

Phasor-Domain Equations

Transmission line equations — time domain:

$$\begin{aligned}-\frac{\partial v(z,t)}{\partial z} &= R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t} \\ -\frac{\partial i(z,t)}{\partial z} &= G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}\end{aligned}$$

Time domain \rightarrow Phasor domain

$$v(z,t) = \operatorname{Re}[\tilde{V}(z)\exp(j\omega t)], \quad i(z,t) = \operatorname{Re}[\tilde{I}(z)\exp(j\omega t)]$$

Transmission line equations — phasor domain:

$$\begin{aligned}-\frac{d\tilde{V}(z)}{dz} &= (R' + j\omega L')\tilde{I}(z) \\ -\frac{d\tilde{I}(z)}{dz} &= (G' + j\omega C')\tilde{V}(z)\end{aligned}$$



4.1

Phasor-Domain Equations

Advantages of Phasor-Domain Representation

- Reduction from partial differential equation to ordinary differential equation
— so that it is easier to solve
- Allows generalization: $L' \rightarrow L'(\omega)$, $C' \rightarrow C'(\omega)$,
 $R' \rightarrow R'(\omega)$, $G' \rightarrow G'(\omega)$

Advantages of Time-Domain Representation

- Allows study of transients



4.2

Phasor-Domain Equations

Second-Order Equations:

$$\frac{d^2 \tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0, \quad \frac{d^2 \tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0$$

with

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

γ = complex propagation constant

$\alpha = \text{Re}(\gamma)$ = attenuation constant (Np/m)

$\beta = \text{Im}(\gamma)$ = phase constant or wavenumber (rad/m)

NOTES: • We pick α and β so that both are positive

- In a passive medium, α is always positive; it can be negative in an active medium

[An active medium, like a laser, has an energy source; a passive medium does not and must always lose energy.]



4.3

Phasor-Domain Equations

The second-order equations have the general solutions

$$\tilde{V}(z) = V_0^+ \exp(-\gamma z) + V_0^- \exp(\gamma z), \quad \tilde{I}(z) = I_0^+ \exp(-\gamma z) + I_0^- \exp(\gamma z)$$

- $\tilde{V}(z)$ and $\tilde{I}(z)$ are not independent
- V_0^+ and V_0^- are arbitrary constants, while I_0^+ and I_0^- are not
- We may also write:

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} \exp(-\gamma z) - \frac{V_0^-}{Z_0} \exp(\gamma z)$$

where

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$



is the characteristic impedance

NOTE: Z_0 (Ulaby et al.'s notation) $\rightarrow Z_C$ (Paul's notation)

4.4

Phasor-Domain Equations

Returning to the time domain:

We first write

$$V_0^+ = |V_0^+| \exp(j\phi^+), \quad V_0^- = |V_0^-| \exp(j\phi^-)$$

from which we obtain

$$\begin{aligned} v(z,t) &= \operatorname{Re}[\tilde{V}(z)\exp(j\omega t)] \\ &= \operatorname{Re}\left\{\left[V_0^+ \exp(-\gamma z) + V_0^- \exp(\gamma z)\right]\exp(j\omega t)\right\} \\ &= \operatorname{Re}\left\{|V_0^+| \exp(j\phi^+) \exp(j\omega t) \exp[-(\alpha + j\beta)z] \right. \\ &\quad \left. + |V_0^-| \exp(j\phi^-) \exp(j\omega t) \exp[(\alpha + j\beta)z]\right\} \\ &= \underbrace{|V_0^+| \exp(-\alpha z) \cos(\omega t - \beta z + \phi^+)}_{\text{forward propagation with attenuation}} + \underbrace{|V_0^-| \exp(\alpha z) \cos(\omega t + \beta z + \phi^-)}_{\text{backward propagation with attenuation}} \end{aligned}$$



$$\text{Phase velocity} = u_p = \omega/\beta$$

4.5

Phasor-Domain Equations

Example: Ulaby et al. Exercise 2.4

Question: A two-wire air line has the following parameters: $R' = 0.404 \text{ m}\Omega/\text{m}$, $L' = 2.00 \text{ }\mu\text{H}/\text{m}$, $G' = 0$, $C' = 5.56 \text{ pF}/\text{m}$. For operation at 5 kHz, determine (a) the attenuation coefficient α , (b) the wavenumber β , (c) the phase velocity u_p , and the characteristic impedance Z_0 .

