# Review of Calculus

This lecture contains a short review of those topics from single-variable calculus that will be needed in later topics. A solid knowledge of calculus is essential for an understanding of the analysis of numerical techniques, and more thorough review might be needed if you have been away from this subject for a while.

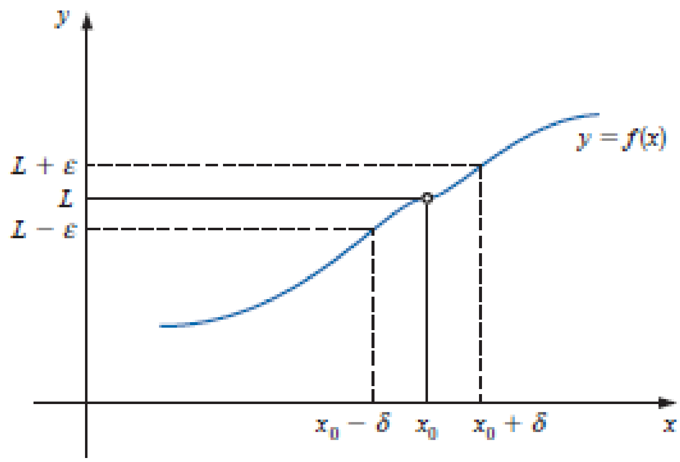
## Limit and Continuity

A function $f$ defined on a set $X$ of real numbers has the **limit** $L$ at $x_0$, written

$$\lim_{x \to x_0} f(x) = L$$

And $f$ is **continuous** at $x_0$ if

$$\lim_{x \to x_0} f(x) = f(x_0)$$

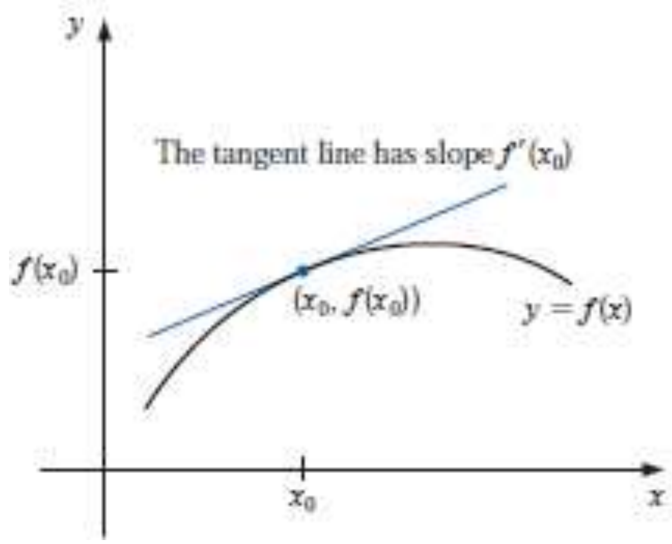



## Differentiability

Let $f$ be a function defined in an open interval containing $x_0$. The function $f$ is **differentiable** at $x_0$ if

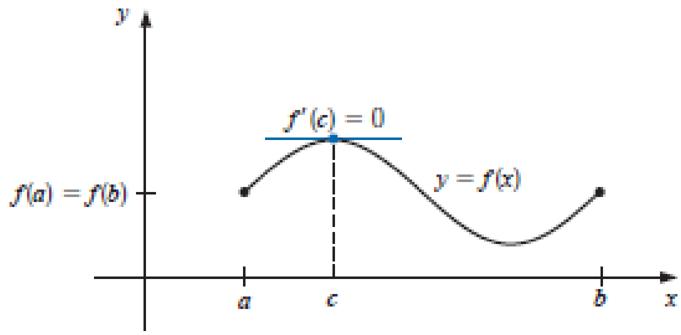
$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists, called the **derivative** of $f$ at $x_0$.



## Rolle's Theorem

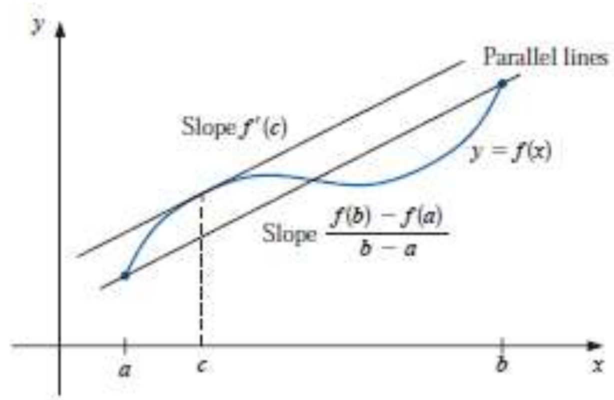
Suppose $f \in C[a, b]$ and $f$ is differentiable on $(a, b)$. If $f(a) = f(b)$, then a number $c$ in $(a, b)$ exists with $f'(c) = 0$.




## Mean Value Theorem

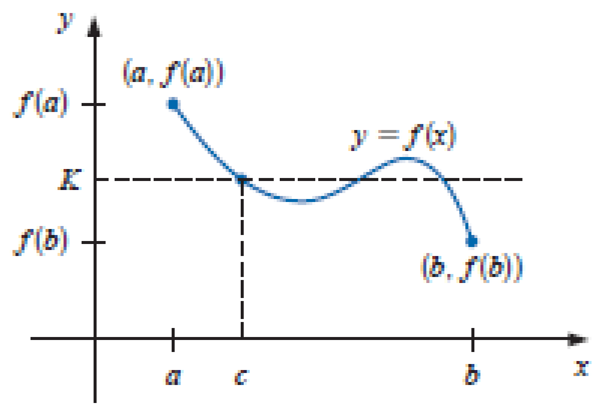
If $f \in C[a, b]$ and $f$ is differentiable on $(a, b)$, then a number $c$ in $(a, b)$ exists with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$




## Intermediate Value Theorem

If $f \in C[a, b]$ and $K$ is any number between $f(a)$ and $f(b)$, then there exists a number $c$ in $(a, b)$ for which $f(c) = K$.




*** Learn the use of calculator, write a function and calculate its values at different points.**

## Example

Show that $x^5 - 2x^3 + 3x^2 - 1 = 0$ has a solution in the interval [0, 1].

**Solution:**

Consider the function defined by

$$f(x) = x^5 - 2x^3 + 3x^2 - 1$$

The function $f$ is continuous on [0, 1]. In addition, $f(0) = -1 < 0$ and $0 < 1 = f(1)$.

The Intermediate Value Theorem implies that a number $x$ exists, with $0 < x < 1$, for which $x^5 - 2x^3 + 3x^2 - 1 = 0$.

## Work to do

## Exercise 1.1

Q1: Show that following equations have at least one solution in the given intervals.

a. $x \cos x - 2x^2 + 3x - 1 = 0$, $[0.2, 0.3]$ and $[1.2, 1.3]$

b. $(x-2)^2 - \ln x = 0$, $[1, 2]$ and $[e, 4]$

c. $x - (\ln x)^x = 0$, $[4, 5]$

Q2: Show that $f'(x)$ is 0 at least once in the given intervals.

a. $f(x) = 1 - e^x + (e-1)\sin(\frac{\pi}{2}x)$, $[0, 1]$

b. $f(x) = (x-1)\tan x + x \sin \pi x$, $[0, 1]$