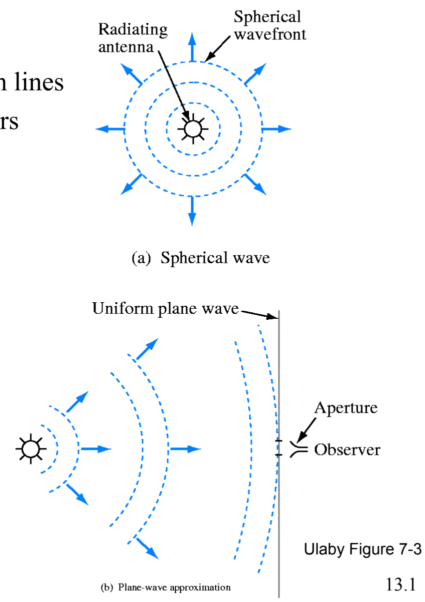
Plane Wave Propagation: Why Study It?

Two kinds of propagation:

- bounded: in waveguides, transmission lines
- unbounded: antennas, flashlights, stars

Unbounded propagation:

Far from the sources, the wave appear planar



(a) Spherical wave

(b) Plane-wave approximation

Ulaby Figure 7-3

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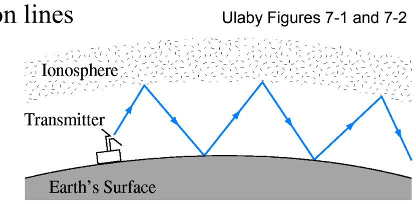
Plane Wave Propagation: Why Study It?

Two kinds of propagation:

- bounded: in waveguides, transmission lines
- unbounded: antennas, stars

Ulaby Figures 7-1 and 7-2



Bounded straight-line propagation:

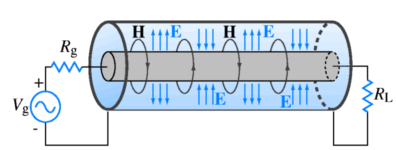
The electric field

$$\mathbf{E}(x,y,z;t) = \mathbf{E}_0(x,y)\cos\left(\omega t - kz + \phi_0\right)$$

and magnetic field

$$\mathbf{H}(x,y,z;t) = \mathbf{H}_0(x,y)\cos\left(\omega t - kz + \phi_0\right)$$

can be treated as transversely modulated plane waves.



The plane wave approximation allows us to separate material effects effects (nonlinearity, dispersion, birefringence) from geometric effects

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Maxwell's Equations: Phasor Domain

Phasor Domain Fields:

$$\mathbf{E}(x,y,z;t) = \mathrm{Re}\left[\tilde{\mathbf{E}}(x,y,z)\exp(j\omega t)\right]$$

Reminder: In this section of his book, Ulaby and Ravaioli use
k and the designation *wavenumber*, instead of
β and the designation *phase constant*.

- Phasors are essentially Fourier transforms at a single frequency
- Any time domain behavior can be described by adding up phasors

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Maxwell's Equations: Phasor Domain

Name of Law	Time Domain	Phasor Domain
Gauss's Law	$\nabla \cdot \mathbf{D} = \rho_V$	$\nabla \cdot \tilde{\mathbf{E}} = \tilde{\rho}_V / \varepsilon$
Faraday's Law	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$
Gauss's Law of Magnetics	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \tilde{\mathbf{H}} = 0$
Ampere's Law	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon\tilde{\mathbf{E}}$

- We have used $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$
- In the phasor domain, we can easily generalize
  $\varepsilon \rightarrow \varepsilon(\omega), \ \mu \rightarrow \mu(\omega)$

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Maxwell's Equations: Phasor Domain

Complex permittivity

When Ohm's law holds, $\mathbf{J} = \sigma\mathbf{E}$, we may write Faraday's law

$$\nabla\times\tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon\tilde{\mathbf{E}} = (\sigma + j\omega\varepsilon)\tilde{\mathbf{E}} = j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\tilde{\mathbf{E}}$$

$$= j\omega\varepsilon_{\mathrm{c}}\tilde{\mathbf{E}}, \quad \text{where} \quad \varepsilon_{\mathrm{c}} = \varepsilon' - j\varepsilon''$$

$$\text{with} \;\; \varepsilon' = \varepsilon, \;\; \varepsilon'' = \sigma/\omega$$

Charge-Free Medium

In a charge-free medium the phasor domain equations become

$$\nabla\cdot\tilde{\mathbf{E}} = 0, \qquad \nabla\times\tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$$
$$\nabla\cdot\tilde{\mathbf{H}} = 0, \qquad \nabla\times\tilde{\mathbf{H}} = j\omega\varepsilon_{\mathrm{c}}\tilde{\mathbf{E}}$$

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Maxwell's Equations: Phasor Domain

