

Time Averages of Time Waveforms

- We are usually interested in the energy contained in a waveform or signal...
- ...as energy is the key to overcoming noise in the system.

$$\mathcal{E} = \int_{-\infty}^{\infty} x(t)x^*(t)dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \stackrel{\substack{\text{real} \\ \text{valued} \\ \text{signal}}}{=} \int_{-\infty}^{\infty} x^2(t)dt$$

- If our signal has units of volts, then \mathcal{E} has units of volt²-sec, which is not joules!
- When computing the mathematical energy (as above) we assume there is a 1Ω resistor as a scale factor, giving volt²-sec/ohm = watt-sec = joules!
- Using this definition, the energy in a periodic signal is infinite!! (Why?)

- For periodic signals, we compute the “average energy per period”

$$\frac{\mathcal{E}}{T} = \frac{1}{T} \int_{\alpha}^{T+\alpha} x(t)x^*(t)dt = \frac{1}{T} \int_{\alpha}^{T+\alpha} |x(t)|^2 dt \stackrel{\substack{\text{real} \\ \text{valued} \\ \text{signal}}}{=} \frac{1}{T} \int_0^T x^2(t)dt$$

- Again using a 1Ω resistor as a scale factor, we wind up with volt²-sec/ohm-sec=watt-sec/sec=watt...
- ...so this is power!
- A signal with finite energy is called an energy signal
- A signal with meaningful power is called a power signal.
- Some signals are neither power or energy signals, but these are rare.

Examples

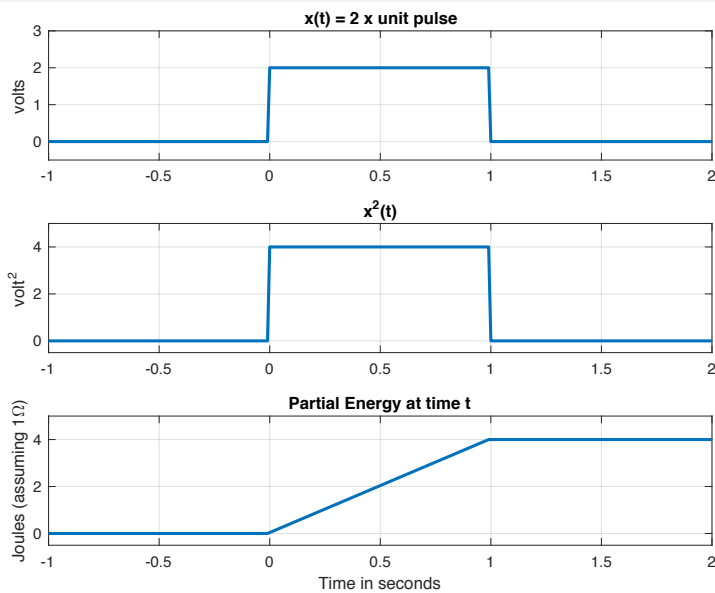
▪ Unit pulse

$$p(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = 2p(t)$$

$$x^2(t) = \begin{cases} 4 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{E} = \int_{-\infty}^{\infty} x^2(t) dt = \int_0^1 4 dt = 4 \text{ joules}$$



