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Steering Angle Control of Car for Dubins Path-tracking Using Model Predictive Control

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Abstract. Car as one of transportation is inseparable from technological developments. About ten years, there are a lot of research and development on lane keeping system(LKS) which is a system that automaticaly controls the steering to keep the vehicle especially car always on track. This system can be developed for unmanned cars. Unmanned system car requires navigation, guidance and control which is able to direct the vehicle to move toward the desired path. The guidance system is represented by using Dubins-Path that will be controlled by using Model Predictive Control. The control objective is to keep the car's movement that represented by dinamic lateral motion model so car can move according to the path appropriately. The simulation control on the four types of trajectories that generate the value for steering angle and steering angle changes are at the specified interval.

1. Introduction

Rapid technological developments affect many aspect of human life, on of them is transportation field. Car as one of transportation vehicle didn't escape from this technological developments. About ten years, there was many research and development about Lane Keeping System (LKS) which is a system that automatically controls the steering to keep the vehicle, especially car always on track[1]. That system can be further developed into unmanned car.

Unmanned car requires navigation, guidance and control system which are able to direct the vehicle to move toward the desired point[2]. Navigation system usually using Global Positioning System (GPS) which gives the position on earth by using satellite signals. Guidance system is a path planning that result desired trajectory based on initial and final position from navigation system. Path planning can be obtained from several methods, one of them is Dubins Path. Dubins path is an optimal path that satisfy maximum curvature bound between two points with a particular orientation in a field of both CLC(*Circle-Line-Circle*) or CCC(*Circle-Circle-Circle*) path, or combination of both where C is circle arc and L is a line that intersect with C([3],[4]). Dubins path have several advantages including path planning using this method can use for angle in all quadrant both at initial or final point, and this path can optimize the time because it can generate the path curvature[2]. Besides that, Dubins path not require a lot of time in its planning so it can be more optimal and efficient[5], it also requires little itme to achieve the target[6].

The next system that requires is control system which aims to makes the car can move according the desired path. This control system also can be obtained from several methods, one

of them is Model Predictive Control (MPC). Basically, MPC use the model explicitly to predict the output process in a horizon time and the objective of control's calculation is to minimize a certain objective function. MPC has several advantages compare to other methods, including its uses to control a multivariable case easily and it can solve the system with the disturbance using simple concept directly during the design process[7].

Research on trajectory-tracking have done before where the track obtained from the GPS that sometimes given waypoint irregularly, so a curve fitting done based on that irregular waypoints. The result of that research is the oscillations value of steering angle decreased significantly using MPC method compared with linear quadratic, especially in the curved trajectory[8].

Based on some of the things that has been said, this research aims to control the car's steering angle in order to move in accordance with the desired trajectory especially in not straight trajectory.

2. Research method
In this chapter are discribed the steps to control the car's steering angle on Dubins-path tracking using Model Predictive Control including determination of Dubins-path, dynamic lateral motion model, discretization of model, formulation of objective function and its constraints.

2.1. Determination of Dubins-path
Determination of Dubins-path required an initial and final position of the car. Both of these positions consists of positions in Cartesian coordinates (x, y), car's heading angle (ψ) and minimum radius of curvature in this case symbolized with R. R obtained from a car's minimum turning radius.
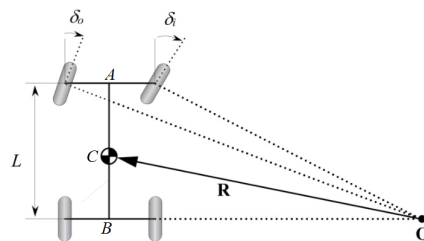

Figure 1. Car's turning geometry[9]

Before obtained the value of R, first thing to do was identifying things that related such as front and rear wheels steering angle that symbolized with δ_f and δ_r, and the center of gravity of the car that given by point C. The outside steering angle (δ_o) and inside steering angle (δ_i) of the front wheels in Fig. 1 is considerered to have same value so it can be symbolized by δ_f. The distance between each point A and B to point C is given by l_f and l_r. So the length of L that defined as the distance between the front and rear wheels is $L = l_f + l_r$. The car's velocity at center of gravity is symbolized by V with angle of slip β. In this paper, assumed that there was not a angle slip or $\beta = 0$ so the car's velocity has same direction with car's heading angle. Suppose that L is very small compared with the R, so the angular velocity can be approximated by[1]

$$\frac{\dot{\psi}}{V} \approx \frac{1}{R} = \frac{\delta}{L}$$

For the steering angle maximum input $\delta = 0.5386 rad$, $l_f = 1.1m$ and $l_r = 1.58m$ result $R = 4.9759 \approx 5m$.

