

Naïve Bayes

Made By Munna Bhai MBBS

Given the dataset

Department	Age	Salary	Status
Sales	31-35	46-50k	Senior
Sales	26-30	26-30k	Junior
Sales	31-35	31-35k	Junior
Systems	21-25	46-50k	Junior
Systems	31-35	66-70k	Senior
Systems	26-30	46-50k	Junior
Systems	41-45	66-70k	Senior
Marketing	36-40	46-50k	Senior
Marketing	31-35	41-45k	Junior
Secretary	46-50	36-40k	Senior
Secretary	26-30	26-30k	Junior

Given the Input : Department = Systems, Age = (26-30), Salary = (46-50k)

Input : Systems, (26-30), (46-50k)

Department	Age	Salary	Status
Sales	31-35	46-50k	Senior
Sales	26-30	26-30k	Junior
Sales	31-35	31-35k	Junior
Systems	21-25	46-50k	Junior
Systems	31-35	66-70k	Senior
Systems	26-30	46-50k	Junior
Systems	41-45	66-70k	Senior
Marketing	36-40	46-50k	Senior
Marketing	31-35	41-45k	Junior
Secretary	46-50	36-40k	Senior
Secretary	26-30	26-30k	Junior

Step: 1

- $P(\text{Status} = \text{Senior}) = \frac{5}{11}$
- $P(\text{Status} = \text{Junior}) = \frac{6}{11}$

Step: 2

- $P(\text{Department} = \text{Systems} \mid \text{Status} = \text{Senior}) = \frac{2}{5}$
- $P(\text{Department} = \text{Systems} \mid \text{Status} = \text{Junior}) = \frac{2}{6} = \frac{1}{3}$
- $P(\text{Age} = (26-30) \mid \text{Status} = \text{Senior}) = \frac{0}{5} = \frac{1}{6}$
- $P(\text{Age} = (26-30) \mid \text{Status} = \text{Junior}) = \frac{3}{6} = \frac{1}{2} = \frac{2}{3}$

- $P(\text{Salary} = (46-50k) \mid \text{Status} = \text{Senior}) = \frac{2}{5}$
- $P(\text{Salary} = (46-50k) \mid \text{Status} = \text{Junior}) = \frac{2}{6} = \frac{1}{3}$

Step: 3

- $P(X \mid \text{Status} = \text{Senior}) = \frac{2}{5} \times \frac{1}{6} \times \frac{2}{5} = \frac{2}{75}$
- $P(X \mid \text{Status} = \text{Junior}) = \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

Department	Age	Salary	Status
Sales	31-35	46-50k	Senior
Sales	26-30	26-30k	Junior
Sales	31-35	31-35k	Junior
Systems	21-25	46-50k	Junior
Systems	31-35	66-70k	Senior
Systems	26-30	46-50k	Junior
Systems	41-45	66-70k	Senior
Marketing	36-40	46-50k	Senior
Marketing	31-35	41-45k	Junior
Secretary	46-50	36-40k	Senior
Secretary	26-30	26-30k	Junior

Input : Systems, (26-30), (46-50k)

Step: 4

- $P(X \mid \text{Status} = \text{Senior}) P(\text{Status} = \text{Senior}) = \frac{2}{75} \times \frac{5}{11} = 0.01212$
- $P(X \mid \text{Status} = \text{Junior}) P(\text{Status} = \text{Junior}) = \frac{2}{27} \times \frac{6}{11} = 0.04040$

Which Bigger One??

Obviously 0.04040, So we can say that for Systems department, Age Limit (26 - 30) and Salary Range (46 – 50k), Status will be Junior.

K- Means Clustering

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Given the dataset

No	X	Y
1	1	1
2	2	3
3	1	2
4	3	3
5	2	2
6	3	1

Given No of Cluster $K = 2$

Iteration : 1

Let, Centroids - $c1 = (1,1)$ and $c2 = (3, 3)$

Now Fill the table using Eq- $\sqrt{(c1_x - x_i)^2 + (c1_y - y_i)^2}$

	(1,1)	(2,3)	(1,2)	(3,3)	(2,2)	(3,1)
C1 = (1,1)	0	2.24	1	2.83	1.41	2
C2 = (3,3)	2.83	1	2.24	0	1.41	2
Cluster	c1	c2	c1	c2	c1	c2

c1	(1,1), (1,2), (2,2)
c2	(2,3), (3,3), (3,1)

