Phasor-Domain Equations

Transmission line equations — time domain:

$$-\frac{\partial v(z,t)}{\partial z} = R'\,i(z,t) + L'\,\frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G'\,v(z,t) + C'\,\frac{\partial v(z,t)}{\partial t}$$

Time domain $\rightarrow$ Phasor domain

$$v(z,t) = \mathrm{Re}\left[\tilde{V}(z)\exp(j\omega t)\right], \quad i(z,t) = \mathrm{Re}\left[\tilde{I}(z)\exp(j\omega t)\right]$$

Transmission line equations — phasor domain:

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

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4.1

Phasor-Domain Equations

Advantages of Phasor-Domain Representation

- Reduction from partial differential equation to ordinary differential equation
  — so that it is easier to solve
- Allows generalization: $L' \rightarrow L'(\omega)$, $C' \rightarrow C'(\omega)$, $R' \rightarrow R'(\omega)$, $G' \rightarrow G'(\omega)$

Advantages of Time-Domain Representation

- Allows study of transients

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4.2

Phasor-Domain Equations

Second-Order Equations:

$$\frac{d^2\tilde{V}(z)}{dz^2}-\gamma^2\tilde{V}(z)=0,\quad \frac{d^2\tilde{I}(z)}{dz^2}-\gamma^2\tilde{I}(z)=0$$

with

$$\gamma=\alpha+j\beta=\sqrt{(R'+j\omega L')(G'+j\omega C')}$$

γ = complex propagation constant

α = $\mathrm{Re}(\gamma)$ = attenuation constant (Np/m)

β = $\mathrm{Im}(\gamma)$ = phase constant or wavenumber (rad/m)

NOTES:
- We pick α and β so that both are positive
- In a passive medium, α is always positive; it can be negative in an active medium

  [An active medium, like a laser, has an energy source; a passive medium does not and must always lose energy.]

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4.3

Phasor-Domain Equations

The second-order equations have the general solutions

$$\tilde{V}(z)=V_0^+\exp(-\gamma z)+V_0^-\exp(\gamma z),\quad \tilde{I}(z)=I_0^+\exp(-\gamma z)+I_0^-\exp(\gamma z)$$

- $\tilde{V}(z)$ and $\tilde{I}(z)$ are not independent
- V_0^+ and V_0^- are arbitrary constants, while I_0^+ and I_0^- are not
- We may also write:

$$\tilde{I}(z)=\frac{V_0^+}{Z_0}\exp(-\gamma z)-\frac{V_0^-}{Z_0}\exp(\gamma z)$$

where

$$Z_0=\sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$$

is the characteristic impedance

NOTE: Z_0 (Ulaby et al.'s notation) $\rightarrow Z_C$ (Paul's notation)

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4.4

Phasor-Domain Equations

Returning to the time domain:

We first write

$$V_0^+ = |V_0^+|\exp(j\phi^+), \quad V_0^- = |V_0^-|\exp(j\phi^-)$$

from which we obtain

$$\begin{aligned} v(z,t) &= \mathrm{Re}\left[\tilde{V}(z)\exp(j\omega t)\right] \\ &= \mathrm{Re}\left\{\left[V_0^+\exp(-\gamma z) + V_0^-\exp(\gamma z)\right]\exp(j\omega t)\right\} \\ &= \mathrm{Re}\left\{|V_0^+|\exp(j\phi^+)\exp(j\omega t)\exp[-(\alpha+j\beta)z]\right. \\ &\qquad \left. + |V_0^-|\exp(j\phi^-)\exp(j\omega t)\exp[(\alpha+j\beta)z]\right\} \\ &= \underbrace{|V_0^+|\exp(-\alpha z)\cos(\omega t - \beta z + \phi^+)}_{\text{forward propagation with attenuation}} + \underbrace{|V_0^-|\exp(\alpha z)\cos(\omega t + \beta z + \phi^-)}_{\text{backward propagation with attenuation}} \end{aligned}$$

Phase velocity $= u_\mathrm{p} = \omega/\beta$

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4.5

Phasor-Domain Equations

Example: Ulaby et al. Exercise 2.4

Question: A two-wire air line has the following parameters: $R' = 0.404$ mΩ/m, $L' = 2.00$ μH/m, $G' = 0$, $C' = 5.56$ pF/m. For operation at 5 kHz, determine (a) the attenuation coefficient α, (b) the wavenumber β, (c) the phase velocity u_p, and the characteristic impedance Z_0.