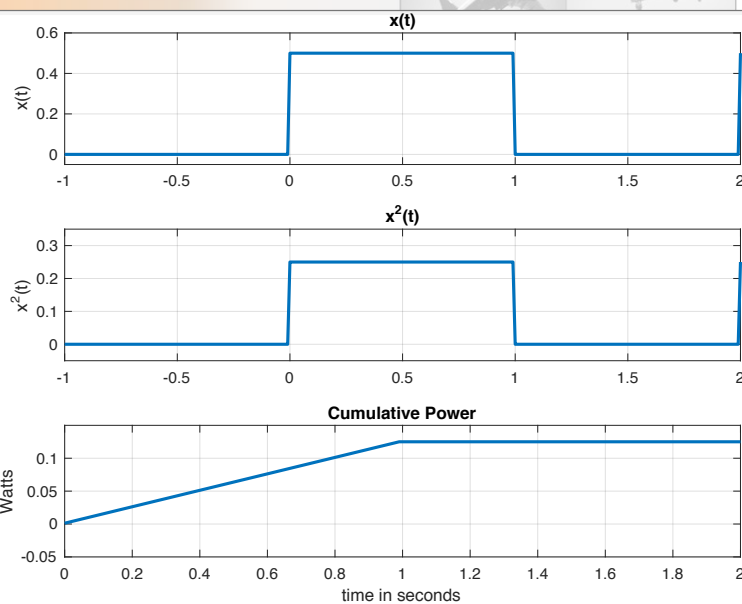
▪ Pulse train

$$p(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} 0.5p(t-2n)$$

$$x^2(t) = \sum_{n=-\infty}^{\infty} 0.25p(t-2n)$$

$$P = \frac{1}{2} \int_0^2 0.25 dt = 0.125 \text{ watts}$$



You try

- Compute the power or energy, as appropriate

$$a(t) = \begin{cases} e^{-0.5t} & 0 \leq t < 2 \\ 0 & \text{o/w} \end{cases} \quad (\text{exponential pulse})$$

$$b(t) = \sum_{k=-\infty}^{\infty} a(t-4k)$$

$$c(t) = 120 \cos(120\pi t + \pi/3)$$

$$d(t) = \begin{cases} 120 \cos(120\pi t) & -\frac{1}{240} \leq t < \frac{1}{240} \\ 0 & \text{o/w} \end{cases}$$

$$f(t) = \begin{cases} 0.25t & 0 \leq t < 4 \\ 0 & \text{o/w} \end{cases}$$

$$g(t) = \sum_{k=-\infty}^{\infty} f(t-16k)$$

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$a(t)$ is an energy signal

$$\mathcal{E} = \int_0^2 (e^{-0.5t})^2 dt = \int_0^2 (e^{-1t}) dt = 1 - e^{-2}$$

$b(t)$ is periodic with period 4, therefore it is a power signal

$$P = \frac{1}{4} \int_0^4 (b(t))^2 dt = \frac{1}{4} \left(\int_0^2 (e^{-0.5t})^2 dt + \int_2^4 (0)^2 dt \right) = \frac{1 - e^{-2}}{4}$$

$c(t)$ is periodic with period $\frac{2\pi}{120\pi} = \frac{1}{60}$. The phase does not affect

the period, $c(t)$ is a power signal

$$\text{Let } 120\pi t_0 = \frac{\pi}{3} \Rightarrow t_0 = \frac{1}{360}, \quad c(t) = 120 \cos(120\pi(t + t_0))$$

$$P = \left(\frac{1}{60} \right) 120^2 \int_{-1/360}^{-1/360+1/60} \cos^2(120\pi(t + t_0)) dt = \left(\frac{1}{60} \right) 120^2 \int_0^{1/60} \cos^2 120\pi\tau d\tau$$

$$= \left(\frac{1}{60} \right) 120^2 \int_0^{1/60} \left(\frac{1}{2} + \frac{1}{2} \cos 240\pi\tau \right) d\tau$$

$$= 120^2 \frac{((1/60) - 0)}{2 \times 60} + 120^2 \frac{\left(\sin \frac{240\pi}{60} - \sin 0 \right)}{2 \times 60 \times 240\pi} = \frac{120^2}{2} = 7200 \text{ watts}$$

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Manipulation of the Time Axis

