 5

Problem Set 5: Some notes.

- There are 8 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Please watch significant digits.
- Get started early!

UMBC AN HONORS UNIVERSITY IN MARYLAND

9.40