Math-Phys Quiz 9 Question:

1. Derive the boundary conditions for $\mathbf{E}$ when crossing a boundary with no currents and charges in which the dielectric constant changes, so that in medium 1 the dielectric constant is ϵ_1 and in medium 2 the dielectric constant is ϵ_2.

2. Derive the boundary condition for $\mathbf{H}$ when going from medium 2 to medium 1 with magnetic permeabilities μ_2 and μ_1 and with no currents and charges.

Exam Quiz 9 Questions:

1. **Charged particle motion: slides 11.1-11.3:** (a) Find the motion of a charged particle in a constant electric field. (b) Find the motion of a charged particle in a constant magnetic flux.

2. **Transmission Line Parameters: Slides 9.18, 9.19, 9.31, 11.29:** Find the conductance per unit length, the capacitance per unit length, and the inductance per unit length for the coaxial cable transmission line.

Math-Physics Quiz 9 Solutions:

1. See slides 9.24 and 9.25 for the derivation. The answer is $E_{1\mathrm{t}} = E_{2\mathrm{t}}$ and $\epsilon_1 E_{1\mathrm{n}} = \epsilon_2 E_{2\mathrm{n}}$, where the subscripts "t" and "n" denote respectively the components tangential and normal to the boundary. Note that because we are going from one dielectric medium to another, i.e., there is no conductor or semi-conductor on either side, there is no free charge. From the statement of the problem, including free charge is a mistake.

2. See slides 11.23 and 11.24 and the discussion in the textbook. The answers are $H_{1\mathrm{t}} = H_{2\mathrm{t}}$ and $\mu_1 H_{1\mathrm{n}} = \mu_2 H_{2\mathrm{n}}$.

Exam Quiz 9 Solutions:

1. We let E and B be the strength of the electric field and the magnetic flux, we let q and m be the charge and mass of the charged particle, we let u_{x0} u_{y0} and u_{z0} be the initial velocities in the x-, y-, and z-directions, and we let x_0, y_0, and z_0 equal the corresponding initial coordinates.

   a. A constant electric field leads to constant acceleration in the direction of the electric field. If we take the direction of the electric field to be in the z-direction, then it has no effect on the motion in the x- and y-directions and leads to constant acceleration in the z-direction. We then find $u_x(t) = u_{x0}$, $u_y(t) = u_{y0}$, and $u_z(t) = u_{z0} + (qE/m)t$. Integrating these equations to find the position of the charged particle as a function of time, we find $x(t) = x_0 + u_{x0}t$, $y(t) = y_0 + u_{y0}t$, and $z(t) = z_0 + u_{z0}t + (qE/2m)t^2$.

   b. A constant magnetic flux leads to cyclotron motion of the charged particle and has no effect in the direction of the field. If we take the direction of the flux to be in the z-direction, we find that $u_z(t) = u_{z0}$ and $z(t) = z_0 + u_{z0}t$. To solve for the motion in the x- and y-directions, we use the equations of motion

   $$\frac{du_x}{dt} = \omega_\mathrm{L} u_y, \qquad \frac{du_y}{dt} = -\omega_\mathrm{L} u_x,$$

   where $\omega_\mathrm{L} = qB/m$ is called the Larmor frequency. The solution to these equations is $u_x(t) = u_{x0}\cos\omega_\mathrm{L}t + u_{y0}\sin\omega_\mathrm{L}t$, $u_y(t) = -u_{x0}\sin\omega_\mathrm{L}t + u_{y0}\cos\omega_\mathrm{L}t$. Integrating these equations, we obtain $x(t) = x_0 + (u_{x0}/\omega_\mathrm{L})\sin\omega_\mathrm{L}t + (u_{y0}/\omega_\mathrm{L})(1 - \cos\omega_\mathrm{L}t)$, $y(t) = y_0 - (u_{x0}/\omega_\mathrm{L})(1 - \cos\omega_\mathrm{L}t) + (u_{y0}/\omega_\mathrm{L})\sin\omega_\mathrm{L}t$.

2. The charge per unit length Q' on the inner conductor of a coaxial cable equals $2\pi a\rho_\mathrm{S}$, where a is the radius of the inner conductor. The field that this conductor generates is given by $\mathbf{E} = \hat{\mathbf{r}}\rho_\mathrm{S}a/\epsilon r$, where ϵ is the permittivity. Integrating this field from the inner to the outer conductor, we find that the voltage change is given by $V = (a\rho_\mathrm{S}/\epsilon)\ln(b/a)$, where b is the inner radius of the outer conductor. The capacitance per unit length is then given by $C' = Q'/V = 2\pi\epsilon/\ln(b/a)$. We now use the relationship $G' = (\sigma/\epsilon)C'$ to find $G' = 2\pi\sigma/\ln(b/a)$ and the relationship $L' = \mu\epsilon/C'$ to obtain $L' = (\mu/2\pi)\ln(b/a)$.