CMPE323



We've been talking about functions of time

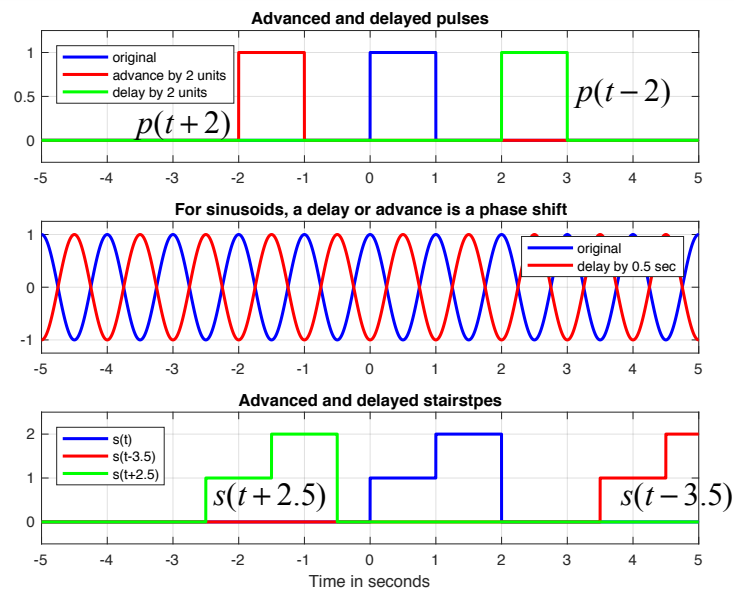
- Unit Step
- Unit Impulse
- Sinusoid
- Complex Exponential
- Unit pulse of duration T $p(t;T) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$
- Now let's examine what happens as we manipulate the time axis...
- ...that is, as we change the *argument* of the function
- We'll use the unit pulse for examples

Basic manipulations

- **Advance** – we advance a waveform when we move all of its features to occur at an *earlier* time (shift to the left), this shifts in the direction of *decreasing* time
- **Delay** – we delay a waveform when we move all of its features to occur at a later time, shift in direction of *increasing* time
- **Reversal** – we reverse a waveform when we change the sign of the time variable (and only the time variable!)
- **Expansion** – we expand a waveform if we increase its duration, that is, if we move its features farther apart in time
- **Contraction** – we contract a waveform if we decrease its duration, that is, if we move its features closer together in time.
- **Linear adjustment** – a linear adjustment in time is a combination of advance/delay and expand/contract

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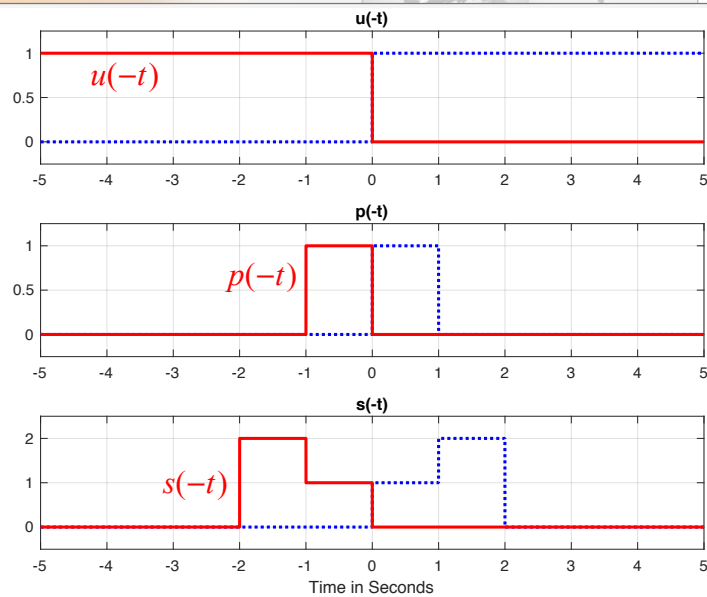


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A reversal does just that

- By changing t to $-t$ in the waveform argument, you make time “increase” from right to left...
- ...that is, you reflect the waveform around the line $t = 0$
- We saw this in CMPE306, when we used $u(-t)$ to turn a voltage source off at $t = 0$
- When we make the $t \rightarrow -t$ swap, it affects *only* t , *not the entire argument*



