CMPE 323: Signals and Systems

Dr. LaBerge

Lab 08 Report

Using the Fast Fourier Transform

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1. Introduction

This lab explores the use of the Fast Fourier Transform (FFT) to do spectrum analysis of sampled data systems. The FFT is an extremely efficient algorithm for computing the Discrete Fourier Transform (DFT).

2. Equipment

A computer with MATLAB installed.

3. Procedure

3.1 Computing the Fourier Transform of the basic pulse

Using a time array from [-4096: 0.001: 4095], the basic anonymous pulse function was used to compute the $pulse(t + \frac{\tau}{2}, \tau)$ for t = 1. Pulses shifted by $-t/2$ (to the right) and $+t/2$ (to the left) were also computed and plotted

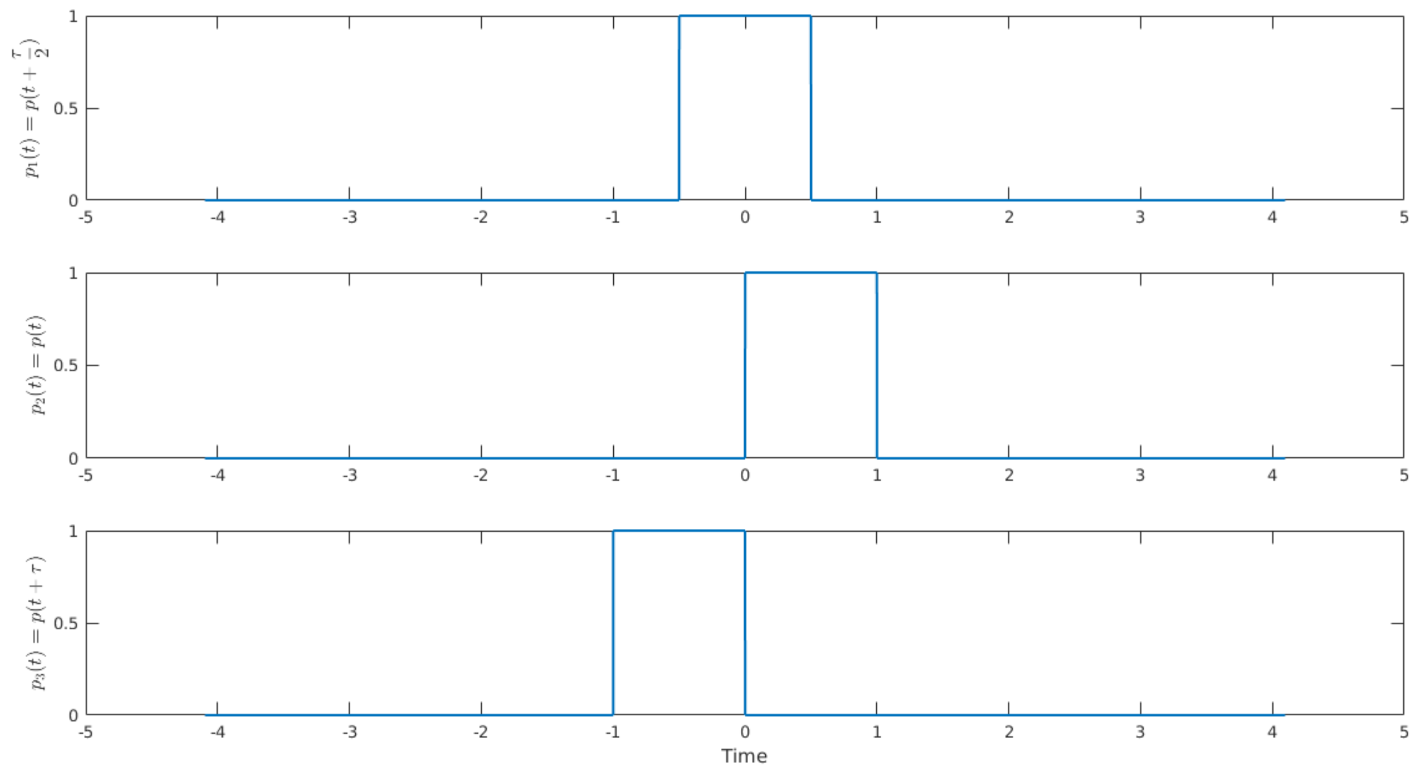

Figure 1: Basic Unit Pulses

An algorithm to estimate the Discrete Time Fourier Transform of the three pulses was developed, where DTFT is defined

$$\hat{X}[k] = T \sum_{n=0}^{n=N-1} x[n] e^{-\frac{j2\pi kn}{N}}$$

with $X[k]$ is the result at $f = \frac{k}{T}$, where $k = 0,1,2, \ldots, N-2, N-1$, and $x[n] = x(n\Delta t)$, where $n = 0,1,2, \ldots, N-2, N-1$.

