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Digital Electronics —

It is an important part of electronics. Due to digital communication in transmission & reception of 'signals', Information technology rapidly increases. Digital watch, digital computer, T.V. Radio, make our daily life effective.

Signal :- (1) Analog Signal (2) Digital Signal

(1) Analog Signal :- It is defined as any physical quantity which varies continuously with respect to time.

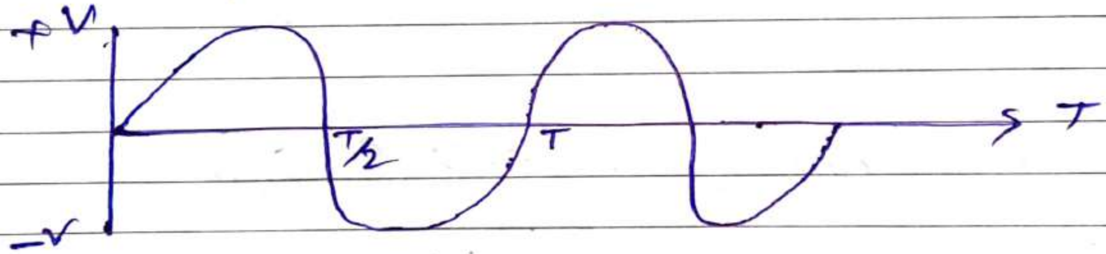

fig:- wave form of Analog Signal

(2) Digital Signal :- It is defined as any physical quantity having discrete values. It is the value of digital signal is one value from any two possible values (0, 1), or (Low, High) or (False, True) or (No, Yes).

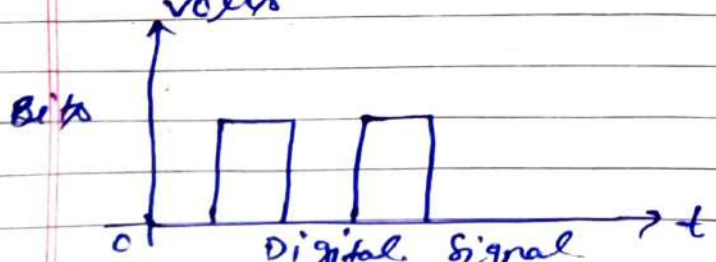

fig:- wave form of a digital signal.

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Advantages of digital Signal :-

(1) Easy to operate (eg. on & OFF)
(2) Easy to store information in digital system.
(3) accuracy is high in digital system.
(4) Digital circuits are more reliable.
(5) Compression of data is possible in digital system

Application of digital electronics :-

(1) modern mobile & computers.
(2) Electronic exchange of telephone.
(3) Power electronic. (4) Traffic, ~~rail~~ railway signal
(5) Communication.

Diff. b/w Analog & digital Signal

Analog Signal	Digital Signal
Continous change	Discontinous change
Not based on Boolean Algebra	Based on Boolean Algebra
maximum effect of Sound	Low effect of sound.
difficult to store information	Easy to store information
Compression of data is impossible	Possible.
Operation is slow	Operation is fast
Low accuracy	High accuracy.

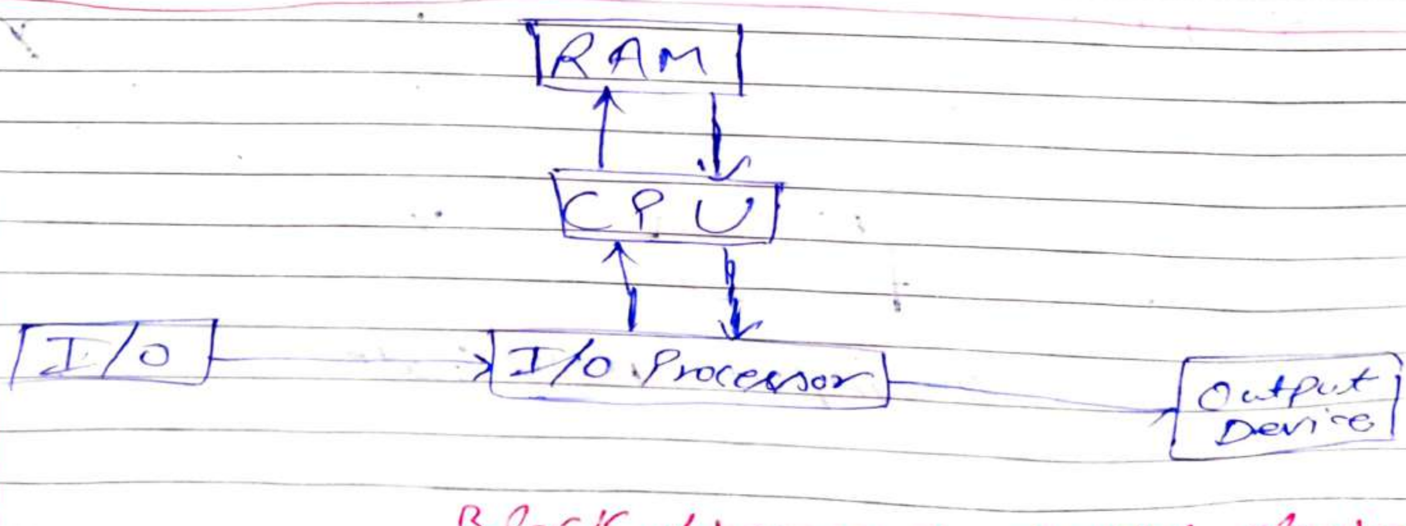
Digital electronics & computer organization

Unit - I

Digital Electronics:- digital Electronics is an area of Computer science. It manages with tools that can transmit out computer software. In digital electronics, we facilitate two-state or binary logic. There are two logic states including "0" (low) & "1" (high).

