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CMPE 320 HW 03

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1. With 3 n-sided rolls, there are n^3 possibilities.
   The probability that either of the pair of persons roll the same face of the die is therefore $n/n^3 = 1/n^2$.
   Therefore

$$P(A_{12}) = P(A_{13}) = P(A_{23}) = \frac{n}{n^3}$$
$$= \frac{1}{n^2}$$

   But if both the events A_{12} and A_{13} takes place, that is both persons 1 and 2 and persons 1 and 3 roll the same face, then that yields A_{23}.
   That is, if both persons 1 and 2 and persons 1 and 3 roll the same face, then that implies persons 1 and 3 rolled the same face.
   But the outcome of person 3's roll is not dependent on the other persons.
   That is, pairwise A_{12} and A_{13}, A_{12} and A_{23}, and A_{13} and A_{23} are independent.
   But if considered individually, they are dependent. □

2. Consider the following counter-example with two independent tosses of a fair coin.
   Let events $B = \{HT, HH\}$ and $C = \{HT, TT\}$ represent tosses where they landed heads and tails respectively.
   Let $A = \{HT, TH\}$ be the event that exactly one toss resulted in heads.
   Then,

$$P(A) = P(B) = P(C) = \frac{1}{2}$$

   And,

$$P(A \cup B) = P(A \cup C) = \frac{1}{4}$$

   Therefore, A and B and A and C are both independent events.
   Therefore, B and C are also independent events.
   However,

$$P(A \cap (B \cup C)) = \frac{1}{4} \neq P(A)P(B \cup C) = \frac{1}{2} \cdot \frac{3}{4}$$

   Therefore, A and $B \cup C$ are dependent □

3. (a) $A_1A_2A_4A_6$ and $A_1A_3A_5A_6$ $\square$

   (b) Given,

   Probability that a_1 is closed $= p$

   Probability that a_2 and a_4 are closed $= p \cdot p = p^2$

   Probability that a_3 and a_5 are closed $= p \cdot p = p^2$

   Therefore, the probability that at least one closed path,

$$\begin{aligned} A_2A_4A_3A_5 &= 1 - P(\text{neither paths are closed}) \\ &= 1 - (1-p^2)(1-p^2) \\ &= 1 - (1-p^2)^2 \\ &= p^2(1-(1-p^2)^2) \end{aligned}$$ $\square$

4. Let p_5 denote the longer path of 5 links from A to B,
   and p_3 denote the shorter path of 3 links.

   Given, the probability of links failing independently is q.
   Therefore, the probability of links not failing is $1-q$.

   For a successful transmission, all of the links have to not fail.
   Therefore, for path p_5 the probability is $P(p_5) = (1-q)^5$
   and for path p_3 the probability is $P(p_3) = (1-q)^3$.
   Since the paths are independent of each other,

$$\begin{aligned} P(p_5 \cap p_3) &= P(p_5) \cdot P(p_3) \\ &= (1-q)^5 \cdot (1-q)^3 \\ &= (1-q)^8 \end{aligned}$$

   That is, the probability of both the paths not failing is $(1-q)^8$.
   Therefore, the probability of either the paths not failing for a successful transmission from terminal A to B is:

$$\begin{aligned} P(p_5 \cup p_3) &= P(p_5) + P(p_3) - P(p_5 \cap p_3) \\ &= (1-q)^5 + (1-q)^3 - (1-q)^8 \end{aligned}$$ $\square$

5. Possible combinations for the sum of the two rolls to be 7:

$$\{\{1,6\},\{2,5\},\{3,4\},\{4,3\},\{5,2\},\{6,1\}\}$$

6 combinations, therefore $P(\text{sum = 7}) = 6/36 = 1/6$.
For 100 repetitions:

$$\begin{aligned} P(\text{sum = 7 10 times}) &= \binom{100}{10}\left(\frac{1}{6}\right)^{10}\left(1-\frac{1}{6}\right)^{100-10} \\ &\approx 0.021 \end{aligned} \qquad \square$$

6. Since the probability of destroying one BM by both the AMMs is
$1 - P(\text{neither of the AMMs destroy}) = 1 - 0.2 * 0.2 = 0.96$

(a) With 6 BMs:

$$\begin{aligned} P(\text{all BMs are destroyed}) &= 0.96^6 \\ &= 0.78275 \end{aligned} \qquad \square$$

(b)

$$\begin{aligned} P(\text{at least one BM gets through}) &= 1 - P(\text{all BMs are destroyed}) \\ &= 1 - 0.96^6 \\ &= 0.21724 \end{aligned} \qquad \square$$

(c)

$$\begin{aligned} P(\text{exactly one BM gets through}) &= 6 \cdot 0.96^5 \cdot 0.04 \\ &= 0.19568 \end{aligned} \qquad \square$$

7. Given:
$P(\text{qualified}) = q$,
$P(\text{not qualified}) = 1 - q$,
$P(\text{correct answer} \mid \text{qualified}) = p$
$P(\text{incorrect answer} \mid \text{not qualified}) = p$

Therefore,

$$P(\text{>15 correct} \mid \text{qualified}) = \frac{P(\text{>15 correct} \cap \text{qualified})}{P(\text{qualified})}$$

$$= \frac{q\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5}{q\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5 + (1-q)\sum_{k=15}^{20}\binom{20}{15}(p)^{15}(1-p)^5} \quad \square$$

8. Let C represent the 20 random distinct cars chosen for a test drive.

   (a) To find $P(K = 0 \mid C)$, without replacement:

$$P(K=0 \mid C) = \frac{P(K=0)P(C \mid K=0)}{\sum_{i=0}^{9} P(K=i)P(C \mid K=i)}$$

$$= \frac{\binom{100}{20}}{\sum_{i=0}^{9}\binom{100-i}{20}}$$

$$\approx 0.227 \quad \square$$

   (b) To find $P(K = 0 \mid C)$, with replacement:

$$P(K=0 \mid C) = \frac{P(K=0)P(C \mid K=0)}{\sum_{i=0}^{9} P(K=i)P(C \mid K=i)}$$

$$= \frac{100^{20}}{\sum_{i=0}^{9}(100-i)^{20}}$$

$$\approx 0.213 \quad \square$$

9. With 6 colors of jelly beans, without replacement, a negative binomial distribution can be simulated:

$$\binom{100+6-1}{6-1} = \binom{105}{5}$$

$$= 96560646 \quad \square$$

10. The permutations of a word is given by:

$$\frac{\text{(length of word)!}}{\text{(repetitions of A)!(repetitions of B)!}\ldots\text{(repetitions of Z)!}}$$

(a) Since there are no repeating characters, the permutation is simply:

$$\begin{aligned}\text{permutations} &= length(\text{children})! \\ &= 8! \\ &= 40320 \qquad \square\end{aligned}$$

(b) Since the characters o repeats 2 times, k repeats 2 times, and e repeats 3 times:

$$\begin{aligned}\text{permutations} &= \frac{length(\text{bookkeeper})!}{\text{(repetitions of o)!(repetitions of k)!(repetitions of e)!}} \\ &= \frac{10!}{2!2!3!} \\ &= 151200 \qquad \square\end{aligned}$$