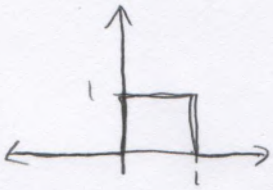
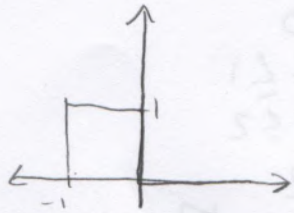
Simple Pulses:

$$x(t) = p(t,1) = u(t) - u(t-1)$$
$$h(t) = p(t,1) = u(t) - u(t-1)$$
$$x(t) * h(t) = y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$\therefore$ $x(t)$

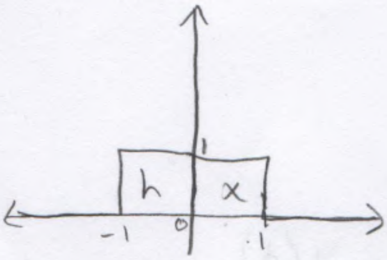


$h(-t)$



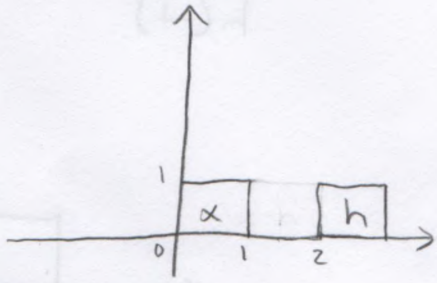
* for $t \leq 0$, $y(t)$:

$$\int_{-\infty}^{0} x(\tau)h(t-\tau)\,d\tau = 0$$



* for $t-1 > 1 \Rightarrow t > 2$:

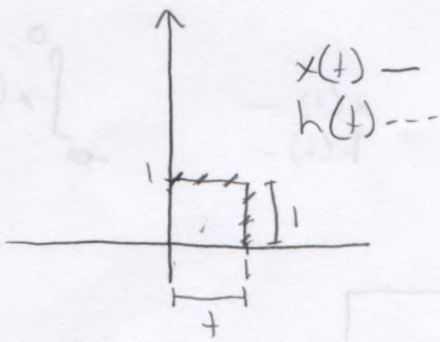
$$\int_{2}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0$$



* for $0 < t \leq 1$:

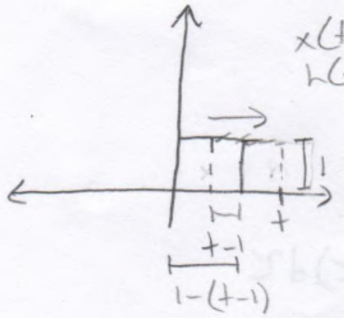
area: $(1)(t) = t$

$$\int_{0}^{1} x(\tau)h(t-\tau)\,d\tau = t$$



*for $1 < t \leq 2$:

area: $(1)(1-(t-1))$
$= 2-t$



$$\int_1^2 x(\tau)h(t-\tau)\,d\tau = 2-t$$

$$\therefore y(t) = \begin{cases} 0, & t \leq 0 \\ t, & 0 < t \leq 1 \\ 2-t, & 1 < t \leq 2 \\ 0, & t > 2 \end{cases}$$ $\square$

Unequal pulses:

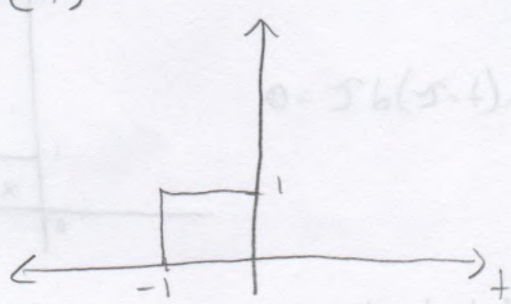
$x(t) = p(t,2) = u(t) - u(t-2)$

$h(t) = p(t,1) = u(t) - u(t-1)$

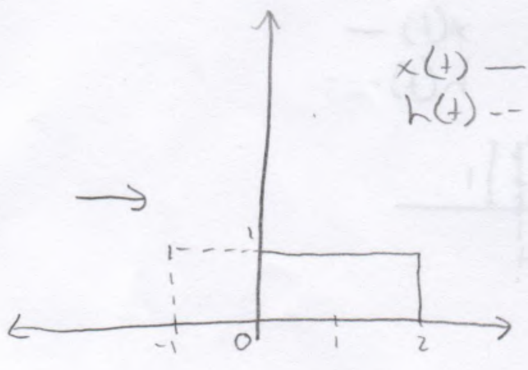
$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = y(t)$$

$\therefore$ $x(t)$

$h(-t)$



*for $t \leq 0$:

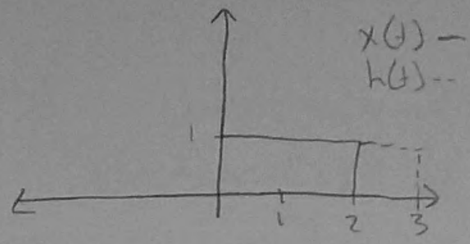


x(t) —
h(t) --

$$\int_{-\infty}^{0} x(\tau)h(t-\tau)\,d\tau = 0$$

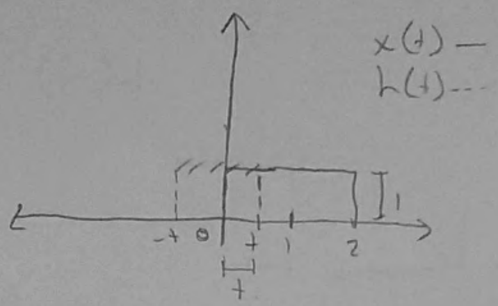
* for $t-1>2 \Rightarrow t>3$

$$\int_{3}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0$$



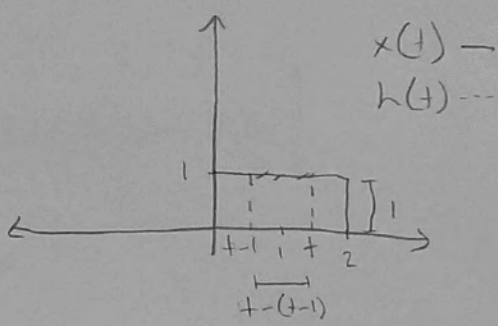
* for $0 \le t \le 1$

$$\int_{0}^{1} x(\tau)h(t-\tau)\,d\tau = (t)(1) = t$$



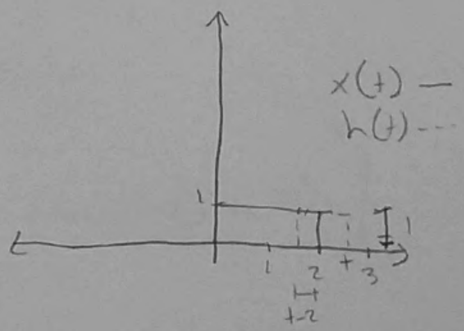
* for $1 < t \le 2$

$$\int_{1}^{2} x(\tau)h(t-\tau)\,d\tau = (1)(1) = 1$$



* for $2 < t \le 3$

area: $(1)(1-(t-2))$
$= 3-t$



$$\therefore y(t) = \begin{cases} 0, & t<0 \\ t, & 0 \le t \le 1 \\ 1, & 1 < t \le 2 \\ 3-t, & 2 < t \le 3 \\ 0, & t > 2 \end{cases}$$

□

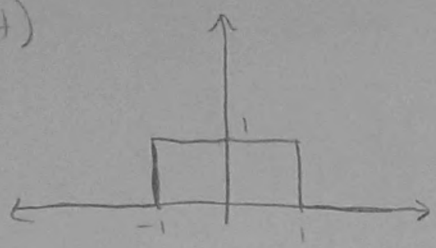
Offset input:

$$x(t) = p(t+1,2) = u(t+1) - u(t+1-2) = u(t+1) - u(t-1)$$

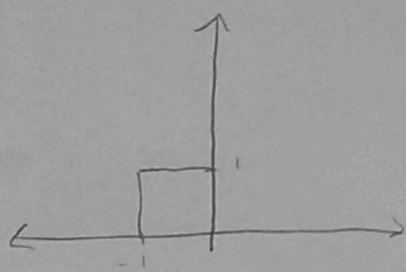
$$h(t) = p(t,1) = u(t) - u(t-1)$$

$$x(t) * h(t) = y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

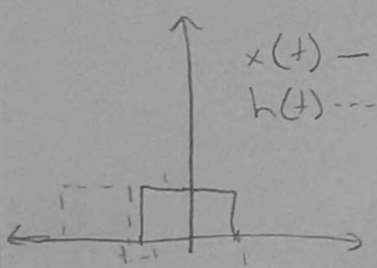
$x(t)$



$h(-t)$

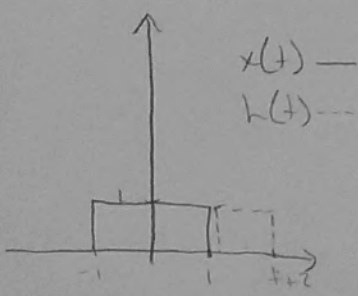


* for $t < -1$



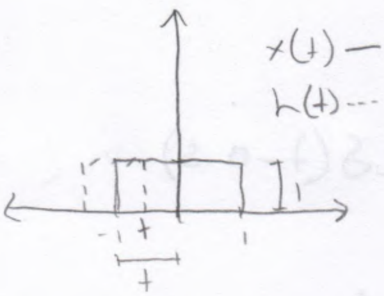
$$\int_{-\infty}^{-1} x(\tau)h(t-\tau)d\tau = 0$$

