STAT 355, Spring 2014 **Practice Exam2–2**

1. Someone claims that the functions $f(x)$ in (1) is pdf and $F(x)$ in (2) are CDF of some continuous random variables. However it is not certain that they really are. Check whether the following functions are pdf(1) and CDF(2). Justify your answer for each case.

   (a) $f(x) = \begin{cases} x^2 & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

   (b) $F(x) = \begin{cases} x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

2. Suppose X is a Uniform random variable in (25, 45).

   (a) Draw the probability density function of X.

   (b) Find $P(X < 40)$.

3. The density function of X is given by

   $f(x) = \begin{cases} b + ax^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

   If $E(X) = \frac{3}{5}$, find a, b.

4. Suppose that verbal GRE score X is known to have normal distribution with mean 500, standard deviation 115, $X \sim N(500, 115^2)$.

   (a) Use Table to find what proportion of scores are less than 300, $P(X < 300)$.

   (b) A graduate school program in English will offer scholarships to students with verbal GRE scores in the top 15%. What is the cut-off score for these scholarships? (Hint : $P(X \ge c) = .15$ Find c.)

5. (20 pts) The joint probability mass function of the two random variables, X and Y is specified by the following table:

		Y		
		1	2	3
X	1	.15	.25	.10
	2	.12	.20	.08
	3	0.03	.05	.02

(a) (10 pts) Compute the correlation of X and Y.

(b) (10 pts) Are X and Y independent? (Explain your answer.)

6. The joint probability mass function of the two random variables, X and Y is specified by the following table:

		Y		
		1	2	3
	1	.10	.25	.10
X	2	.12	?	?
	3	0.05	.05	.02

(a) (7 pts) If $E(Y) = 2$, what are $P(X = 2, Y = 2)$ and $P(X = 2, Y = 3)$?

(b) (5 pts) What is $P(Y \geq 2|X = 1)$?

(c) (5 pts) What is $E(Y|X = 1)$?

7. (7 pts) The lifetime of a certain type of battery is normally distributed with mean value 5 hours and standard deviation 1.5 hour. There are four batteries in a package. What lifetime value is such that the total lifetime of all batteries in a package exceeds that value for only 15% of all packages?

8. (10 pts) Let $\bar{X}$ denote the mean of random sample of size 30 from the distribution whose p.d.f. is $f(x) = x^3/4, 0 \leq x \leq 2$. What is $P(1.5 \leq \bar{X} \leq 1.65)$?

9. In a local coffee shop, the time between ordering a cup of coffee and when it is delivered is $f(x) = \frac{1}{3.4}\ e^{-\frac{x}{3.4}}, \quad x > 0$.

(a) (5 pts) If you order a cup of coffee, what is the probability that you have to wait between 3.5 and 4.5 minutes?

(b) (5 pts) If you order a cup of coffee, what is the shortest amount of time that you can wait and still be in the slowest 15% of the population of customers?

(c) Find $E(X)$.