SEISMIC RISK- ANALYSIS REPORT

ABSTRACT

The report provides a comprehensive seismic risk analysis of Acme Insurance which has 312 locations under its portfolio in the state of California

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Risk Quantification and Insurance – (CEE 209)

Contents

Introduction

The project requires the students to perform a complete risk analysis of Acme's portfolio with properties located in California. The portfolio of the company comprises of 312 locations spread across the Northern and Southern California. There are 287 locations in Southern California and 26 locations in Northern California. Acme insures these locations against earthquake losses. A total of 16 faults are considered as potential sources of hazard and risk estimation. Northern California has three faults and Southern California has 13 faults. The largest of these faults is the San Andreas Fault which is capable of producing an earthquake of Magnitude (Mw) 7.9.

To proceed with the analysis, Monte Carlo simulation is used to sample values and the coding is done in Matlab 2015a. The process is repeated for 50000 years to achieve reasonable convergence. It is assumed a part of the problem statement that the company starts with a capital of 1.5 M dollars. Acme charges a flat price of 1500 dollars as premium from all its insured locations and pays a dividend of 2% to an investment bank from whom it borrowed the money. The cost of replacement is considered constant across all the locations. A deductible of 1% of replacement cost is considered for the analysis.

The magnitude of earthquakes are chosen randomly as uniformly distributed with the Latin Hypercube sampling technique to distribute the samples evenly between the limit points. The lower limit of Mw for all the faults is considered to be 5 since buildings in California are hardly damaged by earthquakes lower magnitudes. Following the process of analysis the, total losses are calculated for every year. This provides the distribution of losses every year, the expected value and the standard deviation. Given the distribution, we find the optimum Capital the company should have had at the beginning and the optimal premium it should have charged from every client. The report also analyzes whether the company's initial capital was enough to sustain a probability of ruin with a return period of 1 in 2000 years. Overall, it summarizes certain conclusions based on the analysis.

Risk Analysis Algorithm

Matlab is used to code the entire analysis. Cartesian Coordinates are used to calculate the shortest distance between the rupture location of a fault and a point. However, the data was provided in terms of latitudes and longitudes which were converted to Cartesian using a set of simplified formulae.

```
x = R * cos(lat) * cos(lon)

y = R * cos(lat) * sin(lon)
```

Where, R is the radius of the earth. The z – coordinates are assumed to be zeros to make all the points fall on a plane.

The magnitude of earthquakes for each fault is sampled using the **Latin Hypercube** technique as a uniformly distributed random variable between a fixed limit. Then, for each such magnitudes, the rate of event occurrence is calculated using Gutenberg-Richter relationship.

$$N = 10^{(a-bM)}$$

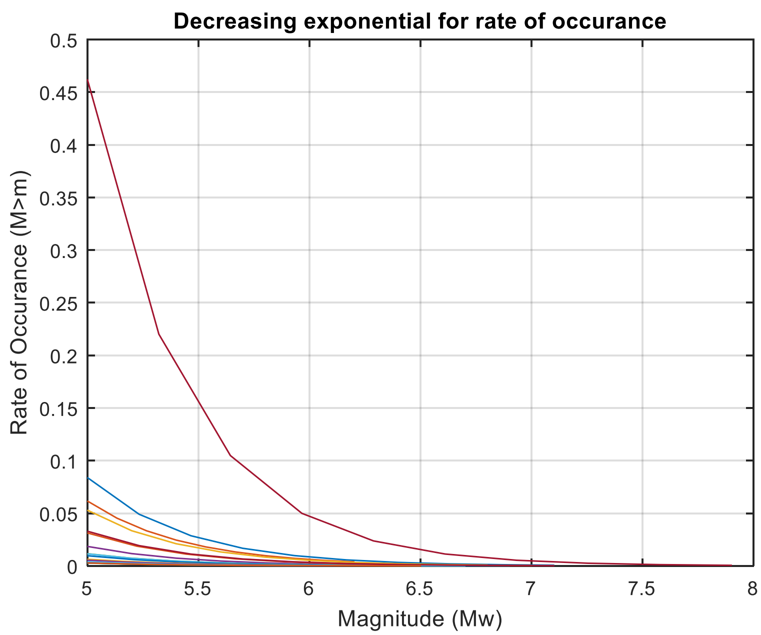


Figure 1: Earthquake Occurrence vs Magnitude Plot

Then, the number of earthquakes for the chosen magnitude is calculated using poisons distribution, following which the rupture length is calculated using Wells and Coppersmith Relationship.

$$\log(SRL) = -3.55 + 0.74*M$$

For each event on a fault, the ground shaking is calculated using GMPE from Boore, Joyner and Fumal relationship.

$$\ln[S_a(\mathbf{g})] = C_1 + C_2(\mathbf{M} - 6) + C_3(\mathbf{M} - 6)^2 + C_4 \ln[\mathbf{R}] + C_5 \ln[500 / C6]$$

For each of these ground shaking, the loss distribution is calculated as lognormal random variables with a coefficient of correlation of 0.2 between and two locations. This is done using the Factor Model.

$$X = \sqrt{\rho}\, z + \sqrt{1-\rho}\, y$$

Where z and y follow standard normal distribution. '*y*' changes with every location and z changes with every event. X follows a standard normal distribution as well. We calculate a quantile for X and use it to get the loss value for one event in one location.

