Population ~~10~~ $\frac{4-10}{}$ } Dn

Sample size = 2-3

Dn-1 } estimate

Mean

- Group.

Closure, Associativity, Identity, Inverse

* if also commutativity — Abelian.

* Mean.
  ↳ Comparision b/w two data set
  ↳ does not give idea about dist of data.

# 43	56	78	82	99	# 58	64	73	81	85
	71.6					72.2			

* Median
  ↳ Half of seq is less than Med.
  ↳ Half of seq is more than Med.

* Mode -
  ↳ Most freq. occuring.

* Variance. — (2nd order statistics)

$$X = x_1, x_2, \ldots x_n$$

$$\bar{X} = \frac{\sum_{k=1}^{n} x_k}{n}$$

dispersion ← $\sigma^2 = \frac{\sum_{k=1}^{n} (x_k - \bar{x})^2}{n-1}$ ↳ for unbaised estimator

$$\frac{\sum_{k=1}^{n} |x_k - \mu|}{n}$$

manhattan dist

* Std. dev - $\sqrt{\sigma^2}$.

"Correlated - Data"

$$\frac{\sum (x-\bar{x})(y-\bar{y})}{n}$$

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- Coefficient of co-relation (ρ) = $\dfrac{\text{Covariance}(x,y)}{\sigma_x \sigma_y}$

$$0 \le |\rho| \le 1$$

0.7 - 1 - Strong

0.5 - 7 - Not very strong

0.3 - 0.5 - Weak

0 - 0.3 - No Relation

let

$V_1 = (x_1, x_2, \ldots, x_n)$

$V_2 = (y_1, y_2, \ldots, y_n)$

$$\cos\theta = \frac{x_1 y_1 + x_2 y_2 + \ldots + x_n y_n}{|V_1||V_2|} = \rho$$

☆ we have to centred the seq. $(-\mu)$

$$\rho = \frac{\sum_{k=1}^{N} x_k y_k - N\bar{x}\bar{y}}{\sqrt{\sum_{k=1}^{N} x_k^2 - N(\bar{x}^2)}\sqrt{\sum_{k=1}^{N} y_k^2 - N(\bar{y}^2)}}$$

1 2 3

quartile quartile

$$\frac{n-1}{1000000} \times 100 = 99.99$$

999900

1 2 3

4 5 6

99 000

$$x_1 - \frac{(x_1 + x_2 + \ldots + x_n)}{n} + x_2 - \frac{(x_1 + x_2 + \ldots + x_n)}{n} + x_3 + - \frac{\bar{x}}{n}$$

$$\left(x_1 - \frac{\bar{x}}{n} + x_2 - \frac{\bar{x}}{n} + \ldots\right.$$

$x_1 x_2 x_3$

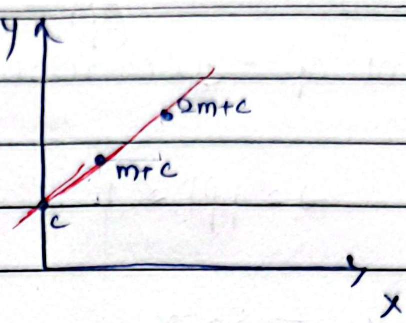
$$x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n - n\bar{x}\bar{y}$$

$n\bar{x} - \bar{x}$

$$\frac{(n-1)\bar{x}(n-1)\bar{y}}{n}$$

Regression

$y = mx + c$

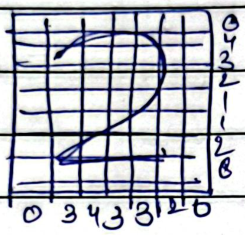


n-D plane

$w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n = 0$ Clustering

"bias" ↳ Hyperplane

similar object form cluster.



$w_0 + [w_1, w_2, w_3, \dots, w_n] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$

$w_0 + W^T x = 0$

deviation of point ($\perp$ distance)

error function / cost function. Sum of squared error
SSE
↓
modern parameter can analytically derived.

Y is estimate T is target.

$L(T, x) = \frac{1}{n} \sum_{k=1}^{n} (T_k - (w_1 x_k + w_0))^2$

$f(x, w_0, w_1) = w_0 + w_1 x$

$L = \frac{1}{N} \sum_{n=1}^{N} f_k(T_k, f(x_k$

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$$\mathcal{L} = \frac{1}{N}\sum_{k=1}^{N}\left(w_1^2 x_k^2 + 2w_1 x_k(w_0 - T_k) + w_0^2 - 2w_0 T_k + T_k^2\right)$$

$$\frac{\partial L}{\partial w_1} = \frac{-2x_k}{n}\sum_{k=1}^{n}\left(T_k - (w_1 x_k + w_0)\right)$$

$$\boxed{\cancel{\frac{\partial L}{\partial w_1} = \frac{2w_1}{n}\sum_{k=1}^{n}\left(T_k - (w_1 x_k + w_0)\right)}}$$

$$\frac{\partial L}{\partial w_0} = \frac{2}{n}\sum_{k=1}^{n}($$

$$\frac{\partial L}{\partial w_1} = \frac{2}{N}\sum_{k=1}^{N}\left(-x_k T_k + w_1 x_k^2 + w_0 x_k\right)$$

$$= \boxed{\frac{2w_1}{N}\sum_{k=1}^{N} x_k^2 + \frac{2}{N}\sum_{k=1}^{N} x_k(w_0 - T_k)} \longrightarrow (1)$$

$$\frac{\partial L}{\partial w_0} = \frac{-2}{N}\sum_{k=1}^{N}\left(T_k - w_1 x_k - w_0\right)$$

$$= \boxed{\frac{2w_1}{N}\sum_{k=1}^{N} x_k + \frac{2}{N}\sum_{k=1}^{N} w_0 - T_k} \longrightarrow (2)$$

let (1) $= 0$ & (2) $= 0$

$$\boxed{w_1 = \frac{\overline{(x \cdot t)} - \bar{x}\bar{t}}{\overline{(x^2)} - (\bar{x})^2}}$$

$$\frac{w_1}{N}\sum_{k=1}^{N} x_k + \frac{1}{N}\sum_{k=1}^{N} w_0 - T_k = 0$$

$$w_1\bar{x} + \frac{1}{N}\sum_{k=1}^{N} w_0 - \frac{1}{N}\sum_{k=1}^{N} T_k = 0$$

$$w_1\bar{x} + w_0 - \bar{T} = 0$$

$$\boxed{w_0 = \bar{T} - w_1\bar{x}}$$

Vector
↳ By array OR linked list.

Matrix

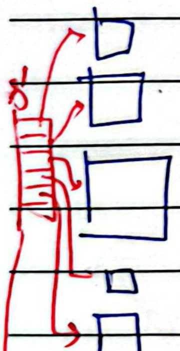
$M = m_{ij} \ \forall \ 0 \le i, j < n$ OR

$M = \{ m_{ij} \mid \forall \ 0 \le i < m, \ 0 \le j < n \}$

- Array -
  Mapping of element to indicies.

