

Project 2

STAT 355

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1 Part 1

1000 random samples of size 40 were generated from normal distribution with mean $\mu = 3$ and standard deviation $\sigma = 2$.

```
# initialize parameters for normal distribution
N <- 40 # size
mu <- 3 # mean
sigma <- 2 # standard deviation
sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
# generate 1000 samples
for (i in 1:NUMSAMPS){
  generatedData <- rnorm(N, mu, sigma)
  # store the sample means in vector
  sampMeans[i] = mean(generatedData)

  if (i == 1) {
    firstMean = mean(generatedData)
    firstStd = sd(generatedData)
  }
}
```

1.1 Output

The first sample mean and standard deviation were computed:

$$E(\bar{X}) = 2.833, \sigma_{\bar{X}} = 0.316$$

All the samples were then used to find the sample mean and standard deviation. The theoretical values were also computed based on the relationships:

$$\begin{aligned}\mu &= \mu \\ E(\bar{X}) &= \mu \\ \sigma &= \sigma \\ \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{n}}\end{aligned}$$

	Actual	Theoretical
μ	3.000	3.000
$E(\bar{X})$	2.990	3.000
σ	2.000	2.000
$\sigma_{\bar{X}}$	0.311	0.316

1.2 Distribution

Distribution of the data was plotted with a histogram using ggplot2 in Figure 1.

```
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.1, color="black", fill="white") +
  labs(y="Count", x="Sample Means")
```

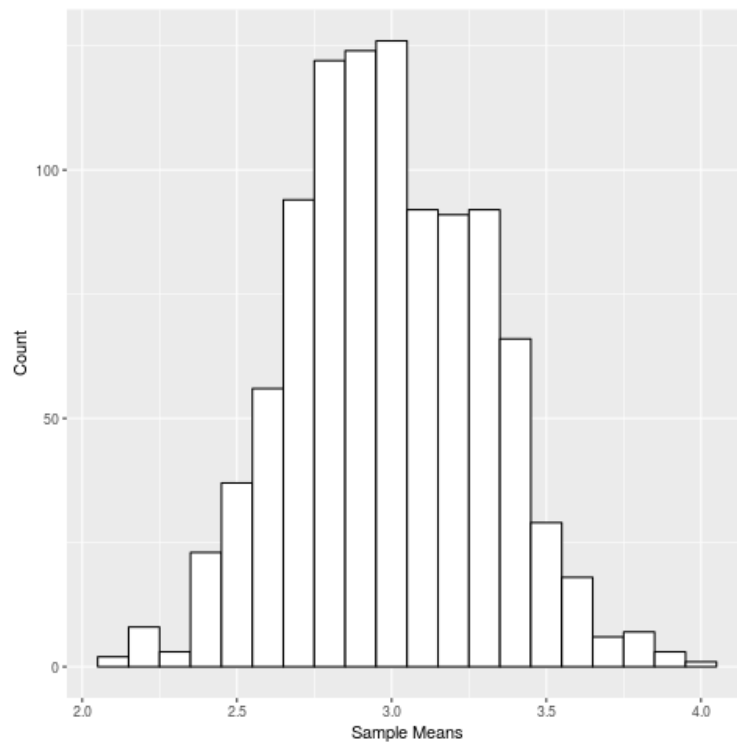


Figure 1: Histogram of the Generated Data

2 Part 2

1000 random samples of size 15 were generated from a binomial distribution with $n = 10$ and standard deviation $p = 0.15$.

```
# initialize parameters for binomial distribution
N <- 15
n <- 10
p <- 0.15
sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
for (i in 1:NUMSAMPS){
```

```

generatedData <- rbinom(N, n, p)
sampMeans[i] = mean(generatedData)

if (i == 1) {
  firstMean = mean(generatedData)
  firstStd = sd(generatedData)
}
}

```

2.1 Output

The first sample mean and standard deviation were computed:

$$E(\bar{X}) = 1.000, \sigma_{\bar{X}} = 0.926$$

All the samples were then used to find the sample mean and standard deviation. The theoretical values were also computed based on the relationships:

$$\begin{aligned}\mu &= np \\ E(\bar{X}) &= np \\ \sigma &= np(1-p) \\ \sigma_{\bar{X}} &= \frac{np(1-p)}{\sqrt{n}}\end{aligned}$$

	Actual	Theoretical
μ	1.500	1.500
$E(\bar{X})$	1.518	1.500
σ	1.275	1.275
$\sigma_{\bar{X}}$	0.292	0.329

2.2 Distribution

Distribution of the data was plotted with a histogram using ggplot2 in Figure 2.

```

# plot a histogram of the data
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.2, color="black", fill="white") +
  labs(y="Count", x="Sample Means")

```

3 Part 3

1000 random samples of size 120 were generated from a binomial distribution with $n = 10$ and standard deviation $p = 0.15$.

```

# initialize parameters for binomial distribution
N <- 120
n <- 10
p <- 0.15
sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
for (i in 1:NUMSAMPS){

