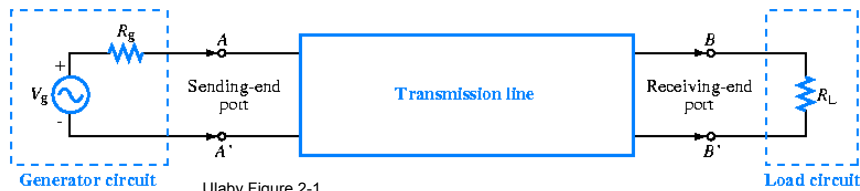


What is a transmission line?

The basic structure — a four port device that connects:

- (1) A Thévenin equivalent input or *generator circuit* with V_g and R_g , as the equivalent voltage source and resistance
- (2) A *load circuit* with a load resistance R_L as the equivalent resistance



Ulaby Figure 2-1

More generally, with a-c inputs:

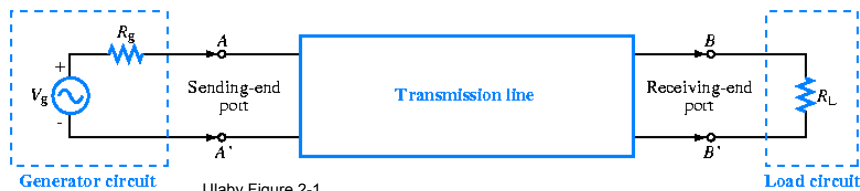
$$V_g \text{ and } R_g \rightarrow \tilde{V}_g \text{ and } Z_g$$

$$R_L \rightarrow Z_L$$

What is a transmission line?

The basic structure — a four port device that connects:

- (1) A Thévenin equivalent input or *generator circuit* with V_g and R_g , as the equivalent voltage source and resistance
- (2) A *load circuit* with a load resistance R_L as the equivalent resistance

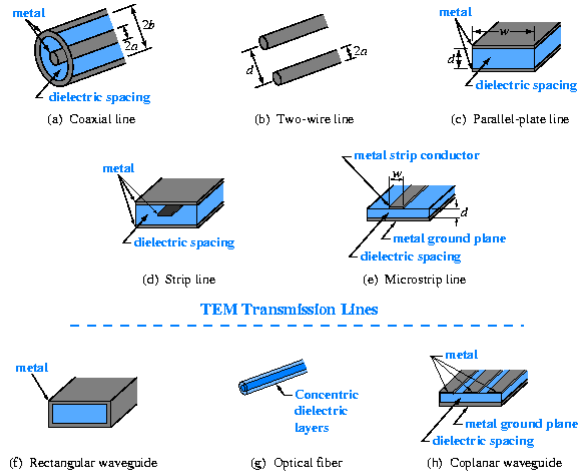


Ulaby Figure 2-1

*We take into account the finite transmission time
from A, A' to B, B'*

What is a transmission line?

There are two basic types of transmission line:



TEM Transmission Lines

Higher Order Transmission Lines

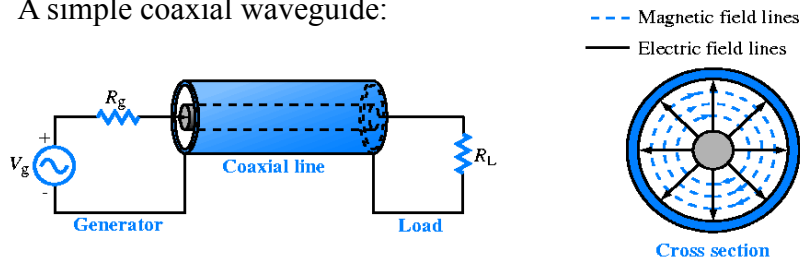
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Ulab
Figure 2-4

3.3

What is a transmission line?

A simple coaxial waveguide:



Ulab Figure 2-5

- There is an electric field caused by charge separation from the inner and outer wires
- There is a magnetic field that is caused by the current flow, in opposite directions, along the inner and outer conductors

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3.4

Lumped Element Model

We replace the detailed physics of the transmission line with lumped circuit parameters:

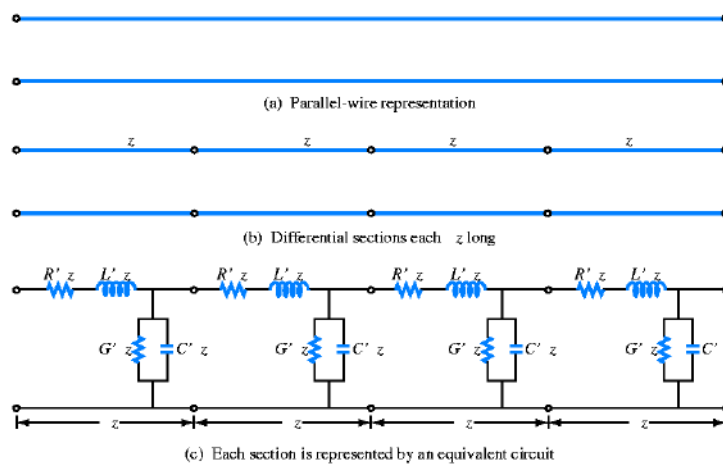
- R' : The combined resistance of both conductors (ohms/meter)
- L' : The combined inductance of both conductors (henrys/meter)
- G' : The conductance of the insulating medium between conductors (siemens/meter)
- C' : The capacitance between the two conductors (farads/meter)