Convergence Analysis

Convergence for Monte Carlo simulation is achieved by running the code for 'n' number of years and comparing Value at Risk (VaR), Tail Value at Risk (TVaR) for both portfolio and location level. The VaR's and the TVaR's are calculated for p = 0.5, 0.75, 0.9, 0.95, 0.99. The comparison charts are provided below with plots illustrating the results.

Table 1 shows the convergence of analysis on the portfolio level. **Fig 2** shows the date in another format. **Table 2** shows the convergence of Southern California location level and **Table 3** shows the convergence of Northern California Location level.

Table-1: VaR, AAL and Ruin at Portfolio Level

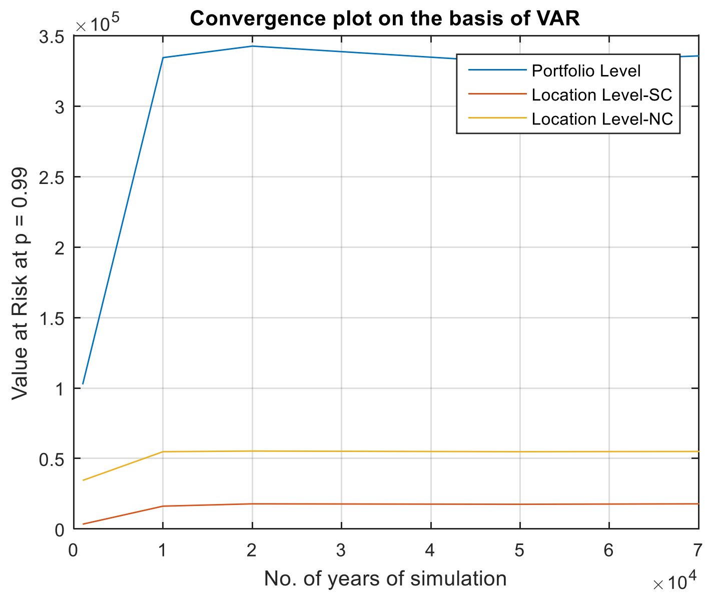
Simulation No	VAR(0.75)	VAR(0.9)	VAR(0.95)	VAR(0.99)	Ruin_P	AAL
1000	0	0	102566.6	3008874.26	0.09	184214.5
10000	0	0	334363.4	6122123.01	0.009	236026.6
20000	0	0	342518	6229809.96	0.0045	248812
50000	0	0	330731.6	5685380.64	0.0018	241398.9
70000	0	0	335574.7	5715652.7	0.001286	241133.3

Table-2: Parameters calculated for South California (Location Level)

Simulation No	**VAR(0.9)**	**VAR(0.95)**	**VAR(0.99)**	**AAL**
1000	0	0	3414.63415	534.6318
10000	0	0	16186.83	652.5831
20000	0	0	17824.2753	697.0802
50000	0	0	17524.9654	680.7138
70000	0	0	17824.2753	679.9869

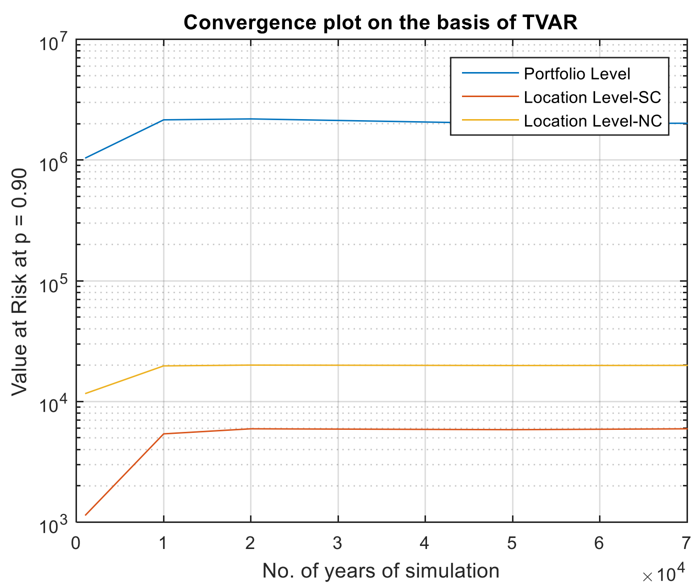
Table-3: Parameters calculated for North California (Location Level)

Simulation No	**VAR(0.9)**	**VAR(0.95)**	**VAR(0.99)**	**AAL**
1000	0	423.529	34442.4936	1276.735
10000	0	4318.862	54837.1108	1936.6
20000	0	4824.136	55255.0595	1938.94
50000	0	4771.997	54825.7578	1834.738
70000	0	4748.855	54980.8891	1830.52



