Maxwell's Equations: Differential and Integral Forms

Name of Law	Differential Form	Integral Form
Gauss's Law	$\nabla \cdot \mathbf{D} = \rho_V$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_v \rho_V \, dv = Q$
Faraday's Law	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$
Gauss's Law of Magnetics	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampere's Law	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$

This version of the integral forms is the most useful for implementation of numerical methods

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12.1

Maxwell's Equations: Alternative Integral Forms

Faraday's Law:

$$\oint_C \mathbf{E}_{\mathrm{EMF}} \cdot d\mathbf{l} = \oint_C \left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right) \cdot d\mathbf{l} = \frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{s} = \frac{d\Phi}{dt}$$

$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$ = magnetic flux

Ampere's Law:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I = I_{\mathrm{c}} + I_{\mathrm{d}}$$

I_{c} = conduction current; I_{d} = displacement current

This version of the integral forms

- ***has historical and conceptual importance***
- ***is the basis for understanding motors and generators***

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12.2

What we refer to here as E_EMF is the E-field in the frame of the moving charge (which follows from relativity).

I_c is the standard current that flows through wires. We will say more about what I_d is shortly.

Faraday's Law

Electromotive Force:

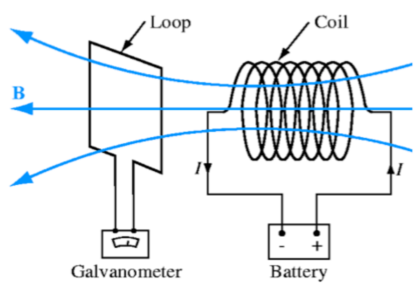
When a changing magnetic flux passes through a wire loop, it induces a loop voltage V_{EMF}, which is called ***the electromotive force***.

There are two possible sources of change:

- The magnetic flux density varies in time, leading to a transformer EMF, $V_{\text{EMF}}^{\text{tr}}$
- The loop area normal to the flux density varies in time, leading to a motional EMF, $V_{\text{EMF}}^{\text{m}}$

We have

$$V_{\text{EMF}} = V_{\text{EMF}}^{\text{tr}} + V_{\text{EMF}}^{\text{m}}$$



Ulaby Figure 6-1

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12.3

The galvanometer has a resistor; so, the voltage V_EMF appears across the resistor and induces a current flow, which is detectable.

Faraday's Law

Electromotive Force:

When the loop has N turns, the effect of the induced EMF is multiplied N times, so we have:

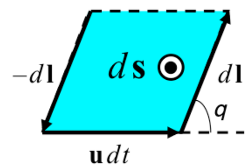
$$V_{\text{EMF}} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$$

$$V_{\text{EMF}}^{\text{tr}} = -N\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$$

$$V_{\text{EMF}}^{\text{m}} = -N\int_S \mathbf{B}\cdot\frac{d\mathbf{s}}{dt} = -N\oint_C \mathbf{B}\cdot(\mathbf{u}\times d\mathbf{l}) = N\oint_C (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$|d\mathbf{s}| = |d\mathbf{l}||\mathbf{u}\,dt|\sin\theta$$

$$\hat{\mathbf{n}} = \frac{(\mathbf{u}\times d\mathbf{l})}{|\mathbf{u}\times d\mathbf{l}|},\quad d\mathbf{s} = \hat{\mathbf{n}}\,|d\mathbf{s}|$$



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Ulaby 2010

12.4

The galvanometer has a resistor; so, the voltage V_EMF appears across the resistor and induces a current flow, which is detectable.

Ulaby et al. obtain V_EMF^m, using force arguments, which we discussed previously. That is fine, but does not demonstrate the connection to the total V_EMF here. The connection is demonstrated in Section 6.5.