Answer: $\omega = 2 \times 3.14159 \times (5\times10^3\ \mathrm{s}^{-1}) = 3.14159\times10^4\ \mathrm{s}^{-1}$. $R' + j\omega L' = (4.04000\times10^{-4} + j\times3.14159\times10^4 \times 2.00000\times10^{-6})\ \Omega/\mathrm{m} = (4.04000\times10^{-4} + j\times6.28318\times10^{-2})\ \Omega/\mathrm{m} = 6.28319\times10^{-2} \times \exp(j\times1.56436)\ \Omega/\mathrm{m}$. $G' + j\omega C' = j\times1.74673\times10^{-7}\ \Omega^{-1}/\mathrm{m} = 1.74673\times10^{-7} \times \exp(j\times1.57080)\ \Omega^{-1}/\mathrm{m}$. Note the small difference in phases! Six digits of accuracy are needed to keep three digits in the attenuation coefficient. $\gamma^2 = 1.09750\times10^{-8} \times \exp(j\times3.13156)\ \mathrm{m}^{-2}$, so that $\gamma = 1.04762\times10^{-4} \times \exp(j\times1.56758)\ \mathrm{m}^{-1} = 3.37\times10^{-7} + j\times1.04761\times10^{-4}$, so that

$\alpha = 3.37\times10^{-7}$ Np/m and $\beta = 1.05\times10^{-4}$ rad/m.

We have $u_\mathrm{p} = \omega/\beta = (3.142/1.048)\times10^8 = 3.00\times10^8$ m/s

and $Z_0 = \{(6.283\times10^{-2} / 1.747\times10^{-7})\exp[j\times(1.56436 - 1.57080)]\}^{1/2}$

$= (600 - j\times1.93)\ \Omega$

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4.6

Lossless Transmission Line

Specializing to the case of no loss ($R' = 0$, $G' = 0$):

$$\alpha = 0,\ \beta = \omega\sqrt{L'C'},\ Z_0 = \sqrt{L'/C'},\ u_{\mathrm{p}} = 1/\sqrt{L'C'}$$

We will later show that for any TEM transmission line:

$$L'C' = \mu\varepsilon$$

μ = magnetic permeability
ε = electrical permittivity

- For any insulating material that would be used in a transmission line, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m, where μ_0 is the vacuum permeability.
  — The value for μ_0 is exact and defines the relation between **B** and **H**.
- By contrast, the values for ε differ significantly for different materials. We write: $\varepsilon = \varepsilon_{\mathrm{r}}\,\varepsilon_0$, where ε_{r} is referred to as the relative permittivity, and $\varepsilon_0 = 1/\mu_0 c^2 \simeq 8.854 \times 10^{-12}$ F/m is the vacuum permittivity

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4.7

Lossless Transmission Line

We now have:

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_{\mathrm{r}}} = (\omega/c)\sqrt{\varepsilon_{\mathrm{r}}},\quad u_{\mathrm{p}} = 1/\sqrt{\mu\varepsilon} = c/\sqrt{\varepsilon_{\mathrm{r}}}$$

In optics, we define an *index of refraction*, $n = c / u_{\mathrm{p}}$, so that $n = \sqrt{\varepsilon_{\mathrm{r}}}$

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4.8

Lossless Transmission Line

The impedance relation:

$$Z_0 = \sqrt{L'/C'} = \sqrt{\mu_0/\varepsilon_0\varepsilon_r}K = \sqrt{\mu_0/\varepsilon_0}\left(K/\sqrt{\varepsilon_r}\right);\ 377\left(K/\sqrt{\varepsilon_r}\right)\Omega$$

is a bit more complex. It involves a geometric factor K.

Table of Geometric Factor K

	Coaxial	Two wire	Parallel plane
K	$\frac{1}{2\pi}\ln(b/a)$	$\frac{1}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\frac{d}{w}$

The vacuum impedance of 377 Ω is a very important number

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- Loads with lower impedance have large magnetic near fields
- Loads with higher impedance have large electric near fields

EMI properties are very different in the two cases!

4.9

Lossless Transmission Line

Voltage Reflection:

When $\alpha = 0$, our phasor relations become

$$\tilde{V}(z) = V_0^+\exp(-j\beta z) + V_0^-\exp(j\beta z),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}\exp(-j\beta z) - \frac{V_0^-}{Z_0}\exp(j\beta z)$$

At the load, we have

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L},\quad \text{with}\quad \tilde{V}_L = V_0^+ + V_0^- \text{ and } \tilde{I}_L = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

which implies

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0}V_0^+ \equiv \Gamma V_0^+$$

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and defines the voltage reflection coefficient Γ

Load impedances and reflection coefficients are usually complex!

