Engineering 90: Senior Design Project

Wavelet Analysis in Economics

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Abstract:

The motivation behind this exploratory project was to add to the increasing literature on the application of wavelet analysis in economic discourse. The limited body of publications on the subject demonstrate wavelets as a promising tool in economic and finance analysis. Generally, wavelet analysis is used in engineering and physics to perform important signal processing applications such as filtering, denoising and compression. The multiresolution property of wavelets can be utilized to understand the correlation and the lead-lag relationship between economic variables across time-scales. This is important since standard economic theories model variables interacting at two time scales, short-run and long-run. Wavelet analysis can also be used with common statistical methods such as Granger-causality tests to further understand hidden relationships that are not observable from aggregated time series. The three variables of interest in this research are interest rate, GDP and inflation in the U.S. The scope of wavelet applications, however, can be expanded to explore several areas or branches of economics although the main focus in this research is macroeconomic variables.

Introduction:

The theoretical framework of this project is based on previous publications on wavelet analysis in economics. Wavelets have been used for business cycle analysis and empirical illustration on the nature of co-movements between several microeconomic variables. For example, Aguiar-Conraria and Soares (2011) employed wavelets in analyzing the relationship between industrial production and oil prices. Rua (2010) investigated the long continuing controversy in economics between money growth and inflation. Other important publications are Crowley and Mayes (2008) and Rua and Silva Lopes (2012) who used wavelets as an alternative tool to traditionals methods such as correlation analysis and spectral analysis to account for stochastic fluctuations of time-series data. These publications demonstrate the value of wavelet analysis and the need for more wavelet based analysis exploration in economics.

Three important variables in macroeconomics that have not been explored with wavelet analysis are inflation, interest rate and GDP in the U.S. Inflation measures the price change of a basket of goods and is derived from the Consumer Price Index. The nominal interest rate is set by the Federal Reserve Bank and measures the cost of borrowing as used by banks and business. GDP is the quarterly output of the economy. Understanding the relationship between these variables is important since it can give an indication about the stability of the economy as a whole and enables policy setters such as the Federal Reserve Board to battle inflation and other economic shocks with a balanced effective policy. The type of correct market policy can be seen depending on several time horizons. One policy can focus on the short run performance of the economy and other on the long-run trajectory of the economy. Wavelet analysis can reveal useful information about these time scales.

Data Source and Code Preparation.

All the data used for this project is from the Federal Reserve Economic Data or FRED in short, and all the computations were done using the popular wavelet packages, pywt and pycwt.

Theory:

Wavelet Decomposition is given by the following equation, with X_t representing the economic time-series data,

$$W_X(\tau, s) = \int_{-\infty}^{+\infty} X(t)\left(\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)\right)^* \mathrm{d}t$$

Where, τ is the time position and s is the scale. Low scales represent high frequencies and high scales capture low frequencies. The selected wavelet for this project was the analytic or complex Morlet wavelet based on its simplicity and the fact that it only gives positive frequencies. It is given by

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_o t} e^{-\frac{t^2}{2}}$$

Where, w_0 has a value of 6 and t is the time parameter.

Frequency in economics is redefined as the period in years and indicates the timescale of the variable of interest. The relationship between scales is given, S = $S_0 * 2^{(J*dj)}$. In this project the starting scale, S_0 was chosen as ¼ years, and the spacing between discrete scales, dj, as ¼. The highest scale was bounded by the number of years in the data and was set to 9. Choosing a maximum scale of J = 9 gives a total of 16 scales that captures the frequency information from 0.172 to 5.5 years, a good window for short-run and long-run.

Methods:

Cross Wavelet Analysis (XWT)

The cross wavelet analysis is defined as the multiplication of the continuous wavelet transforms of two times series. It is given by,

$W^{xy} = W^x W^{y*}$, where W^x and W^{y*} are the respective continuous wavelet transforms of time-series data, calculated using equation 1, and ⋆ denotes complex conjugation. This was calculated for each of the two variables.

The figure below shows the result of the continuous cross wavelet transform of inflation and GDP. It shows the correlation between the two variables across several timescale cycles from 1954 to 2019. The surprising result is there is no observable correlation in the 1970s between inflation and GDP in the inflationary period of the U.S economy between 1970 to 1985 when the inflation rose to a high level of 14%. This is in complete contrast to the relationship between the two variables between 2000 to 2010 in which at any given timescale cycle, they show high common power. This is probably due to some internal structure change within the economy that shifted the relationship between inflation and GDP.

