UNIT-I ROOTS OF EQUATIONS

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1. Bisections Method :-

$$x_2 = \frac{x_0 + x_1}{2}$$

Q.) Find the real root of the given correct to 1 decimal place

$$f(x) = x^3 - x - 1$$

$$f(x) = x^3 - x - 1$$
$$f(0) = -1$$
$$f(1) = 1 - 1 - 1 = -1$$
$$f(2) = 8 - 2 - 1 = 5$$

$f(1) \cdot f(2) < 0$

hence root lies between 1 & 2

Iteration-1

$x_0 = 1$, $x_1 = 2$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1+2}{2}$$

$$\therefore x_2 = 1.5$$

$$f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875$$

-ve +ve ve

1 1.5 2

$f(1) \cdot f(1.5) < 0$

hence root lies between 1 & 1.5

Iteration-2

$x_0 = 1$, $x_1 = 1.5$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1 + 1.5}{2}$$

$$\therefore x_2 = 1.25$$

-ve   -ve   +ve

1   1.25   1.5

$$f(1.25) = (1.25)^3 - 1.25 - 1$$
$$= -0.296875$$

$f(1.25) \cdot f(1.5) < 0$

hence root lies between 1.25 & 1.5

Iteration-3.

$x_0 = 1.25$, $x_1 = 1.5$ by bisection method.

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1.25 + 1.5}{2}$$

$$\therefore x_2 = 1.375$$

-ve   +ve   +ve

1.25   1.375   1.5

$$f(1.375) = (1.375)^3 - 1.375 - 1$$
$$= 0.2246093$$

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$f(1.25) \cdot f(1.375) < 0$

Iteration-4 hence root lies between 1.25 & 1.375

$x_0 = 1.25$, $x_1 = 1.375$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1.25 + 1.375}{2}$$

$$\therefore x_2 = 1.3125$$

-ve	-ve	+ve
1.25	1.3125	1.375

$$f(1.3125) = (1.3125)^3 - 1.3125 - 1$$
$$= -0.05$$

$f(1.3125) \cdot f(1.375) < 0$

Iteration-5 hence root lies between 1.3125 & 1.375

$x_0 = 1.3125$, $x_1 = 1.375$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2} = \frac{1.3125 + 1.375}{2}$$

$$\therefore x_2 = 1.34375$$

-ve	+ve	+ve
1.3125	1.34375	1.375

$$f(1.34375) = (1.34375)^3 - 1.34375 - 1$$
$$= 0.082611$$

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The real root of given equation is 1.3.

2. Regula falsi method or Method of falsi position :-

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Q.) Find the real root of the given correct to 1 decimal place $f(x) = x^3 - x - 1$

$f(x) = x^3 - x - 1$
$f(1) = 1 - 1 - 1 = -1$
$f(2) = 8 - 2 - 1 = 5$

$f(1) \cdot f(2) < 0$
hence root lies between 1 & 2.

Iteration 1

$x_0 = 1, \; x_1 = 2$
$f_0 = -1 ; \; f_1 = 5$

by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1 \times 5 + 2 \times 1}{5 + 6}$$

$$\Rightarrow x_2 = \frac{7}{6} = 1.167$$

-ve   -ve   +ve

1   1.167   2

$f(1.167) = (1.167)^3 - 1.167 - 1$

$= -0.578$

$f(1.167) \cdot f(2) < 0$

hence root lies between 1.167 & 2.

Iteration-2.

$x_0 = 1.167 \quad , \; x_1 = 2$

$f_0 = -0.578 \; , \; f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.167 \times 5 + 2 \times 0.578}{5 + 0.578}$$

$$\Rightarrow x_2 = \frac{6.991}{5.578} = 1.253.$$

-ve   -ve   +ve

1.167   1.253   2

$f(1.253) = (1.253)^3 - 1.253 - 1$

$= -0.286$

$f(1.253) \cdot f(2) < 0$

hence root lies between 1.253 & 2.

Iteration-3

$x_0 = 1.253 \quad , \; x_1 = 2$

$f_0 = -0.286 \; , \; f_1 = 5$

by regula falsi method

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.253 \times 5 + 2 \times 0.286}{5 + 0.286}$$

$$\Rightarrow x_2 = \frac{6.837}{5.286}$$

$$\therefore x_2 = 1.293.$$

-ve 1.253   -ve 1.293   +ve 2

$$f(1.293) = (1.293)^3 - 1.293 - 1$$
$$= -0.131$$

$$f(1.293) \cdot f(2) < 0$$

hence root lies between 1.293 & 2

Iteration 4.

$x_0 = 1.293$, $x_1 = 2$
$f_0 = -0.131$, $f_1 = 5$

by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.293 \times 5 + 2 \times 0.131}{5 + 0.131}$$

$$\Rightarrow x_2 = \frac{6.727}{5.131}$$

$$\therefore x_2 = 1.311$$

-ve 1.293   -ve 1.311   +ve 2

$$f(1.311) = (1.311)^3 - 1.311 - 1$$
$$= -0.058$$

Iteration-5

$f(1.311) \cdot f(2) < 0$

Hence root lies between 1.311 & 2.

$x_0 = 1.311$, $x_1 = 2$

$f_0 = -0.058$, $f_1 = 5$

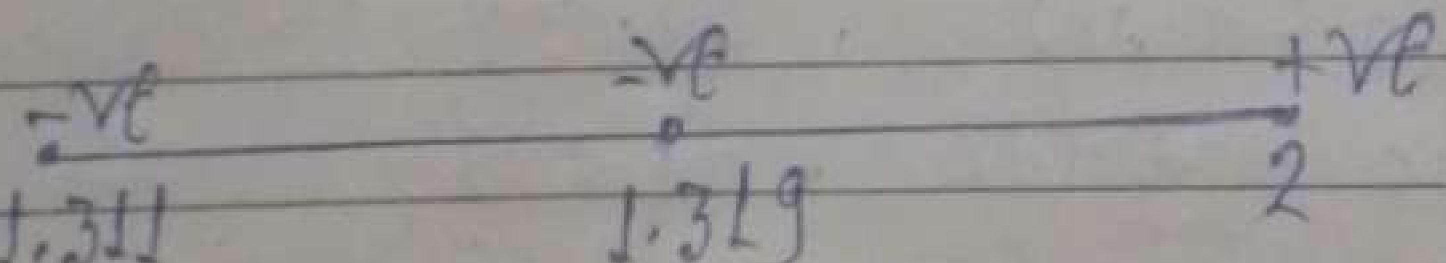
by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.311 \times 5 + 2 \times 0.058}{5 + 0.058}$$

$$\Rightarrow x_2 = \frac{6.671}{5.058}$$

$$\therefore x_2 = 1.319$$



$$f(1.319) = (1.319)^3 - 1.319 - 1$$
$$= -0.024$$

$f(1.319) \cdot f(2) < 0$

Hence root lies between 1.319 & 2.