4.10

Lossless Transmission Line

Example: Ulaby et al. Exercise 2.7

Question: A 50 Ω lossless transmission line is terminated in a load impedance $Z_L = (30 - j200)\ \Omega$. Calculate the voltage reflection coefficient at the load.

Answer: $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (30 - j200 - 50) / (30 - j200 + 50) = (-20 - j200) / (80 - j200) = 201\exp(-j1.67) / 215\exp(-j1.19) = 0.93\exp(-j0.48)$. NOTE: -0.48 rads $= -28^{\circ}$.

Writing $\Gamma = |\Gamma| \exp(j\theta_r)$, we find $\theta_r = -0.48$ rads.

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Lossless Transmission Line

Standing Waves

After using the relation $V_0^- = \Gamma V_0^+$ in the phasor equations,

$$\tilde{V}(z) = V_0^+\left[\exp(-j\beta z) + \Gamma\exp(j\beta z)\right] = V_0^+\left[\exp(-j\beta z) + |\Gamma|\exp(j\theta_r + j\beta z)\right],$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}\left[\exp(-j\beta z) - \Gamma\exp(j\beta z)\right] = \frac{V_0^+}{Z_0}\left[\exp(-j\beta z) - |\Gamma|\exp(j\theta_r + j\beta z)\right]$$

We then find

$$|\tilde{V}(z)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta z + \theta_r)\right]^{1/2},$$

so that the amplitude of the voltage varies sinusoidally with z.

- This pattern is called a *standing wave*.
- It comes from the interference of forward- and backward-propagating waves

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Lossless Transmission Line

Standing Waves

- The voltage and current maxima and minima are 180° out of phase
- The amplitude multiplies a $\cos(\omega t)$ dependence with a complicated but periodic z-variation.
- The maxima are spaced $\lambda/2$ apart, and so are the minima.

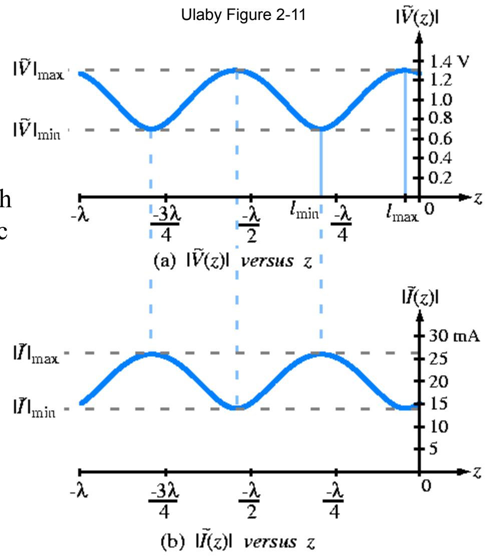

Figure Parameters:
$|\Gamma| = 0.3,\ \theta_r = 30^\circ$
$Z_0 = 50\ \Omega$
$|V_0^+| = 1.3$ V

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4.13

Lossless Transmission Line

Standing Waves

The total dependence of the standing wave voltage is

$$v(z,t) = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta z + \theta_r)\right]^{1/2} \times \cos\left\{\omega t - \tan^{-1}\left[\frac{\sin(\beta z - \phi^+) - |\Gamma|\sin(\beta z + \theta_r + \phi^+)}{\cos(\beta z - \phi^+) + |\Gamma|\cos(\beta z + \theta_r + \phi^+)}\right]\right\}$$

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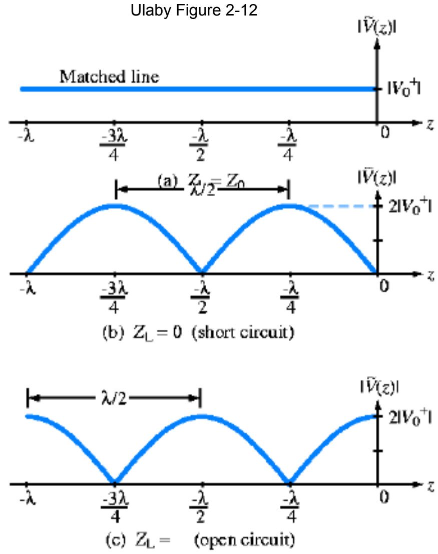
4.14