Go over Ulaby 2007 Demos 6.1 and 6.2. These are the same as Ulaby et al. 2010 CD Modules 6.1 and 6.2

Faraday's Law

Lenz's Law:

The current in a loop is always in such a direction as to oppose the change of the magnetic flux $\Phi(t)$

This law allows us to rapidly determine the direction of the current that is induced by an EMF

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Ulaby 2010

12.5

Go over Ulaby et al. 2007 Modules 6.1 - 6.4

Tech Brief 12: EMF Sensors

EMF Sensors

Generate an induced voltage in response to an external stimulus

Piezoelectric transducers

Certain crystals, such as quartz, become electrically polarized when subjected to mechanical pressure, thereby exhibiting a voltage difference. Under no applied pressure, the polar domains are randomly oriented, but under compressive or tensile stress, the domains align along a principal axis of the crystal.

Compression and stretching generate opposite voltages.

Discovered in 1880 by Curie brothers. In 1881, Lippman predicted converse property (that electrical stimulus would change the shape of the crystal).

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12.6

Tech Brief 12: EMF Sensors

Piezoelectric transducers

Used in microphones, loudspeakers, positioning sensors for scanning tunneling microscopes (can measure deformations as small as nanometers), accelerometers (can measure from 10^{-4}g to 100g), spark generators, clocks and electronic circuits (precision oscillators), medical ultrasound transducers, and numerous other applications

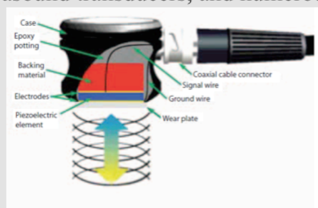
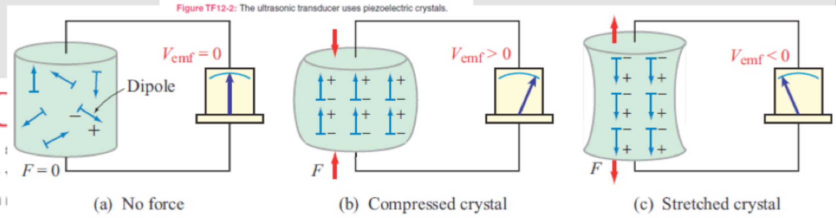


Figure TF12-2: The ultrasonic transducer uses piezoelectric crystals.



Tech Brief 12: EMF Sensors

Faraday Magnetic Flux Sensor

According to Faraday's law, the emf voltage induced across the terminals of a conducting loop is directly proportional to the rate of change of magnetic flux passing through the loop. In the configuration below, Vemf and its derivative directly indicate the velocity and acceleration of the loop.

$$V_{\mathrm{EMF}} = -uB_0 l$$

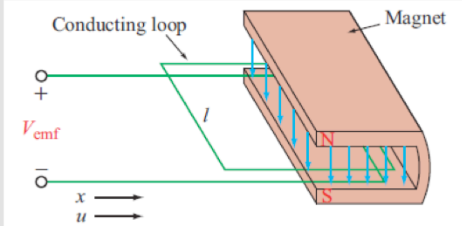


Figure TF12-3: In a Faraday accelerometer, the induced emf is directly proportional to the velocity of the loop (into and out of the magnet's cavity).

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Tech Brief 12: EMF Sensors

Thermocouple

Seebeck discovered in 1821 that a junction of two conducting materials will generate a thermally induced emf (called the Seebeck potential) when heated.

Becquerel in 1826 used this concept to measure an unknown temperature by relative to a cold reference junction. Traditionally, the cold reference is an ice bath, but in modern thermocouples, an electric circuit generates the reference potential.

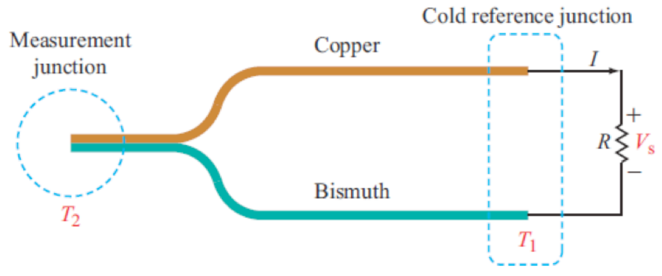

Figure TF12-4: Principle of the thermocouple.

