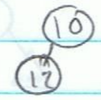
Sabbir Ahmed
Section: 02
HW4, Version A
Username: sabbir1

① Create a min heap with:

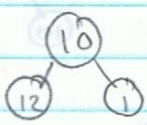
10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, 2

① insert 10:

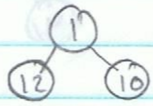
insert 12:



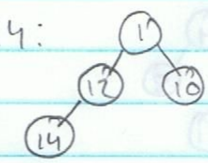
insert 1:



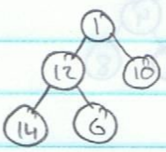
$\because 1 < 10$, swap(1, 10)



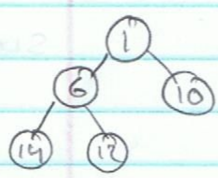
insert 14:



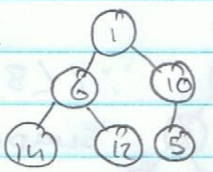
insert 6:



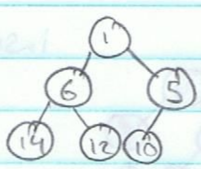
$\because 6 < 12$, swap(6, 12)



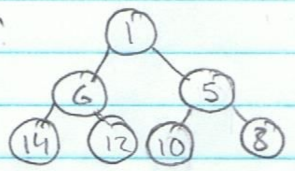
insert 5:



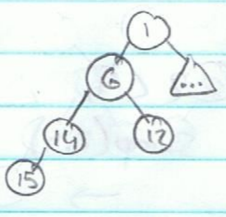
$\because 5 < 10$, swap(5, 10)



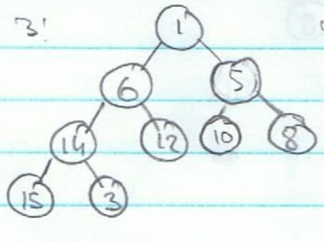
insert 8:



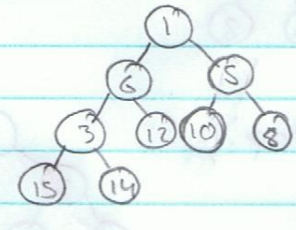
insert 15:



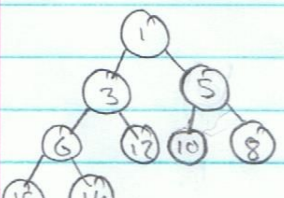
insert 3:



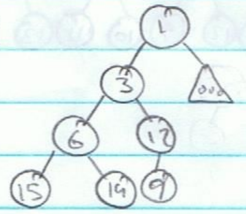
$\because 3 < 14$, swap(3, 14)



$\because 3 < 6$, swap(3, 6)

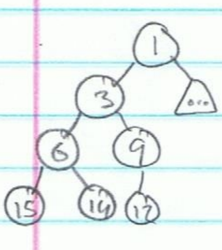


insert 9:

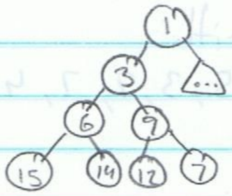


$\because 9 < 12$, swap(9, 12)

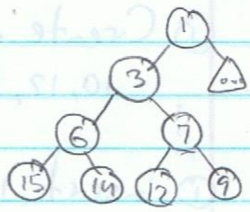
(→)



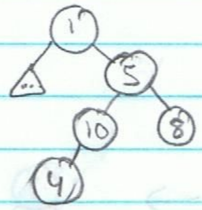
insert 7:



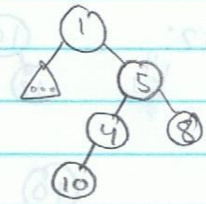
∵ 7<9,
swap(7,9)
→



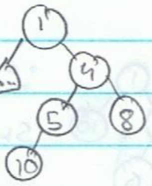
insert 4:



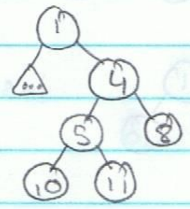
∵ 4<10,
swap(4,10)
→



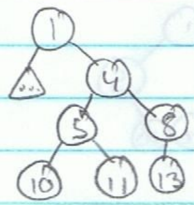
∵ 4<5,
swap(4,5)



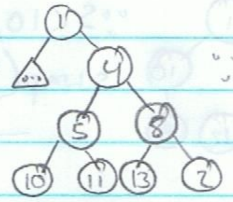
insert 11:



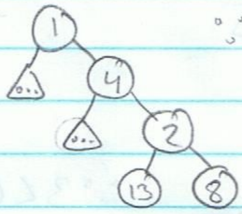
insert 13:



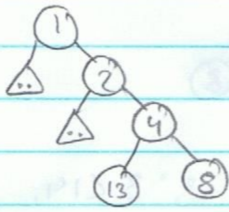
insert 2:



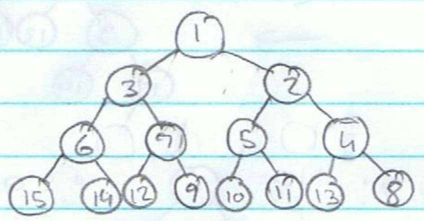
∵ 2<8,
swap(2,8)



∵ 2<4,
swap(2,4)
→



Final heap:



② Prove the following regarding the max item in a min-heap w/ N items.

a) It must be at one of the leaves.

