CMPE 330 **Spring 2017** **Problem Set #5**

NOTE: You must show complete work for full credit. Report numerical solutions to two significant figures unless otherwise specified.

1. Find the total charge on a circular disk defined by $r \le a$ and $z = 0$ if: [modified from Ulaby and Ravaioli 4.5, p. 227.]

   a. $\rho_{\mathrm{s}} = \rho_{\mathrm{s0}} \cos\phi$ (C/m^2)

   b. $\rho_{\mathrm{s}} = \rho_{\mathrm{s0}} \cos^2\phi$ (C/m^2)

   c. $\rho_{\mathrm{s}} = \rho_{\mathrm{s0}} \exp(-2r/b)$ (C/m^2)

   d. $\rho_{\mathrm{s}} = \rho_{\mathrm{s0}} \exp(-2r/b) \sin^2\phi$ (C/m^2)

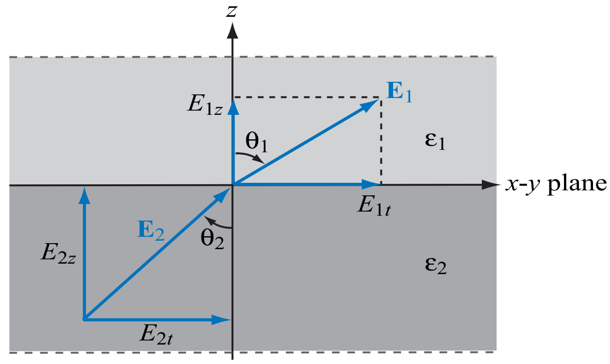
2. Three point charges, each with $q = 6$ nC, are located at the corners of a triangle in the x-y plane, with one corner at the origin, another at (4 cm, 0, 0), and the third at (0, 4 cm, 0). Find the force acting on the charge located at the origin. [modified from Ulaby and Ravaioli 4.12, p. 227.]

3. A line of charge with uniform density ρ_l extends between $z = -L/2$ and $z = L/2$ along the z-axis. Apply Coulomb's law to obtain an expression for the electric field at any point $P(r, \phi, 0)$ on the x-y plane. Show that the result reduces to Ulaby's expression (4.33), which may also be found on slide 9.3, as the length L is extended to infinity. [Ulaby and Ravaioli 4.16, p. 227.]

4. The electric flux density inside a dielectric sphere of radius a centered at the origin is given by $\mathbf{D} = \hat{\mathbf{R}}\rho_0 R$, where ρ_0 is a constant charge density. Find the total charge within the sphere. [Ulaby and Ravaioli 4.25, p. 228.]

5. Find the electric potential V at a location a distance b from the origin in the x-y plane due to a line charge with density ρ_l and length l. The line charge is coincident with the z-axis and extends from $z = -l/2$ to $z = l/2$. Determine the field from the potential and show that the result agrees with what you obtained in problem 3 once the change in the variable names is taken into account. [Modified from Ulaby and Ravaioli 4.34, p. 229.]

6. With reference to the figure shown at the right [Ulaby, et al. Fig. 4-19], find $\mathbf{E}_1$ if $\mathbf{E}_2 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}2$ V/m, $\epsilon_1 = 4\epsilon_0$, $\epsilon_2 = 9\epsilon_0$, and the boundary has a surface charge density $\rho_S = 3.54 \times 10^{-11}$ C/m^2. What angle does $\mathbf{E}_2$ make with the z-axis? [modified from Ulaby and Ravaioli 4.48, p. 231.]

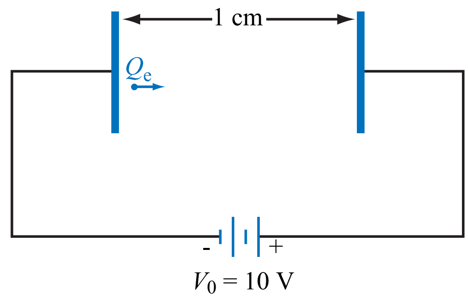


7. An electron with charge $Q_e = -1.6 \times 10^{-19}$ C and mass $m_e = 9.1 \times 10^{-31}$ kg is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled parallel-plate capacitor with separation of 2 cm and rectangular plates each 10 cm^2 in area, as shown in the figure below [Ulaby, et al. Fig. P4.54]. If the voltage across the capacitor is 5 V, find the following [modified from Ulaby and Ravaioli 4.54, p. 231.]

   a. The force acting on the electron

   b. The acceleration of the electron

   c. The time it takes for the electron to reach the positively charged plate, assuming that it starts from rest

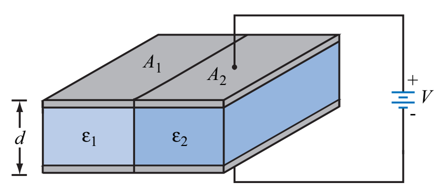


8. The figure below (part a) [Ulaby, et al. Fig. P4.56] depicts a capacitor consisting of two parallel, conducting plates separated by a distance d. The space between the plates contains two adjacent dielectrics, one with permittivity ϵ_1 and surface area A_1 and another with ϵ_2 and A_2. The objective of this problem is to show that the capacitance C of the configuration shown in the figure below (part a) is equivalent to two capacitances in parallel as illustrated in the figure below (part b), with $C = C_1 + C_2$, where $C_1 = \epsilon_1 A_1/d$ and $C_2 = \epsilon_2 A_2/d$. To this end, proceed as follows: [Ulaby and Ravaioli 4.56, p. 232.]

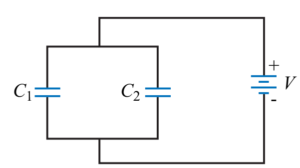
   a. Find the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ in the two dielectric layers.

   b. Calculate the energy stored in each section and use the result to calculate C_1 and C_2.

   d. Use the total energy stored in the capacitor to obtain an expression for C. Show that the expression $C = C_1 + C_2$ is indeed a valid result.



(a)



(b)