Probability and Paradoxes in Baseball

Scott Powers

BRAINS
IN A BAR

The Birthday Problem (von Mises, 1939)

- In a group of n people, what is the probability that at least two will share a birthday?

Q: How big does n need to be for the probability to exceed 50%?
A: $n \geq 23$

Simpson's Paradox (Simpson, 1951)

- Justice had the higher batting average in 1995 *and* 1996, but Jeter had the higher combined batting average

	1995	1996	Combined
Derek Jeter	.250	.314	.310
David Justice	.253	.321	.270

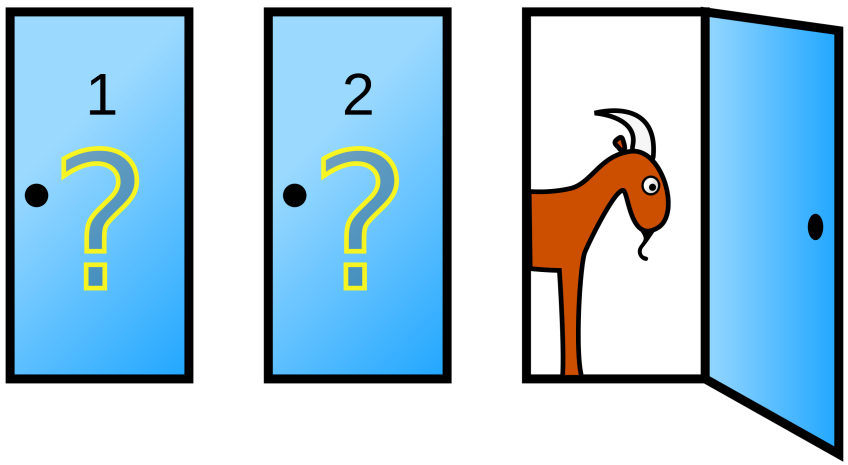
Q: How is this possible?

A:

	1995	1996	Combined
Derek Jeter	12/48	183/582	195/630
David Justice	104/411	45/140	149/551

The Monty Hall Problem (Selman, 1975)

- On a game show, you are given the choice of three doors
  - One door hides a car, and two doors hide a goat
- After you choose door #1, the host opens door #3, revealing a goat (the host alway reveals a goat behind a different door)



Q: Do you want to switch your choice to door #2?
A: Yes! (door #1 = 33% car, door #2 = 67% car)

Stein's Paradox (Efron and Morris, 1977)

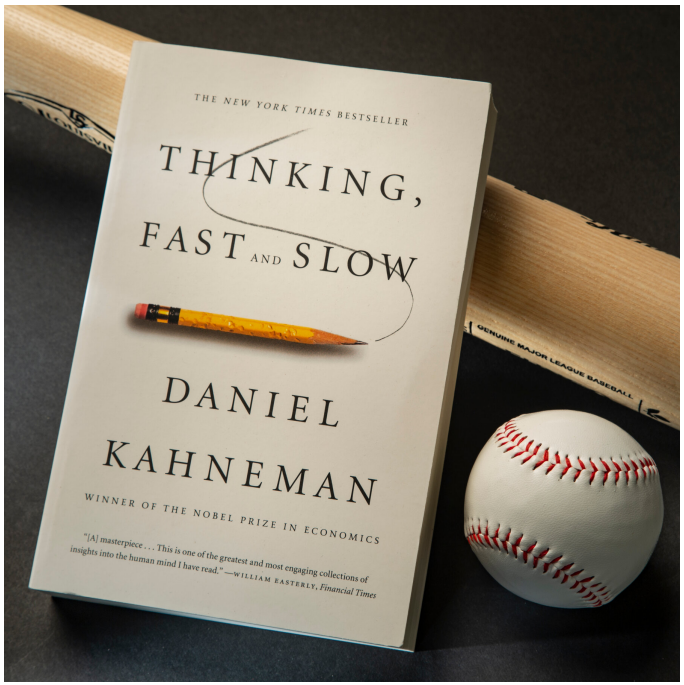
As framed by Brown (2008):

