Math-Phys Quiz 3 Questions: Given 02/21/2017

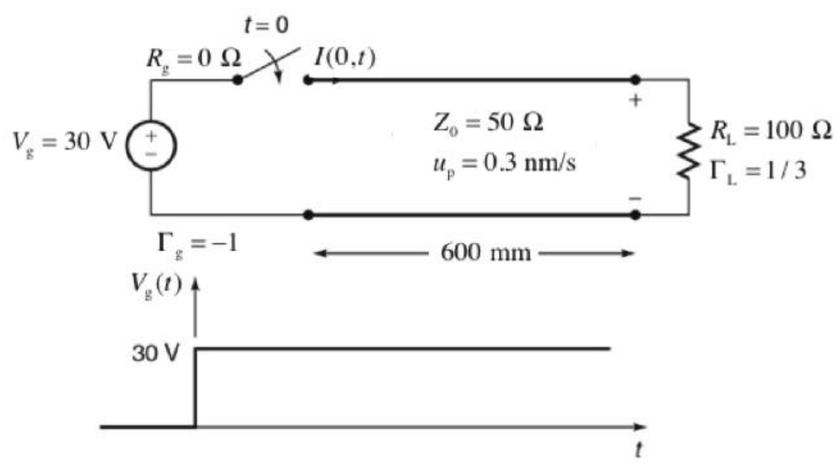
1. Calculate $(1 + \pi \times 10^{-30})^{1/2} - 1$ to three significant figures. Explain why you cannot obtain this result directly from a standard calculator.

2. Show that if the voltage at a load is given by $v(t) = v_L \cos(\omega t + \phi_v)$ and the current is given by $i(t) = i_L \cos(\omega t + \phi_i)$, then the total power dissipated in the load is given by $(1/2)\,\mathrm{Re}(\tilde{V}\tilde{I}^*) = \cos(\phi_v - \phi_i)$, where $\tilde{V}$ and $\tilde{I}$ are the phasors corresponding to $v(t)$ and $i(t)$. What is the condition for zero dissipation?

Exam Quiz 3 Question: Given 02/21/2017

1. **Modified from Menyuk Slides no. 3.27–30:**

   For the transmission line shown below, calculate $V(0,t)$, $I(0,t)$, $V(l,t)$, and $I(l,t)$ for times up to $4T$. In Ulaby's notation, the generator voltage $V_\mathrm{g} = 0$ V For $t < 0$ and $V_\mathrm{g} = 30$ V For $t > 0$. We have $R_\mathrm{g} = 0$, $R_\mathrm{L} = 100\ \Omega$, and $Z_0 = 50\ \Omega$. The line is 400 m long, and the propagation velocity is 2.0×10^8 m/s.



Math-Physics Quiz 3 Solutions:

1. $(1+\pi\times 10^{-40})^{1/2}-1 \simeq [1+(\pi/2)\times 10^{-40}]-1=(\pi/2)\times 10^{-30} \simeq 1.57\times 10^{-40}$. You cannot obtain this result from a standard calculator because only 64 bits are used to store each digit which corresponds to about 15 digits of accuracy. When the difference between two numbers is smaller than 1 part in 10^{15}, as is the case here, then the calculator will return 0. All digits of accuracy are lost in this subtraction.

2. In the time domain, we find that the average power dissipated at the load P_L is given by

$$\begin{aligned} P_L &= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} v_L i_L \cos(\omega t+\phi_v)\cos(\omega t+\phi_i)\,dt \\ &= v_L i_L \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1}{2}\left[\cos(\phi_v-\phi_i)+\cos(2\omega t+\phi_v+\phi_i)\right]\,dt \\ &= \frac{1}{2} v_L i_L \cos(\phi_v-\phi_i). \end{aligned}$$

The phasor domain expression may be written

$$\begin{aligned} \frac{1}{2}\operatorname{Re}(\tilde{V}\tilde{I}^*) &= \frac{1}{2}\operatorname{Re}\{[v_L\exp(j\phi_v)][i_L\exp(-j\phi_i)]\} \\ &= \frac{1}{2} v_L i_L \operatorname{Re}\{\exp[j(\phi_v-\phi_i)]\} = \frac{1}{2} v_L v_i \cos(\phi_v-\phi_i). \end{aligned}$$

These two expressions are evidently equal.

Exam Quiz 3 Solutions

1. The line parameters are given by

$$\Gamma_{\mathrm{g}} = \frac{0-50}{0+50} = -1, \qquad \Gamma_{\mathrm{L}} = \frac{100-50}{100+50} = \frac{1}{3}, \qquad T = \frac{l}{up_{\mathrm{p}}} = 2 \text{ ns}$$

For $t=(0 \text{ ns}, 2 \text{ ns})$, $V(l,t)=0$. For $t=(2 \text{ ns}, 6 \text{ ns})$, $V(l,t)=$ (incoming voltage) + (reflected voltage) $= 30 \text{ V} + (1/3)\times 30 \text{ V} = 40$ V. The corresponding current is $I(l,t)=V(l,t)/R_{\mathrm{L}}=0.4$ A. Since the reflected voltage is 10 V, we find that for $t=(0 \text{ ns}, 4 \text{ ns})$, $I(0,t)=30 \text{ V}/50\ \Omega = 0.6$ A. For $t=(4 \text{ ns}, 8 \text{ ns})$, $I(0,t)=(20 \text{ V} - 10 \text{ V})/50\ \Omega = 0.2$ A. $V(0,t)=30$ V for $t>0$.