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CMPE 320 HW 04

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1. Given,
   probability of not losing the first game: $p_1 = 0.4$
   probability of losing the first game: $p_1^c = 1 - 0.4 = 0.6$
   probability of not losing the second game: $p_2 = 0.7$
   probability of losing the second game: $p_2^c = 1 - 0.7 = 0.3$

   Therefore, the p_X where $X = 0, 1, 2, 4$ represents the number of points earned over the weekend:

$$\begin{aligned} P(X = 0) &= p_1^c \cdot p_2^c \\ &= 0.6 \cdot 0.3 \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} P(X = 1) &= \frac{p_1^c \cdot p_2}{2} + \frac{p_1 \cdot p_2^c}{2} \\ &= \frac{0.6 \cdot 0.7}{2} + \frac{0.4 \cdot 0.3}{2} \\ &= 0.27 \end{aligned}$$

$$\begin{aligned} P(X = 2) &= \frac{p_1^c \cdot p_2}{2} + \frac{p_1 \cdot p_2^c}{2} + \frac{p_1}{2} \cdot \frac{p_2}{2} \\ &= \frac{0.6 \cdot 0.7}{2} + \frac{0.4 \cdot 0.3}{2} + \frac{0.7}{2} \cdot \frac{0.4}{2} \\ &= 0.34 \end{aligned}$$

$$P(X = 3) = \frac{p_1}{2} \cdot \frac{p_2}{2} + \frac{p_1}{2} \cdot \frac{p_2}{2}$$

$$= \frac{0.7}{2} \cdot \frac{0.4}{2} + \frac{0.7}{2} \cdot \frac{0.4}{2}$$

$$= 0.14$$

$$P(X = 4) = \frac{p_1}{2} \cdot \frac{p_2}{2}$$

$$= \frac{0.7}{2} \cdot \frac{0.4}{2}$$

$$= 0.07$$

$$p_X(k) = \begin{cases} 0.18, & \text{if } k = 0, \\ 0.27, & \text{if } k = 1, \\ 0.34, & \text{if } k = 2, \\ 0.14, & \text{if } k = 3, \\ 0.07, & \text{if } k = 4, \\ 0, & \text{otherwise} \end{cases} \qquad \square$$

2. Given $p = 1/649640$. Therefore,

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - \left(\frac{649640 - 1}{649640}\right)^{649640}$$

$$= 1 - \left(1 - \frac{1}{649640}\right)^{649640}$$

If $n = 649640$

$$P(X \geq 1) = 1 - \left(1 - \frac{1}{n}\right)^{n}$$

$$= 1 - \lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^{n}$$

$$= 1 - \frac{1}{e} \qquad \square$$

3. A claim is first filed with the geometric probability

$$\begin{aligned} p(1-p)^{n-1} &= (0.05)(1-0.05)^{n-1} \\ &= (0.05)(0.095)^{n-1} \end{aligned}$$

The total premium is

$$\begin{aligned} 1000 \cdot \sum_{k=0}^{n-1} (0.9)^k &= 1000 \cdot \frac{1-0.9^n}{1-0.9} \\ &= 10000 \cdot (1-(0.9)^n) \end{aligned}$$

Therefore, the PMF is

$$p_X(k) = \begin{cases} 0.05 \cdot (0.095)^{n-1}, & \text{if } k = 10000 \cdot (1-(0.9)^n), n = 1, 2, \ldots, \\ 0, & \text{otherwise} \end{cases} \qquad \square$$

4. (a) $Y = X \ (mod\ 3)$

$$\begin{aligned} P(Y=0) &= P(X = \{0, 3, 6, 9\}) \\ &= \frac{4}{10} \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(Y=0) &= P(X = \{0, 3, 6, 9\}) \\ &= \frac{4}{10} \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(Y=1) &= P(X = \{1, 4, 7\}) \\ &= \frac{3}{10} \\ &= 0.3 \end{aligned}$$

$$P(Y=2) = P(X=\{2,5,8\}) = \frac{3}{10} = 0.3$$

(b) $Y = 5\ (mod\ X+1)$

$$\begin{aligned} P(Y=0) &= P(X=\{0,4\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y=1) &= P(X=\{1,5\}) \\ &= \frac{2}{10} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(Y=2) &= P(X=\{2\}) \\ &= \frac{1}{10} \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} P(Y=5) &= P(X=\{5,6,7,8,9\}) \\ &= \frac{5}{10} \\ &= 0.5 \end{aligned}$$

