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MATH 407: HW 08

3.2 **1** In $GL_2(R)$, find the order of each of the following elements.

b

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Ans $\square$

d

$$\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$$

Ans $\square$

11 Let S be a set, and let a be a fixed element of S. Show that $\{\sigma \in \text{Sym}(S) \mid \sigma(a) = a\}$ is a subgroup of Sym(S).

Ans $\square$

12 For each of the following groups, find all elements of finite order.

a $\mathbb{R}^{\times}$

Ans $\square$

d $\mathbb{C}^{\times}$

Ans $\square$

19 Let G be a group, and let $a \in G$. The set $C(a) = \{x \in G \mid xa = ax\}$ of all elements of G that commute with a is called the **centralizer** of a.

a Show that $C(a)$ is a subgroup of G.

Ans $\square$

b Show that $\langle a \rangle \subseteq C(a)$.

Ans $\square$

c Compute $C(a)$ if $G = S_3$ and $a = (1, 2, 3)$.

Ans $\square$

d Compute $C(a)$ if $G = S_3$ and $a = (1, 2)$.

Ans $\square$

20 Compute the centralizer in $\text{GL}_2(\mathbb{R})$ of the matrix $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

Ans $\square$

25 Let G be a finite group, let $n > 2$ be an integer, and let S be the set of elements of G that have order n. Show that S has an even number of elements.

Ans $\square$

3.3 **4** Find the cyclic subgroup generated by $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ in $\text{GL}_2(\mathbb{Z}_3)$.

Ans $\square$

5 Prove that if G_1 and G_2 are abelian groups, then the direct product $G_1 \times G_2$ is abelian.

Ans $\square$

8 Let G_1 and G_2 be groups, with subgroups H_1 and H_2, respectively. Show that $\{(x_1, x_2) \mid x_1 \in H_1, x_2 \in H_2\}$ is a subgroup of the direct product $G_1 \times G_2$.

Ans $\square$

11 Let G_1 and G_2 be groups, and let G be the direct product $G_1 \times G_2$. Let $H = \{(x_1, x_2) \in G_1 \times G_2 \mid x_2 = e\}$ and let $K = \{(x_1, x_2) \in G_1 \times G_2 \mid x_1 = e\}$.

a Show that H and K are subgroups of G.

Ans $\square$

b Show that $HK = KH = G$.

Ans $\square$

c Show that $H \cap K = \{e, e\}$.

Ans $\square$

13 Let p, q be distinct prime numbers, and let $n = pq$. Show that $HK = \mathbb{Z}_n^\times$, for the subgroups $H = \{[x] \in \mathbb{Z}_n^\times \mid x \equiv 1 \pmod{p}\}$ and $K = \{[y] \in \mathbb{Z}_n^\times \mid y \equiv 1 \pmod{q}\}$ of $\mathbb{Z}_n^\times$.

Ans $\square$

16 Let G be a group of order 6, and suppose that $a, b \in G$ with a of order 3 and b of order 2. Show that either G is cyclic or $ab \neq ba$.

Ans $\square$