Now Update, Centroids -

$$c1 = \left(\frac{1+1+2}{3}, \frac{1+2+2}{3} \right)$$

$$= \left(\frac{4}{3}, \frac{5}{3} \right)$$

$$= (1.33, 1.67)$$

$$c2 = \left(\frac{2+3+3}{3}, \frac{3+3+1}{3} \right)$$

$$= \left(\frac{8}{3}, \frac{7}{3} \right)$$

$$= (2.67, 2.33)$$

Iteration : 2

New Centroids - $c1 = (1.33, 1.67)$ and $c2 = (2.67, 2.33)$

Now Fill the table again using Eq- $\sqrt{(c1_x - x_i)^2 + (c1_y - y_i)^2}$

	(1,1)	(2,3)	(1,2)	(3,3)	(2,2)	(3,1)
C1 = (1.33,1.67)	0.75	1.49	0.47	2.13	0.75	1.79
C2 = (2.67,2.33)	2.13	0.95	1.70	0.75	0.75	1.37
Cluster	c1	c2	c1	c2	c2	c2

c1	(1,1), (1,2)
c2	(2,3), (3,3), (2,2), (3,1)

Now Update, Centroids -

$$c1 = \left(\frac{1+1}{2}, \frac{1+2}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{3}{2} \right)$$

$$= (1, 1.5)$$

$$c2 = \left(\frac{2+3+2+3}{4}, \frac{3+3+2+1}{4} \right)$$

$$= \left(\frac{10}{4}, \frac{9}{4} \right)$$

$$= (2.5, 2.25)$$

Iteration : 3

c1	(1,1), (1,2)
c2	(2,3), (3,3), (2,2), (3,1)

New Centroids - c1 = (1,1.5) and c2 = (2.5, 2.25)