Initial and final position defined after R have been obtained as follows:

$$p_s = \begin{bmatrix} 1100 & 1150 & 180 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 3200 & 2675 & 180 & 5 \end{bmatrix}$$

The length of each path type given in Table 1 can be obtained using Dubins-algortihm.

Table 1. Dubins-path length table

Dubins-path type	Path length (m)
RSR	2626.7
LSL	2626.7
RSL	2614.6
LSR	2664

Table 1 provides RSL as optimal trajectory path with its length is 2664 m. The path form given on Fig. 2. Minimum value of car's turning radius is too small compared to the trajectory length so it's not visible in the initial form of trajectory. Therefore, magnification of image at the initial and final positions done and shown in Fig. 3.
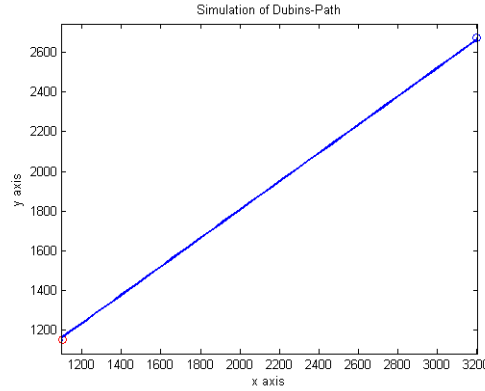

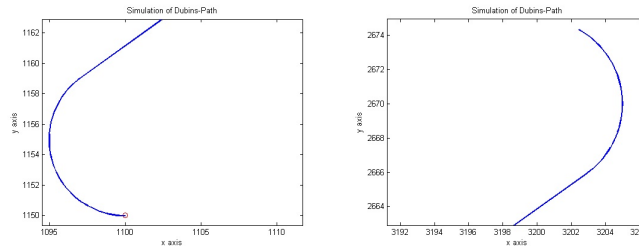**Figure 2.** RSL type of Dubins-path

Figure 3. Image magnification of Dubins-Path at initial position (left) and final position (right)

2.2. Dynamic lateral motion model
Dynamic lateral motion model ([1],[9]) is given as follows:

$$\dot{x} = \begin{bmatrix} -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & -\frac{2C_{\alpha f}-2C_{\alpha r}}{mV_x} - V_x \\ -\frac{2l_f C_{\alpha f}-2l_r C_{\alpha r}}{I_z V_x} & -\frac{2l_f{}^2 C_{\alpha f}+2l_r{}^2 C_{\alpha r}}{I_z V_x} \end{bmatrix} x + \begin{bmatrix} \frac{2C_{\alpha f}}{m} \\ \frac{2l_f C_{\alpha f}}{I_z} \end{bmatrix} u \qquad (1)$$
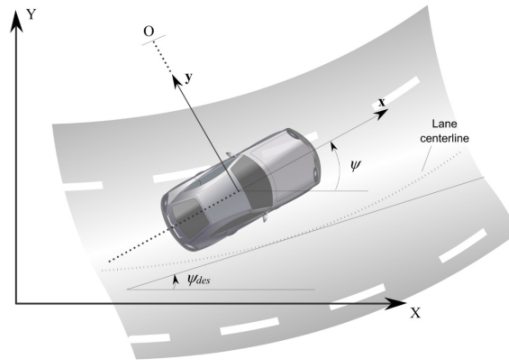
Figure 4. Dynamic lateral motion[9]

with $x = \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix}$, $u = \delta$ and output system is $\dot{\psi}$ which can be expressed in matrix form as follows:

$$\bar{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

and also parameters given as follow:

$\{XYZ\}$: global coordinate plane
$\{xyz\}$: local (vehicle) coordinate plane
m : vehicle total mass
I_z : yaw moment of inertia of vehicle
l : (l_f, l_r) longitudinal distance from c.g. to (front,rear) tires
α : (α_f, α_r) slip angle at (front,rear) wheel tires
C : $(C_{\alpha f}, C_{\alpha r})$ cornering stiffness of (front, rear) tires
δ : steering angle