Pf. Assume the contrary, suppose the maximum item in a min-heap is not one of the leaves.
Let max(N) := maximum item in a min-heap w/ N items.

$\therefore$ if max(N) $\neq$ leaf, then it has to be the root
$\because$ in a min-heap w/ only max(N), max(N) is the root.
when inserting another node $n <$ max(N), n has to be placed on the left of the root
$\because$ By def. of min-heap, if $n <$ max(N), n must be promoted up.
$\therefore$ Doing so, it would demote max(N) to a leaf.
$\therefore$ By contradiction, the maximum item in a min-heap must be a leaf, unless it is the only node $\square$

b) There are exactly $\lceil N/2 \rceil$ leaves.

Pf. Let T := binary min heap, and leaves(T) := number of leaves of T.

$\therefore$ T is a tree w/ N nodes and leaves(T) = $\lceil N/2 \rceil$ leaves, and T_L and T_R are its immediate sub-binary trees.
$\therefore$ $T = T_R + T_L +$ root, for height(T) > 1.
$\because$ when $N = 1$, leaves(T) $= \lceil N/2 \rceil = \lceil 1/2 \rceil = 1$
$\therefore$ Assume leaves(T) $= \lceil N/2 \rceil$ holds $\forall k \in \mathbb{N}$.
$\therefore$ Consider $k+1$:
$\because$ $k \geq 1$, height(T) > 1 and $T = T_R + T_L +$ root
$\therefore$ $T_L \leq \lceil N_L/2 \rceil$ and $T_R \leq \lceil N_R/2 \rceil$
$\therefore$ leaves(T) = leaves(T_L) + leaves(T_R)

$$\leq \lceil N_L/2 \rceil + \lceil N_R/2 \rceil$$
$$\leq \left\lceil \frac{N_L + N_R + 1}{2} \right\rceil = \lceil N_T/2 \rceil$$

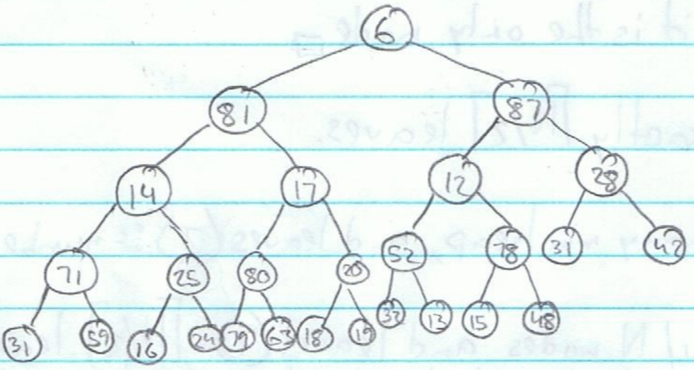
$\therefore$ A min-heap tree w/ N nodes has $\lceil N/2 \rceil$ leaves $\square$

c) Every leaf must be examined to find it.

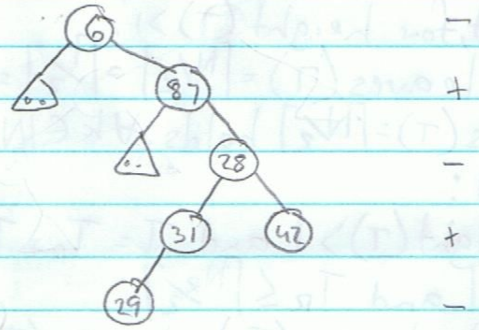
Pf. $\because$ By definition of binary min-heap, the parent node $\leq$ its children node.
$\therefore$ The maximum item in a min-heap w/ N nodes, max(N), is at the h^{th} level of the heap of height h.
$\because$ Only leaves exist at level h
$\therefore$ The entire level h must be scanned to find max(N), (which would scale to $O(n)$).