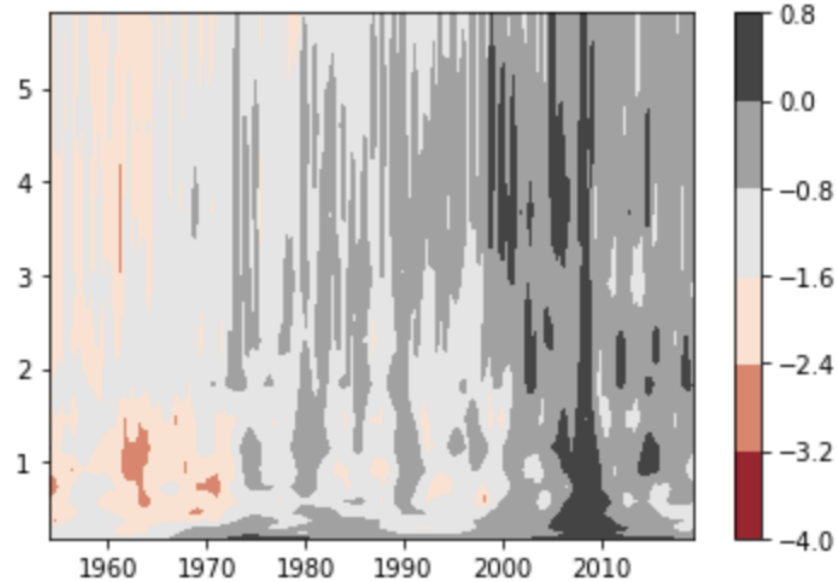

Figure 1: Continuous Cross Wavelet Transform of Inflation and GDP

In the figure 2 below, there is a similar relationship between interest rate and GDP as in the first figure. There appears to be a strong relationship between inflation and GDP in the inflationary period of 1980s that is non-existent in the decades since then. The disappearance of this relationship may be related to the developed relationship between inflation and GDP starting in late 1990s.

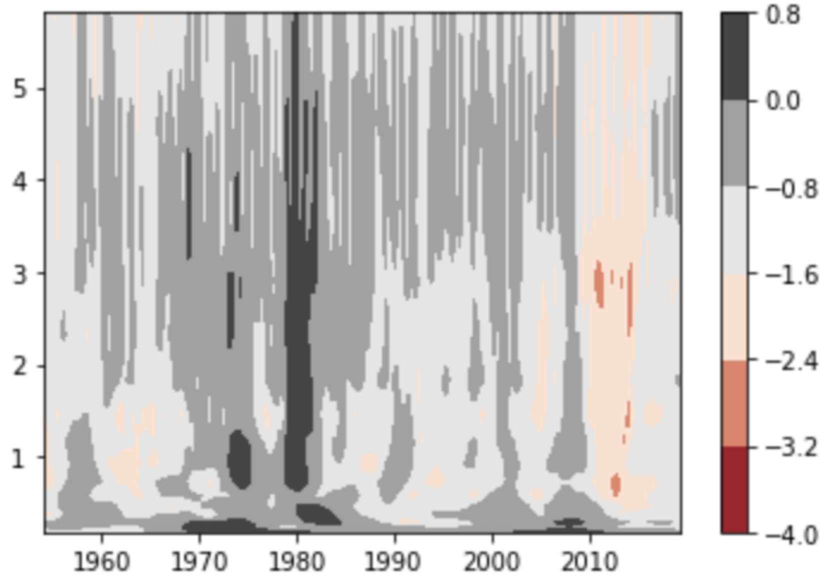

Figure 2: Continuous Cross Wavelet Transform of Interest Rate and GDP.

When it comes to the relationship between inflation and interest rates, there is no constant relationship between the two variables across any time scale cycles. There is some common power between the two variables in 1980 and 2010, but is not as significant as the relationship between GDP and inflation and GDP and interest rate.

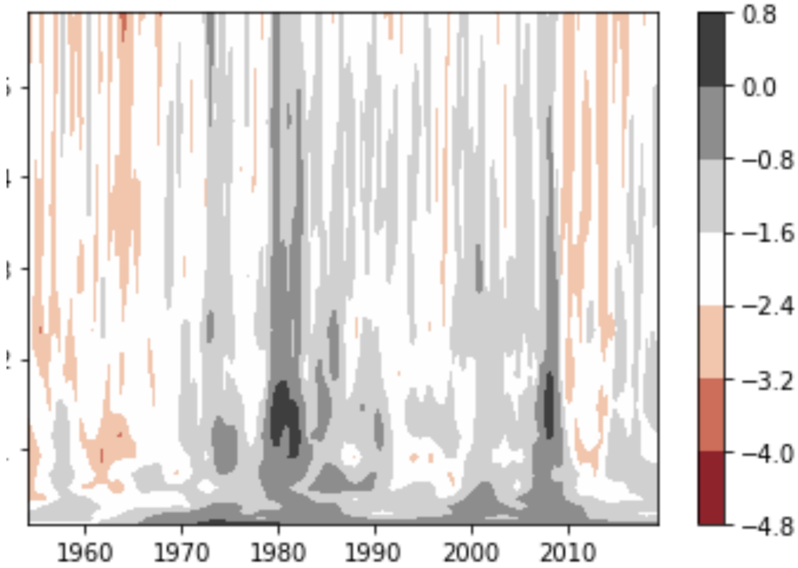

Figure 3: Continuous Cross Wavelet Transform of Inflation and Interest Rate.