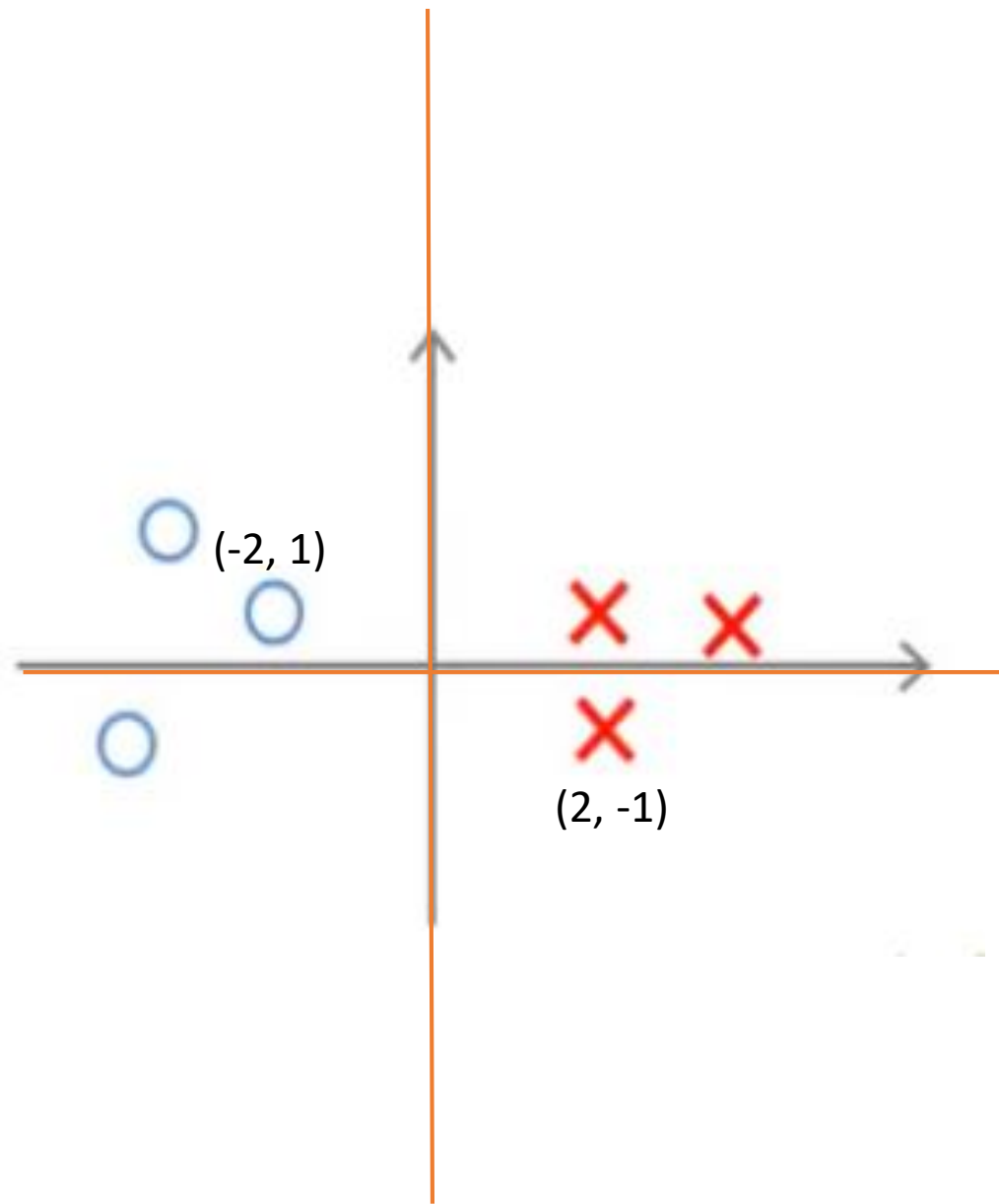
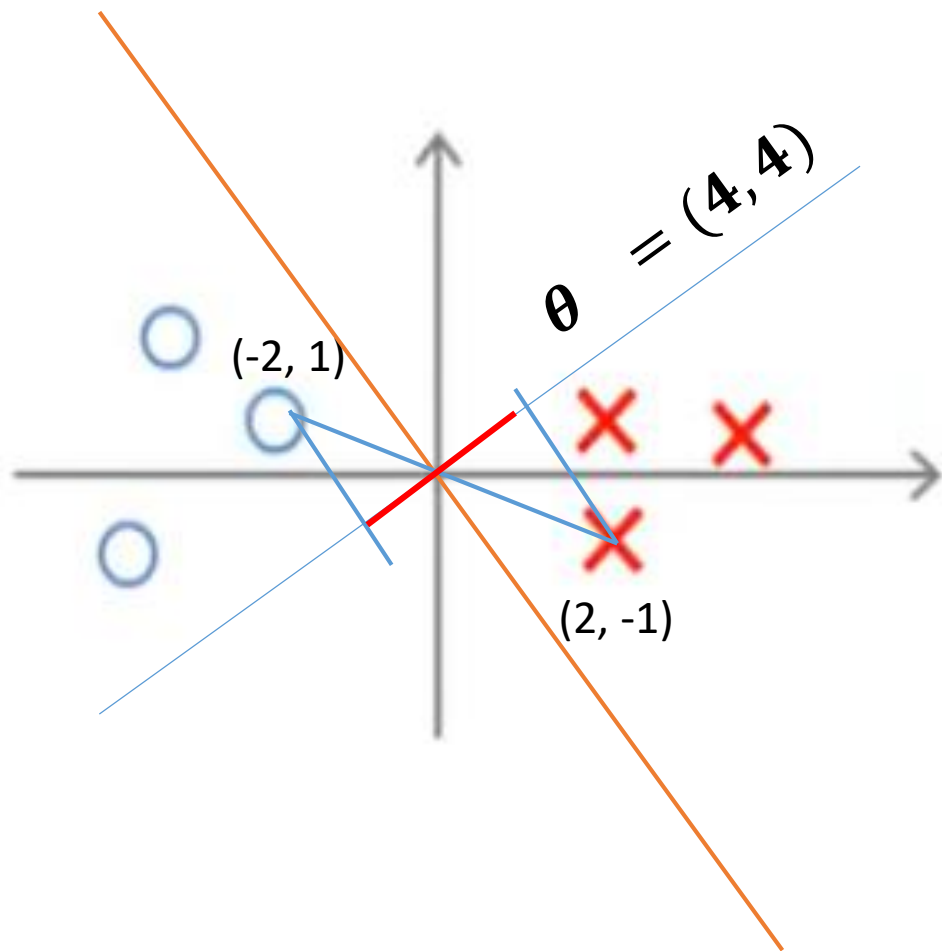
Now Fill the table again using Eq- $\sqrt{(c1_x - x_i)^2 + (c1_y - y_i)^2}$ Same as iteration 2

	(1,1)	(2,3)	(1,2)	(3,3)	(2,2)	(3,1)
C1 = (1,1.5)	0.5	1.80	0.5	2.5	1.12	2.06
C2 = (2.5,2.25)	1.95	0.90	1.52	0.90	0.56	1.35
Cluster	c1	c2	c1	c2	c2	c2

SVM Large Margin

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Given, $\theta_0 = 0$



For Figure 1,

Here, $X = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $Y = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and let, $\theta = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

We know that, $P_x = \theta^T X$

$$\begin{aligned} &= [4 \quad 4] \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ &= \left((4 \times 2) + (4 \times (-1)) \right) \\ &= (8 - 4) \\ &= 4 \end{aligned}$$

Similarly, $P_y = \theta^T Y$

$$\begin{aligned} &= [4 \quad 4] \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\ &= \left((4 \times (-2)) + (4 \times 1) \right) \\ &= (-8 + 4) \\ &= -4 \end{aligned}$$

