1 MATH407

4/18/18

* Symmetries of regular polygons in $\mathbb{R}^2$

T symmetry of A

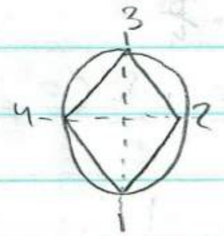
$\|a_1 - a_2\|$ max over A

$\|T(a_1) - T(a_2)\|$ max over A

T maps vertices of polygon P to vertices of P

$T|$ vertices $\in$ Sym(vertices)

T is linear and vertices spanning set in $\mathbb{R}^2$



T must map adjacent vertices to adjacent vertices

If $T(1) = j$, then $T(2) = j+1$

$T(3) = j+2$

$\vdots$

$T(1+k) = j+k \pmod{n}$

Let α = rotation by $\frac{2\pi}{n}$, so $T(1) = 2$

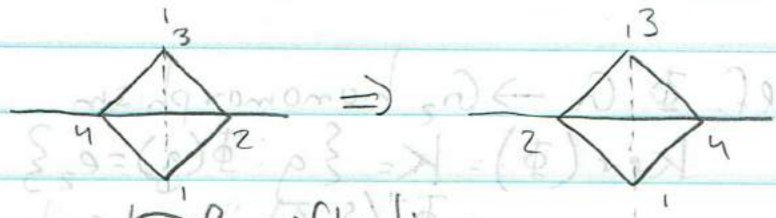
$\alpha^{j-1} = (j, j+1, \dots, n+j)$

$2n$ symmetries of P_n

* D_n = geometric symmetries of P_n (Dihedral Group)

$|D_n| = 2n$

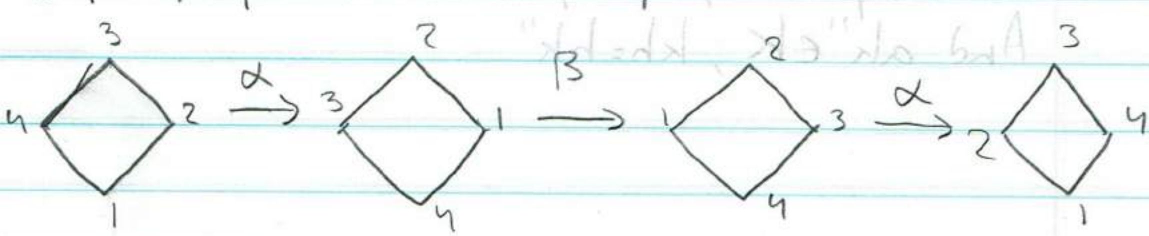
	e	α	α^2	α^3
e	e	α	α^2	α^3
α	α	α^2	α^3	e
α^2	α^2	α^3	e	α
α^3	α^3	e	α	α^2



β reflection

$\Rightarrow$

	e	α	α^2	α^3	β	$\alpha\beta$	$\alpha^2\beta$	$\alpha^3\beta$
e	e	α	α^2	α^3	β	$\alpha\beta$	$\alpha^2\beta$	$\alpha^3\beta$
α	α	α^2	α^3	e	$\alpha\beta$	$\alpha^2\beta$	$\alpha^3\beta$	β
α^2	α^2	α^3	e	α				
α^3	α^3	e	α	α^2				
β	β	$\alpha^3\beta$	$\alpha^2\beta$	$\alpha\beta$				
$\alpha\beta$	$\alpha\beta$							
$\alpha^2\beta$	$\alpha^2\beta$							
$\alpha^3\beta$	$\alpha^3\beta$							



$\alpha\beta\alpha = \beta \Rightarrow \beta\alpha = \alpha^{-1}\beta = \alpha^{n-1}\beta$

$\beta\alpha^2 = (\beta\alpha)\alpha = \alpha^3(\beta\alpha) = \alpha^3(\alpha^3\beta) = \alpha^2\beta$

(3)

In P_n, $\alpha^n = e$
$\beta\alpha = \alpha^{n-1}\beta$

In general, $\beta\alpha^j = \alpha^{n-j}\beta$

(*) Sections 3.7 and 3.8:

Def. $\Phi : G_1 \to G_2$ homomorphism
$\ker(\Phi) = K = \{g : \Phi(g) = e_2\}$
$= \Phi^{-1}(\{e\})$ is a subgroup of G_1

*Ker := kernel

Recap: Prop 3.4.4: Φ is 1-1 iff $\ker(\Phi) = \{e_1\}$

*H, K subgroups of G if $\forall h \in H, k \in K$,
$hk \in Kh$
Then, HK is a subgroup of G (and is KH)

For any $h \in H$, $k \in H$, $\exists k' \in K$ s.t. $hk = k'h$
And a $k'' \in K$, $kh = hk''$