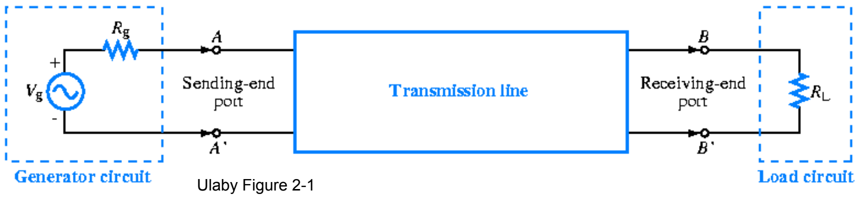
What is a transmission line?

The basic structure — a four port device that connects:

(1) A Thénevin equivalent input or *generator circuit* with V_g and R_g, as the equivalent voltage source and resistance

(2) A *load circuit* with a load resistance R_L as the equivalent resistance



Ulaby Figure 2-1

More generally, with a-c inputs:

V_g and $R_g \rightarrow \tilde{V}_g$ and Z_g

$R_L \rightarrow Z_L$

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3.1

What is a transmission line?

The basic structure — a four port device that connects:

(1) A Thénevin equivalent input or *generator circuit* with V_g and R_g, as the equivalent voltage source and resistance

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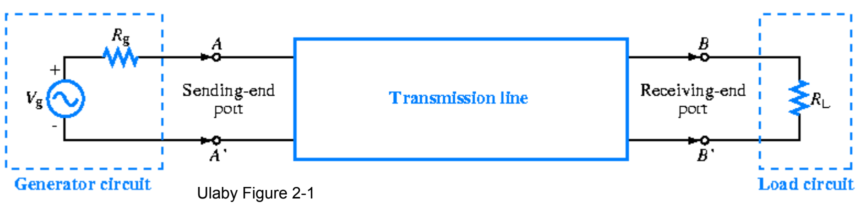

Ulaby Figure 2-1

We take into account the finite transmission time from A, A' to B, B'

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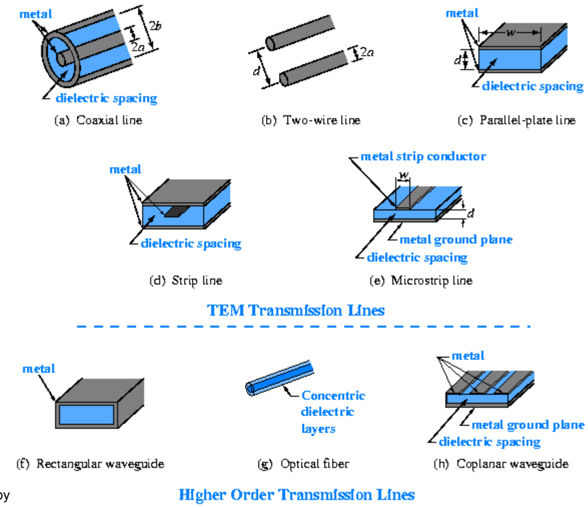
3.2

What is a transmission line?

There are two basic types of transmission line:



Ulaby
Figure 2-4

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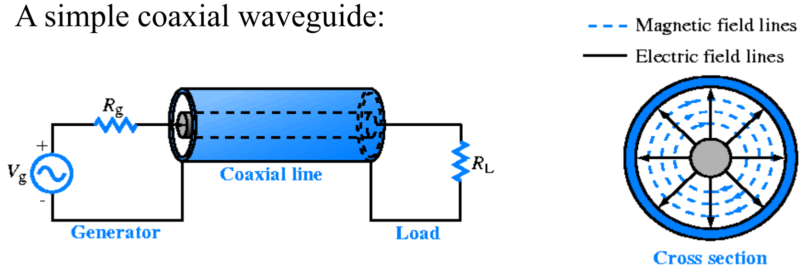
3.3

What is a transmission line?

A simple coaxial waveguide:



Ulaby Figure 2-5

- There is an electric field caused by charge separation from the inner and outer wires
- There is a magnetic field that is caused by the current flow, in opposite directions, along the inner and outer conductions

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3.4

Lumped Element Model

We replace the detailed physics of the transmission line with lumped circuit parameters:

R' : The combined resistance of both conductors (ohms/meter)

L' : The combined inductance of both conductors (henrys/meter)

G' : The conductance of the insulating medium between conductors (siemens/meter)

C' : The capacitance between the two conductors (farads/meter)

- These parameters are all "per unit length." Hence, we put primes.
- Their values are determined by the detailed physics.
- We will use this model to find a simple pair of coupled equations, ***The telegrapher's equation*** that describes propagation through the transmission line