For Figure 2,

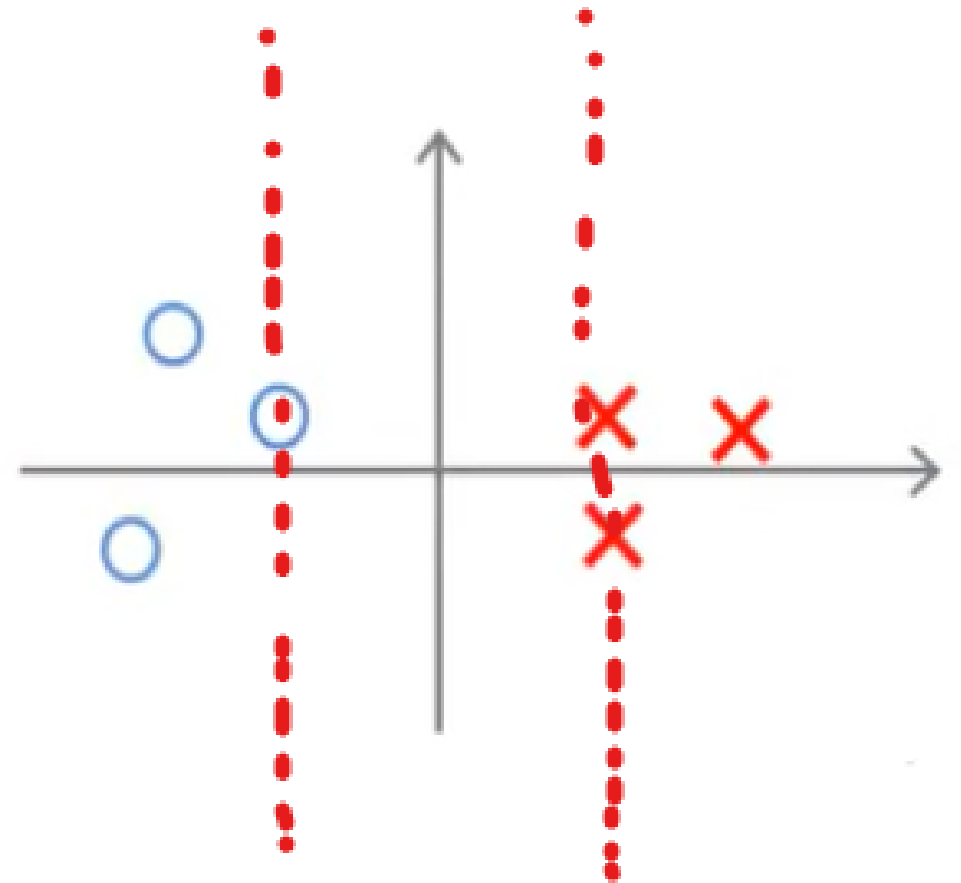
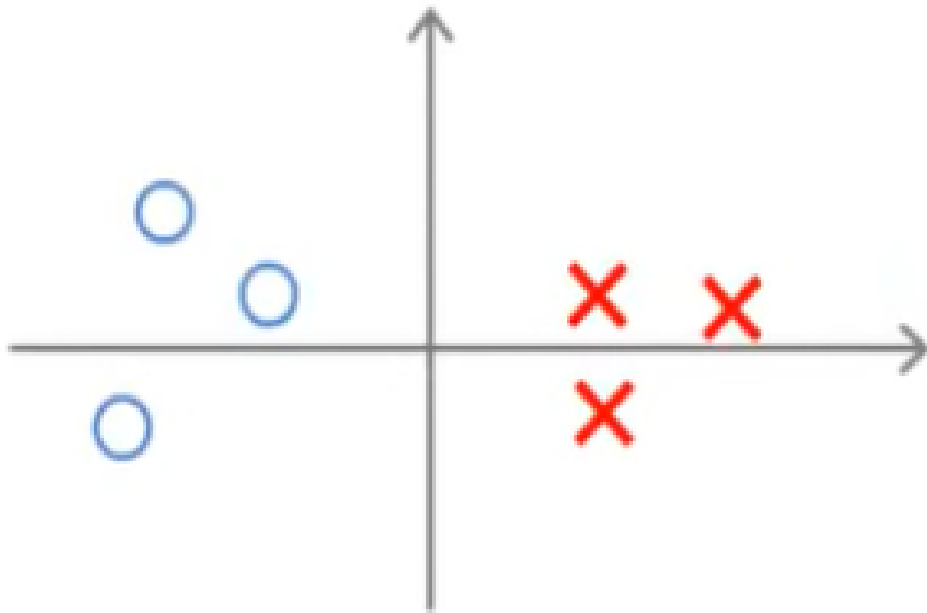
Here, $X = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $Y = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and let, $\theta = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

We know that, $P_x = \theta^T X$

$$\begin{aligned} &= [5 \quad 0] \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ &= \left((5 \times 2) + (0 \times (-1)) \right) \\ &= (10 - 0) \\ &= 10 \end{aligned}$$