Iteration-6

$x_0 = 1.319$, $x_1 = 2$

$f_0 = -0.024$, $f_1 = 5$

by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.319 \times 5 + 2 \times 0.024}{5 + 0.024}$$

$$\Rightarrow x_2 = \frac{6.643}{5.024}$$

$$\therefore x_2 = 1.322$$

-ve	-ve	+ve
1.319	1.322	2

$$f(1.322) = (1.322)^3 - 1.322 - 1$$
$$= -0.012$$

$f(1.322) \cdot f(2) < 0$

hence root lies between 1.322 & 2

Iteration-7

$x_0 = 1.322$, $x_1 = 2$

$f_0 = -0.012$, $f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.322 \times 5 + 2 \times 0.012}{5 + 0.012}$$

$$\Rightarrow x_2 = \frac{6.634}{5.012}$$

$$\therefore x_2 = 1.324$$

-ve	-ve	+ve
1.322	1.324	2

$f(1.324) = (1.324)^3 - 1.324 - 1$
$= -0.003$

$f(1.324) \cdot f(2) < 0$

Iteration-8

hence root lies between 1.324 & 2

$x_0 = 1.324$, $x_1 = 2$
$f_0 = -0.003$, $f_1 = 5$

by regula falsi method

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.324 \times 5 + 2 \times 0.003}{5 + 0.003}$$

$$\Rightarrow x_2 = \frac{6.626}{5.003}$$

$$\therefore x_2 = 1.3244$$

-ve	-ve	+ve
1.324	1.3244	2

$f(1.3244) = (1.3244)^3 - 1.3244 - 1$
$= -0.001$

$f(1.3244) \cdot f(2) < 0$

hence root lies between 1.3244 & 2.

Iteration 9

$x_0 = 1.3244$, $x_1 = 2$
$f_0 = -0.001$, $f_1 = 5$

by regula falsi method

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.3244 \times 5 + 2 \times 0.001}{5 + 0.001}$$

$$\Rightarrow x_2 = \frac{6.624}{5.001}$$

$$\therefore x_2 = 1.3245$$

-ve -ve +ve

1.3244 1.3245 2

$$f(1.3245) = (1.3245)^3 - 1.3245 - 1$$
$$= -0.0009$$

$f(1.3245) \cdot f(2)$

Hence root lies between 1.3245 & 2.

Iteration-10

$x_0 = 1.3245$ $x_1 = 2$
$f_0 = -0.0009$ $f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.3245 \times 5 + 2 \times 0.0009}{5 + 0.0009}$$

$$\Rightarrow x_2 = 1.325$$

-ve +ve +ve

1.3245 1.325 2

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$f(1.325) = (1.325)^3 - 1.325 - 1$
$= 0.001$

The real root of given equation is 1.3 Ans

3. Newton Raphson Method :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

or

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Q.) Find the real root of the given correct to 1 decimal place $f(x) = x^3 - x - 1$

$f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$

$$\Rightarrow x_{n+1} = \frac{3x_n^3 - \cancel{x_n} - x_n^3 + \cancel{x_n} + 1}{3x_n^2 - 1}$$

$$\Rightarrow x_{n+1} = \frac{2x_n^3 + 1}{3x_n^2 - 1} \quad \text{--- (i)}$$

$f(x) = x^3 - x - 1$
$f(0) = -1$
$f(1) = -1$
$f(2) = 5$

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Root lies between 1 & 2

$$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

Root $x = 0$ in eqn (i)

Iteration 1

$$x_1 = \frac{2x_0^3 + 1}{3x_0^2 - 1}$$

$$\Rightarrow x_1 = \frac{2 \times (1.5)^3 + 1}{3 \times (1.5)^2 - 1}$$

$$\Rightarrow x_1 = \frac{6.750 + 1}{6.75 - 1}$$

$$\therefore x_1 = 1.348$$

Iteration-2.

$$x_2 = \frac{2x_1^3 + 1}{3x_0^2 - 1}$$

$$\Rightarrow x_2 = \frac{2 \times (1.348)^3 + 1}{3 \times (1.348)^2 - 1}$$

$$\Rightarrow x_2 = \frac{5.899}{4.451}$$

$$\therefore x_2 = 1.325$$

The real root of given eqn is 1.3

UNIT-II. INTERPOLATION AND EXTRAPOLATION

Finite difference :-

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x+h) = f(x+2h) - f(x+h)$$
$$\Rightarrow \Delta f(x+2h) = f(x+3h) - f(x+2h)$$

$$\Delta^2 f(x) = \Delta[\Delta f(x)]$$
$$= \Delta[f(x+h) - f(x)]$$
$$= \Delta f(x+h) - \Delta f(x)$$

$$\Delta^2 f(x) = [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$$
$$\Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$$

Forward operator (Δ)

$$\Delta f(x) = f(x+h) - f(x)$$

1. x
2. $x+h$
3. $x+2h$
4. $x+3h$.

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Rightarrow \Delta f(x+h) = f(x+2h) - f(x+h)$$
$$\Rightarrow \Delta^2 f(x) = \Delta[\Delta f(x)]$$

$$\Rightarrow \Delta^2 f(x) = \Delta[f(x+h) - f(x)]$$

$$\Rightarrow \Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

$$\Rightarrow \Delta^2 f(x) = [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$$

$\Rightarrow \Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$

$\Rightarrow \Delta^3 f(x) = \Delta[\Delta^2 f(x)]$

$\Rightarrow \Delta^3 f(x) = \Delta[f(x+2h) - 2f(x+h) + f(x)]$

$\Rightarrow \Delta^3 f(x) = \Delta f(x+2h) - 2\Delta f(x+h) + \Delta f(x)$

$\Rightarrow \Delta^3 f(x) = [f(x+3h) - f(x+2h)] - 2[f(x+2h) - f(x+h)] + f(x+h) - f(x)$

$\Rightarrow \Delta^3 f(x) = f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)$

Q.) $\Delta(x^2+1)$ where h is unity $(h=1)$

$f(x) = x^2+1$

$\Rightarrow f(x+h) = x^2+1$

$\Rightarrow f(x+1) = (x+1)^2+1$

$\Rightarrow f(x+1) = x^2+2x+1+1$

$\therefore f(x+1) = x^2+2x+2$

Now,

$\Delta(x^2+1) = f(x+1) - f(x)$

$\Rightarrow \Delta(x^2+1) = x^2+2x+2-x^2-1$

$\therefore \Delta(x^2+1) = 2x+1$ A

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Q.) $\Delta^2(x+4)$ where h is unity $(h=1)$

$f(x) = x+4$

$\Rightarrow f(x+h) = x+4$

$\Rightarrow f(x+1) = x+1+4$

$\therefore f(x+1) = x+5$

$f(x+2h) = x+4$

$\Rightarrow f(x+2) = x+2+4$

$\therefore f(x+2) = x+6$

Now,

$\Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$

$\Rightarrow \Delta^2 f(x) = x+6 - 2(x+5) + x+4$

$\Rightarrow \Delta^2 f(x) = x+6 - 2x - 10 + x + 4$

$\therefore \Delta^2 f(x) = 0$

3. Backward difference operator (∇)

$$\left[\nabla f(x) = f(x) - f(x-h)\right]$$

4. Central difference operator (δ) :-

$$\left[\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)\right]$$

5. Shift operator (E) :-

$$\left[E f(x) = f(x+h)\right]$$

6. Identity Operator (I) :-

$$I f(x) = f(x)$$

7. $\Delta = E - I$

Here,

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Rightarrow \Delta f(x) = E f(x) - I f(x)$$
$$\Rightarrow \Delta f(x) = f(x)[E - I]$$
$$\therefore \Delta = E - I$$

Proved

8. $\nabla = I - E^{-1}$

Here

$$\nabla f(x) = f(x) - f(x-h)$$
$$\Rightarrow \nabla f(x) = I f(x) - E^{-1} f(x)$$
$$\Rightarrow \nabla f(x) = f(x)[I - E^{-1}]$$
$$\therefore \nabla = I - E^{-1}$$