A computer facilitates a binary numbers, including 1 & 0, using two voltage levels in a machine known as a logic gate. Frequently two states can also be defined using boolean logic circuit. A logic gate creates two input & creates an individual output.

It contains the data mechanism, the instruct group, & methods for addressing m/r. The structural design of a computer system is concerned with the descriptions of the multiple functional modules, including processor & memories, & managing them together in to an electronic system.



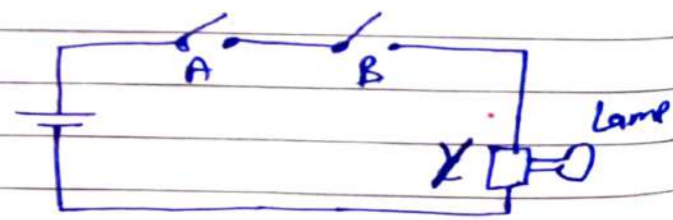
Block diagram of digital electronics

Logic gates:- Logic gates play an important role in circuit design & digital systems. It is a building block of a digital system & an electronic circuit that always have only one output. These gates can have one input or more than one input, but most of the gates have two inputs. On the basis of the relationship b/w the I/O, these gates are named as:-

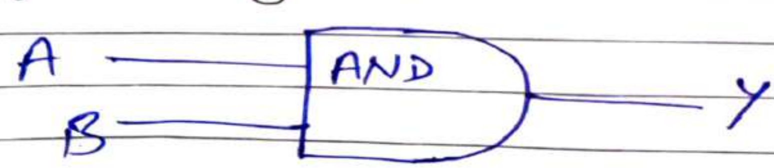
(1) AND Gate (2) OR Gate (3) NOT Gate
(4) NAND Gate (5) NOR Gate (6) X OR Gate
(7) X NOR Gate

(1) **AND Gate :-** This gate works in the same way as the logical operator "and". The AND gate is a circuit that performs the AND operation of this inputs. This gate has a minimum of 2 input values & an output value.

eg:- $Y = A$ AND B
$Y = A \cdot B \cdot$
$Y = AB$



Logic Design:-



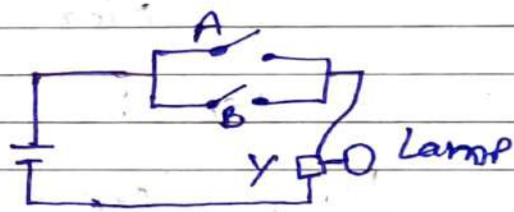
2- input AND Gate

Truth Table :-

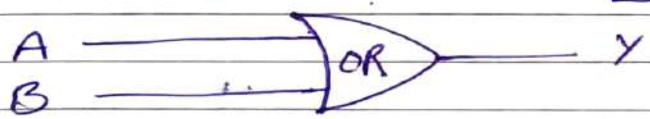
I/P		O/P
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

(2) OR Gate :- This gate works in the same way as the logical operator or.

eg:- Y = A OR B
Y = A + B



Logic Design:-



2- Input OR Gate

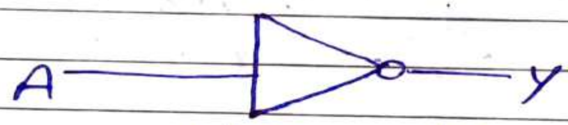
Truth Table

Input		Output
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

(3) NOT Gate :- This is also called inverter. This gate gives the inverse value of the input value as a result. This gate has only one input & one output value.

$Y = \text{NOT } A$
$Y = A'$

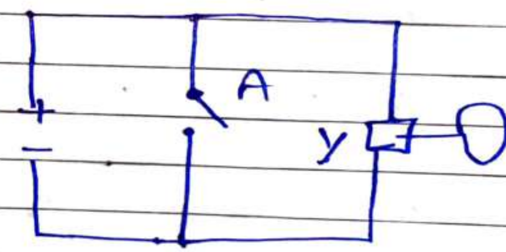
Logic Design:-



Not gate.

Truth Table:-

I/P	O/P
A	B
0	1
1	0



) NAND Gate:- This is the combination of AND Gate & NOT Gate. This gate gives the same result as NOT-AND operation. This gate can have two or more then two input values & only one o/p value.

Eg:- $Y = A \text{ NAND } B$

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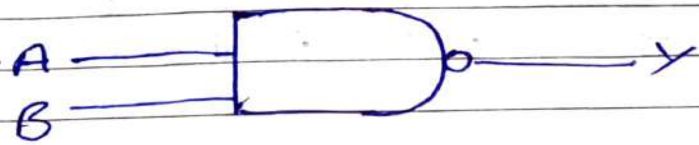
Logic Design :-



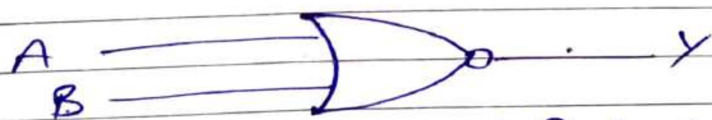
2- input NAND Gate

Truth Table :-

A	B	(AB)'
0	0	1
0	1	1
1	0	1
1	1	0

(5) NOR Gate :- Combination of OR Gate & NOT gate. This gate gives the same result as the NOT-OR operation.