* Homogeneous.
↳ Contigous → Space
Continuous → Time

↳ Random Access. $A + (n-1)s$
↳ $A + (n-1)2^p$
OR $A + [(n-1) << p]$



Sequntial Access → access time depends upon space of all

$A[k]$

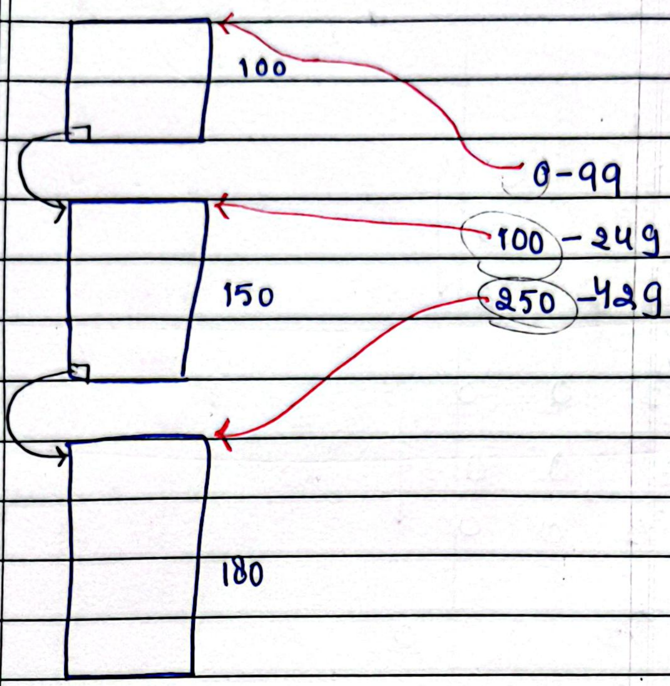
↳ $A + (n-1)s'$ → still fast access.

↱ k+1th element

$*[\beta + ks']$

↳ size of pointer array

"Index"



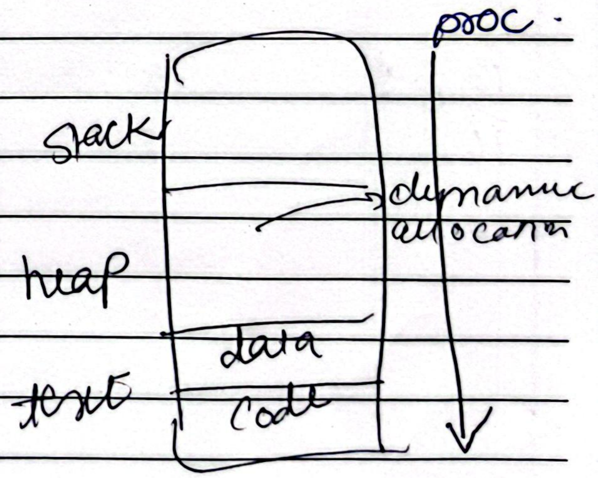
- Two dimensional -

Row major.	Column major
a_{00}	a_{00}
a_{01}	a_{10}
⋮	⋮
$a_{0\,n-1}$	$a_{m-1\,0}$
a_{10}	a_{01}
a_{11}	a_{11}
⋮	⋮
⋮	⋮
$a_{m-1\,0}$	$a_{0\,n-1}$
$a_{m-1\,1}$	$a_{1\,n-1}$
⋮	⋮
$a_{m-1\,n-1}$	$a_{m-1\,n-1}$

* Parallel Programming
* Data is Disjoint
* Parallel reading & writting.

↓

Zig-Zag pattern



④

$$M = \begin{bmatrix} 2 & 2 & 3 \\ 4 & 9 & 0 \\ 3 & 6 & 1 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 1 & 3/2 \\ 4 & 9 & 0 \\ 3 & 6 & 1 \end{bmatrix} \quad \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 1 & 3/2 \\ 0 & 5 & -6 \\ 3 & 6 & 1 \end{bmatrix} \quad \begin{bmatrix} 1/2 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\hookrightarrow$

In Gauss Jordan method
If diagonal entry $\rightarrow$ small
$\Downarrow$
~ 0
$\hookrightarrow$ lead errors.

★ Numerical Recipies in C++

- LUP decomposition-

simutaneously eq^n $\begin{Bmatrix} 2x + 3y = 7 \\ 3x - 2y = 4 \end{Bmatrix}$

$$x - 2y + z = 5$$
$$5x + 3y + z = 4$$
$$2x + 7y + 2z = -1$$

$m \rightarrow$ no. of eq^n $\quad n \rightarrow$ no. of unk.

if $m = n$ then exist a soln.
$\hookrightarrow$ Independent

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if $m < n$ - underdetermined
↳ infinite sol^n or no sol^n.

↳ if $m > n$ - Overdetermined.
errors in accuring the data.

- Lower triangle.
  ↳ ~~Epp~~ forward substitution

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

so,

$$a_{11} x_1 = b_1$$

$$a_{21} x_1 + a_{22} x_2 = b_2 \qquad \to n^2.$$

$-6+6$ n multiplication and $(n-1)$ addition.

$x_1 = 2$

$x_2 = 3$

$x_3 = 1$

$x_4 = -1$

$4+3-2$

- Upper triangular.
  ↳ Backward Substitution.
  ↳ n^2.

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$$\boxed{\text{" } M = \underbrace{L}_{\text{lower}} \times \overbrace{U}^{\text{upper}} \text{ "}}$$

$LUx = B$

let,

$y = Ux \rightarrow$ $\boxed{\cancel{\text{forward}}}$ Backward Substitution

Now

$Ly = B \rightarrow$ $\boxed{\cancel{\text{Backward}}}$ Forward Substitution.

- ~~let~~ Matrix Inverse

Q is inverse of M.

so,

$Q = M^{-1}$

or. $MQ = I$.

$$\boxed{MQ_k = I_k}$$

so,

$LU\,Q_{kth} = I_{kth}$

↳ column.

★ $M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$ & $Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$

so, $MQ = I$

so,

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{21} \\ q_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

n^3 ↳ Inverse

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & m_{33} \end{bmatrix} \begin{bmatrix} q_{12} \\ q_{22} \\ q_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

* LU Decomposition.

