

1. To test the hypothesis $H_0 : \mu = 5$ vs $H_1 : \mu < 5$ about the population mean, a random sample of size $n = 36$ is collected. The sample mean and variance are 4.82 and 0.93, respectively. Compute an appropriate test statistic. Compute the p-value for the observed test statistic. At level $\alpha = 0.01$, what is the decision about the hypotheses? What assumptions did you make to carry out the test?
2. A standard cordless 18V power drill delivers a mean torque of 440 units. A new design claims to have increased the amount of torque produced by 40 units when other specifications are kept similar to the previous design. To test the hypothesis a sample of torques produced by the new design is taken. The sample size is 9 and the sample mean and sample standard deviation are 472 and 60 units, respectively. Based on the sample, test the hypothesis that $H_0 : \mu = 440$ vs $H_1 : \mu > 440$ where μ is the mean of the new design. Assume normality and a level $\alpha = 0.05$. Assuming that the population standard deviation is known to be 60 units, what is the probability of type two error at an alternative value of $\mu_1 = 480$ units?
3. Suppose the sample proportion of positive responses in a sample of 1600 binary responses (positive/negative) is 0.52. Test the hypothesis that $H_0 : p = 0.5$ versus $H_1 : p > 0.5$ at level $\alpha = 0.05$.
4. Calculate the sample size required to have a power of 0.9 at an alternative value of $\mu_1 = 2$ while testing $H_0 : \mu = 1$ vs $H_1 : \mu > 1$ for a normal population with mean μ and variance 4. Assume $\alpha = 0.05$.