MEMO Number CMPE323-Lab04 rev1
DATE: October 7, 2016
TO: CMPE323
FROM: EFC LaBerge
SUBJECT: Convolution

1 INTRODUCTION

We have been talking about convolution. In this lab we will demonstrate that the convolution operation

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau \quad (1)$$

is a filtering operation.

2 EQUIPMENT

For this lab, you need a laptop with MATLAB installed.

For the purpose of CMPE323, please use the following naming conventions for all output files:

CMPE323F16_Lab<Lab#>_<Your Campus ID>

For the purpose of CMPE323, please use the following naming conventions for MATLAB scripts or functions that you are required to submit.

<function name>_<Your Campus ID>

Examples will be given in the lab description. Follow the instructions exactly, or you may not get graded!

3 LAB TASKS

You might find it useful to use the MATLAB function `diary` to capture your inputs and outputs.

3.1 A simple filter

Using a sample rate of 1000 Hz, and a time record from -1 to +10 seconds, create our simple rectangular pulse $p(t,0.25)$ and use it as your system impulse response. Then create system inputs $x(t;f) = e^{j2pft}u(t)$. The notation $x(t;f)$ should be read as “x is a function of t, with a fixed parameter f”. Using MATLAB conv function, compute and plot the *magnitude* of the output for $f = 0,1,4,6.5,7.5,8.5,11.5,19.5$ Hz. (There are eight different frequencies). Remember that I want the output of the convolution integral, not the convolution sum. Comment on whether this is a high pass, low pass or band pass filter.

Now replot the results, discarding the "convolution tail" at the end of the output array. This is due to the impulse response "walking off" of the finite duration complex exponential signal.

Now recompute the convolution and replot the results using only the non-zero elements of h. Is there any change in your outputs? For this exercise, you can again discard the "convolution tail".

3.2 A less simple filter

Now compute a new $h_2(t) = h(t) \times e^{j15\pi t}$. Compute and plot the magnitude of the output of this system to the same set of complex exponentials. Explain your results, including identification of the type of filter.

3.3 A mechanization of this lab

Create the following MATLAB function (*not* script!)

```
function response = cexp_response(h,t,f,ifploton)
% function response = cexp_response(h,t,f,ifploton)
%  This function plots the response of an LTI system described by impulse
% response h sampled at the times in t, at the frequencies given in f.
%
% Calling parameters
%  (1 x N) array of impulse response.
%  (1 x N) array of sample times in seconds
%  (1 x M) array of evaluation frequencies in Hz
%  Note M is not necessarily equal to N
%  ifploton = 1 for plots, 0 otherwise
%   ifploton = 1 will plot the magnitude and phase of the response at the
%   response frequencies (in Hz)
%
% Returned parameters
%  response (1 x M) array of complex outputs representing the steady state output at each %   of the M frequencies.
```

Note that I'm not interested in any transient response in the filter output, just the steady state. Ask yourself how long any transient response might last. To help inform your programming, write an expression for the output of the filter at a time *after* the transient response is complete. Start by assuming that the impulse response is of finite duration, as in 3.1 and 3.2.

You might find the MATLAB function kron helpful.

You might find it helpful to sketch what's going on before you start to code. This is especially true if you're trying to use matrix multiplication.

You might find matrix multiplication to be helpful, but you can certainly solve the problem using nested loops.

Test your function using the inputs and results of 3.1 and 3.2. Remember that I'm only looking for the steady-state response.

3.4 Complex eigenvalues

Your function puts out a single complex number representing the complex value (rectangular or amplitude and phase) at each of your input frequencies. Since the complex exponentials are *eigenfunctions* of LTI systems, these single values are the *eigenvalues*. We generally denote these eigenvalues as $H(w)$ or $H(jw)$. The eigen equation is then

$$\mathcal{H}\left(e^{j2\pi ft}\right) = H(j2\pi f)e^{j2\pi ft} = H(j\omega)e^{j\omega t} \tag{2}$$

where I have used the notation $\mathcal{H}\big(x(t)\big)$ to denote the output of the system $\mathcal{H}$ to the input $x(t)$. I used the funny looking $\mathcal{H}$ to denote the system so that I could follow standard practice and use $H(jw)$ to denote the eigenvalue.

Use your function to compute and plot the responses of $h(t)$ and $h_2(t)$ from parts 3.1 and 3.2 to complex exponentials with circular frequencies $f = \left[0:0.1:50\right]$ Hz. Plot the amplitude and phase of the output as a function of f.

4 LAB SUBMISSIONS

Submit the following via the Blackboard assignment Lab 3.

Using this lab description document as a template, create a single PDF file named in accordance with the output naming conventions given above. The content must include

a. The outputs and discussions generated in 3.1.

b. The outputs and discussions generated in 3.2.

c. Describe your approach to the coding exercise.

d. The outputs and discussions generated in 3.4.

Professional, high quality writing, math, and graphic (that is plots) presentation is expected, and must be provided for you to earn full credit.

In a separate file, but as part of your submission, include you MATLAB script for 3.3. Use good programming practices in your function. I strongly suggest that your function include my function definition section from 3.3.