Answer: $\omega = 2\pi \times 3.14159 \times (5 \times 10^3 \text{ s}^{-1}) = 3.14159 \times 10^4 \text{ s}^{-1}$.
 $R' + j\omega L' = (4.04000 \times 10^{-4} + j \times 3.14159 \times 10^4 \times 2.00000 \times 10^{-6}) \text{ }\Omega/\text{m} = (4.04000 \times 10^{-4} + j \times 6.28318 \times 10^{-2}) \text{ }\Omega/\text{m} = 6.28319 \times 10^{-2} \times \exp(j \times 1.56436) \text{ }\Omega/\text{m}$.
 $G' + j\omega C' = j \times 1.74673 \times 10^{-7} \text{ }\Omega^{-1}/\text{m} = 1.74673 \times 10^{-7} \times \exp(j \times 1.57080) \text{ }\Omega^{-1}/\text{m}$. Note the small difference in phases! Six digits of accuracy are needed to keep three digits in the attenuation coefficient. $\gamma^2 = 1.09750 \times 10^{-8} \times \exp(j \times 3.13156) \text{ m}^{-2}$, so that $\gamma = 1.04762 \times 10^{-4} \times \exp(j \times 1.56758) \text{ m}^{-1} = 3.37 \times 10^{-7} + j \times 1.04761 \times 10^{-4}$, so that $\alpha = 3.37 \times 10^{-7} \text{ Np}/\text{m}$ and $\beta = 1.05 \times 10^{-4} \text{ rad}/\text{m}$.
We have $u_p = \omega/\beta = (3.142/1.048) \times 10^8 = 3.00 \times 10^8 \text{ m}/\text{s}$
and $Z_0 = \{(6.283 \times 10^{-2} / 1.747 \times 10^{-7}) \exp[j \times (1.56436 - 1.57080)]\}^{1/2} = (600 - j \times 1.93) \text{ }\Omega$



4.6

Lossless Transmission Line

Specializing to the case of no loss ($R' = 0, G' = 0$):

$$\alpha = 0, \beta = \omega\sqrt{L'C'}, Z_0 = \sqrt{L'/C'}, u_p = 1/\sqrt{L'C'}$$

We will later show that for any TEM transmission line:

$$L'C' = \mu\epsilon$$

μ = magnetic permeability

ϵ = electrical permittivity

- For any insulating material that would be used in a transmission line,
 $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m, where μ_0 is the vacuum permeability.
 — The value for μ_0 is exact and defines the relation between **B** and **H**.

- By contrast, the values for ϵ differ significantly for different materials.

We write: $\epsilon = \epsilon_r \epsilon_0$, where ϵ_r is referred to as the relative permittivity,
 and $\epsilon_0 = 1/\mu_0 c^2 \approx 8.854 \times 10^{-12}$ F/m
 is the vacuum permittivity

Lossless Transmission Line

We now have:

$$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = (\omega/c)\sqrt{\epsilon_r}, \quad u_p = 1/\sqrt{\mu\epsilon} = c/\sqrt{\epsilon_r}$$

In optics, we define an *index of refraction*, $n = c / u_p$, so that $n = \sqrt{\epsilon_r}$

Lossless Transmission Line

The impedance relation:

$$Z_0 = \sqrt{L'/C'} = \sqrt{\mu_0 / \epsilon_0 \epsilon_r} K = \sqrt{\mu_0 / \epsilon_0} \left(K / \sqrt{\epsilon_r} \right); 377 \left(K / \sqrt{\epsilon_r} \right) \Omega$$

is a bit more complex. It involves a geometric factor K .

Table of Geometric Factor K

	Coaxial	Two wire	Parallel plane
K	$\frac{1}{2\pi} \ln(b/a)$	$\frac{1}{\pi} \ln \left[(d/2a) + \sqrt{(d/2a)^2 - 1} \right]$	$\frac{d}{w}$

The vacuum impedance of 377Ω is a very important number



- Loads with lower impedance have large magnetic near fields
- Loads with higher impedance have large electric near fields

EMI properties are very different in the two cases!