Similarly, $P_y = \theta^T Y$

$$\begin{aligned} &= [5 \quad 0] \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\ &= \left((5 \times (-2)) + (0 \times 1) \right) \\ &= (-10 + 0) \\ &= -10 \end{aligned}$$



SVM Select This

Back Propagation

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Given the dataset

Department	Age	Salary	Status
Sales	31-35	46-50k	Senior
Sales	26-30	26-30k	Junior
Sales	31-35	31-35k	Junior
Systems	21-25	46-50k	Junior
Systems	31-35	66-70k	Senior
Systems	26-30	46-50k	Junior
Systems	41-45	66-70k	Senior
Marketing	36-40	46-50k	Senior
Marketing	31-35	41-45k	Junior
Secretary	46-50	36-40k	Senior
Secretary	26-30	26-30k	Junior

Given the training instance “(sales, senior, 31 . . . 35, 46K . . . 50K)”.

Encoding the Dataset First-

Department	Age	Salary	Status
1	33	48	1
1	28	28	0
1	33	33	0
2	23	48	0
2	33	68	1
2	28	48	0
2	43	68	1
3	38	48	1
3	33	43	0
4	48	38	1
4	28	28	0

Given the training instance “(sales, senior, 31 . . . 35, 46K . . . 50K)”.

Using min-max normalization, normalize the data into (0-1). So, new min = 0 and new max = 1

We know that,

$$V' = \frac{v_i - \min_{vi}}{\max_{vi} - \min_{vi}} (\text{new max} - \text{new min}) + \text{new min}$$

For Department Column

Values: 1, 2, 3, 4 min = 1 and max = 4

$$v_1 = \frac{1 - 1}{4 - 1} (1 - 0) + 0 = 0$$

$$v_2 = \frac{2 - 1}{4 - 1} (1 - 0) + 0 = 0.33$$

$$v_3 = \frac{3 - 1}{4 - 1} (1 - 0) + 0 = 0.67$$

$$v_4 = \frac{4 - 1}{4 - 1} (1 - 0) + 0 = 1$$

For Age Column

Values: 23, 28, 33, 38, 43, 48 min = 23 and max = 48

$$v_{23} = \frac{23 - 23}{48 - 23} (1 - 0) + 0 = 0$$

$$v_{43} = \frac{43 - 23}{48 - 23} (1 - 0) + 0 = 0.8$$

$$v_{28} = \frac{28 - 23}{48 - 23} (1 - 0) + 0 = 0.2$$

$$v_{48} = \frac{48 - 23}{48 - 23} (1 - 0) + 0 = 1$$

$$v_{33} = \frac{33 - 23}{48 - 23} (1 - 0) + 0 = 0.4$$

$$v_{38} = \frac{38 - 23}{48 - 23} (1 - 0) + 0 = 0.6$$

For Salary Column

Values: 28, 33, 38, 43, 48, 68 min = 23 and max = 68

$$v_{28} = \frac{28 - 28}{68 - 28} (1 - 0) + 0 = 0$$

$$v_{48} = \frac{48 - 28}{68 - 28} (1 - 0) + 0 = 0.5$$

$$v_{33} = \frac{33 - 28}{68 - 28} (1 - 0) + 0 = 0.125$$

$$v_{68} = \frac{68 - 28}{68 - 28} (1 - 0) + 0 = 1$$

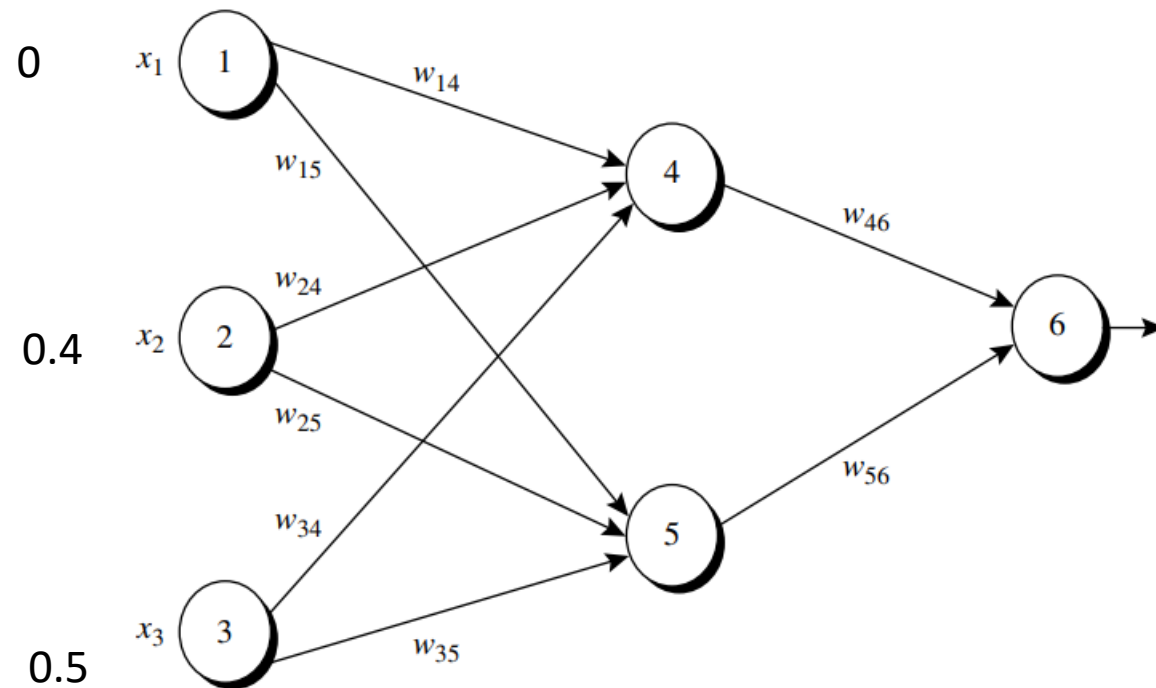
$$v_{38} = \frac{38 - 28}{68 - 28} (1 - 0) + 0 = 0.25$$