Multi-Scale Granger Causality Tests

Given two time-series, X and Y, Granger-causality test shows if the past values of a given variable X are good predictors of Y beyond the explanation power in the past values of X. Mathematically, the bivariate Granger Causality is described by the following regression equations,

$$Y_t = \alpha + \varphi Y_{t-1} + \beta_1 X_{t-1} + U_t \ \ldots (1)$$

$$X_t = \alpha + \varphi X_{t-1} + \beta_1 Y_{t-1} + U_t \ \ldots..(2)$$

where , φ and β_1 are constants, and U_t is an error term.

Using an F test, the significance of β_1 is tested with the null hypothesis that $\beta_1 = 0$, which implies that there is no Granger-causality relationship. Depending on which regression equation is significant at a given p-value (e.g., 5%), the test determines the type of relationship between the variables. If both equations are significant, it implies there is a feedback mechanism between the variables since the leading or lagging variable cannot be conclusively determined. If both equations are not significant, the relationship is inconclusive. In cases where one equation is significant, the leading variable in that equation is statistically significant and provides useful information about forecasting the lagging variable.

The wavelet analysis can be combined with the Granger-causality test to gain useful insight into the lead and lag relationship between macroeconomic variables. Since wavelet analysis divides the time-series into several time-scales, each scale can be applied with Granger-causality test. Figure 4 below shows a hypothesized model of a possible relationship between the three variables. It assumes the Federal Reserve is proactive in its policy and leads the two other variables. The model also assumes that inflation leads GDP, although it could have been the opposite. The important specification is that nominal interest rate leads the two other variables.

Proposed Model.

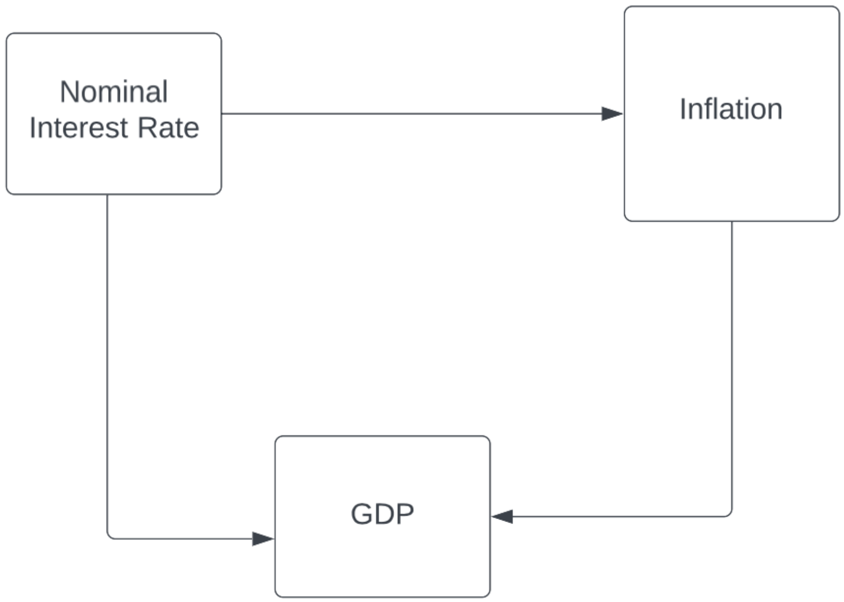

Figure 4: Flowchart for the expected relationship between nominal interest rate, inflation and GDP.

The tables below summarize the results of applying Granger causality test of each scale from the regression equations discussed above.

Table 1: Interest Rate and GDP Multi-scale Granger-causality test

Scale	**Granger Causality Direction Test:** Interest Rate → GDP	**Granger Causality Direction Test:** GDP→Interest Rate	**Classification**
D_1	1.0411 (0.3850)	1.4074 (0.2297)	Inconclusive
D_2	0.5068 (0.6725)	0.1360 (0.9690)	Inconclusive
D_3	0.2789 (0.8916)	4.7718 (0.0008)	GDP→Interest Rate
D_4	0.1451 (0.9652)	7.2718 (0.0000)	GDP→Interest Rate
D_5	1.1309 (0.3406)	24.9781 (0.0000)	GDP→Interest Rate
D_6	1.5689 (0.1807)	22.5609 (0.0000)	GDP→Interest Rate
D_7	8.0762 (0.0000)	276.52 (0.0000)	Feedback
S_8	(267.9) (0.0000)	168.8 (0.0000)	Feedback