- These parameters are all “per unit length.” Hence, we put primes.
- Their values are determined by the detailed physics.
- We will use this model to find a simple pair of coupled equations,

The telegrapher's equation
that describes propagation through the transmission line

Lumped Element Model

Transmission Line Model:



Lumped Element Model

Examples of parameter determination from the physics:

Parameter	Coaxial	Two wire	Parallel plane
R'	$\frac{R_S}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_S}{\pi d}$	$\frac{2R_S}{w}$
L'	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]$	$\frac{\mu h}{w}$
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\sigma w}{h}$
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\epsilon w}{h}$



Ulaby Table 2-1 Slide 3.3 and the notes have definitions of a, b, d
 R_S = surface resistance = $(\pi f \mu_c / \sigma_c)^{1/2}$; μ_c, σ_c are conductor parameters
 μ, ϵ , and σ are insulator parameters

3.7

Microstrip Line

Real transmission lines can be complex

⇒ Curve fitting approximations are useful $s = w/h$
 w = strip width
 h = substrate thickness

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \left(\frac{\epsilon_r - 1}{2} \right) \left(1 + \frac{10}{s} \right)^{-xy}$$

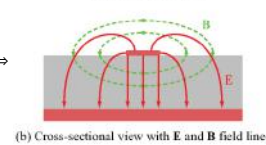
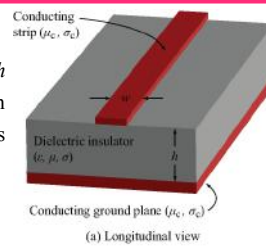
$$x = 0.56 \left(\frac{\epsilon_r - 0.9}{\epsilon_r + 3} \right)^{0.05}$$

$$y = 1 + 0.02 \ln \left(\frac{s^4 + 3.7 \times 10^{-4} s^2}{s^4 + 0.43} \right) + 0.05 \ln(1 + 1.7 \times 10^{-4} s^3)$$

$$Z_0 = \frac{60}{\sqrt{\epsilon_{\text{eff}}}} \ln \left[\frac{6 + (2\pi - 6) \exp(-t)}{s} + \left(1 + \frac{4}{s^2} \right)^{1/2} \right]; \quad t = \left(\frac{30.67}{s} \right)^{0.75}$$

$$R' = 0; \quad G' = 0$$

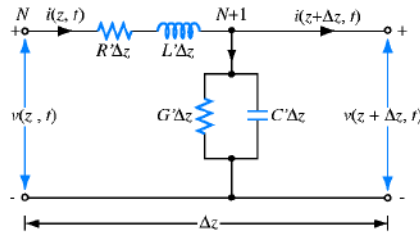
$$C' = \sqrt{\epsilon_{\text{eff}}} / Z_0 c; \quad L' = Z_0 \sqrt{\epsilon_{\text{eff}}} / c$$



Ulaby and Ravaioli 2015, Figure 2-10 3.8

Transmission Line Equations

Our starting point is one section of the transmission line



Ulaby Figure 2-8

Note: We are taking a section that is small enough so that we can assume that the transmission through that section is instantaneous

Kirchoff's voltage law: $v(z, t) - R' \Delta z i(z, t) - L' \Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0$

Kirchoff's current law: (at node $N + 1$)

$$i(z, t) - G' \Delta z v(z + \Delta z, t) - C' \Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0$$

Transmission Line Equations

Rearranging terms

$$-\frac{v(z + \Delta z, t) - v(z, t)}{\Delta z} = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}$$

$$-\frac{i(z + \Delta z, t) - i(z, t)}{\Delta z} = G' v(z + \Delta z, t) + C' \frac{\partial v(z + \Delta z, t)}{\partial t}$$

Take the limit as $\Delta z \rightarrow 0$:

$$-\frac{\partial v(z, t)}{\partial z} = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}$$

$$-\frac{\partial i(z, t)}{\partial z} = G' v(z, t) + C' \frac{\partial v(z, t)}{\partial t}$$