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12.9

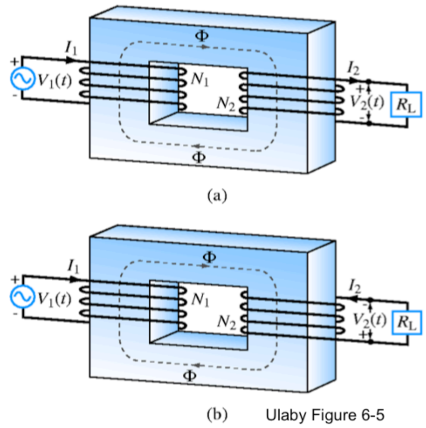
Faraday's Law

Transformers:

These are used to transform voltages, currents, and hence impedances.

In practice, they are made by winding current loops with different numbers of turns around a common magnetic core, which fixes the magnetic flux F.

NOTE: The direction of the winding determines the polarity of the output.



Ulaby Figure 6-5

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12.10

Faraday's Law

Transformers:

Voltage Transformation:

In the primary: The oscillating voltage creates an oscillating flux, found by integrating

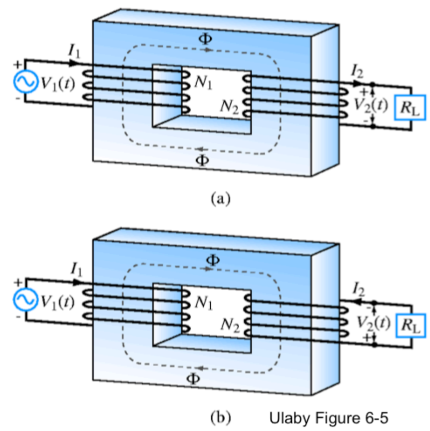
$$V_1 = -N_1 \frac{d\Phi}{dt}$$

In the secondary:

$$V_2 = -N_2 \frac{d\Phi}{dt}$$

We conclude

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$



Ulaby Figure 6-5

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Faraday's Law

Motors and Generators:

Current Transformation:

In an ideal transformer, the power is conserved, so we have $P_1 = P_2$.
Since $P_1 = I_1 V_1$ and $P_2 = I_2 V_2$, we have

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

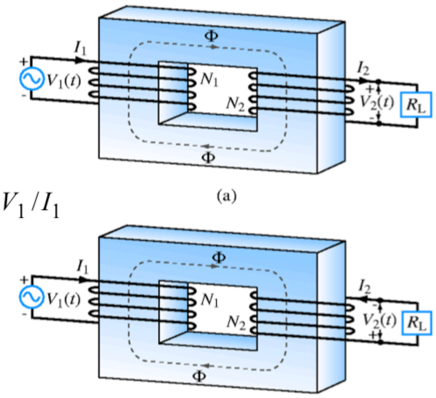
Impedance Transformation:

Ouput resistance, $R_L = V_2 / I_2$

Equivalent input resistance, $R_{in} = V_1 / I_1$

We conclude

$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$



(a) (b) Ulaby Figure 6-5

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12.12

This approach to impedance matching alters the magnitude of the impedance, but not its phase. We need delay, from sort of a transmission line, for a phase change.

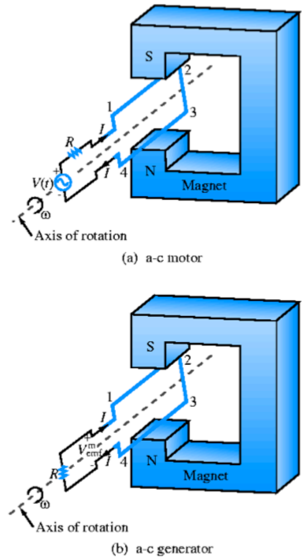
Transformers are widely used in power electronics. High voltages are used to transfer high powers, since increasing voltages and reducing currents reduces the losses. The voltages are then "stepped down" using transformers for local distribution.