The Discrete Fourier Transform of the basic unshifted pulse is known to be

$$\int_{\frac{\tau}{2}}^{-\frac{\tau}{2}} (1)\left(e^{j2\pi f}\right)dt = \tau sinc(\tau f)$$

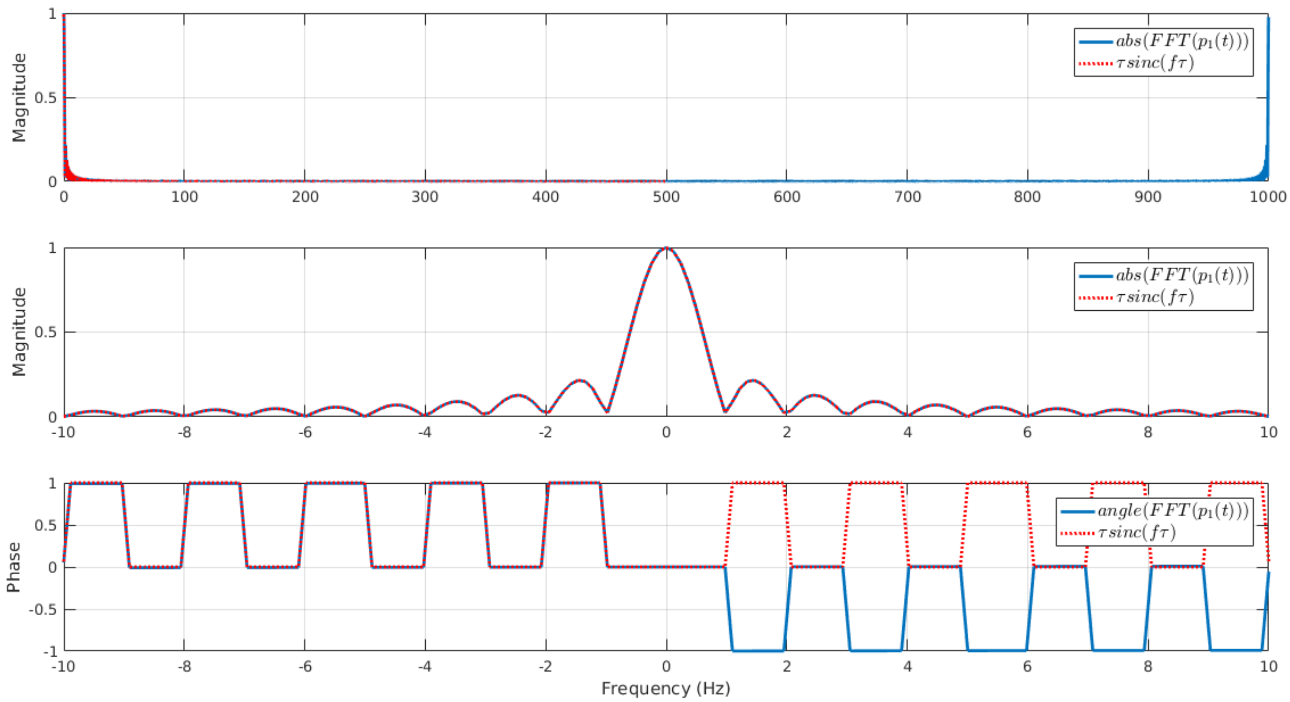

Figure 2: Fast Fourier Transform of $p_1(t)$

Using the time shift property of Fourier Transform:

$$\mathcal{F}\big(x(t-t_0)\big) = e^{-j2\pi f t_0}\mathcal{F}(x(t))$$

$$\because p_2(t) = p_1\left(t-\frac{\tau}{2}\right) \to \mathcal{F}\big(p_2(t)\big) = e^{-j\pi f\tau}\mathcal{F}\big(p_1(t)\big) = e^{-j\pi f\tau}\tau sinc(\tau f)$$

$$\because p_3(t) = p_1\left(t+\frac{\tau}{2}\right) \to \mathcal{F}\big(p_3(t)\big) = e^{j\pi f\tau}\mathcal{F}\big(p_1(t)\big) = e^{j\pi f\tau}\tau sinc(\tau f)$$

