Binary Exponentiation

$$3^{11} = 3^{2^0} \times 3^{2^1} \times 3^{2^3}$$

$$= 3^1 \times 3^2 \times 3^8$$

→ reason ?

Ans: 3^{11} কে manually —

$$3^{11} = 3 \times 3 \times 3 \times \ldots \text{ (11 times)}$$

multiply করা লাগতো।

How binary exponentiation is related?

$$\Rightarrow 3^{11} = 3^{2^0} \times 3^{2^1} \times 3^{2^3} \quad \text{(3 times)}$$

$$= 3^1 \times 3^2 \times 3^8$$

$$= 3 \times 9 \times 6561 = 177,187$$

মান ওটা step এ করলাম।

আসলে, 3^{11} →

8	4	2	1
1	0	1	1

↓

এভাবেই ভ্যালু বের করেছি।

Let's fun with real life AKA massive values

$3^{100} = 3 \times 3 \times 3 \times \ldots$ (100 times) Naive

$100 \rightarrow$ 64 32 16 8 4 2 1

1 1 0 0 1 0 0

$3^{100} = 3^{64} \times 3^{32} \times 3^{4} \times 3^{0}$

$= 3^{2^6} \times 3^{2^5} \times 3^{2^2} \times 3^{2^1}$

Even we can easily do this for $5^{1000000000}$

$= 5^{10^9}$

Representing 10^9 needs 30 bits which is efficient because we need 10^9 operations in the naive version which binary exponentiation reduced it at 30 steps!!

Time complexity : $O(\log N)$

Recap :

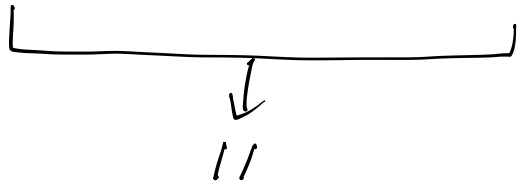
23 → 16 8 4 2 1

1 0 1 1 1

`23 >> = 1;` = `23 /= 2`

16 8 4 2 1

0 1 0 1 1 (1)x



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Proved