□

5. Since X is uniformly distributed over $[a,b]$,

$$p_X(k) = \begin{cases} \frac{1}{b-a+1}, & \text{if } k \in [a,b], \\ 0, & \text{otherwise} \end{cases}$$

and

$$max\{0, X\} = \begin{cases} X, & \text{if } X > 0 \\ 0, & \text{if } X \leq 0 \end{cases}$$

Then,

$$P(max\{0, X\} = 0) = P(X \leq 0)$$

$$= \frac{|a| + 1}{b - a + 1}$$

Similarly, for $min\{0, X\}$

$$P(min\{0, X\} = 0) = P(X \geq 0)$$

$$= \frac{b + 1}{b - a + 1}$$

For $k > 0$,

$$P(max\{0, X\} = k) = P(max\{0, X\} = k)$$

$$= P(X = k)$$

$$= \frac{1}{b - a + 1} \qquad \square$$

6. (a) Find K

$$1 = \sum_{K=-3}^{3} p_X(K)$$
$$1 = K \sum_{x=-3}^{3} x^2$$
$$1 = K(9 + 4 + 1 + 0 + 1 + 4 + 9)$$
$$\Rightarrow K = \frac{1}{28}$$

(b) Find the PMF of Y

Since $Y = |X|$, then $y \in \{0, 1, 2, 3\}$

$$p_Y(0) = p_X(0)$$

$$= 0$$

$$p_Y(1) = p_X(-1) + p_X(1)$$

$$= \frac{1^2}{28} + \frac{1^2}{28}$$

$$= \frac{1}{14}$$

$$p_Y(2) = p_X(-2) + p_X(2)$$

$$= \frac{2^2}{28} + \frac{2^2}{28}$$

$$= \frac{4}{14}$$

$$p_Y(3) = p_X(-3) + p_X(3)$$

$$= \frac{3^2}{28} + \frac{3^2}{28}$$

$$= \frac{9}{14}$$

(c) General formula for p_Y

$$p_Y = \begin{cases} 2p_X(y), & \text{if } y \in \{0, 1, 2, 3\}, \\ 0, & \text{otherwise} \end{cases}$$ □

7. Since $P_x(X) = sin(X\pi) = 0$ for $X \in \mathbb{Z}$:

$$E[sin(X\pi)] = \sum_{k \in \mathbb{Z}} kP_x(k)$$

$$= 0$$

Since $P_x(X) = cos(X\pi) = 1$ for $X \in \mathbb{Z}$:

$$E[cos(X\pi)] = \sum_{k\in\mathbb{Z}} kP_x(k)$$
$$= 1 \qquad \square$$

8. (a) Since the event where Fischer wins is independent, and a win is determined by a win in the $(n+1)$th until n ties:

$$\sum_{n\geq 0}(1-p-q)^{n-1}(p) = \frac{p}{p+q}$$

(b) The PMF of the geometric probability

$$p_X(k) = (1-p-q)^{k-1}(p+q), \text{ for } k \geq 0$$

The mean of the geometric probability

$$E[X] = \frac{1}{p+q}$$

The variance of the geometric probability

$$var[X] = \frac{1-(p+q)}{(p+q)^2} \qquad \square$$

9. Since the distribution is binomial with $n = 10$

$$E[X] = np \geq 10000 - 10$$
$$\Rightarrow np \geq 9990$$
$$\Rightarrow p \geq 0.999 \qquad \square$$

10. Since

$$var(X) = E[X^2] - (E[X])^2 \Rightarrow E[X^2] = var(X) + (E[X])^2$$

Then

$$E[(X_1 + \ldots + X_n)^2] = var(X_1 + \ldots + X_n) + (E[(X_1 + \ldots + X_n)])^2$$

$$= n \cdot var(X_1) + (n \cdot E[X_1])^2, \text{ (since the variables are identical)}$$

$$= n \cdot (E[X_1^2] - (E[X_1])^2) + n^2 \cdot E[X_1]^2$$

$$= n \cdot E[X_1^2] + (n^2 - n) \cdot (E[X_1])^2$$

Let $c = n$ and $d = n^2 - n$, then

$$E[(X_1 + \ldots + X_n)^2] = cE[X_1^2] + d(E[X_1])^2 \qquad \square$$