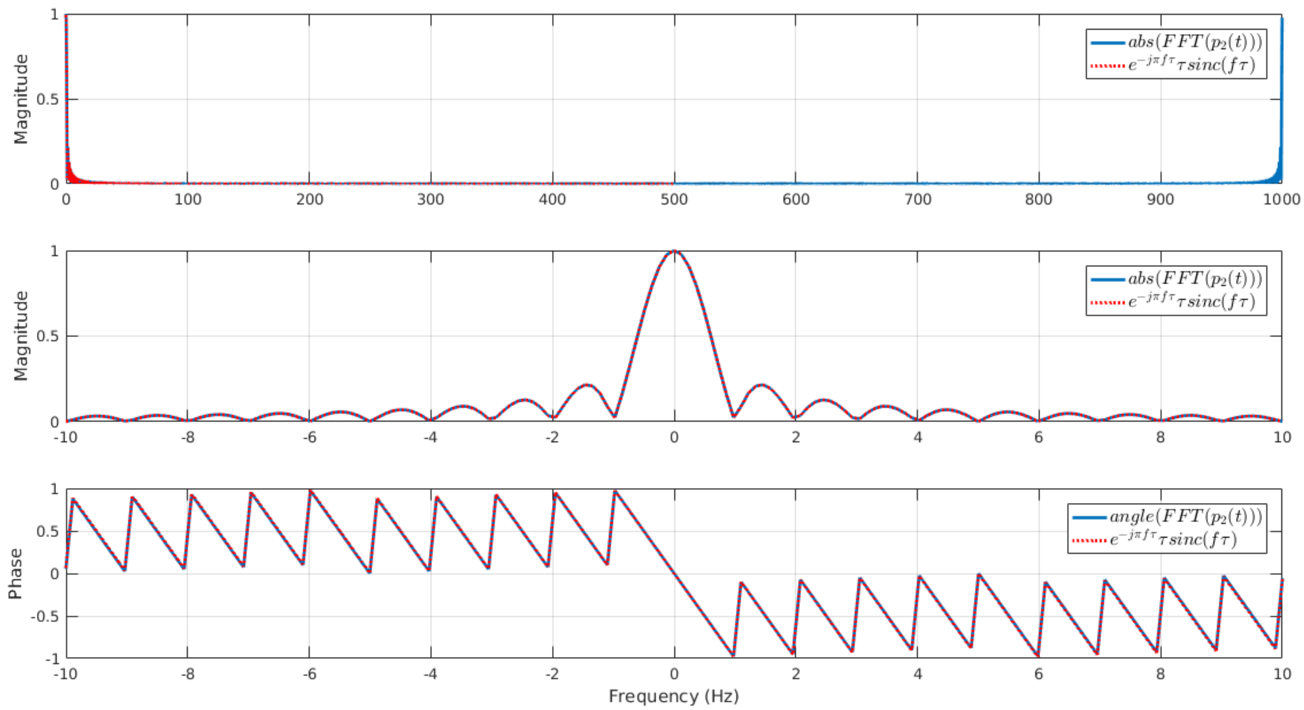

Figure 3: Fast Fourier Transform of p_2(t)

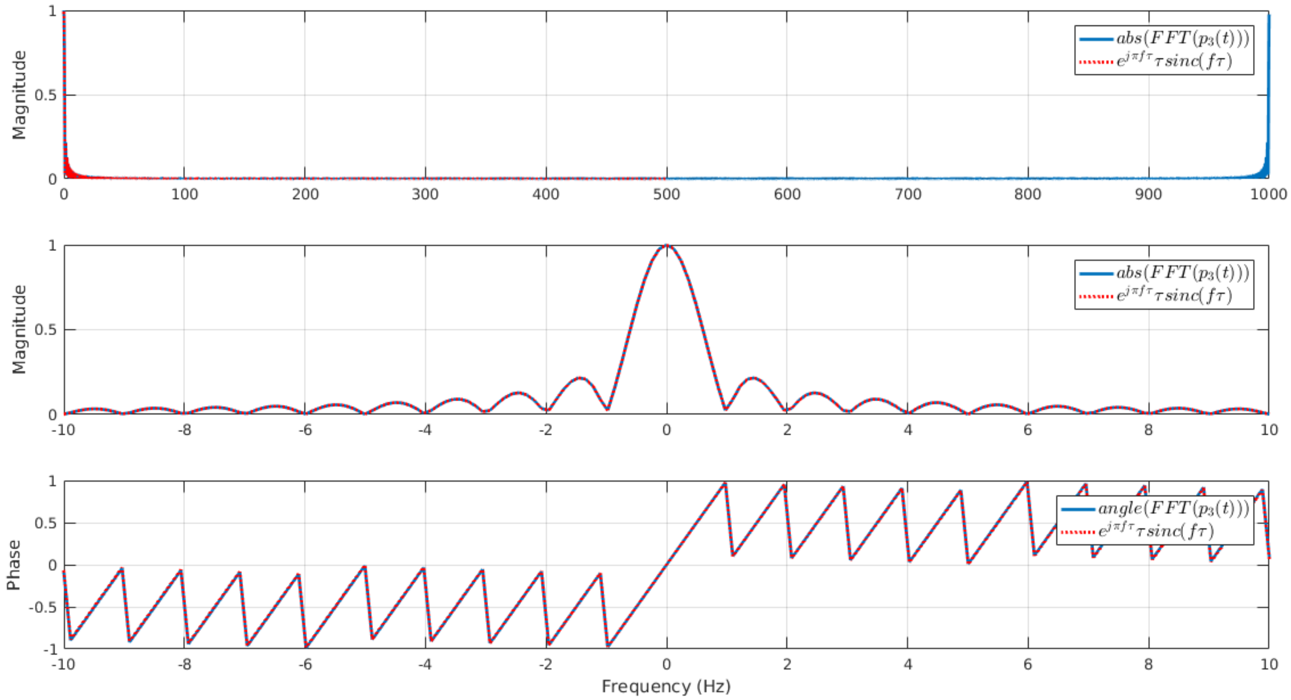

Figure 4: Fast Fourier Transform of p_3(t)