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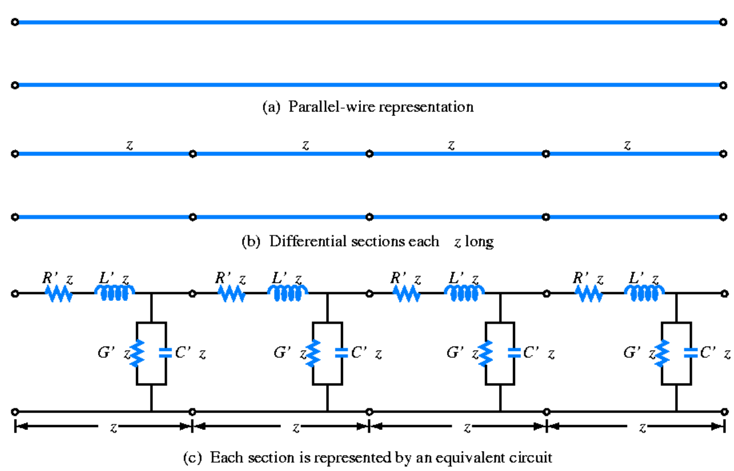
3.5

Lumped Element Model

Transmission Line Model:



Ulaby Figure 2-6

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3.6

Lumped Element Model

Examples of parameter determination from the physics:

Parameter	Coaxial	Two wire	Parallel plane
R'	$\dfrac{R_S}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{2R_S}{\pi d}$	$\dfrac{2R_S}{w}$
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\dfrac{\mu h}{w}$
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\sigma w}{h}$
C'	$\dfrac{2\pi\varepsilon}{\ln(b/a)}$	$\dfrac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\varepsilon w}{h}$

Ulaby Table 2-1 Slide 3.3 and the notes have definitions of a, b, d

R_S = surface resistance = $(\pi f \mu_c/\sigma_c)^{1/2}$; μ_c, σ_c are conductor parameters

μ, ε, and σ are insulator parameters

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3.7

Microstrip Line

Real transmission lines can be complex

$\Rightarrow$ Curve fitting approximations are useful

$s = w/h$

w = strip width

h = substrate thickness

$$\varepsilon_{\mathrm{eff}} = \frac{\varepsilon_r+1}{2}+\left(\frac{\varepsilon_r-1}{2}\right)\left(1+\frac{10}{s}\right)^{xy}$$

$$x = 0.56\left(\frac{\varepsilon_r-0.9}{\varepsilon_r+3}\right)^{0.05}$$

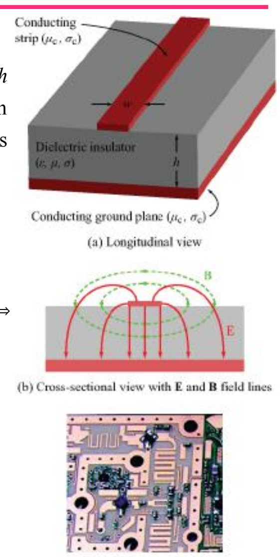
$$y = 1+0.02\ln\left(\frac{s^4+3.7\times10^{-4}s^2}{s^4+0.43}\right)+0.05\ln(1+1.7\times10^{-4}s^3)$$

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\mathrm{eff}}}}\ln\left[\frac{6+(2\pi-6)\exp(-t)}{s}+\left(1+\frac{4}{s^2}\right)^{1/2}\right];\quad t=\left(\frac{30.67}{s}\right)^{0.75}$$

$\Rightarrow$

$$R' = 0;\quad G' = 0$$

$$C' = \sqrt{\varepsilon_{\mathrm{eff}}}/Z_0c;\quad L' = Z_0\sqrt{\varepsilon_{\mathrm{eff}}}/c$$



(a) Longitudinal view

(b) Cross-sectional view with E and B field lines

(c) Microwave circuit

Ulaby, et al. 2010 Figure 2-10

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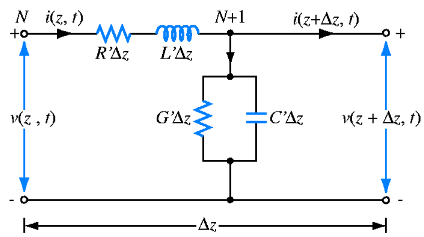
3.8

Transmission Line Equations

Our starting point is one section of the transmission line



Ulaby Figure 2-8

Note: We are taking a section that is small enough so that we can assume that the transmission through that section is instantaneous

Kirchoff's voltage law: $$v(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

Kirchoff's current law: (at node $N+1$)

$$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

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3.9

Transmission Line Equations

Rearranging terms

$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = R'\, i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = G'\, v(z+\Delta z,t) + C' \frac{\partial v(z+\Delta z,t)}{\partial t}$$

Take the limit as $\Delta z \longrightarrow 0$:

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Telegrapher's Equations

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3.10

Transmission Line Equations

Simplification

$$R' = (f\mu_c/\sigma_c)^{1/2} \to 0 \quad \text{(low frequency and high conductivity conductors)}$$

$$G' \to 0 \quad \text{(low conductivity dielectric material)}$$

The transmission equations become

$$-\frac{\partial v(z,t)}{\partial z} = L'\frac{\partial i(z,t)}{\partial t}, \quad -\frac{\partial i(z,t)}{\partial z} = C'\frac{\partial v(z,t)}{\partial t}$$

which imply

$$\frac{\partial^2 v(z,t)}{\partial z^2} = L'C'\frac{\partial^2 v(z,t)}{\partial t^2}, \quad \frac{\partial^2 i(z,t)}{\partial z^2} = L'C'\frac{\partial^2 i(z,t)}{\partial t^2}$$