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$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{12} & l_{22} & 0 \\ l_{13} & l_{23} & l_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$m_{11} = l_{11} u_{11}$ $\quad m_{12} = l_{11} u_{12}$ $\quad m_{13} = l_{11} u_{13}$

$m_{21} = l_{21} u_{11}$ $\quad m_{22} = l_{21} u_{12} + l_{22} u_{22}$ $\quad m_{23} = l_{21} u_{13} + l_{22} u_{23}$

$m_{31} = l_{31} u_{11}$ $\quad m_{32} = l_{31} u_{12} + l_{32} u_{22}$

let $l_{11} = 1$

$\hookrightarrow m_{pq} = l_{p,q-1} \times u_{p-1,q}$

Copy the row to U

Scale the row $\rightarrow$ ~~d~~diagonal (m_{ii}) col

Copy it in the L

$\div 2$

$\star$ $\begin{pmatrix} 2 & 1 & 3 \\ 4 & 5 & 10 \\ 6 & 12 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & -7 \end{pmatrix}$ Copy the row

$3 \quad 4 \quad 9 \quad 5 \quad -7$

Copy Row to U

scale the column

Copy column to L

Now, $m_{pq} = l_{pk} u_{kq}$

→ Divide the column (Not diagonal)

Now subtract (or ~~set~~ set all remaning)

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- Bubble Sort -

```
for h = 1 to n :
    cnt = 0
    for i = 0 to n-2 :
        if (A[i] > A[i+1]) :
            swap( A[i], A[i+1])
            c++
        end if
    end for
    if (cnt==0):
        exit()
```

(in - situ)
(stable)

- Insertion sort. (Insert it correct place)
  $O(n^2)$ in-situ stable

- Radix Sort

unstable sort

				Digit →	Base Table	
3	8	1		0		0
2	3	4	81	1	11	~~1~~ ~~2~~ 3
0	3	9	21	2		
4	2	1	23	3	1	~~3~~ 4
0	0	4	34	4	11	~~4~~ ~~5~~ 6
0	2	3	04	5		
			39	6		
				7		
				8		
				9	1	~~6~~ 7

Max digit
↳ k

so, $O(kn)$
not in situ
unstable

20×20

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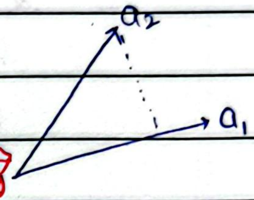
$\langle \vec{a}\ \vec{b} \rangle \rightarrow$ inner product (scalar product)

if $\langle \vec{a}\ \vec{b} \rangle = 0$
↳ Orthogonal

and if $|\vec{a}| = |\vec{b}| = 1$
↳ Orthonormal.

- Gram Schmidt -

let $\vec{u_1}, \vec{u_2}, \dots, \vec{u_n}$ are orthogonal.



$\vec{u_1} = \vec{a_1}$

projection vector $\rightarrow$ $\boxed{\dfrac{\langle \vec{a_2}\ \vec{a_1} \rangle}{\langle \vec{a_1}\ \vec{a_1} \rangle}\ \vec{a_1}}$ $\quad \dfrac{(\vec{a} \cdot \vec{b})\,\hat{b}}{|\vec{b}|^2}$

$\vec{a_1} = 4\vec{a_x} + 3\vec{a_y}$

$\vec{a_2} = 7\vec{a_x} + 5\vec{a_y}$

$$\left(\frac{28 + 15}{25}\right)(4\hat{a}_x + 3\hat{a}_y) = \frac{43}{25}(4\hat{a}_x + 3\hat{a}_y) = a_{2_{\parallel}}$$

$$a_{2\perp} = \left(7 - \frac{43 \times 4}{25}\right)\hat{a}_x + \left(5 - \frac{43 \times 3}{25}\right)a_y$$

$$= 3\hat{a}_x + -4\hat{a}_y$$

$\vec{u_1} = \vec{a_1}$

$\vec{u_2} = \vec{a_2}$ - projection ($\vec{a_2}$ on $\vec{u_1}$)

$\vec{u_3} = \vec{a_3}$ - projection ($\vec{a_3}$ on $\vec{u_1}$) - projection ($\vec{a_3}$ on $\vec{u_2}$)

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$$A\,[x_1 \quad x_2 \quad x_3 \;\ldots\; x_n] = I_n$$

$\Big\downarrow$ m swaps

A'

so, $A' = LU$

$LU x_k = I_k$

$x_k = \alpha_k'$

$A'\,[\alpha_1' \quad \alpha_2' \quad \alpha_3' \;\ldots\; \alpha_n'] = I$

↓

Inverse of A'

↓

Reverse order column swap.

↳ Inverse of A.

$Ax = b$ ↱ swap on LHS & RHS.

Write code

↳

- **Basis**

projection vector $= \dfrac{\langle \vec{a}\,\vec{b} \rangle}{\langle \vec{b}.\vec{b} \rangle}\,\vec{b}$

Given - $\vec{v_1}$ $\vec{v_2}$ $\vec{v_3}$ .... $\vec{v_n}$

let $\vec{u_1} = \vec{u_1}$

So, $\vec{u_2} = \vec{v_2}$ - projection ~~of~~ ($\vec{v_2}$ on $\vec{u_1}$)

$\vec{u_3} = \vec{v_3}$ - projection ($\vec{v_3}$ on $\vec{u_1}$) - projection ($\vec{v_3}$ on $\vec{u_2}$)

modify, $\vec{u_k} = \vec{v_k}$

↳ $u_{k+1} = v_{k+1}$ - projection (v_{k+1} on u_k)

Matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Matrix × v

①. $$A \times V = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix} = \begin{bmatrix} 3\times1 \\ 2\times1 \end{bmatrix} + \begin{bmatrix} 4\times1 \\ 4\times-3 \end{bmatrix}$$

$$= 1\times\begin{bmatrix} 3 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Basis vectors, $\hat{i}$, $\hat{j}$.

↳ matrix linear combination of vector.

wettage. correspon to vector.

②. $\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ → ~~second~~ first column

$\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ → second column.

3Blue ~~one~~ brown

Veritasium

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any vector, $\rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+0 \\ 0+y \end{bmatrix}$.

Matrix multiplying with a vector is producing another vector, (Rotation, scaling).

↳ Depend on Matrix

$$\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x+y \\ 2x-3y \end{bmatrix}$$ → vector having two component.

$$\text{or} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

New vector can defining by basis vector by multiplying the vector.

- $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

ogen vectors are the basis vectors.

(-20, 20)

→ Matrix

→ How all points shifted

↳ Lines.

↳

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- Sparse Matrix -

↳ Matrix with most elements are zero.

$$\begin{array}{c|ccccc|} & 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 & 5 & 0 \\ 2 & 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 2 & 0 & 0 \\ 4 & 0 & 0 & 1 & 4 & 0 \end{array}$$

→ 2D matrix

↳ 3 column. Row idx Col idx val.

(i) $n+1 \times 3$ representation
↳ non zero.

rows r ← 5 5 11 → no. of non zero n

cols c

5	5	11
0	0	1
0	1	2
1	1	3
1	3	5
2	0	2
2	4	1
3	0	1
3	1	1
3	2	2
4	2	1
4	3	4

r_1, c_1 ; r_2, c_2.

```
if (r1 < r2)
    -1
else if (r1 == r2)
    if (c1 < c2)
        -1
    else if (c1 == c2)
        0
    else
        1
else
    1
```

$\langle m_1, n_1, t_1 \rangle$, $\langle m_2, n_2, t_2 \rangle$

if $m_1 != m_2$ or $n_1 != n_2$
-1
else

$m \times n = m \times n \,(n \times n) \quad Q^T A = R$ $\quad A = QR \quad QQ^T = I$

$m \times n \qquad (m \times n)(n \times n)$

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QR decomposition -

(A) $A = \begin{bmatrix} | & | & | & & | \\ c_1 & c_2 & c_3 & \cdots & c_n \\ | & | & | & & | \end{bmatrix}$ let Rank is n.