$$v_{43} = \frac{43 - 28}{68 - 28} (1 - 0) + 0 = 0.375$$

After Normalization of the dataset

Department	Age	Salary	Status
0	0.4	0.5	1
0	0.2	0	0
0	0.4	0.125	0
0.33	0	0.5	0
0.33	0.4	1	1
0.33	0.2	0.5	0
0.33	0.8	1	1
0.67	0.6	0.5	1
0.67	0.4	0.375	0
1	1	0.25	1
1	0.2	0	0

Given the training instance “(0, 0.4, 0.5)”.



Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$x_1 = 0, x_2 = 0.4, x_3 = 0.5$$

Step 1:

We know that,

$$f = w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3 + Bias$$

So,

$$\begin{aligned} f_4 &= w_{14} \times x_1 + w_{24} \times x_2 + w_{34} \times x_3 + \theta_4 \\ &= 0.2 \times 0 + 0.4 \times 0.4 + (-0.5) \times 0.5 - 0.4 \\ &= 0 + 0.16 - 0.25 - 0.4 \\ &= -0.49 \end{aligned}$$

$$\text{Now, final } f_4 = \frac{1}{(1 + e^{-(-0.49)})} = 0.38$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$x_1 = 0, x_2 = 0.4, x_3 = 0.5$$

We know that,

$$f = w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3 + Bias$$

So,

$$\begin{aligned} f_5 &= w_{15} \times x_1 + w_{25} \times x_2 + w_{35} \times x_3 + \theta_5 \\ &= (-0.3) \times 0 + 0.1 \times 0.4 + 0.2 \times 0.5 + 0.2 \\ &= 0 + 0.04 + 0.1 + 0.2 \\ &= 0.34 \end{aligned}$$

$$\text{Now, final } f_5 = \frac{1}{(1+e^{-(0.34)})} = 0.58$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$x_1 = 0, x_2 = 0.4, x_3 = 0.5$$

We know that,

$$f = w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3 + Bias$$

So,

$$\begin{aligned} f_6 &= w_{46} \times f_4 + w_{56} \times f_5 + \theta_6 \\ &= (-0.3) \times 0.38 + (-0.2) \times 0.58 + 0.1 \\ &= -0.114 - 0.116 + 0.1 \\ &= -0.13 \end{aligned}$$

$$\text{Now, final } f_6 = \frac{1}{(1 + e^{-(-0.13)})} = 0.47$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$x_1 = 0, x_2 = 0.4, x_3 = 0.5$$

Step 2: Calculate Error

For Output Layer,

$$Error_{output} = f_6(1 - f_6)(Target - f_6)$$

So,

$$\begin{aligned} Error_{output} &= 0.47(1 - 0.47)(1 - 0.47) \\ &= 0.132 \end{aligned}$$

Let,

Learning Rate = 0.9

$w_{14} = 0.2$

$w_{24} = 0.4$

$w_{34} = -0.5$

$w_{15} = -0.3$

$w_{25} = 0.1$

$w_{35} = 0.2$

$w_{46} = -0.3$

$w_{56} = -0.2$

$\theta_4 = -0.4$

$\theta_5 = 0.2$

$\theta_6 = 0.1$

$x_1 = 0, x_2 = 0.4, x_3 = 0.5$

$F_6 = 0.47$

Target = 1

For Others Layer,

$$Error_{hidden} = f_i(1 - f_i) \times Error_{output} \times w_i)$$

So,

$$\begin{aligned} Error_{f_4} &= 0.38(1 - 0.38) \times 0.132 \times (-0.3) \\ &= -0.0093 \end{aligned}$$

