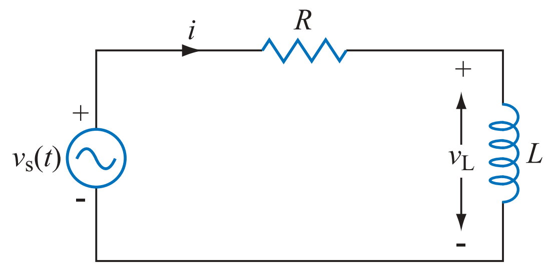
Math-Phys Quiz 4 Question:

1. The voltage at the input to a transmission line is 64 V and is 32 V after 1 m. What is it after 4 m? The current at the input to the transmission line is 4 A. The current attenuates proportional to the voltage, and the power in the transmission line is given by $P = IV$ (Watts). What is the power after 4 m?

Exam Quiz 4 Questions:

1. **Modified from Problem 1.7:**
For the RL circuit shown on the right, show directly (using cosines and sines instead of phasors) that if $v_S(t) = V_0 \sin(\omega t + \phi_0)$, then



$$i(t) = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi_0 - \phi_1)$$

with $\phi_1 = -\tan^{-1}(R/\omega L)$.

2. **Modified from Ulaby et al. Exercise 2.4, Slide 4.6:** A two-wire line has the following parameters: $R' = 0.404$ mΩ/m, $L' = 4.00$ μH/m, $G' = 0$, $C' = 2.78$ pF/m. For operation at 10 kHz, determine (a) the attenuation coefficient α, (b) the wavenumber β, (c) the phase velocity u_p, and the characteristic impedance Z_0.

3. **Modified from Ulaby et al. Exercise 2.11, Slide 4.17:** A 140 Ω lossless line is terminated in a load impedance $Z_L = (280 + j182)$ Ω. If $\lambda = 36$ cm, find (a) the reflection coefficient Γ, (b) the voltage standing wave ratio S, (c) the locations of the voltage maxima and minima.

Math-Phys Quiz 4 Solution:

1. The voltage attenuation is given by $64 \times 2^{-4} = 64/16 = 4$ V. The power attenuation is given by the square of the voltage attenuation, so $P = (64 \times 4)/(16 \times 16) = 1$ W.

Exam Quiz 4 Solutions:

1. We substitute $i(t) = A\cos(\omega t + \phi_0 - \phi_1)$ into the equation

$$Ri(t) + L\frac{di(t)}{dt} = v_S(t),$$

and we obtain

$$RA\cos(\omega t + \phi_0 - \phi_1) - \omega LA\sin(\omega t + \phi_0 - \phi_1) = V_0\sin(\omega t + \phi_0).$$

Writing now

$$\begin{aligned} A\cos(\omega t + \phi_0 - \phi_1) &= A\cos\phi_1\cos(\omega t + \phi_0) + A\sin\phi_1\sin(\omega t + \phi_0), \\ A\sin(\omega t + \phi_0 - \phi_1) &= A\cos\phi_1\sin(\omega t + \phi_0) - A\sin\phi_1\cos(\omega t + \phi_0), \end{aligned}$$

we find

$$\begin{aligned} &[RA\cos\phi_1 + \omega LA\sin\phi_1]\cos(\omega t + \phi_0) \\ &\qquad + [RA\sin\phi_1 - \omega LA\cos\phi_1]\sin(\omega t + \phi_0) = V_0\sin(\omega t + \phi_0). \end{aligned}$$

Separately equating the coefficients of $\cos(\omega t + \phi_0)$ and $\sin(\omega t + \phi_0)$, we obtain

$$RA\cos\phi_1 + \omega LA\sin\phi_1 = 0, \qquad RA\sin\phi_1 - \omega LA\cos\phi_1 = V_0.$$

Solving for $A\cos\phi_1$ and $A\sin\phi_1$, we obtain

$$A\cos\phi = -\frac{\omega L V_0}{R^2 + \omega^2 L^2}, \qquad A\sin\phi = -\frac{RV_0}{R^2 + \omega^2 L^2}$$

From these equations, we obtain

$$\tan\phi_1 = -R/\omega L \qquad \text{or} \qquad \phi_1 = -\tan^{-1}(R/\omega L)$$

and

$$A^2 = \frac{V_0^2}{R^2 + \omega^2 L^2} \qquad \text{or} \qquad A = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}.$$

2. We find $\omega = 2 \times 3.14159 \times (10 \times 10^3\ \text{s}^{-1}) = 6.28319 \times 10^4\ \text{s}^{-1}$. We then find $R' + j\omega L' = (4.04000 \times 10^{-4} + j \times 6.28319 \times 10^4 \times 4.00000 \times 10^{-6}) = 4.040000 \times 10^{-4} + j \times 2.51327 \times 10^{-1} = 0.251328\exp(j \times 1.56919)\ \Omega$/m. We also find $G' + j\omega C' = j \times 1.74673 \times 10^{-7} = 1.74673 \times 10^{-7} \times \exp(j \times 1.570796)\ \Omega^{-1}$/m. Note the small difference in phases. Six digits of accuracy are needed to keep three digits in the attenuation coefficient. We

now have $\gamma^2 = (R' + j\omega L')(G' + j\omega C') = 4.39001 \times 10^{-8} \times \exp(j \times 3.13999)\ \mathrm{m}^{-2}$ so that $\gamma = 2.09523 \times 10^{-4} \times \exp(j \times 1.56999) = 1.684006 \times 10^{-7} + j \times 2.09523^{-4}\ \mathrm{m}^{-1}$, so that $\alpha = 1.68 \times 10^{-7}$ Np/m and $\beta = 2.10 \times 10^{-4}$ rad/m. We have $u_{\mathrm{p}} = \omega/\beta = (6.283 \times 10^4)/(2.095 \times 10^{-4}) = 3.00 \times 10^8$ m/s, and we have $Z_0 = [(R' + j\omega L')/(G' + j\omega C')]^{1/2} = [0.251328 \exp(j \times 1.56919)/1.74673 \times 10^{-7} \times \exp(j \times 1.570796)]^{1/2} = (1.20 \times 10^3 - j \times 0.964)\ \Omega$.

If these calculations are done using MATLAB or a calculator with memory and with 15 digits of accuracy, which is typical, then it is not necessary to make the conversions between the Cartesian and polar representations of the complex numbers.

3. (a) $\Gamma = (Z_{\mathrm{L}} - Z_0)/(Z_{\mathrm{L}} + Z_0) = (280 + j182 - 140)/(280 + j182 + 140) = 230 \exp(j0.915)/485 \exp(j0.409) = 0.50 \exp(j0.51)$. NOTE: 0.51 rads $= 29°$. (b) $S = (1 + |\Gamma|/(1 - |\Gamma|) = (1 + 0.502)/(1 - 0.502) = 3.0$. (c) $l_{\max} = (0.506 \times 36/4\pi + n \times 36/2)$ cm $=$ 1.4 cm, 19 cm, 37 cm,. . .; $l_{\min}(0.506 \times 36/4\pi + 9 + n \times 36/2)$ cm $=$ 10 cm, 28 cm, 36 cm,. . .