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Assignment 1

1Q :-

The perception algorithm updates

$$W = \omega + y_1 x_1$$

ω = weight vector

x_1 = misclassified input vector

y_1 = actual class label (+1 or -1)

Given data,

Initial weight vector $\omega = \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix}$

Training data

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \; y_1 = +1$$

$$x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \; y_2 = +1$$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \; y_3 = -1$$

Rule

$$\operatorname{sign}(\omega^T x_i) = y_i$$

Iteration 1 : Check x_1

$$w^T x_1 = (0 \times 1) + (6 \times 2) + (6 \times 0)$$

$$= 12$$

$$sign(12) = +1$$

Iteration 2 : Check x_2

$$w^T x_2 = (0 \times 1) + (6 \times 2) + (6 \times 2) = 24$$

$$sign(24) = +1$$

Iteration 3 : check x_3

$$w^T x_3 = (0 \times 1) + (6 \times 0) + (6 \times 2) = 12$$

$$sign(12) = +1 \neq -1$$

updated Rule

$$w = w + y_3 x_3$$

$$\begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0-1 \\ 6-0 \\ 6-2 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$$

Iteration 4 : Recheck x_1

$$w^T x_1 = (-1 \times 1) + (6 \times 2) + (4 \times 0) = 11$$

$$sign(11) = +1$$

Iteration 5 : Re-check x_2

$$w^T x_2 = (-1 \times 1) + (6 \times 2) + (4 \times 2) = 19$$

$$sign(19) = +1$$

Iteration 6 : Re-check x_3

$$w^T x_3 = (-1\times1) + (6\times0) + (4\times2) = 7$$

$$\text{sign}(7) = +1 \neq -1$$

updated rule :

$$\begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -1-1 \\ 6-0 \\ 4-2 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 2 \end{bmatrix}$$

2 Ans :-

(a)

$x_1 = (1\ 3\ 2)^T$; $y_1 = 3$

$x_2 = (1\ 2\ 2)^T$; $y_2 = 1$

$x_3 = (1, 1, -1)$; $y_3 = -1$

$x_4 = (1, -1, -3)^T$; $y_4 = -3$

We know Error in Linear Regression

(SSE = sum of squarred Error)

$$SSE = \frac{1}{N} \sum_{n=1}^{N} [h(x) - f(x)]^2$$

$h(x)$ = predicted output

$f(x)$ = Actual output

we know $h(x) = x_n^T w$

and $f(x) = y_n$

$$= \frac{1}{N} \sum_{n=1}^{N} (x_n^T w - y_n)^2$$

$$= \frac{1}{N} ||Xw - y||^2$$

$$= \frac{1}{N} (Xw - y)^T (Xw - y)$$

$$= \frac{1}{N} (w^T X^T Xw - 2y^T Xw + y^T y)$$

we are trying to minimize error

so lets assume $SSE = 0$

upon further simplification, we get

$$X^T X w = X^T y$$

$$w = (X^T X)^{-1} X^T y$$

$(X^T X)^{-1} X^T$ is referred as

" pseudo Inverse of A "

$$\underset{(4\times3)}{X} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & -1 \\ 1 & -1 & -3 \end{bmatrix} ; \quad \underset{(3\times4)}{X^T} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & -1 \\ 2 & 2 & -1 & -3 \end{bmatrix}$$

$(X^T X)^{-1}$ we need to find,

$$\underset{3\times3}{(X^T X)} = \begin{bmatrix} (1+1+1+1) & (3+2+1-1) & (2+2-1-3) \\ (3+2+1-1) & (9+4+1+1) & (6+4-1+3) \\ (2+2-1-3) & (6+4-1+3) & (4+4+1+9) \end{bmatrix}$$

$$\underset{(3\times3)}{(X^T X)} = \begin{bmatrix} 4 & 5 & 0 \\ 5 & 15 & 12 \\ 0 & 12 & 18 \end{bmatrix}$$

$$\underset{3\times 3}{(x^T x)} = \begin{bmatrix} 4 & 5 & 0 \\ 5 & 15 & 12 \\ 0 & 12 & 18 \end{bmatrix}$$

