
New Sub-classifications for Prime and Composite Numbers

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Abstract

A new evidence has emerged suggesting that prime and composite numbers should be sub-classified into additive (primes with prime digit sums or composites with composite digit sums) and non-additive (primes with non-prime digit sums or composites with non-composite digit sums) classes. The numbers 114 and 506 act as twin numbers to produce an unmistakable evidence for the new sub-classification.

Twin Numbers: 114 and 506

114 ÷ 2 = 57
median(1..57) = 29
57 × 29 = 1653
where
16th prime = 53

There are **16** prime numbers with prime digit sums up to 114:
2, 3, 5, 7, 11, 23, 29, 41, 43, 47, 61, 67, 83, 89, 101, 113.

There are **53** composite numbers with composite digit sums up to 114:
4, 6, 8, 9, 15, 18, 22, 24, 26, 27, 28, 33, 35, 36, 39, 40, 42, 44, 45, 46, 48, 51, 54, 55, 57, 60, 62, 63, 64, 66, 68, 69, 72, 75, 77, 78, 80, 81, 82, 84, 86, 87, 88, 90, 91, 93, 95, 96, 99, 105, 108, 112, 114.

Similarly,

506 ÷ 2 = 253
median(1..253) = 127
253 × 127 = 32131
where
32nd prime = 131

There are **42**† prime numbers with non-prime digit sums up to 506:
13, 17, 19, 31, 37, 53, 59, 71, 73, 79, 97, 103, 107, 109, 127, 149, 163, 167, 181, 211, 233, 239, 251, 257, 271, 277, 293, 307, 347, 349, 367, 383, 389, 419, 431, 433, 439, 457, 479, 491, 499, 503.

There are **131** composite numbers with non-composite digit sums up to 506:
10, 12, 14, 16, 20, 21, 25, 30, 32, 34, 38, 49, 50, 52, 56, 58, 65, 70, 74, 76, 85, 92, 94, 98, 100, 102, 104, 106, 110, 111, 115, 119, 120, 122, 124, 128, 133, 140, 142, 146, 148, 155, 160, 164, 166, 175, 182, 184, 188, 200, 201, 203, 205, 209, 210, 212, 214, 218, 221, 230, 232, 236, 238, 245, 247, 250, 254, 256, 265, 272, 274, 278, 287, 289, 290, 292, 296, 298, 300, 302, 304, 308, 319, 320, 322, 326, 328, 335, 340, 344, 346, 355, 362, 364, 368, 371, 377, 380, 382, 386, 388, 391, 395, 403, 407, 410, 412, 416, 418, 425, 427, 430, 434, 436, 445, 452, 454, 458, 469, 470, 472, 476, 478, 481, 485, 490, 494, 496, 500, 502, 506.

What's more,

the 114th prime   number = 619
the 506th composite number = 621
where   114 + 506 = 620

and,

the 114th composite number   = 150
the 114th composite number with a   composite digit sum = 220
the 114th composite number with a non-composite digit sum = 436
where   436 + 220 - 150 = 506

Result

The above interlocked relationships between numbers 114 and 506 directly suggest sub-classing primes and composite numbers into additive and non-additive classes.

Since division by 1 does not divide any number into smaller parts, then division by 1 can be treated as a trivial division (in the same sense as the trivial zeros of the zeta function). Therefore, the number 1 becomes truly indivisible and hence the Unit for all natural numbers without the need for the deliberate "fixing" of the definition of prime numbers to exclude it artificially.
The following definitions arise naturally when excluding the trivial division by 1 and considering only non-trivial divisions:

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The Unit                {U}   = Whole number that is indivisible.
Prime number            {P}   = Whole number that is divisible by
itself only.
Additive Prime          {AP}  = Prime with a prime digit sum.
Non-additive Prime      {XP}  = Prime with a non-prime digit sum.
Composite number        {C}   = Whole number that is divisible by
itself and others.
Additive Composite      {AC}  = Composite with a composite digit
sum.
Non-additive Composite  {XC}  = Composite with a non-composite digit
sum.
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The first few numbers in each number class/sub-class:

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 {U}  = 1
 {P}  = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...
{AP}  = 2, 3, 5, 7, 11,             23, 29,         ...
{XP}  =                 13, 17, 19,         31, 37, ...
 {C}  = 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, ...
{AC}  = 4, 6, 8, 9,             15,     18,         22, 24,     ...
{XC}  =            10, 12, 14,     16,     20, 21,         25, ...
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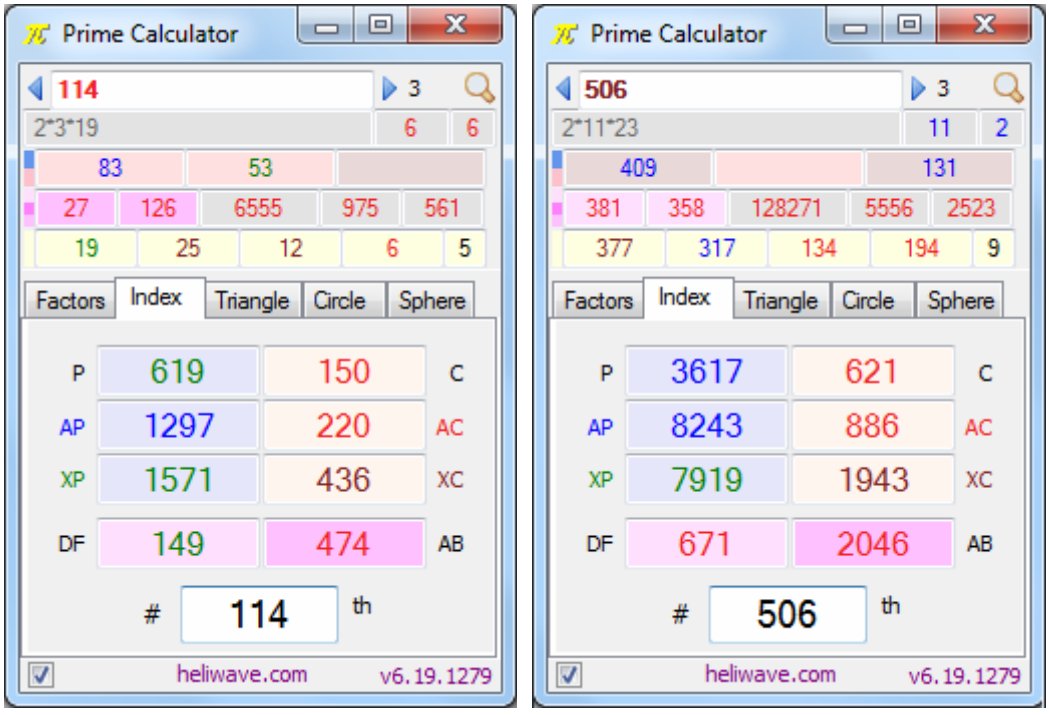
Further Work

A computer program was written to search for natural numbers that exhibit the same pattern as number 114, namely taking half N multiply by median(1...half N) to produce a concatenated prime and its index, but no other number has been found except 506.

And only, the number 13444 was found to exhibit similar pattern (not the same pattern) as number 114 to produce a concatenated composite (not prime) and its index:

13444 ÷ 2 = 6722
6722 ÷ 2 = 3361
6722 × 3361 = 22592642
where
2259th composite = 2642

Up to N = 7,000,000 was tested before realizing that the search was diverging and would never find new such HalfN_MedianHalfN_IndexPrime numbers. Therefore, a mathematical proof is needed to show that only numbers 114 and 506 exhibit this phenomenon.



PrimeCalculator is part of QuranCode software at http://qurancode.com

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† 42 = 32 + 10
32 non-additive primes with a digit sum != 10
and
10 non-additive primes with a digit sum = 10:
19, 37, 73, 109, 127, 163, 181, 271, 307, 433.
where
Sum: 19 + 37 + 73 + 109 + 127 + 163 + 181 + 271 + 307 + 433 = 1720
Sum of the first 31 prime numbers from 2 to 127 = 1720