Y = A NOR B



2- input NOR Gate

A	B	(A+B)'
0	0	1
0	1	0
1	0	0
1	1	0

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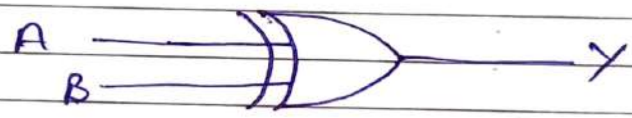
(6) XOR Gate :- The exclusive-OR gate is sometimes called as EX-OR & X-OR gate. XOR gate is used in half & full adder & subtractor.

$Y = A \text{ XOR } B$
$Y = A \oplus B$
$Y = AB' + A'B$

Logic Design :-



2-input XOR Gate

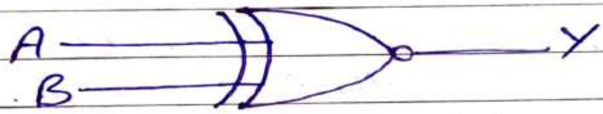
Truth Table :-

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

(7) XNOR Gate:- It is used in half & full adder & Subtractor.

$Y = A \text{ XNOR } B$
$Y = A \ominus B$
$Y = A'B' + AB$

Logic Design



2-input XNOR Gate

Truth Table

A	B	$A \ominus B$
0	0	1
0	1	0
1	0	0
1	1	1

(*) De-morgan's Theorem / Laws :-

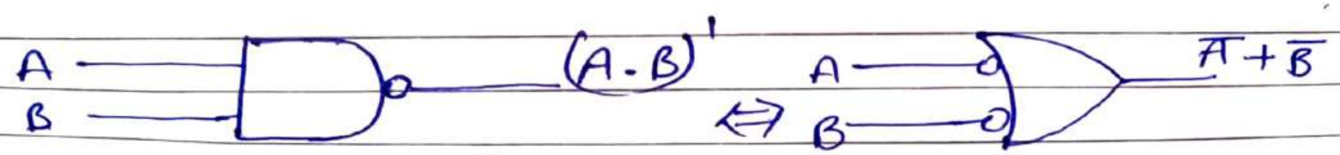
A famous mathematician De.Morgan invented the two most important theorems of boolean algebra. The Demorgan's theorems are used for mathematical verification of the equivalency of the NOR & NAND gates. These theorems play an important role in solving various boolean algebra expressions.

Demorgan's first Law :- According to the first theoram, the complement result of the AND operation is equal to the OR operation of the complement of that variable. Thus, it is equivalent to the NAND function & is a negative-OR function proving that $(A \cdot B)' = A$

$$(A \cdot B)' = (A' + B') \qquad \overline{(A \cdot B)} = \overline{A} + \overline{B}$$

~~NAND~~ ~~NOR~~

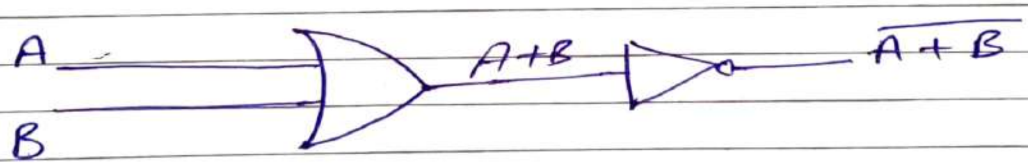
A	B	A·B	(A·B)'	A'	B'	A'+B'
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

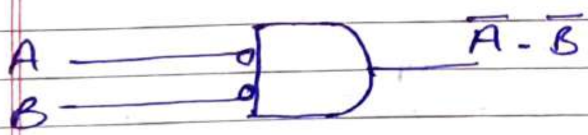


De-Morgan's Second Theorem:- According to the second theorem, the complement result of the OR operation is equal to the AND operation of the complement of that variable.

$$(A+B)' = A'.B'$$

A	B	A+B	(A+B)'	A'	B'	A'.B'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0



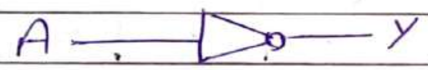


Laws of Boolean Algebra:-

(1) Complementary or NOT law (inversion Law)
(2) AND operation
(3) OR operation law
(4) Commutative law
(5) Associative law
(6) Distributive law.

Note → George Boole develop Boolean algebra.

(1) Not law:- using NOT Gate.

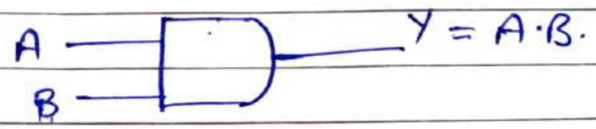


$Y = \text{NOT } A = \overline{A}$

$\overline{0} = 1$
$\overline{1} = 0$

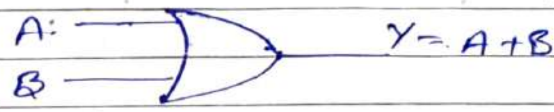
(2) AND operation Law:- AND operation indicates by the dot (.). If on doing AND operation of two input A & B, Y output is obtained the this can be expressed by following equation:-
$Y = A \cdot B$.