Wave Equations

Combining Faraday's and Ampere's laws:

$$\nabla\times(\nabla\times\tilde{\mathbf{E}}) = -j\omega\mu(\nabla\times\tilde{\mathbf{H}}) = -j\omega\mu(j\omega\varepsilon_{\mathrm{c}})\tilde{\mathbf{E}} = \omega^2\mu\varepsilon_{\mathrm{c}}\tilde{\mathbf{E}}$$

Using the vector relation $\nabla\times(\nabla\times\tilde{\mathbf{E}}) = \nabla(\nabla\cdot\tilde{\mathbf{E}}) - \nabla^2\tilde{\mathbf{E}}$,
Gauss's law $\nabla\cdot\tilde{\mathbf{E}} = 0$, and the definition $\gamma^2 \equiv -\omega^2\mu\varepsilon_{\mathrm{c}}$
we obtain the wave equation

$$\nabla^2\tilde{\mathbf{E}} - \gamma^2\tilde{\mathbf{E}} = 0$$

Calculating $\nabla\times(\nabla\times\tilde{\mathbf{H}})$, we find similarly

$$\nabla^2\tilde{\mathbf{H}} - \gamma^2\tilde{\mathbf{H}} = 0$$

We have the same wave equation for $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$
This is analogous to V and I in transmission lines

This analogy is no coincidence!!
Transmission lines can be analyzed using EM waves

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Lossless Plane-Wave Propagation

Dispersion relation and wave equation: $\sigma = 0$

We have: $\varepsilon_c = \varepsilon \quad \Rightarrow \quad \gamma^2 = -\omega^2 \mu\varepsilon = -k^2$ so that $k = \omega\sqrt{\mu\varepsilon}$

and the wave equation becomes

$$\nabla^2 \tilde{\mathbf{E}} + k^2 \tilde{\mathbf{E}} = 0$$

Plane-Wave Properties

(1) By definition, a plane wave only varies in one direction, which we choose to the be the z-direction. Hence,

$$\tilde{\mathbf{E}}(x,y,z) = \tilde{\mathbf{E}}(z) \quad \text{and} \quad \frac{\partial \tilde{\mathbf{E}}}{\partial x} = \frac{\partial \tilde{\mathbf{E}}}{\partial y} = 0$$

Our wave equation becomes

$$\frac{\partial^2 \tilde{\mathbf{E}}}{\partial z^2} + k^2 \tilde{\mathbf{E}} = 0$$

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Lossless Plane-Wave Propagation

Plane-Wave Properties

(2) $\tilde{E}_z = \tilde{H}_z = 0$. **Proof:** From the z-components of Ampere's and Faraday's laws,

$$j\omega\varepsilon \tilde{E}_z = \frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = 0 \quad \text{and} \quad -j\omega\varepsilon \tilde{H}_z = \frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y} = 0$$

Plane-wave fields are always orthogonal to the direction of propagation!

(3) The x- and y-components of $\tilde{\mathbf{E}}$ are uncoupled.

The wave equations for these components becomes

$$\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0 \quad \text{and} \quad \frac{d^2 \tilde{E}_y}{dz^2} + k^2 \tilde{E}_y = 0$$

There are two independent solutions, one with $\tilde{E}_x = 0$ and the other with $\tilde{E}_y = 0$

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13.8

Lossless Plane-Wave Propagation

Plane-Wave Properties

Focusing on the $\tilde{E}_y = 0$ solution, we have

(4) The general solution to the wave equation has forward- and backward-propagating components *(just like in a transmission line)*

$$\tilde{E}_x(z) = \tilde{E}_x^+(z) + \tilde{E}_x^-(z) = \tilde{E}_{x0}^+ \exp(-jkz) + \tilde{E}_{x0}^- \exp(jkz)$$

(5) Writing $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\,\tilde{E}_x$, Faraday's law implies

$$\tilde{H}_x(z) = 0, \quad \tilde{H}_y(z) = \tilde{H}_y^+(z) + \tilde{H}_y^-(z)$$

$$= \frac{k}{\omega\mu}\tilde{E}_{x0}^+ \exp(-jkz) - \frac{k}{\omega\mu}\tilde{E}_{x0}^- \exp(jkz)$$

The analogy with transmission lines is

$$\tilde{V}(z) = \tilde{V}^+(z) + \tilde{V}^-(z) \quad \rightarrow \quad \tilde{E}_x(z) = \tilde{E}_x^+(z) + \tilde{E}_x^-(z)$$

$$\tilde{I}(z) = \frac{\tilde{V}^+(z)}{Z_0} - \frac{\tilde{V}^-(z)}{Z_0} \quad \rightarrow \quad \tilde{H}_y(z) = \frac{\tilde{E}_x^+(z)}{\eta} - \frac{\tilde{E}_x^-(z)}{\eta}, \eta = \frac{\omega\mu}{k}$$

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Lossless Plane-Wave Propagation

Plane-Wave Properties

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} \;(\Omega) = \text{ intrinsic impedance}$$

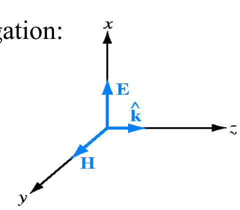
This is a characteristic property of the medium (not the wave)

