Adversarial Search and Game-Playing

Search versus Games

- Search – no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristic techniques can find *optimal* solution
  - Evaluation function: estimate of cost from start to goal through given node
  - Examples: path planning, scheduling activities

- Games – adversary
  - Solution is **strategy** (strategy specifies move for every possible opponent reply).
  - **Optimality depends on opponent.** Why?
  - Time limits force an *approximate* solution
  - Evaluation function: evaluate "goodness" of game position
  - Examples: chess, checkers, Othello, backgammon

Adversarial Search

- Examine the problems that arise when we try to plan ahead in a world where other agents are planning against us.
- A good example is in board games.
- Two agents whose actions alternate
- Utility values for each agent are the opposite of the other
  - creates the adversarial situation
- Fully observable environments
- In game theory terms: Zero-sum games of perfect information.

Types of game in AI

	Deterministic	Chance Moves
Perfect information	Chess,	Backgammon,
Imperfect information	Battleships,	poker

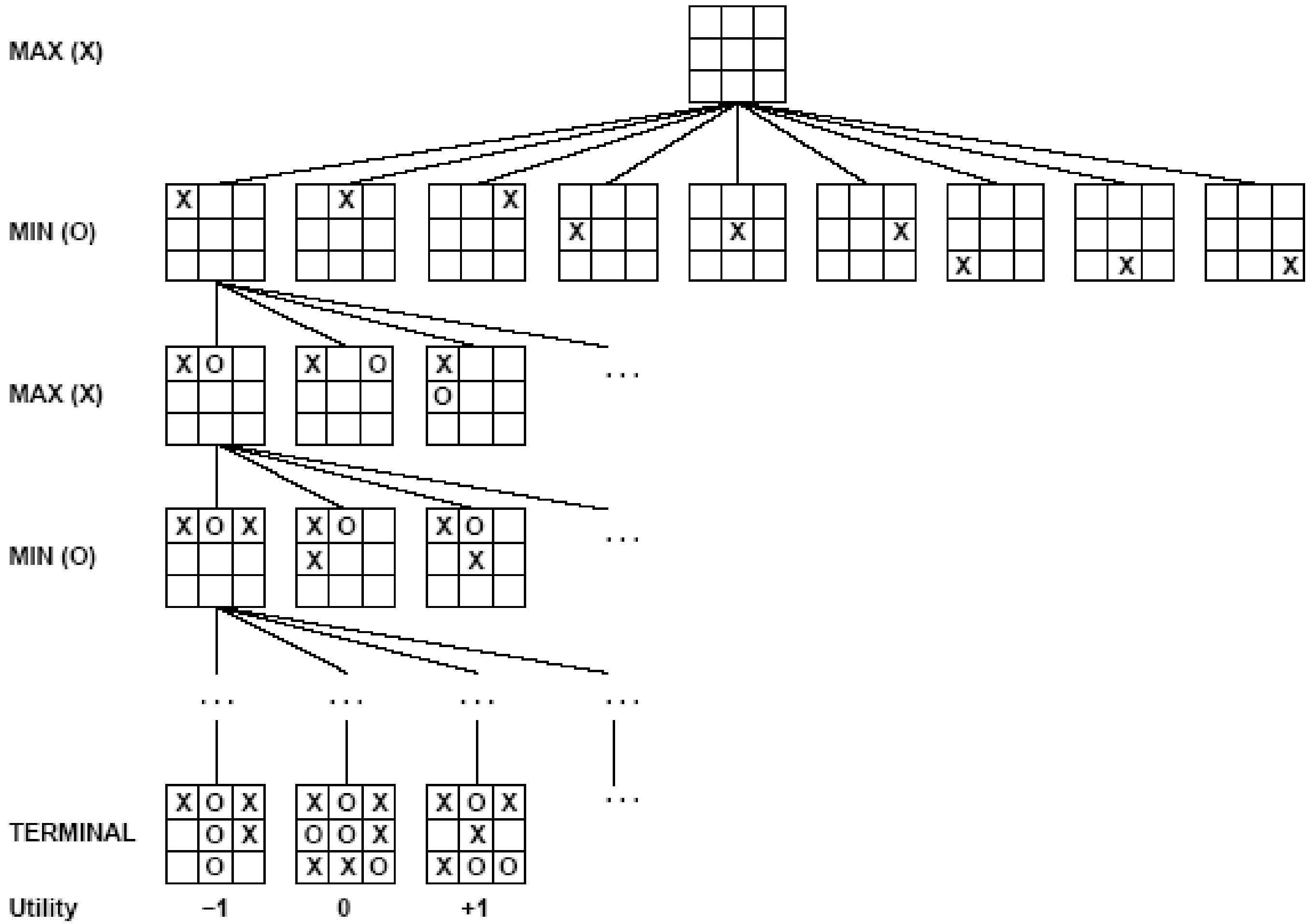
Game Setup

- Two players: MAX and MIN
- MAX moves first and they take turns until the game is over
  - Winner gets award, loser gets penalty.
- Games as search:
  - Initial state: e.g. board configuration of chess
  - Successor function: list of (move,state) pairs specifying legal moves.
  - Terminal test: Is the game finished?
  - Utility function: Gives numerical value of terminal states. E.g. win (+1), lose (-1) and draw (0) in tic-tac-toe or chess