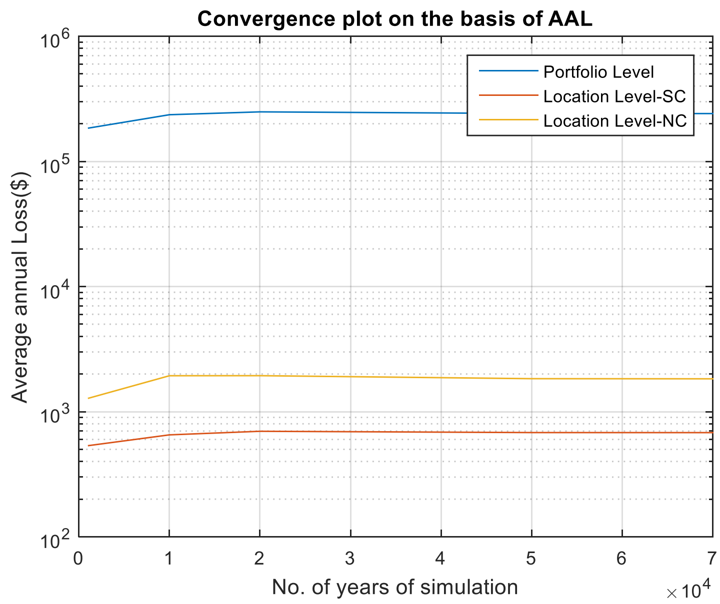
Convergence plot on the basis of VAR
×10^5
3.5
3
2.5
2
1.5
1
0.5
0
Value at Risk at p = 0.99
Portfolio Level
Location Level-SC
Location Level-NC
0
1
2
3
4
5
6
7
No. of years of simulation
×10^4

(a)



Convergence plot on the basis of TVAR
10^7
10^6
10^5
10^4
10^3
Value at Risk at p = 0.90
Portfolio Level
Location Level-SC
Location Level-NC
0
1
2
3
4
5
6
7
No. of years of simulation
×10^4

(b)



(c)

Fig 2: (a) Convergence of VaR at different levels (b) Convergence of TVaR at different levels (c) Convergence of AAL at different levels

It is seen from the above plots that the convergence is pretty strong when a simulation of 50000 year and above is performed. For VaR, the convergence is achieved fastest at the location levels rather than the portfolio level. Similar trends are seen in case of VAR and AAL's. So, it can be concluded that convergence is much faster in location level as compared to the portfolio level for all the three parameters (VaR, TVaR and AAL).

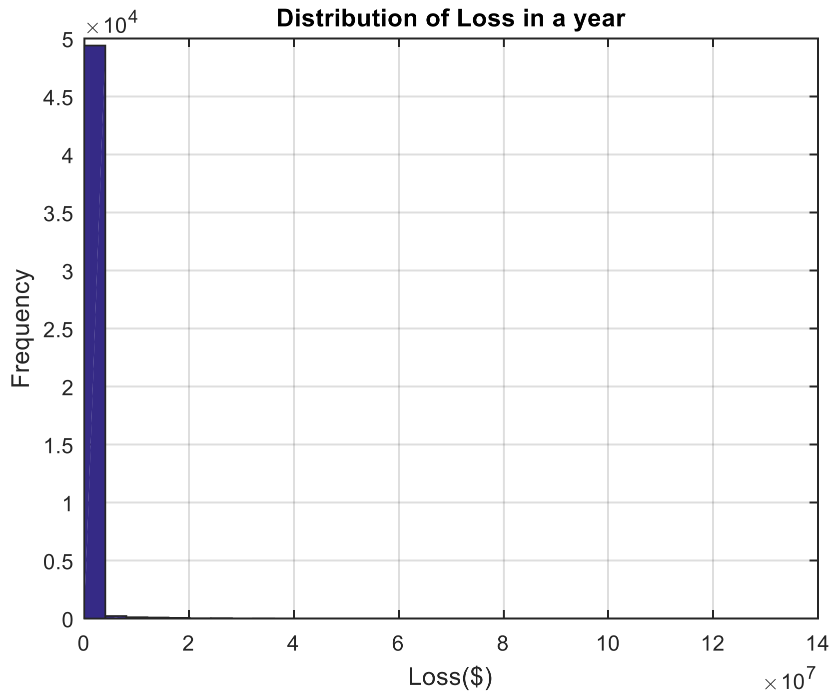
For the purpose of further analysis, a simulation of 50,000 years is performed.

