·21

HW10

1

$m = 129$ $\quad \bar{x} = 107.6$ $\quad s_1 = 1.3$

$n = 129$ $\quad \bar{y} = 123.6$ $\quad s_2 = 2.0$

μ_2 pop mean for Grade 2

(a) $H_0 = \mu_2 - \mu_1 = 10$ vs $H_1 =$ Grade 2 > Grade 1 by 10

$H_1 = \mu_2 - \mu_1 > 10$

large sample: $z = \dfrac{123.6 - 107.6 - 10}{\sqrt{\frac{1.3^2}{129} + \frac{2.0^2}{129}}} \simeq 28.6$

$z_\alpha = z_{0.01} = 2.3$ => we reject H_0 as $z > z_\alpha$

(b) $\mu_1 - \mu_2 \in \left[(107.6 - 123.6) \mp z_{\frac{0.05}{2}} \sqrt{\frac{1.3^2}{129} + \frac{2^2}{129}}\right]$

$\mu_1 - \mu_2 \in \left[-6 \mp 1.96 \sqrt{\frac{5.69}{129}}\right] = [-6.4\ ,\ -5.59]$

(c) $n = m = \dfrac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\Delta' - \Delta_0)^2} = \dfrac{(1.3 + 4)(2.33 + 0.85)^2}{(12 - 10)^2}$

$= 14$

(d) The appropriate test is t-test

(e) $\mu_1 - \mu_2 \in \left[-6 \mp t_{\frac{\alpha}{2}, v} \sqrt{\frac{1.3^2}{14} + \frac{2^2}{22}}\right]$

$.01, 127, 28 \doteq 2.54$

$0.01, 2, 24 = 2.54$

$$v = \frac{\left[\frac{1.69}{14} + \frac{4}{22}\right]^2}{\frac{(1.69/14)^2}{13} + \frac{(4/22)^2}{21}} = 33.96 \simeq 33$$

$$t_{0.005, 33} = 2.738 \Rightarrow \mu_1 - \mu_2 \in \left[.6 \mp 2.738\sqrt{0.303}\right]$$

f. $H_0: \sigma_1^2 = \sigma_2^2$ vs $H_1: \sigma_1^2 < \sigma_2^2$, $\alpha = 0.1$

$$F = 1.3^2/2^2 = 0.4225$$

we reject H_0 if $F \leq F_{1-\alpha, m-1, n-1} = \frac{1}{F_{0.1, 13, 21}} = \frac{1}{2.01} = 0.498$

$\Rightarrow$ reject H_0

g. $H_0: P_1 = P_2$ vs $H_1: P_1 < P_2$ $\alpha = 0.05$

$$\hat{P}_1 = 0.67 \quad \hat{P}_2 = 0.82 \Rightarrow Z = \frac{0.67 - 0.82}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}}$$

$$\hat{p} = \frac{m}{n+m}\hat{P}_1 + \frac{n}{m+n}\hat{P}_2 = 0.745 \Rightarrow Z \simeq -2.76$$

we reject H_0.

$\therefore$ we reject H_0 if $Z < -Z_\alpha \Rightarrow Z_\alpha = 1.645$

$$h - \quad P_1 - P_2 \in \left[\hat{P}_1 - \hat{P}_2 \mp z_{\alpha/2} \sqrt{\frac{\hat{P}_1 \hat{q}_1}{m} + \frac{\hat{P}_2 \hat{q}_2}{n}} \right]$$

$$\in \left[(0.67 - 0.82) \mp 1.96 \sqrt{\frac{0.67(.33)}{129} + \frac{.82(.18)}{129}} \right]$$

$\boxed{2}$ $\quad n_d = 14 \quad \bar{d} = 0.14 \quad s_d = 0.046$

$H_0 = \mu_d = 0 \quad vs \quad H_1 = \mu_d > 0$

$$t = \frac{0.14}{0.046/\sqrt{14}} = 11.39$$

reject H_0 as $t > t_{\alpha, n-1} = t_{0.01, 13} = 2.650$