Proved

9. $\Delta^2 = E^2 - 2E + I$

Here

$$\Delta^2 = f(x+2h) - 2f(x+h) + f(x)$$
$$\Rightarrow \Delta^2 f(x) = E^2 f(x) - 2E f(x) + I f(x)$$
$$\Rightarrow \Delta^2 f(x) = (E^2 - 2E + I) f(x)$$
$$\therefore \Delta^2 = E^2 - 2E + I$$

Proved

Q.) $\left[\frac{\Delta^2}{E}\right] x^2$

$\Rightarrow \left[\frac{E^2 - 2E + I}{E}\right] x^2$

$\Rightarrow (E^1 - 2E + E^{-1})\, x^2$

$\Rightarrow Ex^2 - 2I x^2 + E^{-1}x^2$

$\Rightarrow (x+h)^2 - 2x^2 + (x-h)^2$

$\Rightarrow x^2 + h^2 + 2xh - 2x^2 + x^2 + h^2 - 2xh$

$\Rightarrow \quad 2h^2$ Ans

6. Difference Table for equal interval.

Q.) Find the difference table for the given data

x : 0 1 2 3

$f(x)$: ~~2~~1 2 5 9.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
1	2		2	
		3		-1
2	5		1	
		4		
3	9			

7.) Newton Forward Interpolation Difference.

$$f(x_0+nh) = f(x_0) + \frac{n\,\Delta f(x_0)}{1!} + \frac{n(n-1)}{2!}\Delta^2 f(x_0) + \frac{n(n-1)(n-2)}{3!}\Delta^3 f(x_0) + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f(x_0)$$

8.) Find the value of $f(9)$ where.

x	2	4	6	8	10
$f(x)$	15	20	30	35	50

← Difference Table →

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_0	2	15				
			5			
	4	20		5		
			10		-10	
	6	30		-5		25
			5		15	
	8	35		10		
			15			
	10	50				

$x_n = x_0 + nh$

$\Rightarrow 9 = 2 + n \times 2$

$\Rightarrow 9-2 = 2n$

$\Rightarrow 2n = 7$

$\therefore n = 3.5$

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$$f(9) = f(x_0) + \frac{n\Delta f(x_0)}{1!} + \frac{n(n-1)}{2!}\Delta^2 f x_0$$

$$+ \frac{n(n-1)(n-2)}{3!}\Delta^3 f(x_0) + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f(x_0)$$

$$\Rightarrow f(9) = 15 + (3.5 \times 5) + \frac{3.5(3.5-1)}{2} \times 5$$

$$+ \frac{3.5(3.5-1)(3.5-2)}{6} \times (-10) + \frac{3.5(3.5-1)(3.5-2)(3.5-3)}{24} \times 25$$

$$\Rightarrow f(9) = 15 + 17.5 + 21.875 - 21.875 + 6.836$$

$$\therefore f(9) = 39.336 \text{ Ans}$$

8.) Newton Backward Interpolation Difference :-

$$f(x_n) = f(x_0) + n\nabla f(x_0) + \frac{n(n+1)}{2!}\nabla^2 f(x_0)$$

$$+ \frac{n(n+1)(n+2)}{3!}\nabla^3 f(x_0) + \frac{n(n+1)(n+2)(n+3)}{4!} \times \nabla^4 f(x_0) + \ldots\ldots$$

Q.) Find the value of f(9) where.

x	2	4	6	8	10
$f(x)$	15	20	30	35	50

← Difference Table →

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
2	15				
		5			
4	20		5		
		10		-10	
6	30		-5		25
		5		15	
8	35		10		
		15			
x_0 10	50				

$$x_n = x_0 + nh$$
$$\Rightarrow 9 = 10 + n \times 2$$
$$\Rightarrow -1 = 2n$$
$$\therefore n = -0.5$$

$$f(x_n) = f(x_0) + n\nabla f(x_0) + \frac{n(n+1)}{2!}\nabla^2 f(x_0) + \frac{n(n+1)(n+2)}{3!} \times \nabla^3 f(x_0) + \frac{n(n+1)(n+2)(n+3)}{4!} \times \nabla^4 f(x_0)$$

$$\Rightarrow f(9) = 50 - 0.5 \times 15 - \frac{0.5(-0.5+1)}{2} \times 10 - \frac{0.5(-0.5+1)(-0.5+2)}{6} \times 15 - \frac{0.5(-0.5+1)(-0.5+2)(-0.5+3)}{24} \times 25$$

$$\Rightarrow f(9) = 50 - 7.5 - 1.25 - 0.9375 - 0.9766$$
$$\Rightarrow f(9) = 39.3359$$
$$\therefore f(9) = 39.336$$

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9. CENTRAL DIFFERENCE FORMULA :-

i.) Gauss's forward Interpolation formula :-

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \dots$$

Q. find the value of function at x = 5 where.

x	0	2	4	6	8
$f(x)$	1	10	25	60	70

← Difference Table →

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_{-2}	0	$1_{y_{-2}}$				
			$9_{y_{-2}}$			
x_{-1}	2	$10_{y_{-1}}$		$6_{y_{-2}}$		
			$15_{y_{-1}}$		$14_{y_{-2}}$	
x_0	4	25_{y_0}		$20_{y_{-1}}$		$-59_{y_{-2}}$
			35_{y_0}		$-45_{y_{-1}}$	
x_1	6	60_{y_1}		-25_{y_0}		
			10_{y_1}			
x_2	8	70_{y_2}				

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$$x_n = x_0 + nh$$
$$\Rightarrow 5 = 4 + n \times 2$$
$$\Rightarrow 1 = 2n$$
$$\therefore n = 0.5$$

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!} \times \Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$$\Rightarrow y_n = 25 + 0.5 \times 35 + \frac{0.5(0.5-1)}{2} \times 20 + \frac{(0.5+1)0.5(0.5-1)}{6} \times (-45) + \frac{(0.5+1)\,0.5\,(0.5-1)(0.5-2)}{24} \times (-59)$$

$$\Rightarrow y_n = 25 + 17.5 - 2.5 + 2.8125 - 1.3828$$

$$\therefore y_n = 41.4297 \text{ Ans}$$

ii.) Gauss's Backward Interpolation Formula :-

$$\boxed{\begin{aligned} y_n = {} & y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} \\ & + \frac{(n+2)(n+1)n(n-1)}{4!} \times \Delta^4 y_{-2} \\ & + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!} \times \Delta^5 y_{-3} \end{aligned}}$$

Q.) Find the value of y when x = 3.75 where

x	2.5	3	3.5	4	4.5	5
$f(x)$	24.145	22.043	20.225	18.644	17.262	16.047

← Difference Table →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_{-2}	2.5	24.145 y_{-2}					
			-2.102 y_{-2}				
x_{-1}	3	22.043 y_{-1}		0.284 y_{-2}			
			-1.818 y_{-1}		-0.047 y_{-2}		
x_0	3.5	20.225 y_0		0.237 y_{-1}		0.009 y_{-2}	
			-1.581 y_0		-0.038 y_{-1}		-0.003 y_{-2}
x_1	4	18.644 y_1		0.199 y_0		0.006 y_{-1}	
			-1.382 y_1		-0.032 y_0		
x_2	4.5	17.262 y_2		0.167 y_1			
			-1.215 y_2				
x_3	5	16.047 y_3					

$$x_n = x_0 + nh$$

$$\Rightarrow 3.75 = 3.5 + n \times 0.5$$

$$\Rightarrow 0.25 = n \times 0.5$$

$$\therefore n = 0.5$$

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-3}$$

$$\Rightarrow y_{3.75} = 20.225 + 0.5\times(-1.818) + \frac{0.5(0.5+1)}{2}\times 0.232 + \frac{(0.5+1)0.5(0.5-1)}{6}\times(-0.047) + \frac{(0.5+2)(0.5+1)0.5(0.5-1)}{24}\times 0.009$$

$$\Rightarrow y_{3.75} = 20.225 - 0.909 + 0.08888 + 0.00294 - 0.00035$$

$$\therefore y_{3.75} = 19.40747$$

iii.) Stirling formula :-

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2} + \dots$$

Q.) Compute $10^5 y_{12.2}$ from the following table.

