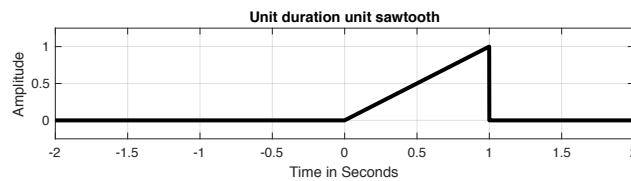## You try

- **Use the unit sawtooth to find the following**

$$f(t) = s(0.5t - 3)$$
$$g(t) = s(2t + 5)$$
$$h(t) = s(-2t + 5)$$
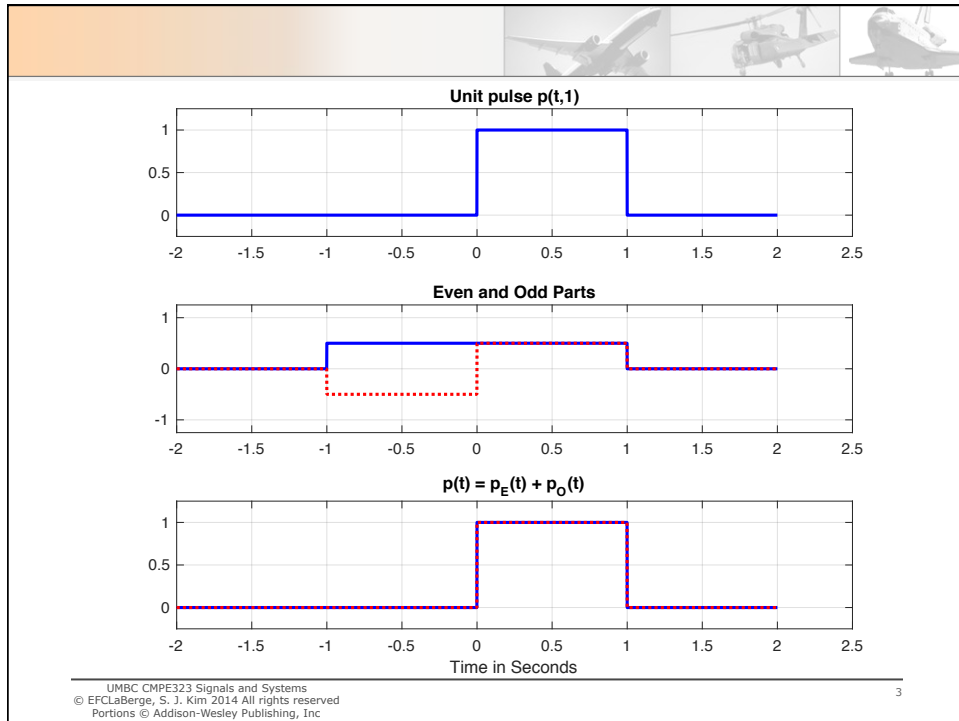
Unit duration unit sawtooth

Even and Odd

- **Finally, let's discuss even and odd functions**
- **For an even function** $f(t) = f(-t)$
  - **Examples** $x(t) = t^2$, $x(t) = \cos(\omega t)$, $x(t) = A$
- **For an odd function** $f(t) = -f(-t)$
  - **Examples** $x(t) = t^3$, $x(t) = at$, $x(t) = \sin(\omega t)$
- **But what about a general function, like** $u(t)$**?**
- **Any function can be decomposed into the sum of an even function and an odd function**

$$f_E(t) = \frac{f(t) + f(-t)}{2}, f_O(t) = \frac{f(t) - f(-t)}{2}$$
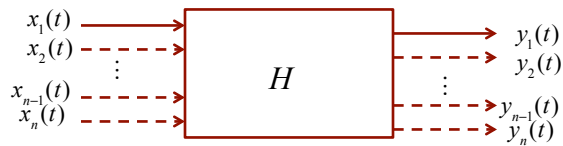$$f(t) = f_E(t) + f_O(t)$$

Unit pulse p(t,1)

