CMPE 330 **Spring 2014** **Problem Set #4**

NOTE: You must show complete work for full credit.

1. In Cartesian coordinates, the three corners of a triangle are $P_1(1,5,5)$, $P_2(5,-3,5)$, and $P_3(3,3,-3)$. Find the area of the triangle. [modified from Ulaby and Ravaioli 3.3, p. 171]

2. Given $\mathbf{A} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}3 + \hat{\mathbf{z}}4$ and $\mathbf{B} = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}B_z$, [modifid from Ulaby and Ravaioli 3.4, p. 171]

   a. Find B_x and B_z if $\mathbf{A}$ is parallel to $\mathbf{B}$.

   b. Find a relation between B_x and B_z if $\mathbf{A}$ is perpendicular to $\mathbf{B}$

3. Show that given two vectors $\mathbf{A}$ and $\mathbf{B}$ [Ulaby and Ravaioli 3.14, pp. 171]

   a. The vector $\mathbf{C}$ defined as the vector component of $\mathbf{B}$ in the direction of $\mathbf{A}$ is given by
   $$\mathbf{C} = \hat{\mathbf{a}}(\mathbf{B}\cdot\hat{\mathbf{a}}) = \frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}$$
   Awhere $\hat{\mathbf{a}}$ is the unit vector of $\mathbf{A}$.

   b. The vector $\mathbf{D}$ defined as the vector component of $\mathbf{B}$ perpendicular to $\mathbf{A}$ is given by
   $$\mathbf{D} = \mathbf{B} - \frac{\mathbf{A}(\mathbf{A}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

4. Convert the coordinates of the following points from cylindrical to Cartesian coordinates, [modified from Ulaby and Ravaioli 3.23, p. 172]

   a. $P_1(2, 3\pi/4, -2)$

   b. $P_2(3, 0, -2)$

   c. $P_3(4, \pi/2, 5)$

5. Using the relations that relate Cartesian coordinates to cylindrical coordinates and to spherical coordinates that are given in Ulaby's Table 3-2 (my slides 7.6 and 7.7), derive the transformations for the unit vectors and the vector components for cylindrical to spherical and spherical to cylindrical transformations.

6. Find the distance between the following pairs of points, [modified from Ulaby and Ravaioli 3.32, p. 173]

   a. $P_1(1,1,2)$ and $P_2(0,3,5)$ in Cartesian coordinates

   b. $P_3(2,\pi/3,1)$ and $P_4(4,\pi/2,3)$ in cylindrical coordinates

   c. $P_5(3,\pi,\pi/2)$ and $P_6(4,\pi/2,\pi)$ in spherical coordinates

7. Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points, [Ulaby 3.34, p. 173]

   a. $\mathbf{A} = \hat{\mathbf{x}}(x + y)$ at $P_1(1, 2, 3)$

   b. $\mathbf{B} = \hat{\mathbf{x}}(y - x) + \hat{\mathbf{y}}(x - y)$ at $P_2(1, 0, 2)$

   c. $\mathbf{C} = \hat{\mathbf{x}}[y^2/(x^2 + y^2)] - \hat{\mathbf{y}}[x^2/(x^2 + y^2)] + \hat{\mathbf{z}}4$ at $P_3(1, -1, 2)$

   d. $\mathbf{D} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta + \hat{\boldsymbol{\phi}} \cos^2\theta$ at $P_4(2, \pi/2, \pi/4)$

   e. $\mathbf{E} = \hat{\mathbf{R}} \cos\phi + \hat{\boldsymbol{\theta}} \sin\phi + \hat{\boldsymbol{\phi}} \sin^2\theta$ at $P_5(3, \pi/2, \pi)$

8. For a vector field $\mathbf{E}(x, y)$ that does depend on z, the divergence in Cartesian coordinates is given by [See Ulaby, Eq. 3.96]

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}.$$

   Derive the divergence in polar (cylindrical) coordinates

$$\nabla \cdot \mathbf{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi}$$

   in two ways.

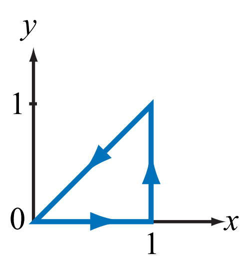
   a. Use partial derivative transformations in the same way that Ulaby's Eqs. (3.81) and (3.82) were derived.

   b. Use the general result on slide 7.22.

9. For the vector field $\mathbf{E} = \hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)$, calculate the following [Ulaby, et al. 3.50, p. 175]

   a. $\oint_C \mathbf{E} \cdot d\mathbf{l}$ around the triangle shown to the right [Ulaby, et al. Fig. P3.50]

   b. $\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s}$ over the area of the triangle



10. Find the Laplacian of the following scalar functions: [modified from Ulaby and Ravaioli 3.57, p. 177]

   a. $V = 2xy^2z^4$

   b. $V = 3xy + 2yz + xz$

   c. $V = 3/(x^2 + y^2)$

   d. $V = 5\exp(-r)\cos\phi$

   e. $V = 10\exp(-R)\sin\theta$