Assignment 4

1. Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

Solution 1

	Unit Shipping cost			Unit	Monthly
	Warehouse 1	Warehouse 2	Warehouse 3	Production Cost	Production Capacity
Plant A	$22	$14	$30	$600	100
Plant B	$16	$20	$24	$625	120
Monthly Demand	80	60	70		

Let

X_{ij} = No. of Units produced on Plant i and shipped to warehouse j where i= A(Plant A) , B(Plant B) and warehouse j=1 (Warehouse 1) ,2 (Warehouse 2) and 3(Warehouse 3).

Minimize the combined cost for production and shipping

$$Z = 622\, X_{A1} + 614X_{A2} + 630X_{A3} + 641X_{B1} + 645X_{B2} + 649X_{B3}$$

Plant A has monthly Production capacity of 100 AEDs and Plant B has monthly Production capacity of 120 AEDs. Hence,

$$X_{A1} + X_{A2} + X_{A3} \leq 100$$
$$X_{B1} + X_{B2} + X_{B3} \leq 120$$

Monthly demand for each warehouse

$$X_{A1} + X_{B1} \leq 80$$
$$X_{A2} + X_{B2} \leq 60$$
$$X_{A3} + X_{B3} \leq 70$$

Hence, the linear programming model should be defined as follows:

Decision variables

X_{ij} = No. of Units produced on Plant i and shipped to warehouse j

where i=A (Plant A) , B(Plant B) and

warehouse j=1 (Warehouse 1) ,2 (Warehouse 2) and 3(Warehouse 3).

Minimize

$$Z = 622\,X_{A1} + 614X_{A2} + 630X_{A3} + 641X_{B1} + 645X_{B2} + 649X_{B3}$$

Subject To

$$X_{A1} + X_{A2} + X_{A3} \le 100$$
$$X_{B1} + X_{B2} + X_{B3} \le 120$$
$$X_{A1} + X_{B1} \le 80$$
$$X_{A2} + X_{B2} \le 60$$
$$X_{A3} + X_{B3} \le 70$$

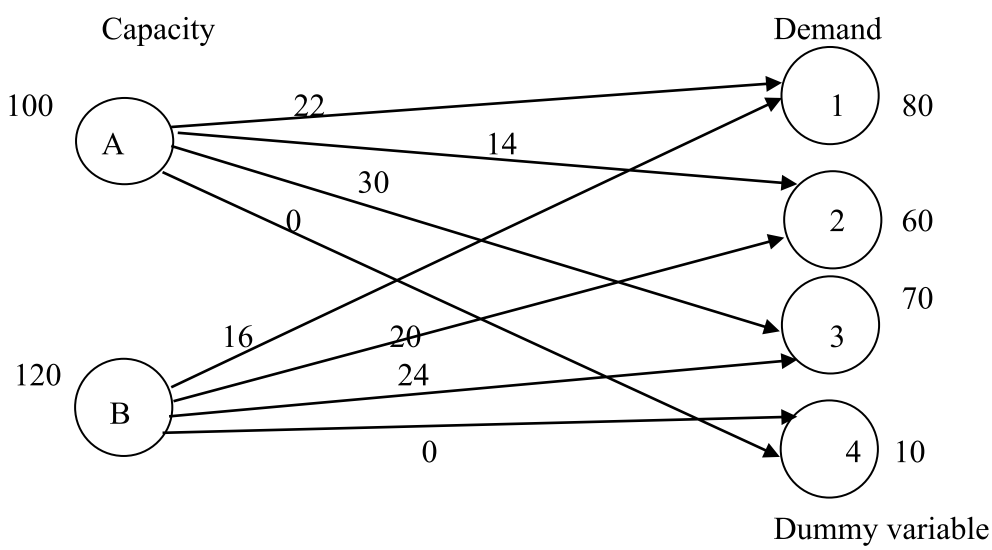
And

$$X_{A1},\ X_{A2},\ X_{A3},\ X_{B1},\ X_{B2},\ X_{B3} \ge 0$$

Using (Dummy Variable)

It is observed that the monthly demand is less than monthly production capacity. Hence, we create a dummy variable called as Warehouse 4 which can accommodate 10 units and the shipping cost will be $0 since it is a dummy warehouse.

	Unit Shipping cost				Unit	Monthly
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Production Cost	Production Capacity
Plant A	$22	$14	$30	$0	$600	100
Plant B	$16	$20	$24	$0	$625	120
Monthly Demand	80	60	70	10		



Let

X_{ij} = No. of Units produced on Plant i and shipped to warehouse j where i= A(Plant A) , B(Plant B) and warehouse j=1 (Warehouse 1) ,2 (Warehouse 2) ,3(Warehouse 3) and 4(Warehouse 4).