Telegrapher's Equations

Transmission Line Equations

Simplification

$$R' = (f\mu_c/\sigma_c)^{1/2} \rightarrow 0 \quad (\text{low frequency and high conductivity conductors})$$

$$G' \rightarrow 0 \quad (\text{low conductivity dielectric material})$$

The transmission equations become

$$-\frac{\partial v(z,t)}{\partial z} = L' \frac{\partial i(z,t)}{\partial t}, \quad -\frac{\partial i(z,t)}{\partial z} = C' \frac{\partial v(z,t)}{\partial t}$$

which imply

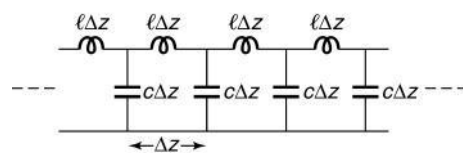
$$\frac{\partial^2 v(z,t)}{\partial z^2} = L'C' \frac{\partial^2 v(z,t)}{\partial t^2}, \quad \frac{\partial^2 i(z,t)}{\partial z^2} = L'C' \frac{\partial^2 i(z,t)}{\partial t^2}$$

Note: In Paul's notation: $v \rightarrow V$, $i \rightarrow I$, $L' \rightarrow l$, and $C' \rightarrow c$

$$\frac{\partial V(z,t)}{\partial z} = -l \frac{\partial I(z,t)}{\partial t}, \quad \frac{\partial I(z,t)}{\partial z} = -c \frac{\partial V(z,t)}{\partial t}$$

Transmission Line Equations

The circuit diagram also simplifies

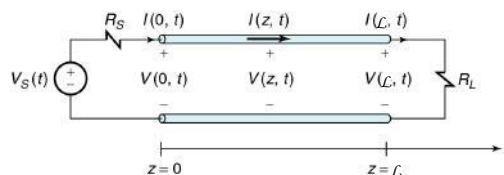


Paul Figure 6.1(c)

Note change in notation
from Ulaby

The second-order wave equations become in Paul's notation

$$\frac{\partial^2 V(z,t)}{\partial z^2} = lc \frac{\partial^2 V(z,t)}{\partial t^2}, \quad \frac{\partial^2 I(z,t)}{\partial z^2} = lc \frac{\partial^2 I(z,t)}{\partial t^2}$$



V_S = source voltage
 R_S = source resistance
 R_L = load resistance
 L = total length of
transmission line

Time-Domain Evolution

The second-order voltage equation has the general solution

$$V(z,t) = \underbrace{V^+\left(t - \frac{z}{v}\right)}_{\text{forward propagation}} + \underbrace{V^-\left(t + \frac{z}{v}\right)}_{\text{backward propagation}}, \quad \text{with } v = 1/\sqrt{lc}$$

- $V^+(x)$ and $V^-(x)$ = two completely arbitrary functions
- v = the velocity of propagation

- (1) Arbitrarily shaped pulses propagate in the transmission line, both forward and backward, *without dispersing!*
- (2) The absence of dispersion is a special feature of TEM modes, which makes direct time-domain analysis possible.



Caveat: We are treating l and c as if they have no frequency dependence. This assumption is not generally true! It will often be approximately true over a limited frequency range

3.13

Time-Domain Evolution

The current is determined from the voltage

— After substitution into the transmission line equations

$$I(z,t) = \underbrace{\frac{V^+\left(t - \frac{z}{v}\right)}{Z_C}}_{\text{forward propagation}} - \underbrace{\frac{V^-\left(t + \frac{z}{v}\right)}{Z_C}}_{\text{backward propagation}}, \quad \text{with } Z_C = \sqrt{l/c}$$

- Z_C = the characteristic impedance of the transmission line
- Note that since $V(z, t)$ and $I(z, t)$ are real, so is Z_C
 - The characteristic impedance is purely resistive in this case.

We have $I^+(z,t) = V^+(z,t)/Z_C$ and $I^-(z,t) = -V^-(z,t)/Z_C$

BUT $|I(z,t)| \neq |V(z,t)|/Z_C$



There is no simple relationship between $V(z,t)$ and $I(z,t)$!

3.14

Time-Domain Evolution

Example: From Paul Quick Review Exercises 6.1 and 6.7

Question: What are the per unit length capacitance and inductance of a two-wire line whose wires have a radius of 7.5 mils and a separation of 50 mils? What is the characteristic impedance and the velocity of propagation? (These dimensions are typical for ribbon cables used to interconnect components.)

Answer: $7.5 \text{ mils} \times 2.54 \times 10^{-5} \text{ m/mil} = 1.91 \times 10^{-4} \text{ m}$,
 $50 \text{ mils} = 1.27 \times 10^{-3} \text{ m}$. $(D/d) = 50.0/15.0 = 3.33$, $[(D/d)^2 - 1] = 3.18$,
 $\ln(3.33 + 3.18) = 1.87$. $L' = (\mu_0 / \pi) \times 1.87 = 4 \times 10^{-7} \text{ H/m} \times 1.87 =$
 $7.46 \times 10^{-7} \text{ H/m}$. $C' = (\pi \epsilon_0 / 1.87) = (\pi / 1.87) \times 8.85 \times 10^{-12} \text{ F/m}$
 $= 1.49 \times 10^{-11} \text{ F/m}$. $Z_0 = (L' / C')^{1/2} = 224 \text{ ohms}$. $u_p = 1 / (7.46 \times 10^{-7} \text{ H/m} \times$
 $1.49 \times 10^{-11} \text{ F/m})^{1/2} = 3.00 \times 10^8 \text{ m/s}$. The velocity equals the speed of light in the vacuum.



