HJ $\longrightarrow$ $0 = \dfrac{\partial V(x,u,t)}{\partial t} + \min_{u} \left\{ g(x,u,t) + \left[\dfrac{\partial V(x,u,t)}{\partial x}\right] f(x,u) \right\}$

$J(x_0, u(\cdot)) = \inf_{t \in \mathbb{R}_{\geq 0}} - s_{\mathcal{X}}(x(t))$ 13

$* \, g(x,u,t)$

$x(t) \longrightarrow$ a trajectory

$S_{\mathcal{X}}(x(t)) \leq 0$

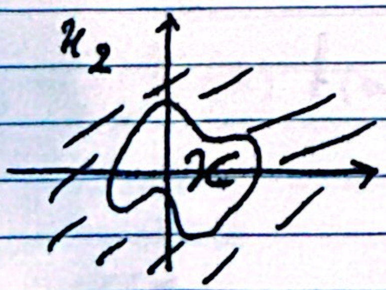
why?

$J(x_0, u(\cdot)) \geq 0$

14) $V(x_0) = \sup_{u(\cdot) \in PC(\mathbb{R}_{\geq 0}, u)} J(x_0, u(\cdot))$ why?

Signed-Distance Function :

$\mathcal{X} \longrightarrow \mathcal{X} \subseteq \mathbb{R}^n$ closed and time invariant



$\longrightarrow$ the Complement of all unacceptable failure states

$\mathbb{R}^m \setminus \mathcal{X}$

$$S_{\mathcal{X}} = \begin{cases} \inf\limits_{y \in \mathcal{X}} \|x(t) - y\| & x(t) \in R^m \setminus \mathcal{X} \\ -\inf\limits_{y \in \mathbb{R}^m \setminus \mathcal{X}} \|y - x(t)\| & x(t) \in \mathcal{X} \end{cases} \quad ?$$

$y \to$ exogenous input $\in Y$ chapter 7 page 127

set theretic methods in Control:

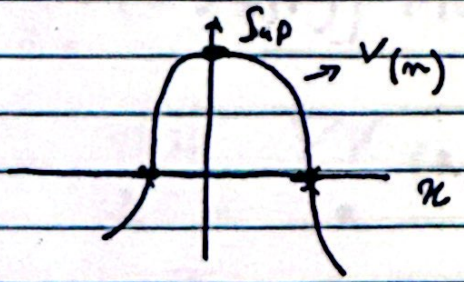
$$\mathrm{dist}(y, S) = \inf_{w \in S} \|y - w\|_*$$

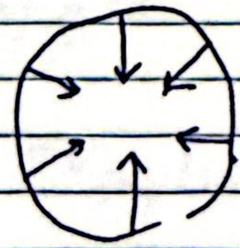
$y \to 0$, vectNorm of $x(t)$

Theorem 2 $\quad S_{\varepsilon} = \{ x \in \mathcal{X} \mid V(x) \geq \varepsilon \}$ (15)

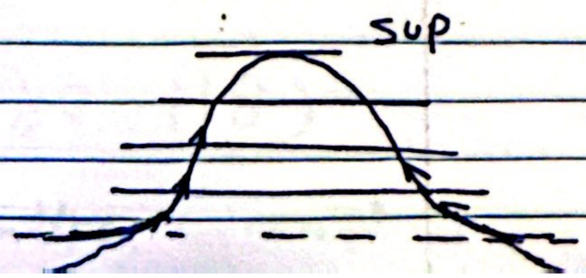
S_0 is maximal invariant set.

$V(x)$ is a sup of $J(x,\cdot)$ it means it has upered bonded





$$\frac{\partial V(x)}{\partial x}(x) \cdot f(x, u_0^*) \geq 0$$



Youtube video

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rewrite HJ $t \to \infty$

$$0 = \min_{u} \{ J(\cdot) \} + \min_{u} \left\{ \left[\frac{\partial V}{\partial x}(x,u,t) \right]^T f(x,u) \right\}$$

$$-\min_{u} \{ J(\cdot) \} = \min_{u} \left\{ \left[\frac{\partial V}{\partial x}(x,u,t) \right]^T f(x,u) \right\}$$

$$J(\cdot) = V(x,t,u) - V(x,T)$$

$$\min_{u} \{ V(x,T) - V(x,t,u) \} = \min_{u} \left\{ \left[\frac{\partial V}{\partial x}(x,u,t) \right]^T f(x,u) \right\}$$

as we want maximiz the invariant set

there must be u Control ~~invariant set~~ action

$$\left\{ \left[\frac{\partial V}{\partial x}(U^{max}, x, t) \right]^T f(x,u) \right\} \geq 0$$

or $$\max_{u \in U} \frac{\nabla V}{\partial x}(x,u,t) f(x,u) \geq 0 \quad (16)$$

$$[\nabla V(x)]^T f(x, K_V^*(x)) = \max_{u \in U} \nabla V(x) f(x,u) \quad (17)$$

$$\dot{V}(x) = \nabla V(x) f(x, K_V^*) \geq 0 \quad (18)$$

3 necessary and sufficient condition nagumo's theorem

S_ϵ forward invarient set under K_V^*

$$K_f(x,u) = \begin{cases} K_V^*(x) & V(x) < \epsilon \text{ or } u \notin \mathcal{U} \\ u & \text{else} \end{cases} \quad (19)$$

20 $$0 = \min \left\{ -S_{\mathcal{X}}(x) - V(x),\ \max_{u \in \mathcal{U}} \nabla V(x) f(x,u) \right\}$$

Sufficent Condition on the boundary or go inside

Nessesarry Condition remain insize the invariant set.

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