Expected Capital and Premium Calculations

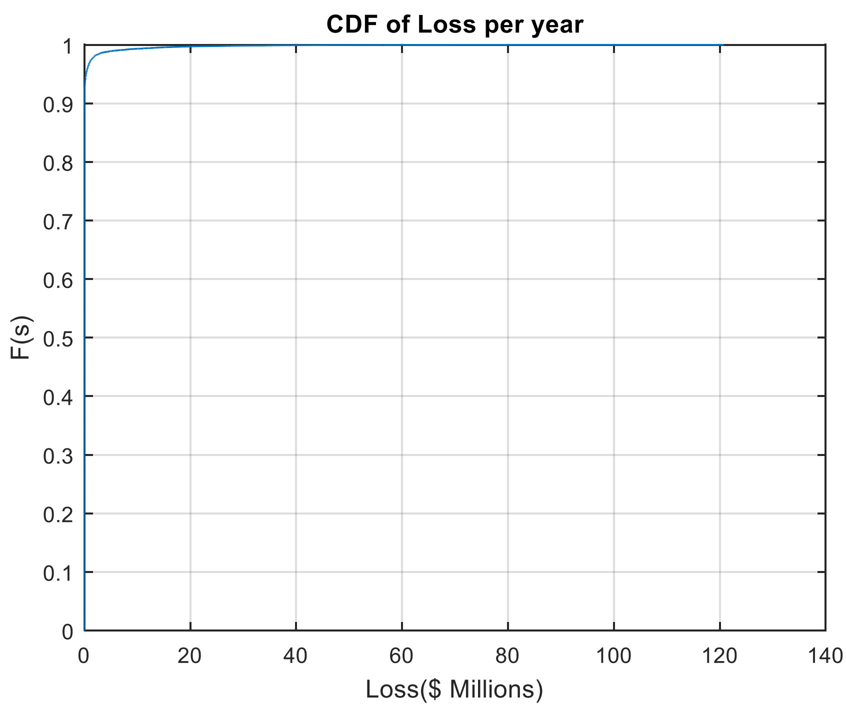
For the purpose of our main analysis the following parameters are considered after multiple runs and are considered to fair the best.

- The replacement value is $1 Million
- Deductible is 0.01 times the replacement value.
- The initial capital is $1.5 Million
- The premium charged per location is $1500.
- Dividend paid by Acme is 2%
- No of simulation years is 50,000.

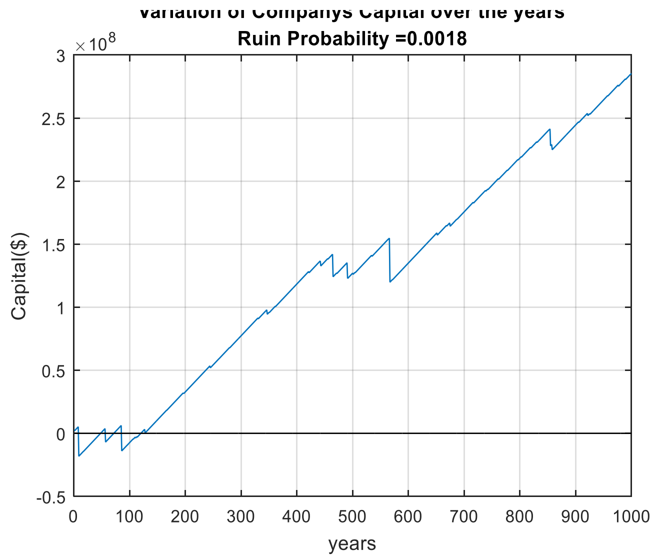
The following plots show the Cumulative distribution function of losses per year, the histogram of losses per year and the Capital verses time plot of Acme respectively. The probability of ruin is calculated as the number of times, the profit sinks below zero to the total number of simulated years.


Distribution of Loss in a year
× 10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Frequency
0
2
4
6
8
10
12
14
Loss($)
× 10^7

(a)


CDF of Loss per year
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
F(s)
0
20
40
60
80
100
120
140
Loss($ Millions)

(b)



(c)

Fig 3: (a) Probability distribution of Loss per year (b) Cumulative distribution of Loss per year (c) Capital vs time graph

From the analysis, it is found out that the probability of ruin is 0.0018 which is quite good for the premium the company is charging.

From the analysis, the following data are worth mentioning.

- Probability of ruin = 0.0018
- Expected loss Portfolio level = $ 241398.9
- Expected loss Location level (NC) = $ 1834.738
- Expected loss Location level (SC) = $ 680.71

Also, given the given the probability of ruin, the expected loss and variation of loss, the value of expected premium for **each location** is calculated using the formulae

$$\pi = E(s) + R(Var[s])$$

$$R = \frac{|log\varepsilon|}{U}$$

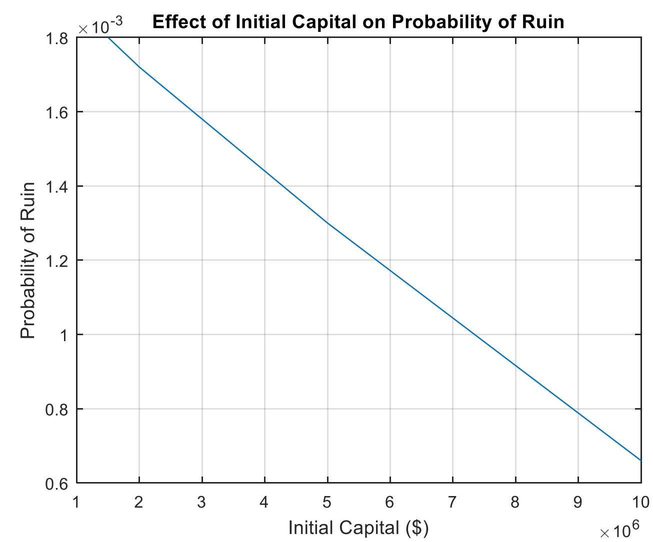
It is seen that mean expected premium calculated for locations in Southern California = $ 731.03
And the mean expected premium for locations in Northern California = $ 1976.9
Evidently it can be deciphered that a flat rate of premium of $1500 is not a very good business plan.