Lossless Transmission Line

Standing Waves

- For a matched load ($Z_L = Z_0$), there is no standing wave
- For a short-circuited load ($Z_L = 0$) or an open-circuited load ($Z_L = \infty$), there are complete reflections and an oscillation depth of 100%
- With $|\Gamma| = 1$, there are points where the voltage is exactly zero, spaced $\lambda / 2$ apart

Ulaby Figure 2-12



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4.15

Lossless Transmission Line

Standing Waves

- We will designate the location of the maxima as $l_{max} = -z$, so that l_{max} is a positive number (since the load is at $z = 0$).

$$-z = l_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}$$

  Only n-values that satisfy $l_{max} \geq 0$ are allowed
- The voltage standing wave ratio (VSWR or SWR) gives the oscillation depth. It is defined:

$$S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

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4.16

Lossless Transmission Line

Example: Ulaby et al. Exercise 2.11

Question: A 140 Ω lossless line is terminated in a load impedance Z_L = (280 + j 182) Ω. If λ = 72 cm, find (a) the reflection coefficient Γ, (b) the VSWR S, (c) the locations of the voltage maxima and minima.

Answer: (a) $\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = (280 + j\,182 - 140) / (280 + j\,182 + 140) = 230 \exp(j0.915) / 458 \exp(j0.409) = 0.50 \exp(j0.51)$. NOTE: 0.51 rads = 29°. (b) $S = (1 + |\Gamma|) / (1 - |\Gamma|) = (1 + 0.502) / (1 - 0.502) = 3.0$. (c) $l_{max} = (0.506 \times 72 / 4\pi + n \times 72 / 2)$ cm = 2.9 cm, 39 cm, 75 cm,…; $l_{min} = (0.506 \times 72 / 4\pi + 18 + n \times 72 / 2)$ cm = 21 cm, 57 cm, 93 cm,…

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4.17

Lossless Transmission Line

Input Impedance

With standing waves, the voltage-to-current ratio, which is referred to as the *input impedance*, varies as a function of position

$$Z_{in}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = \frac{V_0^+\left[\exp(-j\beta z) + \Gamma \exp(j\beta z)\right]}{V_0^+\left[\exp(-j\beta z) - \Gamma \exp(j\beta z)\right]} Z_0 = Z_0 \frac{\left[1 + \Gamma \exp(2j\beta z)\right]}{\left[1 - \Gamma \exp(2j\beta z)\right]},$$

Of particular interest is the input impedance at the generator, $z = -l$

$$Z_{in}(-l) = Z_0 \left[\frac{1 + \Gamma \exp(-2j\beta l)}{1 - \Gamma \exp(-2j\beta l)}\right]$$

$$= Z_0 \left(\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}\right) = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

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4.18

Lossless Transmission Line

Input Impedance

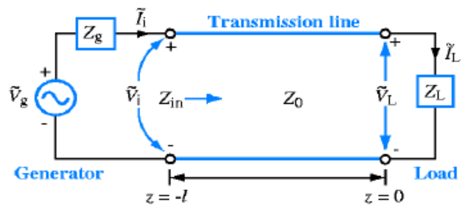
From the standpoint of the generator circuit, the transmission line appears as an input impedance $Z_{in} \equiv Z_{in}(-l)$, so that

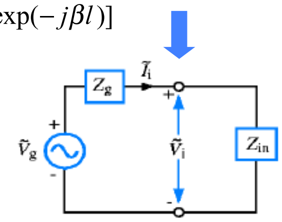
$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}$$

From the standpoint of the transmission line

$$\tilde{V}_i = \tilde{V}(-l) = V_0^+[\exp(j\beta l) + \Gamma \exp(-j\beta l)]$$

Combining these relations…





Ulaby Figure 2-14

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4.19

Lossless Transmission Line

Input Impedance

…we conclude

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{\exp(j\beta l) + \Gamma \exp(-j\beta l)}\right)$$

Thus, we can now relate the wave parameters,

$$V_0^+, \quad V_0^-(=\Gamma V_0^+), \quad I_0^+(=V_0^+/Z_0), \quad I_0^-(=-V_0^-/Z_0 = -\Gamma V_0^+/Z_0),$$

to the transmission line parameters

$$Z_g, \quad Z_L, \quad Z_0 = \sqrt{L'/C'}, \quad u_p = 1/\sqrt{L'C'}, \quad l$$

and the input parameters

$$\tilde{V}_g, \quad f = \omega/2\pi$$

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4.20

Lossless Transmission Line

Paul Example 6.7 (extended)

