Solution to sample exam

1. $P(\text{Line 1}) = 500/1500 = 5/15$, $P(\text{Line 2}) = 4/15$, $P(\text{Line 3}) = 6/15$

a) $P(\text{Crack}) = P(\text{Crack} \mid \text{Line 1})\,P(\text{Line 1}) + P(\text{Crack} \mid \text{Line 2})\,P(\text{Line 2}) + P(Cr \mid L3)\,P(L3)$

$$= (0.5)(5/15) + (0.44)(4/15) + (0.40)(6/15)$$

b) $P(\text{blemish} \mid \text{Line 1}) = 0.15$

c) $$P(\text{Line 1} \mid \text{Surf Def}) = \frac{P(\text{Surf Def} \mid \text{Line 1})\,P(\text{Line 1})}{P(\text{Surf def} \mid \text{Line 1})P(\text{Line 1}) + P(\text{Surf Def} \mid \text{Line 2})P(\text{Line 2}) + P(SD \mid L3)P(L3)}$$

BAYE'S RULE

$$= \frac{(0.10)(5/15)}{(0.10)(5/15) + (0.08)(4/15) + (0.15)(6/15)}$$

2. $$P(B^c \mid A) = \frac{P(B^c \cap A)}{P(A)} = \frac{P(A) - P(B \cap A)}{P(A)} = 1 - \frac{P(B \cap A)}{P(A)} = 1 - P(B \mid A)$$
$$< 1 - P(B) = P(B^c)$$

3. Let N_i denote incorrect supplier at the ith test and
C_i denote correct supplier at the ith test

want $P(N_1 \cap N_2 \cap C_3) + P(N_1 \cap N_2 \cap N_3 \cap C_4) + P(N_1 \cap N_2 \cap N_3 \cap N_4 \cap C_5)$

$P(N_1 \cap N_2 \cap C_3) = P(C_3 \mid N_1 \cap N_2)\,P(N_2 \mid N_1)\,P(N_1)$ Product rule

$$= (1/3)(3/4)(4/5) = 1/5$$

Similarly do the others

P(at least three tests) $= 3/5$

4. a) $\binom{4}{2}\binom{11}{1} / \binom{15}{3}$

b) $\left[\binom{5}{3} + \binom{6}{3} + \binom{4}{3}\right] / \binom{15}{3}$

c) $\binom{4}{1}\binom{5}{1}\binom{6}{1} / \binom{15}{3}$

d) $$1 - \left[\left(\frac{4}{15}\right) + \left(\frac{11}{15}\right)\left(\frac{4}{14}\right) + \left(\frac{11}{15}\right)\left(\frac{10}{14}\right)\left(\frac{4}{13}\right) + \left(\frac{11}{15}\right)\left(\frac{10}{14}\right)\left(\frac{9}{13}\right)\left(\frac{4}{12}\right) + \left(\frac{11}{15}\right)\left(\frac{10}{14}\right)\left(\frac{9}{13}\right)\left(\frac{8}{12}\right)\left(\frac{4}{11}\right)\right]$$

[complement of at least 6 tries is a 23 watt is found either in the first try or the second or ... or 5th]

5) $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - [0.22 + 0.25 - 0.11]$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(A \cap C)] + P(A \cap B \cap C)$
(inclusion – exclusion principle)

$P(C) = P(C \cap (A \cap B)) + P(C \cap (A \cap B)^c)$

$\therefore P(C \cap (A^c \cup B^c)) = P(C \cap (A \cap B)^c) = P(C) - P(A \cap B \cap C)$