(P-values in parentheses at 5% significance)

Table 2: Interest Rate and Inflation Multi-scale Granger-causality test

Scale	**Granger Causality Direction Test:** Interest Rate → Inflation	**Granger Causality Direction Test:** Inflation→Interest Rate	**Classification**
D_1	3.5866 (0.0282)	4.0260 (0.0451)	Feedback
D_2	3.3728 (0.0348)	1.6440 (0.4125)	Interest Rate → Inflation
D_3	0.1294 (0.9426)	0.1851 (0.9965)	Inconclusive
D_4	0.1738 (0.9142)	1.4987 (0.2135)	Inconclusive
D_5	1.6835 (0.01518)	1.1893 (0.3128)	Inconclusive
D_6	0.9929 (0.4105)	1.8195 (0.1424)	Inconclusive
D_7	5.8015 (0.0001)	169.209 (0.0000)	Feedback
S_8	95.046 (0.0000)	57.0299 (0.0000)	Feedback

(P-values in parentheses at 5% significance)

Table 3: Inflation and GDP Multi-scale Granger-causality test

Scale	**Granger Causality Direction Test:** Inflation → GDP	**Granger Causality Direction Test:** GDP→Inflation	**Classification**
D_1	10.8751 (0.000)	143.1896 (0.000)	Feedback
D_2	8.4554 (0.000)	154.6491 (0.0000)	Feedback
D_3	2.9056 (0.0210)	105.6104 (0.000)	Feedback
D_4	15.1305 (0.0000)	56.49 (0.000)	Feedback
D_5	4.5689 (0.0012)	28.53 (0.000)	Feedback
D_6	42.3855 (0.0000)	66.4917 (0.0000)	Feedback
D_7	872.85 (0.0000)	28.0247 (0.0000)	Feedback
S_6	52.8243 (0.0000)	271.97 (0.000)	Feedback

(P-values in parentheses at 5% significance)

The tables above show that there is no consistent relationship between variables on each scale. The Granger-causality produces mainly inconclusive or feedback mechanisms. One possible explanation is the fact that the equations assume a constant lead-lag relationship. However, this may not be the case, as the Fed may be proactive or reactive. To gain a better perspective of the true relationship between variables, it is useful to explore the lead-lag relationship between the variables through a wavelet coherence test.

Wavelet Coherence Test

The wavelet coherence of a time-series is defined as,

$$R^2{}_n(s) = \frac{\left|S\left(s^{-1}W^{XY}{}_n(s)\right)\right|^2}{S\left(s^{-1}\left|W^X{}_n(s)\right|^2\right) \cdot S\left(s^{-1}\left|W^Y{}_n(s)\right|^2\right)}$$

Where, S is a smoothing operator given by the Morlet wavelet and s is a scale factor. The phase between the two variables is given by,

$$\varnothing(\tau, s) = \tan^{-1}\left(\frac{I\left(W_{XY}(\tau, s)\right)}{R\left(W_{XY}(\tau, s)\right)}\right)$$

Wavelet coherence formula can be interpreted as a dynamic correlation. Coherence reveals information about the co-movement between variables and the respective lead-lag relationship. Similar to the cross wavelet transform, the wavelet coherence was computed for each two variables. The results are discussed in the following section below. .

The wavelet coherence for inflation and interest rate shows interesting results about the lead-lag relationship of the two variables. The arrows pointing to the right indicate an in-phase relationship and the interest rate leading inflation. This is consistent with the hypothesized model in figure 4. However, this is only observable at the 4-8 time scale range. The Granger-causality test from Table 2 revealed inconclusive results for this range. The coherence test shows why

Granger-causality tests failed. The lead-lag relationship is not constant. In 1990, there was a change in the lead-lag relationship as indicated by the arrows switching from right to the left. This is consistent with a situation in which inflation leads the interest rate as a result of possible change response by the Fed. Since the inflation leads, this can be seen as a reactive response.

