Unit — 05

Sr. No.______ Date:______

Digraph

Directed graph or Digraph:-

=) A directed graph G consists of two sets.

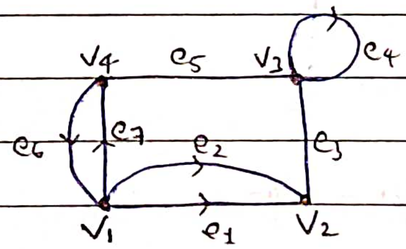
1.) A finite set V of vertex.
2.) A finite set E of edges, contains ordered pair of vertices.

(v_1, v_2) $v_1 \longrightarrow v_2$

$(v_1, v_2) \neq v_2 \longrightarrow v_1$

* if an edge $e_k = (v_i, v_j)$, then this edges is represented ~~betn vi and vj~~ in the digraph by a line segement - betn vi and vj with arrow directed from vi to vj

$v_i \xrightarrow{e_k} v_j$



Note:- Let $e_k = (v_i, v_j)$ is an edge $v_i \xrightarrow{e_k} v_j$ then vi is called initial vertex and vi is Called forminal vertex.

Parallel edge (Digraph) :-

⇒ Two edge are said to be Parallel edge in digraph if their initial and torminal vertex are same.

ex:- In digraph G e_1 and e_2 are Parallel edges but e_6 and G are not Parallel edges.

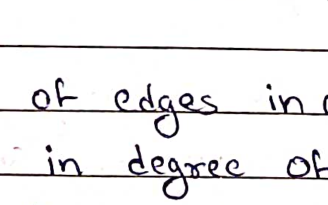
Self Loop (Digraph) :-

⇒ A edges for which the inital and the terminal vertex are same, is called Self loop.

ex:- In digraph G edge e_4 from a self - loop.

NOTE:- Let $e_k = (v_i, v_j)$ be can edge in digraph. Then the edges e_k is said to be incident out of its intitial vertex v_i and incident into its terminal vertex v_j.

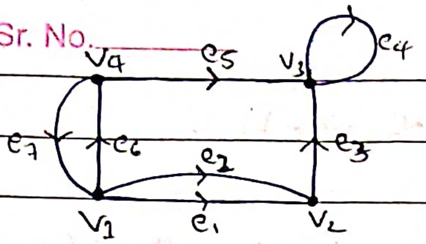
ex:- In digraph G edge e_5 is incident out of v_4 ~~on~~ and incident into v_3.



In degree and out - degree of a vertex.

⇒ The number of edges incident into a vertex v_i is called the in degree of vertex v_i and is denoted ~~dy~~ by $d^-(v_i)$ And the number of edges incident out of a vertex v_i is called out - degree of a vertex v_i and is denoted by $d^+(v_i)$.

Sr. No.   Date:

① 

$d^-(v_1) = 1$

$d^+(v_1) = 3$

$d^-(v_2) = 2$

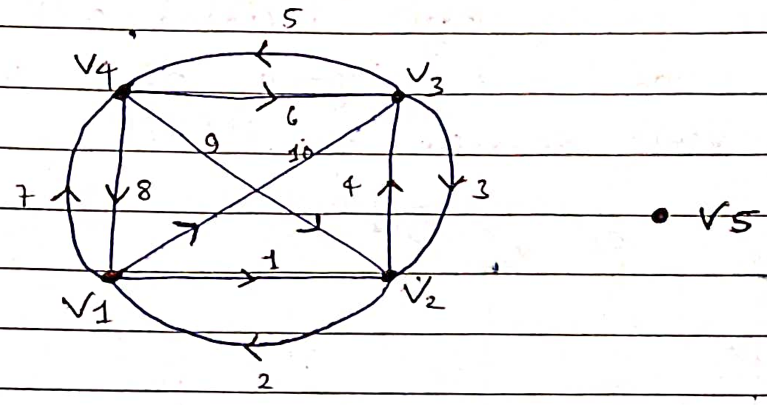
$d^+(v_2) = 1$

$d^-(v_3) = 3$

$d^+(v_3) = 1$

$d^-(v_4) = 1$

$d^+(v_4) = 2$

② 

$d^-(v_1) = 2$

$d^+(v_1) = 3$

$d^-(v_2) = 3$

$d^+(v_2) = 2$

$d^-(v_3) = 3$

$d^+(v_3) = 2$

$d^-(v_4) = 2$

$d^+(v_4) = 3$

$d^-(v_5) = 0$

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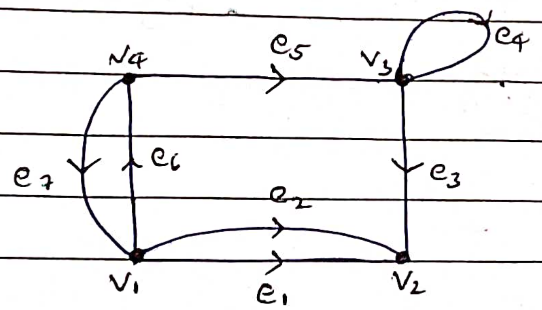
Sr. No.________ Date:________

In degree and out-degree of vertex.

⇒ The no. of edge incident into a vertex v_i is called the in degree of vertex v_i and is denoted by $d^-(v_i)$.
And the no. of edges incident out of a vertex v_i is called out degree of a vertex v_i and is ~~verte~~ denoted by $d^+(v_i)$.

Ex:-

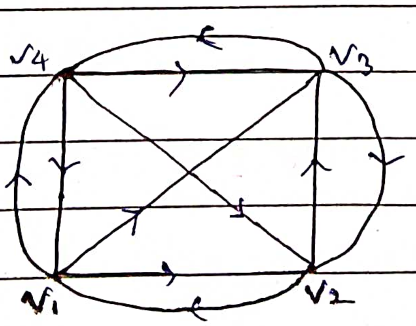


$d^-(v_1) = 1$ $d^-(v_3) = 2$

$d^+(v_1) = 3$ $d^+(v_3) = 2$

$d^-(v_2) = 3$ $d^-(v_4) = 1$

$d^+(v_2) = 0$ $d^+(v_4) = 2$



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Sr. No.______ Date:______

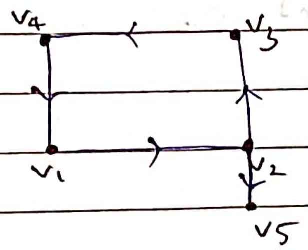
Isolated vertex:-

⇒ A vertex is said to be isolated vertex if $d^-(v_i) = d^+(v_i) = 0$

Pendent vertex:-

⇒ A vertex v is said to be pendent vertex if $d^-(v) + d^+(v) = 1$.

Ex:-



$d^-(v_5) = 1$

$d^+(v_5) = 0$

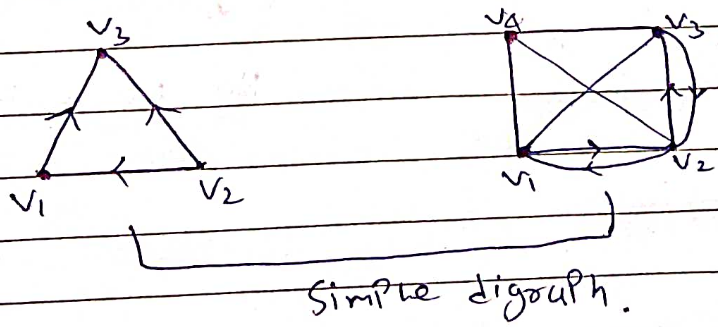
$d^-(v_5) + d^+(v_5) = 1 + 0 = 1$

⇒ v_5 is a pendent vertex!

Some type of Digraph:-

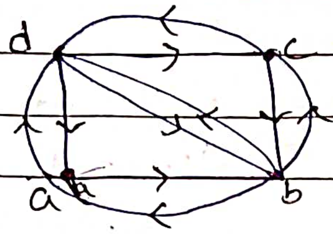
1. Simple digraph:- A digraph that has no self-loop and no parallel edges is called simple digraph.

