UNIT-III. NUMERICAL DIFFERENTIATION & INTEGRATION

1. Newton Forward.

$$y = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0 + \cdots\cdots$$

$$y = y_0 + n\Delta y_0 + \frac{n^2-n}{2}\Delta^2 y_0 + \frac{n^3-3n^2+2n}{6}\Delta^3 y_0 + \frac{n^4-6n^3+11n^2-6n}{24}\Delta^4 y_0 + \cdots\cdots$$

$$\because x = x_0 + nh$$

$$1 = 0 + h\frac{dn}{dx}$$

$$\frac{dn}{dx} = \frac{1}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dn}\left(\frac{dn}{dx}\right) \quad \text{—— (i)}$$

$$\frac{dy}{dx} = \frac{1}{h}\Bigg[0 + \Delta y_0 + \frac{(2n-1)}{2}\Delta^2 y_0 + \frac{3n^2-6n+2}{6}\Delta^3 y_0 + \frac{(4n^3-18n^2+22n-6)}{24} \times \Delta^4 y_0 + \cdots\cdots\Bigg]$$

When

$x = x_0$ $(\therefore n = 0)$

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$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \dots\right]$$

Derrived of equation (i)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h}\left[\Delta^2 y_0 + \left(\frac{6x-6}{6}\right)\Delta^3 y_0 + \left(\frac{12x^2-36x+100}{24}\right)\Delta^4 y_0 + \dots\right]$$

When

$x = x_0 \ (\therefore x = 0)$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \dots\right]$$

Q.)

x :	1	2	3	4	5
y :	1	4	9	16	25

find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $x = 2$

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← Difference Table →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	1	1				
			3			
x_0	2	4		2		
			5		0	
	3	9		2		0
			7		0	
	4	16		2		
			9			
	5	25				

$$x_n = x_0 + nh$$
$$2 = 2 + n \times 1$$
$$\therefore n = 0$$

Now,

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[0 + \Delta y_0 - \frac{1}{2}\Delta^2 y_0\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[0 + 5 - \frac{1}{\cancel{2}} \times \cancel{2}\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{n=0} = 5 - 1$$

$$\therefore \left(\frac{dy}{dx}\right)_{n=0} = 4$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\Delta^2 y_0\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{1} \times 2 = 2$$

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2. Newton Backward

$$y_n = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_0 + \frac{n(n+1)(n+2)(n+3)}{4!} \times \nabla^4 y_0 + \cdots$$

$$y_n = y_0 + n\nabla y_0 + \frac{n^2+n}{2}\nabla^2 y_0 + \frac{n^3+3n^2+2}{6} \times \nabla^3 y_0 + \frac{n^4+6n^3+11n^2+6}{24} \times \nabla^4 y_0$$

Here, $x = x_0 + nh$

$1 = 0 + h\frac{dn}{dx}$

$\therefore \frac{dn}{dx} = \frac{1}{h}$

$$\frac{dy}{dx} = \frac{dy}{dn} \times \frac{dn}{dx} \quad \text{——— (i)}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\left[0 + \nabla y_0 + \frac{2n+1}{2} \times \nabla^2 y_0 + \frac{3n^2+6n}{6} \times \nabla^3 y_0 + \frac{4n^3+18n^2+22n+6}{24} \times \nabla^4 y_0 + \cdots\right]}$$

Where,

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 + \frac{1}{5}\nabla^5 y_0 + \frac{1}{6}\nabla^6 y_0 + \dots\right]$$

Derived of Equation (i)

$$\frac{d^2y}{dx^2} = \frac{d}{dn}\left(\frac{dy}{dx}\right)\frac{dn}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 + \frac{6n+6}{6}\nabla^3 y_0 + \frac{12n^2+36n+22}{24} \times \nabla^4 y_0 + \dots\right]$$

Where,

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \frac{5}{6}\nabla^5 y_0 + \frac{137}{180}\nabla^6 y_0 + \dots\right]$$

Q.) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $x = 5$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
		3			
2	4		2		
		5		0	
3	9		2		0
		7		0	
4	16		2		
		9			
x_0 5	25				

$$x = x_0 + nh$$
$$\Rightarrow 5 = 5 + n \times 1$$
$$\therefore n = 0$$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0\right]$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{1}\left[9 + \frac{1}{2} \times 2\right]$$
$$\therefore \left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{1}\left[9 + 1\right] = 10$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\nabla^2 y_0\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{1} \times 9 = 9$$

3. Derivative by using Starling formula.

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1}$$
$$+ \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2}$$
$$+ \frac{n(n^2-1)(n^2-2^2)}{5!}\left[\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right] + \frac{n^2(n^2-1)(n^2-2^2)}{6!} \times \Delta$$