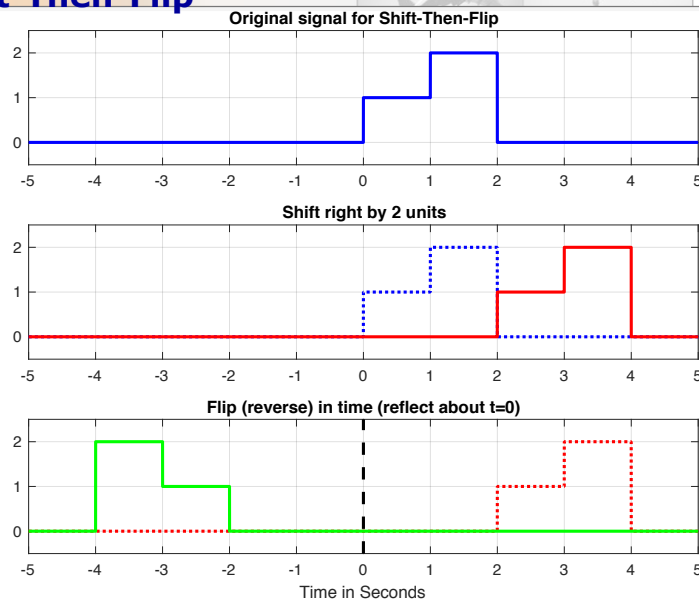
Combined reversal and shift

- Of course we can combine things, but we have to be careful in how we define what we're doing!!
- There are three ways to look at it (I like way 1)
- "Shift then flip"
 - $p(-t-2) = p(\tau-2)_{\tau=-t}$ shift $p(\tau)$ to the right and then reverse, reflecting about $\tau = 0$.
- "Flip then shift"
 - $p(-t-2)$ = reverse to get $p(-t)$, then shift toward increasing values of $-t$ (effectively "left"!)
- "Flip shifted"
 - $p(-t-2) = p(-(t+2))$ shift to left by 2, then reverse around the $t = -2$ axis ($t = -2 \Rightarrow t+2 = 0$)

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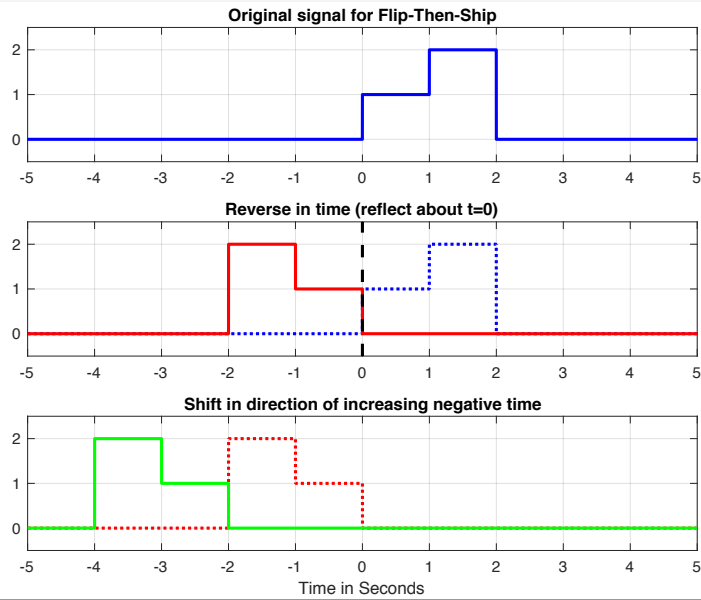
Shift-Then-Flip



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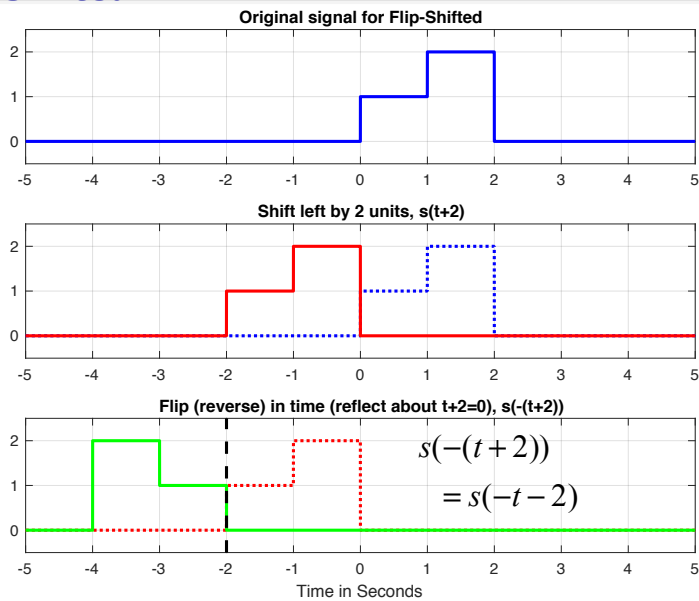
Flip-then-Shift