Ex:-

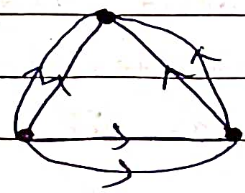


② Symmetric Digraph :-

⇒ A digraph in which for every edge from a to b there is an edge from b to a is called Symmetric digraph.



symmetric.

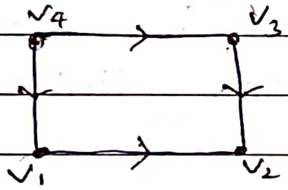


non-symmetric.

Asymmetric Graph :-

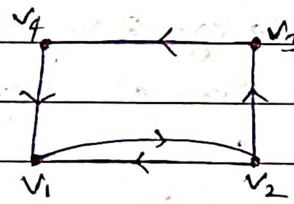
⇒ A digraph which has at most one edge between a pair of vertices is called asymmetric graph.

Ex :- ①



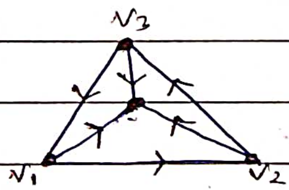
Asymmetric graph

②



Not Asymmetric.

③



Asymmetric graph.

Sr. No.________ Date:________

Complete Digraph :-

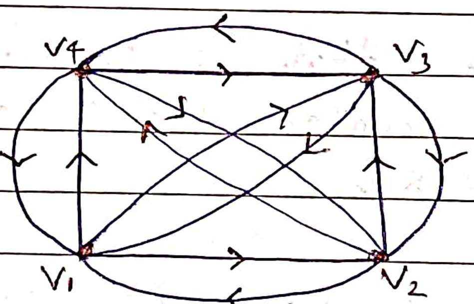
⇒ There are two types of complete graph.

1) Complete symmetric graph.
2) Complete Asymmetric graph.

① Complete symmetric graph.

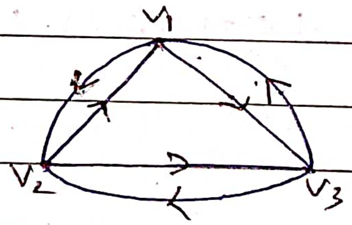
⇒ A complete symmetric digraph is a simple digraph in which there is exactly one edge directed from every vertex to every other vertex.

Ex:- ①



Complete symmetric digraph

②

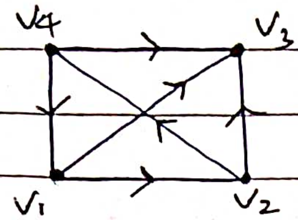


② Complete Asymmetric digraph.

⇒ A complete asymmetric digraph is an asymmetric digraph in which there is exactly one edge between $\boxed{\text{every}}$ pair of vertices.

Sr. No.________ Date:________

Ex!-



complete Asymmetric digraph.

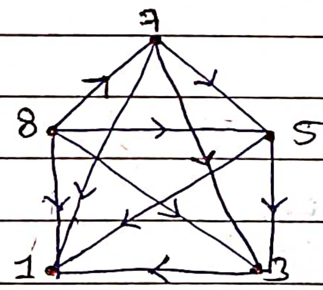
Digraphs and Binary Relation.

⇒ A binary relation on a set $X = \{x_1, x_2, x_3 --- x_n\}$ is a collection of ordered pair of elements of x ordered pair (x_i, x_j) means x_i is related to x_j.