Focusing on forward-going waves, we have

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\,\tilde{E}_x^+(z) = \hat{\mathbf{x}}\,\tilde{E}_{x0}^+ \exp(-jkz), \quad \tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\tilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{\tilde{E}_{x0}^+}{\eta}\exp(-jkz)$$

(6) Plane waves are TEM waves (transverse electromagnetic waves), waves in which the electric and magnetic fields are orthogonal to each other and the direction of propagation:

$(\tilde{\mathbf{E}}, \tilde{\mathbf{H}}, \hat{\mathbf{k}})$ is a right-handed system



Ulaby Figure 7-4

Our solution generalizes to:

$$\tilde{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{k}} \times \tilde{\mathbf{E}}, \quad \tilde{\mathbf{E}} = -\eta\hat{\mathbf{k}} \times \tilde{\mathbf{H}}$$

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13.10

Lossless Plane-Wave Propagation

Plane-Wave Properties

(7) Plane waves (like all waves) are characterized by an amplitude and a phase. We may write

$$\tilde{E}_{x0}^{+} = \left|\tilde{E}_{x0}^{+}\right| \exp(j\phi^{+}),$$

so that $\mathbf{E}(z,t) = \mathrm{Re}\left[\tilde{\mathbf{E}}(z)\exp(j\omega t)\right] = \hat{\mathbf{x}}\left|\tilde{E}_{x0}^{+}\right|\cos(\omega t - kz + \phi^{+}),$

and $\mathbf{H}(z,t) = \mathrm{Re}\left[\tilde{\mathbf{H}}(z)\exp(j\omega t)\right] = \hat{\mathbf{y}}\dfrac{\left|\tilde{E}_{x0}^{+}\right|}{\eta}\cos(\omega t - kz + \phi^{+}),$

The electric and magnetic fields are ***in phase***

(8) In a vacuum

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega, \quad u_{\mathrm{p}} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^{8}\ \mathrm{m/s}$$

The observation that electromagnetic waves propagate at c led led Maxwell to propose the light is made of electromagnetic waves!

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13.11

Lossless Plane-Wave Propagation

Electromagnetic Plane Wave in Air: Ulaby and Ravaioli Ex. 7-1

Question: The electric field of a 1-MHz plane wave traveling in the $+z$-direction in air points along the x-direction. If the peak value of E is 1.2π mV/m and E is a maximum at $t = 0$ and $z = 50$ m, obtain expressions for $E(z,t)$ and $H(z,t)$, and then plot these variations as a function of z at $t = 0$.

Answer: At $f = 1$ MHz, the wavelength in air is given by

$$\lambda = \frac{c}{f} = \frac{3\times 10^{8}}{1\times 10^{6}} = 300\ \mathrm{m}$$

so that the wavenumber is $k = (2\pi / 300) \sim 0.0209$ rad/m. From the general expression for $\mathbf{E}(z,t)$, we have

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\left|\tilde{E}_{x0}^{+}\right|\cos(\omega t - kz + \phi^{+})$$

$$= \hat{\mathbf{x}}\, 1.2\pi \cos\left(2\pi\times 10^{6} t - \frac{2\pi}{300} z + \phi^{+}\right)\ \mathrm{mV/m}.$$

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Lossless Plane-Wave Propagation

Electromagnetic Plane Wave in Air: Ulaby and Ravaioli Ex. 7-1

Answer (continued): The cosine is a maximum when its argument is 0 (or other multiples of 2π), so we have

$$-\frac{2\pi\times 50}{300}+\phi^{+}=0 \quad \text{or} \quad \phi^{+}=\frac{\pi}{3}$$

and

$$\mathbf{E}(z,t)=\hat{\mathbf{x}}1.2\pi\cos\left(2\pi\times 10^{6}t-\frac{2\pi}{300}z+\frac{\pi}{3}\right)\ \text{mV/m}$$

$$\mathbf{H}(z,t)=\frac{\hat{\mathbf{z}}\times\mathbf{E}(z,t)}{\eta_0}=\frac{\hat{\mathbf{z}}\times\mathbf{E}(z,t)}{120\pi\ (\Omega)}=\hat{\mathbf{y}}10\cos\left(2\pi\times 10^{6}t-\frac{2\pi}{300}z+\frac{\pi}{3}\right)\ \mu\text{A/m}$$

which becomes at $t = 0$:

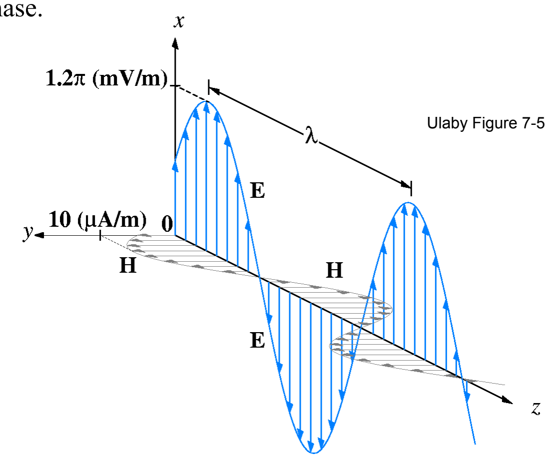
$$\mathbf{E}(z,t=0)=\hat{\mathbf{x}}1.2\pi\cos\left(\frac{2\pi}{300}z-\frac{\pi}{3}\right)\ \text{mV/m}$$

$$\mathbf{H}(z,t=0)=\hat{\mathbf{y}}10\cos\left(\frac{2\pi}{300}z-\frac{\pi}{3}\right)\ \mu\text{A/m}$$

Lossless Plane-Wave Propagation

Electromagnetic Plane Wave in Air: Ulaby and Ravaioli Ex. 7-1

Answer (continued): We show the plot below.
Note that **E** and **H** are in phase.



Ulaby Figure 7-5

Tech Brief 13: RFID Systems

History

1973: Two patents issued –one for active RFID with rewritable memory to Mario Cardullo, second for passive RFID system for keyless entry to Charles Walton.

Overview

System consists of a reader (transceiver) and a tag (transponder). Tag broadcasts information about its identity when polled by the reader.

Active vs. Passive

The reader must generate a strong enough signal to generate the current necessary in a passive tag to transmit the message. Thus, passive devices are limited in range. However, active devices are significantly more expensive to fabricate. Devices with higher frequency of operation generally transmit greater distances, but are also more expensive.

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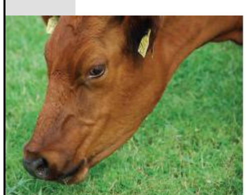
Tech Brief 13: RFID Systems



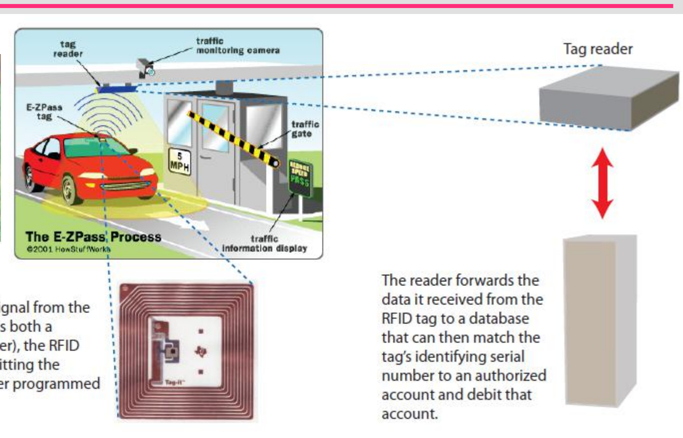


Figure TF13-2: How an RFID system works is illustrated through this EZ-Pass example. (Tag courtesy of Texas Instruments.)

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Tech Brief 13: RFID Systems

Table TT13-1: Comparison of RFID frequency bands.

Band	LF	HF	UHF	Microwave
RFID frequency	125–134 kHz	13.56 MHz	865–956 MHz	2.45 GHz
Read range	≤ 0.5 m	≤ 1.5 m	≤ 5 m	≤ 10 m
Data rate	1 kbit/s	25 kbit/s	30 kbit/s	100 kbit/s
Typical applications	• Animal ID • Automobile key/antitheft • Access control	• Smart cards • Article surveillance • Airline baggage tracking • Library book tracking	• Supply chain management • Logistics	• Vehicle toll collection • Railroad car monitoring

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13.17

Polarization

Two independent solutions

Forward-propagating and at the same ω

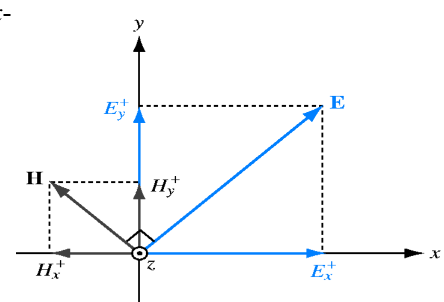
$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\,\tilde{E}_x^+(z) + \hat{\mathbf{y}}\,\tilde{E}_y^+(z); \quad \tilde{\mathbf{H}}(z) = -\hat{\mathbf{x}}\frac{\tilde{E}_y^+(z)}{\eta} + \hat{\mathbf{y}}\frac{\tilde{E}_x^+(z)}{\eta}$$

Each independent solution is referred to as a different ***polarization***

The time variation of the amplitude and phase that an observer sees depends on the phase difference between the x- and y-components, as well as their amplitudes. We may write

$$\tilde{E}_x^+(z) = a_x \exp(-jkz)$$

$$\tilde{E}_y^+(z) = a_y \exp(j\delta)\exp(-jkz)$$



Ulaby Figure 7-6

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13.18

Polarization

Two independent solutions

The vector sum yields $\tilde{\mathbf{E}}(z) = \left[\hat{\mathbf{x}}\, a_x + \hat{\mathbf{y}}\, a_y \exp j\delta\right]\exp(-jkz)$

with the corresponding time-dependent field

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\tilde{\mathbf{E}}(z)\exp(j\omega t)\right] = \hat{\mathbf{x}}\, a_x \cos(\omega t - kz) + \hat{\mathbf{y}}\, a_y \cos(\omega t - kz + \delta)$$