$(x^T x)^{-1}$ we need to find inverse of this matrix

let's assume $(x^T x)$ is called A.

we need to find A inverse

$$A^{-1} = \frac{1}{\det. A} \cdot Adj(A)$$

$$\det. A \Rightarrow 4(15\times18 - 12\times12) - 6(5\times18 - 0\times12) + 0(5\times12 - 15\times0)$$

$$\det A = 4(126) - 5(90) + 0$$

$$= 54$$

$$A_{(1,1)} = (15\times18) - (12\times12) = 270 - 144$$

$$= 126$$

$$A_{(1,2)} = (5\times18) - (0\times12) = 90$$

$$A(1,3) = (5\times12) - (15\times0) = 60$$

$$A(2,1) = (18\times6) - (12\times0) = 90$$

$$A(2,2) = (4\times18) - (0\times0) = 72$$

$$A(2,3) = (4\times12) - (16\times0) = 48$$

$$A(3,1) = 5\times12 - 15\times0 = 60$$

$$A(3,2) = 4\times12 - 5\times0 = 48$$

$$A(3,3) = (4\times15) - (5\times5) = 35$$

$$(x^T x)^{-1} = \frac{1}{54}\begin{bmatrix} 126 & -90 & 60 \\ 90 & 72 & -48 \\ 60 & -48 & 35 \end{bmatrix}$$

$$\omega = \underbrace{(x^T x)^{-1}}_{\text{we know}} \underbrace{x^T y}_{\text{we need to find}}$$

$$\frac{1}{54}\begin{bmatrix} 126 & -90 & 60 \\ -90 & 72 & -48 \\ 60 & -48 & 35 \end{bmatrix}$$

$$\underset{(3\times 1)}{x^T y} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & -1 \\ 2 & 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1-1-3 \\ 9+2-1+3 \\ 6+2+1+9 \end{bmatrix}$$

$$x^T y = \begin{bmatrix} 0 \\ 13 \\ 18 \end{bmatrix}$$

$$\omega = (x^T x)^{-1} \left[(x)^T (y) \right]$$

$$= \frac{1}{54}\begin{bmatrix} 126 & -90 & 60 \\ -90 & 72 & -48 \\ 60 & -48 & 35 \end{bmatrix} \begin{bmatrix} 0 \\ 13 \\ 18 \end{bmatrix}$$

$$= \frac{1}{54}\begin{bmatrix} 126\times 0 & -90\times 13 & 60\times 18 \\ -90\times 0 & 72\times 13 & -48\times 18 \\ 60\times 0 & -13\times 48 & 35\times 18 \end{bmatrix}$$

$$= \frac{1}{54}\begin{bmatrix} (0) + (-1170) + (1080) \\ (0) + (936) + (-864) \\ (0) + (-624) + (630) \end{bmatrix}$$

$$= \begin{bmatrix} 90/54 \\ 72/54 \\ -6/54 \end{bmatrix} \qquad w = \begin{bmatrix} 1.6 \\ 1.33 \\ -0.11 \end{bmatrix}$$

3Q. fisher LDA

maximise ratio of $\left\{ \dfrac{\text{variance b/w class}}{\text{variance within class}} \right.$

$$J(w) = \frac{(m_2-m_1)^2}{s_1^2+s_2^2} = \frac{w^T S_B w}{w^T S_W w}$$

$$S_B = (m_2-m_1)(m_2-m_1)^T$$

$$S_W = \left[\sum_{x_i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{x_j \in C_2} (x_j - m_2)(x_j - m_2)^T \right]$$

upon simplification using

$$w \propto S_W^{-1}(m_1 - m_2)$$

Given

$$x = \begin{bmatrix} 4 & 3 \\ 2 & 2 \\ 1 & 1 \\ -2 & -1 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

x_1 & x_2 $\in C_1 \rightarrow$ 1 $\begin{cases} x_1 \rightarrow 4\ 3 \\ x_2 \rightarrow 2\ 2 \end{cases}$

x_3 and $x_4 \in C_2 \rightarrow -1$ $\left\langle \begin{array}{l} x_3 - 1\ 1 \\ x_4 - -2\ -1 \end{array} \right.$