4.9

Lossless Transmission Line

Voltage Reflection:

When $\alpha = 0$, our phasor relations become

$$\tilde{V}(z) = V_0^+ \exp(-j\beta z) + V_0^- \exp(j\beta z),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} \exp(-j\beta z) - \frac{V_0^-}{Z_0} \exp(j\beta z)$$

At the load, we have

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}, \quad \text{with} \quad \tilde{V}_L = V_0^+ + V_0^- \quad \text{and} \quad \tilde{I}_L = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

which implies

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+ \equiv \Gamma V_0^+$$



and defines the voltage reflection coefficient Γ

Load impedances and reflection coefficients are usually complex!

4.10

Lossless Transmission Line

Example: Ulaby et al. Exercise 2.7

Question: A $50\ \Omega$ lossless transmission line is terminated in a load impedance $Z_L = (30 - j200)\ \Omega$. Calculate the voltage reflection coefficient at the load.

Answer: $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (30 - j200 - 50) / (30 - j200 + 50) = (-20 - j200) / (80 - j200) = 201 \exp(-j1.67) / 215 \exp(-j1.19) = 0.93 \exp(-j0.48)$. NOTE: $-0.48\ \text{rads} = -28^\circ$.

Writing $\Gamma = |\Gamma| \exp(j\theta_r)$, we find $\theta_r = -0.48\ \text{rads}$.

Lossless Transmission Line

Standing Waves

After using the relation $V_0^- = \Gamma V_0^+$ in the phasor equations,

$$\tilde{V}(z) = V_0^+ [\exp(-j\beta z) + \Gamma \exp(j\beta z)] = V_0^+ [\exp(-j\beta z) + |\Gamma| \exp(j\theta_r + j\beta z)],$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} [\exp(-j\beta z) - \Gamma \exp(j\beta z)] = \frac{V_0^+}{Z_0} [\exp(-j\beta z) - |\Gamma| \exp(j\theta_r + j\beta z)]$$

We then find

$$|\tilde{V}(z)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r) \right]^{1/2},$$

so that the amplitude of the voltage varies sinusoidally with z .

- This pattern is called a *standing wave*.
- It comes from the interference of forward- and backward-propagating waves

Lossless Transmission Line

Standing Waves

- The voltage and current maxima and minima are 180° out of phase
- The amplitude multiplies a $\cos(\omega t)$ dependence with a complicated but periodic z -variation.
- The maxima are spaced $\lambda/2$ apart, and so are the minima.

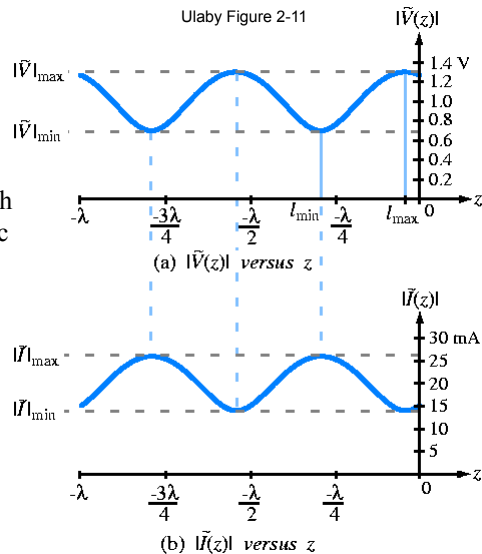
Figure Parameters:

$$|\Gamma| = 0.3, \theta_r = 30^\circ$$

$$Z_0 = 50 \Omega$$

$$|V_0^+| = 1.3 \text{ V}$$

Ulab Figure 2-11



4.13

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Lossless Transmission Line

Standing Waves

The total dependence of the standing wave voltage is

$$v(z, t) = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r) \right]^{1/2} \times \cos \left\{ \omega t - \tan^{-1} \left[\frac{\sin(\beta z - \phi^+) - |\Gamma| \sin(\beta z + \theta_r + \phi^+)}{\cos(\beta z - \phi^+) + |\Gamma| \cos(\beta z + \theta_r + \phi^+)} \right] \right\}$$