At a fixed point in z, the tip of the vector $\mathbf{E}(z,t)$ can trace out a line, a circle, or an ellipse on the x-y plane

In general, light is elliptically polarized

Field magnitude and phase

$$\left|\mathbf{E}(z,t)\right| = \left[a_x^2 \cos^2(\omega t - kz) + a_y^2 \cos^2(\omega t - kz + \delta)\right]^{1/2}$$

$$\psi(z,t) = \tan^{-1}\left[\frac{E_y(z,t)}{E_x(z,t)}\right]$$

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13.19

Polarization

Linear Polarization

For convenience, we will observe the polarization state at $z = 0$

Linear polarization corresponds to $\delta = 0$ or $\delta = \pi$.

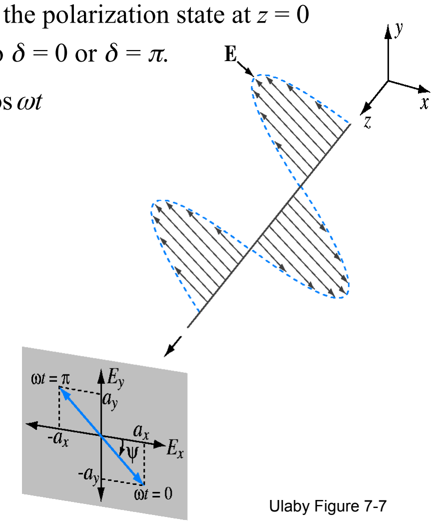
We have $\mathbf{E}(0,t) = (\hat{\mathbf{x}}\, a_x \pm \hat{\mathbf{y}}\, a_y)\cos\omega t$

so that

$$\left|\mathbf{E}(0,t)\right| = \left(a_x^2 + a_y^2\right)^{1/2}\cos\omega t$$

$$\psi(0,t) = \tan^{-1}\left(\pm a_y / a_x\right)$$

Since ψ is constant the tip of the **E**-field vector is a sinusoidally-varying straight line



Ulaby Figure 7-7

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13.20

Polarization

Circular Polarization

This case corresponds to $a_x = a_y,\ \delta = \pm\pi/2$

Left circular polarization (LCP): $\delta = +\pi/2$

Right circular polarization (RCP): $\delta = -\pi/2$

Field variation

$$\tilde{\mathbf{E}}(z) = a\left[\hat{\mathbf{x}} + \hat{\mathbf{y}}\exp(\pm j\pi/2)\right]\exp(-jkz) = a(\hat{\mathbf{x}} \pm j\hat{\mathbf{y}})\exp(-jkz)$$

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\,a\cos(\omega t - kz) \mp \hat{\mathbf{y}}\,a\sin(\omega t - kz)$$

At $z = 0$, we now find

$$|\mathbf{E}(0,t)| = a = \text{constant}$$

$$\psi(0,t) = \mp\omega t$$

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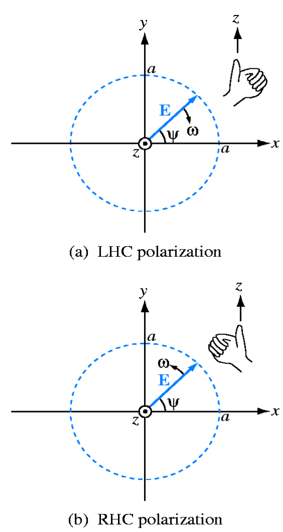
Polarization

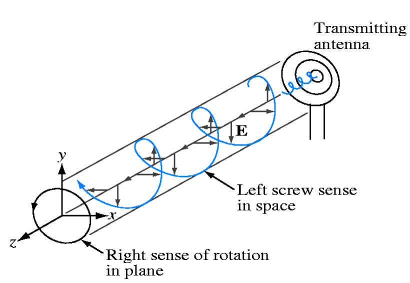
Circular Polarization

Left circular polarization: Left-hand screw points in the direction of propagation

Right circular polarization: Right-hand screw points in the direction of propagation



Ulaby Figure 7-8



Ulaby Figure 7-9

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Polarization

Elliptical Polarization

The general case is somewhat complicated

The polarization ellipse is specified by two angles:

— γ = rotation angle

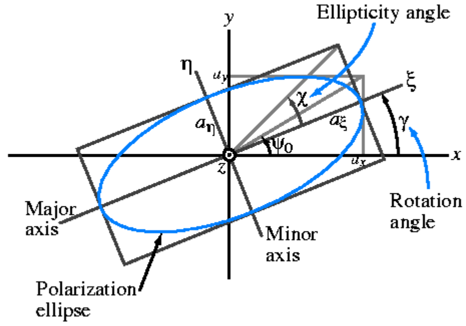
— χ = ellipticity angle

These are related to a_x, a_y, δ by

$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta$$

$$\sin 2\chi = (\sin 2\psi_0)\sin\delta$$

$$\text{with} \quad \tan\psi_0 = a_y / a_x$$



Ulaby Figure 7-11

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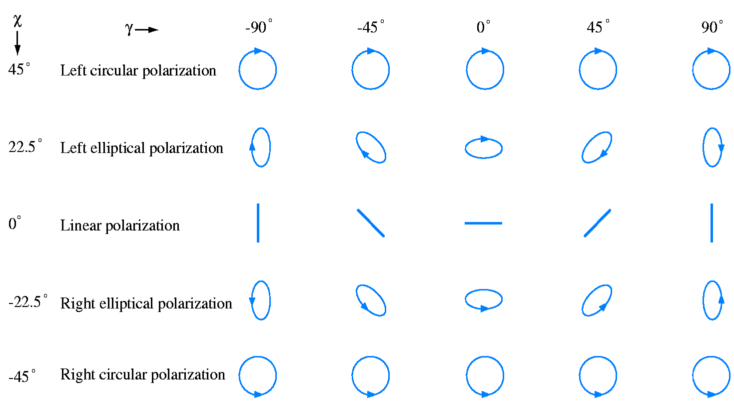
13.23

Polarization

Elliptical Polarization



Ulaby Figure 7-12

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13.24

Polarization

Right-Hand Circular Polarization State: Ulaby and Ravaioli Ex. 7-2

Question: A right-circularly-polarized plane wave with electric field modulus of 3 mV/m is traveling in the $+y$-direction in a dielectric medium with $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 0$. If the wave frequency is 100 MHz, obtain expressions $\mathbf{E}(y,t)$ and $\mathbf{H}(y,t)$.

Answer: Since the wave is traveling in the $+y$-direction, its field components are in the x- and z-directions. When a right-circularly polarized wave travels in the $+z$-direction, the y-component is retarded with respect to the x-component by a phase $\pi/2$. By the cyclic permutation rule, x-component must be retarded with respect to the z-component by a phase $\pi/2$ in this case. We thus conclude,

$$\tilde{\mathbf{E}}(y) = \hat{\mathbf{x}}\,\tilde{E}_x + \hat{\mathbf{z}}\,\tilde{E}_z = a\left[\hat{\mathbf{x}}\exp(-j\pi/2) + \hat{\mathbf{z}}\right]\exp(-jky)$$

$$= 3(-\hat{\mathbf{x}}\,j + \hat{\mathbf{z}})\exp(-jky) \text{ mV/m}$$

$$\tilde{\mathbf{H}}(y) = \frac{1}{\eta}\hat{\mathbf{y}} \times \tilde{\mathbf{E}}(y) = \frac{3}{\eta}(\hat{\mathbf{z}}\,j + \hat{\mathbf{x}})\exp(-jky) \text{ mA/m}$$

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13.25

Polarization

Right-Hand Circular Polarization State: Ulaby and Ravaioli Ex. 7-2

Answer (continued): With $\omega = 2\pi f = 2\pi \times 10^8\,\text{s}^{-1}$, we have

$$k = \frac{\omega\sqrt{\varepsilon_r}}{c} = \frac{\left(2\pi\times 10^8\right)\times 2}{3\times 10^8} = \frac{4}{3}\pi \text{ rad/m}$$

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\,\pi\ \Omega$$

The time-domain fields are then given by

$$\mathbf{E}(y,t) = \mathrm{Re}\left[3(-\hat{\mathbf{x}}\,j + \hat{\mathbf{z}})\exp(-jky)\exp(j\omega t)\right]$$

$$= 3\left[\hat{\mathbf{x}}\sin(\omega t - ky) + \hat{\mathbf{z}}\cos(\omega t - ky)\right] \text{ mV/m}$$

$$\mathbf{H}(y,t) = \mathrm{Re}\left[\frac{3}{\eta}(\hat{\mathbf{z}}\,j + \hat{\mathbf{x}})\exp(-jky)\exp(j\omega t)\right]$$

$$= \frac{1}{20\pi}\left[\hat{\mathbf{x}}\cos(\omega t - ky) - \hat{\mathbf{z}}\sin(\omega t - ky)\right] \text{ mA/m}$$

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13.26

Tech Brief 14: Liquid Crystal Display (LCD)

Physical principal

- Liquid crystals are neither solid nor liquid, but hybrid of both
- Twisted nematic LCs have rod shaped molecules that tend to assume a twisted spiral shape when sandwiched between two glass substrates with orthogonal grooving orientations
- Twisted spiral LCs act as a polarizer since incident light tends to follow the orientation of the spiral
- The LC layer is sandwiched between orthogonal polarization filters. The LC layer rotates the light by 90º when it passes through, allowing light to exit from the other polarization filter
- The twisted nematic LC spiral untwists under the influence of an electric field
- Degree of untwisting is proportional to field strength
- Under an applied voltage, light cannot pass the second filter