Magnetic Forces

Motors and Generators

We previously discussed motors and generators as an application of magnetostatics. That is possible because the magnetic field is fixed and the currents are moving, so that V_{EMF} is purely motional.

We can instead directly use Faraday's law!



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Ulaby Figure 6-11 12.13

See notes 11.34 and 11.35.

Ulaby et al. derive the generator equation both ways to show that they are equal.

Magnetic Forces

Motors and Generators

We can instead directly use Faraday's law!

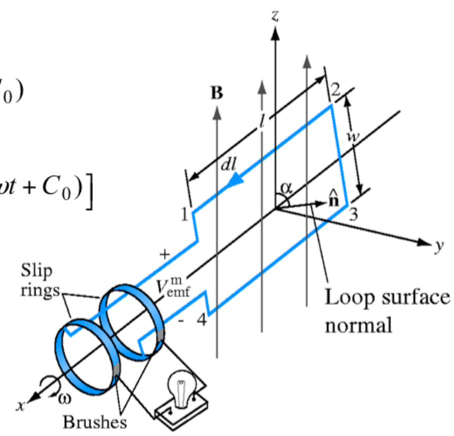
We first calculate the magnetic flux

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_S \hat{\mathbf{z}} B_0 \cdot \hat{\mathbf{n}}\, ds$$
$$= B_0 A \cos\alpha = B_0 A \cos(\omega t + C_0)$$

We thus find

$$V_{\mathrm{EMF}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[B_0 A \cos(\omega t + C_0)\right]$$
$$= \omega B_0 A \sin(\omega t + C_0)$$

which is the same result that is obtained from the motional EMF



Ulaby Figure 6-12

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12.14

See notes 11.34 and 11.35.

Ulaby et al. derive the generator equation both ways to show that they are equal.

Displacement Current

… or Maxwell's great insight!

Original (static) version of Ampere's law: $\oint_C \mathbf{H} \cdot d\mathbf{l} = I_c = \int_S \mathbf{J} \cdot d\mathbf{s}$

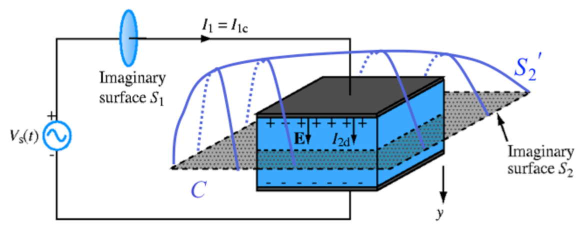
This version of Ampere's law is inconsistent! ***Why?***

Different surfaces attached to same closed curve C yield different results!

— Compare S_2 and S_2'

Maxwell assumed that adding $I_d = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$ to Ampere's law would fix things

…and all experimental evidence indicates that it has.



Modified from Ulaby Figure 6-12

12.15

I think that Ulaby et al. miss the main point here!

This was the only one of Maxwell's equations whose final form was inferred by pure reason, rather than being inspired directly by experiments.

Displacement Current

Example: Current flow onto a parallel plate capacitor

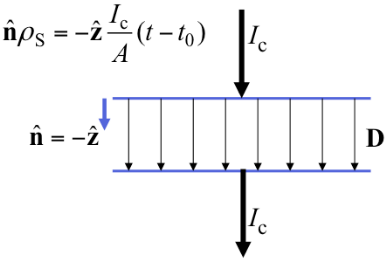
Question: During a time interval t, a steady current I_c flows onto a parallel plate capacitor with area A, separation d, and dielectric constant ϵ, show that the displacement current I_d equals the conduction current.