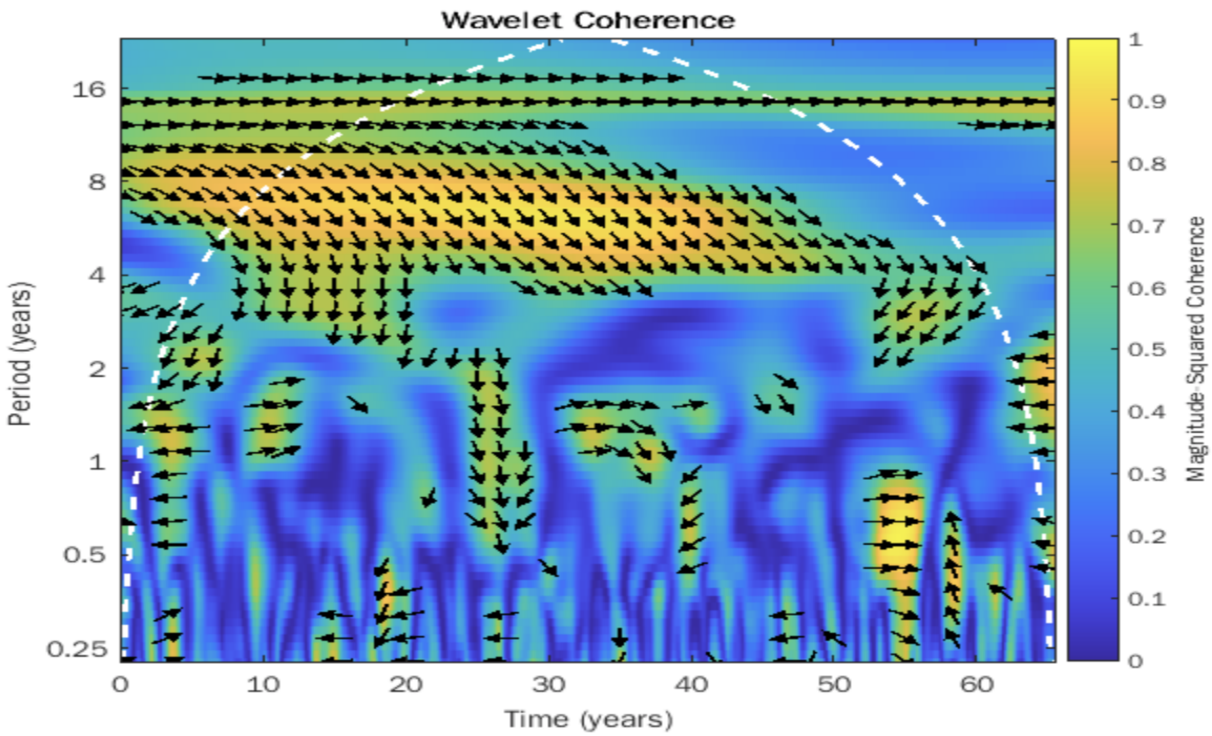

Figure 5: Interest Rate and Inflation Coherence test

The coherence relationship between interest rate and GDP is shown below. The interest rate leads GDP in the 2-8 year scale although this relationship switched direction in 1990. The analysis for this