The theoretical functions appear identical to the values computed by MATLAB's $fft(x)$ function. The magnitudes of the DTFT of the three pulses also appear identical, even if the shift is obvious in the time domain. The phases however show distinction in the frequency domain. The shifted functions (p_2(t) and p_3(t)) have their phases appearing inverted of each other.

3.2 The Complex Modulation Property

Each of the three pulse functions $p_1(t)$, $p_2(t)$, and $p_3(t)$ were taken and multiplied by

$$c(t) = e^{j2\pi f_c}$$

with $f_c = 5.0049\ Hz$. The new pulses were called $x_1(t)$, $x_2(t)$, and $x_3(t)$, and were plotted along with their Discrete Time Fourier Transforms.

Using the modulation property of Fourier Transform:

$$\mathcal{F}\left(e^{j2\pi f_0 t}x(t)\right) = X(f - f_0)$$

$$\therefore \mathcal{F}\left(e^{j2\pi f_c t}p_1(t)\right) = \tau sinc\big(\tau(f - f_c)\big)$$

$$\therefore \mathcal{F}\left(e^{j2\pi f_c t}p_2(t)\right) = e^{-j\pi(f-f_c)\tau}\tau sinc\big(\tau(f - f_c)\big)$$

$$\therefore \mathcal{F}\left(e^{j2\pi f_c t}p_3(t)\right) = e^{j\pi(f-f_c)\tau}\tau sinc\big(\tau(f - f_c)\big)$$