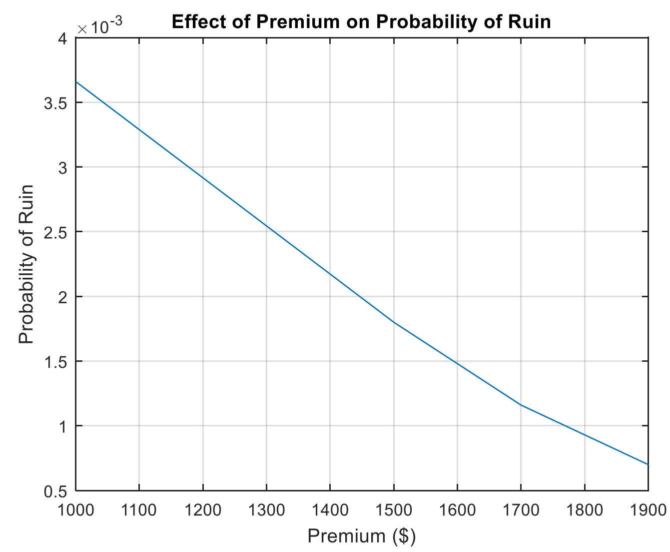
It is seen, that a premium of $ 2000 is charged from every location, the probability of ruin drops down to 0.0004 which is better than a 1 in 2000 year event with an initial capital of $ 1.5 Million.

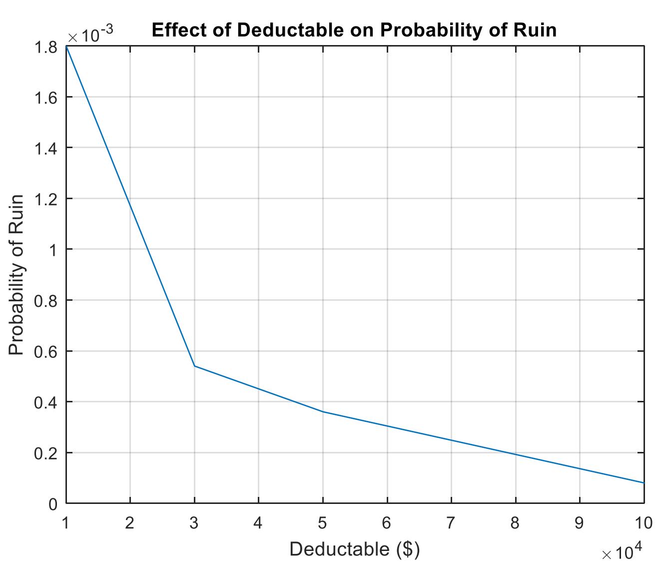
Sensitivity Analysis

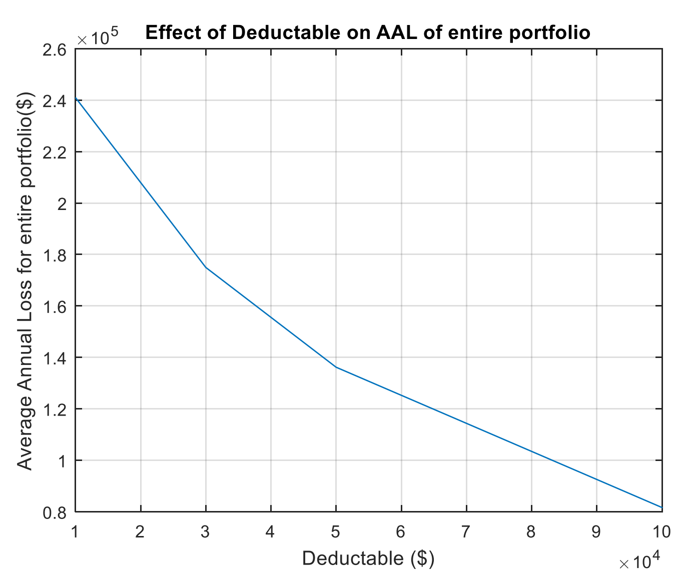
The following parameters are speculated to carry out the sensitivity

- Effect of Initial Capital on Probability of Ruin.
- Effect of Premium on Probability of Ruin
- Effect of Deductible on Probability of Ruin
- Effect of deductible on AAL of portfolio
- Effect of deductible on AAL of NC
- Effect of deductible on AAL of SC
- Effect of simulation years on Probability of ruin
- Effect of Simulation years on number of events