2.3. Discretization of model
Model in the discrete time is given in equation as follow:

$$x(k+1) = A_d(T)x(k) + B_d(T)u(k)$$

with

$$
\begin{aligned}
A_d(T) &= e^{At} \\
B_d(T) &= \left(\int_0^T e^{A\lambda} d\lambda \right) B
\end{aligned}
$$

and

$$e^{AT} = I + At + \frac{1}{2!}A^2 t^2 + \frac{1}{3!}A^3 t^3 + \cdots + \frac{1}{n!}A^n t^n + \cdots$$

Analytic calculation produces value cutting for $n > 6$ so discretization done using a software to obtain more accurate results. Based on state space equation in (1), discrete state space obtained with a sampling time 0.1 seconds as follow:

$$x(k+1) = \begin{bmatrix} 0.4450 & -1.3734 \\ 0.0431 & 0.4402 \end{bmatrix} x(k) + \begin{bmatrix} 1.6503 \\ 4.5607 \end{bmatrix} u(k)$$

with system output is $\bar{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} x$

2.4. Control design using MPC

Control design using MPC was done by determining the objective function and its constraint in quadratic programming form.

A discrete system is given as follow:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) & (2)\\
y(k) &= Cx(k) & (3)
\end{aligned}
$$

Controls carried out on the steering angle so that the steering angle changes need to know from time to time during the journey of the vehicle (car). Therefore, the input increment is defined as follows:

$$
\Delta u\,(k) = u\,(k) - u\,(k-1)
$$

so equation (2) becomes

$$
x\,(k+1) = Ax\,(k) + Bu\,(k-1) + B\Delta u\,(k) \tag{4}
$$

Consecutive given prediction horizon and control horizon, N_p and N_c with $N_c = N_p$ so obtainable

$$
\begin{aligned}
x\,(k+1|k) &= Ax\,(k|k) + Bu\,(k|k)\\
&= Ax\,(k|k) + Bu\,(k-1|k) + B\Delta u\,(k|k)\\
x\,(k+2|k) &= Ax\,(k+1|k) + Bu\,(k|k) +\\
& \quad B\Delta u\,(k+1|k)\\
&\quad\vdots\\
x\,(k+N_p|k) &= Ax\,(k+N_p-1|k) + Bu\,(k-1|k) +\\
& \quad B\Delta u\,(k|k) + \cdots + B\Delta u\,(k+N_p-1|k)
\end{aligned}
$$

with prediction output as follows:

$$
\begin{aligned}
\hat{y}\,(k|k) &= Cx\,(k|k)\\
\hat{y}\,(k+1|k) &= Cx\,(k+1|k)\\
&\quad\vdots\\
\hat{y}\,(k+N_p|k) &= Cx\,(k+N_p|k)
\end{aligned}
$$

Objective function that uses in this research is given as follows:

$$
J_{min} = \sum_{i=1}^{N_p} \|\hat{r}\,(k+i|k) - \hat{y}\,(k+i|k)\|^2_{Q_{(i)}} + \|\Delta\hat{u}\,(k+i-1|k)\|^2_{R_{(i-1)}}
$$

with the first part of objective function is representaion of error between prediction output $(\hat{y}(k+i|k))$ and reference from Dubins-path $(\hat{r}(k+i|k))$ while the seconde part of objective function is representation of required energy when steering angle changes $(\Delta\hat{u}(k+i|k))$ at $k+i$ time that measure at k time. Objective function changes in quadratic programming form and result this equation bellows:

$$
J_{min} = \frac{1}{2}j^T E' j + j^T F \tag{5}
$$

with

$$
j = \begin{bmatrix} \Delta u(k|k) \\ x(k+1|k) \\ \vdots \\ \Delta u(k+N_p-1|k) \\ x(k+N_p|k) \end{bmatrix},
F = \begin{bmatrix} 0 \\ -2C^T Q_1 \hat{r}(k+1|k) \\ \vdots \\ 0 \\ -2C^T Q_{N_p} \hat{r}(k+N_p|k) \end{bmatrix},
E = \begin{bmatrix} R_0 & 0 & \cdots & 0 & 0 \\ 0 & C^T Q_1 C & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & R_{N_p-1} & 0 \\ 0 & 0 & \cdots & 0 & C^T Q_{N_p} C \end{bmatrix}
$$