$A \cdot 0 = 0$
$A \cdot 1 = A$
$A \cdot A = A$
$A \cdot \overline{A} = 0$



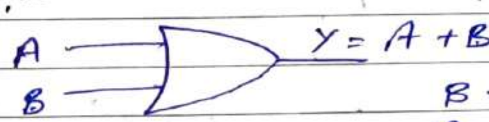
(3) OR operation laws:-

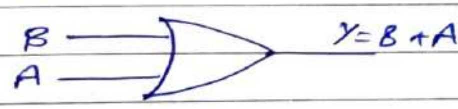
$A + 0 = A$
$A + 1 = 1$
$A + A = A$
$A + \bar{A} = 1$

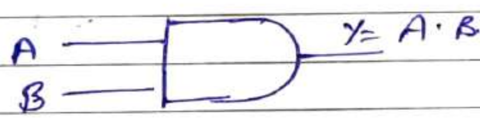


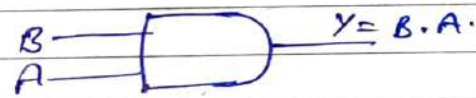
(4) Commutative Law:-

$A + B = B + A$
$A \cdot B = B \cdot A$



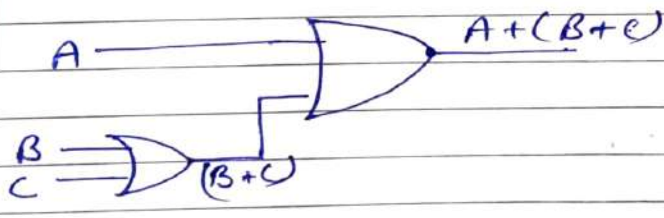


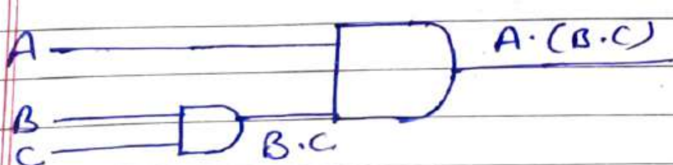


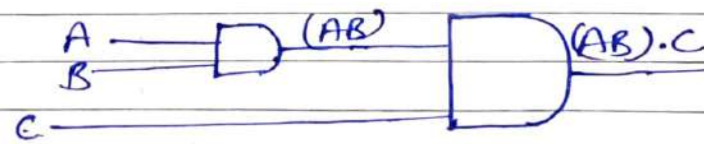


(5) Associative Law:-

$A + (B + C) = (A + B) + C$
$A \cdot (BC) = (AB) \cdot C$







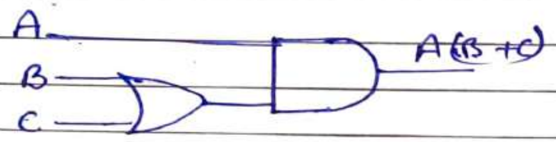
(6) Distributive law :-

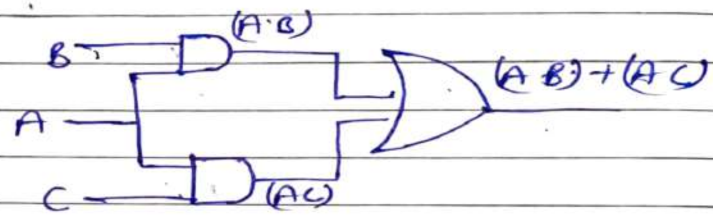
$$A(B+C) = (A\cdot B) + (A\cdot C)$$
$$A + (BC) = (A+B)(A+C)$$

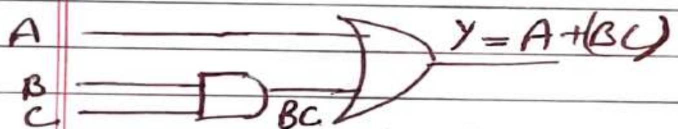
Proof :- $A + (B\cdot C) = (A+B)(A+C)$

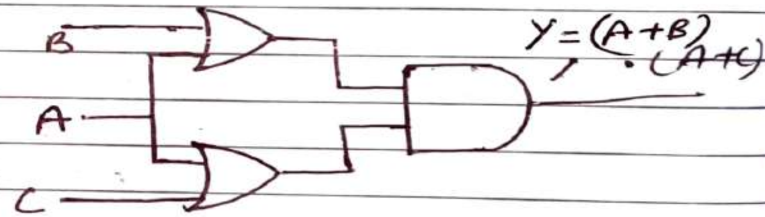
by truth table.

~~A·1+B·C~~









(*) Some other Laws :-

(1) $x + 0 = x$ (10) $x + y = y + x$

(2) $x + 1 = x$ (11) $x\cdot y = y\cdot x$

(3) $x + x = x$ (12) $x + (y+z) = (x+y) + z = (x+y+z)$

(4) $x + \bar{x} = 1$ (13) $x\cdot(y\cdot z) = (z\cdot y)x = x\cdot y\cdot z$

(5) $x\cdot 0 = 0$ (14) $x\cdot(y+z) = x\cdot y + x\cdot z$

(6) $x\cdot 1 = x$ (15) $(w+x)(y+z) = w\cdot y + w\cdot z + x\cdot y + x\cdot z$

(7) $x\cdot x = x$ (16) $x + x\cdot y = x(1+y) = x\cdot 1 = x$

(8) $x\cdot\bar{x} = 0$ (17) $x + y\cdot z = (x+y)(x+z)$

(9) $\bar{\bar{x}} = x$ (18) $x + \bar{x}\cdot y = x + y$

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(19) $\bar{x} + x\bar{y} = \bar{x} + \bar{y}$ (22) $\bar{x} + xy = \bar{x} + y$
(20) $\overline{(x+y)} = \bar{x} \cdot \bar{y}$ (23) $x + x\bar{y} = x$
(21) $\overline{x \cdot y} = \bar{x} + \bar{y}$

(*) Circuit Designing techniques:-

(1) Sum of Product
(2) Product of Sum
(3) K-map

(1) Sum of products (SOP) form:- Group of product terms summed together.