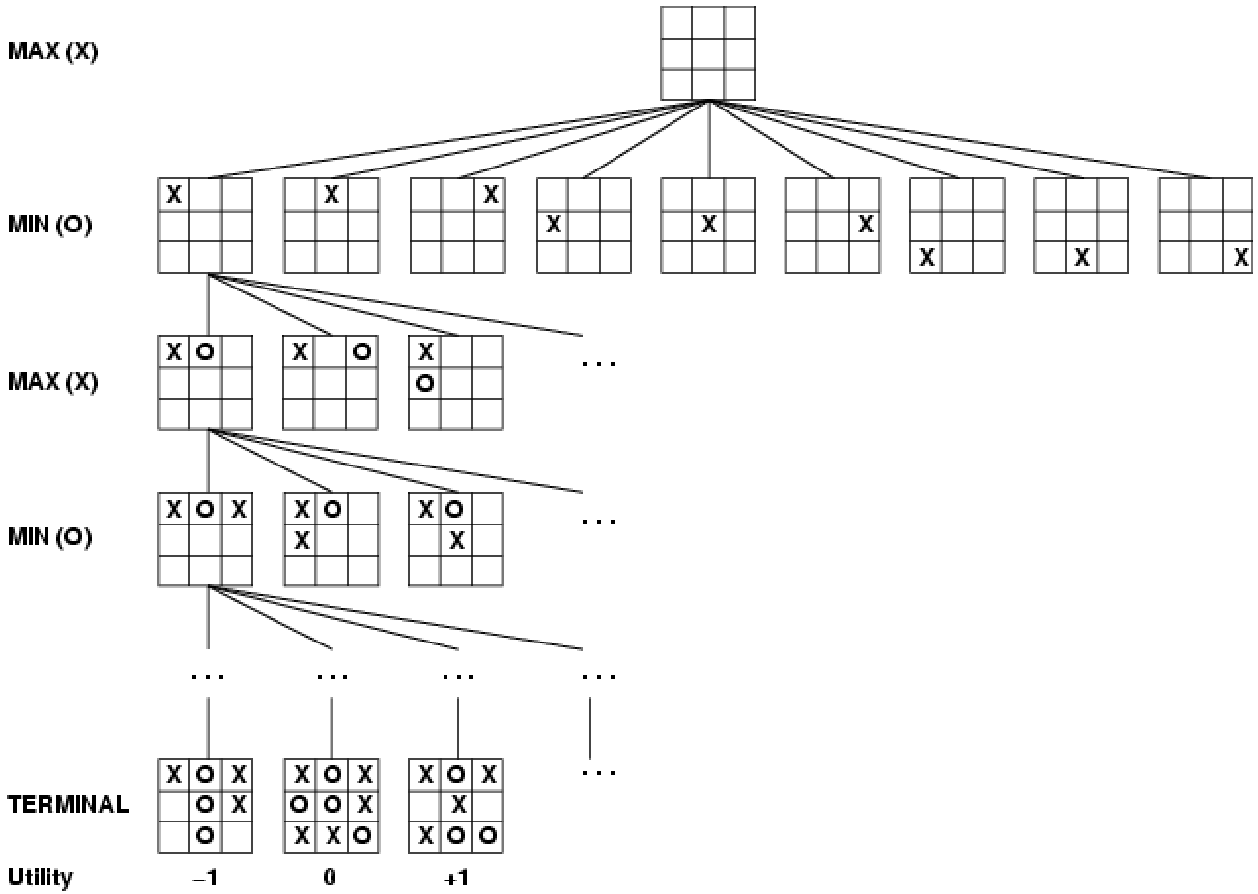
Size of search trees

- b = branching factor
- d = number of moves by both players
- Search tree is $O(b^d)$
- Chess
  - b ~ 35
  - D ~100
    - search tree is ~ 10^{154} (!!)
    - completely impractical to search this
- Game-playing emphasizes being able to make optimal decisions in a finite amount of time
  - Somewhat realistic as a model of a real-world agent
  - Even if games themselves are artificial

Partial Game Tree for Tic-Tac-Toe



Game tree (2-player, deterministic, turns)



How do we search this tree to find the optimal move?

Minimax strategy: Look ahead and reason backwards

- Find the optimal *strategy* for MAX assuming an infallible MIN opponent
  - Need to compute this all the down the tree

- Assumption: Both players play optimally!
- Given a game tree, the optimal strategy can be determined by using the **minimax value of each node.**

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Mini-max algorithm

- Mini-max algorithm is a recursive or backtracking algorithm which is used in decision-making and game theory. It provides an optimal move for the player assuming that opponent is also playing optimally.
- Mini-Max algorithm uses recursion to search through the game-tree.
- Two players play the game, one is called MAX and other is called MIN.
- Both the players fight it as the opponent player gets the minimum benefit while they get the maximum benefit.
- Both Players of the game are opponent of each other, where MAX will select the maximized value and MIN will select the minimized value.
- The minimax algorithm performs a depth-first search algorithm for the exploration of the complete game tree.
- The minimax algorithm proceeds all the way down to the terminal node of the tree, then backtrack the tree as the recursion.

