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CMPE 320 HW 02

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1. The following problem was given to 60 students and doctors at the famous Hevardi Medical School (HMS): Assume there exists a test to detect a disease, say D, whose prevalence is 0.001, that is, the probability, $P[D]$, that a person picked at random is suffering from D, is 0.001 .The test has a false positive rate of 0.005 and a correct detection rate of 1. The correct detection rate is the probability that if you have D, the test will say that you have D. Given that you test positive for D, what is the probability that you actually have it? Many of the HMS experts answered 0.95 and the average answer was 0.56. Show that your knowledge of probability is greater than that of the HMS experts by getting the right answer of 0.17.

Ans Given:

$P(D) = 0.001$, $P(D^C) = 1 - P(D) = 0.999$

$P(\text{positive} \mid D) = 1$, $P(\text{positive} \mid D^C) = 0.005$ Therefore, $P(D \mid \text{positive})$:

$$\begin{aligned} P(D \mid \text{positive}) &= \frac{P(\text{positive} \mid D)P(D)}{P(\text{positive} \mid D)P(D) + P(\text{positive} \mid D^C)P(D^C)} \\ &= \frac{(1)(0.001)}{(1)(0.001) + (0.005)(0.999)} \\ &= 0.1668 \\ &\approx 0.17 \end{aligned}$$

□

2. In the ternary communication channel shown below, a 3 is sent three times more frequently than a 1, and a 2 is sent two times more frequently than a 1. A 1 is observed; what is the conditional probability that a 1 was sent?

Ans Let $P(1) = x$, so $P(2) = 2x$ and $P(3) = 3x$.

$P(1) + P(2) + P(3) = 1$

$x + 2x + 3x = 1 \Rightarrow x = 1/6$

Therefore, $P(1) = x = 1/6$

$P(2) = 2x = 1/3$

$P(3) = 3x = 1/2$

Therefore:

$$
\begin{aligned}
P(\text{1 sent} \mid \text{1 obs}) &= \frac{P(\text{1 sent and 1 obs})}{P(\text{1 obs})} \\
&= \frac{P(\text{1 obs} \mid \text{1 sent})P(\text{1 sent})}{P(\text{1 obs} \mid \text{1 sent})P(\text{1 sent}) + P(\text{1 obs} \mid \text{2 sent})P(\text{2 sent}) + P(\text{1 obs} \mid \text{3 sent})P(\text{3 sent})} \\
&= \frac{(1-\alpha) \cdot \frac{1}{6}}{(1-\alpha) \cdot \frac{1}{6} + \frac{\beta}{2} \cdot \frac{1}{3} + \frac{\gamma}{2} \cdot \frac{1}{2}} \\
&= \frac{\frac{(1-\alpha)}{6}}{\frac{(1-\alpha)}{6} + \frac{\beta}{6} + \frac{\gamma}{4}} \\
&= \frac{2(1-\alpha)}{-2\alpha + 2\beta + 3\gamma + 2}
\end{aligned}
$$

□

3. A large class in probability theory is taking a multiple-choice test. For a particular question on the test, the fraction of examinees who know the answer is p; $1-p$ is the fraction that will guess. The probability of answering a question correctly is unity for an examinee who knows the answer and $1/m$ for a guessee; m is the number of multiple-choice alternatives. Compute the probability that an examinee knew the answer to a question given that he or she has correctly answered it.

Ans Let K represent the examinee knowing the correct answer, and A represent the examinee answering the correct answer.
Therefore, $P(K \cap A) = 1$ since the examinee answered correctly if they knew the correct answer. $P(A)$ consists of the total examinees who know the correct answer and those who guessed with the $1/m$ probability.

$$
\begin{aligned}
P(K \mid A) &= \frac{P(K \cap A)}{P(A)} \\
&= \frac{1}{p + (1-p)\frac{1}{m}} \\
&= \frac{1}{p + \frac{1}{m} - \frac{p}{m}} \\
&= \frac{m}{mp - p + 1}
\end{aligned}
$$

□

4. Assume there are three machines A, B, and C in a semiconductor manufacturing facility that makes chips. They manufacture, respectively, 25, 35, and 40 percent of the total semiconductor chips there. Of their outputs, respectively, 5, 4, and 2 percent of the chips are defective. A chip is drawn randomly from the combined output of the three machines and is found defective. What is the probability that this defective chip is manufactured by

machine A? by machine B? by machine C?