$Y = AB + AC + BC$

Sum → OR / (+)
Product → AND / (·)

eg:- $Y = ABC + BCD + AB\bar{D}$.
$Y = \bar{P}\bar{Q} + PQR + PQ\bar{R}$

A, B, C, D, P, Q, R → input terms / literals

(2) Product of Sum (POS) form:- Group of sum terms multiplied together.

eg:- $Y = (A+B) \cdot (B+C) \cdot (A+C)$
$Y = (A+\bar{B}+\bar{C}) \cdot (A+B) \cdot (A+\bar{C})$
$Y = (\bar{P}+Q) \cdot (P+R) \cdot (\bar{P}+R)$

→ OR

AND

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→ Standard / Canonical SOP form:- In this each product term consists of all the literals in the complemented or uncomplemented form.

eg:- (A, B, C) are literals.

$$Y = ABC + A\bar{B}\bar{C} + \bar{A}BC$$

→ Standard / Canonical POS form:- In this each sum term consists of all the literals in the complemented or uncomplemented form.

eg:- $Y = (A + B + \bar{C}) \cdot (A + \bar{B} + C)\ (\bar{A} + \bar{B} + \bar{C})$

(Q) Check whether the following are standard form or not.

(1) $Y = \underbrace{AB}_{\text{C is missing.}} + AB\bar{C} + \bar{A}BC$ (SOP)] Non-Standard (SOP)

(2) $Y = AB + A\bar{B} + \bar{A}\bar{B}$ → Standard (SOP)

(3) $Y = (\bar{A} + B)\ (A + B)\ (A + \bar{B})$ → Standard (POS)

(4) $Y = \underbrace{(\bar{A} + B)}_{\text{'C' is missing.}}\ (A + B + C)$ → Non-Standard

(*) Convert SOP to standard SOP form:-
(i) for each term find missing literal.
(ii) AND term with the term formed by ORing missing literal & its complement.

eg:- $Y = AB + A\bar{C} + BC$ here is three literals A B C

$AB\cdot(C+\bar{C}) + A\bar{C}(B+\bar{B}) + BC(A+\bar{A})$
$ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}BC$
$ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$
now it is a standard SOP.

eg:- $Y = A + BC + ABC$, 3 literals A,B,C

$Y = A(B+\bar{B})\cdot(C+\bar{C}) + BC(A+\bar{A}) + ABC$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC + ABC$

$Y = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$ Ans.

(*) Convert POS to standard POS term:-
(1) for each term, find missing literal.
(ii) OR each term with the formed by ANDing missing literal in that term with its complement.

eg:- $Y = (A+B)(A+C)(B+\bar{C})$ - 3 literal A,B,C
$\boxed{A+BC = (A+B)(A+C)}$
$Y = (A+B+C\bar{C})(A+C+B\bar{B})\cdot(B+\bar{C}+A\bar{A})$
$\boxed{A\cdot A = A}$
$(A+B+C)(A+B+\bar{C})\cdot(A+\bar{B}+C)(\bar{A}+B+\bar{C})$ Ans.

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$Y = \underbrace{(A+B)}_{C}\ \underbrace{(\bar{B}+C)}_{A}$ — 3 literals – ABC

$Y = (A+B+C\bar{C})(\bar{B}+C+A\bar{A})$

$Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$ Ans

(*) Minterm & maxterm :-

(1) minterm :- Each individual term in the standard SOP form is called minterm.

eg:- $Y = \underbrace{ABC} + \underbrace{A\bar{B}\bar{C}} + \underbrace{\bar{A}BC}$ → minterms (m_i)

(2) Maxterm :- Each individual term in the standard POS form is called maxterm.

eg:- $Y = \underbrace{(A+B)}\ \underbrace{(A+\bar{B})}$ → maxterm (M_i)

Ques write minterms & maxterms for the following Truth Table :-

A	B	S-SOP minterms m_i	S-POS maxterm M_i
0	0	$\bar{A}\bar{B} \to m_0$	$A+B = M_0$
0	1	$\bar{A}B \to m_1$	$A+\bar{B} \to M_1$
1	0	$A\bar{B} \to m_2$	$\bar{A}+B \to M_2$
1	1	$AB \to m_3$	$\bar{A}+\bar{B} \to M_3$

SOP → 1
POS → 0

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(*) How to represent Logical expressions with minterms & maxterms.

(1) $Y = \underset{m_7}{ABC} + \underset{m_3}{\bar{A}BC} + \underset{m_4}{A\bar{B}\bar{C}}$

$Y = m_7 + m_3 + m_4$,

$Y = \Sigma m(3, 4, 7)$ Ans.

(2) $Y = \underset{M_2}{(A+\bar{B}+C)} \underset{M_0}{(A+B+C)} \underset{M_6}{(\bar{A}+\bar{B}+C)}$

$Y = M_2 + M_0 + M_6$

$Y = \Pi M(0, 2, 6)$ Ans.