$(n \times n)$

1. $\vec{u_1} = \vec{c_1}$ and $\hat{e}_1 = \dfrac{\vec{u_1}}{|u_1|}$

2. $\vec{u_2} = \vec{c_2} - \left(\dfrac{\vec{c_2} \cdot \vec{u_1}}{\vec{u_1} \cdot \vec{u_1}}\right)\vec{u_1}$ or $\vec{c_2} = \vec{u_2} + \left(\dfrac{\vec{c_2} \cdot \vec{u_1}}{\vec{u_1} \cdot \vec{u_1}}\right)\vec{u_1}$

3. $\vec{u_3} = \vec{c_3} - \left(\dfrac{c_3 \cdot u_1}{u_1 \cdot u_1}\right)u_1 - \left(\dfrac{c_3 \cdot u_2}{u_2 \cdot u_2}\right)u_2$

$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$

$\frac{1}{4} + \frac{1}{4} + \frac{36}{4}$

$\frac{38}{4} \quad \sqrt{\frac{19}{2}}$

$\vec{u_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{e_1} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$

$\vec{u_2} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \dfrac{2+1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1/2 \\ 3 \end{pmatrix} \quad \hat{e}_2 = \begin{pmatrix} \frac{1}{\sqrt{38}} \\ \frac{-1}{\sqrt{38}} \\ \frac{3\sqrt{2}}{\sqrt{19}} \end{pmatrix}$

$\frac{1}{2} + 1.2 = 1 + 1.2$

$\vec{u_3} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} - \dfrac{3}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \left(\cancel{\dfrac{-1}{\sqrt{38}} + \dfrac{24}{\sqrt{38}}}\right)\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \times \dfrac{23}{2}\begin{pmatrix} 1/2 \\ -1/2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 - 3/2 + 23/4 \\ 2 - 3/2 - 23/4 \\ 4 - 0 + 69/2 \end{pmatrix} = \begin{matrix} 5.25 \\ -5.25 \end{matrix}$

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$u_3 = (1.105, -0.105, 0.368)$

$\cancel{e_3 = (0.808, -0.0767, 0.798)}$

$e_3 = (0.688, -0.688, 0.23)$

$$Q = \begin{bmatrix} 0.7 & 0.162 & 0.69 \\ 0.7 & -0.162 & -0.69 \\ 0 & 0.973 & 0.23 \end{bmatrix}$$

$0.973 \times 3 = 2.919$ $0.162 + 2.919 = 3.081$

$0.973 \times 4 = 3.892$ $0.92 - 0.69 = 0.23$ $3.892 - 0.162 = 3.730$

$$R = Q^T A = \begin{bmatrix} 0.7 & 0.7 & 0 \\ 0.162 & -0.162 & 0.973 \\ 0.69 & -0.69 & 0.23 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$-0.69 + 4 \times 0.23$

$$= \begin{bmatrix} 1.4 & 2.1 & 2.1 \\ 0 & 3.081 & 3.73 \\ 0 & 0 & 0.23 \end{bmatrix}$$

$$Qx = \lambda x \rightarrow (1)$$

$$\Rightarrow (Qx)^T = (\lambda x)^T$$

$$x^T Q^T = \lambda x^T \rightarrow (2)$$

(1) × (2).

$$x^T Q^T Q x = \lambda x^T \lambda x$$

$$x^T x = \lambda^2 x^T x$$

hence,

$$\boxed{\lambda = 1}$$

CR decomp'

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initially -

$$A^0 = A = Q^0 R^0$$

Now again a matrix:

$$A' = R^0 Q^0$$
$$\hookrightarrow Q' R'$$

Similarly,

$$A^2 = R' Q'$$
$$\hookrightarrow Q^2 R^2$$

~~New~~

Q = Square orthogonal matrix

c_i, c_j are orthonormal

$$|c_i| = |c_j| = 1$$

So,

$$Q = [c_1 \quad c_2 \quad c_3 \;\cdots\; c_n]$$

$$Q^T = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}$$

Now,

$\cancel{QQ^T}$ ~~or~~ $Q^T Q = \begin{bmatrix} c_1c_1 & c_1c_2 & \cdots & c_1c_n \\ c_2c_1 & c_2c_2 & \cdots & c_2c_n \\ \vdots & & & \vdots \\ c_nc_1 & c_nc_2 & \cdots & c_nc_n \end{bmatrix} = I_n$

$$M = Q_1 Q_2$$

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$Q^T Q = I \Rightarrow \boxed{Q^T = Q^{-1}}$

$\cancel{Q Q^T Q = Q I}$

$Q Q^T = Q Q^{-1}$

$\cancel{(Q^T Q)^T = I^T}$

$\cancel{Q^T (Q^T)^T = I^T = I}$

$Q Q^T = I = Q^T Q$

Let Q have some eigen value λ.

So, eigen vector

$$\boxed{Qx = \lambda x} \longrightarrow (i)$$

taking transpose

$$x^T Q^T = \lambda x^T \longrightarrow (ii)$$

(ii) $\times$ (i)

$\cancel{Q x\, x^T Q I =}$

$$x^T Q^T Q x = \lambda^2 x^T x$$

$$x^T x = \lambda^2 x^T x$$

$$(\lambda^2 - 1) x^T x = 0$$

$$\Rightarrow (\lambda^2 - 1) \|x\|^2 = 0$$

$$\|x^2\| \neq 0.$$

Therefore, $\boxed{\lambda = \pm 1}$

<u>Consider</u>

$$M = Q_1 Q_2$$

$$M^T = Q_2^T Q_1^T$$

$$M^T M = Q_2^T Q_1^T Q_1 Q_2$$

$$M^T M = Q_2^T Q_2$$

$M^T M = I$ Similarly $M M^T = I$.

so, $\boxed{M^T = M^{-1}}$

M is orthogonal.

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- Let A is a Square matrix having eigen value λ,

$\Rightarrow \quad Ax = \lambda x$

$\Rightarrow \quad A(Ax) = A(\lambda x) \quad$ {multiply by A}

$A(Ax) = \lambda(Ax)$

$A(Ax) = \lambda^2 x$

$A^2(x) = \lambda^2 x$

So,

$A^n x = \lambda^n x$

Proof.

Let $A^n x = \lambda^n x$.