Q. Draw a digraph that represent the relation "is greater then" on a set = {1, 3, 5, 7, 8}.

Soln:

Binary relation :- {(8,7) (8,5) (8,3) (8,1) (7,5) (7,3) (7,1) (5,3) (5,1) (3,1)}



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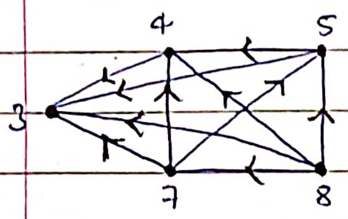
Sr. No.________ Date:________

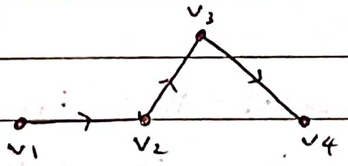
Q. Draw a digraph that ~~that~~ represent the relation "is greater than" on a set $X = \{3, 4, 7, 5, 8\}$

Sol:- Binary relation :- {(8,7) (8,5) (8,4) (8,3), (7,5) (7,4) (7,3) ~~(4,3)~~ (5,4) (5,3) (4,3) }

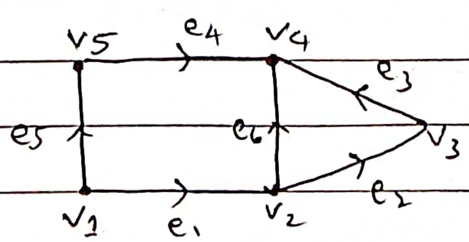


Directed Walk :-

⇒ A directed walk from a vertex v_i to v_j is an alternating sequence of vertex and edges, begining with v_i and ending with v_j such that each edge is directed from the vertex Preceding it to that vertex following it.



Note:- Repetation of edge is not allowed.



$v_1\ e_1\ v_2\ e_2\ v_3\ e_3\ v_4$ is a directed walk

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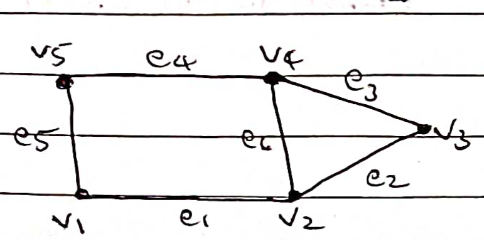
$v_1\ e_5\ v_5\ e_4\ v_4$ is a directed walk.

Sr. No.________ Date:________

Semi walk :-

⇒ A semi-walk in a direted graph is a walk in the corresponding undirected graph but not in directed graph.

Ex:- $v_1 e_1 v_2 e_6 v_4 e_3 v_3$ is a semi walk.



Directed Path :-

⇒ An open directed walk in which no edge appears more than ~~ones~~ once is called directed path.

Ex:- ~~$v_1 v_5$~~ $v_1 e_5 v_5$ is a directed path.
$v_1 e_1 v_2 e_6 v_4$ is a directed path.

Semi-directed Path.

⇒ A semi-directed path in a directed graph is a path in corresponding undirected graph but not in directed graph.

Ex:- $v_1 e_1 v_2 e_6 v_4 e_4 v_5$ is a semi-directed path.

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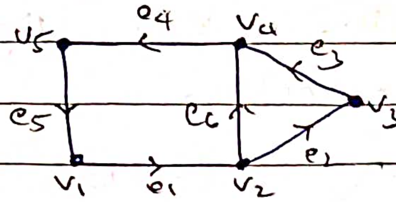
Sr. No.________ Date:________

Directed circuit :-

⇒ A closed directed walk in which no vertex apears more than once ~~~~ except and vertices is called directed circuit.



ex:- $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_1$ is directed circuit.

Semi-directed circuit :-

⇒ A semi-directed circuit in a directed graph is a circuit in corresponding undirected graph but not in directed graph.

Ex:- $v_2 e_6 v_4 e_3 v_3 e_2 v_2$ is a semi-directed circuit.

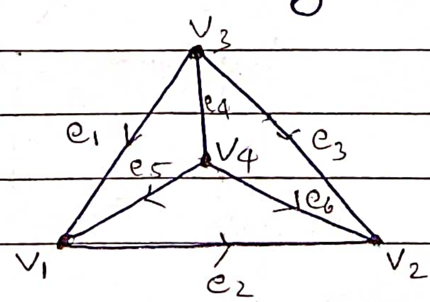
Imp.

