Hw6

1 a- $P(\bar{X} > 51) = P\left(Z \geq \frac{51-50}{1.2/\sqrt{9}}\right) = P(Z \geq 2.5) = 1 - 0.9938$

$= 0.0062$

b- $P(\bar{X} > 51) = P\left(Z \geq \frac{51-50}{1.2/\sqrt{40}}\right) = P(Z \geq 5.27)$

$= 1 - P(Z \leq 5.27) = 0$

2 $X \sim N(np, np(1-p))$

a- $P(X < 125.5) = P\left(Z < \frac{125.5-100}{\sqrt{90}}\right) = P(Z < 2.69)$

b- $X, Y \overset{i.i.d}{\sim} N(100, 90) \Rightarrow X - Y \sim N(0, 180)$

$P(-50.5 < X-Y < 50.5) = P\left(\frac{-50.5-0}{\sqrt{180}} < Z < \frac{50.5-0}{\sqrt{180}}\right)$

$= P(Z \leq 3.76) - P(Z \leq -3.76)$

3 $Cov(X, Y+Z) = E(X(Y+Z)) - E(X)E(Y+Z)$

$= E(XY+XZ) - E(X)[E(Y)+E(Z)] = E(XY) + E(XZ) - E(X)E(Y) - E(X)E(Z)$

$= E(XY) - E(X)E(Y) + E(XZ) - E(X)E(Z)$

$= Cov(X, Y) + Cov(X, Z)$

4 $X \sim N(13, 4)$

$T = X_1 + \cdots + X_{14} \rightarrow T \sim N(B(14), 14(2)) = N(182, 56)$

$P(T < 192) = P\left(Z \leq \frac{192-182}{\sqrt{56}}\right) = P(Z < 1.34)$

6 c. we will have success if the strength exceeds 10 MP

$\hat{p} = \frac{X}{n}$, $n = 12$, $X = \#$ of beams with strength exceeding 10

$\hat{p} = \frac{1}{12}$

5 $X_1 = 2V_1 + 10$, $X_2 = 2V_2$, $V_1 \sim N(520, 100)$ $V_2 \sim N(500, 100)$

$X_1 \sim N(1050, 1400)$ $X_1 - X_2 \sim N(50, 2400)$

$X_2 \sim N(1000, 400)$

a. $P(X_1 - X_2 > 70) = 1 - P(X_1 - X_2 < 0) = 1 - P(Z < \frac{0 - 50}{20}) = 1 - P(Z \leq -2.5)$

b- $P(-10 < X_1 - X_2 < 10) = P(Z < \frac{10 - 50}{20}) - P(Z < \frac{-10 - 50}{20})$

6 a. $\bar{x} = \frac{5.9 + \cdots + 9.0}{12} =$

b- median : 5.9 6.3 6.6 6.8 7.1 7.2 8.1 8.4 8.8 9.0 9.0 18.2

median $= \frac{7.2 + 8.1}{2} = 7.65$