Answer: The charge that accumulates on the upper plate of the capacitor is given by $Q = I_c(t - t_0)$, where t_0 is the initial time at which current starts to flow. Neglecting fringing fields, we have

$$\rho_S = \frac{Q}{A} = \frac{I_c}{A}(t - t_0), \quad \text{which implies } \mathbf{D} = \hat{\mathbf{n}}\rho_S = -\hat{\mathbf{z}}\frac{I_c}{A}(t - t_0)$$

We conclude

$$I_d = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} = \left(-\hat{\mathbf{z}}\frac{I_c}{A}\right) \cdot \left(-\hat{\mathbf{z}}A\right) = I_c$$



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12.16

Note: The parameters d and epsilon are irrelevant!

Charge-Current Continuity

Charge conservation

$$0 = \nabla\cdot(\nabla\times\mathbf{H}) = \nabla\cdot\left(\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\right) = \nabla\cdot\mathbf{J}+\frac{\partial\rho_{\mathrm{V}}}{\partial t} \quad\Rightarrow\quad \nabla\cdot\mathbf{J} = -\frac{\partial\rho_{\mathrm{V}}}{\partial t}$$

Charge cannot be created or destroyed!

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12.17

Again, the point is not to prove that charge is conserved; the point is to demonstrate that Maxwell's equations are consistent with this well-known fact.

This gives the limitation to Kirchoff's current law. When charge accumulates, Kirchoff's law fails.

Charge-Current Continuity

Current Dissipation in Conductors

In real conductors with finite conductivity, excess charge dissipates in a finite time. Using the relations $\mathbf{J} = \sigma \mathbf{E}, \quad \nabla \cdot \mathbf{E} = \rho_{\mathrm{V}} / \varepsilon,$ we obtain

$$\frac{\partial \rho_{\mathrm{V}}}{\partial t} + \frac{\sigma}{\varepsilon} \rho_{\mathrm{V}} = 0 \quad \Rightarrow \quad \rho_{\mathrm{V}}(t) = \rho_{\mathrm{V0}} \exp\left(-t / \tau_{\mathrm{r}}\right)$$

$$\tau_{\mathrm{r}} = \varepsilon / \sigma = \text{material relation time}$$

A good conductor

— copper: $\varepsilon = \varepsilon_0 = 8.9 \times 10^{-12}$ F/m, $\sigma = 5.8 \times 10^{8}$ S/m

$$\Rightarrow \quad \tau_{\mathrm{r}} = 1.5 \times 10^{-19} \text{ s}$$

A good insulator

— mica: $\varepsilon = 6 \varepsilon_0 = 5 \times 10^{-11}$ F/m, $\sigma = 10^{-15}$ S/m

$$\Rightarrow \quad \tau_{\mathrm{r}} = 5 \times 10^{4} \text{ s} = 15 \text{ hours}$$

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12.18

The time scale in copper is shorter than the period of light by a factor of 10,000 and comparable to the period of short x-rays. The corresponding length scale, (tau_r) x (speed of light), is --- not surprisingly --- comparable to the dimension of an atom.

The huge difference in time scales is not due to the difference in dielectric constants. In almost all materials, epsilon is within a factor of 10 of epsilon_0. It is due to the difference in conductivities.

Electromagnetic Potentials

Dynamic Potentials

As before: $\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{B} = \nabla \times \mathbf{A}$

From Faraday's law:

$$0 = \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} + \frac{\partial (\nabla \times \mathbf{A})}{\partial t} = \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right)$$

Hence, we must have

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V \quad \Rightarrow \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

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12.19

This voltage generalizes the notion of an EMF.