* for $t-1 > 1 \Rightarrow t > 2$



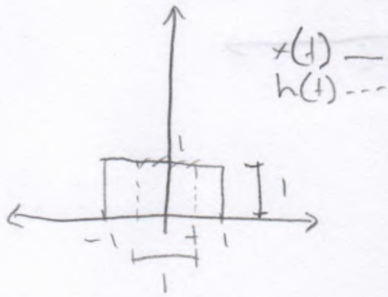
$$\int_{2}^{\infty} x(\tau)h(t-\tau)d\tau = 0$$

* for $-1 \leq t < 0$



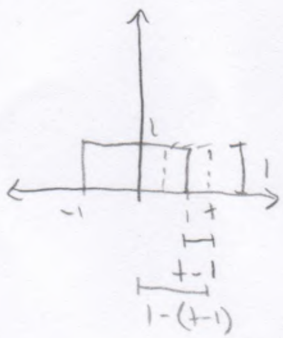
$$\int_{-1}^{0} x(\tau)h(t-\tau)\,d\tau = t$$

* for $0 \leq t < 1$



$$\int_{0}^{1} x(\tau)h(t-\tau)\,d\tau = 1$$

* for $1 \leq t < 2$



$$\int_{1}^{2} x(\tau)h(t-\tau)\,d\tau = (1)(1-(t-1)) = 2-t$$

$$\therefore y(t) = \begin{cases} 0, & t < -1 \\ t, & -1 \leq t \leq 0 \\ 1, & 0 \leq t < 1 \\ 2-t, & 1 \leq t < 2 \\ 0, & t > 2 \end{cases}$$

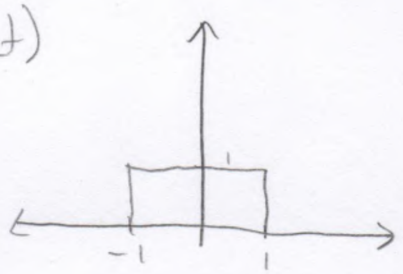
□

Offset input and offset impulse response:

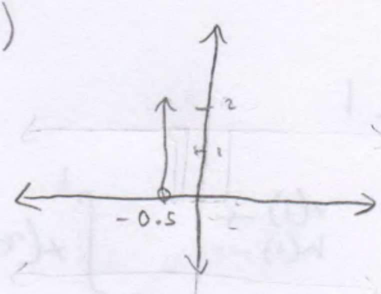
$x(t) = p(t+1, 2) = u(t+1) - u(t-1)$

$h(t) = 2p(t-0.5, 0) = 2u(t-0.5) - 2u(t-0.5) = 2\delta(t-0.5)$

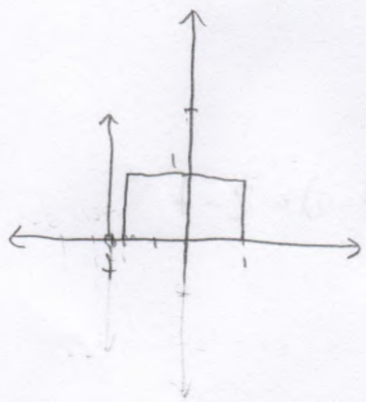
$\therefore x(t)$



$h(t)$

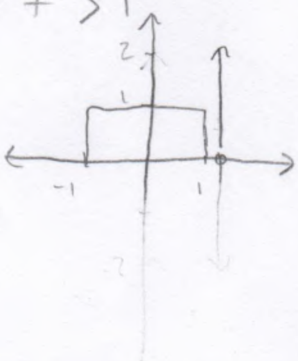


* for $t < -1$



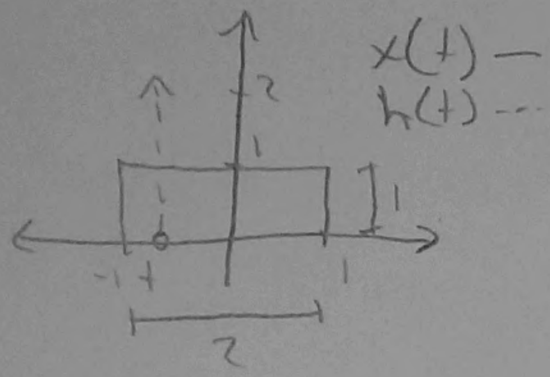
$$\int_{-\infty}^{-1} x(\tau)h(t-\tau)\,d\tau = 0$$

* for $t > 1$



$$\int_{1}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0$$

* for $-1 \le t \le 1$



$$\int_{-1}^{1} x(\tau)\, h(t-\tau)\, d\tau = 2$$

$$\therefore y(t) = \begin{cases} 0, & t < -1 \\ 2, & -1 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

$\square$