Connected Directed Graphs.

⇒ Connected graph is of two types.

1. Strongly connected :-

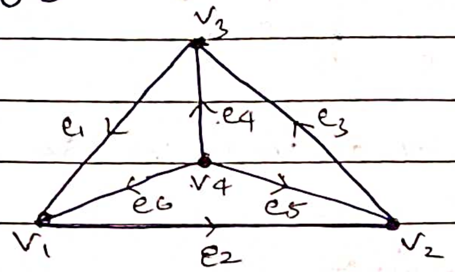
⇒ A digraph G is said to be Strongly connected if there exist at least one directed path from every vertex to every other vertex.



Sr. No.________ Date:________

2) weakly connected:

⇒ A digraph G is said to be strongly connected if the corresponding undirected graph is connected but digraph is not strongly connected.



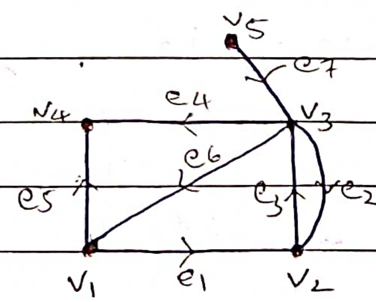
Matrices In digraph.

① ⇒ Incidence matrix:-

⇒ Let G be a digraph with n vertices and e ~~eadges~~ edges and having no self-loop, then incidence matrix is defined as $a_{ij} = 1$ if the j^{th} edge is incident out of i^{th} vertex.

$a_{ij} = -1$ if the j^{th} edge is incident into the i^{th} vertex.

$a_{ij} = 0$ if j^{th} edge is not incident on i^{th} vertex.



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Sr. No.______ Date:______

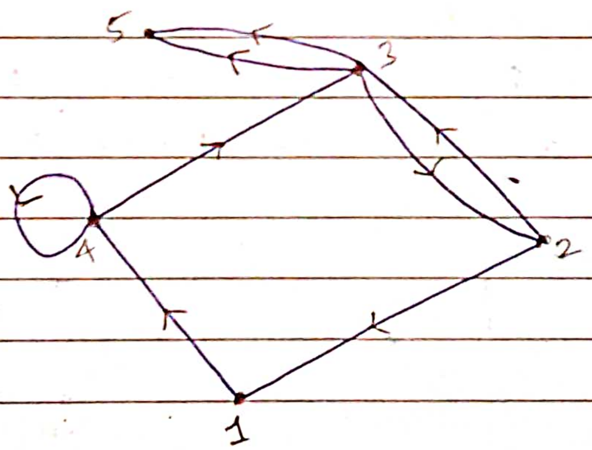
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(2) Adjacency matrix.

⇒ Let G be a digraph with in vertex then adjacency matrix is defined as,

x_{ij} = { k = if there are k edges directed from i^{th} vertex to j^{th} vertex. }

x_{ij} = 0 otherwise.



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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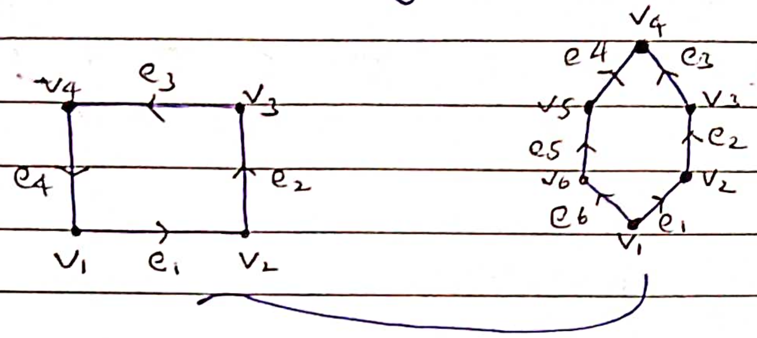
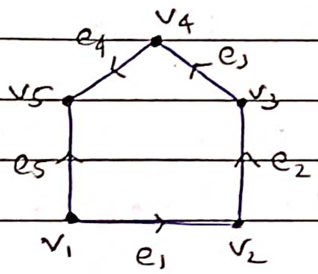
Directed Euler line:

⇒ A directed euler line in a digraph is a close directed walk that traverse every edge of G exactly once.