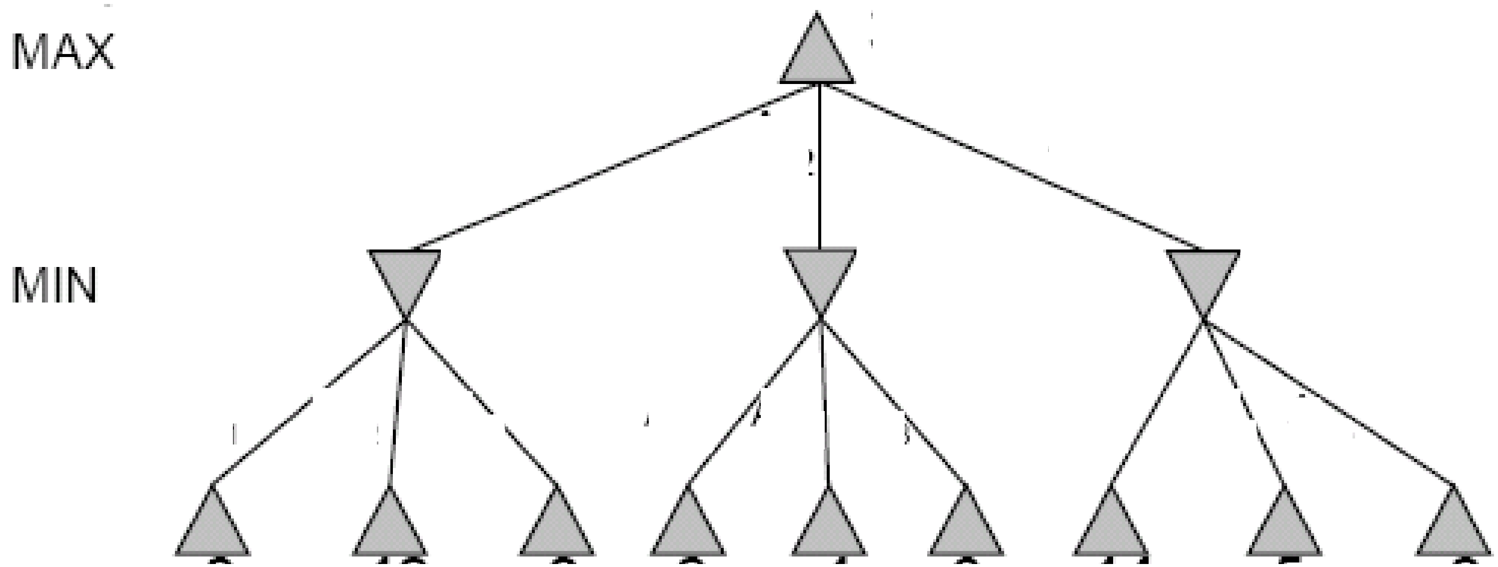
Pseudocode for Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
  inputs: state, current state in game
  v←MAX-VALUE(state)
  return the action in SUCCESSORS(state) with value v
```

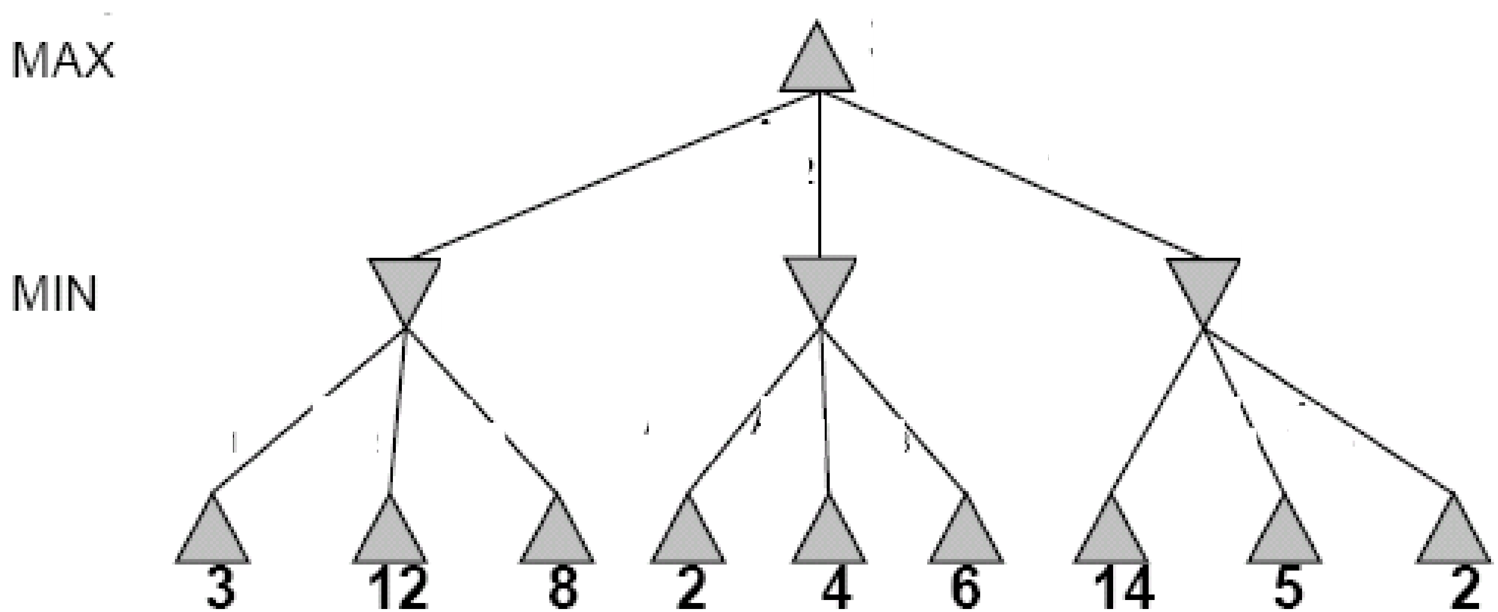
```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← -∞
  for a,s in SUCCESSORS(state) do
    v ← MAX(v,MIN-VALUE(s))
  return v
```