Now, multiply by A

$A(A^n x) = \lambda^n Ax$

$\boxed{A^{n+1} x = \lambda^{n+1} x}$

Now, $\quad Ax = \lambda x$

$\times A^{-1} \Rightarrow \quad A^{-1}Ax = \lambda(A^{-1}x)$

$\Rightarrow \quad x = \lambda(A^{-1}x)$

$\Rightarrow \quad \lambda^{-1}x = A^{-1}x$.

or. $\quad A^{-1}x = \lambda^{-1}x$.

- Symmetric matrix $\boxed{S^T = S}$

$\lambda_1 \neq \lambda_2$

$Sx = \lambda_1 x \quad$ and $\quad Sy = \lambda_2 y$

$y^T S^T = \lambda_2 y^T$

$\boxed{y^T S = \lambda_2 y^T}$

$\Rightarrow \quad y^T S x = \lambda_2 y^T x$

$y^T \lambda_1 x = \lambda_2 y^T x$

$(\lambda_1 - \lambda_2) y^T x = 0$

$\Rightarrow \boxed{y^T x = 0}$

eigen vector of S are orthogonal

$n \times n \quad n \times 1 \qquad 1 \times n \qquad n \times n$

Consider, $S\vec{x} = \lambda\vec{x}$ ~~and~~

let $\vec{y} = k\vec{x}$ multiply by k

$S(k\vec{x}) = \lambda(k\vec{x})$

$S\vec{y} = \lambda\vec{y}$

So, we can scale the vector,

Now, our aim to get unit vector,

$S\vec{q_1} = \lambda\vec{q_1}$ and $S\vec{q_2} = \lambda\vec{q_2}$ so on,

Consider.

$$S[\vec{q_1}\ \vec{q_2}\ \vec{q_3} \cdots \vec{q_n}] = [\vec{q_1}\ \vec{q_2} \cdots \vec{q_n}] \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \cdots & 0 \\ \vdots & & \cdots & & \\ 0 & \cdots & & \cdots & \lambda_n \end{bmatrix}$$

$$\boxed{SQ = Q\Lambda}$$

$SQQ^{-1} = Q\Lambda Q^{-1}$

$\Rightarrow \quad S = Q\Lambda Q^{-1}$

Now,

- $Ax = \lambda x$

$$A[x_1\ x_2 \cdots x_n] = [x_1\ x_2 \cdots x_n] \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \lambda_2 & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix}$$

$AX = X\Lambda$

$$\boxed{A = X\Lambda X^{-1}}$$

$PA = BP$ $\quad$ ~~$PA = BP$~~.

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- Consider,

L or U

eigen values are the diagonal Elements.

- Similar matrix –

let A is matrix and B is invertible matrix.

$Ax = \lambda x$

↳ eigen value for A.

$PA = BP \Rightarrow A = P^{-1}BP$.

if B is similar to A.

$$\boxed{PA = BP}$$

~~$By = \lambda y$~~

$P^{-1}BPx = \lambda x$

$BPx = \lambda Px$

$\boxed{B(y) = \lambda y}$

↳ hence proved.

Now, QR algo.

$A = Q_0 R_0$

$A' = R_0 Q_0$.

$Ax = \lambda x$

$Q_0 R_0 x = \lambda x$

$x^T R_0^T Q_0^T = \lambda x^T$

~~$A = P^{-1}$~~

$B = PAP^{-1}$

$A = P^{-1}BP$.

$A = P^{-1}A'P$

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$A = QR \rightarrow A = Q_0 R_0$

$RA = RQR \qquad A_1 = R_0 Q_0$

$RAR^{-1} = RQ \qquad A_1 = Q_1 R_1$

$A' = RAR^{-1} \qquad A_2 = R_1 Q_1$

$A_2 = Q_2 R_2$

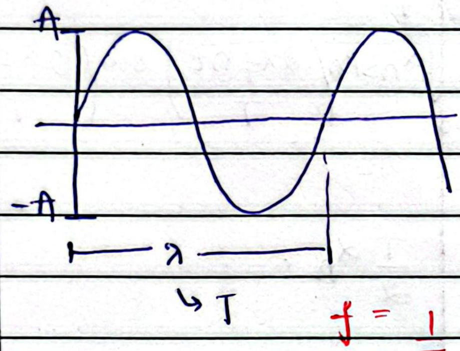
$\vdots$

$A_n = Q_n R_n \approx$ upper Triangular Matrix

- Fourier's Transform

DFT, FFT

Sinusoidal frequency - pure.



Amplitude is A.

$f = \frac{1}{T}, \quad w = 2\pi f = \frac{2\pi}{T}$

$$\boxed{f = A \sin(wt + \phi)}$$

$$s(t) = \sum_{n=0}^{\infty} a_n \sin\left(\frac{2\pi n t}{T} + \phi_n\right)$$

$$s(t) = a_0' + a_1 \sin\left(\frac{2\pi t}{T} + \phi_1\right) + a_2 \sin\left(\frac{4\pi t}{T} + \phi_2\right) + \dots$$

fourier constants − a_n.
coefficients

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Now,

$$\int_0^T s(t)\,dt = \int_0^T a_0'\,dt$$

So,

$$\boxed{a_0 = \frac{1}{T}\int_0^T s(t)\,dt}$$

Now,

$$a_1 \sin\left(\frac{2\pi f t}{T} + \phi_1\right) = \alpha_1 \sin\left(\frac{2\pi f t}{T}\right) + \beta_1 \cos\left(\frac{2\pi f t}{T}\right)$$

So,

$$s(t) = a_0 + \sum_{n=1}^{\infty} \alpha_n \sin\left(\frac{2\pi f n t}{T}\right) + \beta_n \cos\left(\frac{2\pi f n t}{T}\right)$$

$$\int_0^T s(t) \sin\left(\frac{2\pi f n t}{T}\right) = 0 + \int_0^T \alpha_n \sin\left(\frac{2\pi f n t}{T}\right) \sin\left(\frac{2\pi f n t}{T}\right)$$

$$+ \; 0$$

$$\int_0^T s(t) \sin\left(\frac{2\pi f n t}{T}\right) dt = \frac{T}{2}\alpha_n .$$

$$\boxed{\alpha_n = \frac{2}{T}\int_0^T s(t) \sin\left(\frac{2\pi n t}{T}\right) dt}$$

$$\boxed{\beta_n = \frac{2}{T}\int_0^T s(t) \cos\left(\frac{2\pi n t}{T}\right) dt}$$

Higher frequency → low contribution.

$\alpha_n = A\cos\phi_n$, $\beta_n = B\sin\phi_n$

$A = \sqrt{\alpha_n^2 + \beta_n^2}$

$a_0 + \sum b_k e^{\frac{-2\pi k t}{T} i}$

$\phi = \tan^{-1}\frac{\beta}{\alpha}$

- Discrete Fourier Transform -

$$F(k) = \sum_{t=0}^{n-1} f(t)\, e^{\left(-\frac{2\pi k t}{n}\right) i} \qquad k \in \{0, \dots n-1\}$$

for 2 points,

$f = \langle f(0), f(1) \rangle$

$$F(0) = \sum_{t=0}^{1} f(t)\cdot e^{0} = f(0) + f(1)$$

$$F(1) = \sum_{t=0}^{1} f(t)\, e^{-\pi t i} = f(0)e + f(1)e^{-\pi i}$$
$$= f(0) - f(1)$$

8 point transform → 4 point transform + $e^{\left(-\frac{2\pi k}{8}\right)}$ 4 point transform.

