Math-Phys Quiz 10 Questions:

1. From the general expression for the complex for the complex propagation constant γ,

$$\alpha = \omega \left\{ \frac{\mu\epsilon'}{2} \left[\sqrt{1 + \left(\frac{\epsilon'}{\epsilon''} \right)^2} - 1 \right] \right\}^{1/2}, \qquad \beta = \omega \left\{ \frac{\mu\epsilon'}{2} \left[\sqrt{1 + \left(\frac{\epsilon'}{\epsilon''} \right)^2} + 1 \right] \right\}^{1/2},$$

   find the expressions for α and β in the limits of a good conductor and a good insulator.

2. Explain in words why a thin metal sheet is effective at shielding electric fields in frequency ranges where it is a good conductor, but a sheet that is approximately five skin depths thick must be used to shield magnetic fields.

Exam Quiz 10 Question:

1. **A normally incident electromagnetic wave: slides 15.4-15.10:** We consider two lossless media with the boundary at $z = 0$ and that are characterized by permittivities and permabilities ϵ_1, μ_1, ϵ_2, and μ_2. An x-polarized electromagnetic wave traveling in the $+z$-direction from medium 1 into medium 2 is normally incident on the interface.

   a. Given the incident electric field amplitude in the phasor domain $\tilde{\mathbf{E}}^{\mathrm{i}} = E_0^{\mathrm{i}} \exp(-jk_1 z)$, find the incident phasor domain magnetic field, and the reflected and transmitted phasor domain fields. Find the reflection and transmission coefficients.

   b. Finding the Poynting flux for each of the three fields, and show that the power in the incident field that falls on the interface equals the sum of the powers in the reflected and transmitted fields.

2. **Transmission Line Parameters: Slides 14.4 and 14.5:** Find the resistance per unit length for the parallel plate, coaxial cable, and two-wire transmission lines

Math-Physics Quiz 10 Solutions:

1. For a good insulator, we have $\epsilon'' \ll \epsilon'$, so that $[1 + (\epsilon''/\epsilon')^2]^{1/2} \simeq 1 + (1/2)(\epsilon''/\epsilon')^2$. We now find $\alpha \simeq \omega[(\mu\epsilon'/4)(\epsilon''/\epsilon')^2]^{1/2} = (1/2)(\mu/\epsilon')^{1/2}\epsilon''$, and $\beta = \omega(\mu\epsilon')^{1/2}$. For a good conductor, we have $\epsilon'' \gg \epsilon'$, so that $\alpha = \beta = \omega(\mu\epsilon'/2)^{1/2}$.

2. The conductor shorts out the electric field, which is almost completely canceled out. At the same time, a surface current is generated, so that the magnetic field penetrates into the metal. The magnetic field due to this current decays to a negligible fraction of its original value over a length that is approximately five times the skin depth.

Exam Quiz 10 Solutions:

1. We note that $k_1 = \omega(\epsilon_1\mu_1)^{1/2}$, where ω is the angular frequency, and we define $\eta_1 = (\mu_1/\epsilon_1)^{1/2}$, $\eta_2 = (\mu_2/\epsilon_2)^{1/2}$, and $k_2 = \omega(\epsilon_2\mu_2)^{1/2} = k_1(\epsilon_2\mu_2/\epsilon_1\mu_1)^{1/2}$.

   a. We have for the incident wave:

   $$\tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{x}}E_0^{\mathrm{i}}\exp(-jk_1z), \quad \tilde{\mathbf{H}}^{\mathrm{i}}(z) = \hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{i}}(z) = \hat{\mathbf{y}}\frac{E_0^{\mathrm{i}}}{\eta_1}\exp(-jk_1z).$$

   We have for the reflected wave:

   $$\tilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}}E_0^{\mathrm{r}}\exp(jk_1z), \quad \tilde{\mathbf{H}}^{\mathrm{r}}(z) = \hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{r}}(z) = -\hat{\mathbf{y}}\frac{E_0^{\mathrm{r}}}{\eta_1}\exp(jk_1z).$$

