Homework # 4

Problem 1. The MIT soccer team has 2 games scheduled for one weekend. It has a 0.4 probability of not losing the first game. and a 0.7 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie. independent of what happens in the other game. The MIT team will receive 2 points for a win, 1 for a tie. and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend.

Problem 2. The probability of a royal flush in poker is $p = 1/649,740$. Show that approximately 649,740 hands would have to be dealt in order that the probability of getting at least one royal flush is above $1 - 1/e$.

Problem 3. The annual premium of a special kind of insurance starts at \$1000 and is reduced by 10% after each year where no claim has been filed. The probability that a claim is filed in a given year is 0.05, independently of preceding years. What is the PMF of the total premium paid up to and including the year when the first claim is filed?

Problem 4. Let X be a random variable that takes values from 0 to 9 with equal probability 1/10.

(a) Find the PMF of the random variable $Y = X \bmod(3)$.

(b) Find the PMF of the random variable $Y = 5 \bmod(\mathrm{X}+1)$.

Problem 5. Let X be a discrete random variable that is uniformly distributed over the set of integers in the range $[a, b]$, where a and b are integers with $a < 0 < b$. Find the PMF of the random variables $\max\{0, X\}$ and $\min\{0, X\}$.

Problem 6. Let X be a discrete random variable, and let $Y = |X|$.

(a) Assume that the PMF of X is

$$p_X(x) = \begin{cases} Kx^2 & \text{if } x = -3, -2, -1, 0, 1, 2, 3, \\ 0 & \text{otherwise,} \end{cases}$$

where K is a suitable constant. Determine the value of K.

(b) For the PMF of X given in part (a) calculate the PMF of Y.

(c) Give a general formula for the PMF of Y in terms of the PMF of X.

Problem 7. Let X be a random variable that takes integer values and is symmetric, that is, $\mathbf{P}(X = k) = \mathbf{P}(X = -k)$ for all integers k. What is the expected value of $Y = \cos(X\pi)$ and $Y = \sin(X\pi)$?

Problem 8. Fischer and Spassky play a sudden-death chess match whereby the first player to win a game wins the match. Each game is won by Fischer with probability p, by Spassky with probability q, and is a draw with probability $1 - p - q$.

(a) What is the probability that Fischer wins the match?

(b) What is the PMF, the mean, and the variance of the duration of the match?

Problem 9. A particular binary data transmission and reception device is prone to some error when receiving data. Suppose that each bit is read correctly with probability p. Find a value of p such that when 10,000 bits are received, the expected number of errors is at most 10.

Problem 10. Let $X_1, \ldots, X_n$ be independent, identically distributed random variables with common mean and variance. Find the values of c and d that will make the following formula true:

$$\mathbf{E}\big[(X_1 + \cdots + X_n)^2\big] = c\mathbf{E}[X_1]^2 + d\big(\mathbf{E}[X_1]\big)^2.$$