Even and Odd Parts

$p(t) = p_E(t) + p_O(t)$

Time in Seconds

Mathematical Description of Systems

CMPE323

We've been talking about functions of time

- **Functions of time (or space, or temp, or something) are our *signals*...**

- **...and we now know some properties and how to change them.**

- **Now let's turn to the mathematical description of a *system***

- **A *system*, H, takes one or more input signals, $x_n(t)$, and turns it/them into one or more output signals $y_n(t)$**

$$x_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t)$$
$$x_2(t) \dashrightarrow$$
$$\vdots$$
$$x_{n-1}(t) \dashrightarrow \quad H \quad \dashrightarrow y_2(t)$$
$$\vdots$$
$$x_n(t) \dashrightarrow \quad \quad \dashrightarrow y_{n-1}(t)$$
$$\dashrightarrow y_n(t)$$

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## We might know what's in the box!

- **...but again, we might not**
- **We can know (by test) some properties**
  - **Linearity**
  - **Time Invariance**
  - **Causality**
  - **"Left/Right sidedness"**
  - **Stabilty**
  - **Discrete time or continuous time**
  - **Discrete amplitude (quantized) or analog**
  - **Static/Dynamic**
  - **Feedback/No Feedback**

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Linearity

- **Many, even most, systems we deal with are linear...**

- **...or at least linear over some range of input signals**

- **A system is *linear* if it has two properties (same as in CMPE306!)**

  - **Superposition: the output of the sum is the sum of the outputs**

  - **Scaleability: the output of a constant times the input is the same constant times the output**
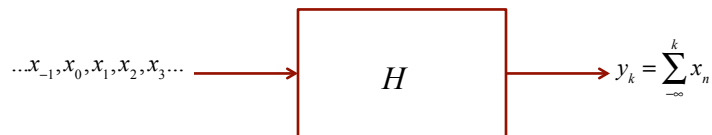
- **What this means is that a system H is linear iff**
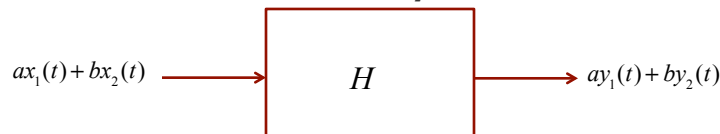
$$ax_1(t) + bx_2(t) \longrightarrow \boxed{H} \longrightarrow ay_1(t) + by_2(t)$$

- **Where** $H(x_1) = y_1$ and $H(x_2) = y_2$

Example

- **A perfect summation**

- **Inputs are discrete in time and discrete in amplitude, so we can treat them like integers**

$$\ldots x_{-1}, x_0, x_1, x_2, x_3 \ldots \longrightarrow \boxed{H} \longrightarrow y_k = \sum_{-\infty}^{k} x_n$$

- **Is it linear?**

- **Test for the properties. Let there be two input sequences:** $\ldots x_{-1}, x_0, x_1, x_2, x_3 \ldots$, and $\ldots w_{-1}, w_0, w_1, w_2, w_3 \ldots$

- **Use the** $\{x_k\}$ **sequence and observe** $y_k^{(x)} = \sum_{-\infty}^{k} x_n$

- **Use the** $\{w_k\}$ **sequence and observe** $y_k^{(w)} = \sum_{-\infty}^{k} w_n$

- **Use** $v_k = ax_k + bw_k$: $y_k^{(v)} = \sum_{-\infty}^{k} v_k = \sum_{-\infty}^{k}(ax_k + bw_k) = a\sum_{-\infty}^{k} x_k + b\sum_{-\infty}^{k} w_k$

$$= ay_k^{(x)} + by_k^{(w)} \implies \text{This system is linear!}$$

Non-linear example

$$y(t) = x^2(t)$$

Time Invariance (TI)

