Hw7: Stat355, F 2016, Due November 7

1. Assume thatHelium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with true standard deviation 0.75.

   (a) Compute a 95% CI for true average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85.

   (b) Compute a 98% CI for the true average porosity if the average porosity based on 16 specimens is 4.56.

   (c) What sample size is necessary if the width of the 95% CI is to be 0.40?

   (d) By what factor the sample size should be increased if the width of the 95% CI is to be halved?

2. Let $X_1, \ldots, X_n$ be a random sample from $Uniform[0, \theta]$. We have already shown in the class that the MLE for θ is the maximum of the sample, $\hat{\theta}_{MLE} \equiv Y = \max\{X_1, \ldots, X_n\}$.

   (a) Using the fact that $Y \leq y$ if and only $X_i \leq y$ for each i, derive the CDF $P(Y \leq y)$ for the sample max.

   (b) Then differentiating the CDF with respect to y show that the pdf $f(y)$ of the sample max from $uniform[0, \theta]$ is

   $$f(y) = \frac{ny^{n-1}}{\theta^n}, \quad 0 \leq y \leq \theta.$$

   (c) Thus, show that $E(\hat{\theta}_{MLE}) \neq \theta$ and hence the MLE is biased. Find a c such that $c\hat{\theta}_{MLE}$ is unbiased for θ.

   (d) What is the MLE for the 90^{th} population percentile?

   (e) What is the MLE of $P(X < r)$ where $0 \leq r \leq \theta$ and X is also $Uniform[0, \theta]$.