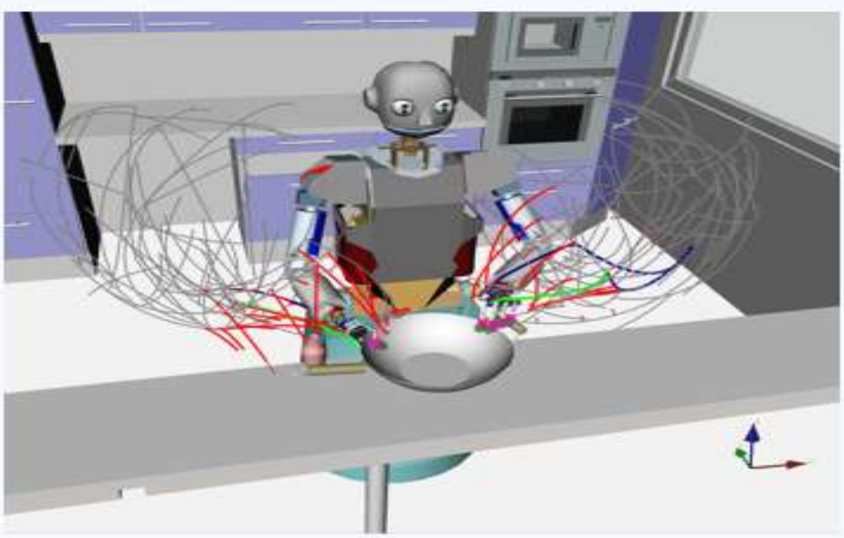
Interpolation and Polynomial Approximation

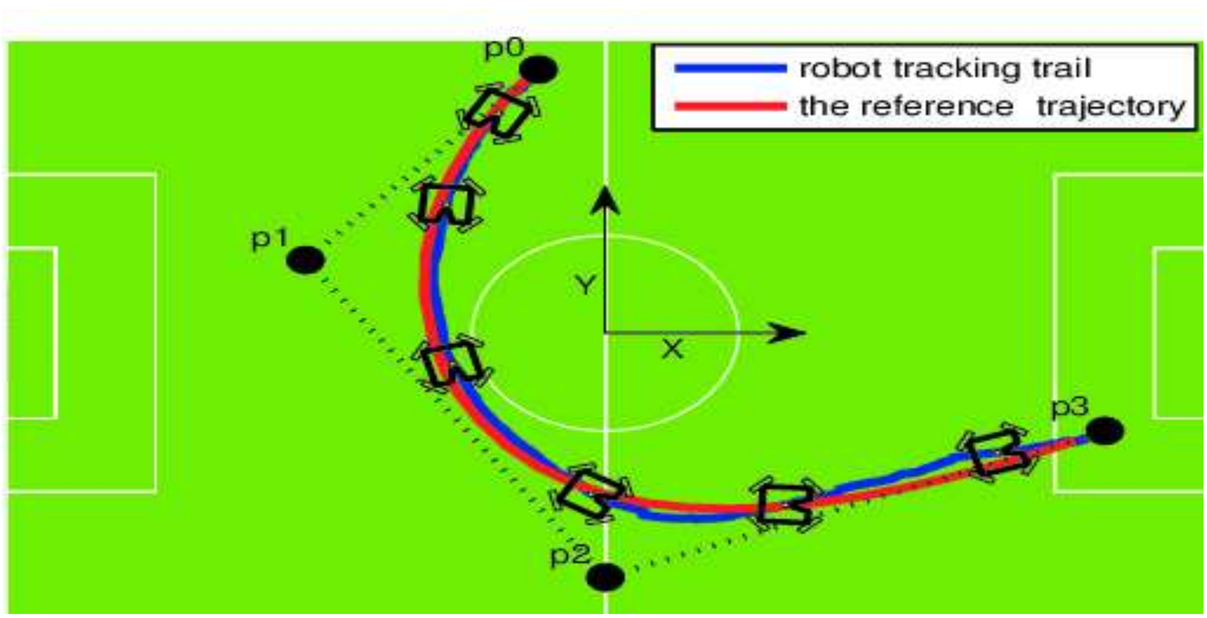
Aspect	Interpolation	Polynomial Approximation
Definition	A method of constructing new data points within the range of known data points.	A method of approximating a function using polynomials, often over a broader range.
Goal	To find an exact function that passes through all given data points.	To find a polynomial that approximates a function, but may not pass through all points exactly.
Methods Used	Lagrange Interpolation, Newton's Divided Differences, Cubic Splines	Least Squares Approximation, Taylor Series Approximation
Accuracy at Data Points	Interpolated polynomial passes exactly through the given points.	The polynomial approximates the data but may not pass through all given points.



ROBOTIC ARM – PATH PLANNING

In robotics, ***interpolation*** is used to control the smooth movement of a robotic arm. Imagine a robotic arm trying to move between two points, like picking up an object from one spot and placing it at another. The arm needs to calculate positions in between these two points to ensure smooth and precise motion. Interpolation helps the arm generate a continuous path by calculating intermediate positions based on a set of known points, ensuring it moves fluidly and accurately from start to finish. Techniques like Lagrange interpolation or cubic splines are used to create this path, making sure the arm doesn’t jerk or stop suddenly but moves seamlessly between points.

On the other hand, ***polynomial approximation*** comes into play when the robotic arm is moving in a more complex or unpredictable environment. Instead of generating an exact path through specific points, polynomial approximation helps the arm follow a smooth, generalized curve that approximates its motion over a broader range. This is useful when the arm has to approximate the shape of a path or when small errors in position are acceptable. For example, if the arm is tracing the shape of a curve, it doesn’t need to pass through every single point but can approximate the curve to keep its movements efficient and responsive. Polynomial approximation allows the arm to perform tasks that don't need pinpoint accuracy at every point but still require smooth, overall motion.



In Taylor approximation, we studied how to approximate a function near a specific point using derivatives, but one major limitation is that it works best when we're close to that point. As we move further away, the accuracy of the approximation tends to drop. This makes Taylor series less reliable when we need a good fit over a wide range of values.

Now, we're shifting towards interpolation because it allows us to work with multiple known data points and create a function that passes exactly through all of them. This way, we can ensure higher accuracy across the entire range of interest, not just near a single point, making interpolation better suited for cases where precision over a broader domain is needed.