NOTE: Paul uses a different notation for all quantities

3.15

Time-Domain Evolution

We now consider the behavior at the load

$$V(\mathcal{L}, t) = V^+(t - T) + V^-(t + T), \quad T = \mathcal{L}/v$$

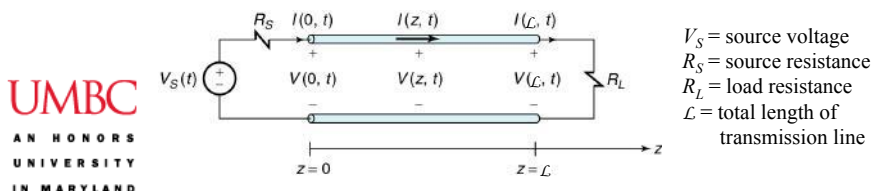
= one-way time delay

$$I(\mathcal{L}, t) = \frac{V^+(t - T)}{Z_C} - \frac{V^-(t + T)}{Z_C}$$

From Ohm's Law: $V(\mathcal{L}, t) / I(\mathcal{L}, t) = R_L$

From the telegrapher's equations: $V^+(t - T) / I^+(t - T) = Z_C$

Unless $R_L = Z_C$, there must be reflected waves!



3.16

Time-Domain Evolution

We now consider the behavior at the load

$$V(\mathcal{L}, t) = V^+(t - T) + V^-(t + T), \quad T = \mathcal{L}/v = \text{one-way time delay}$$

$$I(\mathcal{L}, t) = \frac{V^+(t - T)}{Z_C} - \frac{V^-(t + T)}{Z_C}$$

From Ohm's Law: $V(\mathcal{L}, t) / I(\mathcal{L}, t) = R_L$

From the telegrapher's equations: $V^+(t - T) / I^+(t - T) = Z_C$

Unless $R_L = Z_C$, there must be reflected waves!

When $R_L = Z_C$, we say that the line is *impedance matched*



The concept of impedance matching is one of the most important in this course.

Impedance matching is never perfect, giving rise to transients.

3.17

Time-Domain Evolution

Behavior at the load: Reflection Coefficient Γ_L

$$\Gamma_L = V^-(t + T) / V^+(t - T)$$

which implies

$$V(\mathcal{L}, t) = V^+(t - T)[1 + \Gamma_L], \quad I(\mathcal{L}, t) = \frac{V^+(t - T)}{Z_C}[1 - \Gamma_L]$$

from which

$$\frac{V(\mathcal{L}, t)}{I(\mathcal{L}, t)} = R_L = Z_C \frac{1 + \Gamma_L}{1 - \Gamma_L} \quad \text{and} \quad \Gamma_L = \frac{R_L - Z_C}{R_L + Z_C}$$

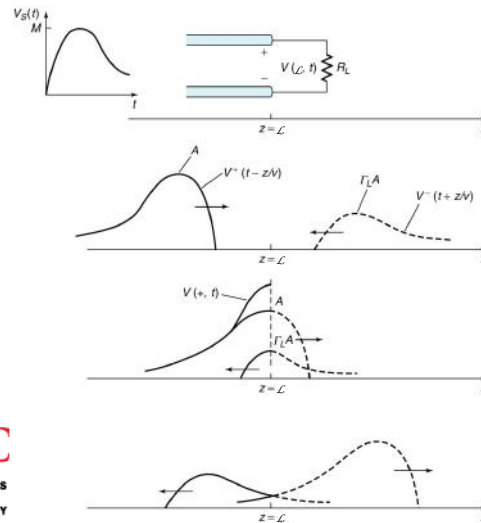
- Schematically, we may understand the behavior at the load as a combination of a forward-propagating wave and a backward-going wave that is a reflected copy of the first wave, multiplied by Γ_L .



3.18

Time-Domain Evolution

Behavior at the load



Paul Figure 6-9

Illustration of the reflection of voltage waves at the load of a transmission line.

