Sabbir Ahmed

DATE: April 18, 2018

MATH 407: HW 10

3.6 **5** Show that no proper subgroup of S_4 contains both $(1, 2, 3, 4)$ and $(1, 2)$.

Pf. $\square$

9 A rigid motion of a cube can be thought of either as a permutation of its eight vertices or as a permutation of its six sides. Find a rigid motion of the cube that has order 3, and express the permutation that represents it in both ways, as a permutation on eight elements and as a permutation on six elements.

Pf. $\square$

10 Show that the following matrices form a subgroup of $GL_2(C)$ isomorphic to D_4:

$$\pm\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \pm\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \pm\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \pm\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$

Pf. $\square$

15 **(a)** Show that $A_4 = \{\sigma \in S_4 \mid \sigma = \tau^2 \text{ for some } \tau \in S_4\}$

Pf. $\square$

(b) Show that $A_5 = \{\sigma \in S_5 \mid \sigma = \tau^2 \text{ for some } \tau \in S_5\}$

Pf. $\square$

(c) Show that $A_6 = \{\sigma \in S_6 \mid \sigma = \tau^2 \text{ for some } \tau \in S_6\}$

Pf. $\square$

(d) What can you say about A_n if $n > 6$?

Pf. $\square$

17 For any elements $\sigma, \tau \in S_n$, show that $\sigma\tau\sigma^{-1}\tau^{-1} \in A_n$.

Pf. □

21 Find the center of the dihedral group D_n.

Hint: Consider two cases, depending on whether n is odd or even.

Pf. □

24 Show that the product of two transpositions is one of (i) the identity; (ii) a 3-cycle; (iii) a product of two (nondisjoint) 3-cycles. Deduce that every element of A_n can be written as a product of 3-cycles.

Pf. □