Note: In Paul's notation: $v \to V,\ i \to I,\ L' \to l,$ and $C' \to c$

$$\frac{\partial V(z,t)}{\partial z} = -l\frac{\partial I(z,t)}{\partial t}, \quad \frac{\partial I(z,t)}{\partial z} = -c\frac{\partial V(z,t)}{\partial t}$$

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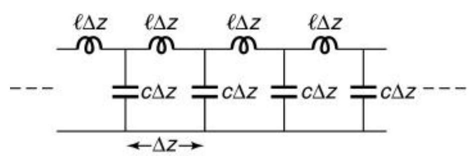
3.11

Transmission Line Equations

The circuit diagram also simplifies

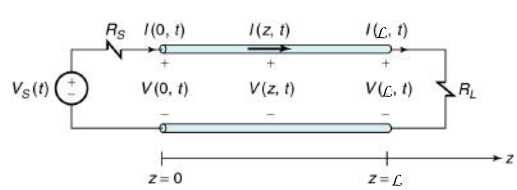


Paul Figure 6.1(c)

Note change in notation from Ulaby

The second-order wave equations become in Paul's notation

$$\frac{\partial^2 V(z,t)}{\partial z^2} = lc\frac{\partial^2 V(z,t)}{\partial t^2}, \quad \frac{\partial^2 I(z,t)}{\partial z^2} = lc\frac{\partial^2 I(z,t)}{\partial t^2}$$



V_S = source voltage
R_S = source resistance
R_L = load resistance
$\mathcal{L}$ = total length of transmission line

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3.12

Time-Domain Evolution

The second-order voltage equation has the general solution

$$V(z,t) = \underbrace{V^+\left(t - \frac{z}{v}\right)}_{\substack{\text{forward}\\\text{propagation}}} + \underbrace{V^-\left(t + \frac{z}{v}\right)}_{\substack{\text{backward}\\\text{propagation}}}, \quad \text{with} \quad v = 1/\sqrt{lc}$$

- $V^+(x)$ and $V^-(x)$ = two completely arbitrary functions
- v = the velocity of propagation

(1) Arbitrarily shaped pulses propagate in the transmission line, both forward and backward, *without dispersing!*

(2) The absence of dispersion is a special feature of TEM modes, which makes direct time-domain analysis possible.

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Caveat: We are treating l and c as if they have no frequency dependence.
This assumption is not generally true!
It will often be approximately true over a limited frequency range

3.13

Time-Domain Evolution

The current is determined from the voltage

— After substitution into the transmission line equations

$$I(z,t) = \underbrace{\frac{V^+\left(t - \frac{z}{v}\right)}{Z_C}}_{\substack{\text{forward}\\\text{propagation}}} - \underbrace{\frac{V^-\left(t + \frac{z}{v}\right)}{Z_C}}_{\substack{\text{backward}\\\text{propagation}}}, \quad \text{with} \quad Z_C = \sqrt{l/c}$$

- Z_C = the characteristic impedance of the transmission line
- Note that since $V(z, t)$ and $I(z, t)$ are real, so is Z_C
  — The characteristic impedance is purely resistive in this case.

We have $I^+(z,t) = V^+(z,t)/Z_C$ and $I^-(z,t) = -V^-(z,t)/Z_C$

BUT $|I(z,t)| \neq |V(z,t)|/Z_C$

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There is no simple relationship between V(z,t) and I(z,t)!

3.14

Time-Domain Evolution

Example: From Paul Quick Review Exercises 6.1 and 6.7

Question: What are the per unit length capacitance and inductance of a two-wire line whose wires have a radius of 7.5 mils and a separation of 50 mils? What is the characteristic impedance and the velocity of propagation? (These dimensions are typical for ribbon cables used to interconnect components.)

Answer: 7.5 mils × 2.54×10^{-5} m/mil = 1.91×10^{-4} m, 50 mils = 1.27×10^{-3} m. $(D/d) = 50.0/15.0 = 3.33$, $[(D/d)^2 - 1] = 3.18$, $\ln(3.33 + 3.18) = 1.87$. $L' = (\mu_0/\pi) \times 1.87 = 4\times10^{-7}$ H/m × 1.87 = 7.46×10^{-7} H/m. $C' = (\pi\varepsilon_0 / 1.87) = (\pi/1.87) \times 8.85\times10^{-12}$ F/m = 1.49×10^{-11} F/m. $Z_0 = (L'/C')^{1/2} = 224$ ohms. $u_p = 1 / (7.46\times10^{-7}$ H/m × 1.49×10^{-11} F/m$)^{1/2} = 3.00\times10^8$ m/s. The velocity equals the speed of light in the vacuum.

