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Week 1

1.1

Linear, Time Invariant Systems

- LTI is not everything!
- There are many non-linear, non time invariant systems.
- But LTI is so common that it is the larges class of systems
- Even non linear systems (think GPS) often act like linear systems in the neighborhood of an operating point...
- ...through use of a first-order Taylor Series expansion

$$\hat{y}(t;c) \approx y(t;c_0) + \frac{\partial y(t;c)}{\partial c} \Delta c + \mathcal{O}(\Delta c^2)$$

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Week 1

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Convolution



$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

- ...where we frequently have to evaluate the integral differently in different regions of t
- Remember that the integration is with respect to τ , and t is a parameter when doing the integration!
- There should be no dependence on $\, au \,$ after the integration...
- ullet ...although there will obviously be a dependence on t
- Convolution is a filtering operation

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Week 1

1.1

Laplace Domain

- The Laplace domain (s-domain) is frequently used for characterizing the performance of linear systems
- ...but not the characteristics of linear signals
- $\mathcal{L}(x(t)) = X(s) = \int x(t)e^{-st} dt$
- The transfer function H(s) is the Laplace Transform of the impulse response h(t) of that system
- The Laplace transform includes specification of the Region of Convergence.
- Many Laplace transforms are rational polynomials in

$$X(s) = \frac{\prod_{k=1}^{M} (s - z_k)}{\prod_{k=1}^{B} (s - p_k)}$$

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Week 1

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Inverse Laplace Transforms

 We generally invert Laplace Transforms by means of evaluating the Residues, also known as Partial Fraction Expansion

$$X(s) = \frac{\prod_{k=1}^{M} (s - z_k)}{\prod_{k=1}^{N} (s - p_k)} = \sum_{m=1}^{N} \frac{R_m}{(s - p_m)}$$

- ... with explict process for dealing with roots of higher order
- Poles and residuals must occur as either real or in complex conjugate pairs

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Week 1

1.1

Right-sided and left-sided signals

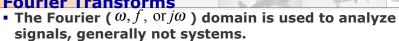
- A right sided signal exists for $t > t_0$
- A left sided signal exists for $t < t_0$
- A two sided signal exists for $t_1 < t < t_2$
- For right sided signals, RoC is $\operatorname{Re}[s] > a$
- For left sided signals, RoC is Re[s] < a
- For two sided signals, RoC is a < Re[s] < b
- There are no poles in the Region of Convergence
- For causal signals, all the poles must be in the left half plane
- For anti causal signals, all the poles must be in the right half plane

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Week 1

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Fourier Transforms



A periodic (power) signal has a Fourier Series

$$c_k = \frac{1}{T} \int_{a}^{a+T} x(t)e^{-j\frac{2\pi k}{T}t} dt, \ x(t) = \sum_{n=-\infty}^{\infty} c_k e^{j\frac{2\pi n}{T}t}$$

A non-periodic (energy) signal has a Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t} dt = X(f)$$
$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t} df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

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The convolution theorem

The convolution theorem ties the time and Laplace/ Fourier domains together

$$\mathcal{L}(x(t)*h(t)) = X(s)H(s)$$



- Sampling in one domain indicates that the signal is periodic in the other domain
- Periodicity in one domain indicates that the signal is sampled (discrete) in the other domain.
- If a signal is periodic and sampled in one domain, it is sampled and periodic in the other domain.

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Week 1

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 The DFT is a numerical computation that approximates a scaled version of the Fourier Transform

$$X_{DFT}[n] = \sum_{k=0}^{N-1} x[k]e^{-j\frac{2\pi kn}{N}}$$

 The DTFT is the Fourier transform of the impulsesampled piecewise continuous time signal and is continuous in frequency

$$X_{DTFT}(\omega) = \sum_{k=-\infty}^{\infty} x(t)e^{-j\omega k\Delta t}$$

• The FFT is an efficient algorithm for computing the DFT. It is most efficient if $N=2^M$ It implicitly assumes both the time and frequency domain signals are periodic, but has no explicit identification with time of frequency

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Analog Filters

- A Butterworth filter has a transfer function $H(j\omega) = H(s)_{s=j\omega}$
- ... that is monotonically decreasing away from the center of the passband.
- A Chebyshev Type 1 filter has a transfer function that has ripples in the pass band, but is monotonically decreasing outside of the band
- A Chebyshev Type 2 filter has a transfer function that has ripples in the stop band, but is monotonically decreasing inside the pass band.

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