- **A system is *time invariant* (TI) iff a delay in the input causes a corresponding delay in the output, without otherwise changing the output signal**

$$y(t) = H\big(x(t)\big) = x^2(t)$$

Let $w(t) = x(t - T)$ (a simple delay)
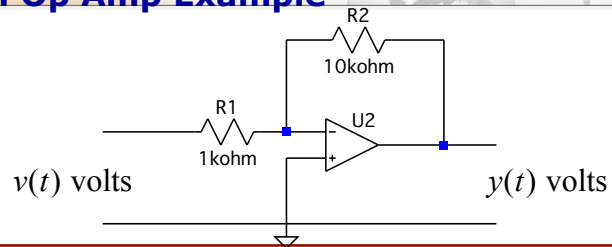
$$y^{(w)}(t) = w^2(t) = x^2(t - T) = y^{(x)}(t - T)$$

Not linear, but time invariant

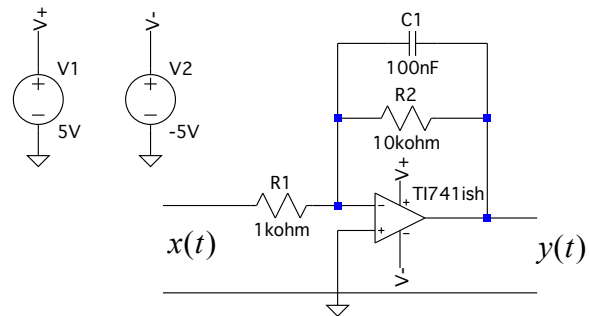- **Example** $y(t) = kx(t)$ with k a known constant

Is it TI?

Ideal Op Amp Example

R2
10kohm

R1
1kohm

U2

$v(t)$ volts

$y(t)$ volts

Linear? TI? Static?

Non-ideal op amp example: L? TI? Static?

V1
5V

V2
-5V

C1
100nF

R2
10kohm

R1
1kohm

TI741ish

$x(t)$

$y(t)$

Linear? TI? Static?

Impulse response

- **One of the most useful characterizations of a system is its "impulse" response**

$$\delta(t) \rightarrow \boxed{H} \rightarrow h(t)$$
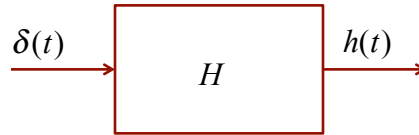
- **Consider a discrete time, discrete amplitude, "N-point average"**

$$y_k = H(x_k) = \frac{1}{N} \sum_{n=k-N+1}^{k} x_n$$

- **Linear? TI? Static? Causal?**

- **What's the impulse response?**

Stability

- **We'll define stability here...**

- **...and come back to it when we discuss Laplace techniques**

- **A system (not necessarily LTI, causal, or static) is STABLE if the application of a bounded input introduces a bounded output**

$$\text{BIBOS: } 0 \le x(t) \le K \Rightarrow |y(t)| \le M$$

$$\text{UBIBOS: } |x(t)| \le K \Rightarrow |y(t)| \le M$$

- **Is our previous op amp circuit stable?**

Using the properties

- **Why do we care?**
- **Remember that we're interested in finding the output from a known (but arbitrary) input**
- **If the system is LTI, and if we can break the input into pieces whose output is known …**
- **…we can apply the pieces one at a time…**
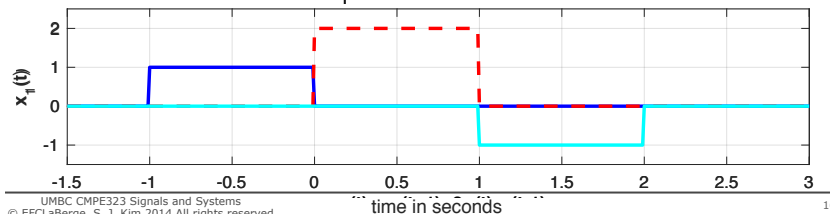- **…and then sum the outputs to get the total output**
- **Let's see**