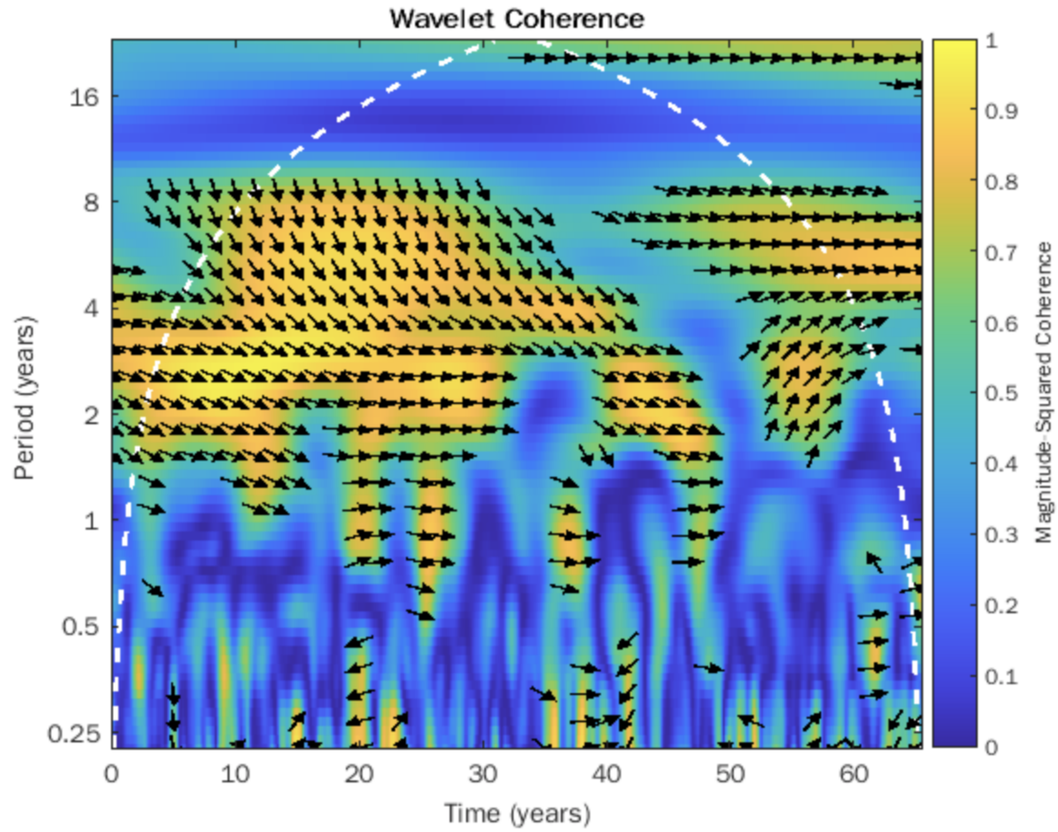

Figure 6: Interest and GDP Coherence Test

Finally, the coherence test for inflation and GDP reveals interesting results about their relationship. The arrows pointing to the left show inflation leading GDP between 1954-1990 in an out-phase relationship, but switches to an in-phase relationship starting in the 1990. This could be the result of an unknown fundamental change in the U.S economy that affected the co-movements between all variables, although more in-depth analysis is needed for the actual reason.

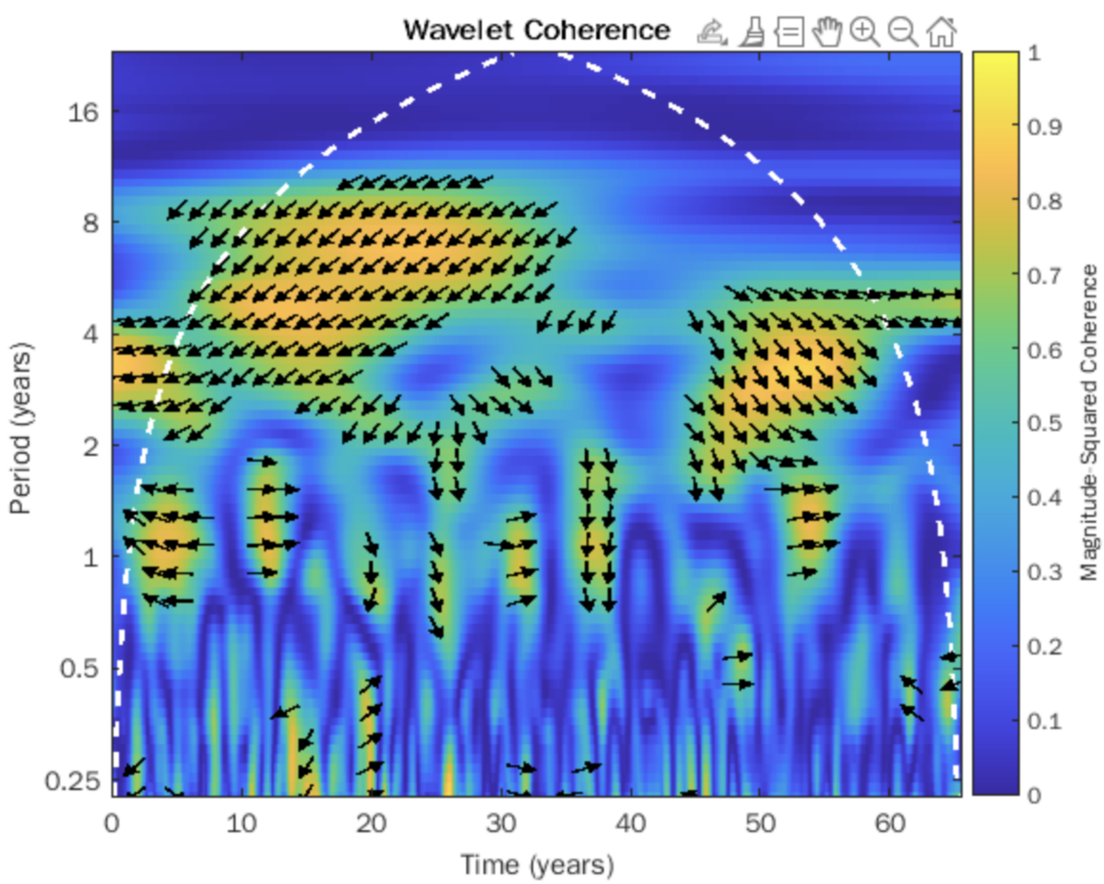

Figure 7 : Inflation and GDP Coherence Result.

The results above can be summarized with the flowchart shown below. It modifies the original hypothesis by taking into account the policy response by the Fed as either proactive or reactive. The model also confirms inflation leading GDP, either as an in-phase or outphase relationship.

