Sabbir Ahmed

DATE: February 10, 2018

CMPE 320 HW 01

---

1. Let A and B be two sets. Under what conditions is the set $A \cap (A \cup B)^c$ empty?

   Answer If $A = B$, then $A \cap (A \cup B)^c = A \cap (A \cup A)^c = A \cap A^c = \varnothing$.

2. Prove:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

   Answer Let $D = B \cup C$ such that $A \cup B \cup C = A \cup D$. Then:

   $$P(A \cup B \cup C) = P(A \cup D)$$
   $$= P(A) + P(D) - P(A \cap D)$$
   $$= P(A) + P(B \cup C) - P(A \cap D)$$
   $$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap D)$$

   Since $P(A \cap D) = P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$ by the distributive law:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap D)$$
   $$= P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C))$$

   Since $P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - (P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C)))$$
   $$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

   $$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

3. We are given that $P(A^c) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$. Determine $P(A \cup B)$.

   Answer

$$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\because P(A^c) = 1 - P(A)$$

$$\therefore P(A \cup B) = (1 - 0.6) + 0.3 - 0.2$$

$$= 0.5$$

4. We roll a four-side die once and then we roll it as many times as is necessary to obtain a different face than the one obtained in the first roll. Let the outcome of the experiment be (r_1, r_2) where r_1 and r_2 are the results of the first and the last rolls, respectively. Assume that all possible outcomes have equal probability. Find the probability that:

   (a) r_1 is even.

   Answer

   (b) Both r_1 and r_2 are even.

   Answer

   (c) $r_1 + r_2 < 5$.

   Answer

5. You enter a special kind of chess tournament, whereby you play one game with each of three opponents, but you get to choose the order in which you play your opponents. You win the tournament if you win two games in a row. You know your probability of a win against each of the three opponents. What is your probability of winning the tournament, assuming that you choose the optimal order of playing the opponents?

   Answer

6. Alice and bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

   (a) The magnitude of the difference of the two numbers is greater than 1/3.

   Answer

   (b) At least one of the numbers is greater than 1/3.

   Answer

   (c) The two numbers are equal.

   Answer

   (d) Alice's number is greater than 1/3.

   Answer

Find the probabilities $P(A)$, $P(B)$, $P(A \cap B)$, $P(C)$, $P(D)$, $P(A \cap D)$,

7. The disc containing the only copy of your term project just got corrupted, and the disc got mixed up with three other corrupted discs that were lying around. It is equally likely that any of the four discs holds the corrupted remains of your term project. Your computer expert friend offers to have a look, and you know from past experience that his probability of finding your term project from any disc is 0.4 (assuming the term project is there). Given that he searches on disc 1 but cannot find your thesis, what is the probability that your thesis is on disc i for $i = 1, 2, 3, 4$?

Answer

8. A person has forgotten the last digit of a telephone number, so he dials the number with the last digit randomly chosen. How many times does he have to dial (not counting repetitions) in order that the probability of dialing the correct number is more than 0.5

Answer

9. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 85% accurate if the student is in fact overstressed. It is known that 99.5% of all students are overstressed. Given that a particular student tests negative for stress, what is the probability that the test results are correct, and that this student is not overstressed?

Answer

10. A hiker starts by taking one of n available trails, denoted $1, 2, \ldots, n$. An hour into the hike, trail i subdivides into $1 + i$ subtrails, only one of which leads to the hiker's destination. The hiker has no map and makes random choices of trail and subtrail. What is the probability of reaching the destination?

Answer