```

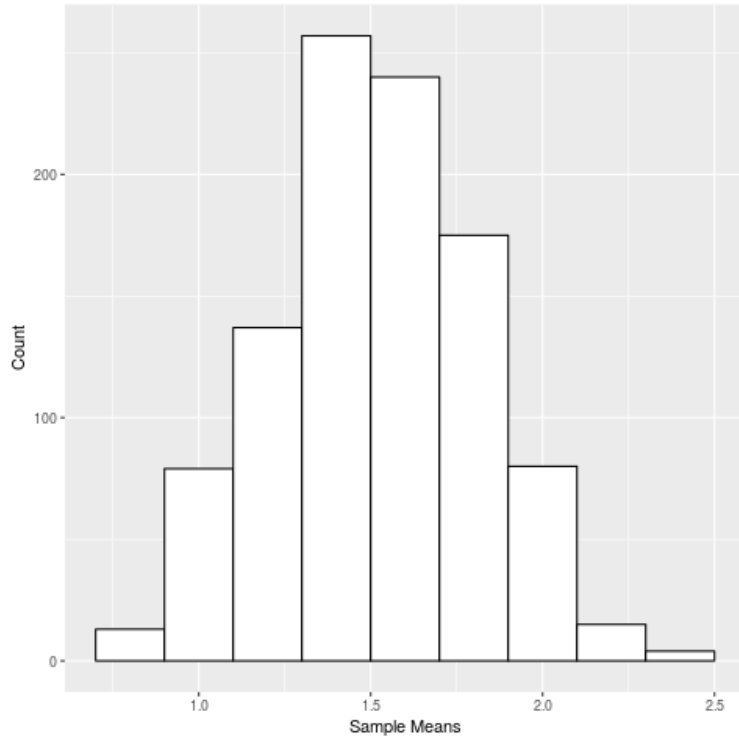


Figure 2: Histogram of the Generated Data

```
generatedData <- rbinom(N, n, p)
sampMeans[i] = mean(generatedData)

if (i == 1) {
  firstMean = mean(generatedData)
  firstStd = sd(generatedData)
}

}
```

3.1 Output

The first sample mean and standard deviation were computed:

$$E(\bar{X}) = 1.492, \sigma_{\bar{X}} = 0.011$$

All the samples were then used to find the sample mean and standard deviation. The theoretical values were also computed based on the relationships:

$$\begin{aligned}\mu &= np \\ E(\bar{X}) &= np \\ \sigma &= np(1-p) \\ \sigma_{\bar{X}} &= \frac{np(1-p)}{\sqrt{n}}\end{aligned}$$

	Actual	Theoretical
μ	1.500	1.500
$E(\bar{X})$	1.502	1.500
σ	1.275	1.275
$\sigma_{\bar{X}}$	0.101	0.116

3.2 Distribution

Distribution of the data was plotted with a histogram using ggplot2 in Figure 3.

```
# plot a histogram of the data
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.1, color="black", fill="white") +
  labs(y="Count", x="Sample Means")
```

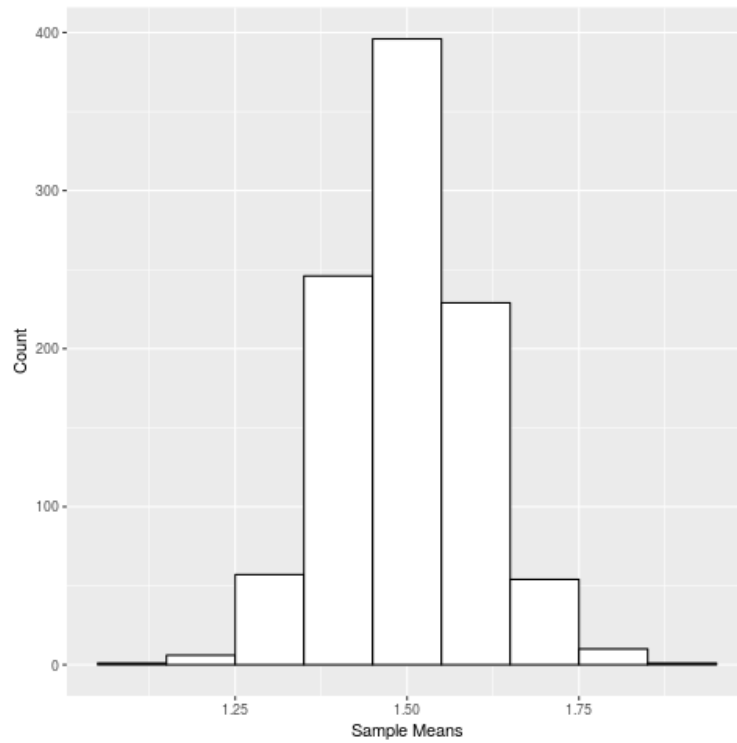


Figure 3: Histogram of the Generated Data

4 Conclusion

```
# main.R
# This file contains the implementation of the functions in the Project 2
# NOTE: THIS SCRIPT WAS COMPILED ON A LINUX MACHINE - SOME STATEMENTS MAY THROW
# WARNINGS OR ERRORS IN OTHER SYSTEMS
```

```
library(ggplot2) # for generating high quality plots
set.seed(124) # seed the random generators
```

```
# LaTeX template for the output
outputTemplate <- "\\subsection{Output}"
```