x :	10	11	12	13	14
y :	23967	28060	31788	35209	38368

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⟵ Difference Table ⟶

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	23967_{-2}				
		4093_{-2}			
11	28060_{-1}		-365_{-2}		
		3728_{-1}		58_{-2}	
12	31788_{0}		-307_{-1}		-13_{-2}
		3421_{0}		45_{-1}	
13	35209_{1}		-362_{0}		
		3159_{1}			
14	38368_{2}				

$$x_n = x_0 + nh$$
$$\Rightarrow 12.2 = 12 + n \times 1$$
$$\therefore n = 0.2.$$

$$10^5 y_{12.2} = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2}$$

$$\Rightarrow 10^5 y_{12.2} = 31788 + 0.2\left[\frac{3421 + 3728}{2}\right] + \frac{(0.2)^2}{2} \times (-307) + \frac{0.2\left[(0.2)^2 - 1\right]}{6}\left[\frac{45 + 58}{2}\right] + \frac{(0.2)^2\left[(0.2)^2 - 1\right]}{24} \times (-1$$

$$\Rightarrow 10^5 y_{12.2} = 31788 + 714.9 - 6.14 + 1.648 + 0.0208$$
$$\Rightarrow 10^5 y_{12.2} = 32498.4288$$
$$\therefore y_{12.2} = 0.32498$$

iv.) Bessel's formula :-

$$y_n = \left[\frac{y_0+y_1}{2}\right] + \left(n-\frac{1}{2}\right)\Delta y_0 + \frac{n(n-1)}{2!}\left[\frac{\Delta^2 y_0+\Delta^2 y_{-1}}{2}\right] + \frac{n\left(n-\frac{1}{2}\right)(n-1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{n(n+1)(n-1)(n-2)}{4!}\left[\frac{\Delta^4 y_{-1}+\Delta^4 y_{-2}}{2}\right] + \frac{n(n+1)\left(n-\frac{1}{2}\right)(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \dots$$

Q.) Compute $10^5 y_{12.2}$ from the following table.

x : 10 11 12 13 14
y : 23967 28060 31788 35209 38368

⟵ Difference Table. ⟶

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	10	23967				
		x_{-2}	4096			
x_{-1}	11	28060	x_{-2}	-365		
		x_{-1}	3728	x_{-2}	58	
x_0	12	31788	x_{-1}	-307	x_{-2}	-13
		x_0	3421	x_{-1}	45	x_{-2}
	13	35209	x_0	-262	x_{-1}	
			3159	x_0		
	14	38368				

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$$x_n = x_0 + nh$$
$$\Rightarrow 12.2 = 12 + n \times 1$$
$$\therefore n = 0.2$$

$$10^5 y_{12.2} = \left[\frac{35209 + 31788}{2}\right] + (0.2 - 0.5) \times 3421$$
$$+ \frac{0.2(0.2-1)}{2}\left[\frac{-262-307}{2}\right] +$$
$$\frac{0.2(0.2-0.5)(0.2-1)}{6} \times 45$$

$$\Rightarrow 10^5 y_{12.2} = 33498.5 - 1026.3 + (-0.08)(-284.5)$$
$$+ 0.36$$

$$\Rightarrow 10^5 y_{12.2} = 33498.5 - 1026.3 + 22.76 + 0.36$$
$$\Rightarrow 10^5 y_{12.2} = 32495.32$$
$$\therefore y_{12.2} = 0.32495 \text{ Ans}$$

v.) Laplace - Everett formula :-

$$y_n = u y_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-1^2)}{5!}$$
$$\times \Delta^4 y_{-1} + \dots\dots\dots$$
$$+ v y_0 + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-2^2)}{5!}$$
$$\times \Delta^4 y_{-2} + \dots\dots$$

Where

$$v = 1 - u$$

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Q.) Apply Everett's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$

← Difference Table →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-1}	20	2854_{-1}			
			308_{-1}		
x_0	24	3162_0		74_{-1}	
			382_0		-8_{-1}
x_1	28	3544_1		66_0	
			448_1		
x_2	32	3992_2			

$$x_n = x_0 + uh$$
$$\Rightarrow 25 = 24 + 4u$$
$$\therefore u = 0.25$$

$$v = 1 - u$$
$$\Rightarrow v = 1 - 0.25$$
$$\therefore v = 0.75$$

$$y_{25} = uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + vy_0 + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1}$$

$$\Rightarrow y_{25} = 0.25 \times 3544 + \frac{0.25[(0.25)^2-1]}{6} \times 66 + 0.75 \times 3162 + \frac{0.75[(0.75)^2-1]}{6} \times 74$$

$$\Rightarrow y_{25} = 886 - 2.578125 + 2371.5 - 4.046875$$
$$\therefore y_{25} = 3250.875 \text{ Ans}$$

10. Interpolation with unequal interval

→ Divided difference Table:-

Q.) Construct a divided difference Table:-

x = 4 5 7 10 11 13

y = 48 100 294. 900. 1210 2028

← Divided difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0
10	900		27		0	
		310		1		
11	1210		33			
		409				
13	2080					

11.) Newton Divided Difference Table for unequal Interval:-

$$f(x) = f(x_0) + (x-x_0)\, f(x_0, x_1) + (x-x_0)(x-x_1)\, f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2)\, f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\, f(x_0, x_1, x_2, x_3, x_4) + \dots$$

Q.) find the value of y when $x = 10$.

x = 5 6 9 11

y = 12 13 14 16

← Divided Difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12			
		1		
6	13		-0.167	
		0.333		0.05
9	14		0.133	
		1		
11	16			

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) + f(x_0, x_1, x_2, x_3)$$

$$\Rightarrow f(10) = 12 + (10 - 5) \times 1 + (10 - 5)(10 - 6) \times (+ (10 - 5)(10 - 6)(10 - 9) \times (0.05)$$

$$\Rightarrow f(10) = 12 + 5 + 20 \times (-0.167) + 20 \times ($$

$$\Rightarrow f(10) = 12 + 5 - 3.34 + 1$$

$$\therefore f(10) = 14.66$$

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Q.) Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, −21 when x has the values 3, 2, 1, −1 respectively by using Newton divided difference table.

