Project 2: Multithread LCS Milestone 2 Report

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1 Description

The longest common subsequence (LCS) problem is to be implemented for the as-
signment. The objective of this project is to design, analyze, and implement a multi-
threaded version of the LCS length algorithm. A memoized or bottom-up approach.
In either case, the goal is to design an algorithm that makes efficient use of multiple
processors.

Analysis is required once the LCS length algorithm is designed. The work $T_1(m, n)$ and
the span $T_\infty(m, n)$ are to be computed, where m and n are the lengths of the input
sequences X and Y, respectively. Finally, the parallelism is to be computed along with
an estimate of a range of parameters (m, n, and P) for which linear or near-linear
speed-up may be expected.

The last part of the project is to implement the LCS length algorithm in C or C++ using
OpenMP on UMBC's maya cluster and measure its performance empirically using 1, 2,
4, and 8 processors (optionally, it may be tested on 16 processors; most maya nodes
have eight cores, but some have 16). Test data and LCS lengths of various sizes are
to be generated to demonstrate the performance characteristics of the algorithm.
The algorithm to recover the LCS must also be implemented, but this need not be
multithreaded.

2 Initialization

The matrix is initialized to store the LCS before computation. Since the implementation
utilizes a two-dimensional array allocated on the heap to represent the matrix, initializa-
tion was possible in $\Theta(m)$ time. Since Milestone 2, the initialization loop was parallelized
to bring the runtime down to $\Theta(\lg(m))$. This method is faster than iterating through all
the cells of the matrix to assign each to a placeholder. Algorithm 2.1 provides the
implementation used to initialize the matrix.

Reading the sequences from the one-line input files, truncating them if necessary and
storing them into buffers all take constant time operations.

Algorithm 2.1: Initialization of the Longest Common Subsequence Matrix

```
1  lcs_matrix = pointer array(m + 1);
2  parallel for i = 0 to m + 1
3      lcs_i = pointer array(n + 1);
```

3   Serial Algorithm

Milestone 1 required implementation of the serial LCS length algorithm and its analysis. Table 1 visualizes the serial implementation of computing the LCS length of sample strings using the algorithm described in equation (15.9) of *Introduction to Algorithms* (1).

Table 1: LCS matrix constructed when comparing GCGTCA and ACGAA

	Ø	G	C	G	T	C	A
Ø	0	0	0	0	0	0	0
A	0	0	0	0	0	0	1
C	0	0	1	1	1	1	1
G	0	1	1	2	2	2	2
A	0	1	1	2	2	2	3
A	0	1	1	2	2	2	3

In Milestone 2, it was observed that the value of the cells depended on the cells above, left or top-left of itself. The cells in the other directions have no effect. This property allowing for subproblems to be independently solvable builds the foundation for the algorithm for the parallel implementation to find the LCS length.

Algorithm 3.2 provides the pseudocode for computing the length of the LCS. The serial algorithm utilizes the bottom-up approach to compute the LCS.

Algorithm 3.1: Serial Longest Common Subsequence Length

```
1   function LCS-LENGTH(X, Y, m, n)
2       // allocate (m + 1) × (n + 1) LCS matrix
3       lcs = new matrix[1, 2, .., m + 1][1, 2, .., n + 1]
4       for i = 0 to m
5           for j = 0 to n
6               // if upper-leftmost cell, content is 0
7               if (i == 0 or j == 0)
8                   lcs_{i,j} = 0
9               // if equal, content is the left diagonal value incremented by 1
10              else if (X_{i-1} == Y_{j-1})
11                  lcs_{i,j} = lcs_{i-1,j-1} + 1
12              // maximum between previous row and previous column values
13              else
14                  lcs_{i,j} = max(lcs_{i-1,j}, lcs_{i,j-1})
15      return lcs_{m,n} // the last value of the matrix is the length
```

3.1 Time Complexity - Work

The running time of the serial implementation provides the work, T_1, of the algorithm. Simple analysis of the algorithm provided in the snippet Algorithm 3.2 suggests a non-linear running time from the nested loop. The inner loop (line 5) iterates n times with several constant conditional checks, while the outer loop (line 4) iterates m times. The runtime is therefore:

$$T_1(m,n) = \Theta(mn)$$

The work for the algorithm, $\Theta(mn)$, is quadratic if $m = n$.

3.2 Printing the Longest Common Subsequence

After computing the length, the matrix may be used to print the LCS itself. Printing the subsequence is done in a serial implementation since the project does not emphasize its analysis. Since its implementation is not necessary for the scope of the project,

the LCS printing functionality will be used for debugging purposes only. Algorithm 3.3 provides the pseudocode used to print the LCS.

Algorithm 3.2: Serial Longest Common Subsequence Printing

```
1   function SERIAL-LCS-PRINT(X, Y, m, n, lcs)
2       lcsstr = new string
3       cursor = lcs_{m,n} // cursor of the matrix
4       i = m, j = n // init from the bottom-rightmost cell
5       while (i > 0 and j > 0)
6           // if current character in X[] and Y[] are same
7           if (X_{i-1} == Y_{j-1})
8               lcsstr_{cursor-1} = X_{i-1} // result gets current character
9               i--, j--, cursor-- // decrement i, j and cursor
10          // find the larger of two and go to that direction
11          else if (lcs_{i-1,j} > lcs_{i,j-1})
12              i--
13          else
14              j--
15      print lcsstr
```

4   Parallel Algorithm

Milestone 2 required implementation of the parallel LCS length algorithm. This version of the implementation uses the same LCS matrix to generate the same cell values. Table 2 revisits the matrix generated by the serial implementation and highlights the independent subproblems.

