1. possible values of $X = 0,1,2,3,4$ examples $X=0 \Leftrightarrow 0000$; $X=2 \Leftrightarrow 2009$

2. $Y = 1,2,3,4$ $P(Y=1) = P(a \text{ or } b \text{ are typified}) = \frac{2}{5} = 0.4$

$P(Y=2) = P(c,d \text{ or } e \text{ typified first, then } a \text{ or } b) = P(a \text{ or } b \text{ in second typification} \mid c,d \text{ or } e \text{ in } 1^{st}) \times P(c,d,e \text{ in first})$

$= \left(\frac{2}{4}\right)\left(\frac{3}{5}\right) = 6/20$

continue as the problem in the sample exam (problem 3)

3. a) $1 = \sum_{y=1}^{5} f(y) = \sum_{y=1}^{5} ky = k[1+2+3+4+5] = 15k \Rightarrow k = 1/15$

b) $P(Y \leq 3) = \sum_{y=1}^{3} y/15$ (In the exam I will accept answers in this form)

c) $f(y) \geq 0$, $\sum_{y=1}^{5} f(y) = \frac{1}{55}[1^2+2^2+3^2+4^2+5^2] = 55/55 = 1$

d) $E(Y) = \sum_{y=1}^{5} y f(y) = \frac{1}{55}[1^3+2^3+3^3+4^3+5^3]$

$V(Y) = E(Y^2) - (E(Y))^2$; $E(Y^2) = \sum_{y=1}^{5} y^2 f(y) = \frac{1}{55}[1^4+2^4+3^4+4^4+5^4]$

4. $E(X) = \sum_{x=1}^{\infty} x f(x) = c \sum_{x=1}^{\infty} x \cdot \frac{1}{x^2} = c \cdot \sum_{x=1}^{\infty} \frac{1}{x} = \infty$

If $E(X)$ is ∞, so is $E(X^2)$ so $V(X)$ does not exist

5. $E(X) = \sum_{x=1}^{n} x f(x) = \frac{1}{n} \sum_{x=1}^{n} x = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}$

$E(X^2) = \sum_{x=1}^{n} x^2 f(x) = \frac{1}{n} \sum_{x=1}^{n} x^2 = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$

$V(X) = E(X^2) - (E(X))^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{(n+1)}{12}[4n+2-3n-3] = \frac{n^2-1}{12}$

6. $27.5 = E(X(X-1)) = E(X^2 - X) = E(X^2) - E(X) = E(X^2) - 5 \Rightarrow E(X^2) = 32.5$

$V(X) = E(X^2) - (E(X))^2 = 32.5 - 5^2 = 32.5 - 25 = 7.5$

7. The problem has two parts. First part is to figure out the probability that a randomly selected flashlight will work

$P[\text{flashlight works}] = P[\text{Battery 1 works} \cap \text{Battery 2 works}] = P[\text{Battery 1 works}]\,P[\text{Battery 2 works}]$

$= (0.9)^2$ [Here we have assumed batteries work INDEPENDENTLY]

Second part is to get probability distribution of total # of flashlights working, say X, in 10 randomly selected lights. Each flashlight is like a BERNOULLI TRIAL with probability of success (i.e working) equal to $p = (0.9)^2$

Then $X \sim \text{BINOMIAL}(10, p)$ [assuming independence of flashlights]

$P[\text{at least nine}] = P[X \geq 9] = \sum_{x=9}^{10} \binom{10}{x} p^x (1-p)^{10-x}$ with $p = (0.9)^2$

8. Group CASH or DEBIT together. So each payment is a Bernoulli trial with success if paid by cash or debit, Failure if paid by credit

$p = \text{Prob}[\text{Success}] = P[\text{cash or Debit}] = P(\text{cash}) + P(\text{Debit}) = 0.2 + 0.3 = 0.5$

so X = # of customers in next 100 trials who pay by cash or Debit ~~BINOMIAL~~

$\sim \text{BINOMIAL}(100, p)$

$E(X) = np = (100)(0.5) = 50$, $V(X) = np(1-p) = (100)(0.5)(1-0.5)$

$= 25$