```
function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for a,s in SUCCESSORS(state) do
    v ← MIN(v,MAX-VALUE(s))
  return v
```

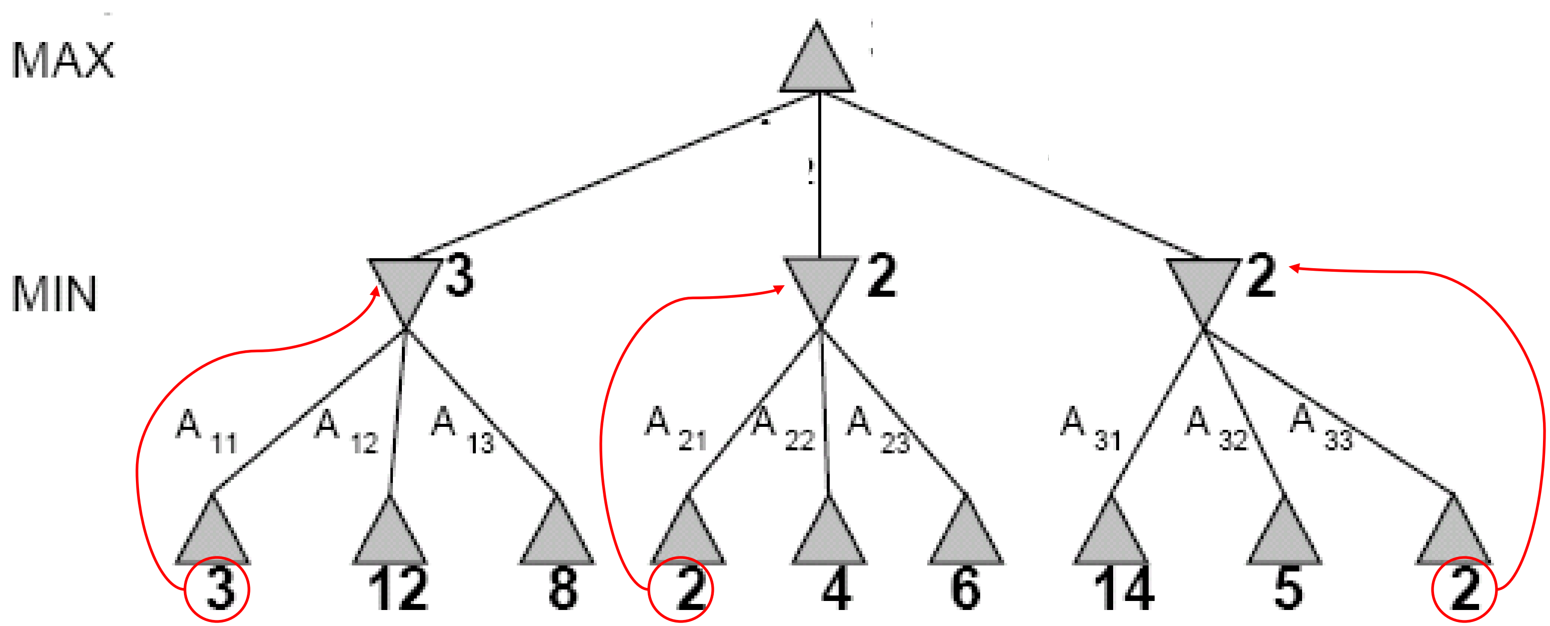
Two-Ply Game Tree



Two-Ply Game Tree

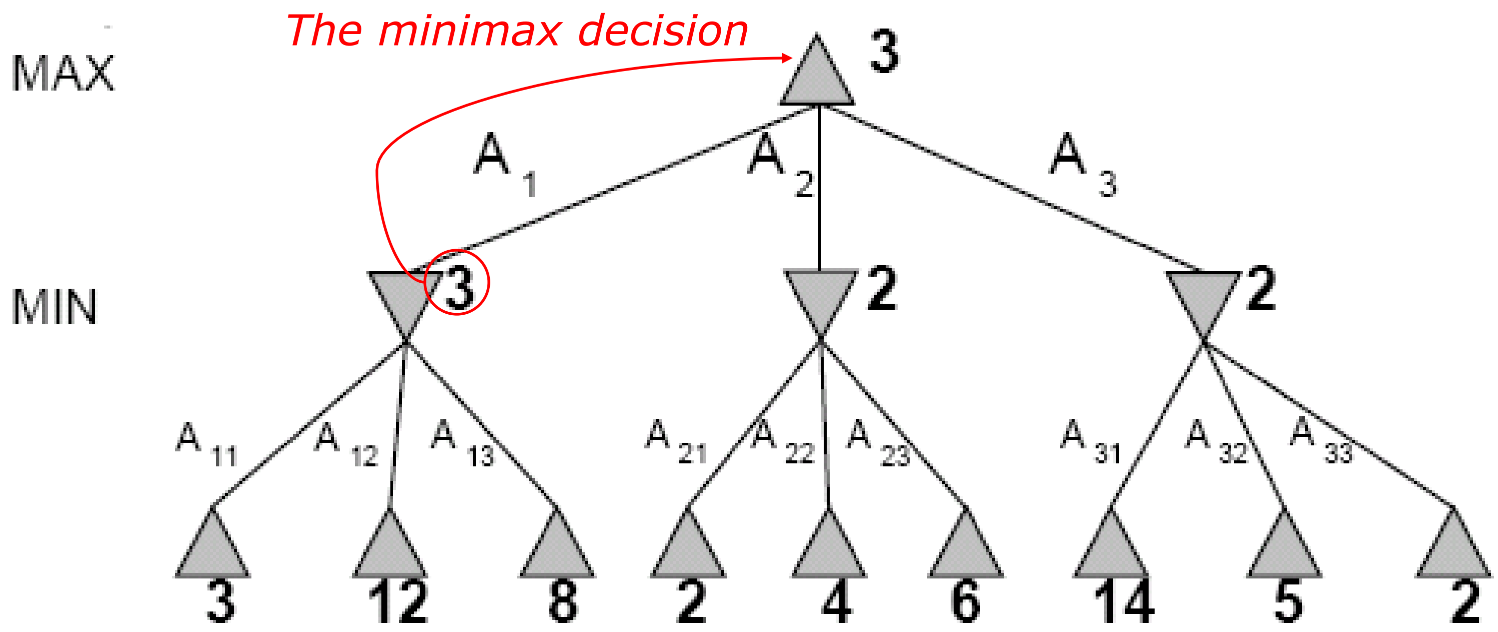


Two-Ply Game Tree



Two-Ply Game Tree

Minimax maximizes the utility for the worst-case outcome for max