← Divided Difference Table →

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
−1	−21			
		18		
			−7	
1	15			1
		−3		
			−3	
2	12			
		−9		
3	3			

$f(x) = -21 + (x+1)\,18 + (x+1)(x-1) \times (-7)$
$\quad + (x+1)(x-1)(x-2)$

$\Rightarrow f(x) = -21 + 18 + 18 + (x^2-1)(-7) + (x^2-1)(x-2)$
$\Rightarrow f(x) = -3 + 18x - 7x^2 + 7 + x^3 - x - 2x^2 + 2$
$\therefore f(x) = x^3 - 9x^2 + 11x + 6$

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12. Lagrange's Interpolation formula for unequal Intervals.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} \times f(x_0)$$
$$+ \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \times f(x_1)$$
$$+ \cdots \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} \times f(x_n)$$

Q.) find the value $\log_{10} 656$ when
$\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$
$\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$.
Here,

$x_0 = 654$ $\quad f(x_0) = 2.8156$
$x_1 = 658$ $\quad f(x_1) = 2.8182$
$x_2 = 659$ $\quad f(x_2) = 2.8189$
$x_3 = 661$ $\quad f(x_3) = 2.8202$

$x = 656.$

Now

$$y = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \times 2.8156$$
$$+ \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} \times 2.8182$$
$$+ \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} \times 2.8189$$
$$+ \frac{(656-654)(656-658)(656-659)}{(661-654)(661-658)(661-659)} \times 2.8202$$

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$$\Rightarrow y = \frac{(-2)\times(-3)\times(-5)}{(-4)\times(-5)\times(-7)} \times 2.8156 + \frac{2\times(-3)\times(-5)}{4\times(-1)\times(-3)} \times 2.818$$

$$+ \frac{(-2)\times 2\times(-5)}{5\times 1\times(-2)} \times 2.8189 + \frac{2\times(-2)\times(-3)}{7\times 3\times 2} \times 2.8202$$

$$\Rightarrow y = 0.603 + 7.0455 - 5.6378 + 0.806$$

$$\therefore y = 2.8167$$

Ans

UNIT-III. NUMERICAL DIFFERENTIATION & INTEGRATION

1. Newton Forward.

$$y = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0 + \cdots\cdots$$

$$y = y_0 + n\Delta y_0 + \frac{n^2-n}{2}\Delta^2 y_0 + \frac{n^3-3n^2+2n}{6}\Delta^3 y_0 + \frac{n^4-6n^3+11n^2-6n}{24}\Delta^4 y_0 + \cdots\cdots$$

$\because\ x = x_0 + nh$

$1 = 0 + h\frac{dn}{dx}$

$\frac{dn}{dx} = \frac{1}{h}$

$$\frac{dy}{dx} = \frac{dy}{dn}\left(\frac{dn}{dx}\right) \quad \text{——— (i)}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[0 + \Delta y_0 + \frac{(2n-1)}{2}\Delta^2 y_0 + \frac{3n^2-6n+2}{6}\Delta^3 y_0 + \frac{(4n^3-18n^2+22n-6)}{24} \times \Delta^4 y_0 + \cdots\cdots\right]$$

When

$x = x_0 \ (\therefore n = 0)$

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$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

Derived of equation (i)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h}\left[\Delta^2 y_0 + \left(\frac{6x-6}{6}\right)\Delta^3 y_0 + \left(\frac{12x^2-36x+100}{24}\right)\Delta^4 y_0 + \cdots\right]$$

When

$x = x_0$ $(\therefore x=0)$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \cdots\right]$$

Q.) x : 1 2 3 4 5
y : 1 4 9 16 25

find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $x=2$

⟵ Difference Table ⟶

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	1	1				
			3			
x_0	2	4				
				2		
			5			
	3	9			0	
				2		
						0
			7			
	4	16			0	
				2		
			9			
	5	25				

$$x_n = x_0 + nh$$
$$2 = 2 + n \times 1$$
$$\therefore n = 0$$

Now,

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[0 + \Delta y_0 - \frac{1}{2}\Delta^2 y_0\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{\cancel{h}}\left[0 + 5 - \frac{1}{\cancel{2}} \times \cancel{2}\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 5 - 1$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 4$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_0\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{1} \times 2 = 2$$

2. Newton Backward

$$y_n = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_0 + \frac{n(n+1)(n+2)(n+3)}{4!} \times \nabla^4 y_0 + \cdots$$

$$y_n = y_0 + n\nabla y_0 + \frac{n^2+n}{2}\nabla^2 y_0 + \frac{n^3+3n^2+2}{6} \times \nabla^3 y_0 + \frac{n^4+6n^3+11n^2+6}{24} \times \nabla^4 y_0$$

Here, $x = x_0 + nh$

$1 = 0 + h\frac{dn}{dx}$

$\therefore \frac{dn}{dx} = \frac{1}{h}$

$$\frac{dy}{dx} = \frac{dy}{dn} \times \frac{dn}{dx} \quad \text{——— (i)}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\left[0 + \nabla y_0 + \frac{2n+1}{2} \times \nabla^2 y_0 + \frac{3n^2+6n}{6} \times \nabla^3 y_0 + \frac{4n^3+18n^2+22n+6}{24} \times \nabla^4 y_0 + \cdots\right]}$$

Where,

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\nabla^4 y_0 + \frac{1}{5}\nabla^5 y_0 + \frac{1}{6}\nabla^6 y_0 + \dots\right.$$

Derived of Equation (i)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 + \frac{6n+6}{6}\nabla^3 y_0 + \frac{12n^2+36n+22}{24} \times \nabla^4 y_0 + \dots\right]$$

Where,

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \frac{5}{6}\nabla^5 y_0 + \frac{137}{180}\nabla^6 y_0 + \dots\right]$$

Q.) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $x = 5$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	1	1				
			3			
	2	4		2		
			5		0	
	3	9		2		0
			7		0	
	4	16		2		
			9			
x_0	5	25				

$$x = x_0 + nh$$
$$\Rightarrow 5 = 5 + n \times 1$$
$$\therefore n = 0$$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0\right]$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{1}\left[9 + \frac{1}{2} \times 2\right]$$
$$\therefore \left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{1}\left[9 + 1\right] = 10$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\nabla^2 y_0\right]$$
$$\therefore \left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{1} \times 9 = 9$$

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3. Derivative by using Stirling formula.

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2} + \frac{n(n^2-1)(n^2-2^2)}{5!}\left[\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right] + \frac{n^2(n^2-1)(n^2-2^2)}{6!} \times \Delta^6 y_{-3}$$

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2}\Delta^2 y_{-1} + \frac{n^3-n}{6}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^4-n^2}{24} \times \Delta^4 y_{-2} + \frac{n^5-5n^3+4n}{120}\left[\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right] + \frac{n^6-5n^4+4n^2}{720}\Delta^6 y_{-3}$$

$$x = x_0 + nh$$

$$\Rightarrow 1 = 0 + h\frac{dn}{dx}$$

$$\therefore \frac{dn}{dx} = \frac{1}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dn} \times \frac{dn}{dx} \qquad \text{——— (i)}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[0 + \left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + n\Delta^2 y_{-1} + \frac{3n^2-1}{6}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{4n^3-2n}{24} \times \Delta^4 y_{-2} + \frac{5n^4-15n^2+4}{120}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \frac{6n^5-20n^3+8n}{720} \times \Delta^6 y_{-3} + \cdots\right]$$

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Where

$x = x_0 \quad (\therefore n = 0)$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \cdots\right]$$

Derived of equation (i)

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[0+0+\Delta^2 y_{-1} + n\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{12n^2-2}{24} \times \Delta^4 y_{-2} + \frac{20n^3-30n}{120}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \frac{30n^4-60n^2+8}{720} \times \Delta^6 y_{-3} + \cdots\right.$$

Where:

$x = x_0 \quad (\therefore n = 0)$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3} + \cdots\right]$$

Q.) find the value of $f'(x)$ & $f''(x)$ at $x = 0.04$ where.

