Homework #5

Problem 1. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B. respectively. over a certain time period. and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-2 \leq x \leq 4, \qquad -1 \leq y - x \leq 1.$$

(a) Find the marginal PMFs and the means of X and Y.

(b) Find the mean of the trader's profit.

Problem 2. The MIT football team wins any one game with probability p, and loses it with probability $1 - p$. Its performance in each game is independent of its performance in other games. Let L_1 be the number of losses before its first win, and let L_2 be the number of losses after its first win and before its second win. Find the joint PMF of L_1 and L_2.

Problem 3. A class of n students takes a test in which each student gets an A with probability p, a B with probability q, and a grade below B with probability $1 - p - q$, independently of any other student. If X and Y are the numbers of students that get an A and a B, respectively, calculate the joint PMF $p_{X,Y}$.

Problem 4. Your probability class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let X be the number of students that get an A in your class.

(a) Calculate $\mathbf{E}[X]$ by first finding the PMF of X.

(b) Calculate $\mathbf{E}[X]$ by viewing X as a sum of random variables, whose mean is easily calculated.

Problem 5. A scalper is considering buying tickets for a particular game. The price of the tickets is \$75, and the scalper will sell them at \$150. However, if she can't sell them at \$150, she won't sell them at all. Given that the demand for tickets is a binomial random variable with parameters $n = 10$ and $p = 1/2$, how many tickets should she buy in order to maximize her expected profit?

Problem 6. Suppose that X and Y are independent discrete random variables with the same geometric PMF:

$$p_X(k) = p_Y(k) = p(1-p)^{k-1}, \qquad k = 1, 2, \ldots,$$

where p is a scalar with $0 < p < 1$. Show that for any integer $n \geq 2$, the conditional PMF

$$\mathbf{P}(X = k \,|\, X + Y = n)$$

is uniform.

Problem 7. **Consider four independent rolls of a 6-sided die. Let X be the number of 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?**

Problem 8. Alvin shops for probability books for K hours, where K is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N \,|\, K}(n \,|\, k) = \frac{1}{k}, \qquad \text{for } n = 1, \ldots, k.$$

(a) Find the joint PMF of K and N.

(b) Find the marginal PMF of N.

(c) Find the conditional PMF of K given that $N = 2$.

(d) Find the conditional mean and variance of K, given that he bought at least 2 but no more than 3 books.

(e) The cost of each book is a random variable with mean \$30. What is the expected value of his total expenditure? *Hint:* Condition on the events $\{N = 1\}, \ldots, \{N = 4\}$, and use the total expectation theorem.