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Flip-Shifted

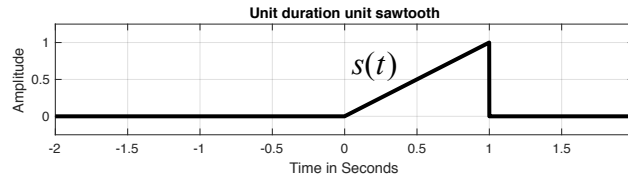


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Your turn

- In your groups, try the following. Use the unit duration unit sawtooth $s(t)$ shown below.



- Use different techniques to verify your answers

$$a(t) = s(-t + 2)$$

$$b(t) = s(-(t - 2))$$

$$c(\tau) = s(\tau - 3)$$

$$d(\tau) = s(3 - \tau)$$

$$e(\tau) = s(t - \tau), \text{ for some fixed value of } t.$$

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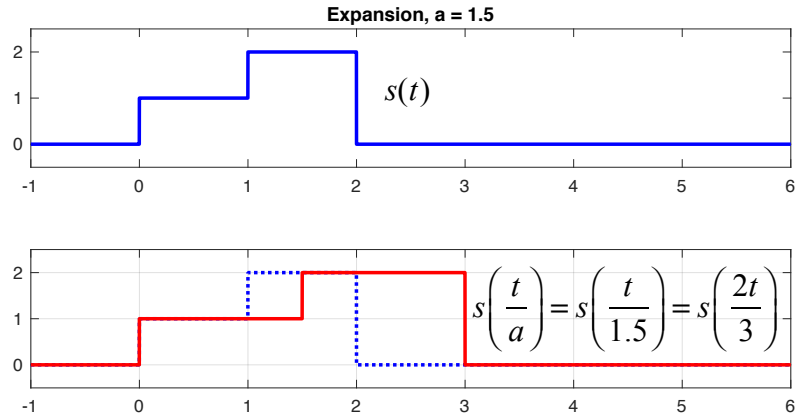
Compression and expansion

- We now *scale* the time axis by a multiplicative constant, creating $x(at)$
- If $0 < |a| < 1$, this creates an expansion of the waveform
- If $|a| > 1$ this creates a compression of the waveform
- If $a < 0$ we get both a reversal and an expansion/contraction
- $a = 1$ does nothing (obviously!)
- $a = -1$ is a simple reversal.

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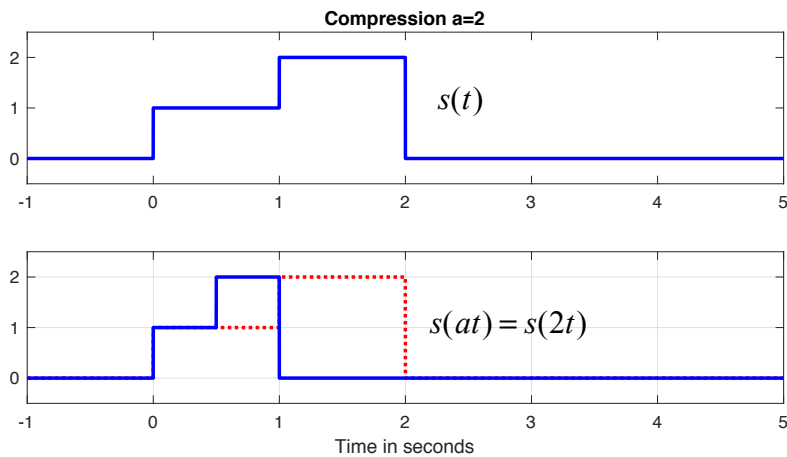
Expansion example