NOTE: Paul uses a different notation for all quantities

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3.15

Time-Domain Evolution

We now consider the behavior at the load

$$V(\mathcal{L},t) = V^+(t-T) + V^-(t+T), \quad T = \mathcal{L}/v$$

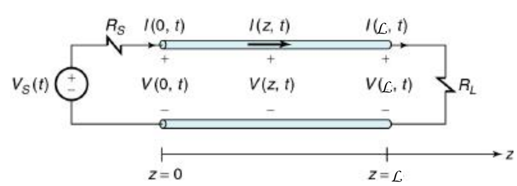
= one-way time delay

$$I(\mathcal{L},t) = \frac{V^+(t-T)}{Z_C} - \frac{V^-(t+T)}{Z_C}$$

From Ohm's Law: $V(\mathcal{L},t) / I(\mathcal{L},t) = R_L$

From the telegrapher's equations: $V^+(t-T) / I^+(t-T) = Z_C$

Unless $R_L = Z_C$, there must be reflected waves!



V_S = source voltage
R_S = source resistance
R_L = load resistance
$\mathcal{L}$ = total length of transmission line

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3.16

Time-Domain Evolution

We now consider the behavior at the load

$$V(\mathcal{L},t) = V^{+}\left(t-T\right) + V^{-}\left(t+T\right), \quad T = \mathcal{L}/v$$

= one-way time delay

$$I(\mathcal{L},t) = \frac{V^{+}\left(t-T\right)}{Z_C} - \frac{V^{-}\left(t+T\right)}{Z_C}$$

From Ohm's Law: $V(\mathcal{L},t)\,/\,I(\mathcal{L},t) = R_L$
From the telegrapher's equations: $V^{+}(t-T)\,/\,I^{+}(t-T) = Z_C$

Unless $R_L = Z_C$, there must be reflected waves!

When $R_L = Z_C$, we say that the line is ***impedance matched***

The concept of impedance matching is one of the most important in this course.

Impedance matching is never perfect, giving rise to transients.

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3.17

Time-Domain Evolution

Behavior at the load: Reflection Coefficient Γ_L

$$\Gamma_L = V^{-}(t+T)\,/\,V^{+}(t-T)$$

which implies

$$V(\mathcal{L},t) = V^{+}\left(t-T\right)\left[1+\Gamma_L\right], \quad I(\mathcal{L},t) = \frac{V^{+}\left(t-T\right)}{Z_C}\left[1-\Gamma_L\right]$$

from which

$$\frac{V(\mathcal{L},t)}{I(\mathcal{L},t)} = R_L = Z_C\frac{1+\Gamma_L}{1-\Gamma_L} \quad \text{and} \quad \Gamma_L = \frac{R_L - Z_C}{R_L + Z_C}$$

- Schematically, we may understand the behavior at the load as a combination of a forward-propagating wave and a backward-going wave that is a reflected copy of the first wave, multiplied by Γ_L.

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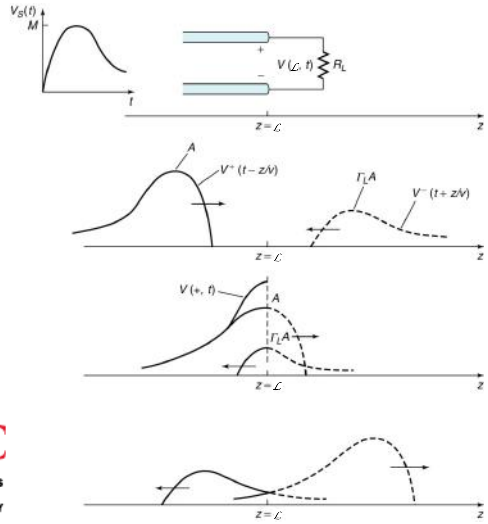
3.18

Time-Domain Evolution

Behavior at the load



Paul Figure 6-9
Illustration of the reflection of voltage waves at the load of a transmission line.

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3.19

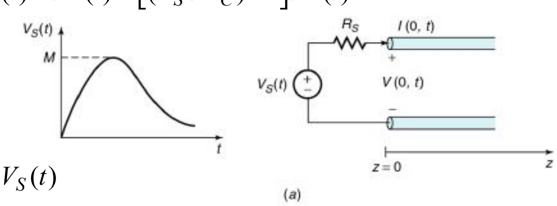
Time-Domain Evolution

Behavior at the source $[0 < t < 2T]$

$$V_S(t) = 0 \ \text{ when } \ t < 0 \quad \Rightarrow \quad V^-(0,t) = 0 \ \text{ when } \ 0 < t < 2T$$

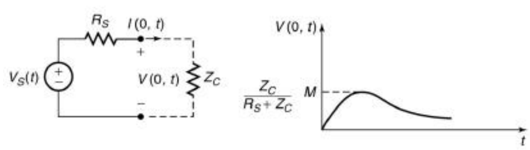
From Kirchoff's voltage law:

$$V_S(t) = R_S I^+(t) + V^+(t) = \left[(R_S / Z_C) + 1\right] V^+(t)$$



$$V(0,t) = V^+(t) = \frac{Z_C}{R_S + Z_C} V_S(t)$$

$$I(0,t) = \frac{V_S(t)}{R_S + Z_C}$$



(b) $t < 2\frac{\mathcal{L}}{v}$ Paul Figure 6-10

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3.20

Time-Domain Evolution

Behavior at the source $[2T < t < 4T;\ V^{-}(t) \neq 0]$

The wave reflected from the load returns to the source where it reflects again, with a coefficient

$$\Gamma_S = \frac{R_S - Z_C}{R_S + Z_C}; \quad V^{+}(t) = \frac{Z_C}{R_S + Z_C}\left[V_S(t) + \Gamma_S \Gamma_L V_S(t - 2T)\right]$$

$$V^{-}(t) = \Gamma_L V_S(t - 2T)$$

We thus find

$$V(0,t) = \frac{Z_C}{R_S + Z_C}\left[V_S(t) + (1 + \Gamma_S)\Gamma_L V_S(t - 2T)\right]$$