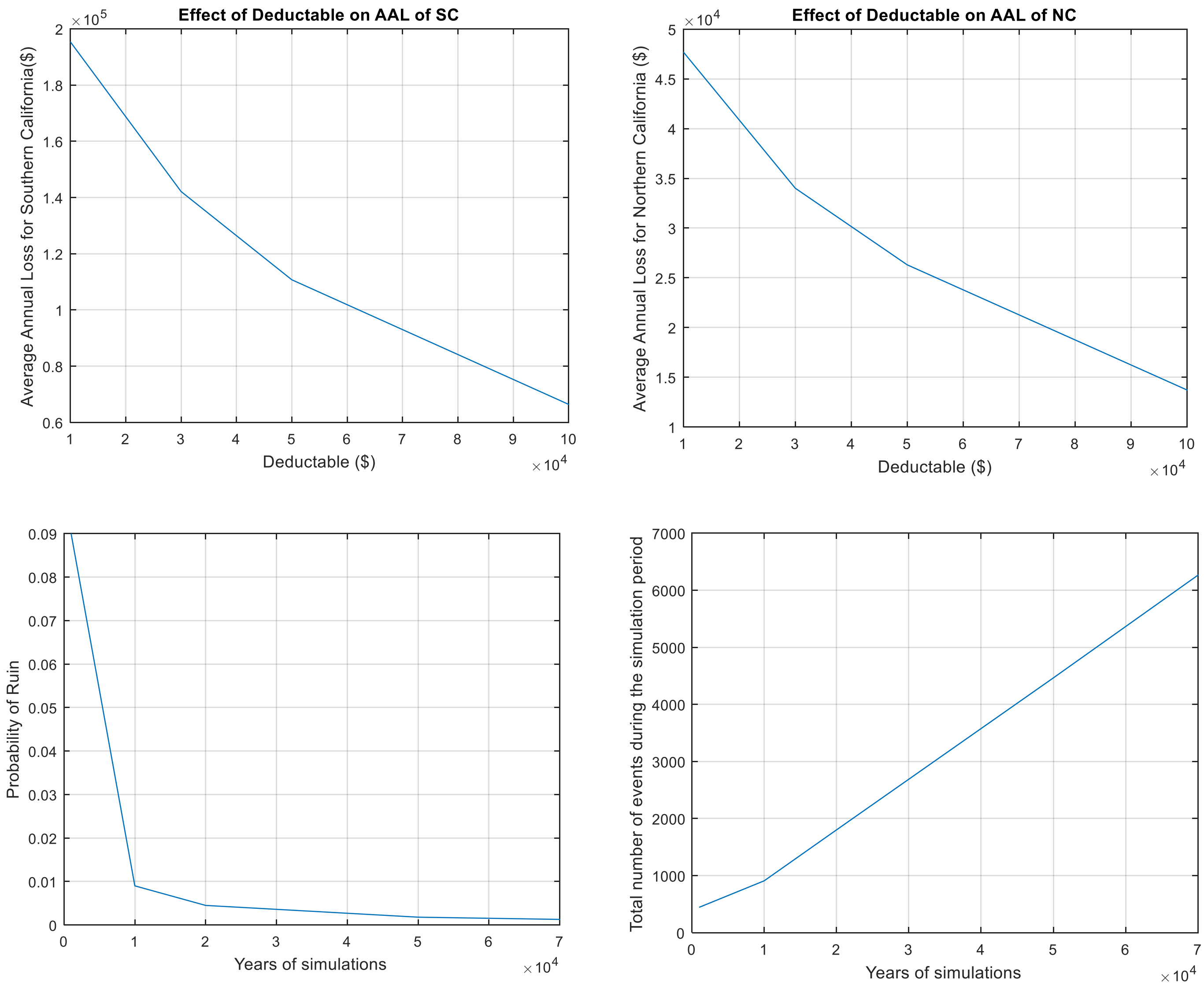


Fig 4: Plots show variation sensitivity analysis conducted through analysis

Result Interpretation

The AAL in Southern California is $680 as compared to Northern California, which is $1835. Hence, rather than choosing a flat rate of $1500 dollars as premium from all locations, we should try to decrease the premium for people in Southern California and increase it for people in Northern California.

An extensive sensitivity study is done on the parameters and the following conclusions are obtained.

- It is seen from the plots that the sensitivity of 'P_ruin' is much less with variations in Initial Capital.

- P_ruin is quite sensitive to variations of Premium.
- P_ruin decreases exponentially with Deductable.
- The AAL's decrease quite steeply with Deductable.
- P_ruin decreases sharply with number of years as expected.
- Number of events rise linearly with number of years of simulation.

Conclusion

- Monte Carlo has reached convergence in 50,000 years of simulation. The convergence is faster in location level as compared to Portfolio level.
- VaR, TVaR and AAL are calculated for both location and portfolio levels to test convergence.
- The AAL in Southern California is $680 as compared to Northern California, which is $1835. Hence, rather than choosing a flat rate of $1500 dollars as premium from all locations, we should try to decrease the premium for people in Southern California and increase it for people in Northern California. This will increase the company's business in Southern California
- Expected premium in Southern California is $ 731 and in Northern California it is $1976.9. Hence, SC locations are over charged and NC locations are under charged. Redistribution of premium in future between these locations would be a good business plan.
- The Capital is enough to obtain a P_ruin of 0.0005 if we increase the premium to $ 2000 instead of $1500 dollars.
- From the expected premium calculations, it is seen that the premium charged is low
- The company should invest more on Southern California locations with lower premium to attract business.