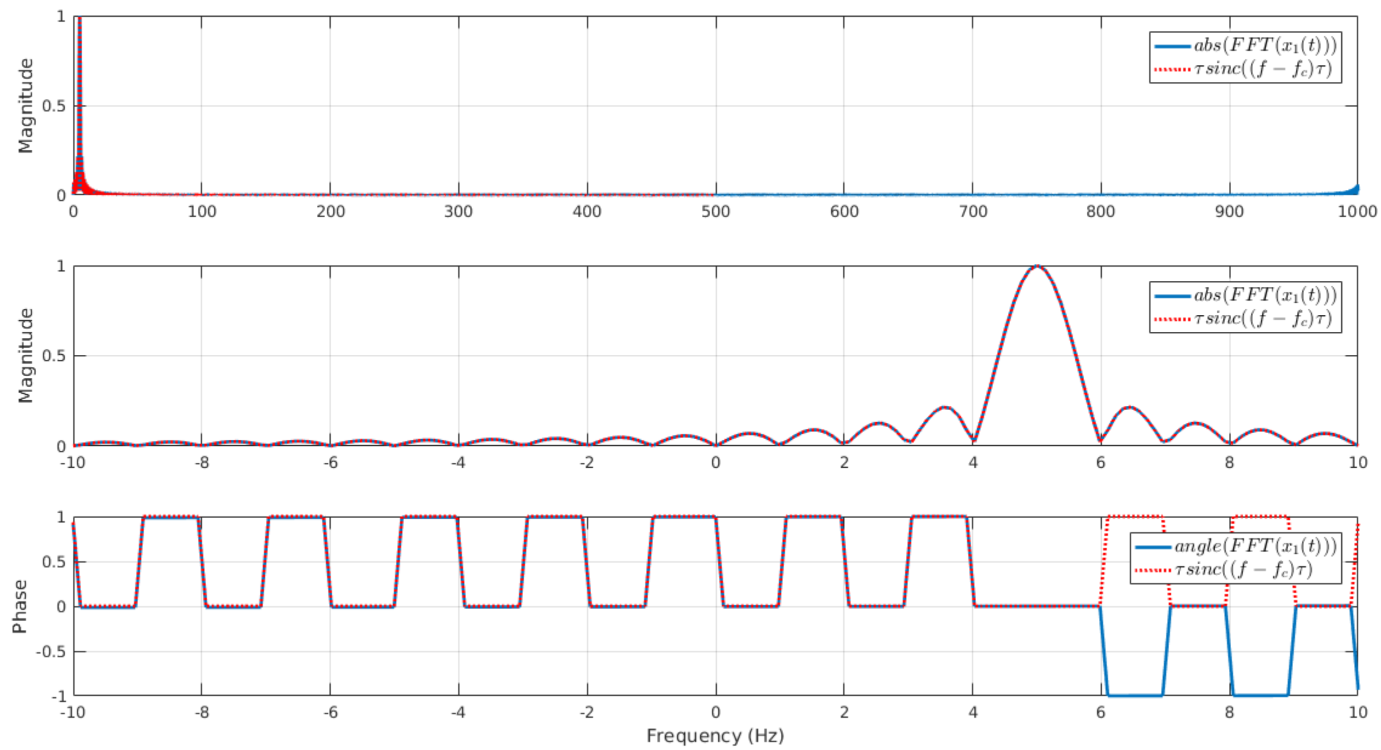

Figure 5: Fast Fourier Transform on $x_1(t)$

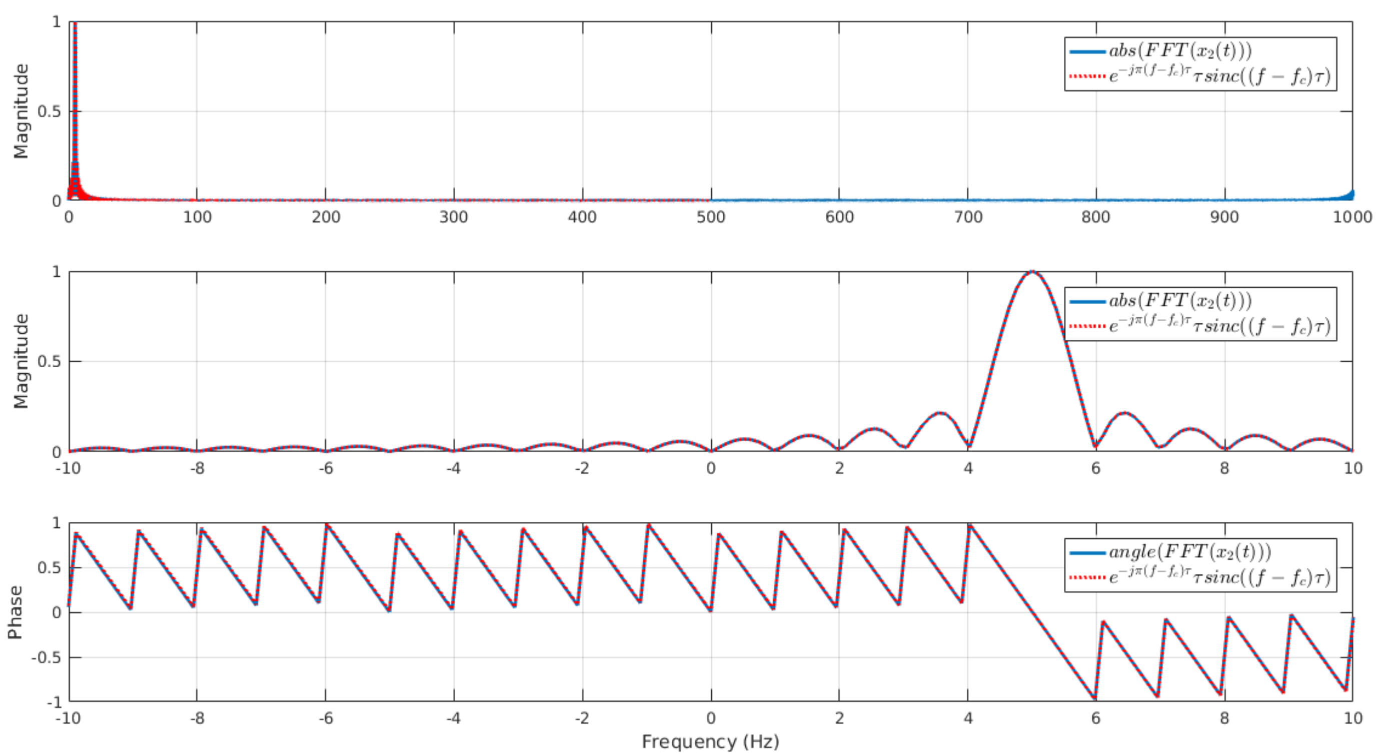

Figure 6: Fast Fourier Transform on $x_2(t)$

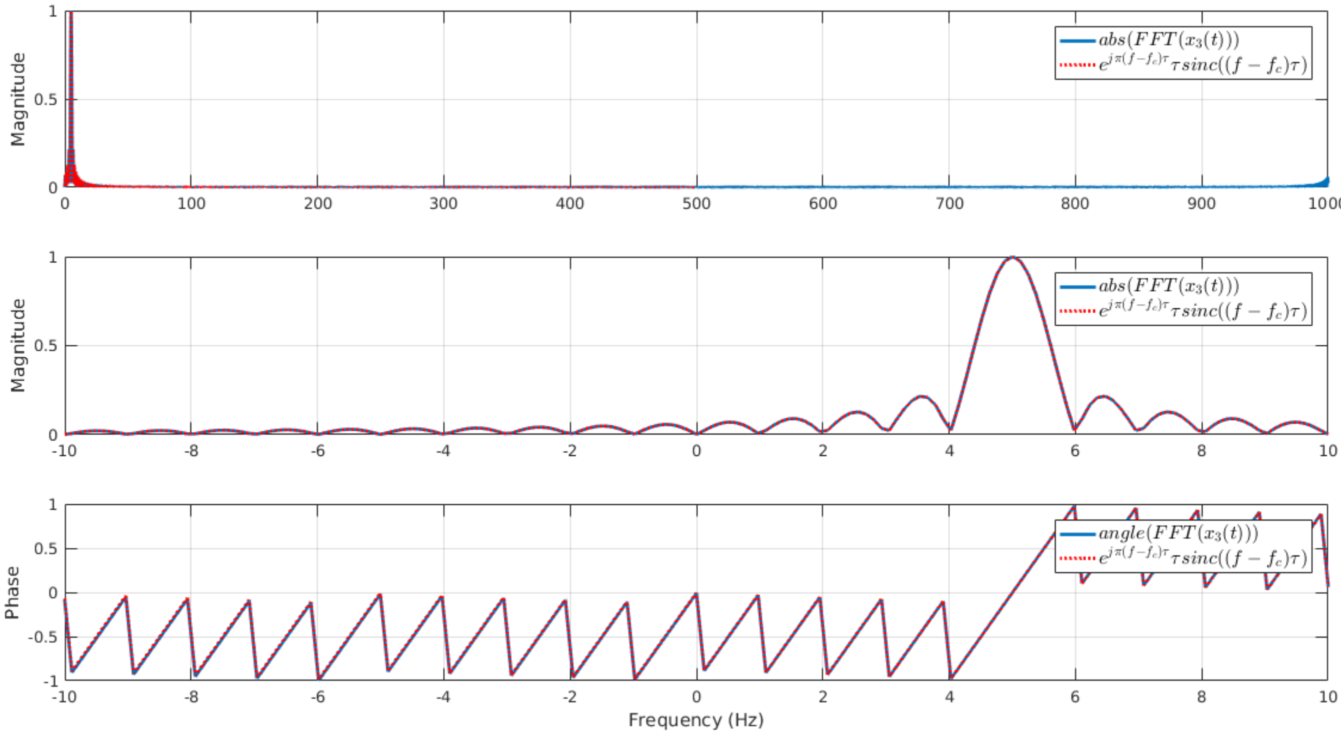

Figure 7: Fast Fourier Transform on $x_3(t)$

Both the theoretical and computed functions appear identical to the Fourier Transforms of the original pulses, with their magnitudes and phases shifted to the right by $f_c = 5.0049\ Hz$.

3.3 The Cosine Modulation Property

A corollary to the Complex Modulation Theorem is the Cosine Modulation Property, which uses the Euler expansion of the cosine and applies the Complex Modulation Property. Each of the original pulses were taken and multiplied by

$$m(t) = \cos(2\pi f_c t)$$

with $f_c = 5.0049\ Hz$. The new pulses were called $w_1(t)$, $w_2(t)$, and $w_3(t)$, and were plotted along with their Discrete Time Frequency Transforms.

Using the modulation property of Fourier Transform:

$$\mathcal{F}\left(e^{j2\pi f_0 t}x(t)\right) = X(f - f_0)$$

And Euler's Identity:

$$\cos(x) = \frac{e^{jx} - e^{-jx}}{2}$$

$\because \mathcal{F}\left(e^{j2\pi f_0 t}x(t)\right) = X(f - f_0)$ and $\mathcal{F}\left(e^{-j2\pi f_0 t}x(t)\right) = X(f + f_0)$

$$\therefore\ \mathcal{F}\big(\cos(2\pi f_c t)x(t)\big) = \frac{1}{2}\big(X(f-f_0) + X(f+f_0)\big)$$

$$\therefore\ \mathcal{F}\big(\cos(2\pi f_c t)\,p_1(t)\big)$$

$$= \frac{1}{2}\Big(\tau sinc\big(\tau(f-f_c)\big) + \tau sinc\big(\tau(f+f_c)\big)\Big)$$

