Problem 1:

Consider randomly selecting a student at a certain university, and let A denote the event that the selected student has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = 0.5, P(B) = 0.4$ and $P(A \cap B) = 0.25$

a. What is the probability that the selected student has at least one of the two types of cards?

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
&= 0.50 + 0.40 - 0.25 \\
&= \textbf{0.65}
\end{aligned}
$$

b. What is the probability that the selected student has neither type of card?

$$
\begin{aligned}
P[(A \cup B)^c] &= 1 - P(A \cup B) \\
&= 1 - 0.65 \\
&= \textbf{0.35}
\end{aligned}
$$

c. Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

$$
\begin{aligned}
P(A \cap B^c) &= P(A) - P(A \cap B) \\
&= 0.50 - 0.25 \\
&= \textbf{0.25}
\end{aligned}
$$

d. Calculate the following:

i. $P(B \mid A)$
(*probability that a randomly selected student has MasterCard **given** that the student has a Visa card*)

$$
\begin{aligned}
P(B \mid A) &= \frac{P(A \cap B)}{P(A)} \\
&= \frac{0.25}{0.50} \\
&= \textbf{0.50}
\end{aligned}
$$

ii. $P(A \mid B)$
(*probability that a randomly selected student has Visa card **given** that the student has a MasterCard*)

$$
\begin{aligned}
P(A \mid B) &= \frac{P(A \cap B)}{P(B)} \\
&= \frac{0.25}{0.40} \\
&= \textbf{0.625}
\end{aligned}
$$

Problem 2:

Let V be the event that a computer contains a virus, and let W be the event that a computer contains a worm. Suppose $P(V) = 0.10, P(W) = 0.15$ and $P(V \cup W) = 0.2$.

a. Find the probability that the computer contains neither a virus nor a worm (the computer is clean).

$$
\begin{aligned}
P[(V \cup W)^c] &= 1 - P(V \cup W) \\
&= 1 - 0.20 \\
&= \mathbf{0.80}
\end{aligned}
$$

b. Find the probability that the computer contains a virus but no worm.

$$
\begin{aligned}
P(V \cap W^c) &= P(V \cup W) - P(W) \\
&= 0.20 - 0.15 \\
&= \mathbf{0.05}
\end{aligned}
$$

Problem 3:

A UMBC student has two different email accounts. Suppose that 70% of her messages come into Account 1. Of the messages that come into Account 1, 5% are spam. Meanwhile, of the messages that come into Account 2, 8% are spam.

a. If a message is randomly selected, what is the probability that it is **NOT** a spam?

$$
\begin{aligned}
P(S^c) &= 1 - P(S) \\
&= 1 - [0.7(0.05) + 0.3(0.08)] \\
&= \mathbf{0.941}
\end{aligned}
$$

b. If a randomly selected message is a spam, what is the probability that it came from Account 1?

$$
\begin{aligned}
P(A_1 \mid S) &= \frac{P(A_1 \cap S)}{P(S)} \\
&= \frac{0.7(0.05)}{0.7(0.05) + 0.3(0.08)} \\
&= \mathbf{0.5932}
\end{aligned}
$$