Appendix

The code used for the entire analysis Consisted of a main function(), one point_to_line() function and one shortest_distance() function.

```
%% ------RISK ANALSYS FOR ACME INSURANCE------
%% ------ ABHISHEK SARKAR ------------
%% ------ CEE - 209 Course Project ------------

clear all; close all;
```

```
load fault.mat;
load location.mat;

tic;

% Initial Parameters
loc_no = length(location(:,1));
Mag(:,2) = fault(:,2);
Mag(:,1) = 5;
n_flt = 16;
a = fault(:,1);
Num_div = 10;
yrs = 1000;
R = 6371;

% Calculation of fault length and coordinate tranformation
for i = 1:n_flt
    M(i,:) = linspace(Mag(i,1),Mag(i,2),Num_div);
    Nm(i,:) = 10.^(a(i) - M(i,:));
    N(i,:) = abs(diff(Nm(i,:)));
    fault_pos(i,:) = [R*cos(fault(i,4)*pi/180)*cos(fault(i,3)*pi/180)
R*cos(fault(i,4)*pi/180)*sin(fault(i,3)*pi/180) R*sin(fault(i,4)*pi/180)...
                      R*cos(fault(i,6)*pi/180)*cos(fault(i,5)*pi/180)
R*cos(fault(i,6)*pi/180)*sin(fault(i,5)*pi/180) R*sin(fault(i,6)*pi/180)];


     pos_xy(i,:) =  [fault_pos(i,1) fault_pos(i,2) fault_pos(i,4)
fault_pos(i,5)];
     fault_length(i) = sqrt((pos_xy(i,3)-pos_xy(i,1))^2 + (pos_xy(i,4)-
pos_xy(i,2))^2);
end

% Coordinate tranformation for location points
for i=1:loc_no
     place_loc(i,:) = [R*cos(location(i,1)*pi/180)*cos(location(i,2)*pi/180)
R*cos(location(i,1)*pi/180)*sin(location(i,2)*pi/180)
R*sin(location(i,1)*pi/180)];
     loc_xy(i,:) = [place_loc(i,1) place_loc(i,2)];
end

% Determining location of each point w.r.t faults
for i = 1:loc_no

    pt = loc_xy(i,:);

    for j=1:n_flt
        v1 = [pos_xy(j,1) pos_xy(j,2)];
        v2 = [pos_xy(j,3) pos_xy(j,4)];

        [projv2,projv1]=point_to_line(pt,v1,v2);

        if projv1> fault_length(j) || projv2> fault_length(j)
            fault_pnt_pos(i,j) = 1;                          % 1-for out; 0-
for in
```

```
        else
            fault_pnt_pos(i,j) = 0;
        end

    end
end

%Declaring V, Deductable, Loss Matrix
V = 10^6;
D = 0.01*V;
Loss = zeros(loc_no,yrs);
ro = 0.2;
rng(1);
counter = 1;

% Main Simulation for Loss Calculation
for i=1:yrs
    u = lhsdesign(n_flt,1);
    for j=1:n_flt   % number of faults

        m(j) = u(j)*(Mag(j,2) - Mag(j,1)) + Mag(j,1);
        [c index] = min(abs(M(j,:)-m(j)));

         if m(j) > M(j,index)
             n(i,j) = poissrnd(N(j,index));
         else
             n(i,j) = poissrnd(N(j,index-1));
         end

         SRL(j) = exp(-3.55 + 0.74*m(j));

         for p=1:n(i,j)
             z = norminv(rand());
            for  k = 1:loc_no

                  y = norminv(rand());
                  X = sqrt(ro)*z + sqrt(1-ro)*y;
                  quantile = normcdf(X);

                  pt = loc_xy(k,:);

                 % Shortest distance calculation
                 d(j) =
shortest_distance(pt,fault_length(j),fault_pnt_pos(k,j),pos_xy(j,:),SRL(j));
% a vector   1 TO BE MODIFIED
                 R = sqrt(d(j)^2 + 4.91^2);
                 mu_Sa(j) =0.212 + 0.831*(m(j)-6) - 0.120*(m(j)-6)^2 -
0.867*log(R) - 0.487*log(500/1954);
                 sigma_Sa = 0.538;
                 Sa = logninv(rand(),mu_Sa(j),sigma_Sa);

                 if Sa ~=0
                     mu_L = V*normcdf(log(Sa/3)/1.5);
```

```
                    sigma_L = min(mu_L*(1/(sqrt(mu_L/V)) - 1), 4*mu_L);
                    sigma_lnL = sqrt(log((sigma_L/mu_L)^2 + 1));
                    mu_lnL = log(mu_L) - 0.5*(sigma_lnL)^2;
                else
                    sigma_lnL=0;
                    mu_lnL=0;
                end
                loss = logninv(quantile,mu_lnL,sigma_lnL);
                loss = min(V-D,loss-D);
                loss = max(0,loss-D);
                Loss(k,i) = Loss(k,i) + loss;

            end
            Loss_per_event(counter) = sum(Loss(:,i));
            counter = counter + 1;               % No of events
        end
    end
end


% Calculation of profit of a company.
Prem_per_yr = 1500;

Premium = length(location(:,1))*Prem_per_yr;
PremiumNCal = length(location(287:312,1))*Prem_per_yr;
PremiumSCal = length(location(1:287,1))*Prem_per_yr;

Total_loss = sum(Loss);
NCal_loss = sum(Loss(287:312,:));
SCal_loss = sum(Loss(1:287,:));

Initial_capital =1.5*10^6;
Capital(1) = Initial_capital + Premium - Total_loss(1);

for i=2:yrs
    Capital(i) = Capital(i-1) + Premium - Total_loss(i);
end

% Finding the value at risk at p = 0.75, 0.9, 0.95 and 0.99
total_loss_sort = sort(Total_loss);
SCal_loss_sort = sort(SCal_loss);
NCal_loss_sort = sort(NCal_loss);
F_loss = (1:yrs)/yrs;

% Plotting PDF, CDF and Capital vs time graph
figure;
plot(M',Nm');
grid on;
xlabel('Magnitude (Mw)');
ylabel('Rate of Occurance (M>m)')
title('Decreasing exponential for rate of occurance');

figure;
stairs(total_loss_sort/10^6,F_loss);
grid on;
```

```
xlabel('Loss($ Millions)');
ylabel('F(s)');
title('CDF of Loss per year');

figure;
hist(Total_loss,30);
grid on;
xlabel('Loss($)');
ylabel('Frequency');
title('Distribution of Loss in a year');

figure;
plot(1:yrs,Capital);
hold on
plot(1:yrs,zeros(1,yrs),'k');
grid on
xlabel('years');
ylabel('Capital($)');
ruin_p = length(Capital(Capital<0))/yrs;
title({'Variation of Companys Capital over the years';strcat('Ruin
Probability =',num2str(ruin_p))});



% Value at Risk, TVAR and AAL Calculations;
p = [0.5 0.75 0.9 0.95 0.99];
for i=1:length(p)
    [c index] = min(abs(F_loss-p(i)));
    if p(i) > F_loss(index)
                VAR(i) = total_loss_sort(index);
                VAR_SC(i) = SCal_loss_sort(index);
                VAR_NC(i) = NCal_loss_sort(index);
    else
                VAR(i) = total_loss_sort(index-1);
                VAR_SC(i) = SCal_loss_sort(index-1);
                VAR_NC(i) = NCal_loss_sort(index-1);
    end
end

A = cumsum(VAR(length(p):-1:1));
A_SC = cumsum(VAR_SC(length(p):-1:1));
A_NC = cumsum(VAR_NC(length(p):-1:1));
A = A(length(p):-1:1);
A_SC = A_SC(length(p):-1:1);
A_NC = A_NC(length(p):-1:1);
TVAR = A./(length(p):-1:1);
TVAR_SC = A_SC./(length(p):-1:1);
TVAR_NC = A_NC./(length(p):-1:1);

AAL = mean(Total_loss);
AAL_SC = mean(SCal_loss);
AAL_NC = mean(NCal_loss);

% Calculating Minumum Capital Required and expected Premium.
```

```
dividend = 0.02;
sigma_loss = std(Total_loss);
sigma_loss_SC = std(SCal_loss);
sigma_loss_NC = std(NCal_loss);
U = sigma_loss*sqrt(-log(ruin_p)/(2*dividend));
U_SC = sigma_loss_SC*sqrt(-log(ruin_p)/(2*dividend));
U_NC = sigma_loss_NC*sqrt(-log(ruin_p)/(2*dividend));
E_prem = (AAL + sigma_loss*sqrt(-2*dividend*log(ruin_p)))/312;
E_prem_SC = (AAL_SC + sigma_loss_SC*sqrt(-2*dividend*log(ruin_p)))/287;
E_prem_NC = (AAL_NC + sigma_loss_NC*sqrt(-2*dividend*log(ruin_p)))/26;

% Calculation of Expected Premium per location
for i = 1: loc_no

    L = Loss(i,:);
    mu_L_loc = mean(L);
    sigma_L_loc = std(L);
    E_prem_loc(i) =  mu_L_loc + R*(sigma_L_loc^2);
end

toc;
```