Electromagnetic Potentials

Retarded Potentials

In the static case

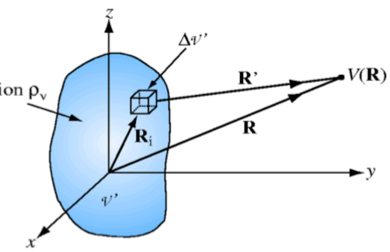
$$V(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})}{R'}\,dv', \quad R' = |\mathbf{R} - \mathbf{R}_{\mathrm{i}}|$$

In the dynamic case, we must take into account the finite time delay:

$$V(\mathbf{R},t) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}}, t - R'/u_{\mathrm{p}})}{R'}\,dv'$$

For the vector potential, we have

$$\mathbf{A}(\mathbf{R},t) = \frac{\mu}{4\pi}\int_{v'} \frac{\mathbf{J}(\mathbf{R}_{\mathrm{i}}, t - R'/u_{\mathrm{p}})}{R'}\,dv'$$



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12.20

Proving this result is non-trivial, but its physical meaning when u_p = c = speed of light in a vacuum is clear. No information about what is happening at point R_i can propagate to the point R faster than the speed of light. This form is a consequence of Maxwell's equations and was known by the late 19-th century. It was one of the facts that pointed Einstein to his theory of relativity.

From a practical standpoint, the use of retarded potentials is very important in antenna problems, radar, etc.

Phasor Fields

Time Harmonic Potentials

The building blocks for understanding dynamical systems are sinusoidally varying signals (phasors).

- In a linear system, *any* time variation can be found by adding up the phasors
- Maxwell's equations are linear, assuming linear material relations

We thus write $\rho_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}},t) = \rho_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\cos\omega t = \mathrm{Re}\left[\tilde{\rho}_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\exp(j\omega t)\right]$

For the retarded charge density, we have

$$\rho_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}},t - R'/u_{\mathrm{p}}) = \mathrm{Re}\left[\tilde{\rho}_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\exp\left(j\omega t - j\frac{\omega R'}{u_{\mathrm{p}}}\right)\right]$$

$$= \mathrm{Re}\left[\tilde{\rho}_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\exp\left(-jkR'\right)\exp(j\omega t)\right], \quad \text{with } k = \omega / u_{\mathrm{p}}$$

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NOTE: In this section of his book, Ulaby et al. use k and the designation *wavenumber*, instead of β and the designation *phase constant*.

12.21

We recall that the phasors are essentially the Fourier amplitudes.

Phasor Fields

Time Harmonic Potentials

For the voltage field, we have

$$V(\mathbf{R},t) = \mathrm{Re}\left[\tilde{V}(\mathbf{R})\exp(j\omega t)\right]$$
$$= \mathrm{Re}\left[\frac{1}{4\pi\varepsilon}\int_{v'}\frac{\tilde{\rho}_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\exp\left(-jkR'\right)}{R'}\exp(j\omega t)\,dv'\right]$$

which implies

$$\tilde{V}(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{v'}\frac{\tilde{\rho}_{\mathrm{V}}(\mathbf{R}_{\mathrm{i}})\exp\left(-jkR'\right)}{R'}dv'$$

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12.22

Phasor Fields

Time Harmonic Potentials

For the vector potential and current density fields, we have similarly

$$\mathbf{A}(\mathbf{R},t) = \mathrm{Re}\left[\tilde{\mathbf{A}}(\mathbf{R})\exp(j\omega t)\right],$$

$$\mathbf{J}(\mathbf{R}_\mathrm{i}, t - R'/u_\mathrm{p}) = \mathrm{Re}\left[\tilde{\mathbf{J}}(\mathbf{R}_\mathrm{i})\exp\left(-jkR'\right)\exp(j\omega t)\right]$$

which imply

$$\tilde{\mathbf{A}}(\mathbf{R}_\mathrm{i}) = \frac{\mu}{4\pi}\int_{v'} \frac{\tilde{\mathbf{J}}(\mathbf{R}_\mathrm{i})\exp\left(-jkR'\right)}{R'} dv'$$

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12.23

Phasor Fields

Electric and Magnetic Fields

— in a non-conducting medium ($\mathbf{J} = 0$)

From the definition of the vector potential, we have

$$\mathbf{H} = \frac{1}{\mu}\nabla\times\mathbf{A} \quad \Rightarrow \quad \tilde{\mathbf{H}} = \frac{1}{\mu}\nabla\times\tilde{\mathbf{A}}$$