4

Table 2: LCS matrix constructed when comparing `GCGTCA` and `ACGAA` with the diagonal subproblems highlighted

	Ø	G	C	G	T	C	A
Ø	0	0	0	0	0	0	0
A	0	0	0	0	0	0	1
C	0	0	1	1	1	1	1
G	0	1	1	2	2	2	2
A	0	1	1	2	2	2	3
A	0	1	1	2	2	2	3

(1)   (2)

The parallel implementation must generate the same values in the cells using these subproblems to avoid race conditions between threads.

To iterate through the cells in these diagonal independent subproblems, the algorithm must consider the constraint that the lengths of the diagonal subproblems do not monotonically increase. The lengths increase until the diagonal label as (1) (highlighted in ▢) and decreases after the diagonal labeled as (2) (highlighted in ▢).

To avoid issues with bounds of the lengths of the diagonals, the parallel algorithm splits the matrix into two halves visualized in Table 3.

Table 3: LCS matrix constructed when comparing `GCGTCA` and `ACGAA` highlighting its two halves used for the parallel algorithm.

	∅	G	C	G	T	C	A
∅	0	0	0	0	0	0	0
A	0	0	0	0	0	0	1
C	0	0	1	1	1	1	1
G	0	1	1	2	2	2	2
A	0	1	1	2	2	2	3
A	0	1	1	2	2	2	3

Algorithm 4.4 provides the pseudocode for computing the length of the LCS.

Algorithm 4.1: Parallel Longest Common Subsequence Length

```
1   function P-LCS-LENGTH(X, Y, m, n)
2       // first half of the matrix
3       for i = 1 to n
4           parallel for j = 1 to i
5               if Y_{i-j} == X_{j-1}
6                   lcs_{i-j+1,j} = lcs_{i-j,j-1} + 1
7               else if lcs_{i-j,j} ≥ lcs_{i-j+1,j-1}
8                   lcs_{i-j+1,j} = lcs_{i-j,j}
9               else
10                  lcs_{i-j+1,j} = lcs_{i-j+1,j-1}
11      // second half of the matrix
12      diagonal_len = 0
13      for i = 2 to m
14          // if the diagonal length is not at its maximum
15          if diagonal_len < (m - n)
16              diagonal_len++   // increment the maximum diagonal length
17          parallel for j = i to (n + diagonal_len)
18              if Y_{n-j+i-1} == X_{j-1}
19                  lcs_{n-j+i,j} = lcs_{n-j+i-1,j-1} + 1
20              else if lcs_{n-j+i-1,j} ≥ lcs_{n-j+i,j-1}
21                  lcs_{n-j+i,j} = lcs_{n-j+i-1,j}
```

```
22              else
23                  lcs_{n-j+i,j} = lcs_{n-j+i,j-1}
24       return lcs_{m,n} // the last value of the matrix is the length
```

4.1 Time Complexity - Span

The running time for the parallel implementation provides the span, T_∞ of the algorithm. Computing the runtime of the parallel implementation with an undefined number of processors will yield the span of the algorithm.

The two loops handling their corresponding half of the LCS matrix consist of symmetric operations with identical runtimes of varying sizes, m and n. Therefore, computing the runtime for one of these loops is sufficient.

The inner **parallel** loop spawns all its $\text{ITER}_\infty(j)$ concurrently with an unbounded number of processors. Since $\max\limits_{1\le j\le i} \text{ITER}(j) = \Theta(i)$, (congruently $\max\limits_{i\le j\le n+d} \text{ITER}(j) = \Theta(i)$ for the second half of the matrix), the **parallel** loop is $\Theta(i)$.

The overhead for the loops come from the outer loops iterating for the total number of diagonals. The number of diagonals is congruent to the sum of the lengths of the sequences minus the $\varnothing$ character, since the implementation skips the first iterations to optimize space. The runtime is therefore:

$$T_\infty(m,n) = \Theta(m+n-1)$$
$$= \Theta(m+n)$$

The span for the algorithm, $\Theta(m+n)$, is linear if $m = n$.

4.2 Parallelism and Speedup

Parallelism is the ratio of the work to the span of the algorithm, T_1/T_∞. It represents the maximum possible speedup on P processors. Computing the parallelism for the LCS

algorithm:

$$\frac{T_1}{T_\infty} = \frac{\Theta(mn)}{\Theta(m+n)}$$

$$= \Theta\left(\frac{mn}{m+n}\right)$$

If $m = n$,

$$\rightarrow \lim_{n\to\infty} \Theta\left(\frac{n^2}{2n}\right)$$

$$\approx \Theta(n)$$

Speedup is similar to parallelism, where it is the ratio of runtime of the serial execution to the parallel execution with P processors, T_1/T_P. Speedup is bounded by the parallelism of the algorithm,

$$\frac{T_1}{T_P} \leq \frac{T_1}{T_\infty}$$

$$\leq \Theta\left(\frac{mn}{m+n}\right)$$

$$\leq \Theta(n)$$

The speedup for the algorithm appears near-linear for P processors.

5   Testing and Debugging

5.1   Milestone 1

The source code was initially developed in C++, but later translated to C. The Intel C++ Compiler, `icpc`, did not appear to properly compile the source code. The executable built without any warnings or errors, but the algorithm during its execution appeared to skip steps. The current implementation in C compiles with both `gcc` and the Intel C Compiler, `icc`.

5.2 Milestone 2

Measurement of the empirical running times began using Slurm on the Maya clusters. Initially, the batch submission script did not have sufficient memory to compute the LCS of 100,000 × 100,000 character length sequences. The memory had to be increased by declaring `#SBATCH -mem=50000` on the script's preprocessor directives. The mean of the execution times for 1, 2, 4, 8 and 16 cores on the `hpcf2013` nodes were computed successfully.

References

(1) T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms, Third Edition*. The MIT Press, 3rd edition, 2009. ISBN 0262033844, 9780262033848.