$$= \frac{1}{2}\Big(sinc\big(\tau(f-f_c)\big) + sinc\big(\tau(f+f_c)\big)\Big)$$

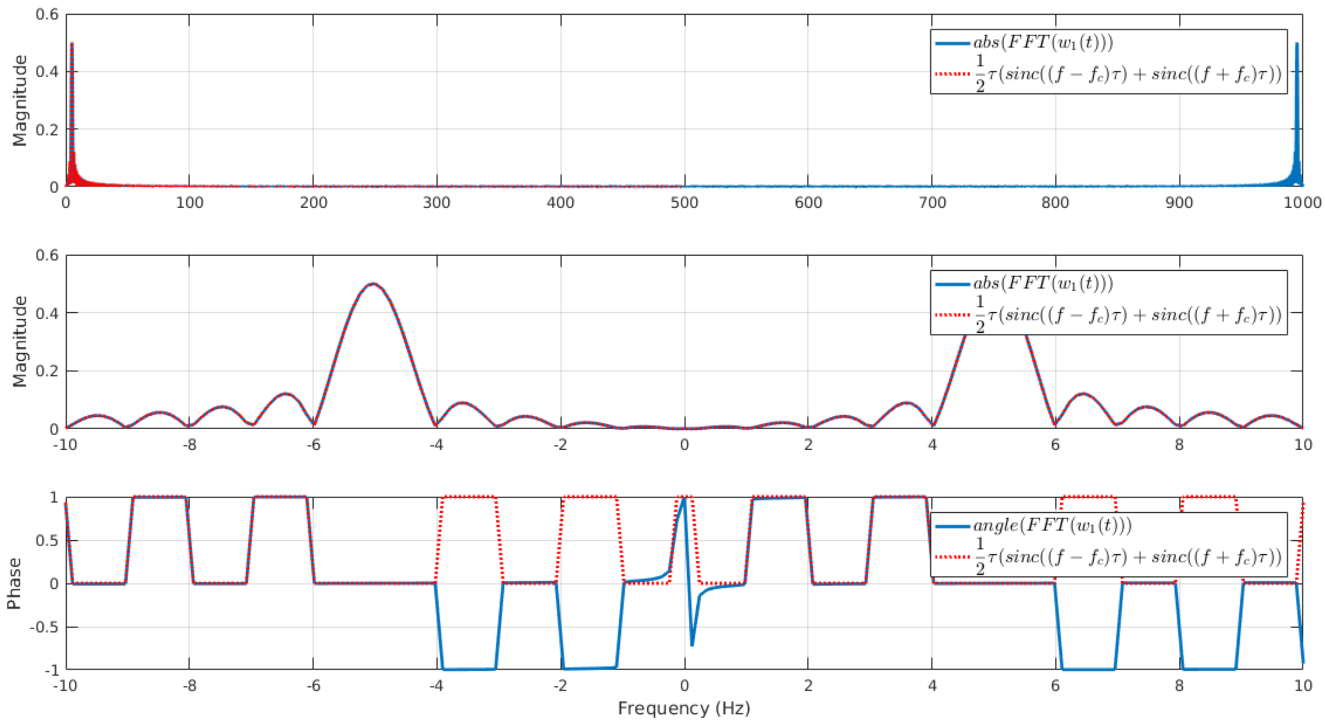

Figure 8: Discrete Time Fourier Transform on w_1(t)

$$\therefore\ \mathcal{F}\big(\cos(2\pi f_c t)\,p_2(t)\big) = \frac{1}{2}\tau\left(e^{-j\pi(f-f_c)\tau} sinc\big(\tau(f-f_c)\big) + e^{-j\pi(f+f_c)\tau} sinc\big(\tau(f+f_c)\big)\right)$$

$$\therefore\ \mathcal{F}\big(\cos(2\pi f_c t)\,p_3(t)\big) = \frac{1}{2}\tau\left(e^{j\pi(f-f_c)\tau} sinc\big(\tau(f-f_c)\big) + e^{j\pi(f+f_c)\tau} sinc\big(\tau(f+f_c)\big)\right)$$