Time-Domain Evolution

Behavior at the source $[0 < t < 2T]$

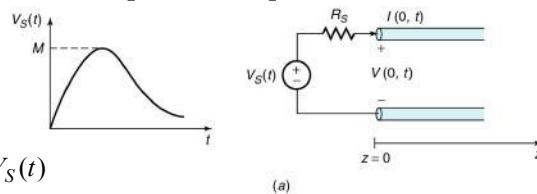
$$V_S(t) = 0 \text{ when } t < 0 \Rightarrow V^-(0,t) = 0 \text{ when } 0 < t < 2T$$

From Kirchoff's voltage law:

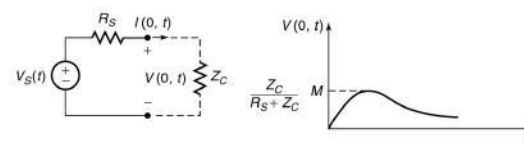
$$V_S(t) = R_S I^+(t) + V^+(t) = [(R_S / Z_C) + 1] V^+(t)$$

$$V(0,t) = V^+(t) = \frac{Z_C}{R_S + Z_C} V_S(t)$$

$$I(0,t) = \frac{V_S(t)}{R_S + Z_C}$$



(a)



(b) $t < 2 \frac{L}{v}$ Paul Figure 6-10

Time-Domain Evolution

Time evolution

$$V(0,t) = \frac{Z_C}{R_S + Z_C} \left[V_S(t) + (1 + \Gamma_S) \Gamma_L V_S(t - 2T) \right. \\ \left. + (1 + \Gamma_S)(\Gamma_S \Gamma_L) \Gamma_L V_S(t - 4T) + (1 + \Gamma_S)(\Gamma_S \Gamma_L)^2 \Gamma_L V_S(t - 6T) + \dots \right]$$

$$V(L,t) = \frac{Z_C}{R_S + Z_C} \left[(1 + \Gamma_L) V_S(t - T) + (1 + \Gamma_L)(\Gamma_S \Gamma_L) V_S(t - 3T) \right. \\ \left. + (1 + \Gamma_L)(\Gamma_S \Gamma_L)^2 V_S(t - 5T) + (1 + \Gamma_L)(\Gamma_S \Gamma_L)^3 V_S(t - 7T) + \dots \right]$$

NOTE: This series continues forever **IF**
we define $V_S(t) = 0$ when $t < 0$

Time-Domain Evolution

Geometric series:

$$1 + x + x^2 + \dots + x^m = \sum_{i=0}^m x^i = \frac{1 - x^{m+1}}{1 - x}$$

Summing all the bounces, we find

$$V(0,t) = \frac{V_S Z_C}{R_S + Z_C} \left[\frac{1 - (\Gamma_S \Gamma_L)^{m_S+1} + \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_S}]}{1 - \Gamma_S \Gamma_L} \right]$$

$$I(0,t) = \frac{V_S}{R_S + Z_C} \left[\frac{1 - (\Gamma_S \Gamma_L)^{m_S+1} - \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_S}]}{1 - \Gamma_S \Gamma_L} \right]$$

$$V(L,t) = \frac{V_S Z_C}{R_S + Z_C} \left[\frac{(1 + \Gamma_L) [1 - (\Gamma_S \Gamma_L)^{m_L}]}{1 - \Gamma_S \Gamma_L} \right]$$

$$I(L,t) = \frac{V_S}{R_S + Z_C} \left[\frac{(1 - \Gamma_L) [1 - (\Gamma_S \Gamma_L)^{m_L}]}{1 - \Gamma_S \Gamma_L} \right]$$

$$m_S = \lfloor t / 2T \rfloor, \quad m_L = \lfloor (t + T) / 2T \rfloor$$

Time-Domain Evolution

In Ulaby et al.'s notation, we find

$$V(0,t) = \frac{V_g Z_0}{R_g + Z_0} \left[\frac{1 - (\Gamma_g \Gamma_L)^{m_g+1} + \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_g}]}{1 - \Gamma_g \Gamma_L} \right]$$

$$I(0,t) = \frac{V_g}{R_g + Z_0} \left[\frac{1 - (\Gamma_g \Gamma_L)^{m_g+1} - \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_g}]}{1 - \Gamma_g \Gamma_L} \right]$$

$$V(l,t) = \frac{V_g Z_0}{R_g + Z_0} \left[\frac{(1 + \Gamma_L) [1 - (\Gamma_g \Gamma_L)^{m_l}]}{1 - \Gamma_g \Gamma_L} \right]$$

$$I(l,t) = \frac{V_g}{R_g + Z_0} \left[\frac{(1 - \Gamma_L) [1 - (\Gamma_g \Gamma_L)^{m_l}]}{1 - \Gamma_g \Gamma_L} \right]$$

$$m_g = \lfloor t / 2T \rfloor, \quad m_l = \lfloor (t + T) / 2T \rfloor$$

Time-Domain Evolution

When m_S and m_L become infinite, these become:

$$V(0,t) = \frac{V_S Z_C}{R_S + Z_C} \left[\frac{1 + \Gamma_L}{1 - \Gamma_S \Gamma_L} \right], \quad I(0,t) = \frac{V_S}{R_S + Z_C} \left[\frac{1 - \Gamma_L}{1 - \Gamma_S \Gamma_L} \right]$$

$$V(\mathcal{L},t) = \frac{V_S Z_C}{R_S + Z_C} \left[\frac{1 + \Gamma_L}{1 - \Gamma_S \Gamma_L} \right], \quad I(\mathcal{L},t) = \frac{V_S}{R_S + Z_C} \left[\frac{1 - \Gamma_L}{1 - \Gamma_S \Gamma_L} \right]$$

which is the same at both the source and the load

In Ulaby et al.'s notation, we have

$$V(0,t) = V(l,t) = \frac{V_g Z_0}{R_g + Z_0} \left[\frac{1 + \Gamma_L}{1 - \Gamma_g \Gamma_L} \right],$$

$$I(0,t) = I(l,t) = \frac{V_g}{R_g + Z_0} \left[\frac{1 - \Gamma_L}{1 - \Gamma_g \Gamma_L} \right]$$