Objective function is given as equation (5) with constraint given as follows:

$$\Delta u^{min} \leq \quad \Delta u \quad \leq \Delta u^{max}$$
$$u^{min} \leq \quad u \quad \leq u^{max}$$

That constraint are formulated in quadratic programming form

$$Pj \leq h$$

and matrices that corresponding to the boundary constraints was obtained as follow:

$$P = \begin{bmatrix} M_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_1 & 0 \\ M_2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, h = \begin{bmatrix} N_1 \\ N_1 \\ N_1 \\ \vdots \\ N_1 \\ N_2 \end{bmatrix}$$

The constraints will be formulated from equation (4) with prediction horizon N_p

$$Yj = b$$

then obtained the corresponding matrices as follows:

$$Y = \begin{bmatrix} -B & I & 0 & 0 & 0 & 0 & \cdots & 0 \\ -B & -A & -B & I & 0 & 0 & \cdots & 0 \\ -B & 0 & -B & -A & -B & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -B & 0 & -B & 0 & -B & 0 & \cdots & I \end{bmatrix}, b = \begin{bmatrix} Ax\,(k|k) + Bu(k-1|k) \\ Bu(k-1|k) \\ Bu(k-1|k) \\ \vdots \\ Bu(k-1|k) \end{bmatrix}$$

In this paper, given the constraints values as follows:

$$-0.4987(rad) \leq \quad \Delta u \quad \leq 0.4987(rad)$$
$$-0.5386(rad) \leq \quad u \quad \leq 0.5386(rad)$$

3. Analysis of the experimental results

In this section, there will be simulation using weighting coeffisients $Q = 100$ and $R = 1$. It's mean that objective function more proritize on prediction output that have been controlled accordance with reference which given and ignoring driver comfort despite the large steering angle changes in a short time. Next will be simulated some cases the steering angle control problems on the car.In the simulation, the initial value is defined $x_1(1) = 0.5$ m/s and $x_2(1) = 0$ rad. It also defined the initial value of the steering angle is $u(1) = 0$ rad.

3.1. The influence of trajectory type with same value of prediction horizon
In order To determine the effect of trajectories on the results of MPC, simulation test conducted with a different trajectory and the other parameter in the same value. Simulations were performed for 60 seconds with a sampling time 0.1 seconds.

Table 2. Dubins-path length table on simulation 1

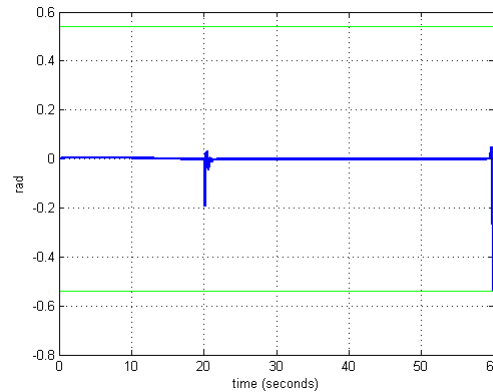| Dubins-path type	Path length (m)
RSR	1796.3
LSL	1844.4
RSL	1826.2
LSR	1814.5

Simulation 1   The first trajectory using initial and final position that defined as follows:

$$p_s = \begin{bmatrix} 10 & 10 & 180 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 1000 & 1500 & 0 & 5 \end{bmatrix}$$

The length of each path type given in Table 2 can be obtained using Dubins-algortihm.

Table 2 provides RSR (Right-Straight-Right) as optimal trajectory path with its length is 1796.3 m.The simulation results for prediction horizon $N_p = 10$ indicates that the steering angle and steering angle changes do not exceed the constraint limits which shown in Fig. 5 and Fig. 6. Fig. 7 shows that difference value between reference of car's heading angle changes and car's heading angle which was controlled is small or can be sai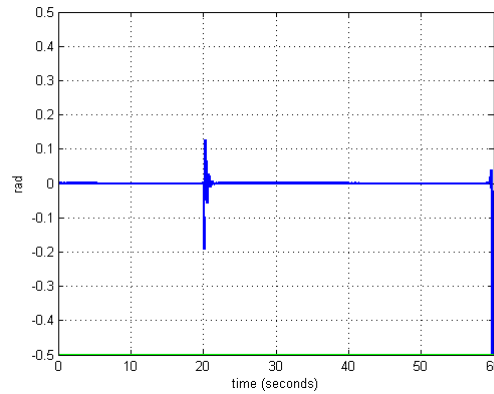d almost same but at some point in the beginning, there is a considerable difference between reference of car's heading angle changes and car's heading angle which was controlled because of trajectory tracking influence.