00 / 00	10 / 01	01 / 10	11 / 11
1	3	2	5
1	2	3	5

(+1, -1)

$\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$\begin{bmatrix} +1 \\ -i \\ -1 \\ i \end{bmatrix}$ $\boxed{\begin{matrix} 11 \\ -1+2i \\ -5 \\ -1-2i \end{matrix}}$

- ## Single Value Decomposition -

Variable elimination Algorithim (Exact Interference)

$A = X \Lambda X^T$

for symmetric matrix

$S = Q \Lambda Q^T$

└ Orthonormal matrix.

Now,

$U, V \rightarrow$ Orthogonal matrix

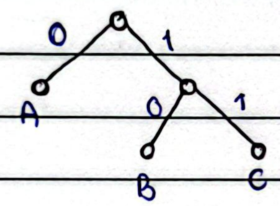
$\Sigma \rightarrow$ Diagonal matrix

Any A (Assymetric matrix) $= U \Sigma V^T$

- ## Pseudo Inverse -

Info Theory.

$$\text{Entropy} = -\sum_{i=1}^{t} p_i \log_2 p_i$$



$\rightarrow$ Bit Representation.

- ## Encoding -

	Acending order			
A − 0.3	H − 0.01			0.15, 0.17 → 0.32
B − 0.2	G − 0.02	H+G → 0.03	0.08, 0.09 → 0.17	0.17
C − 0.15	E − 0.06	0.03 + E → 0.09	0.09	0.18
D − 0.08	D − 0.08		0.15	0.2
E − 0.06	C − 0.15		0.18	0.3
F − 0.18	F − 0.18		0.2	0.18, 0.2 → 0.38
G − 0.02	B − 0.2		0.3	0.3
H − 0.01	A − 0.3			0.32

re

0.3, 0.32 → 0.62 (0.2, 0.3)

0.32

0.38

0.32, 0.38 → 0.5 ... 0.62, ~~0.38~~

1 ← 0.38, 0.62

0.5

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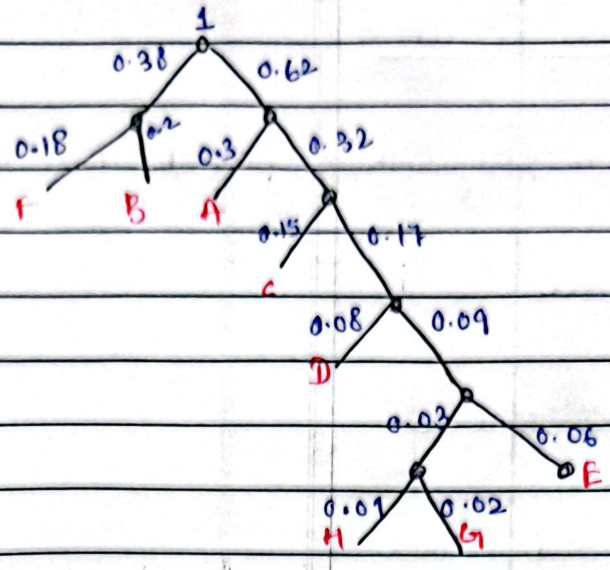
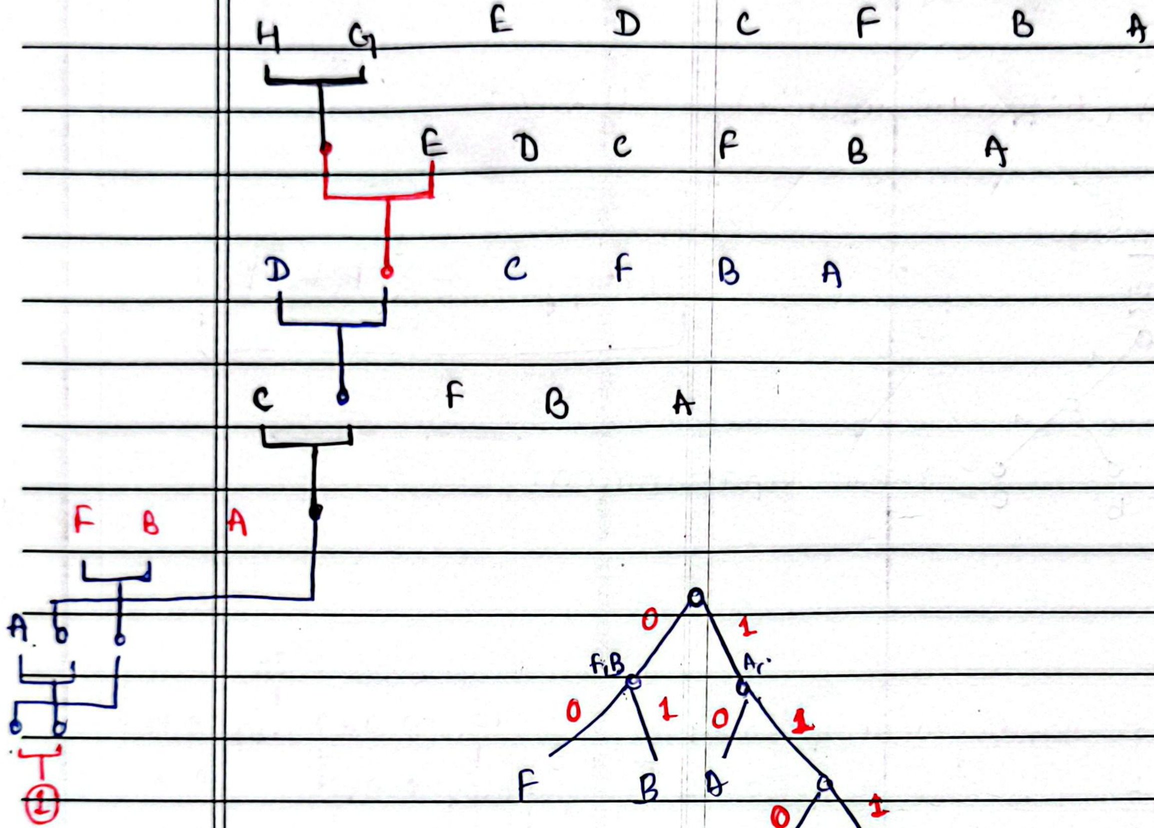
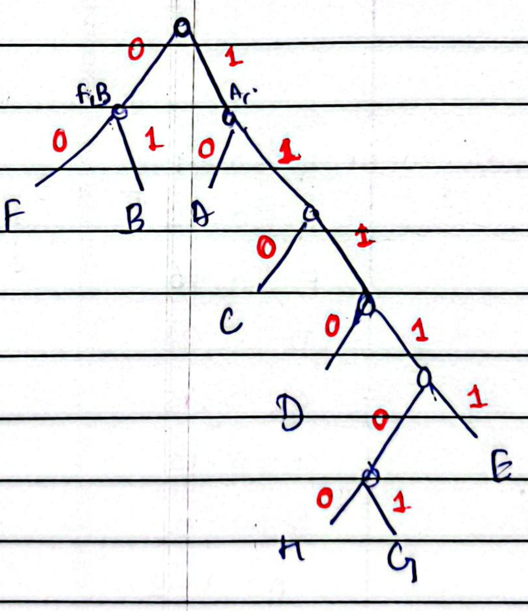