The first sample mean and standard deviation were computed:

```
\\[ E(\\overline{X}) = %.3f, \\ \\sigma_{\\overline{X}} = %.3f \\]
```

All the samples were then used to find the sample mean and standard deviation. The theoretical values were also computed based on the relationships:

```
\\[ \\mu = %s \\]
\\[ E(\\overline{X}) = %s \\]
\\[ \\sigma = %s \\]
\\[ \\sigma_{\\overline{X}} = %s \\]
```

```
\\begin{table}[h]
  \\centering
  \\begin{tabular*}{200pt}{@{\\extracolsep{\\fill}} c c c}

    & \\textbf{Actual} & \\textbf{Theoretical} & \\\\
    \\hline
    $\\mu$ & %.3f & %.3f & \\\\
    E($\\overline{X}$) & %.3f & %.3f & \\\\
    $\\sigma$ & %.3f & %.3f & \\\\
    $\\sigma_{\\textsubscript{$\\overline{X}$}}$ & %.3f & %.3f & \\\\

  \\end{tabular*}
\\end{table}
```

```
"
```

```
# global variables
NUMSAMPS <- 1000
firstMean <- 0
firstStd <- 0
```

```
# ----- Part 1 -----
```

```
# initialize parameters for normal distribution
N <- 40 # size
mu <- 3 # mean
sigma <- 2 # standard deviation
```

```
sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
firstMean <- 0
firstStd <- 0
```

```
# generate 1000 samples
for (i in 1:NUMSAMPS){
  generatedData <- rnorm(N, mu, sigma)
  # store the sample means in vector
  sampMeans[i] = mean(generatedData)

  if (i == 1) {
    firstMean = mean(generatedData)
    firstStd = sigma/sqrt(N)
  }
}
```

```

    }
}

# save output
sink("part1.tex", append=FALSE, split=FALSE)
cat(
  sprintf(
    outputTemplate,
    firstMean, firstStd,
    "\\mu", "\\mu", "\\sigma", "\\frac{\\sigma}{\\sqrt{n}}",
    mu, mu,
    mean(sampMeans), mu,
    sigma, sigma,
    sd(sampMeans), sigma/sqrt(N)
  )
)
sink()

png(filename="figures/hist1.png")
# plot a histogram of the data
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.1, color="black", fill="white") +
  labs(y="Count", x="Sample Means")

dev.off()

# ----- Part 2 -----

# initialize parameters for binomial distribution
N <- 15
n <- 10
p <- 0.15

sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
for (i in 1:NUMSAMPS){
  generatedData <- rbinom(N, n, p)
  sampMeans[i] = mean(generatedData)

  if (i == 1) {
    firstMean = mean(generatedData)
    firstStd = sd(generatedData)
  }
}

# save output
sink("part2.tex", append=FALSE, split=FALSE)
cat(
  sprintf(
    outputTemplate,
    firstMean, firstStd,
    "np", "np", "np(1-p)", "\\frac{np(1-p)}{\\sqrt{n}}",
    n*p, n*p,
    mean(sampMeans), n*p,
    n*p*(1-p), n*p*(1-p),
    sd(sampMeans), n*p*(1-p)/sqrt(N)
  )
)
sink()

png(filename="figures/hist2.png")

# plot a histogram of the data
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.2, color="black", fill="white") +
  labs(y="Count", x="Sample Means")

```

```

dev.off()

# ----- Part 3 -----

# initialize parameters for binomial distribution
N <- 120
n <- 10
p <- 0.15

sampMeans <- rep(0, times=NUMSAMPS) # initialize empty array
for (i in 1:NUMSAMPS){
  generatedData <- rbinom(N, n, p)
  sampMeans[i] = mean(generatedData)

  if (i == 1) {
    firstMean = mean(generatedData)
    firstStd = n*p*(1-p)/N
  }
}

# save output
sink("part3.tex", append=FALSE, split=FALSE)
cat(
  sprintf(
    outputTemplate,
    firstMean, firstStd,
    "np", "np", "np(1-p)", "\\frac{np(1-p)}{\\sqrt{n}}",
    n*p, n*p,
    mean(sampMeans), n*p,
    n*p*(1-p), n*p*(1-p),
    sd(sampMeans), n*p*(1-p)/sqrt(N)
  )
)
sink()

png(filename="figures/hist3.png")

# plot a histogram of the data
ggplot() + aes(sampMeans) +
  geom_histogram(binwidth=0.1, color="black", fill="white") +
  labs(y="Count", x="Sample Means")

dev.off()

```