<u>class (1) mean</u> class (2) mean

$\left(\frac{4+2}{2}\right) \left(\frac{3+2}{2}\right)$ $\frac{1-2}{2}$ $\frac{1+(-1)}{2}$

$\boxed{3, 2.5}$ $\boxed{-0.5, 0}$

$$S_W = \sum_{x_i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{x_j \in C_2} (x_j - m_2)(x_j - m_2)^T$$

$C_1 =$ $(x_1 - m_1) = \begin{bmatrix} 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 4-3 \\ 3-2.5 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$C_1\ (x_2 - m_1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 2-3 \\ 2-2.5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}$$

$$C_2\ (x_3 - m_2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0.5 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$$

$$C_2\ (x_4 - m_2) = \begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -2+0.5 \\ -1-1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}$$

$$(x_1 - m_1) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}; (x_1 - m_1)^T = (1 \ \ 0.5)$$

$$(x_2 - m_1) = \begin{bmatrix} -1 \\ -0.5 \end{bmatrix}; (x_2 - m_1)^T = [-1 \ \ -0.5]$$

$$(x_3 - m_2) = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}; (x_3 - m_2)^T = [1.5 \ \ 1]$$

$$(x_4 - m_2) = \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}; (x_4 - m_2)^T = [-1.5 \ \ -1]$$

$$S_W = \left(\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} [1 \ \ 0.5] + \begin{bmatrix} -1 \\ -0.5 \end{bmatrix} [-1 \ \ -0.5] + \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} [1.5 \ \ 1] + \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} [-1.5 \ \ -1] \right)$$

$$S_W = \left(\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix} + \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix} + \begin{bmatrix} 2.25 & 1.5 \\ 1.5 & 1 \end{bmatrix} + \begin{bmatrix} 2.25 & 1.5 \\ 1.5 & 1 \end{bmatrix} \right)$$

$$S_W = \begin{bmatrix} 6.5 & 4 \\ 4 & 2.5 \end{bmatrix}$$

Now we need to calculate

$$S_W^{-1} = \frac{1}{(6.5)(2.5) - (4)(4)} \cdot \begin{bmatrix} 2.5 & -4 \\ -4 & 6.5 \end{bmatrix}$$

$$S_W^{-1} = \frac{1}{16.25 - 16} \begin{bmatrix} 2.5 & -4 \\ -4 & 6.5 \end{bmatrix}$$

$$S_W^{-1} = \frac{1}{0.25}\begin{bmatrix} 2.5 & -4 \\ -4 & 6.5 \end{bmatrix}$$

$$S_W^{-1} = \begin{bmatrix} 10 & -16 \\ -16 & 26 \end{bmatrix}$$

$$w \propto \begin{bmatrix} 10 & -16 \\ -16 & 26 \end{bmatrix} (m_1 - m_2)$$

$$w \propto \begin{bmatrix} 10 & -16 \\ -16 & 26 \end{bmatrix} \left[\begin{pmatrix} 3 \\ 2.5 \end{pmatrix} - \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} \right]$$

$$w \propto \begin{bmatrix} 10 & -16 \\ -16 & 26 \end{bmatrix} \begin{bmatrix} 3.5 \\ 2.5 \end{bmatrix}$$

$$\underset{2\times 1}{w} \propto \begin{bmatrix} 35 - 40 \\ -56 + 65 \end{bmatrix}$$

$$\underset{2\times 1}{w} \propto \begin{bmatrix} -5 \\ 9 \end{bmatrix}.$$