1
0.38
0.62
0.18
0.2
0.3
0.32
F
B
A
0.15
0.17
C
0.08
0.09
D
0.03
0.06
E
0.01
0.02
H
G


H G E D C F B A
E D C F B A
D C F B A
C F B A
F B A
A
1


0
1
F,B
A,C
0
1
0
1
F
B
A
0
1
C
0
1
D
0
1
E
0
1
H
G

Shannon-Fano

Shanon - encoding

Accending order → Partition

Same Probability or one element.

A	B	C	D	E	F
0.2	0.3	0.1	0.1	0.25	0.05

① | F | C | D | A | E | B |
|---|---|---|---|---|---|
| 0.05 | 0.1 | 0.1 | 0.25 | 0.25 | 0.3 |

|F C D| ↔ |A| |E| |B|

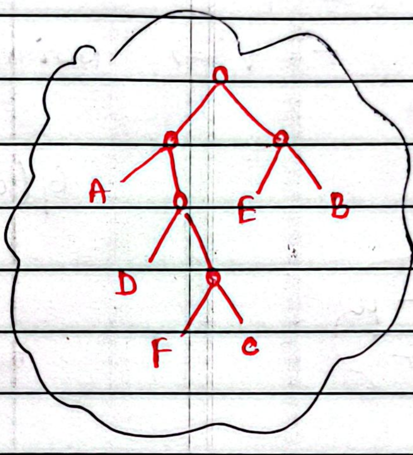
Stop Stop Stop

|F C| ↔ |D|

Stop

|F| |C|

Stop Stop.



- Shannon Capacity.

$$C = B \cdot \log_2 \left(1 + \frac{S}{N}\right)$$

C → Channel Capacity in bits per second

B → Bandwith of channel in Hz

S → Average signal power

N → Average noise power (white Gaussian Noise)

S/N - SNR signal-to-Noise ratio

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- Gaussian Curve - Properties -

A = area of curve (full)
A_k = area of curve from, $\mu - k$ to $\mu + k$

for $k = \sigma$, $\frac{A_k}{A} = 68.2\%$

$k = 2\sigma$, $\frac{A_k}{A} = 95.4\%$

$k = 3\sigma$, $\frac{A_k}{A} = 99.7\%$

max value $= \frac{1}{\sigma\sqrt{2\pi}}$

if a Gaussian curve $\mu = 0$ & $\sigma = 1 \Rightarrow$ Normal curve

z- score,

$$G(\sigma, \mu, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$z = \frac{x - \mu}{\sigma}$$

$\mu = 0$, $\sigma = 1$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2}$$

→ ~~m~~ =

$\mu_m = 60$ if $m_1 = 84$.
$\sigma_m = 5$

$\hookrightarrow m_z = \frac{84-60}{5} = 24 \div 5$

$m_z = 4.8$

$\mu_m = 75$
$\sigma_m = 5$
$m_z = 1.8 \longrightarrow 0.96407$

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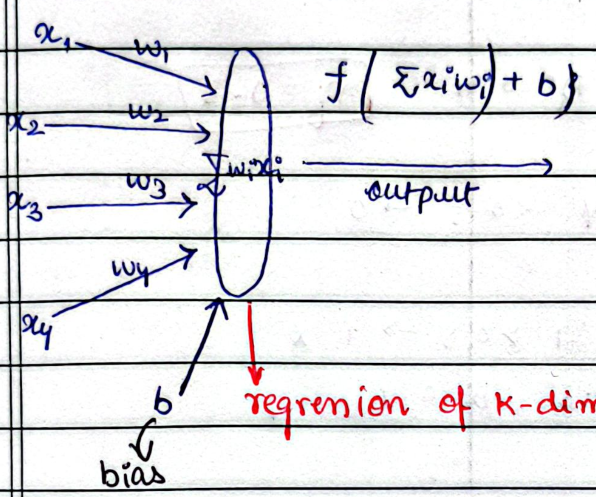
$\mu = 25000, \quad \sigma = 8000, \quad m = 20{,}000 \qquad m = 30{,}000$

$z_1 = \frac{\cancel{300}\ \cancel{5000}}{800}$

$z_1 = \frac{-5000}{800} \qquad z_2 = \frac{5000}{800}$

$\frac{12.5}{2} \qquad -6.25, \qquad 6.25$

- <u>Linear Preceptron</u> (Learning Rule)



$\begin{cases} 1 & \text{If output > threshold} \\ 0 & \text{If output < threshold} \end{cases}$

regresion of K-dimension plane.

Vector Representaiton.

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_k \end{bmatrix}$ $\boxed{f = x^T \cdot w + b}$

~~if x is +ve an~~

	for reducing	for increasing
x +ve	if w is +ve or -ve decrease w	if w is +ve or -ve increase w
x -ve	increase w	decrese w.

EPOCH

- Objective is minimize

Error = Target − Prediction

update rule,

Target → predicted

$$w_k = w_k \mp \eta\,(y_k - \hat{y}_k)$$

$$b = b \mp \eta\,(y_k - \hat{y}_k)$$

↳ learning Rate.

Example.

