CMPE 330 **Spring 2015** **Problem Set #3**

NOTE: You must show complete work for full credit. When nothing else is stated, report two significant figures.

1. In addition to not dissipating power, a lossless line has two important features: (1) It is dispersionless (u_{p} is independent of frequency); and (2) its characteristic impedance is purely real. Sometimes it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the material properties and dimensions so as to satisfy the condition $R'C' = L'G'$. Such a line is called a *distortionless* line, because it still possesses features (1) and (2) of the lossless line. Show that for a distortionless line,

$$\alpha = R'\sqrt{C'/L'} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{L'/C'}.$$

   [Ulaby et al. 2.13, p. 122.]

2. Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 and a minimum magnitude of 0.8 V. Find the magnitude of the load's reflection coefficient. [modified from Ulaby et al. 2.17, p. 122.]

3. Consider a transmission line operating at 1.5 GHz with the following parameters, $Z_{\mathrm{g}} = 50\ \Omega$, $Z_0 = 50\ \Omega$, $Z_{\mathrm{L}} = (25 - 25j)\ \Omega$, $u_{\mathrm{p}} = 1.5 \times 10^8$ m/s, $l = 24$ cm, $v_g(t) = 10\cos(\omega t)$ (V). These correspond to the paramaters in Ulaby's Module 2.1B – 2.4B in the 2001–2007 editions. In this problem, please report three significant figures.

   a. Find the values for ω and ϵ_{r}. Find the normalized load impedance $z_{\mathrm{L}} = Z_{\mathrm{L}}/Z_0$.

   b. Find $|\Gamma|$, θ_{r}, S (the standing wave ratio), and $l_{\max}$ and $l_{\min}$ (the locations of the first voltage maximum and minimum from the load).

   c. Find the input impedance Z_{in}, the incident wave voltage V_0^+, and $V_{\max}$ and $V_{\min}$ (the maximum and minimum voltages on the transmission line).

   d. Show that the complete expression for $v(z,t)$ in the transmission line may be written

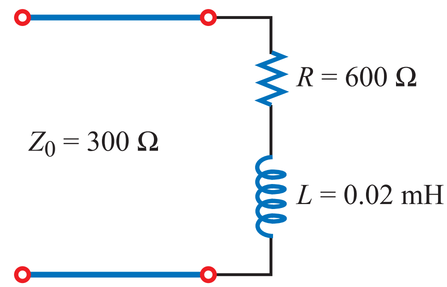
$$A\cos(3\pi \times 10^9 t - 20\pi z + \phi_1) + B\cos(3\pi \times 10^9 + 20\pi z + \phi_2),$$

   and find A, B, ϕ_1, and ϕ_2.

   e. You can study this transmission line using Ulaby et al.'s 2010 module 2.4. Substitute the appropriate parameters into this module and use it to verify that you have obtained the correct values for S, A, ϕ_1, $l_{\max}$, and $l_{\min}$. Provide a screen printout that includes "Transmission Line Data 2."

   f. Derive the expression for $v(z,t)$ on slide 4.14.

4. A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in the figure to the right [Ulaby et al., Figure P2.20]. At 5 MHz, determine: (a) Γ, (b) S, (c) location of the voltage maximum closest to the load, and (d) location of the current maximum closest to the load. Use Ulaby et al. module 2-5 to verify your results and give a screen printout [modified from Ulaby et al. 2.20, p. 123].



5. At an operating frequency of 300 MHz, it is desired to use a short section of a lossless 50-Ω transmission line terminated in a short circuit to construct an equivalent load with a reactance $X = 40\ \Omega$. If the phase velocity of the line $0.75c$, what is the shortest possible line length that would exhibit the desired reactance at its input? Verify your results using Ulaby et al. module 2.5 and give a screen printout. [Ulaby et al. 2.36.]

6. Prove that the time-averaged power incident on a load is given by $(1/2)\,\mathrm{Re}(\tilde{V}_{\mathrm{i}}\tilde{I}_{\mathrm{i}}^*)$ and that similarly the time-averaged reflected power is given by $(1/2)\,\mathrm{Re}(\tilde{V}_{\mathrm{r}}\tilde{I}_{\mathrm{r}}^*)$. [Hint: It is easiest to solve this problem if you use the complex number identity, $\mathrm{Re}(A) = (1/2)(A + A^*)$.]

7. A generator $\tilde{V}_{\mathrm{g}} = 100$ V and $Z_{\mathrm{g}} = 50\ \Omega$ is connected to a load $Z_{\mathrm{L}} = 75\ \Omega$ through a $50\ \Omega$ lossless line of length $l = 0.15\lambda$. [modified from Ulaby 2.31, p. 104.]

   a. Compute Z_{in}, the input impedance of the line at the generator end.

   b. Compute $\tilde{I}_i$ and $\tilde{V}_i$.

   c. Compute the time-average power delivered to the line, $P_{\mathrm{in}} = (1/2)\,\mathrm{Re}(\tilde{V}_i\tilde{I}_i^*)$.

   d. Compute $\tilde{V}_{\mathrm{L}}$, $\tilde{I}_{\mathrm{L}}$, and the time average power delivered to the load, $P_{\mathrm{L}} = (1/2)\,\mathrm{Re}(\tilde{V}_L\tilde{I}_L^*)$. How does P_{in} compare to P_{L}? Explain.

   e. Compute the time-average power delivered by the generator, P_{g}, and the time-average power dissipated in Z_{g}. Is conservation of power satisfied?