CMPE 330 **Spring 2017** **Problem Set #6**

NOTE: You must show complete work for full credit. Report numerical solutions to two significant figures unless otherwise specified.

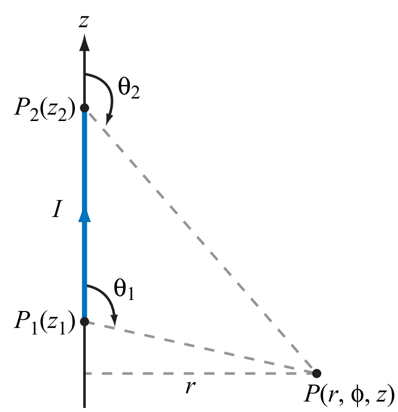
1. In a cylindrical coordinate system, a 2-m-long straight wire carrying a current of 5 A in the positive z-direction is located at $r = 4$ cm, $\phi = \pi/2$, and -1 m $\leq z \leq 1$ m. [Ulaby and Ravaioli 5.5, p. 274.]

   a. If $\mathbf{B} = \hat{\mathbf{r}}0.2\cos\phi$ (T), what is the magnetic force acting on the wire?

   b. How much work is required to rotate the wire once about the z-axis in the negative ϕ-direction (while maintaining $r = 4$ cm)?

   c. At what angle ϕ is the force a maximum?

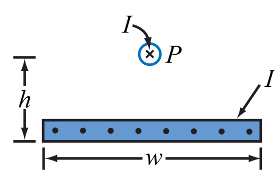
2. Use the approach outlined in Ulaby and Ravaioli's Example 5-2 to develop an expression for the magnetic field $\mathbf{H}$ at an arbitrary point P due to the linear conductor defined by the geometry to the right [Ulaby, et al. Fig. P5.8]. If the conductor extends between $z_1 = 1$ m and $z_2 = 6$ m and carries a current $I = 10$ A, find $\mathbf{H}$ at $P(3, \phi, 0)$. [modified from Ulaby and Ravaioli 5.8, p. 275.]



3. An infinitely long, thin conducting sheet of width w along the x-direction lies in the x-y plane and carries a current I in the $-y$-direction. Determine the following: [Ulaby and Ravaioli 5.18, p. 277.]

   a. The magnetic field at a point P midway between the edges of the sheet and at a height h above it, as shown in the figure to the right. [Ulaby, Fig. P5.18].

   b. The force per unit length exerted on an infinitely long wire passing through the point P and parallel to the sheet if the current through the wire is equal in magnitude but opposite in direction to the carried by the sheet.



4. Consider two parallel, circular loops of radius r, oriented normal to the z-axis and whose centers are at $z = \pm a/2$. Each loop carries a current I, so that the both coils generate a magnetic field in the $+z$-direction along the z-axis (when $x = y = 0$). These loops are referred to as Helmholtz rings.

   a. Plot the magnitude of the magnetic field along the z-axis in the three cases, $a = r/2$, $a = r$, and $a = 2r$. Please generate the solution numerically and indicate how you did it. Include a copy of the computer code that you create to solve the problem.

b. For what ratio r/a is the variation of the magnetic field flattest at the origin? Hint: Show that the first and third derivatives of $\mathbf{H}$ are zero in the z-direction at the origin. Find the conditions that make the second derivative zero. In that case, the first derivative that appears will be the fourth derivative.

5. A uniform current density $\mathbf{J} = \hat{\mathbf{z}}4J_0$ gives rise to a vector magnetic potential

$$\mathbf{A} = -\hat{\mathbf{z}}\mu_0 J_0(x^2 + y^2) \text{ Wb/m}$$

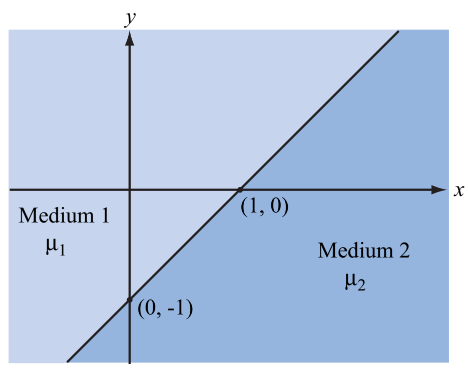
Note that the representation of the vector Poisson's equation, Eq. (5.60), as three scalar equations, Eq. (5.61), *is only valid in Cartesian coordinates!* [modified from Ulaby and Ravaioli 5.28, p. 279.]

a. Apply the vector Poisson's equation to confirm this statement

b. Use the expression for $\mathbf{A}$ to find $\mathbf{H}$.

c. Use the expression for $\mathbf{J}$ in conjunction with Ampere's law to find $\mathbf{H}$. Compare your result with that obtained in part (b).

6. In the figure on the right [Ulaby, Fig. P5.34], the plane defined by $x - y = 1$ separates medium 1 of permeability μ_1 from medium 2 of permeability μ_2. If no surface current exists on the boundary and $\mathbf{B}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3$ T, find $\mathbf{B}_2$ and then evaluate your result for $\mu_1 = 3\mu_2$. Hint: Start by deriving the equation for the unit vector normal to the given plane. [modified from Ulaby and Ravaioli 5.34, p. 279.]



7. How much magnetic energy is stored in the insulating medium of a coaxial transmission line with current I, and with inner radius R_1, outer radius R_2, and length L? Assuming that the coaxial transmission line is air-filled, is 5-m-long, and that the radius of the inner conductor is 2 cm, the radius of the outer conductor is 10 cm, and the current is 10 A, find the energy. [modified from Ulaby and Ravaioli 5.39, p. 280.]