Figure 5. Steering angle which was controlled on RSR type of Dubins-path

Simulation 2   The second trajectory using initial and final position that defined as follows::

$$p_s = \begin{bmatrix} 1100 & 1150 & 180 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 2600 & 2065 & 180 & 5 \end{bmatrix}$$

The length of each path type given in Table 3 can be obtained using Dubins-algortihm.

Table 3 provides RSL (Right-Straight-Left) as optimal trajectory path with its length is 1777.9 m.The simulation results for prediction horizon $N_p = 10$ indicates that the steering angle and steering angle changes do not exceed the constraint limits which shown in Fig. 8 and Fig. 9. Fig. 10 shows that difference value between reference of car's heading angle changes and car's heading angle which was controlled is small or can be said almost same but at some point in the

Figure 6. The changes of steering angle which was controlled on RSR type of Dubins-path
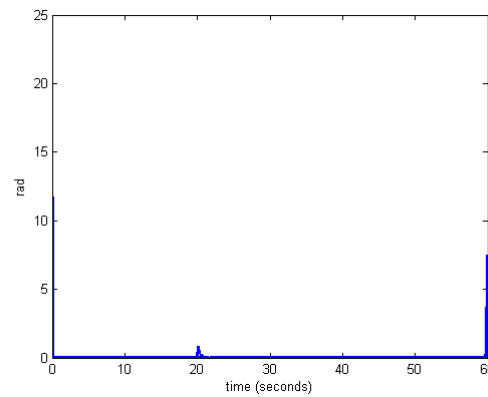

Figure 7. The difference between heading angle of car's which was controlled and its reference on RSR Type of Dubins-path

Table 3. Dubins-path length table on simulation 2

Dubins-path type	Path length (m)
RSR	1788.5
LSL	1788.5
RSL	1777.9
LSR	1825.1

beginning, there is a considerable difference between reference of car's heading angle changes and car's heading angle which was controlled because of trajectory tracking influence.

Simulation 3   The third trajectory using initial and final position that defined as follows:

$$p_s = \begin{bmatrix} 10 & 1200 & 120 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 200 & 10 & 45 & 5 \end{bmatrix}$$

The length of each path type given in Table 4 can be obtained using Dubins-algortihm.

Table 4 provides LSL (Left-Straight-Left) as optimal trajectory path with its length is 1224.1 m.The simulation results for prediction horizon $N_p = 10$ indicates that the steering angle and
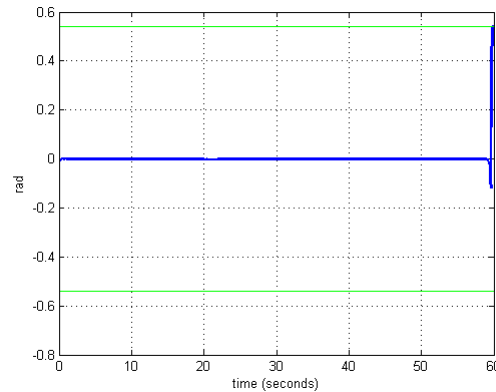
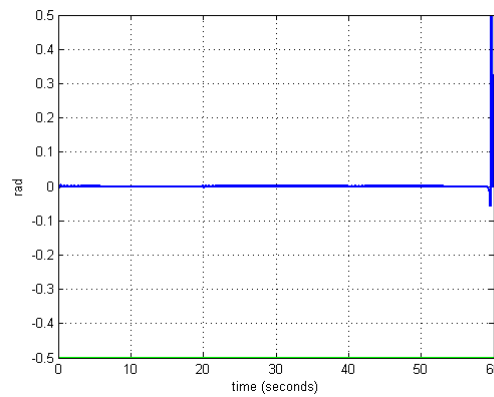**Figure 8.** Steering angle which was controlled on RSL type of Dubins-path