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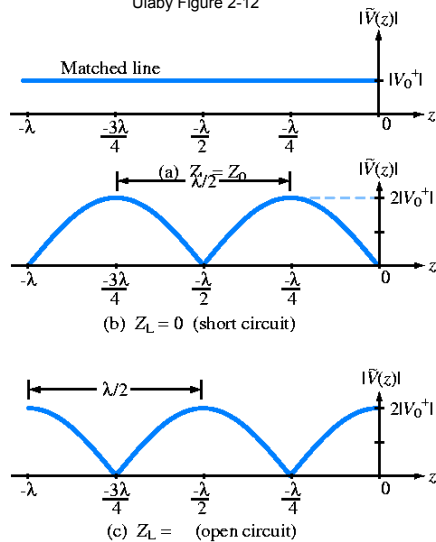
4.14

Lossless Transmission Line

Standing Waves

- For a matched load ($Z_L = Z_0$), there is no standing wave
- For a short-circuited load ($Z_L = 0$) or an open-circuited load ($Z_L = \infty$), there are complete reflections and an oscillation depth of 100%
- With $|\Gamma| = 1$, there are points where the voltage is exactly zero, spaced $\lambda / 2$ apart

Ulaby Figure 2-12



4.15

Lossless Transmission Line

Standing Waves

- We will designate the location of the maxima as $l_{\max} = -z$, so that l_{\max} is a positive number (since the load is at $z = 0$).

$$-z = l_{\max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}$$

Only n -values that satisfy $l_{\max} \geq 0$ are allowed

- The voltage standing wave ratio (VSWR or SWR) gives the oscillation depth. It is defined:

$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

4.16

Lossless Transmission Line

Example: Ulaby et al. Exercise 2.11

Question: A $140\ \Omega$ lossless line is terminated in a load impedance $Z_L = (280 + j182)\ \Omega$. If $\lambda = 72\text{ cm}$, find (a) the reflection coefficient Γ , (b) the VSWR S , (c) the locations of the voltage maxima and minima.

Answer: (a) $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (280 + j182 - 140) / (280 + j182 + 140) = 230 \exp(j0.915) / 458 \exp(j0.409) = 0.50 \exp(j0.51)$.
 NOTE: $0.51\text{ rads} = 29^\circ$. (b) $S = (1 + |\Gamma|) / (1 - |\Gamma|) = (1 + 0.502) / (1 - 0.502) = 3.0$. (c) $l_{\max} = (0.506 \times 72 / 4\pi + n \times 72 / 2)\text{ cm} = 2.9\text{ cm}, 39\text{ cm}, 75\text{ cm}, \dots$;
 $l_{\min} = (0.506 \times 72 / 4\pi + 18 + n \times 72 / 2)\text{ cm} = 21\text{ cm}, 57\text{ cm}, 93\text{ cm}, \dots$

Lossless Transmission Line

Input Impedance

With standing waves, the voltage-to-current ratio, which is referred to as the *input impedance*, varies as a function of position

$$Z_{\text{in}}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = \frac{V_0^+ [\exp(-j\beta z) + \Gamma \exp(j\beta z)]}{V_0^+ [\exp(-j\beta z) - \Gamma \exp(j\beta z)]} Z_0 = Z_0 \frac{[1 + \Gamma \exp(2j\beta z)]}{[1 - \Gamma \exp(2j\beta z)]},$$

Of particular interest is the input impedance at the generator, $z = -l$

$$\begin{aligned} Z_{\text{in}}(-l) &= Z_0 \frac{[1 + \Gamma \exp(-2j\beta l)]}{[1 - \Gamma \exp(-2j\beta l)]} \\ &= Z_0 \left(\frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l} \right) = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \end{aligned}$$

Lossless Transmission Line

Input Impedance

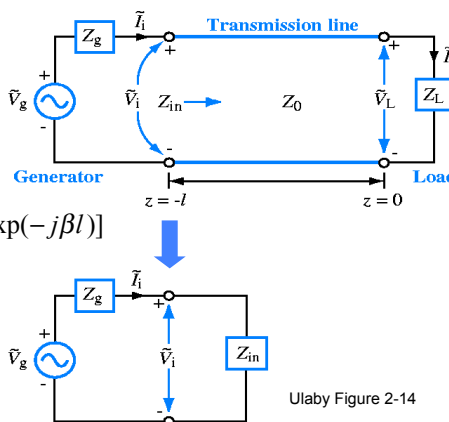
From the standpoint of the generator circuit, the transmission line appears as an input impedance $Z_{in} \equiv Z_{in}(-l)$, so that

$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}$$

From the standpoint of the transmission line

$$\tilde{V}_i = \tilde{V}(-l) = V_0^+ [\exp(j\beta l) + \Gamma \exp(-j\beta l)]$$

Combining these relations...