Hence,

$$Z = 622 X_{A1} + 614X_{A2} + 630X_{A3} + 0X_{A4} + 641X_{B1} + 645X_{B2} + 649X_{B3} + 0X_{B4}$$

Plant A has monthly Production capacity of 100 AEDs and Plant B has monthly Production capacity of 120 AEDs. Hence,

$$X_{A1} + X_{A2} + X_{A3} + X_{A4} \leq 100$$
$$X_{B1} + X_{B2} + X_{B3} + X_{B4} \leq 120$$

Monthly demand for each warehouse

$$X_{A1} + X_{B1} \leq 80$$
$$X_{A2} + X_{B2} \leq 60$$
$$X_{A3} + X_{B3} \leq 70$$
$$X_{A4} + X_{B4} \leq 10$$

Hence, the linear programming model should be defined as follows:

Decision variables

X_{ij} = No. of Units produced on Plant i and shipped to warehouse j
where i=A (Plant A) , B(Plant B) and
warehouse j=1 (Warehouse 1) ,2 (Warehouse 2), 3(Warehouse 3) and 4(Warehouse 4)

Minimize

$$Z = 622 X_{A1} + 614X_{A2} + 630X_{A3} + 0X_{A4} + 641X_{B1} + 645X_{B2} + 649X_{B3} + 0X_{B4}$$

Subject To

$$X_{A1} + X_{A2} + X_{A3} \leq 100$$
$$X_{B1} + X_{B2} + X_{B3} \leq 120$$
$$X_{A1} + X_{B1} \leq 80$$
$$X_{A2} + X_{B2} \leq 60$$
$$X_{A3} + X_{B3} \leq 70$$
$$X_{A4} + X_{B4} \leq 10$$

And

$$X_{A1}, X_{A2}, X_{A3}, X_{A4}, X_{B1}, X_{B2}, X_{B3}, X_{B4} \geq 0$$

2. **Oil Distribution** Texxon Oil Distributors, Inc., has three active oil wells in a west Texas oil field. Well 1 has a capacity of 93 thousand barrels per day (TBD), Well 2 can produce 88 TBD, and Well 3 can produce 95 TBD. The company has five refineries along the Gulf Coast, all of which have been operating at stable demand levels. In addition, three pump stations have been built to move the oil along the pipelines from the wells to the refineries. Oil can flow from any one of the wells to any of the pump stations, and from

any one of the pump stations to any of the refineries, and Texxon is looking for a minimum cost schedule. The refineries' requirements are as follows.

Refinery	R1	R2	R3	R4	R5
Requirement (TBD)	30	57	48	91	48

The company's cost accounting system recognizes charges by the segment of pipeline that is used. These daily costs are given in the tables below, in dollars per thou- sand barrels.

To		Pump A	Pump B	Pump C
From	Well 1	1.52	1.60	1.40
	Well 2	1.70	1.63	1.55
	Well 3	1.45	1.57	1.30

To		R1	R2	R3	R4	R5
From	Pump A	5.15	5.69	6.13	5.63	5.80
	Pump B	5.12	5.47	6.05	6.12	5.71
	Pump C	5.32	6.16	6.25	6.17	5.87

1) What is the minimum cost of providing oil to the refineries? Which wells are used to capacity in the optimal schedule? Formulation of the problem is enough.

Solution:

1) Here ,the supply is 276 TBD and the demand is 274 TBD

Since, the demand is not equal to supply we create a dummy variable in demand side of 2TBD in order to make sure that demand=supply

Objective function:

$Z_{min} = 1.52\ X_{1A} + 1.60\ X_{1B} + 1.40\ X_{1C} + 1.70\ X_{2A} + 1.63\ X_{2B} + 1.55\ X_{2C} + 1.45\ X_{3A} + 1.57\ X_{3B} + 1.30\ X_{3C} + 5.15\ X_{AR_1} + 5.69\ X_{AR_2} + 6.13\ X_{AR_3} + 5.63 X_{AR_4} + 5.80\ X_{AR_5} + 0\ X_{AR_6} + 5.12\ X_{BR_1} + 5.47\ X_{BR_2} + 6.05\ X_{BR_3} + 6.12\ X_{BR_4} + 5.71\ X_{BR_5} + 0 X_{BR_6} + 5.32 X_{CR_1} + 6.16\ X_{CR_2} + 6.25\ X_{CR_3} + 6.17\ X_{CR_4} + 5.87\ X_{CR_5} + 0\ X_{CR_6}$

Constraints:

Supply Constraints:

$X_{1A} + X_{1B} + X_{1C} = 93$
$X_{2A} + X_{2B} + X_{2C} = 88$
$X_{3A} + X_{3B} + X_{3C} = 95$

Demand Constraints:
$X_{AR_1} + X_{BR_1} + X_{CR_1} = 30$
$X_{AR_2} + X_{BR_2} + X_{CR_2} = 57$
$X_{AR_3} + X_{BR_3} + X_{CR_3} = 48$
$X_{AR_4} + X_{BR_4} + X_{CR_4} = 91$
$X_{AR_5} + X_{BR_5} + X_{CR_5} = 48$
$X_{AR_6} + X_{BR_6} + X_{CR_6} = 2$

Constraints from pump to Refineries:
$X_{1A} + X_{2A} + X_{3A} = X_{AR_1} + X_{AR_2} + X_{AR_3} + X_{AR_4} + X_{AR_5} + X_{AR_6}$
$X_{1B} + X_{2B} + X_{3B} = X_{BR_1} + X_{BR_2} + X_{BR_3} + X_{BR_4} + X_{BR_5} + X_{BR_6}$
$X_{1C} + X_{2C} + X_{3C} = X_{CR_1} + X_{CR_2.} + X_{CR_3} + X_{CR_4} + X_{CR_5} + X_{CR_6}$

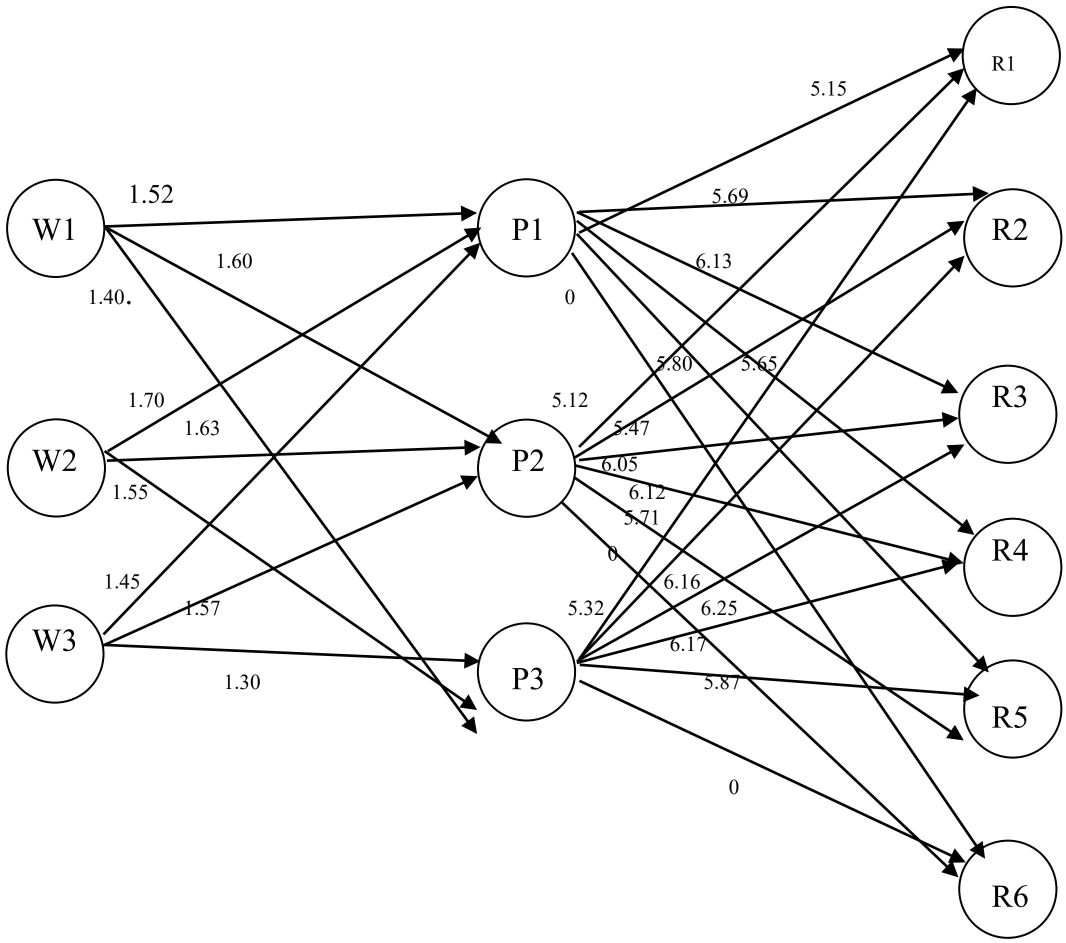
Where $X_{ij} \geq 0$ (Wells =1,2,3 ; Pumps = A,B,C ; Refineries =R_1 , R_2 , R_3 ,R_4 ,R_5 ,R_6)

The optimal solution for the above problem is 1966.68.
Well 3 is used to capacity in the optimal solution.

