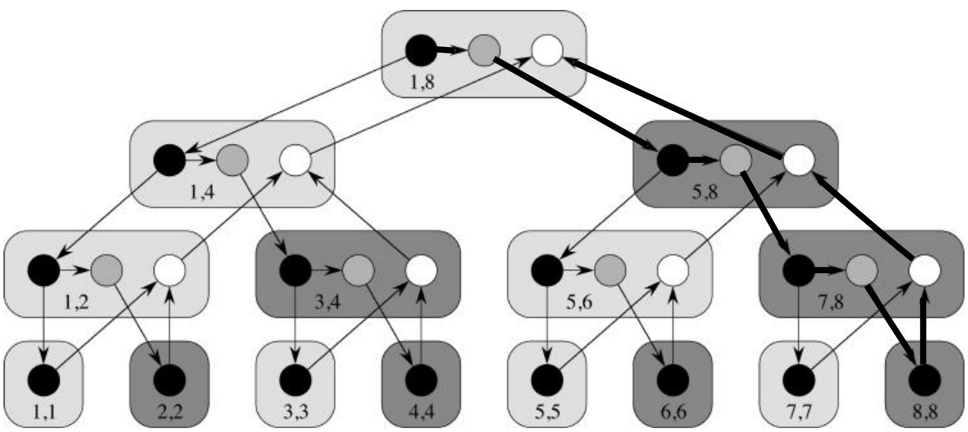
Name: ____

1. (10 points) The algorithm P-Sum computes the sum of the elements of an array L of length n. Draw the DAG for P-Sum when L is an array of length 8. Determine the work, span, and parallelism from the DAG. Show all work.

P-Sum(L)
$n = L.length$
if $n == 1$
    return $L[1]$
$c = \lfloor n/2 \rfloor$
$x =$ **spawn** P-Sum($L[1 \mathinner{..} c]$)
$y =$ P-Sum($L[c+1 \mathinner{..} n]$)
sync
return $x + y$

Solution



The critical path is bolded. The span is the length of the critical path (in strands, not edges), so $T_\infty = 10$. The work is the number of strands, so $T_1 = 29$.

The parallelism is $T_1/T_\infty = 2.9$.

(continued on other side)

2. (10 points) The algorithm MAT-VEC computes the product of an n-by-n matrix A and an n-long vector x:

MAT-VEC(A)
1 $n = A.rows$
2 let y be a new vector of length n
3 **parallel for** $i = 1$ **to** n
4     $y_i = 0$
5 **parallel for** $i = 1$ **to** n
6   **for new** $j = 1$ **to** n
7         $y_i = y_i + a_{ij} \cdot x_j$
8 **return** y

Determine the work, span, and parallelism of MAT-VEC. What is the *parallel slackness* when $n = 256$ and $P = 16$? Show all work and any formulas used.

Solution

The work is just the serial running time of the algorithm. If we ignore the "parallel" keywords, we see that the loop on lines 3 – 4 is $\Theta(n)$ and the nested loops on lines 5 – 7 are $\Theta(n^2)$, so the work is $T_1(n) = \Theta(n) + \Theta(n^2) = \Theta(n^2)$.

To determine the span of the parallel loops, we use the formula

$$T_\infty(n) = \Theta(\lg n) + \max_k \text{iter}_\infty(k).$$

For lines 3 – 4, we see that each iteration is $\Theta(1)$ since it consists of a single variable assignment, so the span of the loop is $\Theta(\lg n) + \Theta(1) = \Theta(\lg n)$. For lines 5 – 7, a single iteration of the parallel loop is the loop over j, which is length n. The inner most computation ($y_i = y_i + a_{ij} \cdot x_j$) is $\Theta(1)$, so the span of the inner loop is $\Theta(n)$. Therefore, using the formula for the span of a parallel loop, the span of lines 5 – 7 is $\Theta(\lg n) + \Theta(n) = \Theta(n)$. All other work is $\Theta(1)$, so we have

$$T_\infty(n) = \Theta(\lg n) + \Theta(n) + \Theta(1) = \Theta(n).$$

The parallelism is, therefore, $T_1(n)/T_\infty(n) = \Theta(n)$.

The parallel slackness when $n = 256$ and $P = 16$ is

$$\frac{T_1(n)}{P \cdot T_\infty(n)} = \frac{256^2}{16 \cdot 256} = 16.$$