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MATH 407 HW 03

1.1 **4** Use the Euclidean algorithm to find the following greatest common divisors.

a $(6643, 2873)$

Ans □

c $(26460, 12600)$

Ans □

e $(12091, 8439)$

Ans □

6 For each part of Exercise 4, find integers m and n such that (a, b) is expressed in the form $ma + nb$.

7 Let a, b, c be integers. Give a proof for these facts about divisors:

a If $b|a$, then $b|ac$.

Ans □

b If $b|a$ and $c|b$, then $c|a$.

Ans □

c If $b|a$ and $c|b$, then $c|(ma + nb)$ for any integers m, n.

Ans □

11 Show that if $a > 0$, then $(ab, ac) = a(b, c)$

Ans □

14 For what positive integers n is it true that $(n, n + 2) == 2$? Prove your claim.

Ans □

17 Let a, b, n be integers with $n > 1$. Suppose that $a = nq_1 + r_1$ with $0 \leq r_1 < n$ and $b = nq_2 + r_2$ with $0 \leq r_2 < n$. Prove that $n|(a - b)$ if and only if $r_1 = r_2$.

Ans $\square$

19 Let a, b, q, n be integers such that $b \neq 0$ and $a = bq + r$. Prove that (a, b) == (b, r) by showing that (b, r) satisfies the definition of the greatest common divisor of a and b.

Ans $\square$

1.2 **7** Let m and n be positive integers such that $m + n = 57$ and $[m, n] = 680$. Find m and n.

Ans $\square$

10 Show that $a\mathbf{Z} \cap b\mathbf{Z} = [a, b]\mathbf{Z}$.

Ans $\square$

16 A positive integer a is called a **square** if $a = n^2$ for some $n \in \mathbb{Z}$. Show that the integer $a > 1$ is an integer if and only if every exponent in its prime factorization is even.

Ans $\square$

20 A positive integer is called **square- free** if it is a product of distinct primes. Prove that every positive integer can be written uniquely as a product of a square and a square-free integer.

Ans $\square$