For LTI systems

- **If we know the input output relationship for a signal if interest, like a pulse…**
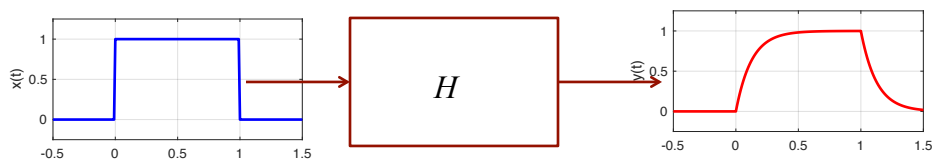
- **…we can find the input output relationship for signals that can be decomposed into sums of delayed versions of the input signal**
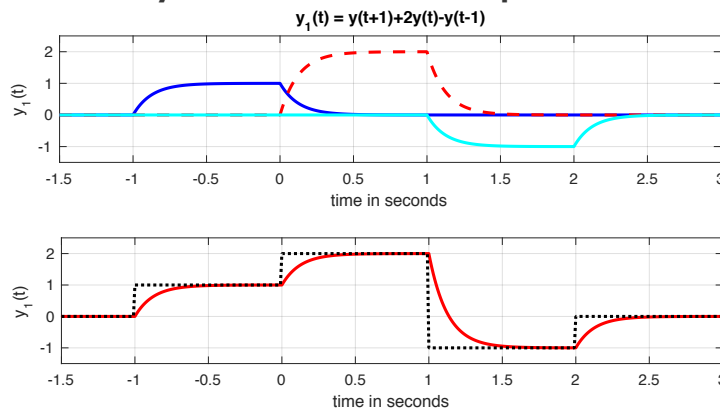
$$x_1(t) = p(t+1) + 2p(t) - p(t-1)$$

time in seconds

...continued

- **By linearity, the output of the sum is the sum of the outputs...**

- **...by TI, a delay or advance in the input results in the same delay or advance in the output**
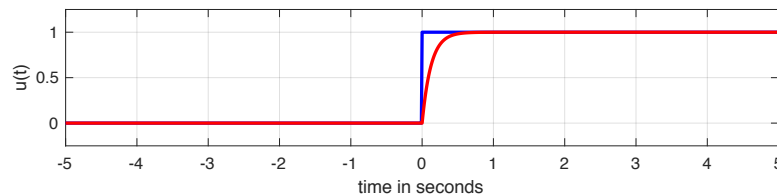
$$y_1(t) = y(t+1)+2y(t)-y(t-1)$$

So what is the impulse response?

- **We might (should) ask what the impulse response is**

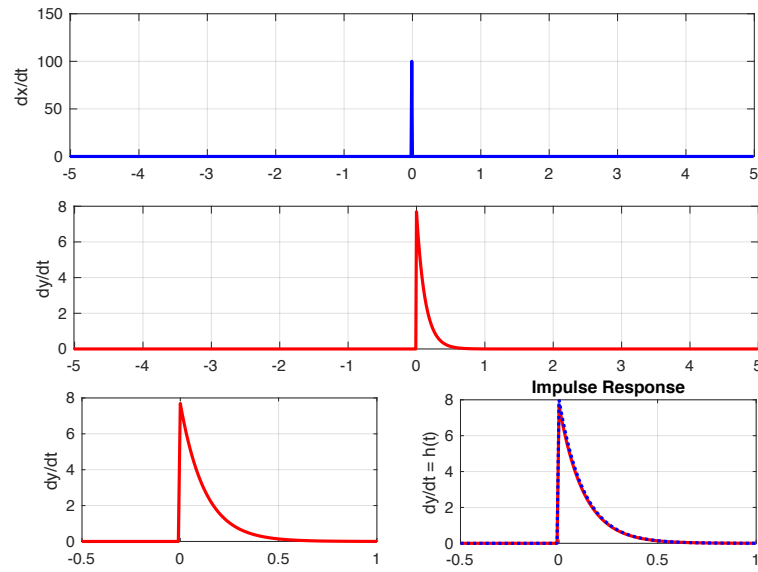- **Imagine the input is a very long pulse...**

- **...which is essential a step, so we have the step response...**

- **The impulse is the derivative of the step, and the system is LTI, so the output of the derivative is the derivative of the output (check this out for yourselves)**

Impulse Response

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