This process of bouncing back and forth continues indefinitely

— illustrated in a bounce diagram

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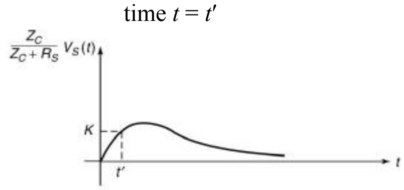
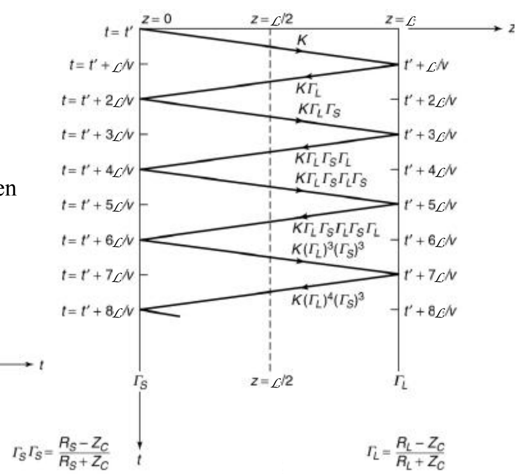
Time-Domain Evolution

Bounce Diagram

K is the voltage at a given time $t = t'$





Paul Figure 6-11

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Time-Domain Evolution

Time evolution

$$V(0,t)=\frac{Z_C}{R_S+Z_C}\Big[V_S(t)+(1+\Gamma_S)\Gamma_L V_S(t-2T)$$
$$+(1+\Gamma_S)(\Gamma_S\Gamma_L)\Gamma_L V_S(t-4T)+(1+\Gamma_S)(\Gamma_S\Gamma_L)^2\Gamma_L V_S(t-6T)+\cdots\Big]$$

$$V(\mathcal{L},t)=\frac{Z_C}{R_S+Z_C}\Big[(1+\Gamma_L)V_S(t-T)+(1+\Gamma_L)(\Gamma_S\Gamma_L)V_S(t-3T)$$
$$+(1+\Gamma_L)(\Gamma_S\Gamma_L)^2 V_S(t-5T)+(1+\Gamma_L)(\Gamma_S\Gamma_L)^3 V_S(t-7T)+\cdots\Big]$$

NOTE: This series continues forever **IF**
we define $V_S(t)=0$ when $t<0$

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3.23

Time-Domain Evolution

Geometric series:

$$1+x+x^2+\cdots+x^m=\sum_{i=0}^{m}x^i=\frac{1-x^{m+1}}{1-x}$$

Summing all the bounces, we find

$$V(0,t)=\frac{V_S Z_C}{R_S+Z_C}\left[\frac{1-(\Gamma_S\Gamma_L)^{m_S+1}+\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_S}]}{1-\Gamma_S\Gamma_L}\right]$$

$$I(0,t)=\frac{V_S}{R_S+Z_C}\left[\frac{1-(\Gamma_S\Gamma_L)^{m_S+1}-\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_S}]}{1-\Gamma_S\Gamma_L}\right]$$

$$V(\mathcal{L},t)=\frac{V_S Z_C}{R_S+Z_C}\left[\frac{(1+\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_L}]}{1-\Gamma_S\Gamma_L}\right]$$

$$I(\mathcal{L},t)=\frac{V_S}{R_S+Z_C}\left[\frac{(1-\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_L}]}{1-\Gamma_S\Gamma_L}\right]$$

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$$m_S=\lfloor t/2T\rfloor,\quad m_L=\lfloor (t+T)/2T\rfloor$$

3.24

Time-Domain Evolution

In Ulaby et al.'s notation, we find

$$V(0,t) = \frac{V_g Z_0}{R_g + Z_0}\left[\frac{1-(\Gamma_g\Gamma_L)^{m_g+1} + \Gamma_L[1-(\Gamma_g\Gamma_L)^{m_g}]}{1-\Gamma_g\Gamma_L}\right]$$

$$I(0,t) = \frac{V_g}{R_g + Z_0}\left[\frac{1-(\Gamma_g\Gamma_L)^{m_g+1} - \Gamma_L[1-(\Gamma_g\Gamma_L)^{m_g}]}{1-\Gamma_g\Gamma_L}\right]$$

$$V(l,t) = \frac{V_g Z_0}{R_g + Z_0}\left[\frac{(1+\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_L}]}{1-\Gamma_g\Gamma_L}\right]$$