Ans Given: $P(A) = 0.25, P(B) = 0.35, P(C) = 0.40.$

Let D represent the event of finding a defective chip.

Therefore $P(D \mid A) = 0.05, P(D \mid B) = 0.04, P(D \mid C) = 0.02.$

$$\begin{aligned}
P(A \mid D) &= \frac{P(D \mid A)P(A)}{P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)} \\
&= \frac{(0.05)(0.25)}{(0.05)(0.25) + (0.04)(0.35) + (0.02)(0.40)} \\
&= 0.3623
\end{aligned}$$

$$\begin{aligned}
P(B \mid D) &= \frac{P(D \mid B)P(B)}{P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)} \\
&= \frac{(0.04)(0.35)}{(0.05)(0.25) + (0.04)(0.35) + (0.02)(0.40)} \\
&= 0.4058
\end{aligned}$$

$$\begin{aligned}
P(C \mid D) &= \frac{P(D \mid C)P(C)}{P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)} \\
&= \frac{(0.02)(0.40)}{(0.05)(0.25) + (0.04)(0.35) + (0.02)(0.40)} \\
&= 0.2319
\end{aligned}$$

□

5. A card is randomly selected from a standard deck of 52 cards. Let A be the event of selecting an ace and let B be the event of selecting a red card. There are 4 aces and 26 red cards in the normal deck. Are A and B independent?

Ans Given: $P(A) = 4/52$, $P(B) = 26/52$.

If $P(A \cap B) = P(A) \cdot P(B)$, then A and B are independent.

Since $A \cap B$ is equivalent to selecting red aces, $A \cap B = 2$.

Therefore,

$$\begin{aligned}
P(A \cap B) &= \frac{2}{52} \\
&= \frac{1}{26}
\end{aligned}$$

□

6. A fair die is tossed three times. Given that a 2 appears on the first toss, what is the proba-

bility of obtaining the sum 7 on the three tosses?

Ans Since the first toss resulted in a 2, the other tosses have to total to $7 - 2 = 5$ for the sum of the three tosses to be 7.

Therefore the combinations: $(1,4),(4,1),(2,3),(3,2)$ are the only ones that would work.

$\therefore P(\text{sum = 7}) = 4/(3 \cdot 12) = 4/36 = 1/9$ $\square$

7. An Internet access provider (IAP) owns two servers. Each server has a 50% chance of being "down" independently of the other. Fortunately, only one server is necessary to allow the IAP to provide service to its customers, i.e., only one server is needed to keep the IAP's system up. Suppose a customer tries to access the Internet on four different occasions, which are sufficiently spaced apart in time, so that we may assume that the states of the system corresponding to these four occasions are independent. What is the probability that the customer will only be able to access the Internet on 3 out of the 4 occasions?

Ans Since $P(\text{Internet access}) = 1 - P(\text{no Internet access})$

$\Rightarrow 1 - P(\text{both servers not working})) = 1 - (0.5)(0.5) = 0.75$

Therefore, $P(\text{any one server not working})$:

$$\begin{aligned} P(\text{any one server not working}) &= \binom{4}{3} \cdot P(\text{no Internet access}) \cdot P(\text{Internet access}) \\ &= \binom{4}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^1 \end{aligned}$$ $\square$

8. A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are 1 × 1.5 inches, the faces showing 2 or 5 are 1 × 0.4 inches, the faces showing 3 or 4 are 0.4 × 1.5 inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?

Ans Area of faces showing 1 or 6: (1×1.5) in$^2 = 1.5$ in^2

Area of faces showing 2 or 5: (1×0.4) in$^2 = 0.4$ in^2

Area of faces showing 3 or 4: (0.4×1.5) in$^2 = 0.6$ in^2

Total area: $2(1.5 + 0.4 + 0.6) = 5$ in^2.

$$\begin{aligned} P(\text{1 or 6}) &= \frac{1.5}{5} = 0.3 \\ P(\text{2 or 5}) &= \frac{0.4}{5} = 0.08 \end{aligned}$$

$$P(\text{1 or 6}) = \frac{0.6}{5} = 0.12$$

$$\begin{aligned}
P(\text{doubles}) &= P(1)P(1) + P(2)P(2) + P(3)P(3) + P(4)P(4) + P(5)P(5) + P(6)P(6) \\
&= 2(0.3^2) + 2(0.08^2) + 2(0.12^2) \\
&= 0.2216
\end{aligned}$$

$\therefore P(\text{doubles}) = 0.2216$ $\square$