Time-Domain Evolution

Example (Paul 6.1):

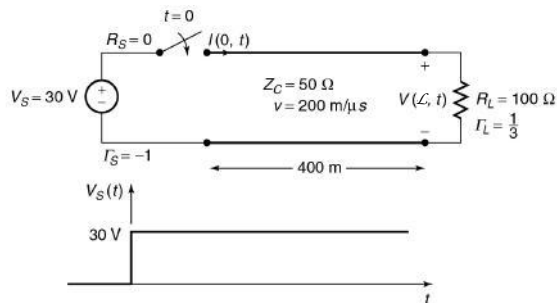
Question: A thirty volt battery is switched onto a line of length 400 m. The line has a characteristic impedance of $50\ \Omega$ and a propagation velocity of $2.00 \times 10^8\text{ m/s}$. The source resistance is zero, and the load resistance is $100\ \Omega$. What is the current at the input and the voltage at the load as a function of time?

Answer: System Parameters:

$$\Gamma_S = \frac{0 - 50}{0 + 50} = -1$$

$$\Gamma_L = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$T = \frac{L}{v} = 2\ \mu\text{s}$$



Paul Figure 6-12(a)

3.27

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Time-Domain Evolution

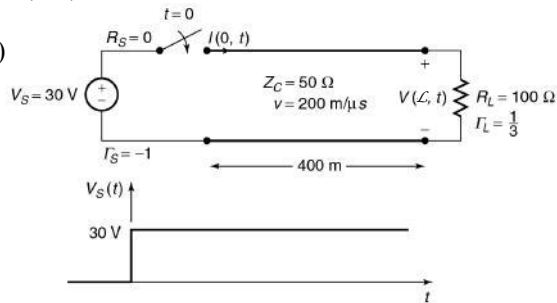
Example (Paul 6.1):

Answer (continued): For $t = (0\ \mu\text{s}, 2\ \mu\text{s})$, $V(L, t) = 0$. For $t = (2\ \mu\text{s}, 6\ \mu\text{s})$, $V(L, t) = (\text{incoming voltage}) + (\text{reflected voltage}) = 30\text{ V} + (1/3) \times 30\text{ V} = 40\text{ V}$. Since the reflected voltage is 10 V , we find that for $t = (4\ \mu\text{s}, 8\ \mu\text{s})$, $V^+(t) = 30\text{ V} - 10\text{ V} = 20\text{ V}$, and, for $t = (6\ \mu\text{s}, 10\ \mu\text{s})$, $V(L, t) = (4/3) \times 20\text{ V} = 26.67\text{ V}$. And so on...

For $t = (0\ \mu\text{s}, 4\ \mu\text{s})$, $I(0, t) = 30\text{ V} / 50\ \Omega = 0.6\text{ A}$.

For $t = (4\ \mu\text{s}, 8\ \mu\text{s})$,
 $I(0, t) = (20\text{ V} - 10\text{ V}) / 50\ \Omega = 0.2\text{ A}$.

And so on...



Paul Figure 6-12(a)

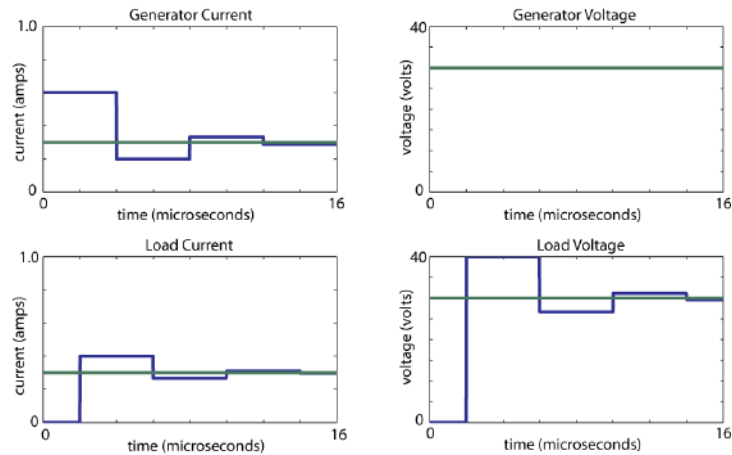
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Time-Domain Evolution

Example (Paul 6.1):