$$I(l,t) = \frac{V_g}{R_g + Z_0}\left[\frac{(1-\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_L}]}{1-\Gamma_g\Gamma_L}\right]$$

$$m_g = \lfloor t/2T \rfloor, \quad m_L = \lfloor (t+T)/2T \rfloor$$

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3.25

Time-Domain Evolution

When m_S and m_L become infinite, these become:

$$V(0,t) = \frac{V_S Z_C}{R_S + Z_C}\left[\frac{1+\Gamma_L}{1-\Gamma_S\Gamma_L}\right], \quad I(0,t) = \frac{V_S}{R_S + Z_C}\left[\frac{1-\Gamma_L}{1-\Gamma_S\Gamma_L}\right]$$

$$V(\angle,t) = \frac{V_S Z_C}{R_S + Z_C}\left[\frac{1+\Gamma_L}{1-\Gamma_S\Gamma_L}\right], \quad I(\angle,t) = \frac{V_S}{R_S + Z_C}\left[\frac{1-\Gamma_L}{1-\Gamma_S\Gamma_L}\right]$$

which is the same at both the source and the load

In Ulaby et al.'s notation, we have

$$V(0,t) = V(l,t) = \frac{V_g Z_0}{R_g + Z_0}\left[\frac{1+\Gamma_L}{1-\Gamma_g\Gamma_L}\right],$$

$$I(0,t) = I(l,t) = \frac{V_g}{R_g + Z_0}\left[\frac{1-\Gamma_L}{1-\Gamma_g\Gamma_L}\right]$$

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3.26

Time-Domain Evolution

Example (Paul 6.1):

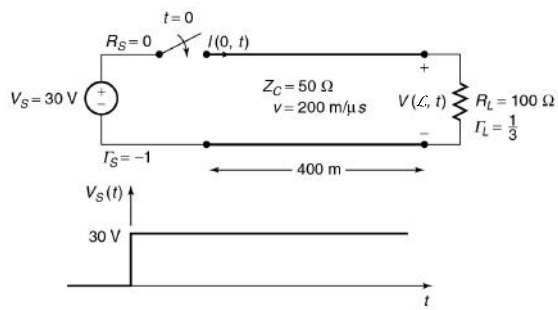
Question: A thirty volt battery is switched onto a line of length 400 m. The line has a characteristic impedance of 50 Ω and a propagation velocity of 2.00×10^8 m/s. The source resistance is zero, and the load resistance is 100 Ω. What is the current at the input and the voltage at the load as a function of time?

Answer: System Parameters:

$$\Gamma_S = \frac{0-50}{0+50} = -1$$

$$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$$

$$T = \frac{\mathcal{L}}{v} = 2\,\mu\text{s}$$



Paul Figure 6-12(a)

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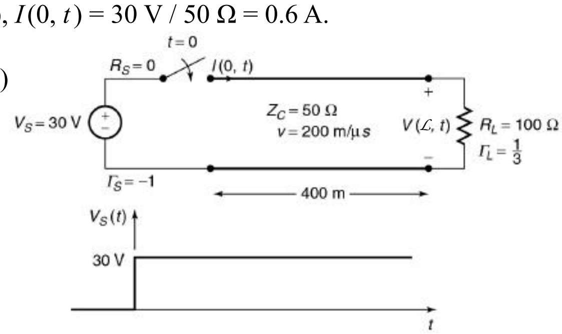
3.27

Time-Domain Evolution

Example (Paul 6.1):

Answer (continued): For $t = (0\,\mu\text{s}, 2\,\mu\text{s})$, $V(\mathcal{L}, t) = 0$. For $t = (2\,\mu\text{s}, 6\,\mu\text{s})$, $V(\mathcal{L}, t)$ = (incoming voltage) + (reflected voltage) = 30 V + (1/3)×30 V = 40 V. Since the reflected voltage is 10 V, we find that for $t = (4\,\mu\text{s}, 8\,\mu\text{s})$, $V^+(t)$ = 30 V – 10 V = 20 V, and, for $t = (6\,\mu\text{s}, 10\,\mu\text{s})$, $V(\mathcal{L}, t)$ = (4/3)×20 V = 26.67 V. And so on…

For $t = (0\,\mu\text{s}, 4\,\mu\text{s})$, $I(0, t)$ = 30 V / 50 Ω = 0.6 A.
For $t = (4\,\mu\text{s}, 8\,\mu\text{s})$,
$I(0, t)$ = (20 V – 10 V) / 50 Ω = 0.2 A.
And so on…



Paul Figure 6-12(a)

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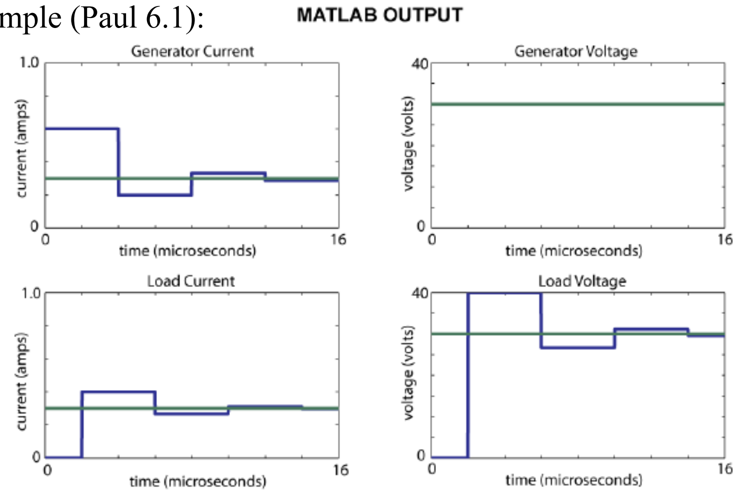
3.28