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2}\Delta^2 y_{-1} + \frac{n^3-n}{6}$$
$$\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^4-n^2}{24} \times \Delta^4 y_{-2} + \frac{n^5-5n^3+4n}{120}$$
$$\left[\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right] + \frac{n^6-5n^4+4n^2}{720}\Delta^6 y_{-3}$$

$$x = x_0 \pm nh$$
$$\Rightarrow 1 = 0 + h\frac{dn}{dx}$$
$$\therefore \frac{dn}{dx} = \frac{1}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dn} \times \frac{dn}{dx} \quad \text{—— (i)}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[0 + \left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + n\Delta^2 y_{-1} + \right.$$
$$\frac{3n^2-1}{6}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{4n^3-2n}{24} \times \Delta^4 y_{-2} +$$
$$\frac{5n^4-15n^2+4}{120}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \frac{6n^5-20n^3+8n}{720} \times \Delta^6 y_{-3} + \dots$$

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Where

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{dy}{dx}\right)_{n=0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \cdots\right]$$

Derived of equation (i)

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[0 + 0 + \Delta^2 y_{-1} + n\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{12n^2 - 2}{24} \times \Delta^4 y_{-2} + \frac{20n^3 - 30n}{120}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \frac{30n^4 - 60n^2 + 8}{720} \times \Delta^6 y_{-3} + \cdots\right]$$

Where.

$$x = x_0 \quad (\therefore n = 0)$$

$$\left(\frac{d^2y}{dx^2}\right)_{n=0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3} + \cdots\right]$$

Q.) find the value of $f'(x)$ & $f''(x)$ at $x = 0.04$ where.

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	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_3	0.01	0.1023					
			0.0024				
x_2	0.02	0.1047		0			
			0.0024		0.0001		
x_1	0.03	0.1071		0.0001		-0.0001	
			0.0025		0		0
x_0	0.04	0.1096		0.0001		-0.0001	
			0.0026		-0.0001		
x_1	0.05	0.1122		0			
			0.0026				
x_2	0.06	0.1148					

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01}\left[\left(\frac{0.0026+0.0025}{2}\right) - \frac{1}{6}\left[\frac{-0.0001}{2}\right.\right.$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01}\left[0.00255 + 0.000008\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.01} \times 0.002558 = 0.2558$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001}\left[0.0001 + \frac{1}{12} \times 0.0001\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001}\left[0.0001 + 0.000008\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.0001} \times 0.000108$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=0} = 1.08$$

Derivative by using Newton divided difference formula :-

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x_0) + \dots$$

$$f'(x) = 0 + \Delta f(x_0) + [(x-x_0)+(x-x_1)]\Delta^2 f(x_0) + [(x-x_0)(x-x_1) + (x-x_2)[(x-x_0)+(x-x_2)(x-x_1)]]\Delta^3 f(x_0) + [(x-x_0)(x-x_1)(x-x_2) + (x-x_3)[(x-x_0)(x-x_1)(x-x_2)(x-x_0) + (x-x_2)(x-x_1)]]\Delta^4 f(x_0)$$

$$\boxed{\begin{aligned} f'(x) &= \Delta f(x_0) + [(x-x_0)+(x-x_1)]\Delta^2 f(x_0) + [(x-x_0)(x-x_1) + (x-x_0)(x-x_2) + (x-x_2)(x-x_1)] \\ &\times \Delta^3 f(x_0) + [(x-x_0)(x-x_1)(x-x_2) + (x-x_0)(x-x_1)(x-x_3) + (x-x_0)(x-x_2)(x-x_3) + (x-x_2)(x-x_1)(x-x_3)]\Delta^4 f(x_0) \\ &+ [(x-x_0)(x-x_1)(x-x_2)(x-x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_4) + (x-x_0)(x-x_1)(x-x_3)(x-x_4) + (x-x_2)(x-x_0)(x-x_3)(x-x_4) \\ &+ (x-x_1)(x-x_2)(x-x_3)(x-x_4)]\Delta^5 f(x_0) + \dots \end{aligned}}$$

$$f''(x) = 0 + 2\Delta^2 f(x_0) + [(x-x_0)+(x-x_1)+(x-x_0)+(x-x_2)+(x-x_2)+(x-x_1)]\Delta^3 f(x_0) + [(x-x_0)(x-x_1)+(x-x_0)(x-x_2)+(x-x_1)(x-x_2) + (x-x_0)(x-x_1)+(x-x_0)(x-x_3)+(x-x_1)(x-x_3) - (x-x_2)(x-x_0) + (x-x_2)(x-x_3) + (x-x_0)(x-x_3) + (x-x_2)(x-x_1) + (x-x_2)(x-x_3) + (x-x_1)(x-x_3)]\Delta f(x_0)$$