   We have for the transmitted wave:

   $$\tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}}E_0^{\mathrm{t}}\exp(-jk_2z), \quad \tilde{\mathbf{H}}^{\mathrm{t}}(z) = \hat{\mathbf{z}} \times \tilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{y}}\frac{E_0^{\mathrm{t}}}{\eta_2}\exp(-jk_2z).$$

   From the continuity of the electric and magnetic fields across the boundary, we infer

   $$E_0^{\mathrm{i}} + E_0^{\mathrm{r}} = E_0^{\mathrm{t}}, \qquad \frac{E_0^{\mathrm{i}}}{\eta_1} - \frac{E_0^{\mathrm{r}}}{\eta_1} = \frac{E_0^{\mathrm{t}}}{\eta_2}.$$

   We now find

   $$E_0^{\mathrm{r}} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)E_0^{\mathrm{i}} \equiv \Gamma E_0^{\mathrm{i}}, \qquad E_0^{\mathrm{t}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right)E_0^{\mathrm{i}} \equiv \tau E_0^{\mathrm{i}},$$

   where Γ is the reflection coefficient and τ is the transmission coefficent. We note that $\tau = 1 + \Gamma$.

   b. The average Poynting flux in medium 1 is given by

   $$\begin{aligned}\mathbf{S}_1 &= \frac{1}{2}\operatorname{Re}\left[\tilde{\mathbf{E}}_1(z) \times \tilde{\mathbf{H}}_1^*(z)\right]\\ &= \frac{1}{2}\operatorname{Re}\left\{\hat{\mathbf{x}}E_0^{\mathrm{i}}\left[\exp(-jk_1z) + \Gamma\exp(jk_1z)\right] \times \hat{\mathbf{y}}\frac{E_0^{\mathrm{i}*}}{\eta_1}\left[\exp(jk_1z) - \Gamma\exp(-jk_1z)\right]\right\}\\ &= \hat{\mathbf{z}}\frac{|E_0^{\mathrm{i}}|^2}{2\eta_1}(1 - \Gamma^2) = \mathbf{S}^{\mathrm{i}} + \mathbf{S}^{\mathrm{r}}\end{aligned}$$

with $\mathbf{S}^{\mathrm{i}} = \hat{\mathbf{z}}(|E_0^{\mathrm{i}}|^2/2\eta_1)$ and $\mathbf{S}^{\mathrm{r}} = -\hat{\mathbf{z}}\Gamma^2(|E_0^{\mathrm{i}}|^2/2\eta_1)$. The average Poynting flux in medium 2 is given by

$$\mathbf{S}_2 = \frac{1}{2}\,\mathrm{Re}\left[\tilde{\mathbf{E}}_2(z) \times \mathbf{H}_2^*(z)\right] = \frac{1}{2}\,\mathrm{Re}\left[\hat{\mathbf{x}}\tau E_0^{\mathrm{i}} \exp(-jk_2 z) \times \hat{\mathbf{y}}\frac{E_0^{\mathrm{i}*}}{\eta_2} \exp(jk_2 z)\right]$$
$$= \hat{\mathbf{z}}\tau^2 \frac{|E_0^{\mathrm{i}}|^2}{2\eta_2}.$$

Using the relation

$$\frac{\tau^2}{\eta_2} = \frac{2}{\eta_1 + \eta_2} = \frac{1 - \Gamma^2}{\eta_1},$$

we conclude $|\mathbf{S}|^{\mathrm{i}} = |\mathbf{S}|^{\mathrm{r}} + |\mathbf{S}|^{\mathrm{t}}$.

2. The resistance per unit length is given by the sum of the resistances per unit length on each of the two conductors in each geometry. For each surface, the resistance is given by $R_{\mathrm{S}} = \sqrt{\pi f \mu/\sigma}$ divided by the cross-sectional surface length. For the parallel plate, the surface lengh is w for both surfaces, where w is the plate width, so that $R' = 2R_{\mathrm{S}}/w$. For the coaxial cables the lengths are $2\pi a$ for the inner surface and $2\pi b$for the outer surface, where a and b are the radii of the inner and outer conductors, so that $R' = (R_{\mathrm{S}}/2\pi)[(1/a) + (1/b)]$. Finally, the diameter of both wires in the two wire line are d, so that $R' = 2R_{\mathrm{S}}/\pi d$.