Properties of Mini-Max algorithm:

- **Complete-** Min-Max algorithm is Complete. It will definitely find a solution (if exist), in the finite search tree.
- **Optimal-** Min-Max algorithm is optimal if both opponents are playing optimally.
- **Time complexity-** As it performs DFS for the game-tree, so the time complexity of Min-Max algorithm is **$O(b^m)$**, where b is branching factor of the game-tree, and m is the maximum depth of the tree.
- **Space Complexity-** Space complexity of Mini-max algorithm is also similar to DFS which is **O(bm)**.

Practical problem with minimax search

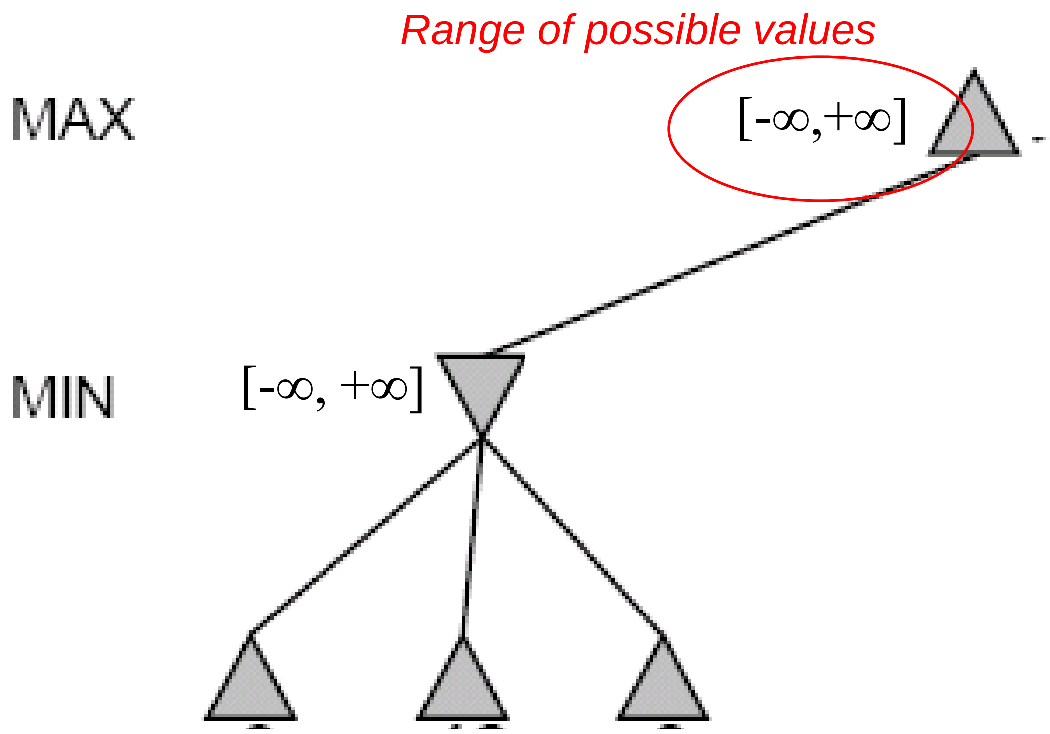