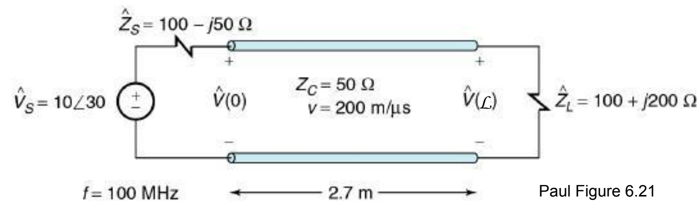
Question: A line 2.7 m in length is excited by a 100 MHz source as shown in the figure. Determine the source and load voltages. Determine the voltages and the current everywhere in the transmission line

Answer: We will work in Ulaby et al.'s notation, and our first task is to translate Paul's problem specification into that notation:

$$\hat{V}_S \to \tilde{V}_{\rm g} = 10\exp(j30^{\rm o}) = 10\exp(j\pi/6) = 10\exp(j0.524) = (8.66 + 5.00j)\ \text{V}$$

$$\hat{V}(0) \to \tilde{V}_{\rm i} = \tilde{V}(-l);\quad \hat{V}(\mathcal{L}) \to \tilde{V}_{\rm L} = \tilde{V}(0);\quad v \to u_{\rm p} = 200\ \text{m}/\mu\text{s} = 2\times 10^8\ \text{m/s}$$

$$\hat{Z}_S \to Z_{\rm g} = 100 - j50\ \Omega;\quad \hat{Z}_L \to Z_{\rm L} = 100 + j200\ \Omega;\quad Z_C \to Z_0 = 50\ \Omega;$$



Paul Figure 6.21

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4.21

Lossless Transmission Line

Paul Example 6.7

Answer (continued):*

(1) *Find the reflection coefficient:*

$$\Gamma = \frac{Z_{\rm L} - Z_0}{Z_{\rm L} + Z_0} = \frac{50 + j200}{150 + j200} = \frac{206\exp(j1.326)}{250\exp(j0.927)} = 0.825\exp(j0.399)$$

(2) *Find the propagation factor*

$$\beta l = (2\pi f/u_{\rm p})l = (6.28\times 10^8 / 2\times 10^8)\times 2.7 = 8.48;$$

$$\exp(j\beta l) = \exp(j0.7\pi) = \exp(j2.20);$$

$$\exp(-j\beta l) = \exp(-j0.7\pi) = \exp(-j2.20);$$

$$\exp(-j2\beta l) = \exp(-j1.4\pi) = \exp(-j4.40) = \exp(j1.88)$$

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*NOTE: I am using MATLAB to calculate values. So, the calculations are good to 15 places, although I only report three.

4.22

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(3) *Find the input impedance:*

$$\Gamma\exp(-j2\beta l) = 0.825\exp[j(0.399+1.885)]$$
$$= 0.825\exp(j2.283) = -0.539 + j0.624;$$
$$1+\Gamma\exp(-j2\beta l) = 0.461 + j0.624;$$
$$1-\Gamma\exp(-j2\beta l) = 1.539 - j0.624;$$

$$Z_{\text{in}} = Z_0\frac{1+\Gamma\exp(-j2\beta l)}{1-\Gamma\exp(-j2\beta l)} = 50\frac{0.461+j0.624}{1.539-j0.624} = 50\frac{0.776\exp(j0.935)}{1.661\exp(-j0.385)}$$
$$= 23.35\exp(j1.320) = 5.80 + j22.62$$

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Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(4) *Find the input voltage:*

$$Z_{\text{g}} + Z_{\text{in}} = (100 - j50) + (5.80 + j22.6)$$
$$= 106 - j27.4 = 109\exp(-j0.253)$$

$$\tilde{V}_{\text{i}} = \tilde{V}_{\text{g}}\frac{Z_{\text{in}}}{Z_{\text{g}}+Z_{\text{in}}} = 10\exp(j0.524)\frac{23.4\exp(j1.320)}{109\exp(-j0.253)}$$
$$= 2.14\exp(j2.10) = 2.14\angle 120^{\circ}$$