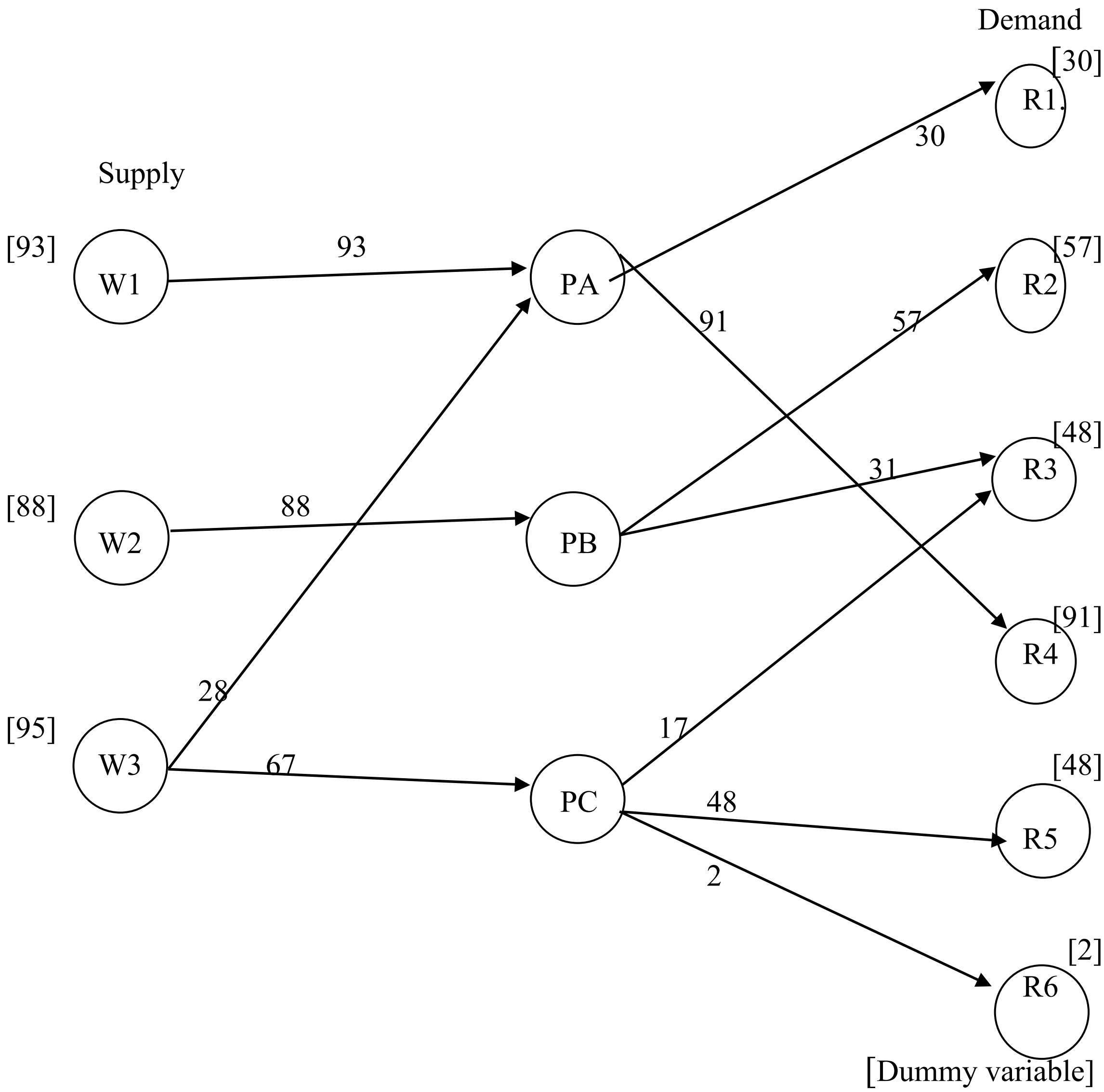
The code is in the .Rmd and pdf file.

2) Show the network diagram corresponding to the solution in (a). That is, label each of the arcs in the solution and verify that the flows are consistent with the given information.

   The network diagram



The network diagram corresponding to the solution in (a) is as follows :



Here ,W (wells) ,P(Pumps) and R (Refineries).