From Ampere's law, we have

$$\nabla\times\mathbf{H} = \varepsilon\frac{\partial\mathbf{E}}{\partial t} \quad \Rightarrow \quad \nabla\times\tilde{\mathbf{H}} = j\omega\varepsilon\,\tilde{\mathbf{E}} \;\text{ or }\; \tilde{\mathbf{E}} = \frac{1}{j\omega\varepsilon}\nabla\times\tilde{\mathbf{H}}$$

From Faraday's law, we have

$$\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t} \quad \Rightarrow \quad \nabla\times\tilde{\mathbf{E}} = -\,j\omega\mu\,\tilde{\mathbf{H}} \;\text{ or }\; \tilde{\mathbf{H}} = -\frac{1}{j\omega\mu}\nabla\times\tilde{\mathbf{E}}$$

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12.24

Phasor Fields

Field Amplitudes and Dispersion Relations: Ulaby et al. Example 6-8

Question: In a nonconducting medium with $\varepsilon = 16\varepsilon_0$ and $\mu = \mu_0$, the electric field intensity is given by

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}10\sin\left(10^{10}t - kz\right) \text{ V/m.}$$

Determine the associated magnetic field intensity **H** and the value of k.

Answer: We have

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}10\cos\left(\omega t - kz - \pi/2\right) \text{ V/m, with } \omega = 10^{10} \text{ s}^{-1}$$

so that $\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}10\exp[-j(kz+\pi/2)] = -\hat{\mathbf{x}}\, j10\exp(-jkz)$

From Faraday's law: $\tilde{\mathbf{H}} = -\dfrac{1}{j\omega\mu}\nabla\times\tilde{\mathbf{E}} = -\hat{\mathbf{y}}\, j\dfrac{10k}{\omega\mu}\exp(-jkz)$

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From Ampere's law: $\tilde{\mathbf{E}} = \dfrac{1}{j\omega\varepsilon}\nabla\times\tilde{\mathbf{H}} = -\hat{\mathbf{x}}\, j\dfrac{10k^2}{\omega^2\mu\varepsilon}\exp(-jkz)$

12.25

Ulaby et al. refer to this process of substitution in Faraday's and Ampere's laws as "performing a circle". We did the same thing to derive the wave equations and the dispersion relations in transmission lines.

Phasor Fields

Field Amplitudes and Dispersion Relations: Ulaby et al. Example 6-8

Answer (continued): Equating the two expressions for $\tilde{\mathbf{E}}(z)$, we obtain the dispersion relation

$$k^2 = \omega^2 \mu\varepsilon$$

This is the same dispersion relation that we found with transmission lines!
This is no coincidence!!

Explicitly, we have

$$k = \omega\sqrt{\mu\varepsilon} = 4\omega\sqrt{\mu_0 \varepsilon_0} = \frac{4\omega}{c} = \frac{4\times 10^{10}}{3\times 10^{8}} = 130 \text{ rad/m.}$$

We now find for the magnetic field

$$\mathbf{H}(z,t) = \mathrm{Re}\left[\tilde{\mathbf{H}}(z)\exp(j\omega t)\right]$$
$$= \mathrm{Re}\left[-\hat{\mathbf{y}}\, j\frac{10k}{\omega\mu}\exp(-jkz)\exp(j\omega t)\right] = \hat{\mathbf{y}}\, 0.11\sin\left(10^{10}t - 130z\right) \text{ A/m}$$

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12.26

Transmission lines can be analyzed in terms of voltages and currents --- or they can be analyzed in terms of H-fields and E-fields. They are two complementary ways of looking at the same physical object, and they must yield the same result. The dispersion relation must be the same with either analysis.

Assignment

Reading: Ulaby, Chapter 7

Problem Set 7: Some notes.

- There are 6 problems. As always, YOU MUST SHOW YOUR WORK TO GET FULL CREDIT!
- All problems come from Ulaby et al.
- Please watch significant digits.
- Get started early!

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