- Number of game states is exponential in the number of moves.
  - Solution: Do not examine every node

  => pruning
    - Remove branches that do not influence final decision

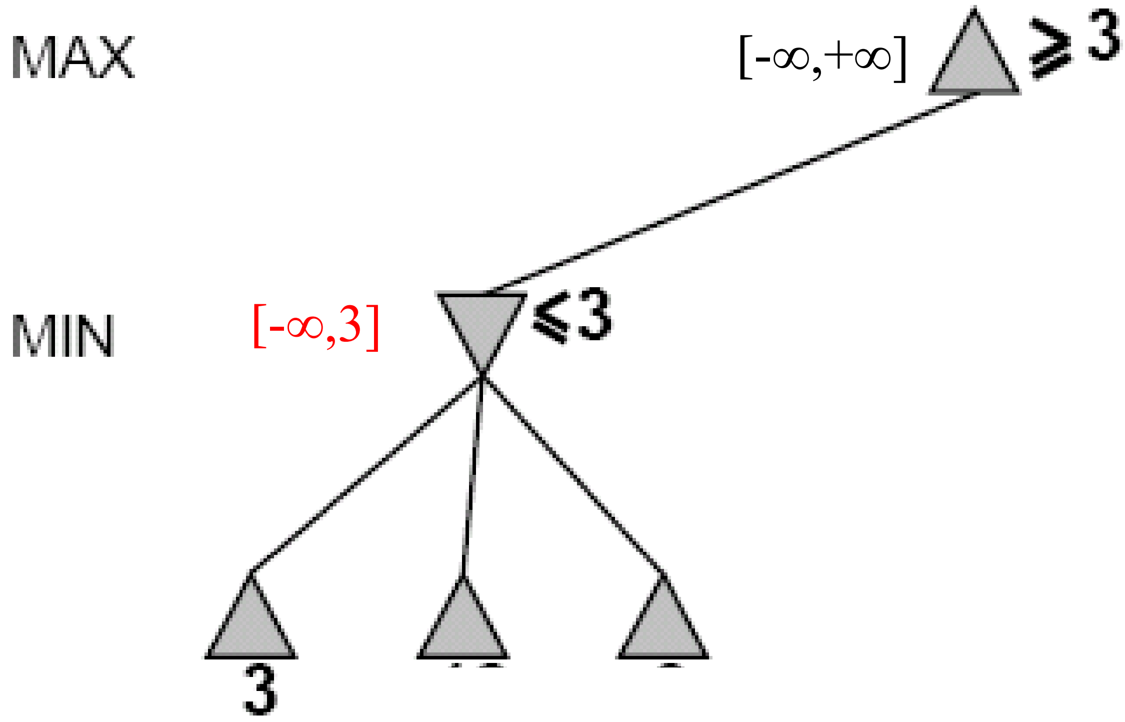
- Revisit example …

Alpha-Beta Example

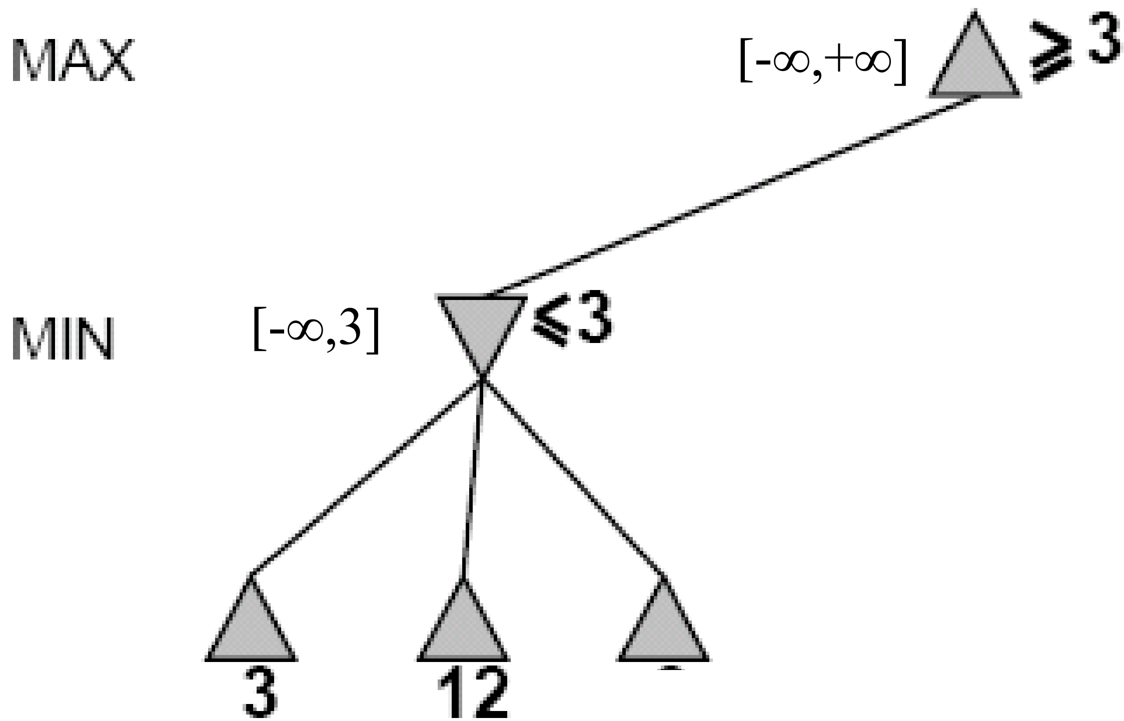
Do DF-search until first leaf



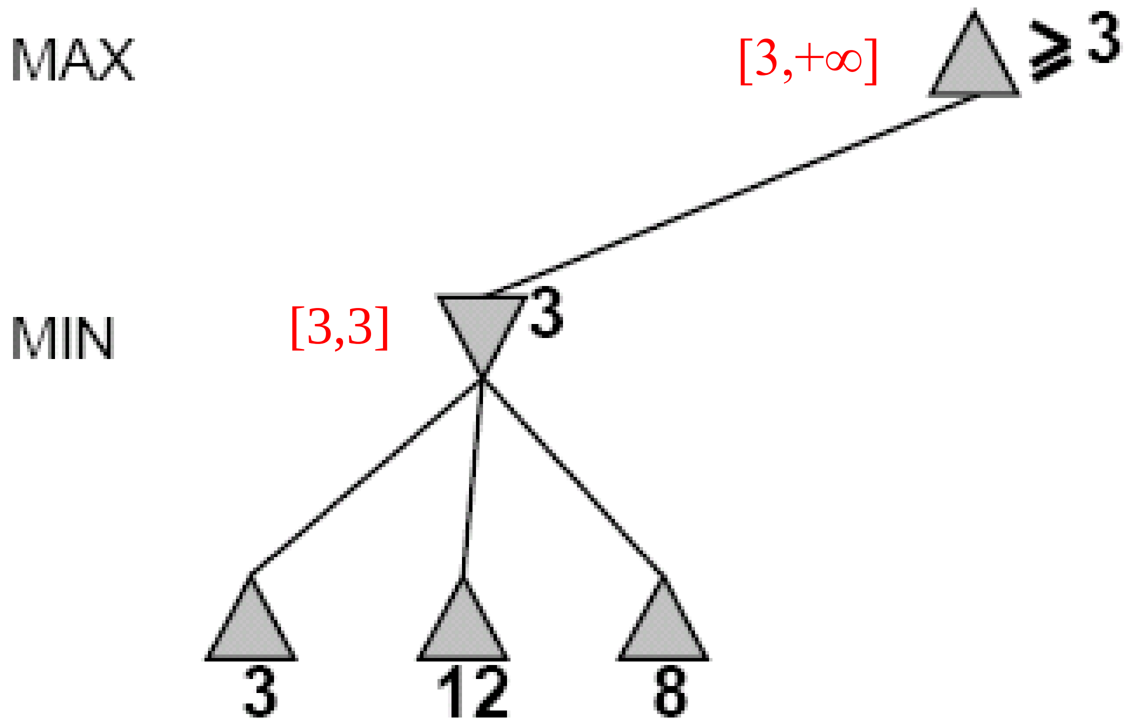
