CMPE 320: Probability, Statistics, and Random Processes

Lecture 14: Cumulative distribution functions

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Cumulative distribution function (CDF)

- CDF "accumulates" probability "up to" the value x

$$F_X(x) = P(X \le x)$$

notation for CDF

- CDF can describe both discrete and continuous RVs

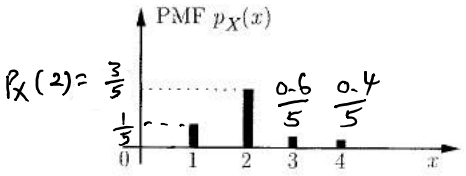
Discrete RV: $F_X(x) = P(X \le x) = \sum_{k \le x} P_0(X = k) = \sum_{k \le x} P_X(k)$

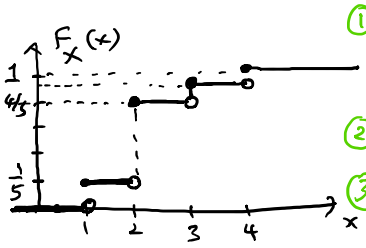
Continuous RV: $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$

- Any specification of probability of events $\{X \le x\}$ (be it through PMF, PDF, or CDF) is probability law of RV X

PMF to CDF

$$F_X(x) = \sum_{k \le x} p_X(k)$$

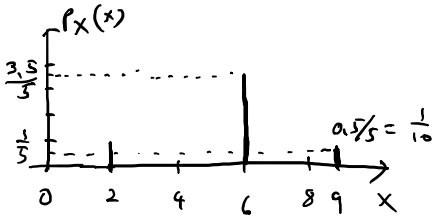


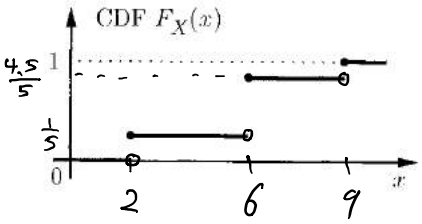


① CDF must be monotonically nondecreasing

② $F_X(-\infty) = 0$

③ $F_X(\infty) = 1$

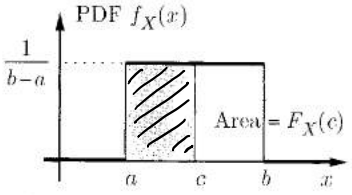


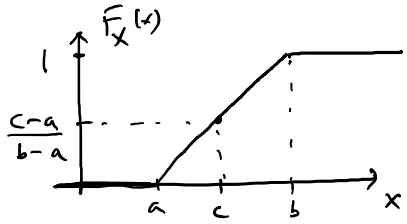


$$p_X(k) = F_X(k) - F_X(k-1)$$

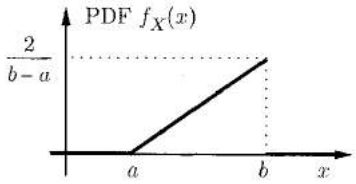
PDF to CDF

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$





PDF to CDF (2)



Properties of CDF

PMF or PDF from CDF

- Discrete RV X

- Continuous RV X

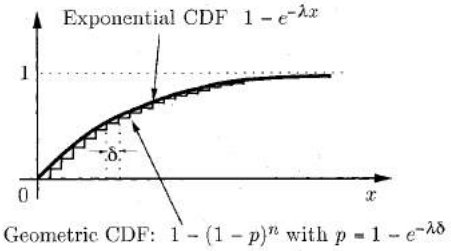
Example 3.6. The Maximum of Several Random Variables. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Thus,

$$X = \max\{X_1, X_2, X_3\},$$

where X_1, X_2, X_3 are the three test scores and X is the final score. Assume that your score in each test takes one of the values from 1 to 10 with equal probability $1/10$, independently of the scores in other tests. What is the PMF p_X of the final score?

Geometric and exponential CDFs

- Find the CDF of a geometric RV X. Repeat with an exponential RV X.



Problem 6. Calamity Jane goes to the bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter λ. What is the CDF of Jane's waiting time?