Two methods for predicting second-half batting average:

1. Use each player's first-half batting average
2. Ignore the data and predict league average (.250) for everyone

Q: Which method is more accurate?
A: Method #2

Thinking, Fast and Slow (Kahneman, 2011)



The New York Times

- **System 1** is fast, intuitive, emotional. **System 2** is slower, more deliberative, more logical.
- Statistical thinking is hard because of System 1 heuristics and biases
- Relevant examples:
  - Availability heuristic
  - Confirmation bias
  - Framing effect

The Hot Hand Fallacy (Gilovich *et al.*, 1985)

- 1980-81 Philadelphia 76ers field goal data
  - Shooting percentage following a make: 51%
  - Shooting percentage following a miss: 54%
- 1980-82 Boston Celtics free throw data
  - No correlation between making 1^{st} FT and making 2^{nd} FT
- Controlled experiment with college players (from Cornell) yielded no evidence of streaky shooting

"Who is this guy? So he makes a study. I couldn't care less."

— Red Auerbach

"The hot hand is a massive and widespread cognitive illusion."

— Daniel Kahneman

Takeaways (2017)

1. Paradoxes show us that human intuition can be incorrect
2. Human judgment can be impaired by heuristics and biases
3. Mathematical modeling is a superior way to make decisions

The Streak of Heads Paradox (Miller and Sanjurjo, 2018)

Flip a coin 100 times. Whenever you flip heads, write down the *next* outcome on a scrap of paper.

Q: What is the expected proportion of heads on the paper?
A: 49.5%

- This paradox is related to the Monty Hall problem through the "principle of restricted choice"
- After adjusting Gilovich *et al.* (1985) for this bias, evidence supports the existence of the hot hand

Replication Crisis (2010s, ongoing)


Essay

Why Most Published Research Findings Are False

John P. A. Ioannidis

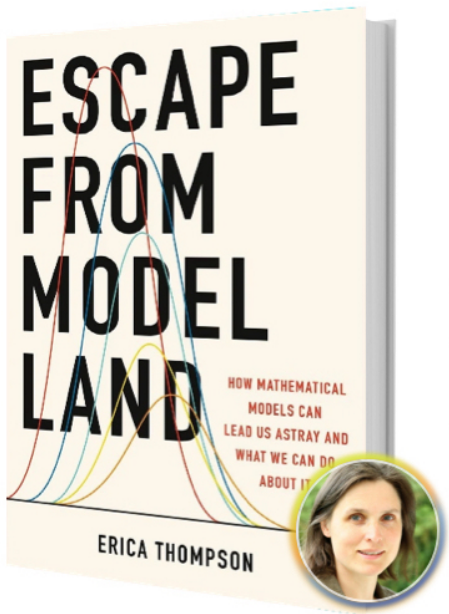
- Because of selection bias in scientific publications, many research findings fail to be replicated
- Shimmack (2020) estimated that only half of the results cited in *Thinking, Fast and Slow* would be replicated

"Readers of *Thinking, Fast and Slow* should read the book as a subjective account by an eminent psychologists, rather than an objective summary of scientific evidence."

— Ulrich Schimmack

Escape from Model Land (Thompson, 2022)

- All models make assumptions that are not literally true in real life
- Escaping from Model Land means validating adequacy-for-purpose
- Functionality as a tool to aid thinking can be more important than predictive accuracy



Takeaways (2024)

1. ~~Paradoxes show us that human intuition can be incorrect~~
2. ~~Human judgment is impaired by heuristics and biases~~
3. ~~Mathematical modeling is a superior way to make decisions~~

1. We can make mistakes when applying math, too
2. Be careful spending too much time and energy in Model Land
3. Validation is king

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