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Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(5) *Find the load voltage:*

$$V_0^+ = \tilde{V}_\mathrm{i}\frac{\exp(-j\beta l)}{1+\Gamma\exp(-j2\beta l)} = 2.14\exp(j2.10)\frac{\exp(-j2.20)}{0.776\exp(j0.935)}$$
$$= 2.75\exp(-j1.037)$$

$$1+\Gamma = 1+(0.760+j0.320) = 1.79\exp(j0.180)$$

$$\tilde{V}_\mathrm{L} = V_0^+(1+\Gamma) = [2.75\exp(-j1.037)][1.79\exp(j0.180)]$$
$$= 4.93\exp(-j0.857) = 4.93\angle -49^\mathrm{o}$$

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4.25

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(6) *Write the time domain voltages:*

$$v_\mathrm{i}(t) = \mathrm{Re}\left[\tilde{V}_\mathrm{i}\exp(j\omega t)\right] = 2.14\cos(6.28\times 10^8 t + 120^\mathrm{o})$$
$$v_\mathrm{L}(t) = \mathrm{Re}\left[\tilde{V}_\mathrm{L}\exp(j\omega t)\right] = 4.93\cos(6.28\times 10^8 t - 49^\mathrm{o})$$

(7) *Find the phasor domain voltage and current in the transmission line*

$$\tilde{V}(z) = V_0^+\exp(-j\beta z) + \Gamma V_0^+\exp(j\beta z)$$
$$= 2.75\exp(-j\beta z - j1.037) + 2.27\exp(j\beta z - j0.638)\ \ \mathrm{(V)}$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}\exp(-j\beta z) - \frac{\Gamma V_0^+}{Z_0}\exp(j\beta z)$$
$$= 55.1\exp(-j\beta z - j1.037) + 45.4\exp(j\beta z + j2.503)\ \ \mathrm{(mA)}$$

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4.26

Lossless Transmission Line

Paul Example 6.7

Answer (continued):

(8) *Find the time domain voltage and current in the transmission line:*

$$v(z,t) = \mathrm{Re}\left[2.75\exp(j\omega t - j\beta z - j1.037) + 2.27\exp(j\omega t + j\beta z - j0.638)\right]$$
$$= 2.75\cos(\omega t - \beta z - 1.037) + 2.27\cos(\omega t + \beta z - 0.638)\ \text{(V)}$$
$$i(z,t) = \mathrm{Re}\left[55.1\exp(j\omega t - j\beta z - j1.037) + 45.4\exp(j\omega t + j\beta z + j2.503)\right]$$
$$= 55.1\cos(\omega t - \beta z - 1.037) + 45.4\cos(\omega t + \beta z + 2.503)\ \text{(mA)}$$

(9) *Checks:*

$$v(0,t) = 2.75\cos(\omega t - 1.037) + 2.27\cos(\omega t - 0.638)$$
$$= 4.93\cos(\omega t - 0.857)\ \text{(V)}$$
$$v(-l,t) = 2.75\cos(\omega t + 8.48 - 1.037) + 2.27\cos(\omega t - 8.48 - 0.638)$$
$$= 2.14\cos(\omega t + 2.097)\ \text{(V)}$$

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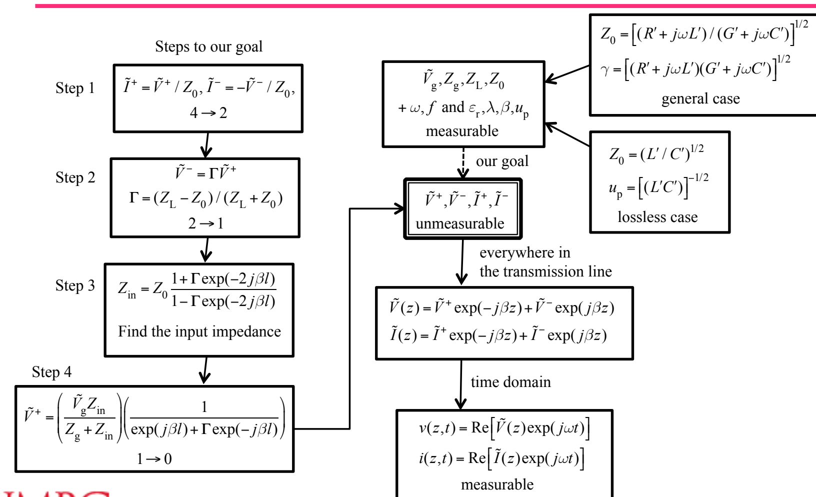
Map to Finding the Transmission Line Voltages and Currents



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