Time-Domain Evolution

Example (Paul 6.1):



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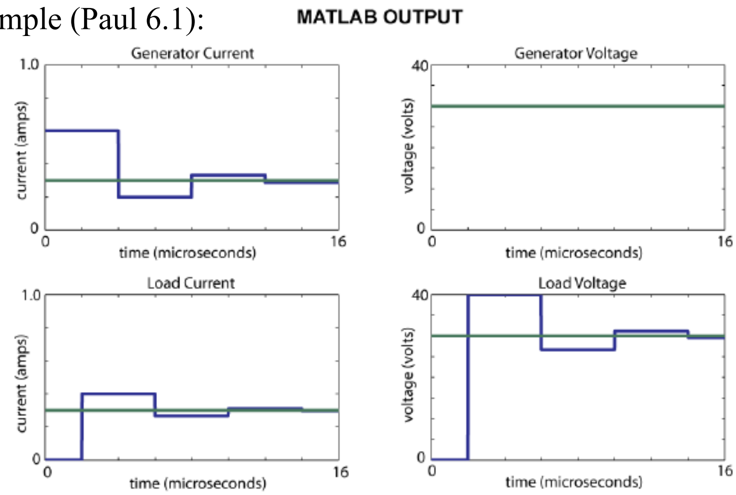
Key Point: Without good impedance matching, transients can exist for exist for a long time, leading to poor system performance!

3.29

Time-Domain Evolution

Example (Paul 6.1):



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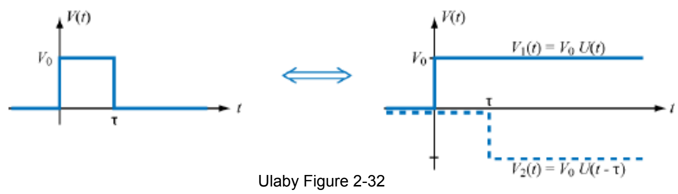
But transients can be useful for TDR in a matched line

3.30

Time-Domain Evolution

When a pulse is present, superpose two step functions



(a) Pulse of duration τ

Ulaby Figure 2-32

(b) $V(t) = V_1(t) + V_2(t)$

3.31

Time-Domain Evolution

Example (Paul 6.2):

Question: Consider a line of length 0.2 m. The source voltage is a pulse of 20 V amplitude and 1 ns duration t_D. The line has a characteristic impedance of 100 Ω and a velocity of propagation of 2 × 10^8 m/s. The source resistance is 300 Ω and the load is open-circuited. Find the source and generator currents and voltages.

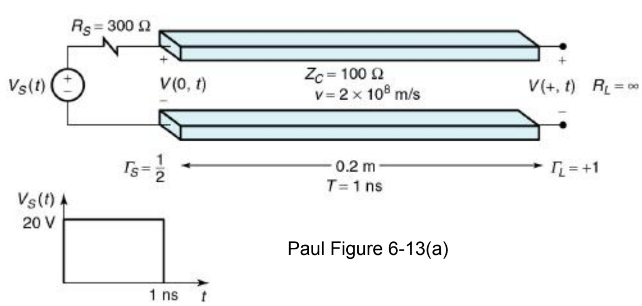
Answer: System Parameters:

$$\Gamma_S = \frac{300-100}{300+100} = \frac{1}{2}$$

$$\Gamma_L = \frac{\infty-100}{\infty+100} = 1$$

$$T = \frac{\mathcal{L}}{v} = 1\,\text{ns};$$

$$t_D = 1\,\text{ns}$$



Paul Figure 6-13(a)

3.32

Time-Domain Evolution

Example (Paul 6.2):

Answer (continued): We use the formulae on slide 3.24, but we must subtract a negative contribution for the back end of the pulse

$$V(0,t)=\frac{V_S Z_C}{R_S+Z_C}\left[\frac{1-(\Gamma_S\Gamma_L)^{m_{S1}+1}+\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_{S1}}]}{1-\Gamma_S\Gamma_L}-\frac{1-(\Gamma_S\Gamma_L)^{m_{S2}+1}+\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_{S2}}]}{1-\Gamma_S\Gamma_L}\right]$$

$$I(0,t)=\frac{V_S}{R_S+Z_C}\left[\frac{1-(\Gamma_S\Gamma_L)^{m_{S1}+1}-\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_{S1}}]}{1-\Gamma_S\Gamma_L}-\frac{1-(\Gamma_S\Gamma_L)^{m_{S2}+1}-\Gamma_L[1-(\Gamma_S\Gamma_L)^{m_{S2}}]}{1-\Gamma_S\Gamma_L}\right]$$