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_3	0.01	0.1023					
			0.0024				
x_2	0.02	0.1047		0			
			0.0024		0.0001		
x_1	0.03	0.1071		0.0001		-0.0001	
			0.0025		0		0
x_0	0.04	0.1096		0.0001		-0.0001	
			0.0026		-0.0001		
x_1	0.05	0.1122		0			
			0.0026				
x_2	0.06	0.1148					

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01}\left[\left(\frac{0.0026+0.0025}{2}\right) - \frac{1}{6}\left[\frac{-0.000}{2}\right.\right.$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01}\left[0.00255 + 0.000008\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01} \times 0.002558 = 0.2558$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001}\left[0.0001 + \frac{1}{12} \times 0.0001\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001}\left[0.0001 + 0.000008\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001} \times 0.000108$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=0} = 1.08$$

Derivative by using Newton divided difference formula :-

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x_0) + \dots$$

$$f'(x) = 0 + \Delta f(x_0) + [(x-x_0)+(x-x_1)]\Delta^2 f(x_0) + [(x-x_0)(x-x_1) + (x-x_2)[(x-x_0)+(x-x_2)(x-x_1)]]\Delta^3 f(x_0) + [(x-x_0)(x-x_1)(x-x_2)+(x-x_3)]] [(x-x_0)(x-x_1)(x-x_2)(x-x_6) + (x-x_2)(x-x_1)]] \Delta^4 f(x_0)$$

$$\boxed{\begin{aligned} f'(x) &= \Delta f(x_0) + [(x-x_0)+(x-x_1)]\Delta^2 f(x_0) + [(x-x_0)(x-x_1)+(x-x_0)(x-x_2)+(x-x_2)(x-x_1)] \\ &\times \Delta^3 f(x_0) + [(x-x_0)(x-x_1)(x-x_2)+(x-x_0)(x-x_1)(x-x_3) + (x-x_0)(x-x_2)(x-x_3) + (x-x_2)(x-x_1)(x-x_3)]\Delta^4 f(x_0) \\ &+ [(x-x_0)(x-x_1)(x-x_2)(x-x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_4) + (x-x_0)(x-x_1)(x-x_3)(x-x_4) + (x-x_2) \\ &\quad (x-x_0)(x-x_3)(x-x_4) + (x-x_1)(x-x_2)(x-x_3)(x-x_4)]\Delta^5 f(x_0) + \dots \end{aligned}}$$

$$f''(x) = 0 + 2\Delta^2 f(x_0) + [(x-x_0)+(x-x_1)+(x-x_0) + (x-x_2)+(x-x_2)+(x-x_1)]\Delta^3 f(x_0) + [(x-x_0)(x-x_1)+(x-x_0)(x-x_2)+(x-x_1)(x-x_2) + (x-x_0)(x-x_1)+(x-x_0)(x-x_3)+(x-x_1)(x-x_3) - (x-x_2)(x-x_0) + (x-x_2)(x-x_3) + (x-x_0)(x-x_3) + (x-x_2)(x-x_1) + (x-x_2)(x-x_3) + (x-x_1)(x-x_3)]\Delta f(x_0)$$

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$$f''(x) = 2\Delta^2 f(x_0) + 2[(x-x_0)+(x-x_1)+(x-x_2)] \Delta^3 f(x_0) + 2[(x-x_0)(x-x_1)+(x-x_0)(x-x_2) + (x-x_0)(x-x_3) + (x-x_1)(x-x_2) + (x-x_1)(x-x_3) + (x-x_2)(x-x_3)] \Delta^4 f(x_0)$$

$$f'''(x) = 0 + 6\Delta f(x_0) + \ldots$$

Q.) Find the first three derivatives of the function at $x = 2.5$.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1.5	3.375					
		6.71				
1.9	6.059		5.9			
		12.61		1		
2.5	13.625		7.6		0	
		22.49		1		0
3.2	29.368		10		0	
		40.49		1		
4.3	73.907		13.4			
		76.67				
5.9	196.579					

$f'(x) = 6.71 + [(2.5-1.5)+(2.5-1.9)] \times 5.9$
$+ [(2.5-1.5)(2.5-1.9) + (2.5-1.5)(2.5-2.5) +$
$(2.5-2.5)(2.5-1.9)] \times 1$

$\Rightarrow f'(x) = 6.71 + [1+0.6] \times 5.9 + [1 \times 0.6 + 1 \times 0 + 0 \times 0.6] \times 1$

$\Rightarrow f'(x) = 6.71 + 9.44 + 1.6$
$\therefore f'(x) = 17.75$

$f''(x) = 2 \times 5.9 + 2[(2.5-1.5) + (2.5-1.9)(2.5-2.5)$
$] \times 1$

$\Rightarrow f''(x) = 11.8 + 2[1+0.6+0] \times 1$

$\Rightarrow f''(x) = 11.8 + 2 \times 1.6$
$\therefore f''(x) = 15$

$f'''(x) = 6 \times 6.71$
$\therefore f'''(x) = 40.26$

5. Trapezoidal Rule :-

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1+y_2+\dots y_{n-1})\right]$$

Q.) $\int_0^1 x^2\,dx$

let $h = \frac{1}{6}$

$x =$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y =$	0	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	1

$$\int_0^1 x^2\,dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{6\times 2}\left[(0+1) + 2\left(\frac{1}{36}+\frac{4}{36}+\frac{9}{36}+\frac{16}{36}+\frac{25}{36}\right)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\left[1 + 2\times\frac{55}{36}\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\left[1 + 3.056\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\times 4.056$$

$$\therefore \int_0^1 x^2\,dx = 0.338$$

6. Simpson's one third (1/3) rule :-

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots y_{n-1}) + 2(y_2+y_4+\cdots y_{n-2})\right]$$

Q.) $\int_0^1 x^2\,dx$

Let $h = \frac{1}{6}$

$x = 0 \quad \frac{1}{6} \quad \frac{2}{6} \quad \frac{3}{6} \quad \frac{4}{6} \quad \frac{5}{6} \quad 1$

$y = 0 \quad \frac{1}{36} \quad \frac{4}{36} \quad \frac{9}{36} \quad \frac{16}{36} \quad \frac{25}{36} \quad 1$

$$\int_0^1 x^2\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{6\times3}\left[(0+1) + 4\left(\frac{1}{36}+\frac{9}{36}+\frac{25}{36}\right) + 2\left(\frac{4}{36}+\frac{16}{36}\right)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{18}\left[1 + 3.889 + 1.111\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{6}{18}$$

$$\therefore \int_0^1 x^2\,dx = 0.333$$

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7. Simpson's three-eight (3/8) rule:

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3h}{8}\Big[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8+\dots y_{n-1})+2(y_3+y_6+y_9+\dots y_{n-3})\Big]$$

Q.) $\int_3^5 \frac{4}{2+x^2}\,dx$ interval 8

let $h = \frac{5-3}{8} = \frac{2}{4} = \frac{1}{4}$

x :	3	$\frac{13}{4}$	$\frac{14}{4}$	$\frac{15}{4}$	$\frac{16}{4}$	$\frac{17}{4}$	$\frac{18}{4}$	$\frac{19}{4}$	5
y :	0.364	0.318	0.281	0.249	0.222	0.199	0.180	0.163	0.148

$$\int_3^5 \frac{4}{2+x^2}\,dx = \frac{3h}{8}\Big[(y_0+y_8)+3(y_1+y_2+y_4+y_5+y_7)+2(y_3+y_6)\Big]$$

