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MATH 407 HW 03

1.2 **7** Let m and n be positive integers such that $m + n = 57$ and $[m, n] = 680$. Find m and n.

Ans $m + n = 57$ and $[m, n] = 680$

Let n be represented by m, such that,

$$m \cdot (57 - m) = 680$$

$$\Rightarrow 0 = m^2 - 57m + 680$$

$$= \frac{57 \pm \sqrt{57^2 - 4 \cdot 680}}{2}$$

$$= \frac{57 \pm 23}{2}$$

$$m = 40, 17$$

$\therefore m = 40$, $n = 17$ □

10 Show that $a\mathbf{Z} \cap b\mathbf{Z} = [a, b]\mathbf{Z}$.

Ans □

16 A positive integer a is called a **square** if $a = n^2$ for some $n \in \mathbb{Z}$. Show that the integer $a > 1$ is an integer if and only if every exponent in its prime factorization is even.

Ans □

20 A positive integer is called **square-free** if it is a product of distinct primes. Prove that every positive integer can be written uniquely as a product of a square and a square-free integer.

Ans □