Ulaby Figure 2-14

4.19

Lossless Transmission Line

Input Impedance

...we conclude

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{1}{\exp(j\beta l) + \Gamma \exp(-j\beta l)} \right)$$

Thus, we can now relate the wave parameters,

$$V_0^+, \quad V_0^- (= \Gamma V_0^+), \quad I_0^+ (= V_0^+ / Z_0), \quad I_0^- (= -V_0^- / Z_0 = -\Gamma V_0^+ / Z_0),$$

to the transmission line parameters

$$Z_g, \quad Z_L, \quad Z_0 = \sqrt{L'/C'}, \quad u_p = 1/\sqrt{L'C'}, \quad l$$

and the input parameters

$$\tilde{V}_g, \quad f = \omega / 2\pi$$

4.20

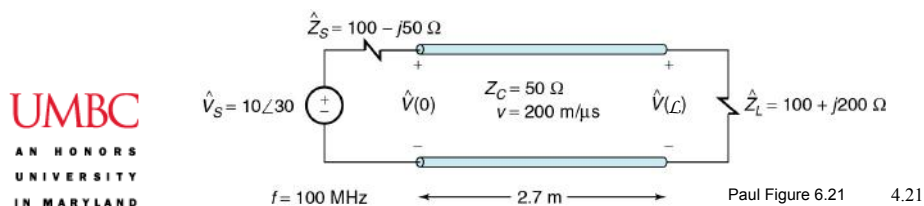
Lossless Transmission Line

Paul Example 6.7 (extended)

Question: A line 2.7 m in length is excited by a 100 MHz source as shown in the figure. Determine the source and load voltages. Determine the voltages and the current everywhere in the transmission line

Answer: We will work in Ulaby et al.'s notation, and our first task is to translate Paul's problem specification into that notation:

$$\begin{aligned}\hat{V}_S &\rightarrow \tilde{V}_g = 10\exp(j30^\circ) = 10\exp(j\pi/6) = 10\exp(j0.524) = (8.66 + j5.00) \text{ V} \\ \hat{V}(0) &\rightarrow \tilde{V}_i = \tilde{V}(-l); \quad \hat{V}(L) \rightarrow \tilde{V}_L = \tilde{V}(0); \quad v \rightarrow u_p = 200 \text{ m}/\mu\text{s} = 2 \times 10^8 \text{ m/s} \\ \hat{Z}_S &\rightarrow Z_g = 100 - j50 \Omega; \quad \hat{Z}_L \rightarrow Z_L = 100 + j200 \Omega; \quad Z_C \rightarrow Z_0 = 50 \Omega;\end{aligned}$$



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Lossless Transmission Line

Paul Example 6.7

Answer (continued):*

(1) Find the reflection coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j200}{150 + j200} = \frac{206 \exp(j1.326)}{250 \exp(j0.927)} = 0.825 \exp(j0.399)$$

(2) Find the propagation factor

$$\begin{aligned}\beta l &= (2\pi f / u_p)l = (6.28 \times 10^8 / 2 \times 10^8) \times 2.7 = 8.48; \\ \exp(j\beta l) &= \exp(j0.7\pi) = \exp(j2.20); \\ \exp(-j\beta l) &= \exp(-j0.7\pi) = \exp(-j2.20); \\ \exp(-j2\beta l) &= \exp(-j1.4\pi) = \exp(-j4.40) = \exp(j1.88)\end{aligned}$$

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*NOTE: I am using MATLAB to calculate values. So, the calculations are good to 15 places, although I only report three.