Threshold − 0.5

$w_1 = 0.6$, $w_2 = 0.6$, $b = 0.4$ $\boxed{\eta = 0.3}$

(i) $A=0$, $B=0$, $Y=0$

$f = 0\times0 + 0\times0 + 0.4 = 0.4 < 0.5 \rightarrow 0$

(ii) $A=0$, $B=1$, $Y=0$

$f = 0.6\times0 + 0.6\times1 + 0.4 = 1 > 0.5 \rightarrow 1$ ✗

NW, $w_1 = 0.3$, $w_2 = 0.3$, $b = 0.1$

- EPOCH

  ↳ Machine seen all inputs → 1 EPOCH

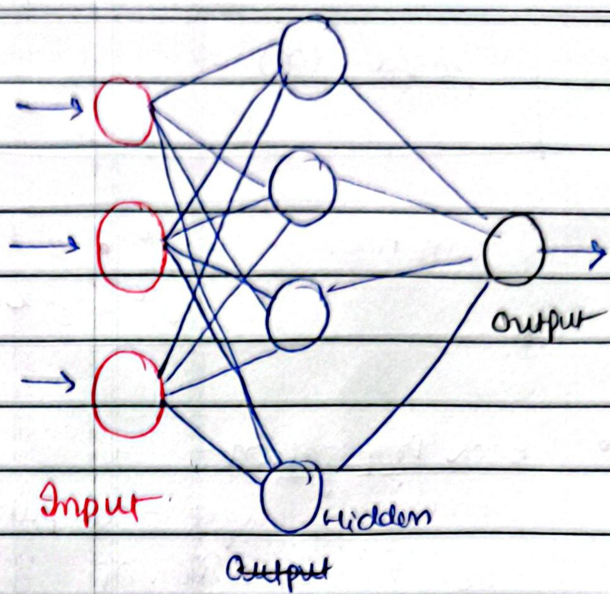
if we get EPOCH → Randomize the input parameter

- convergence

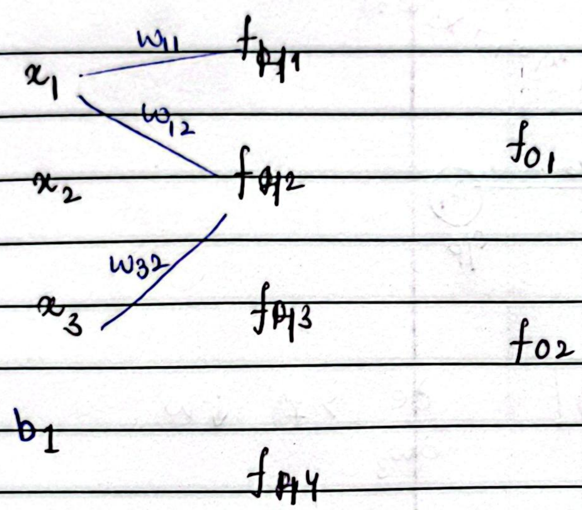
  ↳ Machine learns the input

Multilayer Preceptron -

MLP — 3 layers
↳ Input layer
Hidden layer
Output layer



★ 1 Hot Encoding -



$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \times \begin{array}{c} 1 \\ \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{14} \\ w_{21} & w_{22} & \cdots & w_{24} \\ \vdots & \vdots & & \vdots \\ w_{31} & w_{32} & \cdots & w_{34} \end{bmatrix} \end{array} + \begin{bmatrix} b_{11} & \cdots & b_{14} \end{bmatrix}$$

$$= \left[\sum_{k=1}^{3} x_k w_{k1} + b_{11} + \cdots b_{14} \cdots \sum_{k=1}^{3} x_k w_{k4} + \right]$$

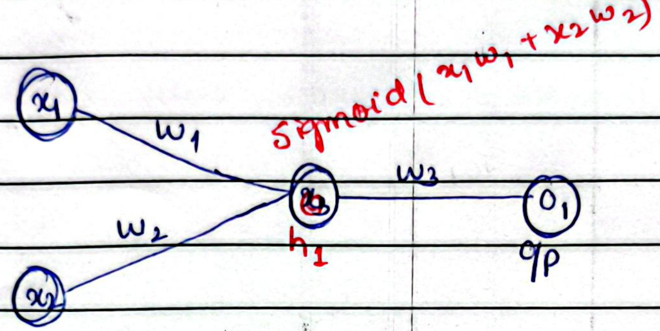
$$\boxed{\text{Sigmoid}(x) = \frac{1}{1+e^{-x}}}$$

$$\boxed{\tanh(x) = \frac{e^{+x} - e^{-x}}{e^{x} + e^{-x}}}$$

- **Back Propagation** -

Error = Target - Output.

$$\frac{\partial e}{\partial w} = \frac{\partial e}{\partial o} \cdot \frac{\partial o}{\partial i} \cdot \frac{\partial i}{\partial w}$$



$$\boxed{op = \text{sigmoid}(w_3 h_1)}$$

$$\frac{\partial e}{\partial w_3} > 0 \quad \downarrow w$$

$$e = (T - op)^2$$

$$\frac{\partial e}{\partial w_3} < 0 \quad \uparrow w$$

$$\frac{\partial e}{\partial w_3} = \frac{\partial e}{\partial op} \frac{\partial op}{\partial w_3}$$

$$\hookrightarrow w_3 \mathrel{-}= \eta\left(\frac{\partial e}{\partial w_3}\right)$$

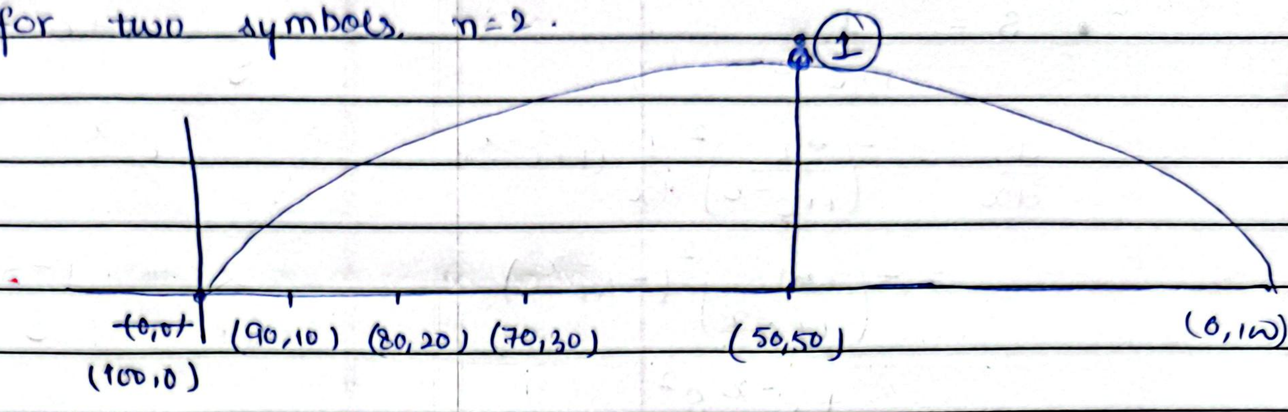
★ Vanishing Gradient Problem

- Relu → Solve VGP

- **Entropy**

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

for two symbols, $n = 2$.



magic Number — 4 bytes.

- Assg.

(Text image exe)

└ check entropy

- **Information Gain.**

Colour	Shape	class.
1 ← Y	R → 1	A
2 ← R	E → 2	M
1 ← G	R → 1	A
1 ← R	R → 1	A
2 ← R	E → 2	M
2 ← Y	R → 2	M
2 ← G	R → 2	M

	A	M
R	1	2
Y	1	1
G	1	1

	A	M
R	3	2
E	0	2

$$H = -\left[\frac{3}{7}\log_2\frac{3}{7} + \frac{4}{7}\log_2\frac{4}{7}\right]$$

0.9852