CMPE 330 **Spring 2015** **Problem Set #4**

NOTE: You must show complete work for full credit.

1. Given vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}4$, and $\mathbf{C} = \hat{\mathbf{y}}3 - \hat{\mathbf{z}}4$, find the following, [modified from Ulaby et al. 3.5, p. 169]
   a. A and $\hat{\mathbf{a}}$
   b. The component of $\mathbf{B}$ along $\mathbf{C}$
   c. θ_{AC}
   d. $\mathbf{A} \times \mathbf{C}$
   e. $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$
   f. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$
   g. $\hat{\mathbf{x}} \times \mathbf{B}$
   h. $(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}}$
2. Prove that the absolute value of the scalar triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ equals the volume of the parallelepiped whose edges correspond to the distance vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$.
3. A plane is described by $2x + 3y + 4z = 16$, find the unit vector normal to the surface in the direction away from the origin. [Ulaby et al. 3.15, p. 169]
4. Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates, [modified from Ulaby et al. 3.22, p. 170.]
   a. $P_1(1, 2, 0)$
   b. $P_2(0, 0, 3)$
   c. $P_3(1, 1, 2)$
   d. $P_4(-3, 3, -3)$
5. Use the appropriate expression for the surface area $d\mathbf{s}$ to determine the area of each of the following surfaces, [Ulaby et al. 3.25, p. 171]
   a. $r = 3$; $0 \le \phi \le \pi/3$; $-2 \le z \le 2$
   b. $2 \le r \le 5$; $\pi/2 \le \phi \le \pi$; $z = 0$
   c. $2 \le r \le 5$; $\phi = \pi/4$; $-2 \le z \le 2$
   d. $R = 2$; $0 \le \theta \le \pi/3$; $0 \le \phi \le \pi$
   e. $0 \le R \le 5$; $\theta = \pi/3$; $0 \le \phi \le 2\pi$

6. Find the distance between the following pairs of points, [Ulaby et al. 3.31, p. 171]
   a. $P_1(1, 2, 3)$ and $P_2(-2, -3, 2)$ in Cartesian coordinates
   b. $P_3(1, \pi/4, 2)$ and $P_4(3, \pi/4, 4)$ in cylindrical coordinates
   c. $P_5(2, \pi/2, 0)$ and $P_6(3, \pi, 0)$ in spherical coordinates

7. Find the gradient of the following scalar functions, [modified from Ulaby et al. 3.32, p. 172]
   a. $T = 2/(x^2 + z^2)$
   b. $V = xy^2z^3$
   c. $U = z\cos\phi/(1 + r^2)$
   d. $W = \exp(-R)\sin\theta$
   e. $S = x^2\exp(-z) + y^2$
   f. $N = r^2\cos\phi$
   g. $M = R\cos\theta\sin\phi$

8. Vector field $\mathbf{E}$ is characterized by the following properties: (a) $\mathbf{E}$ points along $\hat{\mathbf{R}}$; (b) the magnitude of $\mathbf{E}$ is a function only of the distance from the origin; (c) $\mathbf{E}$ vanishes at the origin, and (d) $\nabla \cdot \mathbf{E} = 6$, everywhere. Find an expression for $\mathbf{E}$ that satisfies these properties. [Ulaby 3.45, p. 174]

9. A vector field $D = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylinder surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Verify the divergence theorem by evaluating the following, [Ulaby 3.48, p. 174]
   a. $\oint_S \mathbf{D} \cdot d\mathbf{s}$
   b. $\int_V \nabla \cdot \mathbf{D}\, dv$

10. For a scalar field $V(x, y, z)$, the gradient in Cartesian coordinates is given by [See Ulaby, Eq. 3.71]

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}.$$

Derive the gradient in spherical coordinates

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

in two ways.

   a. Use partial derivative transformations in the same way that Ulaby et al.'s Eqs. (3.82) and (3.83) were derived.
   b. Use the general result on slide 7.24.