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$$\Rightarrow \int_3^{5.1} \frac{4}{2+x^2}\,dx = \frac{3}{4\times 8}\Big[(0.364+0.148)+3(0.318+0.281+0.222+0.199+0.163)+2(0.249+0.180)\Big]$$

$$\Rightarrow \int_3^{5.1} \frac{4}{2+x^2}\,dx = \frac{3}{32}\Big[0.512+3.549+0.858\Big]$$

$$\Rightarrow \int_3^{5.1} \frac{4}{2+x^2}\,dx = \frac{14.757}{32}$$

$$\therefore \int_3^{5.1} \frac{4}{2+x^2}\,dx = 0.461 \quad \text{Ans}$$

Q.) Find the approximate area of cross section of river 80 m wide and the depth y (m) to a distance x from one bank being given by the following table.

x =	0	10	20	30	40	50	60	70	80
y =	0	4	7	9	12	15	14	8	3

→ Here,

$h = 10$

$$\int_0^{80} y\,dx = \frac{h}{3}\Big[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\Big]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{10}{3}\Big[(0+3)+4(4+9+15+8)+2(7+12+14)\Big]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{10}{3}\left[3 + 144 + 66\right]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{2130}{3}$$

$$\therefore \int_0^{80} y\,dx = 710\ m^2$$

A

Q.) A slider is a machine move along a fix straight rod, Its distance x cm along the rod is given below for the various value for time t sec. find the velocity and acceleration of the slider when $t = 0.3$ sec.

t : 0 0.1 0.2 0.3 0.4 0.5 0.6

x : 30.13 31.62 32.87 33.64 33.95 33.81 33.24

← Difference Table. →

	t	x	Δx	$\Delta^2 x$	$\Delta^3 x$	$\Delta^4 x$	$\Delta^5 x$	$\Delta^6 x$
x_{-3}	0	30.13						
			1.49					
x_{-2}	0.1	31.62		-0.24				
			1.25		-0.24			
x_{-1}	0.2	32.87		-0.48		0.26		
			0.77		0.02		-0.027	
x_0	0.3	33.64		-0.46		-0.01		0.29.
			0.31		0.01		0.02	
	0.4	33.95		-0.45		0.01		
			-0.14		0.02			
	0.5	33.81		-0.43				
			-0.57					
	0.6	33.24						

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$$x_n = x_0 + nh$$

$$\Rightarrow 0.3 = 0.3 + n \times 0.1$$

$$\therefore n = 0$$

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{6}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right]\right.$$

$$\cancel{\Rightarrow \left(\frac{dy}{dx}\right)_{x=0}} \quad \left. + \frac{1}{30}\left[\Delta^5 y_{-2} + \Delta^5 y_{-3}\right]\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.1}\left[\left(\frac{0.31 + 0.77}{2}\right) - \frac{1}{6}\left(\frac{0.01 + 0.02}{2}\right) + \frac{1}{30}\left(\frac{0.02 - 0.27}{2}\right)\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.1}\left[0.54 - 0.0025 - 0.0042\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 5.333 \text{ cm/s}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3}\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.01}\left[-0.46 + \frac{1}{12} \times 0.08 + \frac{1}{90} \times 0.29\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.01}\left[-0.46 + 0.0068 + 0.003\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=0} = -45.62 \text{ cm/s}^2$$

8. Maxima and Minima :-

$$y = f(x)$$

i.) $\frac{dy}{dx} = 0$

ii.) $\frac{d^2y}{dx^2} > 0$ (minima) · or

iii.) $\frac{d^2y}{dx^2} < 0$ (maxima)

iv.) $f(x_1)$

v.) Newton forward different.

Q.) From the given table for what value x, y is minimum & what is the minimum value

x :	3	4	5	6	7	8
y :	0.205	0.240	0.259	0.262	0.250	0.224

← Difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
3	0.205					
		0.035				
4	0.240		-0.016			
		0.019		0		
5	0.259		-0.016		0.001	
		0.003		0.001		-0.001
6	0.262		-0.015		0	
		-0.012		0.001		
7	0.250		-0.014			
		-0.026				
8	0.224					

Newton forward.

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2n-1}{2}\Delta^2 y_0\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1}\left[0.035 - \frac{2n-1}{2} \times 0.016\right]$$

$$\Rightarrow \frac{dy}{dx} = 0.035 - \frac{0.032n - 0.016}{2}$$

$$\Rightarrow 0 = 0.035 - 0.016n - 0.008$$

$$\Rightarrow -0.016n = -0.0027$$

$$\Rightarrow n = 1.6878$$

$$\therefore n = 1.7$$

$$x_n = x_0 + nh$$

$$\Rightarrow x_n = 3 + 1.7 \times 1$$

$$\therefore x_n = 4.7.$$

Starling formula.

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right]$$

$$\Rightarrow y_n = 0.240 + 1.7\left[\frac{0.019 + 0.035}{2}\right]$$

$$\Rightarrow y_n = 0.240 \times 1.7 \times 0.027$$

$$\therefore y_n = 0.2859 \quad \text{Ans}$$

UNIT-IV. SOLUTION OF LINEAR EQUATION

1. Gauss's Elimination method :-

Q.) $x + 4y - z = -5$ —— (i)
$x + y - 6z = -12$ —— (ii)
$3x - y - z = 4$ —— (iii)

Elimination x from equation (ii) & (iii) by using equation (i)

Apply :-

eqn (ii) − eqn (i)

$$\begin{array}{r} \cancel{x} + y - 6z = -12 \\ \cancel{x} + 4y - z = -5 \\ - \quad - \quad + \qquad + \\ \hline -3y - 5z = -7 \end{array}$$

$\Rightarrow 3y + 5z = 7$ —— (iv)

eqn (iii) − 3 eqn (i)

$$\begin{array}{r} 3\cancel{x} - y - z = 4 \\ \cancel{3x} + 12y - 3z = -15 \\ - \quad - \quad + \qquad + \\ \hline -13y + 2z = +19 \end{array}$$

$\Rightarrow 13y - 2z = -19$ —— (v)

Eliminate y from eqn (v) by using eqn (iv)

Apply

$$\text{eqn (v)} - \frac{13}{3}\,\text{eqn (iv)}$$

$$13y - 2z = -19$$
$$13y + \frac{65}{3}z = \frac{91}{3}$$
$$- \quad - \quad -$$

$$-2z - \frac{65z}{3} = -19 - \frac{91}{3}$$

$$\Rightarrow \frac{-6z - 65z}{3} = \frac{-57 - 91}{3}$$

$$\Rightarrow \frac{-71z}{3} = \frac{-148}{3}$$

$$\therefore z = \frac{148}{71}$$

put z in eqn (iv)

$$13y - 2z = -19$$

$$\Rightarrow 13y - 2 \times \frac{148}{71} = -19$$

$$\Rightarrow 13y = -19 + \frac{296}{71}$$

$$\Rightarrow 13y = \frac{-1349 + 296}{71}$$

$$\Rightarrow y = \frac{-1053}{923}$$

$$\therefore y = -\frac{81}{71}$$

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Put y & z in eqn (i)

$$x + 4y - z = -5$$

$$\Rightarrow x + 4\times\left(\frac{-81}{71}\right) - \frac{148}{71} = -5$$

$$\Rightarrow x = -5 + \frac{324}{71} + \frac{148}{71}$$

$$\therefore x = \frac{117}{71}$$

Here,

$$x = \frac{117}{71}, \quad y = \frac{-81}{71}, \quad z = \frac{148}{71}$$

2. Gauss's Seidal iterative method.

Q.)
$$2 0x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Firstly select x.y.z

$$x = \frac{1}{20}(17 - y + 2z) \text{ ---- (i)}$$

$$y = \frac{1}{20}(-18 - 3x + z) \text{ ---- (ii)}$$

$$z = \frac{1}{20}(25 - 2x + 3y) \text{ ---- (iii)}$$

Iteration 1.

