Stat355, F 2016, SAMPLE Questions for MIDTERM 2

In the exam I will supply the necessary sample quantities such as mean variance min, max etc that will be needed for method of moments estimator or maximum likelihood estimator. For any given problem, the formula sheet will have the expression for the pmf or pdf and expression for $E(X)$ and $Var(X)$.

1. Consider the following joint distribution table.

X↓ /Y →	0	2	4	P(X = x)
0				
1	0.1		0.1	
P(Y = y)	0.2			1

   (a) If $E(X) = 0.2$ and $E(Y) = 2$, find $Cov(X, Y)$.

   (b) Find $P(Y > X|X > 0)$

2. Suppose the weight of a manufactured steel bar is a random quantity W and when it is measured the measurement adds a mean zero error ϵ to the actual weight and produces a random measurement weight $X = W + \epsilon$. Suppose two independent measurements give values $X_1 = W + \epsilon_1$ and $X_2 = W + \epsilon_2$. If the measurement errors are independent of each other and independent of W and have variances $V(\epsilon_1) = V(\epsilon_2) = 0.01$, and the variance of the actual weight is W is 1, find the correlation coefficient ρ between the two measurement weights X_1 and X_2.

3. The mean weight of luggage checked by a randomly selected economy class passenger flying between two cities in a certain airline is 40 lb with a standard deviation of 10 lb. The same for a randomly selected business class passenger on the same airline flying between the two cities is 30 lb and 6 lb, respectively.

   (a) If there are 12 business class passengers and 50 economy class passengers, what are the expected value and standard deviation of the total luggage weight?

   (b) If the individual luggage weights are independent and normally distributed, what is the probability that the total weight is at most 2500 lb?

4. Suppose the proportion of a certain material in a metal alloy is distributed as $Uniform[\theta, 0.5]$ You are given a random sample of 7 measurements of the proportion, 0.22, 0.45, 0.21, 0.39, 0.42, 0.28, 0.28. What is the maximum likelihood estimator of θ? What is the method of moments estimator of θ?

5. The run time (in hours) of a simulation experiment in a controlled environment is distributed as $Gamma(\alpha, \beta)$ with pdf

$$f(x; \alpha, \beta) = \frac{e^{x/\beta} x^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)}.$$

   The last 40 run times are given in the following table (Mean = 15.57, Std Dev = 8.42)

8.7	11.6	20.9	11.0	18.9	10.8	20.6	6.1	5.2	9.5
46.9	17.0	19.3	17.9	10.6	14.0	15.0	19.0	16.4	35.8
14.8	10.9	8.0	7.1	15.6	10.9	27.9	2.4	8.1	17.5
16.0	13.1	22.6	8.1	24.3	9.6	20.0	14.5	24.7	11.7

(a) If, based on previous knowledge, one knows that $\alpha = 3$ then what is a good estimate of β?

(b) An additional 36 observations will be taken for the next set of simulation runs. What is the probability that the mean run-time for the 36 runs will exceed 15?

6. The measurements of Young's modulus of a certain variety of glass is distributed according to some distribution with mean 7.9 Mpsi and standard deviation 0.5 Mpsi. If 60 independent measurements are taken, what is the probability that the mean of the measurements will exceed 8 Mpsi? If the measurements exceeded the 90^{th}-percentile of the population for K samples, what is the probability that K will be less than 10?

7. Suppose the number of times a component will fail in a given number of operating hours, say T hours, is distributed according to a Poisson distribution with mean failure rate of μ failures per T hours of operation. If in $2T$ hours of operation, the component failed X_1 times in the first T hours and X_2 times in the next T hours, compute the MLE and method of moments estimator for μ (assuming the failures are independent of each other). Is the MLE an unbiased estimator of μ in this situation? What is your estimate of the probability that the component will not fail in the next $2T$ hours of operation?