CMPE 320: Probability, Statistics, and Random Processes

Lecture 1: Set operations

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Sets

- Set is a collection of elements $S = \{x_1, x_2, \cdots, x_n\}$

  $x \in S$, $x \notin S$

  $\{x_1, x_2, \cdots\}$: countably infinite

  $\{x \mid 0 \leq x < 1\}$: uncountable

- Empty set and universal set

  ϕ : empty set Ω : universal set

Subsets

- S is a subset of T means every element in S is also an element of T

$S = \{1, 2, 3\}, \quad T = \{1, 2, 3, 6\}$

$S \subset T \qquad T \supset S \qquad S, T \subset \Omega$

- Two sets S and T are equal means

$S = T \iff S \subset T$ and $T \subset S$

Set operations

- Set complement $S^c = \{x \mid x \notin S, x \in \Omega\}$

$\Omega = \{1, 2, 3, 4, 5\}, \quad S = \{1, 2, 4\}, \quad \underline{S^c = \{3, 5\}}$

$(S^c)^c = S$

- Union and intersection

$S \cup T = \{x \mid x \in S \text{ or } x \in T\} \qquad S \cup S^c = \Omega$

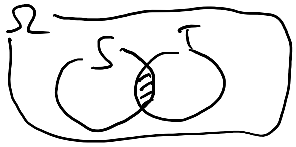
$S \cap T = \{x \mid x \in S \text{ and } x \in T\} \qquad S \cap S^c = \emptyset$

- Infinite union and intersection

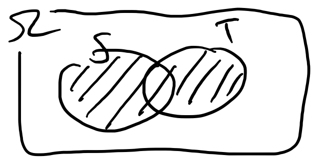
$S_1 \cup S_2 \cup \cdots = \bigcup_{n=1}^{\infty} S_n = \{x \mid x \in S_n \text{ for } \underline{\text{some}}\ n\}$

$S_1 \cap S_2 \cap \cdots = \bigcap_{n=1}^{\infty} S_n = \{x \mid x \in S_n \text{ for } \underline{\text{all}}\ n\}$

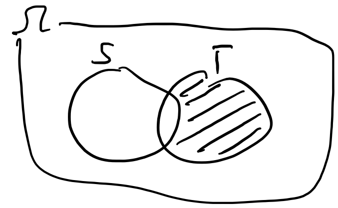
Venn Diagram



$S \cap T$



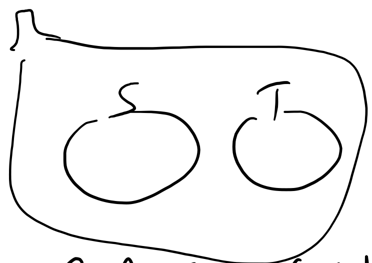
$S \cup T$



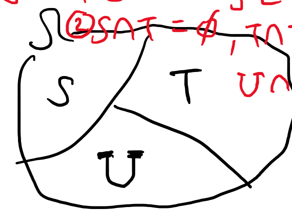
$T \cap S^c$

① $S \cup T \cup U = \Omega$
② $S \cap T = \phi$, $T \cap U = \phi$, $U \cap S = \phi$

$T \subset S$ $\quad S^c$



$S \cap T = \phi$ (disjoint)



S, T, U are forming a partition

Set algebra

- Commutative

$$S \cup T = T \cup S, \quad S \cap T = T \cap S$$

- Associative

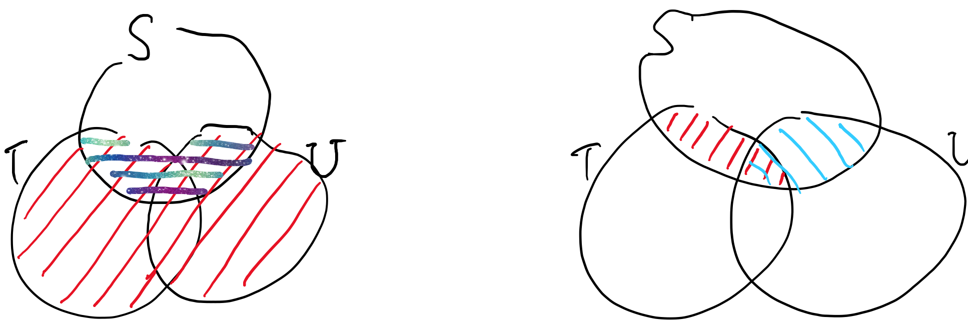
$$(S \cup T) \cup U = S \cup (T \cup U)$$
$$(S \cap T) \cap U = S \cap (T \cap U)$$

- Distributive

$$S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$$
$$S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$$

Distributive law using Venn diagram

$$S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$$



De Morgan's laws

$$(S \cup T)^c = S^c \cap T^c$$

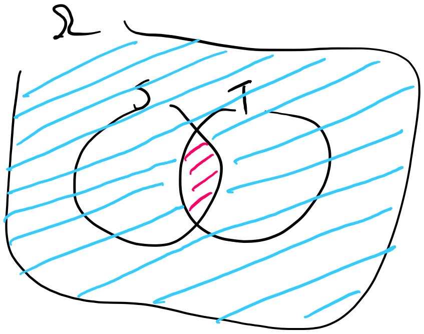
$$(S \cap T)^c = S^c \cup T^c$$

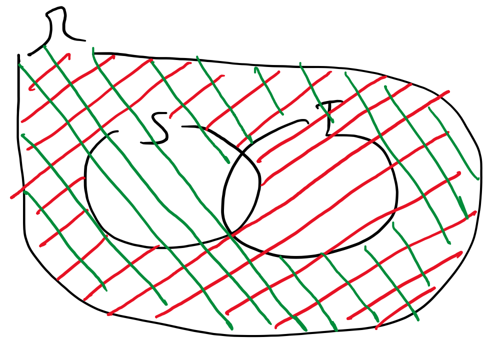
$$(S \cup T \cup U)^c = S^c \cap T^c \cap U^c$$

$$(S \cap T \cap U)^c = S^c \cup T^c \cup U^c$$

$$\left(\bigcup_{n=1}^{\infty} S_n\right)^c = \bigcap_{n=1}^{\infty} S_n^c \qquad \left(\bigcap_{n=1}^{\infty} S_n\right)^c = \bigcup_{n=1}^{\infty} S_n^c$$

De Morgan's laws in Venn diagrams





$$(S \cap T)^c = S^c \cup T^c$$

De Morgan's law proof

Want to prove $(S \cup T)^c = S^c \cap T^c$

① First show $(S \cup T)^c \subset S^c \cap T^c$

② Then show $S^c \cap T^c \subset (S \cup T)^c$

①: For $x \in (S \cup T)^c \Rightarrow x \notin S \cup T$

$\Rightarrow x \notin S$ and $x \notin T$

$\Rightarrow x \in S^c$ and $x \in T^c$

$\Rightarrow \underline{x \in S^c \cap T^c}$

② For $x \in S^c \cap T^c \Rightarrow x \in S^c$ and $x \in T^c$

$\Rightarrow x \notin S$ and $x \notin T \Rightarrow x \notin S \cup T \Rightarrow x \in (S \cup T)^c$