A	B	C	(Σ) m_i	(Π) M_i
0	0	0	$\bar{A}\bar{B}\bar{C} \to m_0$	$A+B+C \to M_0$
0	0	1	$\bar{A}\bar{B}C \to m_1$	$A+B+\bar{C} \to M_1$
0	1	0	$\bar{A}B\bar{C} \to m_2$	$A+\bar{B}+C \to M_2$
0	1	1	$\bar{A}BC \to m_3$	$A+\bar{B}+\bar{C} \to M_3$
1	0	0	$A\bar{B}\bar{C} \to m_4$	$\bar{A}+B+C \to M_4$
1	0	1	$A\bar{B}C \to m_5$	$\bar{A}+B+\bar{C} \to M_5$
1	1	0	$AB\bar{C} \to m_6$	$\bar{A}+\bar{B}+C \to M_6$
1	1	1	$ABC \to m_7$	$\bar{A}+\bar{B}+\bar{C} \to M_7$

- Truth table to Caculate the minterm & maxterm for given expression.

(b) How to write standard SOP Expression for a given Truth Table.

(1) Consider only input combinations whose o/p 'Y' is 1.
(2) Write product term for each such combination.
(3) 'OR' all these product terms.

eg:-

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$\left.\begin{matrix}\bar{A}B \\ A\bar{B}\end{matrix}\right\}$ Product terms

$Y = \bar{A}B + A\bar{B}$

$Y = m_1 + m_2 \Rightarrow Y = \Sigma m(1,2)$ Ans

eg:-

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\bar{A}\bar{B}C \rightarrow m_1$

$A\bar{B}\bar{C} \rightarrow m_4$

$ABC \rightarrow m_7$

$Y = m_1 + m_4 + m_7$

$Y = \Sigma m(1,4,7)$ Ans.

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*) How to write standard POS expression of given truth table:-

(1) Consider only those combinations of input which produces low o/p ($Y=0$).

(2) write maxterms only for such input combination.

(3) 'AND' these maxterms.

eg:-

A	B	C	Y	
0	0	0	0	$A+B+C \rightarrow M_0$
0	0	1	1	
0	1	0	1	
0	1	1	0	$A+\bar{B}+\bar{C} \rightarrow M_3$
1	0	0	1	
1	0	1	0	$\bar{A}+B+\bar{C} \rightarrow M_5$
1	1	0	0	$\bar{A}+\bar{B}+C \rightarrow M_6$
1	1	1	1	

$$(A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

$$\boxed{\begin{aligned} Y &= \prod M(0,3,5,6) \\ Y &= M_0 M_3 M_5 M_6 \end{aligned}}$$ Ans.

(*) Algebric Simplification of Boolean Expression :-

(i) Bring the given expression into SOP form by Boolean laws & Demorgan's theorams.

(ii) Simplify SOP expression by checking the product terms for common factors.

Q $Y = AB + (AB)(\bar{A}+B)$

$Y = AB + (A\bar{A} + AB + \bar{A}B + BB)$

$\because AB + AB = AB$
$\because B \cdot B = B$
$\because A \cdot \bar{A} = 0$

$= AB + B + \bar{A}B$

$= B(A+1) + \bar{A}B$ $\quad \because A+1 = 1$

$= B + \bar{A}B$

$= B(1+\bar{A})$ $\quad 1+\bar{A} = 1$

$\boxed{Y = B}$ Ans

Q2 $Y = \Sigma m(2,4,6)$

↳ min term

$m_2 \rightarrow 2 \rightarrow 010$
$\bar{A}B\bar{C}$

$Y = m_2 + m_4 + m_6$

$= \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{A}B\bar{C} + A\bar{C}(\bar{B}+B)$ $\quad \because \bar{B}+B = 1$

$= \bar{A}B\bar{C} + A\bar{C}$

$= \bar{C}(\bar{A}B + A)$ $\quad \because A+BC = (A+B)(A+C)$

$= \bar{C}(A+\bar{A})(A+B)$ $\quad \because A+\bar{A} = 1$

$\boxed{Y = \bar{C}(A+B)}$

Ans.

K-MAP (KARNAUGH-MAP)

K-map is a graphical method of simplifying a Boolean expression.
for 'n' input variable → 2^n boxes in K-map.
K-map is based on Gray code (Unit distance) co

2-variable K-map :-

A \ B	0	1
0	0	1
1	2	3

3-variable K-map :-

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

BC \ A	0	1
00		
01		
11		
10		

4-variable K-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Relation b/w Truth Table & K-map Entries:-