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Compression Example



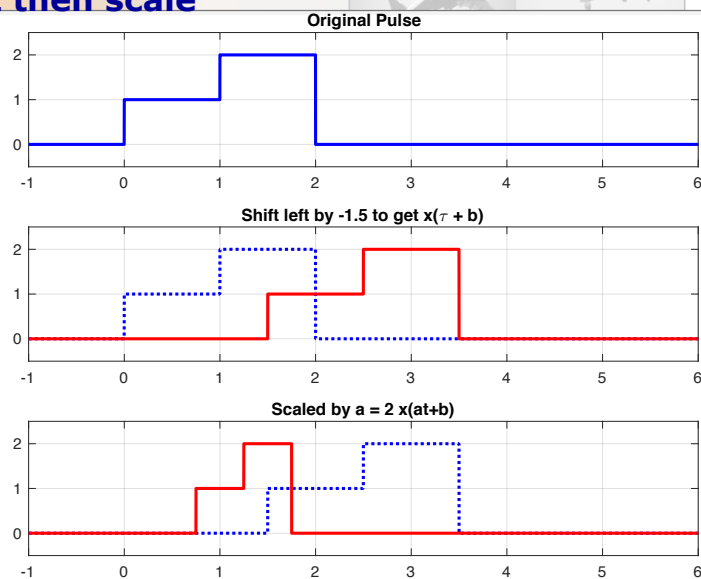
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Linear transformation of the time

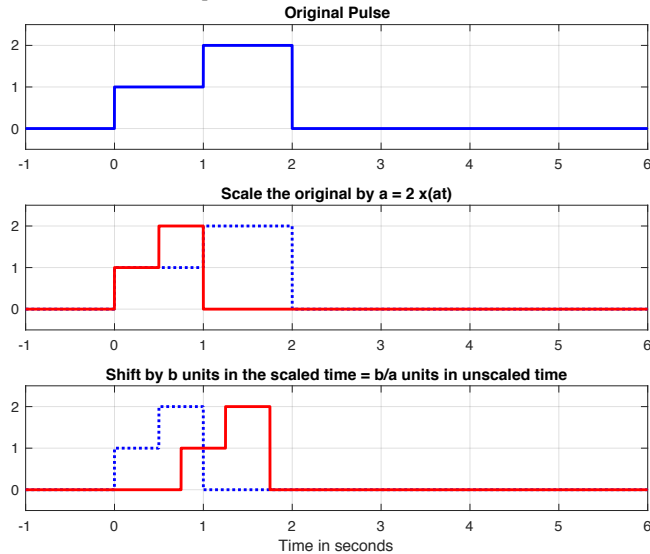
- Now we combine expansion/compression/reversal with offset $y(t) = x(at + b)$
- Use the similar techniques as with reversal + offset
 - Shift then scale = shift by b unscaled units then scale the result
 - Scale then shift = scale by a , then shift by b scaled units
- You must be very careful about what unit to use for the shift!

Shift then scale



Scale then shift

- You must shift by units in the scaled time axis!!!!



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You try

- Use the unit sawtooth to find the following

$$f(t) = s(0.5t - 3)$$

$$g(t) = s(2t + 5)$$

$$h(t) = s(-2t + 5)$$

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Even and Odd

- Finally, let's discuss even and odd functions
- For an even function $f(t) = f(-t)$
 - Examples $x(t) = t^2$, $x(t) = \cos(\omega t)$, $x(t) = A$
- For an odd function $f(t) = -f(-t)$
 - Examples $x(t) = t^3$, $x(t) = at$, $x(t) = \sin(\omega t)$
- But what about a general function, like $u(t)$?
- Any function can be decomposed into the sum of an even function and an odd function

$$f_E(t) = \frac{f(t) + f(-t)}{2}, f_O(t) = \frac{f(t) - f(-t)}{2}$$

$$f(t) = f_E(t) + f_O(t)$$