Let $y=0$, $z=0$ Put in equation (i)

$$x^{(1)} = \frac{1}{20}(17-0+0) = \frac{1}{20}\times 17 = 0.85$$

Now,

$x^{(1)} = 0.85$, $z=0$ Put in eqn (ii)

$$y^{(1)} = \frac{1}{20}(-18-3\times 0.85+0)$$

$$\Rightarrow y^{(1)} = \frac{1}{20}\times(-20.55)$$

$$\therefore y^{(1)} = -1.0275$$

Now,

$x^{(1)} = 0.85$, $y^{(1)} = -1.0275$

$$z^{(1)} = \frac{1}{20}\left[25 - 2\times 0.85 + 3\times(-1.0275)\right)$$

$$\Rightarrow z^{(1)} = \frac{1}{20}\times 20.2175$$

$$\therefore z^{(1)} = 1.010875 = 1.0109$$

Iteration-2

$x^{(1)} = 0.85$, $y^{(1)} = -1.0275$, $z^{(1)} = 1.0109$

Put in eqn (i)

$$x^{(2)} = \frac{1}{20}(17+1.0275+2\times 1.0109)$$

$$\Rightarrow x^{(2)} = \frac{1}{20}\times 20.0493$$

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$\therefore x^{(2)} = 1.0025$

Now,

$x^{(2)} = 1.0025$, $z^{(1)} = 1.0109$

Put in eqn (ii)

$$y^{(2)} = \frac{1}{20}\left(-18 - 3 \times 1.0025 + 1.0109\right)$$

$$\Rightarrow y^{(2)} = \frac{1}{20} \times (-19.9966)$$

$$\therefore y^{(2)} = -0.9998$$

Now,

$x^{(2)} = 1.0025$, $y^{(2)} = -0.9998$

Put in eqn (iii)

$$z^{(2)} = \frac{1}{20}\left(25 - 2 \times 1.0025 - 3 \times 0.9998\right)$$

$$\Rightarrow z^{(2)} = \frac{1}{20} \times 19.9956$$

$$\therefore z^{(2)} = 0.99978$$

Iteration

$x^{(2)} = 1.0025$, $y^{(2)} = -0.9998$, $z^{(2)} = 0.9998$

Put in eqn (i)

$$x^{(3)} = \frac{1}{20}\left(17 + 0.9998 + 2 \times 0.9998\right)$$

$$\Rightarrow x^{(3)} = \frac{1}{20} \times 19.9994$$

$$\therefore x^{(3)} = 0.99997$$

Now,
$$x^{(3)} = 0.9997 \quad , \quad z^{(2)} = 0.9998$$
Put the eqn (ii)

$$y^{(3)} = \frac{1}{20}\left(-18 - 3\times 0.9997 + 0.9998\right)$$
$$\Rightarrow y^{(3)} = \frac{1}{20}\times(-19.9993)$$
$$\therefore y^{(3)} = -0.9996.$$

Now,
$$x^{(3)} = 0.99997 \quad , \quad y^{(3)} = -0.99996$$
Put in eqn (iii)

$$z^{(3)} = \frac{1}{20}\left(25 + 2\times 0.99997 - 3\times 0.99996\right)$$
$$\Rightarrow z^{(3)} = \frac{1}{20}\times 20.00018$$
$$\therefore z^{(3)} = 1.000009$$

Here,
$$x = 1$$
$$y = -1$$
$$z = 1$$

1. Picard's Method.

$$y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{n-1})\, dx$$

Q.) Find the solution of $\frac{dy}{dx} = 1 + xy$ which passes through $(0,1)$ at $x = 0.2$.

$f(x,y) = 1 + xy$, $x_0 = 0$, $y_0 = 1$, $x = 0.2$

Iteration 1

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y^{(0)}) \cdot dx$$

$$\Rightarrow y^{(1)} = 1 + \int_0^x f(x, 1)\, dx$$

$$\Rightarrow y^{(1)} = 1 + \int_0^x (1+x)\, dx$$

$$\Rightarrow y^{(1)} = 1 + \left[x + \frac{x^2}{2} \right]_0^x$$

$$\Rightarrow y^{(1)} = 1 + x + \frac{x^2}{2}$$

$$\Rightarrow y^{(1)}(0.2) = 1 + 0.2 + \frac{0.04}{2}$$

$$\therefore y^{(1)}(0.2) = 1.22$$

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Iteration-2

$$y^{(2)} = y_0 + \int_0^x f(x, y^{(1)})\,dx$$

$$\Rightarrow y^{(2)} = y_0 + \int_0^x f\left(x, 1 + x + \frac{x^2}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left[1 + x\left(1 + x + \frac{x^2}{2}\right)\right] dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left(1 + x + x^2 + \frac{x^3}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]_0^x dx$$

$$\Rightarrow y^{(2)} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

$$\Rightarrow y^{(2)}(0.2) = 1 + 0.2 + 0.02 + 0.003 + 0.0002$$

$$\therefore y^{(2)}(0.2) = 1.2232$$

Here,

$$y = 1.22$$

A

2.) Euler's Method :-

$$\left[\, y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}) \,\right]$$

Q.) Find the solution of differential equation $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0$, $y_0 = 1$ by using Euler method at $x = 0.5$ taking $h = 0.1$.

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$f(x,y) = x+y$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

by Euler's Method.

$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

Now,

$y_1 = y_0 + hf(x_0, y_0)$

$\Rightarrow y_1 = 1 + 0.1 \cdot f(0, 1)$

$\Rightarrow y_1 = 1 + 0.1 \times 1$

$\therefore y_1 = 1.1$

$x_1 = x_0 + h$

$\Rightarrow x_1 = 0 + 0.1$

$\therefore x_1 = 0.1$

$y_2 = y_1 + hf(x_1, y_1)$

$\Rightarrow y_2 = 1.1 + 0.1 f(0.1, 1.1)$

$\Rightarrow y_2 = 1.1 + 0.1 \times 1.2$

$\therefore y_2 = 1.22.$

$x_2 = x_1 + h$

$\Rightarrow x_2 = 0.1 + 0.1$

$\therefore x_2 = 0.2$

$y_3 = y_2 + hf(x_2, y_2)$

$\Rightarrow y_3 = 1.22 + 0.1 f(0.2, 1.22)$

$\Rightarrow y_3 = 1.22 + 0.1 \times 1.42$

$\therefore y_3 = 1.362$

$x_3 = x_2 + h$
$\Rightarrow x_3 = 0.2 + 0.1$
$\therefore x_3 = 0.3$

$y_4 = y_3 + hf(x_3, y_3)$
$\Rightarrow y_4 = 1.362 + 0.1 f(0.3, 1.362)$
$\Rightarrow y_4 = 1.362 + 0.1 \times 1.662$
$\therefore y_4 = 1.5282$

$x_4 = x_3 + h$
$\Rightarrow x_4 = 0.3 + 0.1$
$\therefore x_4 = 0.4$

$y_5 = y_4 + hf(x_4, y_4)$
$\Rightarrow y_5 = 1.5282 + 0.1 (0.4, 1.5282)$
$\Rightarrow y_5 = 1.5282 + 0.1 \times 1.9282$
$\therefore y_5 = 1.72102$

$x_5 = x_4 + h$
$\Rightarrow x_5 = 0.4 + 0.1$
$\therefore x_5 = 0.5$

Now,

x	y
0	1
0.1	1.1
0.2	1.22
0.3	1.362
0.4	1.5282
0.5	1.72102