4.22

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(3) *Find the input impedance:*

$$\begin{aligned}\Gamma \exp(-j2\beta l) &= 0.825 \exp[j(0.399 + 1.885)] \\ &= 0.825 \exp(j2.283) = -0.539 + j0.624; \\ 1 + \Gamma \exp(-j2\beta l) &= 0.461 + j0.624; \\ 1 - \Gamma \exp(-j2\beta l) &= 1.539 - j0.624;\end{aligned}$$

$$\begin{aligned}Z_{\text{in}} &= Z_0 \frac{1 + \Gamma \exp(-j2\beta l)}{1 - \Gamma \exp(-j2\beta l)} = 50 \frac{0.461 + j0.624}{1.539 - j0.624} = 50 \frac{0.776 \exp(j0.935)}{1.661 \exp(-j0.385)} \\ &= 23.35 \exp(j1.320) = 5.80 + j22.62\end{aligned}$$



4.23

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(4) *Find the input voltage:*

$$\begin{aligned}Z_g + Z_{\text{in}} &= (100 - j50) + (5.80 + j22.6) \\ &= 106 - j27.4 = 109 \exp(-j0.253)\end{aligned}$$

$$\begin{aligned}\tilde{V}_i &= \tilde{V}_g \frac{Z_{\text{in}}}{Z_g + Z_{\text{in}}} = 10 \exp(j0.524) \frac{23.4 \exp(j1.320)}{109 \exp(-j0.253)} \\ &= 2.14 \exp(j2.10) = 2.14 \angle 120^\circ\end{aligned}$$



4.24

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(5) Find the load voltage:

$$V_0^+ = \tilde{V}_i \frac{\exp(-j\beta l)}{1 + \Gamma \exp(-j2\beta l)} = 2.14 \exp(j2.10) \frac{\exp(-j2.20)}{0.776 \exp(j0.935)}$$

$$= 2.75 \exp(-j1.037)$$

$$1 + \Gamma = 1 + (0.760 + j0.320) = 1.79 \exp(j0.180)$$

$$\tilde{V}_L = V_0^+ (1 + \Gamma) = [2.75 \exp(-j1.037)][1.79 \exp(j0.180)]$$

$$= 4.93 \exp(-j0.857) = 4.93 \angle -49^\circ$$



4.25

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(6) Write the time domain voltages:

$$v_i(t) = \text{Re}[\tilde{V}_i \exp(j\omega t)] = 2.14 \cos(6.28 \times 10^8 t + 120^\circ)$$

$$v_L(t) = \text{Re}[\tilde{V}_L \exp(j\omega t)] = 4.93 \cos(6.28 \times 10^8 t - 49^\circ)$$

(7) Find the phasor domain voltage and current in the transmission line

$$\tilde{V}(z) = V_0^+ \exp(-j\beta z) + \Gamma V_0^+ \exp(j\beta z)$$

$$= 2.75 \exp(-j\beta z - j1.037) + 2.27 \exp(j\beta z - j0.638) \quad (\text{V})$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} \exp(-j\beta z) - \frac{\Gamma V_0^+}{Z_0} \exp(j\beta z)$$

$$= 55.1 \exp(-j\beta z - j1.037) + 45.4 \exp(j\beta z + j2.503) \quad (\text{mA})$$



4.26

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(8) Find the time domain voltage and current in the transmission line:

$$\begin{aligned}v(z,t) &= \operatorname{Re}[2.75 \exp(j\omega t - j\beta z - j1.037) + 2.27 \exp(j\omega t + j\beta z - j0.638)] \\&= 2.75 \cos(\omega t - \beta z - 1.037) + 2.27 \cos(\omega t + \beta z - 0.638) \text{ (V)}\end{aligned}$$

$$\begin{aligned}i(z,t) &= \operatorname{Re}[55.1 \exp(j\omega t - j\beta z - j1.037) + 45.4 \exp(j\omega t + j\beta z + j2.503)] \\&= 55.1 \cos(\omega t - \beta z - 1.037) + 45.4 \cos(\omega t + \beta z + 2.503) \text{ (mA)}\end{aligned}$$

(9) Checks:

$$\begin{aligned}v(0,t) &= 2.75 \cos(\omega t - 1.037) + 2.27 \cos(\omega t - 0.638) \\&= 4.93 \cos(\omega t - 0.857) \text{ (V)}\end{aligned}$$

$$\begin{aligned}v(-l,t) &= 2.75 \cos(\omega t + 8.48 - 1.037) + 2.27 \cos(\omega t - 8.48 - 0.638) \\&= 2.14 \cos(\omega t + 2.097) \text{ (V)}\end{aligned}$$