$$V(\mathcal{L},t)=\frac{V_S Z_C}{R_S+Z_C}\left[\frac{(1+\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_{L1}}]}{1-\Gamma_S\Gamma_L}-\frac{(1+\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_{L2}}]}{1-\Gamma_S\Gamma_L}\right]$$

$$I(\mathcal{L},t)=\frac{V_S}{R_S+Z_C}\left[\frac{(1-\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_{L1}}]}{1-\Gamma_S\Gamma_L}-\frac{(1-\Gamma_L)[1-(\Gamma_S\Gamma_L)^{m_{L2}}]}{1-\Gamma_S\Gamma_L}\right]$$

$$m_{g1}=\lfloor t/2T\rfloor,\quad m_{g2}=\lfloor (t-t_D)/2T\rfloor,$$
$$m_{L1}=\lfloor (t+T)/2T\rfloor,\quad m_{L2}=\lfloor (t+T-t_D)/2T\rfloor,\quad t>t_D$$

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You only subtract for $t > t_D$

3.33

Time-Domain Evolution

Example (Paul 6.2):

Answer (continued): In Ulaby's notation, this is

$$V(0,t)=\frac{V_g Z_0}{R_g+Z_0}\left[\frac{1-(\Gamma_g\Gamma_L)^{m_{g1}+1}+\Gamma_L[1-(\Gamma_g\Gamma_L)^{m_{g1}}]}{1-\Gamma_g\Gamma_L}-\frac{1-(\Gamma_g\Gamma_L)^{m_{g2}+1}+\Gamma_L[1-(\Gamma_g\Gamma_L)^{m_{g2}}]}{1-\Gamma_g\Gamma_L}\right]$$

$$I(0,t)=\frac{V_g}{R_g+Z_0}\left[\frac{1-(\Gamma_g\Gamma_L)^{m_{g1}+1}-\Gamma_L[1-(\Gamma_g\Gamma_L)^{m_{g1}}]}{1-\Gamma_g\Gamma_L}-\frac{1-(\Gamma_g\Gamma_L)^{m_{g2}+1}-\Gamma_L[1-(\Gamma_g\Gamma_L)^{m_{g2}}]}{1-\Gamma_g\Gamma_L}\right]$$

$$V(l,t)=\frac{V_g Z_0}{R_g+Z_0}\left[\frac{(1+\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_{L1}}]}{1-\Gamma_g\Gamma_L}-\frac{(1+\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_{L2}}]}{1-\Gamma_g\Gamma_L}\right]$$

$$I(l,t)=\frac{V_g}{R_g+Z_0}\left[\frac{(1-\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_{L1}}]}{1-\Gamma_g\Gamma_L}-\frac{(1-\Gamma_L)[1-(\Gamma_g\Gamma_L)^{m_{L2}}]}{1-\Gamma_g\Gamma_L}\right]$$

$$m_{S1}=\lfloor t/2T\rfloor,\quad m_{S2}=\lfloor (t-t_D)/2T\rfloor,$$
$$m_{L1}=\lfloor (t+T)/2T\rfloor,\quad m_{L2}=\lfloor (t+T-t_D)/2T\rfloor,\quad t>t_D$$

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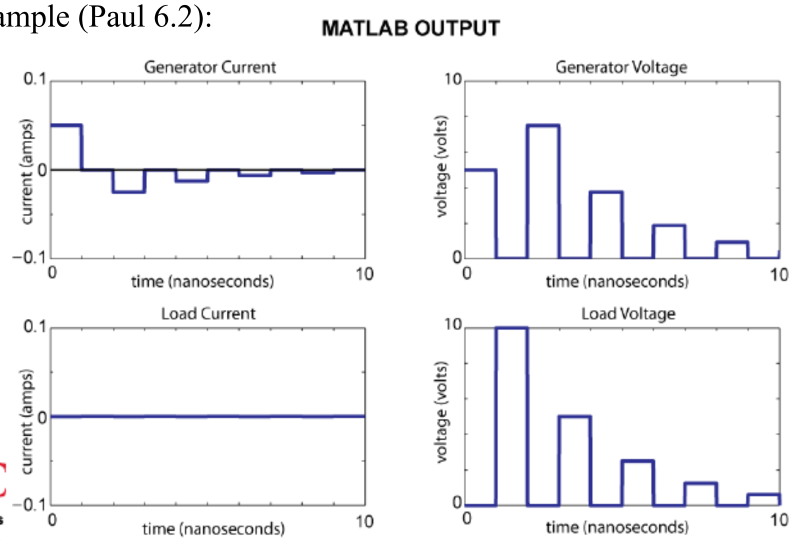
You only subtract for $t > t_D$

3.34

Time-Domain Evolution

Example (Paul 6.2):



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3.35

Assignment

Problem Set 2: Some notes

- There are 6 problems. Many of the answers to these problems have been provided by either Ulaby and Ravaioli or by me. YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- Watch significant figures. Report the number that I ask for.
- These problems are difficult. GET STARTED EARLY!

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3.36