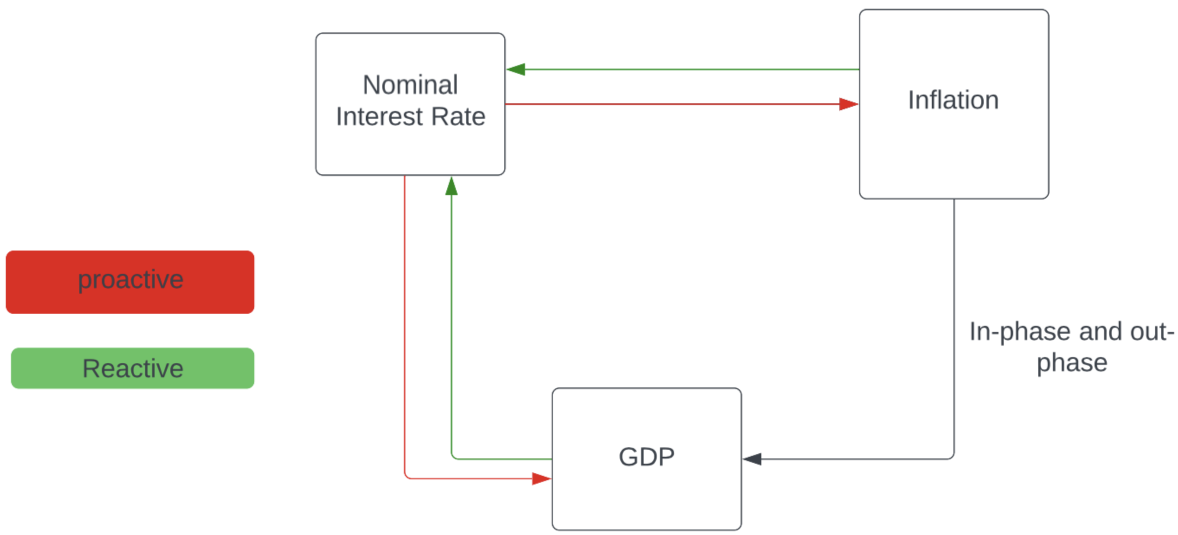

Figure 7: Revised Model of the Inflation, GDP and Interest Relationship

The relationship between variables can be summarized with the following equations.

$$I_t = \beta I_{t-d(x)} + U_t \qquad \ldots (1)$$

$$GDP_t = \beta I_{t-d(x)} + U_t \qquad \ldots (2)$$

$$GDP_t = \beta IN_{t-d(x)} + U_t \qquad \ldots .(3)$$

Where, I_t represents interest rate, GDP_t is the gdp and I_t is the inflation. $d(x)$ is a variable that captures the phase between the variables.

Future Work:

There are multitudes of ways to expand this project. One way is to explore the sensitivity of the results to preprocessing methods. Granger-causality test requires stationary data, and the type of transformation used on the raw data could affect the result. This is due to the information lost through making the data stationary by standard methods such as polynomial fitting or log-difference. The result could also be sensitive to the time-period used for analysis Although a longer-time period is always better since it contains more information, it is also difficult to make it stationary. Other ways to expand this project to include the analysis with more explanatory variables and analyze other macroeconomic variables such as oil prices and dollar.

Acknowledgement:

I would like to express my appreciation to Professor Allan Moser for his valuable and constructive guidance and suggestions during the project.

