CMPE 330 Spring 2017 First Midterm Examination

(Each problem is worth 10 points)

1. A wave in a transmission line has a frequency of $f = 10$ GHz and $\epsilon_\mathrm{r} = 9/4 = 2.25$.

   a. What are ω, λ, β, and u_p?

   b. At a fixed time, what length must you go for the wave to change its phase by $180°$? 3π radians? 20 cycles?

   c. Suppose that that at the input to the transmission line the voltage is 100 V and suppose that it falls to 25 V after 1 m. After what length, to within a meter, will the voltage be less than 1 V?

   Please give quantitative answers with units and no unexpressed variables like π to at least two significant figures, i.e., you must use 3.1 or 3.14 for π.

2. The speed of light transmission in an optical fiber is $u_\mathrm{p} = 2.0 \times 10^8$ m/s, and its wavelength is $\lambda = 1.5$ μm. The birefringence $\Delta n/n = \Delta\epsilon_\mathrm{r}/2\epsilon_\mathrm{r} = 2 \times 10^{-6}$. The attenuation length over which half the power is lost equals 30 km.

   a. The light power at the transmitter is 1 mW. What is the power at 90 km?

   b. What is the difference in the propagation velocity along the two axes of birefringence? Over what distance is there a 2π phase slip between the waves along each of the axes of birefringence.

3. A 50 Ω lossless line is to be matched to a resistive load impedance at 400 Ω. The operation frequency is 1 GHz and $u_\mathrm{p} = c = 2 \times 10^8$ m/s.

   a. What is the length of the impedance transformer?

   b. What is its impedance?

4. Consider a transmission line with $Z_0 = 100$ Ω and $Z_\mathrm{L} = 75$ Ω. What is the standing wave ratio? Measured in wavelengths, how close are the first maximum and the first minimum to the load?

5. Consider a transmission line with a characteristic impedance of 50 Ω and a load impedance of 50 Ω. Suppose that the generator impedance is also 50 Ω and $\tilde{V}_\mathrm{s} = 100$ V. How much power is dissipated in the load resistor and how much power is dissipated in the source resistor?

Midterm Examination Solutions

1. a. $f = 10$ GHz $= 10^{10}$ s^{-1}; $\omega = 2\pi \times f = 6.28 \times 10^{10}$ s^{-1}; $u_{\rm p} = c/\sqrt{\epsilon_{\rm r}} = [3 \times 10^8$ m/s$]/1.5 = 2.0 \times 10^8$ m/s; $\lambda = u_{\rm p}/f = 2.0 \times 10^8/10^{10} = 2.0 \times 10^{-2}$ m $= 2$ cm; $\beta = 2\pi/\lambda = (6.28/0.02)$ m^{-1} $= 314$ m^{-1}.

   b. We have that 180° corresponds to $\lambda/2 = 0.01$ m $= 1.0$ cm; 3π rads corresponds to $1.5\lambda = 0.03$ m $= 3.0$ cm; 5 cycles corresponds to $5\lambda = 0.1$ m $= 10$ cm.

   c. The voltage falls by a factor of 4 in every meter. A factor of 100 is between $4^3 = 64$ and $4^4 = 256$. So, the voltage will fall below 1 V somewhere between 3 meters and 4 meters.

2. a. After 90 km, the power will be decreased by a factor of $2^3 = 8$. If we start with 1 mW, then we end up with $(1000/8)$ μW $= 125$ μW, which equals 130 W to two significant figures.

   b. We have that $\Delta u_{\rm p} = u_{\rm p}\Delta n/n = (2 \times 10^8) \times (2 \times 10^{-6}) = 400$ m/s. The length over which the phase between the two light waves slips by 2π is given by $L = \lambda(u_p/\Delta u_p) = (1.5 \times 10^{-6}) \times [(2 \times 10^8)/400] = 300/400 = 0.75$ m.

3 a. We need a $\lambda/4$ transformer. In this case, we have $\lambda = c/f = (2 \times 10^8)/(10^9) = 0.2$ m. Hence, we have $\lambda/4 = 0.05$ m $= 5$ cm.

   b. The impedance is $(50 \times 400)^{1/2}$ $\Omega = (\sqrt{2} \times 100) = 141$ Ω.

4. We have $\Gamma = (75 - 100)/(75 + 100) = -1/7$, and since Γ is purely real, we have

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 - \Gamma}{1 + \Gamma} = 8/6 = 1.33.$$

   Because Γ is purely real and negative, the voltage has a minimum at the load and every distance $-z = l = n\lambda/2$ away from the load, where $n = 1, 2, 3, \ldots$ So, the first minimum away from the load is at $l = \lambda/2$. The maxima occur in between the minima when $-z = l = (\lambda/4) + n\lambda/2$, where $n = 0, 1, 2, \ldots$ Hence, the first maximum occurs at $l = \lambda/4$.

5. Since the load impedance and the characteristic impedance are matched, there is only a forward-propagating wave, and the input impedance must therefore equal the load impedance, which is 50 Ω. Since the source impedance is also 50 Ω, half the power must be dissipated in the source impedance, and the other half must be dissipated in the load impedance. The power dissipated in the generator is $P_{\rm g} = (1/2)\,{\rm Re}(VI^*)$, and we have $I^* = V^*/(Z_{\rm in}^* + Z_{\rm source}^*) = (100/100)$ A $= 1$ A. Hence, we have $P_{\rm g} = 25$ W and $P_{\rm L} = 25$ W.