Code for shortest Distance:

```
function r = shortest_distance(pt,fault_length,fault_pnt_pos,pos_xy,SRL)

     v1 = [pos_xy(1) pos_xy(2)];
     v2 = [pos_xy(3) pos_xy(4)];
     a = v1 - v2;
     b = pt - v2;
     c = pt - v1;
     X1 = [pt;v1];
     X2 = [pt;v2];
     r0 = norm(cross([a,0],[b,0]))/norm(a);
     r1 = min([pdist(X1,'euclidean') pdist(X2,'euclidean')]);
     r2 = max([pdist(X1,'euclidean') pdist(X2,'euclidean')]);

 if fault_pnt_pos == 1

     d = sqrt(r1^2 - r0^2);
     r = sqrt((rand()*(fault_length - SRL) + d)^2 + r0^2);

 else

     L1 = sqrt(r1^2 - r0^2);
     L2 = sqrt(r2^2 - r0^2);

     if SRL > L1

         x = rand()*(fault_length - SRL);

         if x <= L1
             r = r0;
```

```
        else
            r = sqrt((x - L1)^2 + r0^2);
        end
    else

        x = rand()*(fault_length - SRL);

        if x < (L1 - SRL)

            r = sqrt((L1 - x - SRL)^2 + (r0)^2);

        elseif x>= (L1 - SRL) && x<= L1
            r = r0;
        else
            r = sqrt((x - L1)^2 + (r0)^2);
        end

    end

end
```
