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- I'm confused!
 We have these things that sound similar, but aren't the same.
- How do I keep it straight?
- Let's start with a piecewise continuous in time waveform that is not periodic x(t)
- To analyze this is the frequency domain, we use the Fourier Transform, which essentially gives us the "voltage spectral density"

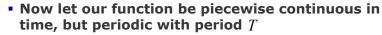
$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt = X(f)$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

 Piecewise continuous in time but not periodic → **Fourier Transform**

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- In this case, the analysis is the Fourier Series
- In the frequency domain, we talk about the Fourier Coefficients, $\,c_{_k}\,$, which are related to harmonics of the period of the time waveform

$$c_{k} = \frac{1}{T} \int_{\alpha}^{\alpha + T} x(t)e^{-j\omega_{0}kt} dt = \frac{1}{T} \int_{\alpha}^{\alpha + T} x(t)e^{-j\frac{2\pi}{T}kt} dt$$
$$x(t) = \sum_{k = -\infty}^{\infty} c_{k}e^{j\omega_{0}t} = \sum_{k = -\infty}^{\infty} c_{k}e^{j\frac{2\pi}{T}kt}$$

• Periodic in time results in discrete in frequencies, with spacing of $\Delta f = 1/T$

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I'm confused, part 3

- Now let's assume that we have a waveform that we will assume is not periodic in time, but is sampled in time at constant intervals Δt
- In the frequency domain, this gives us the Discrete Time Fourier Transform (DTFT), which is a (piecewise) continuous, periodic waveform as a function of frequency

$$X_{S}(\omega) = \int_{-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} x(t)\delta(t-n\Delta t) \right) e^{-j\omega t} dt$$

$$= \sum_{n=-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \left(x(t)e^{-j\omega t} \right) \delta(t-n\Delta t) dt \right] = \sum_{n=-\infty}^{\infty} x(n\Delta t)e^{-j\omega n\Delta t} = DTFT$$

$$x_{S}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{S}(\omega)e^{j\omega t} d\omega$$



- Now let's consider a periodic, piecewise continuous time waveform that is sampled at Δt
- We start by computing the coefficients of the Fourier Series, using \hat{c}_k to remind ourselves that these are the coefficients from the sampled waveform
- Furthermore, assume that Δt is chosen such that

$$N = \frac{T}{\Delta t} > 2 \in \mathbb{N}$$

$$\hat{c}_k = \frac{1}{T} \int_{\alpha}^{\alpha + T} x(t) \sum_{n = -\infty}^{\infty} \delta(t - n\Delta t) e^{-j\frac{2\pi}{T}kt} dt = \frac{1}{T} \sum_{n = -\infty}^{\infty} \int_{\alpha}^{\alpha + T} \left(x(t) e^{-j\frac{2\pi}{T}kt} \right) \delta(t - n\Delta t) dt$$

Let $\alpha = 0^-$, then $\int_{-\infty}^{\infty} picks out <math>n = 0, 1, 2, ..., N - 1$, all other terms are 0

$$\hat{c}_{k} = \frac{1}{N\Delta t} \sum_{n=0}^{N-1} x(n\Delta t) e^{-j\frac{2\pi}{N\Delta t}kn\Delta t} = \frac{1}{N\Delta t} \sum_{n=0}^{N-1} x(n\Delta t) e^{-j\frac{2\pi kn}{N}}$$

This expression is the Discrete Fourier Series (DFS)

continued

By analogy with

$$c_k = \frac{1}{T} X \left(\frac{2\pi k}{T} \right)$$
 (frequency in rad/s) or $\frac{1}{T} X \left(\frac{k}{T} \right)$ (frequency in Hz)

$$\hat{c}_k = \frac{1}{T} X_S \left(\frac{k}{T} \right) \Longrightarrow X_S \left(\frac{k}{T} \right) = \sum_{n=0}^{N-1} x(n\Delta t) e^{-j\frac{2\pi kn}{N}}$$

• Where $X_s\left(\frac{k}{T}\right)$ is the Discrete Fourier Transform

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