And,

$$\begin{aligned} Error_{f_5} &= 0.58(1 - 0.58) \times 0.132 \times (-0.2) \\ &= -0.0064 \end{aligned}$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$X_1 = 0, x_2 = 0.4, x_3 = 0.5$$

$$F_6 = 0.47$$

$$\text{Target} = 1$$

$$F_4 = 0.38$$

$$F_5 = 0.58$$

$$\text{Error } f_6 = 0.132$$

Step 3: Weight and Bias

For Weight,

$$w_{new} = w_{old} + LR \times Error \times Input$$

So,

$$\begin{aligned} w_{14} &= w_{14} + LR \times Error \times Input \\ &= 0.2 + 0.9 \times (-0.0093) \times 0 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} w_{24} &= w_{24} + LR \times Error \times Input \\ &= 0.4 + 0.9 \times (-0.0093) \times 0.4 \\ &= 0.397 \end{aligned}$$

$$\begin{aligned} w_{34} &= w_{34} + LR \times Error \times Input \\ &= -0.5 + 0.9 \times (-0.0093) \times 0.5 \\ &= -0.504 \end{aligned}$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$X_1 = 0, x_2 = 0.4, x_3 = 0.5$$

$$F_6 = 0.47$$

$$\text{Target} = 1$$

$$F_4 = 0.38$$

$$F_5 = 0.58$$

$$\text{Error } f_6 = 0.132$$

$$\text{Error } f_4 = -0.0093$$

$$\text{Error } f_5 = -0.0064$$

Step 3: Weight and Bias

For Weight,

$$w_{new} = w_{old} + LR \times Error \times Input$$

So,

$$\begin{aligned} w_{15} &= w_{15} + LR \times Error \times Input \\ &= -0.3 + 0.9 \times (-0.0064) \times 0 \\ &= -0.3 \end{aligned}$$

$$\begin{aligned} w_{25} &= w_{25} + LR \times Error \times Input \\ &= 0.1 + 0.9 \times (-0.0064) \times 0.4 \\ &= 0.098 \end{aligned}$$

$$\begin{aligned} w_{35} &= w_{35} + LR \times Error \times Input \\ &= 0.2 + 0.9 \times (-0.0064) \times 0.5 \\ &= 0.197 \end{aligned}$$

Let,

Learning Rate = 0.9

$$w_{14} = 0.2$$

$$w_{24} = 0.4$$

$$w_{34} = -0.5$$

$$w_{15} = -0.3$$

$$w_{25} = 0.1$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

$$X_1 = 0, x_2 = 0.4, x_3 = 0.5$$

$$F_6 = 0.47$$

$$\text{Target} = 1$$

$$F_4 = 0.38$$

$$F_5 = 0.58$$

$$\text{Error } f_6 = 0.132$$

$$\text{Error } f_4 = -0.093$$

$$\text{Error } f_5 = -0.064$$

Step 3: Weight and Bias

For Weight,

$$w_{new} = w_{old} + LR \times Error \times Input$$

So,

$$\begin{aligned} w_{46} &= w_{46} + LR \times Error \times Input \\ &= -0.3 + 0.9 \times (0.132) \times 0.38 \\ &= -0.255 \end{aligned}$$

$$\begin{aligned} w_{56} &= w_{56} + LR \times Error \times Input \\ &= -0.2 + 0.9 \times (0.132) \times 0.58 \\ &= -0.131 \end{aligned}$$

Let,

Learning Rate = 0.9

$w_{14} = 0.2$

$w_{24} = 0.4$

$w_{34} = -0.5$

$w_{15} = -0.3$

$w_{25} = 0.1$

$w_{35} = 0.2$

$w_{46} = -0.3$

$w_{56} = -0.2$

$\theta_4 = -0.4$

$\theta_5 = 0.2$

$\theta_6 = 0.1$

$X_1 = 0, x_2 = 0.4, x_3 = 0.5$

$F_6 = 0.47$

Target = 1

$F_4 = 0.38$

$F_5 = 0.58$

Error $f_6 = 0.132$

Error $f_4 = -0.093$

Error $f_5 = -0.064$

Step 3: Weight and Bias

For Bias,

$$\theta_{new} = \theta_{old} + LR \times Error$$

So,

$$\begin{aligned}\theta_4 &= \theta_4 + LR \times Error \\ &= -0.4 + 0.9 \times (-0.0093) \\ &= -0.41\end{aligned}$$

$$\begin{aligned}\theta_5 &= \theta_5 + LR \times Error \\ &= 0.2 + 0.9 \times (-0.0064) \\ &= 0.194\end{aligned}$$

$$\begin{aligned}\theta_6 &= \theta_6 + LR \times Error \\ &= 0.1 + 0.9 \times (0.132) \\ &= 0.22\end{aligned}$$