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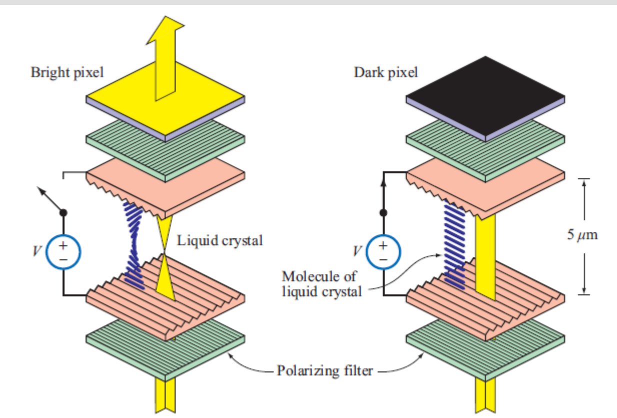
Discovery: 1880s by a botanist Frierich Reinitzer

Uses: Digital clocks, cell phones, desktop and laptop computers, televisions, etc.

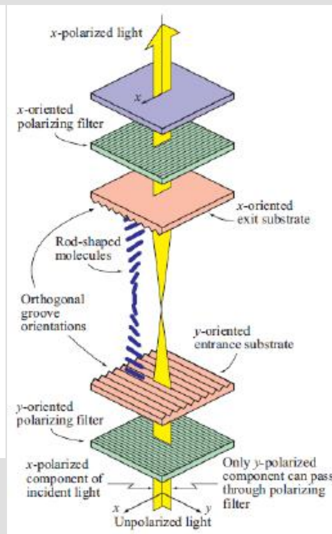
13.27

Tech Brief 14: Liquid Crystal Display (LCD)



(a) ON state (switch open)

(b) OFF state (switch closed)



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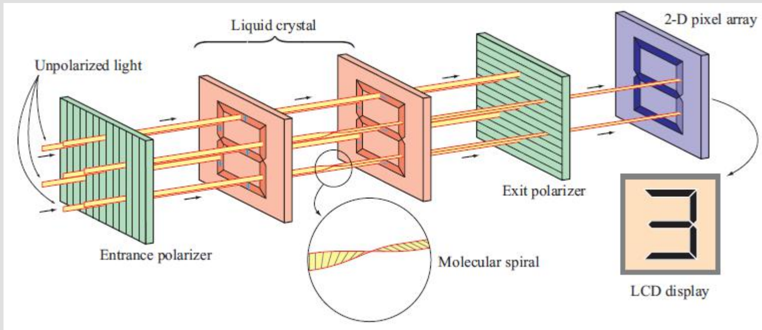
Tech Brief 14: Liquid Crystal Display (LCD)



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Lossy Propagation

Attenuation Constant and Phase Constant

Starting from the wave equation, $\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0$

with $\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu(\varepsilon' - j\varepsilon'')$ $[\varepsilon' = \varepsilon, \varepsilon'' = \sigma / \omega]$

and defining $\gamma = \alpha + j\beta$

— α = attenuation coefficient; β = phase constant

we find $(\alpha + j\beta)^2 = (\alpha^2 - \beta^2) + j2\alpha\beta = -\omega^2 \mu\varepsilon' + j\omega^2 \mu\varepsilon''$

Equating real and imaginary parts and solving for α and β

$$\alpha = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2}, \quad \beta = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]\right\}^{1/2}$$

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Lossy Propagation

Solution (+z-direction)

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\,\tilde{E}_x(z) = \hat{\mathbf{x}}\,E_{x0}\exp(-\gamma z) = \hat{\mathbf{x}}\,E_{x0}\exp(-\alpha z)\exp(-j\beta z)$$

$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\,\tilde{H}_y(z) = \hat{\mathbf{y}}\,\frac{\tilde{E}_x(z)}{\eta_c} = \hat{\mathbf{y}}\,\frac{E_{x0}}{\eta_c}\exp(-\alpha z)\exp(-j\beta z)$$

with

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$

The electric and magnetic fields are no longer in phase!

This is analogous to what happens in lossy transmission lines with the voltage and the current.

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Lossy Propagation

Solution (+z-direction)

The electric and magnetic fields attenuate

$$\left|\tilde{E}_x(z)\right| = \left|E_{x0}\right|\exp(-\alpha z), \quad \left|\tilde{H}_y(z)\right| = \left|\frac{E_{x0}}{\eta_c}\right|\exp(-\alpha z)$$

The corresponding attenuation length is called ***the skin depth*** δ_s

$$\delta_s = 1/\alpha$$

Two important limits

- low-loss dielectric: $\varepsilon''/\varepsilon' \ll 1$ (in practice less than 1/100)
- good conductor: $\varepsilon''/\varepsilon' \gg 1$ (in practice greater than 100)

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Lossy Propagation

Low-Loss Dielectric

From the expression for γ,

$$\gamma = j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2} \simeq j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{2\varepsilon'}\right)$$

so that

$$\alpha \simeq \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}, \quad \beta = \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu\varepsilon}$$