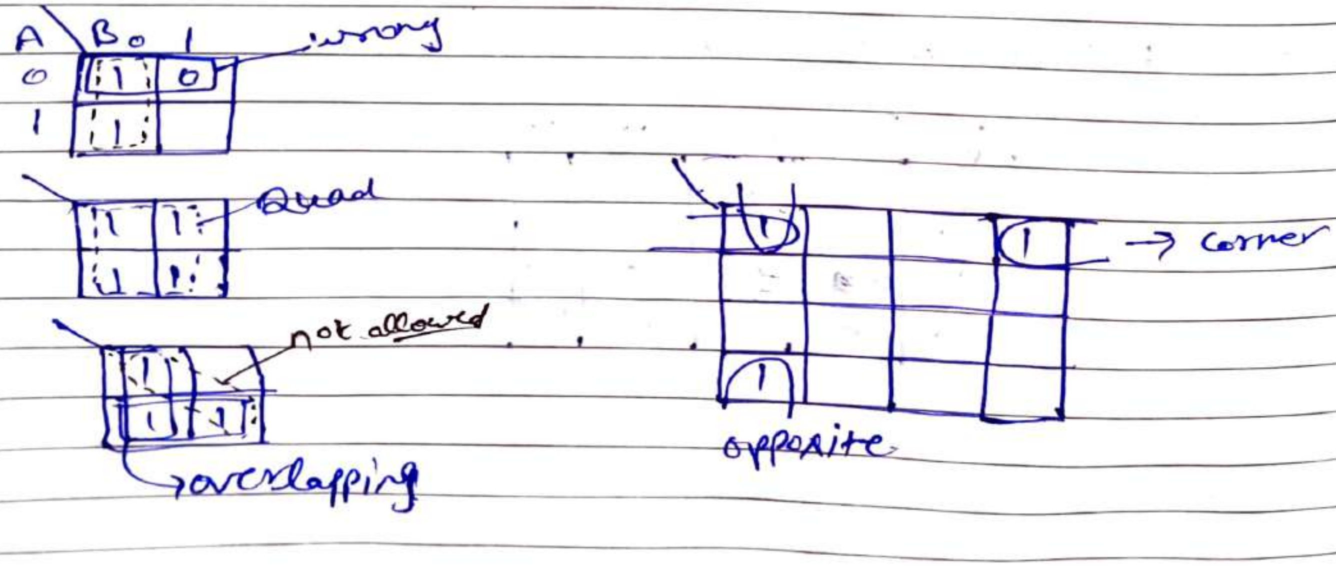
Truth Table

A	B	Y (O/P)
0	0	0
0	1	0
1	0	0
1	1	1

A \ B	0	1
0	0	0
1	0	1

K-map Simplification Rules:-
[SOP - 1 , POS - 0]

(1) Groups may not contain zero.
(2) Grouping in 2^n cells. ∴ Adjacent cells $\langle 2^0, 2^1, 2^n$
(3) making group as large as possible.
(4) Cells ~~counting~~ containing 1 must be grouped.
(5) Groups may overlap.
(6) opposite & corner Grouping allowed.
(7) Diagonal grouping not allowed.



(*) Standard SOP form on K-map:-

→ Enter 1's in the cells (Boxes) of K-map corresponding to each minterm present in expression.

→ Remaining boxes with 0's.

eg:- $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC.$

A, B, C – 3 terms → $2^3 \to 8$

Soln:-

A \ BC	00	01	11	10
0	1 $_0$	1 $_1$	0 $_3$	0 $_2$
1	1 $_4$	0 $_5$	ϕ $_7$	0 $_6$

(*) Standard POS form on K-map:-

→ Enter 0's in the cells of K-map corresponding to each maxterm present in given expression.

→ Remaining boxes with 1's. $\begin{bmatrix} 0 - A \\ 1 - \bar{A} \end{bmatrix}$

eg:- $Y = (A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$

M_0 M_2 M_6

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	1	0
A	1	1	1	0

A \ BC	00	01	11	10
0	0	1	1	0
1	1	1	1	0

Imp

Simplify Boolean Expression with K-map:-

(i) Use Grouping Technique
(ii) Grouping means combining terms in adjacent cells.
(iii) Grouping adjacent 1's → SOP
(iv) Grouping adjacent 0's → POS

Example of Grouping (Pairs):-

(1) $Y = \bar{A}BC + \bar{A}B\bar{C}$

$\bar{A}B(C+\bar{C}) \Rightarrow \bar{A}B$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	1
A	0	0	0	0

→ Now, in groups look for variables whose value is not changing.

$(\bar{A}B)$ Ans.

(2) $Y = \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	0	1	0	0

$(\bar{B}\bar{C}D)$ Ans

Q.3 $Y = \Sigma m(1, 5, 7, 9, 11, 13, 15)$

means here is 4 variable (0 to 15)

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 0	1 1	3 0	2 0
$\bar{A}B$	4 0	5 1	8 1	7 0
AB	12 0	13 1	15 1	14 0
$A\bar{B}$	8 0	9 1	11 1	10 0

Quad-3 → BD

Quad 1 → $\bar{C}D$

Quad 2 → AD

$\bar{C}D + AD + BC$

$\boxed{D(\bar{C} + A + B)}$ Ans

Q.4 $Y = \Sigma m(1, 3, 5, 9, 11, 13)$, 4 variable.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	0
$\bar{A}B$	0	1	0	0
AB	0	1	0	0
$A\bar{B}$	0	1	1	0

Quad 2 $\bar{B}D$

Quad 1 $\bar{C}D$

$\bar{B}D + \bar{C}D \Rightarrow \boxed{D(\bar{C} + \bar{B})}$

Ans.

Q.5 $Y = \Sigma m(1,2,3,4,5,7,9,11,13,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 0	1 1	3 1	2 1
$\bar{A}B$	4 1	5 1	7 1	6 0
AB	12 0	13 1	15 1	14 0
$A\bar{B}$	8 0	9 1	11 1	10 0

Pair 1 $\bar{A}\bar{B}C$

Pair 2 $\bar{A}B\bar{C}$

Octet D

$\boxed{\bar{A}B\bar{C} + D + \bar{A}\bar{B}C}$ Ans

6 $Y = \Sigma m(1,2,9,10,11,14,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	3	2 1
$\bar{A}B$	4	5	7	6
AB	12	13	15 1	14 1
$A\bar{B}$	8	9 1	11 1	10 1

Pair 2 $\bar{B}C\bar{D}$

Pair 1 $\bar{B}\bar{C}D$

Quad AC

$$Y = \bar{B}\bar{C}D + \bar{B}C\bar{D} + AC$$
$$= \bar{B}(\bar{C}D + C\bar{D}) + AC$$

Elimination of Redundant Group:-

(1) if all the 1's in a group are already involved in some other groups.