Appendix:

```
from __future__ import division
from matplotlib import pyplot
from pycwt.helpers import find
from statsmodels.tsa.stattools import adfuller
import statsmodels.api as sm
from pandas.plotting import lag_plot


import matplotlib.pyplot as plt

import pandas as pd
import numpy as np
import pywt
import pycwt as wavelet

from statsmodels.tsa.stattools import grangercausalitytests
from statsmodels.graphics.tsaplots import plot_acf


# inter is the Interest rate variable
# infl is the inflation rate variable
# gdp represents the industrial production index

infl = pd.read_csv('CPIAUCSL.csv')
inter = pd.read_csv('FEDFUNDS.csv')
gdp = pd.read_csv('IPB50001N.csv')

infl.rename(columns = {'CPIAUCSL':'CPI'}, inplace = True)
inter.rename(columns = {'FEDFUNDS':'I'}, inplace = True)
gdp.rename(columns = {'IPB50001N':'gdp'}, inplace = True)
t0 = 1954
dt = 1/12
N = infl['CPI'].size
t = np.arange(0, N) * dt + t0

# Plot the graphs of these macroeconomic variables
```

```
fig, (ax1, ax2, ax3) = plt.subplots(3, 1, figsize=(7,7))
fig.suptitle('Three Variables of Interest')

ax1.plot(t,gdp['gdp'], 'r-', linewidth=3.0)
ax1.set_ylabel('GDP')

ax2.plot(t, infl['CPI'], 'b-', linewidth=3.0)
ax2.set_ylabel('INFLATION')

ax3.plot(t, inter['I'],'k-', linewidth=3.0)
ax3.set_xlabel('TIME(years)')
ax3.set_ylabel('INTEREST RATE')


plt.show()
# After making the data stationary by first differencing
newinfl = (np.log(infl['CPI'])).diff(1)
newinfl.fillna(0,inplace = True)

#newgdp = gdp['gdp'].diff(1)

#newgdp.fillna(0,inplace = True)
newgdp = (np.log(gdp['gdp'])).diff(1)

newgdp.fillna(0,inplace = True)

newinter = (np.log(inter['I'])).diff(1)


newinter.fillna(0,inplace = True)

fig, (ax1, ax2, ax3) = plt.subplots(3, 1, figsize=(7,7))
fig.suptitle('Log Differenced Data')

ax1.plot(t,newgdp, 'r-', linewidth=3.0)
ax1.set_ylabel('GDP')

ax2.plot(t, newinfl, 'b-', linewidth=3.0)
ax2.set_ylabel('INFLATION')
```

```
ax3.plot(t, newinter,'k-', linewidth=3.0)
ax3.set_xlabel('TIME(years)')
ax3.set_ylabel('INTEREST RATE')
plt.show()


# Use the Augmented Dickey Fuller Test (Unit Root) to check

# the stationarity of the mean and variance of the time series data

# Inflation
df_stationarityTest = adfuller(newinfl , 40, autolag='AIC')

print(f'ADF Statistic: {df_stationarityTest[0]}')
print(f'n_lags: {df_stationarityTest[1]}')
print(f'p-value: {df_stationarityTest[1]}')
if df_stationarityTest[1] < 0.05:
   print('The new inflation data is stationary')
else:
   print('The new inflalation data not stationary')

# Interest Rate

df_stationarityTest = adfuller(newinter , 40, autolag='AIC')

print(f'ADF Statistic: {df_stationarityTest[0]}')
print(f'n_lags: {df_stationarityTest[1]}')
print(f'p-value: {df_stationarityTest[1]}')
if df_stationarityTest[1] < 0.05:
   print('The new interst rate data is stationary')
else:
   print('The new interest rate data is not stationary')

# GDP

df_stationarityTest = adfuller(newgdp , 40, autolag='AIC')

print(f'ADF Statistic: {df_stationarityTest[0]}')
print(f'n_lags: {df_stationarityTest[1]}')
print(f'p-value: {df_stationarityTest[1]}')
```

```
if df_stationarityTest[1] < 0.05:
    print('The new gdp data is stationary')
else:
    print('The new gdp data is not stationary')
# Perform Cross Wavelet Transform Analysis

# Smooth the data using the morlet wavelet with centered omega of 6

mother = wavelet.Morlet(6)

s0 = 2 * dt  # Starting scale, in this case 2 * 1/12 years = 1/4 years
dj = 12 / 12  # Spacing between discrete scales
J = 5 / dj  # Seven powers of two with dj spacing sclaes


W12, cross_coi12, freq12, signif12 = wavelet.xwt(v1, v2, dt, dj, s0, J,
                                     significance_level=0.80, wavelet=mother,
                                     normalize=True)

W13, cross_coi13, freq13, signif13 = wavelet.xwt(v1, v3, dt, dj, s0, J,
                                    significance_level=0.80, wavelet=mother,
                                     normalize=True)

W23, cross_coi23, freq23, signif23 = wavelet.xwt(v2, v3, dt, dj, s0, J,
                                     significance_level=0.80, wavelet=mother,
                                    normalize=True)

# Inflation and interest rate at 5 year scale

data1 = pd.DataFrame({'Interest Rate':wave2[15],'Inflation':wave1[15]})

# Interest rate and Gdp at 8 year scale

data2 = pd.DataFrame({'Interest Rate':wave2[15],'GDP':wave3[15]})

# inflation and GDP at 8 year scale
data3 = pd.DataFrame({'Interest Rate':wave2[15],'GDP':wave3[15]})

grangercausalitytests(data3,maxlag=[1])
```

```
WCT1, aWCT, corr_coi, freq, sig = wavelet.wct(newgdp,newinter, dt, dj,s0,J,
                            significance_level=0.80,
                            wavelet='morlet', normalize=True,
                            cache=False)
WCT2, aWCT, corr_coi, freq, sig = wavelet.wct(newgdp,newinfl, dt, dj,s0,J,
                            significance_level=0.80,
                            wavelet='morlet', normalize=True,
                            cache=False)

WCT3, aWCT, corr_coi, freq, sig = wavelet.wct(newinfl,newinter, dt, dj,s0,J,
                            significance_level=0.80,
                            wavelet='morlet', normalize=True,
                            cache=False)
```

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