HW 4

[1] a- $p(X=x) = \dfrac{\binom{10}{x}\binom{10}{15-x}}{\binom{20}{15}}$; $x = 5, 6, 7, 8, 9, 10$

b- $p(X=5) + p(X=10)$

c- $E(X) = 7.5$ $\quad var(X) \overset{u}{=} (6.99)^2$

within one SD from mean $\Rightarrow \mu \pm \sigma = 7.5 \pm 0.99 \Rightarrow$ we want

$p(6.51 < X < 8.49)$

[2] please see question 2 from the worksheet 5

[3] $X \sim Bin(1000, \frac{1}{200}) \Rightarrow$ using poisson approximation

$X \sim Poi(5)$

a- $p(5 \leq x \leq 8) = p(X=5) + p(X=6) + p(X=7) + p(X=8)$

$= 0.492$

b- $p(x \geq 8) = 1 - p(X \leq 7) = 0.133$ { use cumulative table. }

[4] $0.99 = p(X \geq 1) \Rightarrow 0.01 = p(X=0)$

$p(X=0) = \dfrac{e^{-\alpha \pi R^2} (\alpha \pi R^2)^0}{0!} \Rightarrow 0.01 = e^{-\alpha \pi R^2}$

$0.01 = e^{-2\pi R^2} \Rightarrow R^2 = -\frac{1}{2\pi} \log 0.01 = \frac{1}{\pi}$

5 a- $c\int_{-2}^{2} 4-x^2\,dx = 1 \Rightarrow \frac{1}{c} = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

b. $P(X<-0.5 \text{ or } X>1) = P(X<-0.5) + P(X>1)$

$$= \int_{-2}^{-0.5} f(x) + \int_{1}^{2} f(x)$$

c. $V(X) = E(X^2) - (E(X))^2$

$$V(X^2) = E(X^4) - (E(X^2))^2$$

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a- $F_y(y) =$

$$\begin{cases} 0 & y<0 \\ \frac{1}{50}y^2 & 0 \le y < 5 \\ \frac{2}{5}y - \frac{y^2}{50} - \frac{3}{2} + \frac{1}{2} & 5 \le y \le 10 \\ 1 & y > 10 \end{cases}$$

b- The 100^{th} percentile $= \pi_e(p)$

$$\begin{cases} \frac{1}{50}\pi_e^2(p) & 0 \le p < .5 \\ \frac{2}{5}\pi_e - \frac{\pi_e^2(p)}{50} - 1 & .5 \le p < 1 \end{cases}$$