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CMPE 320 HW 04

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1. Given,
   Probability of not losing the first game: $p_1 = 0.4$
   Probability of losing the first game: $p_1^c = 1 - 0.4 = 0.6$
   Probability of not losing the second game: $p_2 = 0.7$
   Probability of losing the second game: $p_2^c = 1 - 0.7 = 0.3$

   Therefore, the $pmf(X)$ where $X = 0, 1, 2, 4$ represents the number of points earned over the weekend:

$$\begin{aligned} P(X = 0) &= p_1^c \cdot p_2^c \\ &= 0.6 \cdot 0.3 \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} P(X = 1) &= \frac{p_1^c \cdot p_2}{2} + \frac{p_1 \cdot p_2^c}{2} \\ &= \frac{0.6 \cdot 0.7}{2} + \frac{0.4 \cdot 0.3}{2} \\ &= 0.27 \end{aligned}$$

□

2. Given $p = 1/649640$. Therefore,

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \left(\frac{649640 - 1}{649640} \right)^{649640} \\ &= 1 - \left(1 - \frac{1}{649640} \right)^{649640} \end{aligned}$$

If $n = 649640$

$$P(X \geq 1) = 1 - \left(1 - \frac{1}{n}\right)^n$$

$$= 1 - \lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^n$$

$$= 1 - \frac{1}{e} \qquad \square$$

3. A claim is first filed with the probability
   $pq = (0.05)(1 - 0.05)^{n-1} = (0.05)(0.095)^{n-1}$ $\square$

4. (a) $Y = X\ (mod\, 3)$

$$P(Y = 0) = P(X = \{0, 3, 6, 9\})$$
$$= \frac{4}{10}$$
$$= 0.4$$

$$P(Y = 0) = P(X = \{0, 3, 6, 9\})$$
$$= \frac{4}{10}$$
$$= 0.4$$

$$P(Y = 1) = P(X = \{1, 4, 7\})$$
$$= \frac{3}{10}$$
$$= 0.3$$

$$P(Y = 2) = P(X = \{2, 5, 8\})$$
$$= \frac{3}{10}$$
$$= 0.3$$

(b) $Y = 5 \; (mod \; X + 1)$

$$\begin{aligned} P(Y = 0) &= P(X = \{0, 4\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y = 1) &= P(X = \{1, 5\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y = 2) &= P(X = \{2\}) \\ &= \frac{1}{10} \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} P(Y = 5) &= P(X = \{5, 6, 7, 8, 9\}) \\ &= \frac{5}{10} \\ &= 0.5 \end{aligned}$$

□

5. Since X is uniformly distributed over $[a, b]$,

$$p_X(k) = \begin{cases} \frac{1}{b-a+1}, & \text{if } k \in [a, b], \\ 0, & \text{otherwise} \end{cases}$$

and

$$max\{0, X\} = \begin{cases} X, & \text{if } X > 0 \\ 0, & \text{if } X \leq 0 \end{cases}$$

Then,

$$P(max\{0, X\} = 0) = P(X \leq 0)$$

$$= \frac{|a|+1}{b-a+1}$$

Similarly, for $min\{0, X\}$

$$P(min\{0, X\} = 0) = P(X \geq 0)$$

$$= \frac{b+1}{b-a+1}$$

For $k > 0$,

$$P(max\{0, X\} = k) = P(max\{0, X\} = k)$$

$$= P(X = k)$$

$$= \frac{1}{b-a+1} \qquad \square$$

6. $\square$

7. Since $P_x(X) = sin(X\pi) = 0$ for $X \in \mathbb{Z}$:

$$E[sin(X\pi)] = \sum_{k \in \mathbb{Z}} kP_x(k)$$

$$= 0$$

Since $P_x(X) = cos(X\pi) = 1$ for $X \in \mathbb{Z}$:

$$E[cos(X\pi)] = \sum_{k \in \mathbb{Z}} kP_x(k)$$

$$= 1 \qquad \square$$

8. (a) Since the event where Fischer wins is independent, and a win is determined by a win in the $(n+1)$th until n ties:

$$\sum_{n \geq 0} (1-p-q)^{n-1}(p) = \frac{p}{p+q}$$

(b) The PMF of the geometric probability

$$p_X(k) = (1-p-q)^{k-1}(p+q), \text{ for } k \geq 0$$

The mean of the geometric probability

$$E[p_X] = \frac{1}{p+q}$$

The variance of the geometric probability

$$var[p_X] = \frac{1-(p+q)}{(p+q)^2} \qquad \square$$

9. $\square$

10. $\square$