Figure 9. The changes of steering angle which was controlled on RSL type of Dubins-path
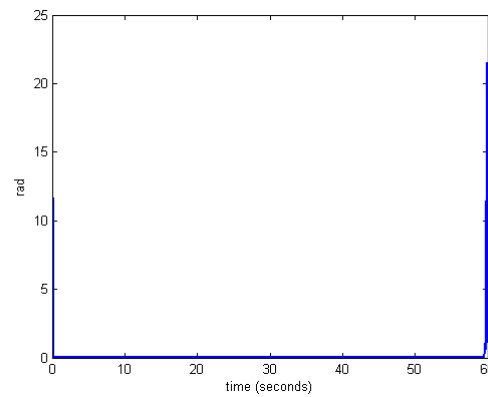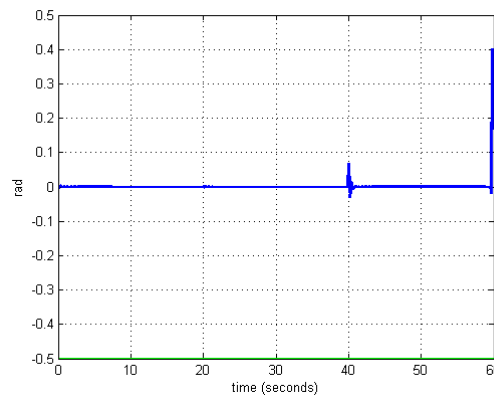

Figure 10. The difference between heading angle of car's which was controlled and its reference on RSL Type of Dubins-path

steering angle changes do not exceed the constraint limits which shown in Fig. 11 and Fig. 12. Fig. 13 shows that difference value between reference of car's heading angle changes and car's heading angle which was controlled is small or can be said almost same but at some point in the beginning, there is a considerable difference between reference of car's heading angle changes and car's heading angle which was controlled because of trajectory tracking influence.

Table 4. Dubins-path length table on simulation 3

Dubins-path type	Path length (m)
RSR	1248.9
LSL	1224.1
RSL	1231.5
LSR	1263.6
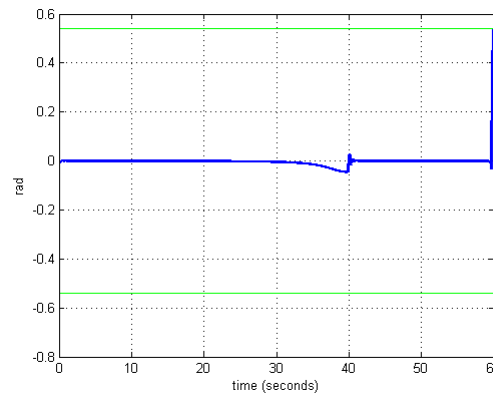
Figure 11. Steering angle which was controlled on LSL type of Dubins-path

Figure 12. The changes of steering angle which was controlled on LSL type of Dubins-path

Simulation 4   The fourth trajectory using initial and final position that defined as follows::

$$p_s = \begin{bmatrix} 1500 & 0 & 90 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 0 & 0 & 30 & 5 \end{bmatrix}$$

The length of each path type given in Table 5 can be obtained using Dubins-algortihm.

Table 5 provides LSR (Left-Straight-Right) as optimal trajectory path with its length is 1513.5 m.The simulation results for prediction horizon $N_p = 10$ indicates that the steering angle and steering angle changes do not exceed the constraint limits which shown in Fig. 14 and Fig. 15. Fig. 16 shows that difference value between reference of car's heading angle changes and car's heading angle which was controlled is small or can be said almost same but at point $t = 40$, there is a large difference because of trajectory tracking influence.
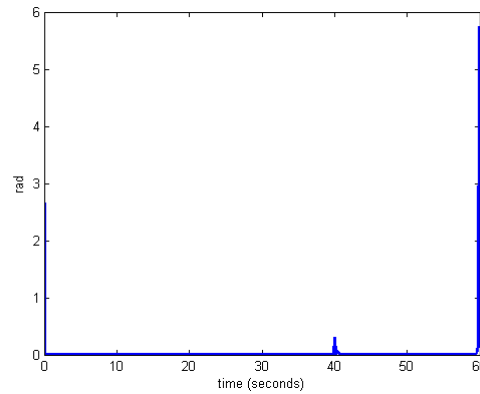
Figure 13. The difference between heading angle of car's which was controlled and its reference on LSL Type of Dubins-path