Let,

Learning Rate = 0.9

$w_{14} = 0.2$

$w_{24} = 0.4$

$w_{34} = -0.5$

$w_{15} = -0.3$

$w_{25} = 0.1$

$w_{35} = 0.2$

$w_{46} = -0.3$

$w_{56} = -0.2$

$\theta_4 = -0.4$

$\theta_5 = 0.2$

$\theta_6 = 0.1$

$X_1 = 0, x_2 = 0.4, x_3 = 0.5$

$F_6 = 0.47$

Target = 1

$F_4 = 0.38$

$F_5 = 0.58$

Error $f_6 = 0.132$

Error $f_4 = -0.0093$

Error $f_5 = -0.0064$

Patter Evaluation

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	Milk	! Milk	Sum of Row
Coffee	1000 (108.9)	100 (991.1)	1100
! Coffee	10000 (10891.1)	100000 (99108.9)	110000
Sum of Col	11000	100100	111100

Calculate the Expected Values :

We know that, Expected Value = $\frac{\text{Total of Column} \times \text{Total of Row}}{\text{Overall Total}}$

For Example, Expected Value(1,1) = $\frac{11000 \times 1100}{111100} = \frac{11000}{101} = 108.9$

	Milk	! Milk	Sum of Row
Coffee	1000 (108.9)	100 (991.1)	1100
! Coffee	10000 (10891.1)	100000 (99108.9)	110000
Sum of Col	11000	100100	111100

Calculate the probability of Milk, Coffee and (Milk,Coffee) :

$$P_{milk} = \frac{11000}{111100} = 0.099, P_{coffee} = \frac{1100}{111100} = 0.0099, P_{milk,coffee} = \frac{1000}{111100} = 0.009$$

$$\text{sup}_{\text{milk}} = 0.099$$

$$\text{sup}_{\text{coffee}} = 0.0099$$

$$\text{sup}_{\text{milk} \cup \text{coffee}} = 0.009$$

$$\begin{aligned} \text{milk} \rightarrow \text{coffee} &= \frac{\text{sup}_{\text{milk} \cup \text{coffee}}}{\text{sup}_{\text{milk}}} & \text{coffee} \rightarrow \text{milk} &= \frac{\text{sup}_{\text{milk} \cup \text{coffee}}}{\text{sup}_{\text{coffee}}} \\ &= \frac{0.009}{0.099} = 0.091 & &= \frac{0.009}{0.0099} = 0.91 \end{aligned}$$

	Milk	! Milk	Sum of Row
Coffee	1000 (108.9)	100 (991.1)	1100
! Coffee	10000 (10891.1)	100000 (99108.9)	110000
Sum of Col	11000	100100	111100

Calculate lift:

$$P_{milk} = 0.099, P_{coffee} = 0.0099, P_{milk,coffee} = 0.009$$

$$lift = \frac{P_{milk,coffee}}{P_{milk} \times P_{coffee}} = \frac{0.009}{0.099 \times 0.0099} = 9.183$$

	Milk	! Milk	Sum of Row
Coffee	1000 (108.9)	100 (991.1)	1100
! Coffee	10000 (10891.1)	100000 (99108.9)	110000
Sum of Col	11000	100100	111100

Calculate χ^2 :

$$\begin{aligned}
 \chi^2 &= \frac{(1000-108.9)^2}{108.9} + \frac{(100-991.1)^2}{991.1} + \frac{(10000-10891.1)^2}{10891.1} + \frac{(100000-99108.9)^2}{99108.9} \\
 &= 7291.64 + 801.19 + 72.91 + 8.012 \\
 &= 8173.75
 \end{aligned}$$

$$\begin{aligned}
 All_{conf} &= \min(milk \rightarrow coffee, \quad coffee \rightarrow milk) \\
 &= \min(0.091, 0.91) \\
 &= 0.091
 \end{aligned}$$

$$milk \rightarrow coffee = 0.091$$

$$coffee \rightarrow milk = 0.91$$

$$\begin{aligned}
 Max_{conf} &= \max(milk \rightarrow coffee, \quad coffee \rightarrow milk) \\
 &= \max(0.091, 0.91) \\
 &= 0.91
 \end{aligned}$$

$$\begin{aligned}
 Kulc. &= \frac{milk \rightarrow coffee + coffee \rightarrow milk}{2} \\
 &= \frac{0.091 + 0.91}{2} \\
 &= 0.5
 \end{aligned}$$

$$\begin{aligned}
 cosine &= \sqrt{milk \rightarrow coffee \times coffee \rightarrow milk} \\
 &= \sqrt{0.091 \times 0.91} \\
 &= 0.29
 \end{aligned}$$