MATLAB OUTPUT



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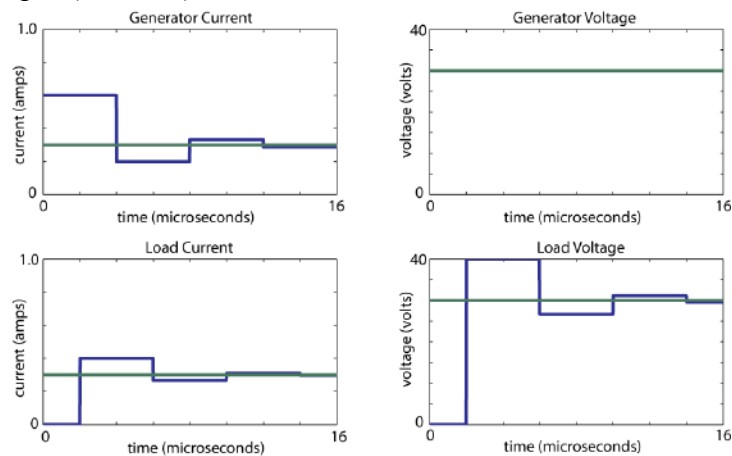
Key Point: Without good impedance matching, transients can exist for a long time, leading to poor system performance!

3.29

Time-Domain Evolution

Example (Paul 6.1):

MATLAB OUTPUT



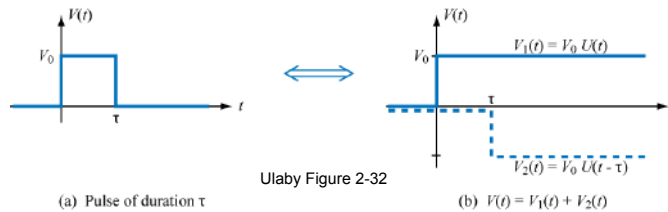
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But transients can be useful for TDR in a matched line

3.30

Time-Domain Evolution

When a pulse is present, superpose two step functions



Time-Domain Evolution

Example (Paul 6.2):

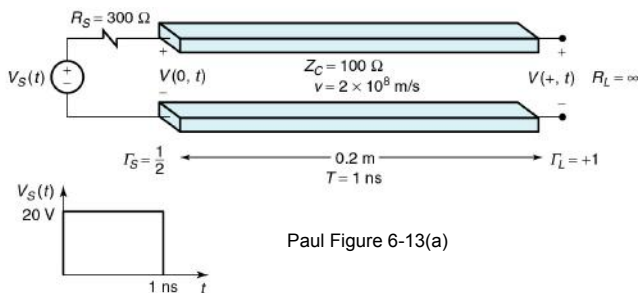
Question: Consider a line of length 0.2 m. The source voltage is a pulse of 20 V amplitude and 1 ns duration t_D . The line has a characteristic impedance of 100 Ω and a velocity of propagation of 2×10^8 m/s. The source resistance is 300 Ω and the load is open-circuited. Find the source and generator currents and voltages.

Answer: System Parameters:

$$\Gamma_S = \frac{300 - 100}{300 + 100} = \frac{1}{2}$$

$$\Gamma_L = \frac{\infty - 100}{\infty + 100} = 1$$

$$T = \frac{\mathcal{L}}{v} = 1 \text{ ns}; \quad t_D = 1 \text{ ns}$$



Time-Domain Evolution

Example (Paul 6.2):

Answer (continued): We use the formulae on slide 3.24, but we must subtract a negative contribution for the back end of the pulse

$$\begin{aligned}
 V(0,t) &= \frac{V_S Z_C}{R_S + Z_C} \left[\frac{1 - (\Gamma_S \Gamma_L)^{m_{S1}+1} + \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_{S1}}]}{1 - \Gamma_S \Gamma_L} - \frac{1 - (\Gamma_S \Gamma_L)^{m_{S2}+1} + \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_{S2}}]}{1 - \Gamma_S \Gamma_L} \right] \\
 I(0,t) &= \frac{V_S}{R_S + Z_C} \left[\frac{1 - (\Gamma_S \Gamma_L)^{m_{S1}+1} - \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_{S1}}]}{1 - \Gamma_S \Gamma_L} - \frac{1 - (\Gamma_S \Gamma_L)^{m_{S2}+1} - \Gamma_L [1 - (\Gamma_S \Gamma_L)^{m_{S2}}]}{1 - \Gamma_S \Gamma_L} \right] \\
 V(\mathcal{L},t) &= \frac{V_S Z_C}{R_S + Z_C} \left[\frac{(1 + \Gamma_L)[1 - (\Gamma_S \Gamma_L)^{m_{L1}}]}{1 - \Gamma_S \Gamma_L} - \frac{(1 + \Gamma_L)[1 - (\Gamma_S \Gamma_L)^{m_{L2}}]}{1 - \Gamma_S \Gamma_L} \right] \\
 I(\mathcal{L},t) &= \frac{V_S}{R_S + Z_C} \left[\frac{(1 - \Gamma_L)[1 - (\Gamma_S \Gamma_L)^{m_{L1}}]}{1 - \Gamma_S \Gamma_L} - \frac{(1 - \Gamma_L)[1 - (\Gamma_S \Gamma_L)^{m_{L2}}]}{1 - \Gamma_S \Gamma_L} \right]
 \end{aligned}$$