From the expression for η_c,

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \simeq \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right) = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) \simeq \sqrt{\frac{\mu}{\varepsilon}}$$

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Lossy Propagation

Good Conductor

From the expression for γ,

$$\gamma \simeq \omega\sqrt{j\mu\varepsilon''} = \frac{1+j}{\sqrt{2}}\omega\sqrt{\frac{\mu\sigma}{\omega}} = \sqrt{\frac{\mu\sigma\omega}{2}} + j\sqrt{\frac{\mu\sigma\omega}{2}}$$

so that

$$\alpha \simeq \sqrt{\frac{\mu\sigma\omega}{2}} = \sqrt{\pi f\mu\sigma}, \quad \beta = \alpha \simeq \sqrt{\pi f\mu\sigma}$$

From the expression for η_c,

$$\eta_c \simeq \sqrt{j\frac{\mu}{\varepsilon''}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma}\Bigg\} \begin{array}{l} \textbf{\textit{As } } \boldsymbol{\sigma \to \infty, \eta_c \to 0} \\ \mathbf{|E| / |H| \to 0} \end{array}$$

Some consequences

- The **E**-field and **H**-field are 45° out of phase
- A good conductor shorts out the **E**-field, but not the **H**-field

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Lossy Propagation

Plane Wave in Seawater: Ulaby and Ravaioli Example 7-4

Question: A plane wave travels in the +z-direction downward in sea water, with the x-y plane at the sea surface and $z = 0$ denoting a point just below the sea surface. The constitutive parameters are $\varepsilon = 80\,\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 4$ S/m. If the magnetic field at $z = 0$ is given by $\mathbf{H}(0,t) = \hat{\mathbf{y}}\,100\cos\left(2\pi\times 10^3 t + 15°\right)$ mA/m, (a) Obtain expressions $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$, and (b) determine the depth at which the amplitude of $\mathbf{E}$ has fallen to 1% of its value at $z = 0$.

Answer: Seawater is a good conductor at $f = 1$ kHz, so we have

$$\beta = \alpha\,; \quad \sqrt{\pi f \mu\sigma} = \sqrt{\pi\times 10^3 \times 4\pi\times 10^{-7}\times 4} = 0.126\ \mathrm{m}^{-1}$$

$$\eta_c = (1+j)\alpha/\sigma = 0.044\exp(j\pi/4)$$

In phasor form, we have

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\,\tilde{E}_{x0}\exp(-\alpha z)\exp(-j\beta z)$$

$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\tilde{E}_{x0}}{\eta_c}\exp(-\alpha z)\exp(-j\beta z)$$

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Lossy Propagation

Plane Wave in Seawater: Ulaby and Ravaioli Example 7-4

Answer (continued): We may write the initial field as $\tilde{E}_{x0} = \left|\tilde{E}_{x0}\right|\exp(j\phi_0)$ so that in the time domain

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\hat{\mathbf{x}}\left|\tilde{E}_{x0}\right|\exp(j\phi_0)\exp(-\alpha z)\exp(-j\beta z)\exp(j\omega t)\right]$$

$$= \hat{\mathbf{x}}\left|\tilde{E}_{x0}\right|\exp(-0.126\,z)\cos\left(2\pi\times 10^3 t - 0.126\,z + \phi_0\right)$$

$$\mathbf{H}(z,t) = \mathrm{Re}\left[\hat{\mathbf{y}}\frac{\left|\tilde{E}_{x0}\right|}{0.44\exp(j\pi/4)}\exp(j\phi_0)\exp(-\alpha z)\exp(-j\beta z)\exp(j\omega t)\right]$$

$$= \hat{\mathbf{y}}\,22.5\left|\tilde{E}_{x0}\right|\exp(-0.126\,z)\cos\left(2\pi\times 10^3 t - 0.126\,z + \phi_0 - 45°\right)$$

Comparing to the initial conditions, we have

$$22.5\left|\tilde{E}_{x0}\right| = 100\times 10^{-3} \quad \Rightarrow \quad \left|\tilde{E}_{x0}\right| = 4.44\ \mathrm{mV/m}$$

$$\phi_0 - 45° = 15° \quad \Rightarrow \quad \phi_0 = 60°$$

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Lossy Propagation

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Answer (continued): Our final result is

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\ 4.44\exp(-0.126\ z)\cos\left(2\pi\times10^3 t - 0.126\ z + 60°\right)\ \text{mV/m}$$

$$\mathbf{H}(z,t) = \hat{\mathbf{y}}\ 100\exp(-0.126\ z)\cos\left(2\pi\times10^3 t - 0.126\ z + \phi_0 + 15°\right)\ \text{mA/m}$$

The depth at which the amplitude of **E** has decreased to 1% of its initial value at $z = 0$ is obtained from

$$0.01 = \exp(-0.126 z) \quad \Rightarrow \quad z = \frac{\ln(0.01)}{-0.126} = 36.5\ \text{m}$$

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