Table 5. Dubins-path length table on simulation 4

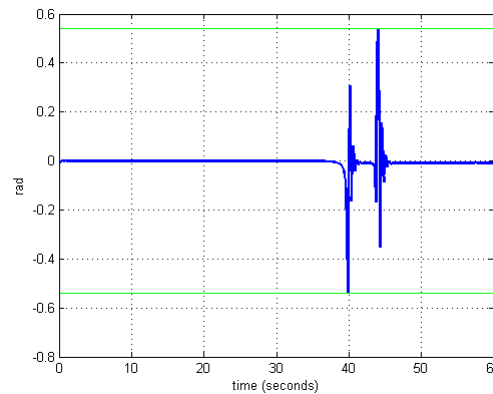| Dubins-path type	Path length (m)
RSR	1539.2
LSL	1523.7
RSL	1549.5
LSR	1513.5

Figure 14. Steering angle which was controlled on LSR type of Dubins-path

3.2. The influence of prediction horizon with the same trajectory type
In order to determine the effect of prediction horizon on the results of MPC, simulation test conducted with a different value of prediction horizon that is 10, 50 and 100. Each value of prediction horizon will be simulated with the same trajectory type which intial and final position defined as follows:

$$p_s = \begin{bmatrix} 1100 & 1150 & 180 & 5 \end{bmatrix}$$
$$p_f = \begin{bmatrix} 2600 & 2065 & 180 & 5 \end{bmatrix}$$

which produces the type and length of the path as in the previous simulations 2. The other parameter at the same value. Simulations were performed for 60 seconds with a sampling time 0.1 seconds. The value of Root-Mean-Square-Error(RMSE) and its computation time for each
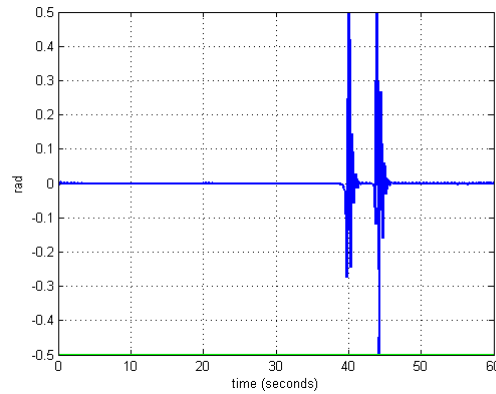
11

Figure 15. The changes of steering angle which was controlled on LSR type of Dubins-path
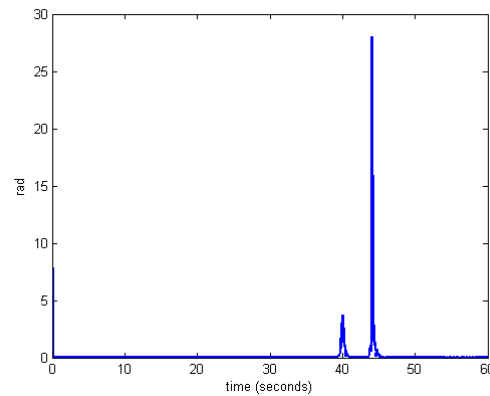

Figure 16. The difference between heading angle of car's which was controlled and its reference on LSR Type of Dubins-path

value of N_p are calculated to determine the effect of N_p value changes.

The result is given on Table 6 which is shown the differences between value of RSME and

Table 6. Error value between reference of car's heading angle and car's heading angle which was controlled and each computation time

Np	RMSE	Computation time (seconds)
10	0.00704210296467999	17.6487602346906
50	0.00706729399940004	244.029521069138
100	0.00706729399939593	357.740033058028

computation time on $N_p = 10$ and $N_p = 50$,while on $N_p = 50$ and $N_p = 100$ the differences between value of RSME is small but there was a large difference on camputation time. $N_p = 10$ is better than $N_p = 50$ and $N_p = 100$ when viewed from the value of RSME and computation time for each N_p.

4. Conclusion
Based on the analysis and simulation of control using MPC, obtained the following conclusions:

(i) Model Predictive Control (MPC) can be applied well in steering angle control of car for Dubins path-tracking. It can be seen from the simulation control on the four types of

trajectories that generate the value for steering angle and steering angle changes are at the specified interval.

(ii) The simulation results showed that in this paper which is using coefficient weights $Q = 100$ and $R = 1$, $N_p = 10$ is better than $N_p = 50$ and $N_p = 100$ when viewed from the value of RSME and computation time for each N_p.

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