MATH 407

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* $\gcd(2x^4+x^3-6x^2+7x-2,\ 2x^3-7x^2+8x-4)$

Doing long division $\Rightarrow q_1=(x+4),\ r_1=(14x^2-21x+14)$

Doing long division $\Rightarrow q_2=(x-2),\ r_2=0$

$\therefore \gcd=(x^2-\frac{3}{2}x+1)$ (make it monic by dividing by 2)

* Prop 4.2.8 Let $p(x), f(x), g(x) \in F[x]$
If $\gcd(p,f)=1$ and $p|fg$ then $p|g$

Pf. $ap+bf=1$

Thus, $pag+b(fg)=g$

$fg=h\cdot p$, then $pag+p(hb)=g$

or $p(ag+hb)=g$

$p|g$

* Thm. 4.2.9 (Prime Factorization Thm)

If $f\in F[x]$ is non-constant then it is uniquely expressed as $p_1^{r_1}p_2^{r_2}\cdots p_k^{r_k}$, where:

a) each p_i is irreducible

b) each $r_i \in \mathbb{N}$

* Assume $\deg(f)=k$ and that Thm. 4.2.9 valid all polynomials lesser degree

Let p_1 irreducible $p_1|f$ so $f=p_1g=p_1(p_1^{r_1-1}p_2^{r_2}\cdots p_k^{r_k})$

$(\longrightarrow)$

Suppose $f = q_1^{s_1} q_2^{s_2} \cdots q_\ell^{s_\ell}$

Assume $p_1 | q_1^{s_1}$

Then $p_1 = q_1 \Rightarrow p_1 \left(p_1^{r_1-1} p_2^{r_2} \cdots p_k^{r_k}\right) = p_1 \left(p_1^{s_1-1} q_2^{s_2} \cdots q_\ell^{s_\ell}\right)$

$$\Rightarrow p_1^{r_1-1}\left(\cdots p_k^{r_k}\right) = p_1^{s_1-1}\left(q_2^{s_2} \cdots q_\ell^{s_\ell}\right)$$

$$\Rightarrow r_1 - 1 = s_1 - 1$$

Each p_i is a q_j and $r_i = s_j$

* Let $D : F[x] \to F[x]$ be linear defined by $D(x^k) = kx^{k-1}, \forall k \in \mathbb{Z}^+$

$$\Rightarrow D\left(\sum_{k=0}^{n} a_k x^k\right) = \sum_{k=0}^{n} a_k D(x^k) = \sum_{k=1}^{n} a_k \left(kx^{k-1}\right)$$

* Lemma $\dfrac{d(x^i x^j)}{dx} = \dfrac{dx^{i+j}}{dx} = (i+j)x^{i+j-1}$

$$= \left(ix^{i-1}\right)x^j + x^i\left(jx^{j-1}\right)$$

$$= \frac{dx^i}{dx} x^j + x^i \frac{dx^j}{dx}$$

Look at $D(x^i g(x))$

$$= D\left(x^i \sum_j b_j x^j\right) = D\left(\sum_j b_j x^i x^j\right)$$

$$= \sum_j b_j \left(D(x^i) \cdot x^j + x^i D(x^j)\right)$$

$$= D(x^i)\left(\sum_j b_j x^j\right) + x^i D\left(\sum_j b_j x^j\right)$$

$(\to)$

$= D(x^i)g(x) + x^i D(g(x))$ (product rule)

$* D\left(\sum_i a_i x^i \sum_j b_j x^j\right) = D\left(\sum a_i x^i\right)\sum b_j x^j + \left(\sum a_i x^i\right) D\left(\sum b_j x^j\right)$

$*$ Cor. $D(f^n) = nf^{n-1} D(f)$ (pf: calculus)