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$$f''(x) = 2\Delta^2 f(x_0) + 2[(x-x_0)+(x-x_1)+(x-x_2)]\Delta^3 f(x_0) + 2[(x-x_0)(x-x_1)+(x-x_0)(x-x_2) + (x-x_0)(x-x_3)+(x-x_1)(x-x_2)+(x-x_1)(x-x_3)+(x-x_2)(x-x_3)]\Delta^4 f(x_0)$$

$$f'''(x) = 0 + 6\Delta f(x_0) + \ldots$$

Q.) Find the first three derivatives of the function at $x = 2.5$.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1.5	3.375					
		6.71				
1.9	6.059		5.9			
		12.61		1		
2.5	13.625		7.6		0	
		22.49		1		0
3.2	29.368		10		0	
		40.49		1		
4.3	73.907		13.4			
		76.67				
5.9	196.579					

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$f'(x) = 6.71 + [(2.5-1.5)+(2.5-1.9)] \times 5.9$
$+ [(2.5-1.5)(2.5-1.9) + (2.5-1.5)(2.5-2.5) +$
$(2.5-2.5)(2.5-1.9)] \times 1$

$\Rightarrow f'(x) = 6.71 + [1+0.6] \times 5.9 + [1 \times 0.6 + 1 \times 0 + 0 \times 0.6] \times 1$

$\Rightarrow f'(x) = 6.71 + 9.44 + 1.6$

$\therefore f'(x) = 17.75$

$f''(x) = 2 \times 5.9 + 2[(2.5-1.5) + (2.5-1.9)(2.5-2.5)] \times 1$

$\Rightarrow f''(x) = 11.8 + 2[1 + 0.6 + 0] \times 1$

$\Rightarrow f''(x) = 11.8 + 2 \times 1.6$

$\therefore f''(x) = 15$

$f'''(x) = 6 \times 6.71$

$\therefore f'''(x) = 40.26$

5. Trapezoidal Rule :-

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1+y_2+\cdots y_{n-1})\right]$$

Q.) $\int_0^1 x^2\,dx$

let $h = \frac{1}{6}$

$x =$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y =$	0	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	1

$$\int_0^1 x^2\,dx = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{6\times 2}\left[(0+1)+2\left(\frac{1}{36}+\frac{4}{36}+\frac{9}{36}+\frac{16}{36}+\frac{25}{36}\right)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\left[1+2\times\frac{55}{36}\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\left[1+3.056\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{12}\times 4.056$$

$$\therefore \int_0^1 x^2\,dx = 0.338$$

6. Simpson's one third (1/3) rule :-

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots y_{n-1}) + 2(y_2+y_4+\cdots y_{n-2})\right]$$

Q.) $\int_0^1 x^2\,dx$

Let $h = \frac{1}{6}$

$x =$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$y =$	0	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	1

$$\int_0^1 x^2\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{6\times 3}\left[(0+1) + 4\left(\frac{1}{36}+\frac{9}{36}+\frac{25}{36}\right) + 2\left(\frac{4}{36}+\frac{16}{36}\right)\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{18}\left[1 + 3.889 + 1.111\right]$$

$$\Rightarrow \int_0^1 x^2\,dx = \frac{6}{18}$$

$$\therefore \int_0^1 x^2\,dx = 0.333$$

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7. Simpson's three-eight (3/8) rule:

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3h}{8}\Big[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8+\cdots y_{n-1})+2(y_3+y_6+y_9+\cdots y_{n-3})\Big]$$

Q.) $\int_3^5 \frac{4}{2+x^2}\,dx$ interval 8

Let $h = \frac{5-3}{8} = \frac{2}{4} = \frac{1}{4}$

x	3	$\frac{13}{4}$	$\frac{14}{4}$	$\frac{15}{4}$	$\frac{16}{4}$	$\frac{17}{4}$	$\frac{18}{4}$	$\frac{19}{4}$	5
y	0.364	0.318	0.281	0.249	0.222	0.199	0.180	0.163	0.148

$$\int_3^5 \frac{4}{2+x^2}\,dx = \frac{3h}{8}\Big[(y_0+y_8)+3(y_1+y_2+y_4+y_5\ldots)+2(y_3+y_6)\Big]$$