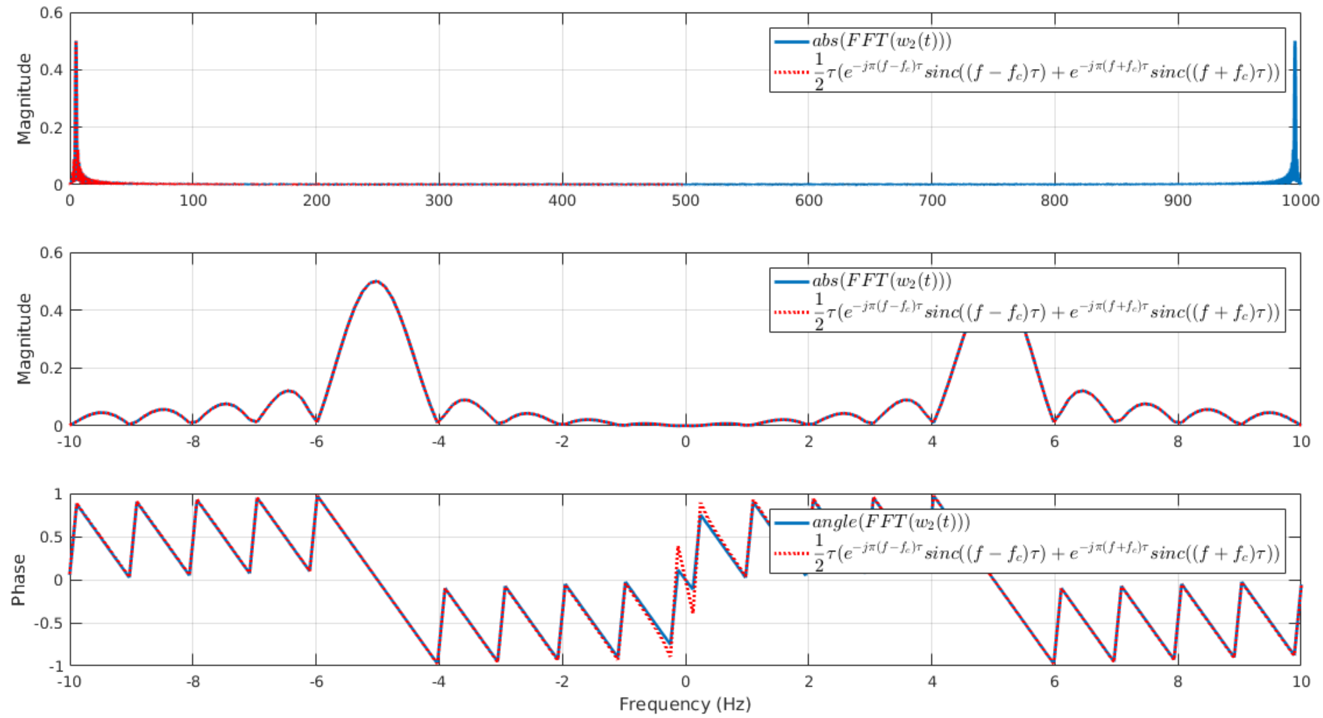

Figure 9: Fast Fourier Transform on $w_2(t)$

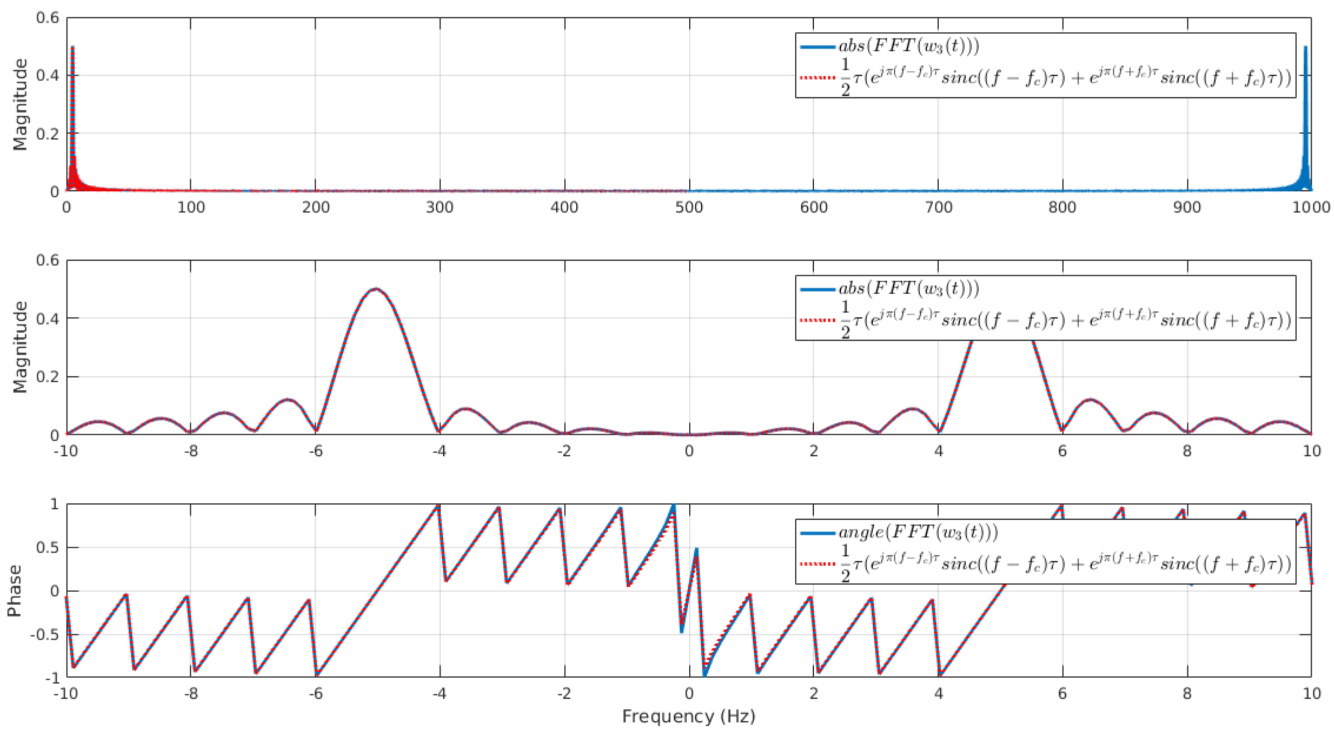

Figure 10: Fast Fourier Transform on $w_3(t)$

Both the theoretical and computed functions appear identical to each other except some discrepancies in the phases.