CMPE 320: Probability, Statistics, and Random Processes

Lecture 17: Review

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Problem 26. PMF of the minimum of several random variables. On a given day, your golf score takes values from the range 101 to 110, with probability 0.1, independent of other days. Determined to improve your score, you decide to play on three different days and declare as your score the minimum X of the scores X_1, X_2, and X_3 on the different days.

(a) Calculate the PMF of X.

(b) By how much has your expected score improved as a result of playing on three days?

$X = \min\{X_1, X_2, X_3\}$ $\quad$ X_i : uniform over $101, \cdots, 110$

(a) $p_X(k) = P(X=k) = F_X(k) - F_X(k-1) = P(X \le k) - P(X \le k-1)$

$= 1 - P(X>k) - [1 - P(X>k-1)] = P(X>k-1) - P(X>k)$

$= P(X_1 > k-1, X_2 > k-1, X_3 > k-1) - P(X_1>k, X_2>k, X_3>k)$

$= P(X_1 > k-1)P(X_2>k-1)P(X_3>k-1) - P(X_1>k)P(X_2>k)P(X_3>k)$

$= \left(\frac{111-k}{10}\right)^3 - \left(\frac{110-k}{10}\right)^3$

$\underbrace{k, k+1, k+2, \cdots, 110}_{110-k+1}$

(b) $E[X] = \sum_{k=101}^{110} k \cdot p_X(k) = 103.025$

$E[X_i] = 105.5$

Problem 31. Consider four independent rolls of a 6-sided die. Let X be the number of 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?

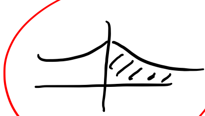
$$p_{X,Y}(x,y) = P(X=x, Y=y) = \frac{4!}{x!\,y!\,(4-x-y)!}\left(\frac{1}{6}\right)^{x}\left(\frac{1}{6}\right)^{y}\left(\frac{4}{6}\right)^{4-x-y} \qquad 0 \le x+y \le 4$$

$$\binom{4}{x\ \ y\ \ 4-x-y}$$

Alternatively, $$P(X=x, Y=y) = \underbrace{P(X=x \mid Y=y)}\ \underbrace{P(Y=y)}$$
$$= \binom{4-y}{x}\left(\frac{1}{5}\right)^{x}\left(\frac{4}{5}\right)^{4-y-x}\binom{4}{y}\left(\frac{1}{6}\right)^{y}\left(\frac{5}{6}\right)^{4-y}$$

Problem 2. Laplace random variable. Let X have the PDF

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|},$$



where λ is a positive scalar. Verify that f_X satisfies the normalization condition, and evaluate the mean and variance of X.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow 2\int_0^{\infty} \frac{\lambda}{2} \cdot e^{-\lambda x}\,dx = 1$$

$$E[X] = \int x f_X(x)\,dx = \underbrace{\int_0^{\infty} x \cdot \frac{\lambda}{2} e^{-\lambda x}\,dx} + \int_{-\infty}^{0} x\,\frac{\lambda}{2} e^{+\lambda x}\,dx \qquad -\frac{1}{\lambda}$$

$$= \Big[\underbrace{x \cdot \frac{\lambda}{2} \cdot \left(-\frac{1}{\lambda}\right) e^{-\lambda x}\Big|_0^{\infty}}_{0} + \frac{1}{2}\int_0^{\infty} e^{-\lambda x}\,dx\Big]$$

$$= \Big[\ 0 + \frac{-1}{2\lambda} e^{-\lambda x}\Big|_0^{\infty}\Big] = 2 \cdot \frac{1}{2\lambda} \qquad = \cancel{\frac{1}{\lambda}}\ 0$$

$$Var[X] = E[(X - E[X])^2]$$
$$= E[X^2] - (E[X])^2$$

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