(2) Redundant Group increase the no. of gates required.

Q.1 $Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 **1**	3	2
$\bar{A}B$	4	5 **1**	7 **1**	6 **1**
AB	12 **1**	13 **1**	15 **1**	14
$A\bar{B}$	8	9	11 **1**	10

This Quad is a redundant Group

$Y = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD + BD$

redundant. (to be removed)

Q.2 $Y = \Sigma m(0, 1, 2, 5, 13, 15)$

Redundant groups.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 **1**	1 **1**	3	2 **1**
$\bar{A}B$	4	5 **1**	7	6
AB	12	13 **1**	15 **1**	14
$A\bar{B}$	8	9	11	10

$Y = \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}D + ABD$ Ans.

(x) Don't Care Condition:-

denoted by (x) or (d) & may be assumed 0 or 1 as per requirement for simplication.

Q.1 $Y = \Sigma m(1, 3, 7, 11, 15) + d(0, 2, 5)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 x	1 1	3 1	2 x
$\bar{A}B$	4 0	5 x	7 1	6 0
AB	12 0	13 0	15 1	14 0
$A\bar{B}$	8 0	9 0	11 1	10 0

Q2 – $\bar{A}\bar{B}$

Q1 – CD

$$\boxed{Y = \bar{A}\bar{B} + CD}$$

Q.2 $Y = \Sigma m(0, 1, 5, 9, 13, 14, 15) + d(3, 4, 7, 10, 11)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 1	1 1	3 X	2
$\bar{A}B$	4 X	5 1	7 X	6
AB	12	13 1	15 1	14 1
$A\bar{B}$	8	9 1	11 X	10 X

Q2 $\bar{A}\bar{C}$

Q1 = AC

octet – D

$$\boxed{Y = \bar{A}\bar{C} + AC + D}$$ Ans.

Q. No. of Product terms in the minimized SOP expression obtained through the following K-map. is –

(a) 2 ✓
(b) 3
(c) 4
(d) 5

1	0	0	1
0	d	0	0
0	0	d	1
1	0	0	1

Product terms
↳ Quad
↳ Pair

Pair

Quad

Q. $F(A,B,C) = \Sigma m(2,3,4,5) + \Sigma d(6,7)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$_0$ 0	$_1$ 0	$_3$ 1	$_2$ 1
A	$_4$ 1	$_5$ 1	$_7$ X	$_6$ X

Soln:- with out don't care condition

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$			1	1
A	1	1		

$Y = \bar{A}B + A\bar{B}$

OR $\boxed{Y = A \oplus B}$

↳ XOR

With don't care condition.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$			1	1
A	1	1	x	X

$\boxed{Y = A + B}$ Ans.

(*) POS Simplification – K map :-

(1) $Y = \prod M(0, 2, 3, 5, 7)$

∵ Grouping → 0
0 → x
1 → $\bar{x}$

A \ BC	00	01	11	10
0	0 (0)	1 (1)	3 (0)	2 (0)
1	4 (1)	5 (0)	7 (0)	6 (1)

Pair 3 = $(A + C)$

Pair 1 = $(\bar{B} + \bar{C})$

Pair 2 = $(\bar{A} + \bar{C})$

$$\boxed{Y = (\bar{B} + \bar{C})(\bar{A} + \bar{C})(A + C)}$$

(2) $Y = \prod M(0, 2, 3, 7)$

A \ BC	00	01	11	10
0	0 (0)	1	3 (0)	2 (0)
1	4	5	7 (0)	6

Pair 1 = $(\bar{B} + \bar{C})$

Pair 2 = $(A + C)$

$\boxed{Y = (\bar{B} + \bar{C})(A + C)}$ Ans.

Q.3

AB \ CD	00	01	11	10
00	0			0
01				
11				
10	0			0

Solⁿ :- Quad $= (B + D)$. Ans

(x) K-map using MAX Terms [POS] :- $0 \rightarrow x, 1 \rightarrow \bar{x}$
Grouping $\rightarrow$ 0's

Q.1

A \ BC	00	01	11	10
0	0	0	0	1
1	1	1	1	1

$\rightarrow G_1$, $\rightarrow G_2$

Method 1 $\rightarrow$ $\bar{F} = G_1 + G_2$ ∵ using SOP

$$\bar{F} = (\bar{A}\bar{B}) + (\bar{A}C)$$

$$\bar{\bar{F}} = \overline{(\bar{A}\bar{B})} + \overline{(\bar{A}C)}$$

$$\boxed{F = (A+B) \cdot (A+\bar{C})} \text{ Ans}$$

∵ using Demorgans law

Method 2 :- $G_1 = (A+B)$

$$G_2 = (A+\bar{C})$$

$$Y = G_1 \cdot G_2$$

$$\boxed{Y = (A+B) \cdot (A+\bar{C})} \text{ Ans}$$

Q.2 $F(ABCD) = \prod M(0,2,6,7,8,10,12,13)$
↳ maxterms

AB \ CD	00	01	11	10
00	0: 0	1: 1	3: 1	2: 0
01	4: 1	5: 1	7: 0	6: 0
11	12: 0	13: 0	15: 1	14: 1
10	8: 0	9: 1	11: 1	10: 0

Quad 1 = $(B+D)$
Pair 1 = $(A+\overline{B}+\overline{C})$
Pair 2 = $(\overline{A}+\overline{B}+C)$

$$\boxed{Y = (B+D)(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+C)}$$ Ans.