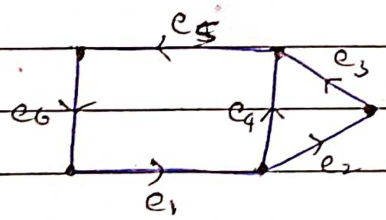


euler line

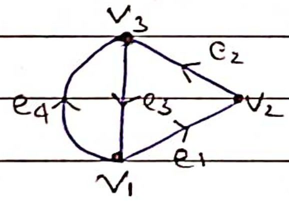


Euler's Digraph :-

⇒ A digraph is called Euler's digraph, if it contains a directed Euler line.



Not euler Digraph.

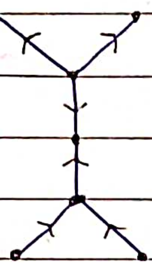


$v_1 e_1 v_2 e_2 v_3 e_3 v_1 e_4 v_3$

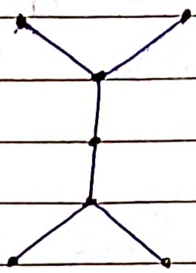
Sr. No.________ Date:________

Directed tree :-

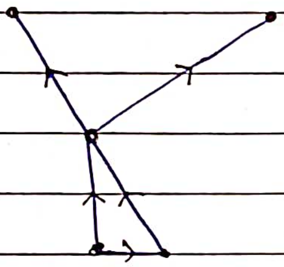
⇒ A directed graph is said to be directed tree if the corresponding undirected graph is a tree.



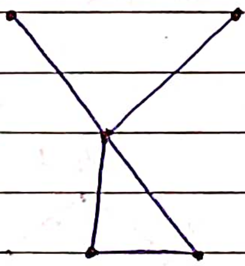
Directed Tree



Tree



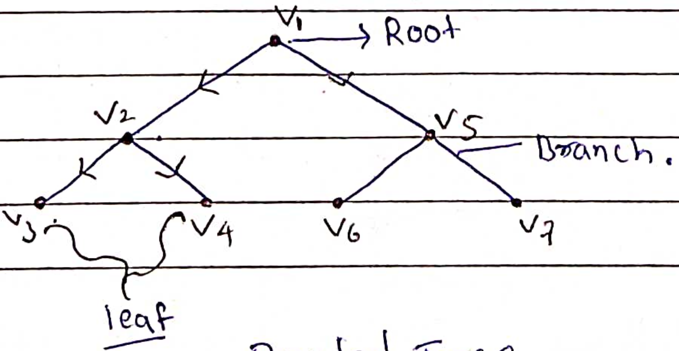
not a directed Tree



Not Tree

Rooted tree:

→ A directed tree is called rooted tree if there is only one vertex, whose in-degree is zero and the degree of all other vertices is equal to 1.



Rooted Tree

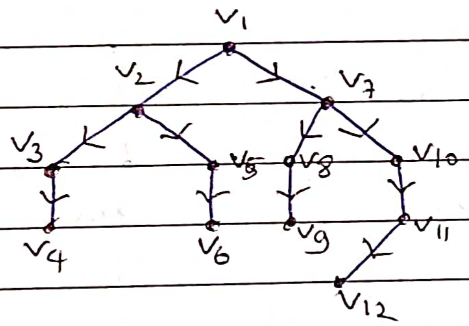
Sr. No.________ Date:________

* The vertex whose in-degree is '0' is called "root" of tree.

* A vertex whose out degree zero is called "leaf" of the tree and the vertex whose out-degree is non-zero is called branches or internal node.

Binary tree.

=> A rooted tree in which out degree of the root is 2 and the remaining interned vertices at out degree at most 2 is called binary tree.



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