# Homework #3

**Problem 1.** Three persons roll a fair $n$-sided die once. Let $A_{ij}$ be the event that person $i$ and person $j$ roll the same face. Show that the events $A_{12}$, $A_{13}$, and $A_{23}$ are pairwise independent but are not independent.

**Problem 2.** We are told that events $A$ and $B$ are independent. In addition, events $A$ and $C$ are independent. Is it true that $A$ is independent of $B \cup C$? Provide a proof or counterexample to support your answer.

**Problem 3.**
The switch network shown in Figure 3 represents a digital communication link. Switches $\alpha_i$ $i = 1, \ldots, 6$, are open or closed and operate independently. The probability that a switch is closed is $p$. Let $A_i$ represent the event that switch $i$ is closed.

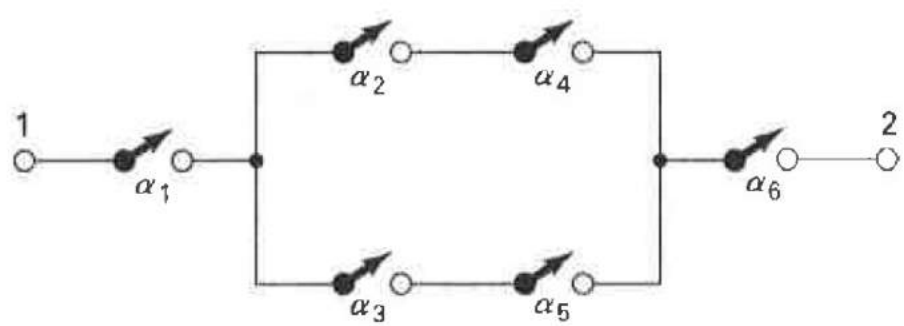


**Figure 3** Switches in telephone link.

(a) In terms of the $A_i$'s write the event that there exists at least one closed path from 1 to 2.

(b) Compute the probability of there being at least one closed path from 1 to 2.

**Problem 4.**

In a ring network consisting of eight links as shown in Figure 4, there are two paths connecting any two terminals. Assume that links fail independently with probability $q$, $0 < q < 1$. Find the probability of successful transmission of a packet from terminal A to terminal B. (Note: Terminal A transmits the packet in *both* directions on the ring. Also, terminal B removes the packet from the ring upon reception. Successful transmission means that terminal B received the packet from either direction.)

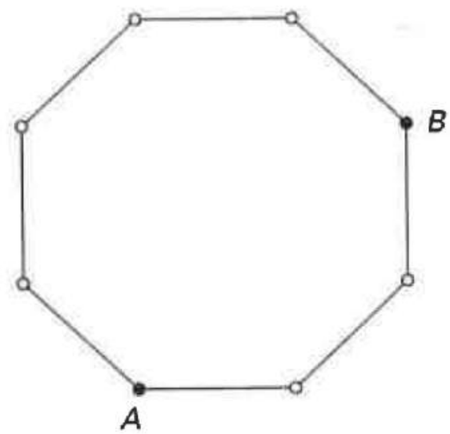


**Figure** 4. A ring network with eight stations.

**Problem 5.**

Toss two unbiased dice (each with six faces: 1 to 6), and write down the sum of the two face numbers. Repeat this procedure 100 times. What is the probability of getting 10 readings of value 7?

**Problem 6.**

War-game strategists make a living by solving problems of the following type. There are 6 incoming ballistic missiles (BMs) against which are fired 12 antimissile missiles (AMMs). The AMMs are fired so that two AMMs are directed against each BM. The single-shot-kill probability (SSKP) of an AMM is 0.8. The SSKP is simply the probability that an AMM destroys a BM. Assume that the AMM's don't interfere with each other and that an AMM can, at most, destroy only the BM against which it is fired. Compute the probability that (a) all BMs are destroyed, (b) at least one BM gets through to destroy the target, and (c) exactly one BM gets through.

**Problem 7.** A company is interviewing potential employees. Suppose that each candidate is either qualified, or unqualified with given probabilities $q$ and $1 - q$, respectively. The company tries to determine a candidate's qualifications by asking 20 true-false questions. A qualified candidate has probability $p$ of answering a question correctly, while an unqualified candidate has a probability $p$ of answering incorrectly. The answers to different questions are assumed to be independent. If the company considers anyone with at least 15 correct answers qualified, and everyone else unqualified, give a formula for the probability that the 20 questions will correctly identify someone to be qualified or unqualified.

**Problem 8.** A parking lot contains 100 cars that all look quite nice from the outside. However, $K$ of these cars happen to be lemons. The number $K$ is known to lie in the range $\{0, 1, \ldots, 9\}$, with all values equally likely.

(a) We testdrive 20 distinct cars chosen at random, and to our pleasant surprise, none of them turns out to be a lemon. Given this knowledge, what is the probability that $K = 0$?

(b) Repeat part (a) when the 20 cars are chosen with replacement; that is, at each testdrive, each car is equally likely to be selected, including those that were selected earlier.

**Problem 9.** A candy factory has an endless supply of red, orange, yellow, green, blue, and violet jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each. One possible color distribution, for example, is a jar of 58 red, 22 yellow, and 20 green jelly beans. As a marketing gimmick, the factory guarantees that no two jars have the same color distribution. What is the maximum number of jars the factory can produce?

**Problem 10.** Count the number of distinguishable ways in which you can arrange the letters in the words:

(a) children

(b) bookkeeper