Quiz 2, Stat355 fall 2016 Name:

1. The table below gives information on the percentages of different types of products manufactured in a particular manufacturing process and the percentage of production during the day and night shifts. Consider randomly selecting an item manufactured during a particular day

	Nuts	Rods	Bearing
Day	35%	5%	5%
Night	35%	5%	15%

(including day and night shifts). From the table $P(Nut \cap Day) = .35$; $P(Nut \cap Night) = 0.35$; $P(Rod \cap Day) = 0.05$; $P(Rod \cap Night) = 0.05$; $P(Bearing \cap Day) = 0.05$ and $P(Bearing \cap Night) = 0.15$. [Hint: Recall that if you have mutually exclusive and exhaustive events $B_1, \ldots, B_k$, then for any event A the law of total probability states $P(A) = \sum_{i=1}^{k} P(A \cap B_i$.]

(a) [3] What is the probability that the item is a rod?

$P(rod) = P(rod \cap Day) + P(rod \cap Night) = 0.1$

(b) [3] If the randomly selected item was manufactured during the day shift, what is the probability that it is a Nut?

$$P(Nut \mid Day) = \frac{P(Nut \cap Day)}{P(Day)} = \frac{P(Nut \cap Day)}{P(Nut \cap Day) + P(rods \cap Day) + P(Bearing \cap Day)}$$

$$= \frac{0.35}{0.35 + 0.05 + 0.05} \simeq 0.778$$

2. [4] For any three events A, B and C on a sample space, show that

$$P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$$

$$P(A|C) + P(B|C) - P(A \cap B|C) = \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)} = \frac{P(A \cap B) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} \quad (*)$$

note $P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C)) = P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))$

$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$

$$P(A \cup B|C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} \Rightarrow (*)$$