$\therefore$ Yes, every leaf must be examined $\square$

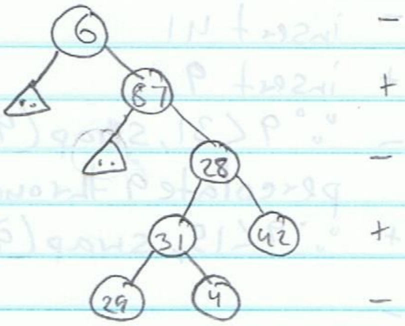
③ Insert 29, 4, 90, 5, 41, 9, 93, 67 into the min-max heap:



insert 29:



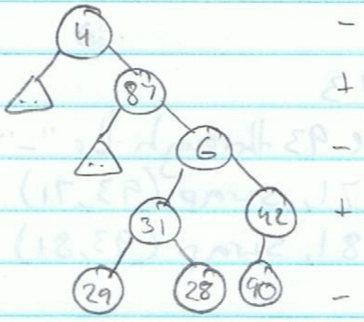
insert 4:



percolate 4 through the "-" levels

$\because 4 < 28$, swap(4,28)

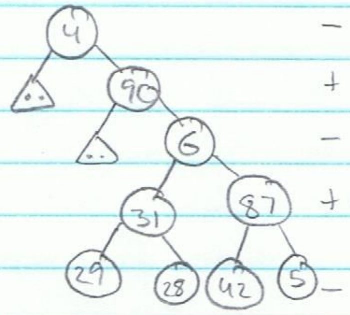
$\because 4 < 6$, swap(4,6)



insert 90

$\because 90 < 42$, swap(90,42)

percolate 90 through the "+" levels

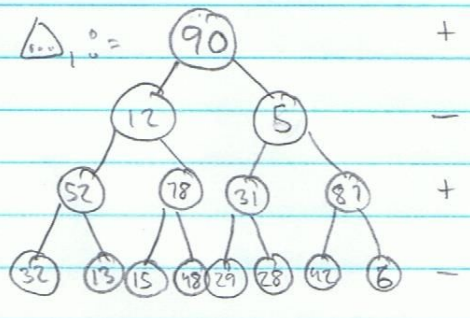


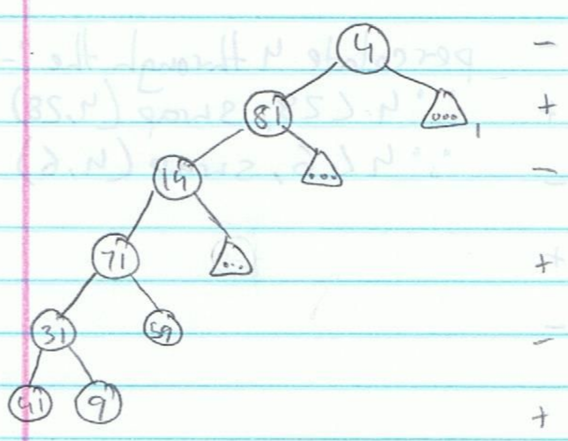
insert 5

percolate 5 through the "-" levels

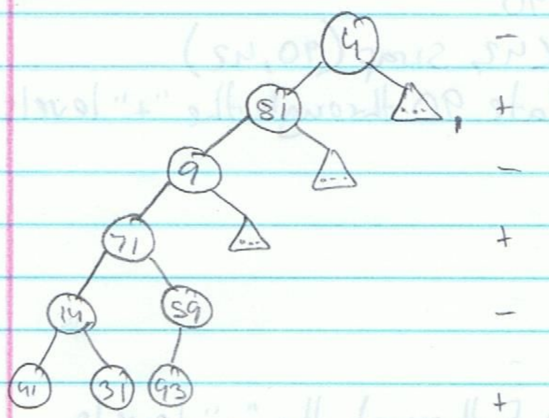
$\because 5 < 6$, swap(5,6)

Let $\triangle_1 :=$

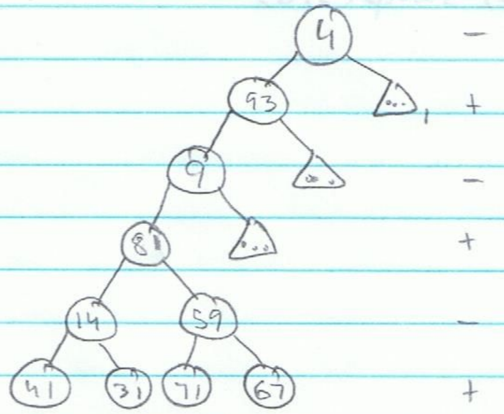




insert 41
insert 9
∵ 9 < 31, swap(9,31)
percolate 9 through the "–" levels
∵ 9 < 15, swap(9,14)



insert 93
percolate 93 through the "–" levels
∵ 93 > 71, swap(93,71)
∵ 93 > 81, swap(93,81)



insert 67

Final min-max heap!