Homework #2 (Due: 4pm, 2/19/18)

Problem 1.
The following problem was given to 60 students and doctors at the famous Hevardi Medical School (HMS): Assume there exists a test to detect a disease, say D, whose prevalence is 0.001, that is, the probability, $P[D]$, that a person picked at random is suffering from D, is 0.001. The test has a false-positive rate of 0.005 and a correct detection rate of 1. The correct detection rate is the probability that if you have D, the test will say that you have D. Given that you test positive for D, what is the probability that you actually have it? Many of the HMS experts answered 0.95 and the average answer was 0.56. Show that your knowledge of probability is greater than that of the HMS experts by getting the right answer of 0.17.

Problem 2.
In the ternary communication channel shown in Figure 2, a 3 is sent three times more frequently than a 1, and a 2 is sent two times more frequently than a 1. A 1 is observed; what is the conditional probability that a 1 was sent?

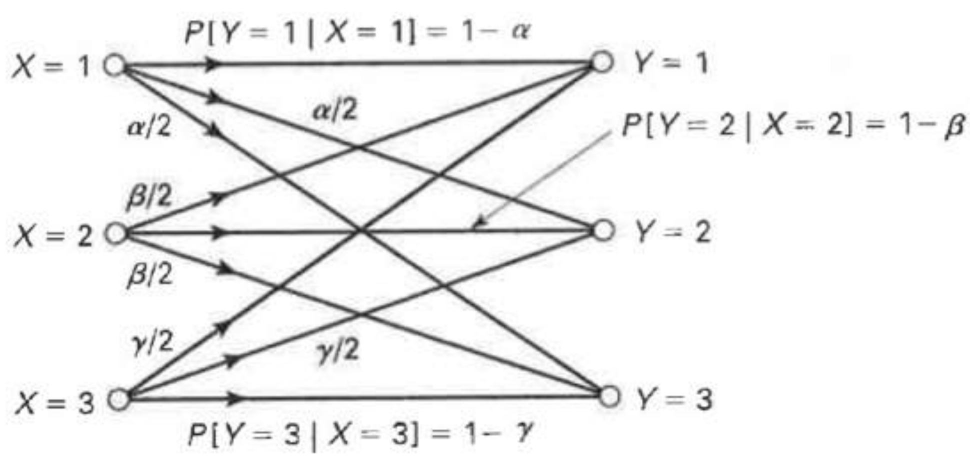

Figure 2. Ternary communication channel.

Problem 3.
A large class in probability theory is taking a multiple-choice test. For a particular question on the test, the fraction of examinees who know the answer is p; $1-p$ is the fraction that will guess. The probability of answering a question correctly is unity for an examinee who knows the answer and $1/m$ for a guessee; m is the number of multiple-choice alternatives. Compute the probability that an examinee knew the answer to a question given that he or she has correctly answered it.

Problem 4.
Assume there are three machines A, B, and C in a semiconductor manufacturing facility that make chips. They manufacture, respectively, 25, 35, and 40 percent of the total semiconductor chips there. Of their outputs, respectively, 5, 4, and 2 percent of the chips are defective. A chip is drawn randomly from the combined output of the three machines and is found defective. What is the probability that this defective chip was manufactured by machine A? by machine B? by machine C?

Problem 5.
A card is selected at random from a standard deck of 52 cards. Let A be the event of selecting an ace and let B be the event of selecting a red card. There are 4 aces and 26 red cards in the normal deck. Are A and B independent?

Problem 6.
A fair die is tossed three times. Given that a 2 appears on the first toss, what is the probability of obtaining the sum 7 on the three tosses?

Problem 7. An internet access provider (IAP) owns two servers. Each server has a 50% chance of being "down" independently of the other. Fortunately, only one server is necessary to allow the IAP to provide service to its customers, i.e., only one server is needed to keep the IAP's system up. Suppose a customer tries to access the internet on four different occasions, which are sufficiently spaced apart in time, so that we may assume that the states of the system corresponding to these four occasions are independent. What is the probability that the customer will only be able to access the internet on 3 out of the 4 occasions?

Problem 8. A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are 1×1.5 inches, the faces showing 2 or 5 are 1×0.4 inches, and the faces showing 3 or 4 are 0.4×1.5 inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?