
WORKSHEET 4

1. Given the following pmf, provide the $E(X)$ and variance of the random variable X :

X	1	2	4	8	16
$P(X)$	0.05	0.1	0.35	0.4	0.1

$$E(X) = \sum x_i p(x_i) = 1(0.05) + \dots + 16(0.1) = 6.45$$

$$Var(X) = \sum (x_i - E(X))^2 P(x_i) = E(X^2) - E(X)^2 \simeq 15.65$$

2. Prove $var(ax + b) = a^2 var(X)$

$$\begin{aligned} var(ax + b) &= E((ax + b)^2) - (E(ax + b))^2 = E(a^2 x^2 + 2abx + b^2) - (aE(x) + b)^2 \\ &= a^2 E(x^2) + 2abE(x) + b^2 - a^2 (E(x))^2 - 2abE(x) - b^2 = a^2 (E(x^2) - (E(x))^2) = a^2 var(x) \end{aligned}$$

3. For $X \sim Bin(10, 0.6)$, find $P(3 \leq X \leq 5)$: first using Binomial

$$P(3 \leq X \leq 5) = P(X \leq 5) - P(X \leq 2) = 0.367 - 0.012 = 0.355$$

Now using the pmf of Binomial:

$$\begin{aligned} P(3 \leq X \leq 5) &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= \binom{10}{3} 0.6^3 0.4^7 + \binom{10}{4} 0.6^4 0.4^6 + \binom{10}{5} 0.6^5 0.4^5 = 0.355 \end{aligned}$$

We know that $E(x) = np = 6$ and $var = np(1 - p) = 2.4$

4. For X a random variable with the following pmf:

x	4	6	8
$P(x)$	0.5	0.3	0.2

Find the $E(h(x))$, $h(x) = 0.5x^2 + 3x + 20$. We can do this in two ways, first:

$h(x)$	40	56	76
$P(h(x))$	0.5	0.3	0.2

$$E(h(x)) = 0.5(40) + 0.3(56) + 76(0.2) = 52$$

a second solution:

$$\begin{aligned} E(h(x)) &= 0.5E(x^2) + 3E(x) + 20 \\ &= 0.5(31.6) + 3(5.4) + 20 = 52 \end{aligned}$$