$$\begin{aligned}
 m_{g1} &= \lfloor t / 2T \rfloor, & m_{g2} &= \lfloor (t - t_D) / 2T \rfloor, \\
 m_{L1} &= \lfloor (t + T) / 2T \rfloor, & m_{L2} &= \lfloor (t + T - t_D) / 2T \rfloor, \quad t > t_D
 \end{aligned}$$

You only subtract for $t > t_D$

3.33

Time-Domain Evolution

Example (Paul 6.2):

Answer (continued): In Ulaby's notation, this is

$$\begin{aligned}
 V(0,t) &= \frac{V_g Z_0}{R_g + Z_0} \left[\frac{1 - (\Gamma_g \Gamma_L)^{m_{g1}+1} + \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_{g1}}]}{1 - \Gamma_g \Gamma_L} - \frac{1 - (\Gamma_g \Gamma_L)^{m_{g2}+1} + \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_{g2}}]}{1 - \Gamma_g \Gamma_L} \right] \\
 I(0,t) &= \frac{V_g}{R_g + Z_0} \left[\frac{1 - (\Gamma_g \Gamma_L)^{m_{g1}+1} - \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_{g1}}]}{1 - \Gamma_g \Gamma_L} - \frac{1 - (\Gamma_g \Gamma_L)^{m_{g2}+1} - \Gamma_L [1 - (\Gamma_g \Gamma_L)^{m_{g2}}]}{1 - \Gamma_g \Gamma_L} \right] \\
 V(l,t) &= \frac{V_g Z_0}{R_g + Z_0} \left[\frac{(1 + \Gamma_L)[1 - (\Gamma_g \Gamma_L)^{m_{L1}}]}{1 - \Gamma_g \Gamma_L} - \frac{(1 + \Gamma_L)[1 - (\Gamma_g \Gamma_L)^{m_{L2}}]}{1 - \Gamma_g \Gamma_L} \right] \\
 I(l,t) &= \frac{V_g}{R_g + Z_0} \left[\frac{(1 - \Gamma_L)[1 - (\Gamma_g \Gamma_L)^{m_{L1}}]}{1 - \Gamma_g \Gamma_L} - \frac{(1 - \Gamma_L)[1 - (\Gamma_g \Gamma_L)^{m_{L2}}]}{1 - \Gamma_g \Gamma_L} \right]
 \end{aligned}$$



$$\begin{aligned}
 m_{S1} &= \lfloor t / 2T \rfloor, & m_{S2} &= \lfloor (t - t_D) / 2T \rfloor, \\
 m_{L1} &= \lfloor (t + T) / 2T \rfloor, & m_{L2} &= \lfloor (t + T - t_D) / 2T \rfloor, \quad t > t_D
 \end{aligned}$$

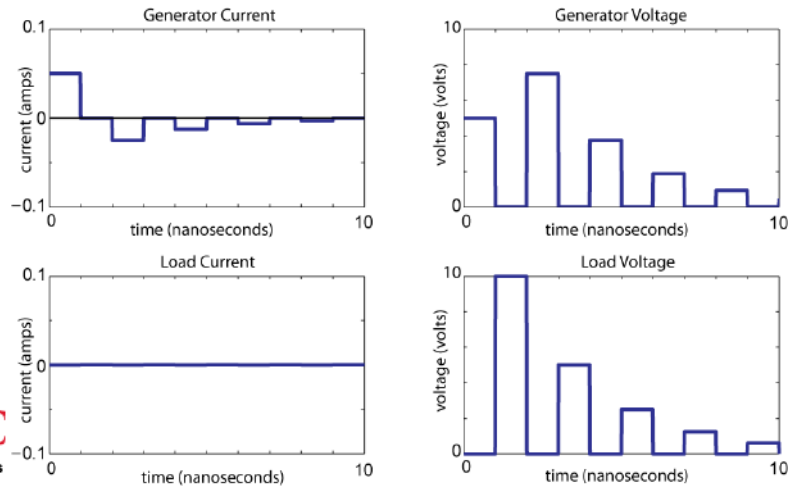
You only subtract for $t > t_D$

3.34

Time-Domain Evolution

Example (Paul 6.2):

MATLAB OUTPUT



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3.35

Assignment

Problem Set 2: Some notes

- There are 6 problems. Many of the answers to these problems have been provided by either Ulaby and Ravaoli or by me. YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Watch significant figures. Report the number that I ask for.
- These problems are difficult. GET STARTED EARLY!

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