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$$\Rightarrow \int_3^{5'} \frac{4}{2+x^2}\,dx = \frac{3}{4\times 8}\Big[(0.364+0.148)+3(0.318+0.281+0.222+0.199+0.163)+2(0.249+0.180)\Big]$$

$$\Rightarrow \int_3^{5'} \frac{4}{2+x^2}\,dx = \frac{3}{32}\Big[0.512+3.549+0.858\Big]$$

$$\Rightarrow \int_3^{5'} \frac{4}{2+x^2}\,dx = \frac{14.757}{32}$$

$$\therefore \int_3^{5'} \frac{4}{2+x^2}\,dx = 0.461 \quad Ans$$

Q.) Find the approximate area of cross section of river 80 m wid and the depth y (m) to a distance x from one bank being given by the following table.

x =	0	10	20	30	40	50	60	70	80
y =	0	4	7	9	12	15	14	8	3

→ Here,

$h = 10$

$$\int_0^{80} y\,dx = \frac{h}{3}\Big[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\Big]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{10}{3}\Big[(0+3)+4(4+9+15+8)+2(7+12+14)\Big]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{10}{3}\left[3 + 144 + 66\right]$$

$$\Rightarrow \int_0^{80} y\,dx = \frac{2130}{3}$$

$$\therefore \int_0^{80} y\,dx = 710\ m^2$$

Ans

Q.) A slider is a machine move along a fix straight rod, Its distance x cm along the rod is given below for the various value for time t sec. find the velocity and acceleration of the slider when $t = 0.3$ sec.

t :	0	0.1	0.2	0.3	0.4	0.5	0.6
x :	30.13	31.62	32.87	33.64	33.95	33.81	33.24

← Difference Table. →

	t	x	Δx	$\Delta^2 x$	$\Delta^3 x$	$\Delta^4 x$	$\Delta^5 x$	$\Delta^6 x$
x_{-3}	0	30.13						
			1.49					
x_{-2}	0.1	31.62		-0.24				
			1.25		-0.24			
x_{-1}	0.2	32.87		-0.48		0.26		
			0.77		0.02		-0.027	
x_0	0.3	33.64		-0.46		-0.01		0.29
			0.31		0.01		0.02	
	0.4	33.95		-0.45		0.01		
			-0.14		0.02			
	0.5	33.81		-0.43				
			-0.57					
	0.6	33.24						

$$x_n = x_0 + nh$$
$$\Rightarrow 0.3 = 0.3 + n \times 0.1$$
$$\therefore n = 0$$

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{6}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right]\right.$$

$$\left. + \frac{1}{30}\left[\Delta^5 y_{-2} + \Delta^5 y_{-3}\right]\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.1}\left[\left(\frac{0.31+0.77}{2}\right) - \frac{1}{6}\left(\frac{0.01+0.02}{2}\right) + \frac{1}{30}\left(\frac{0.02-0.27}{2}\right)\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{0.1}\left[0.54 - 0.0025 - 0.0042\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 5.333 \text{ cm/s}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3}\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.01}\left[-0.46 + \frac{1}{12} \times 0.08 + \frac{1}{90} \times 0.29\right]$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{0.01}\left[-0.46 + 0.0068 + 0.003\right]$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=0} = -45.62 \text{ cm/s}^2$$

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8. Maxima and Minima :-

$$y = f(x)$$

i.) $\frac{dy}{dx} = 0$

ii.) $\frac{d^2y}{dx^2} > 0$ (minima) · or

iii.) $\frac{d^2y}{dx^2} < 0$ (maxima)

iv.) $f(x_1)$

v.) Newton forward differend.

Q.) From the given table for what value x, y is minimum & what is the minimum value

x :	3	4	5	6	7	8
y :	0.205	0.240	0.259	0.262	0.250	0.224

⟵ Difference Table ⟶

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
3	0.205					
		0.035				
4	0.240		-0.016			
		0.019		0		
5	0.259		-0.016		0.001	
		0.003		0.001		-0.001
6	0.262		-0.015		0	
		-0.012		0.001		
7	0.250		-0.014			
		-0.026				
8	0.224					

Newton forward

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2n-1}{2}\Delta^2 y_0\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1}\left[0.035 - \frac{2n-1}{2} \times 0.016\right]$$

$$\Rightarrow \frac{dy}{dx} = 0.035 - \frac{0.032n - 0.016}{2}$$

$$\Rightarrow 0 = 0.035 - 0.016n - 0.008$$

$$\Rightarrow -0.016n = -0.0027$$

$$\Rightarrow n = 1.6878$$

$$\therefore n = 1.7$$

$$x_n = x_0 + nh$$

$$\Rightarrow x_n = 3 + 1.7 \times 1$$

$$\therefore x_n = 4.7.$$

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Starling formula.

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right]$$

$$\Rightarrow y_n = 0.240 + 1.7\left[\frac{0.019 + 0.035}{2}\right]$$

$$\Rightarrow y_n = 0.240 \times 1.7 \times 0.027$$

$$\therefore y_n = 0.2859$$