Alpha-Beta Example (continued)



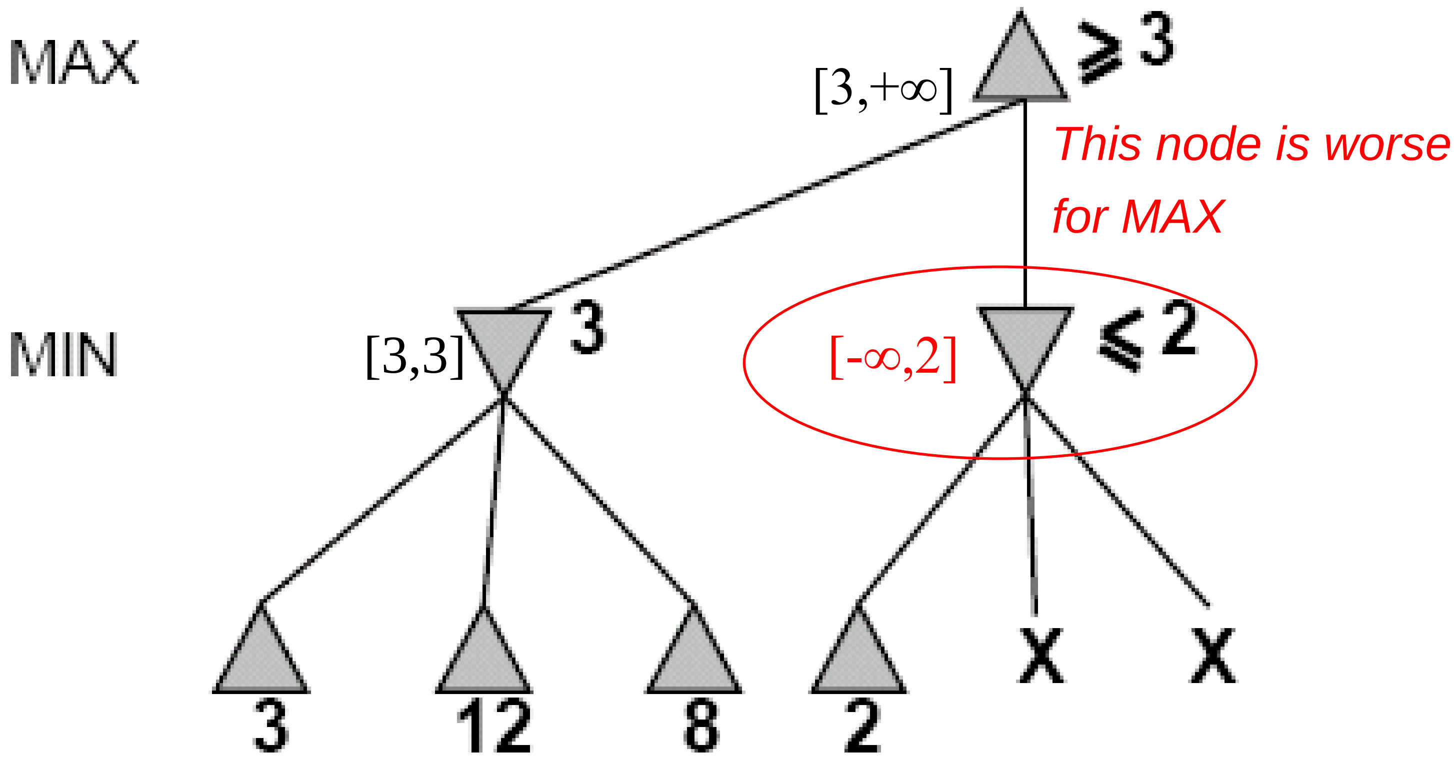
Alpha-Beta Example (continued)



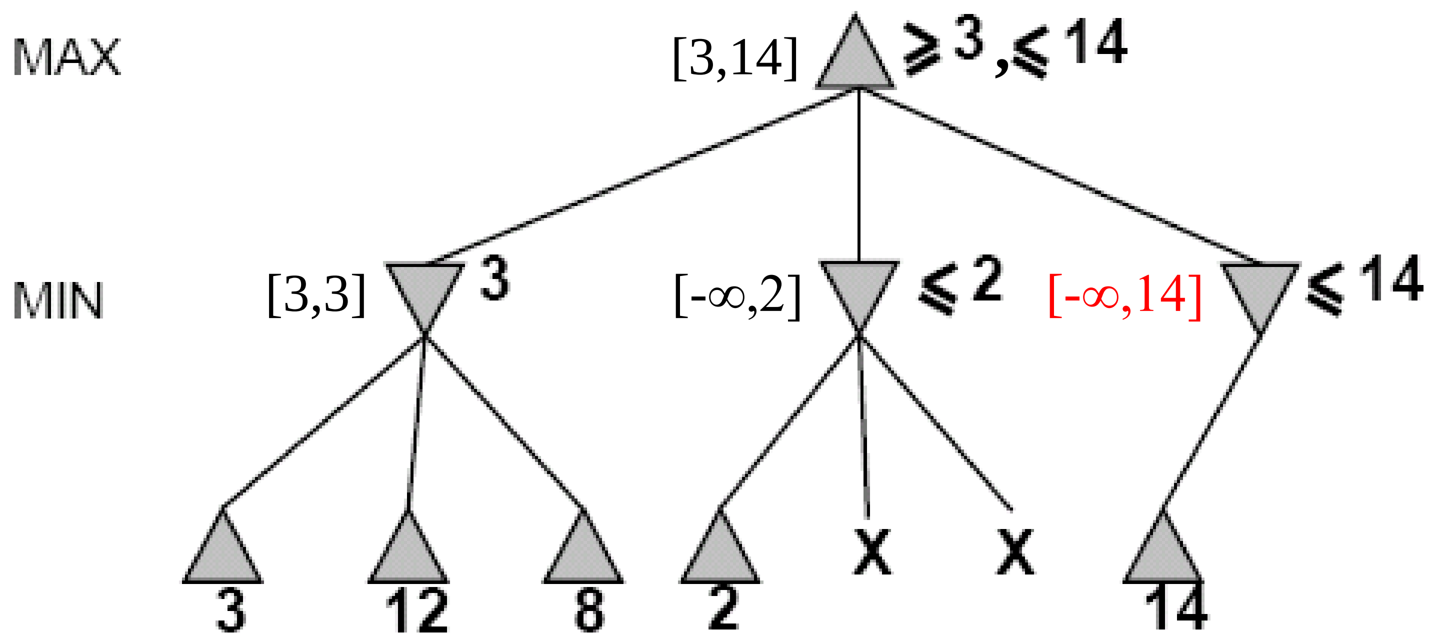
Alpha-Beta Example (continued)



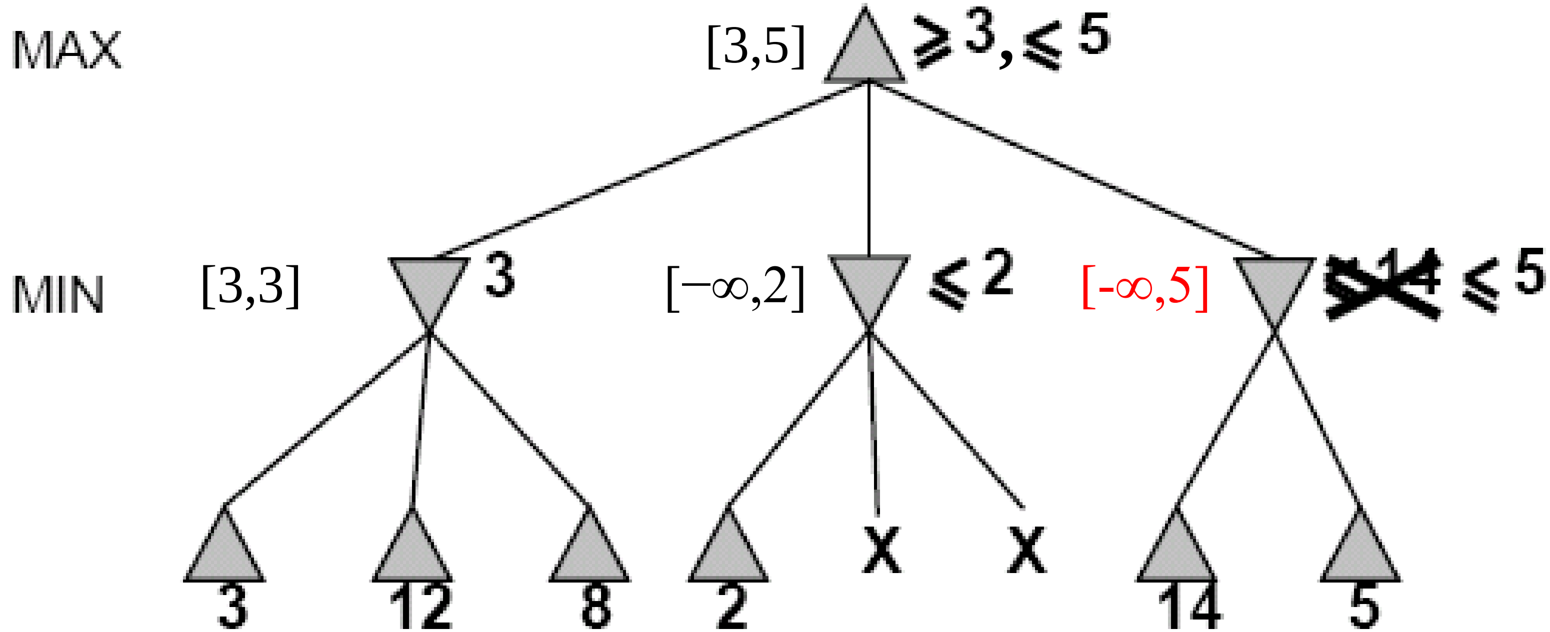
Alpha-Beta Example (continued)



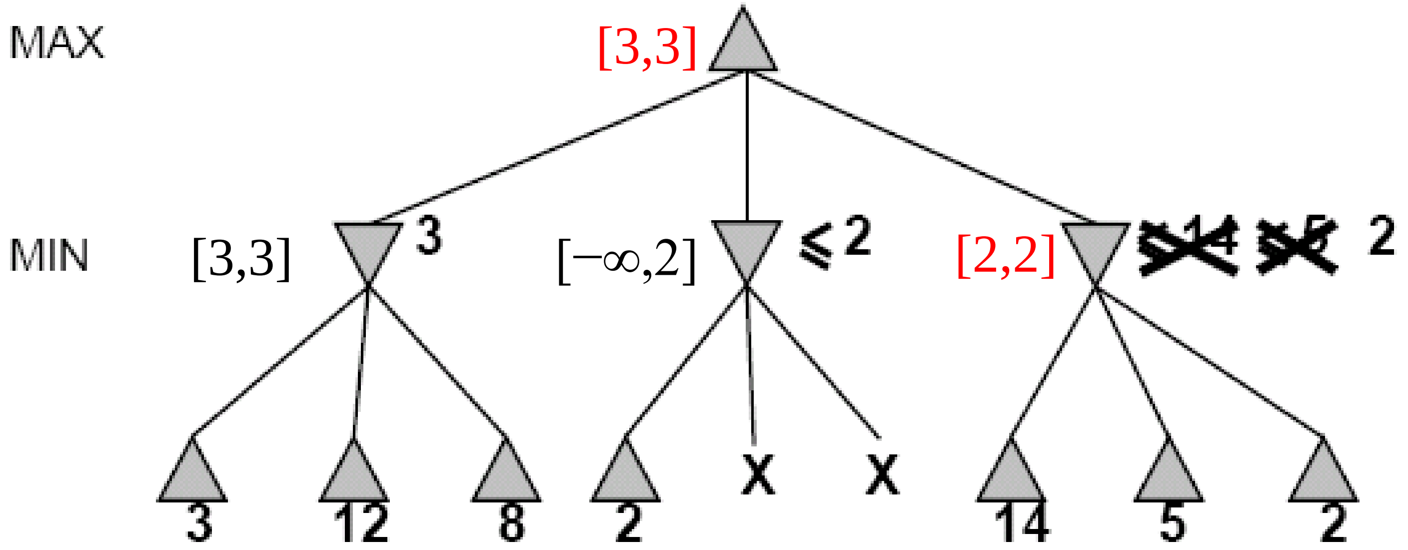
Alpha-Beta Example (continued)



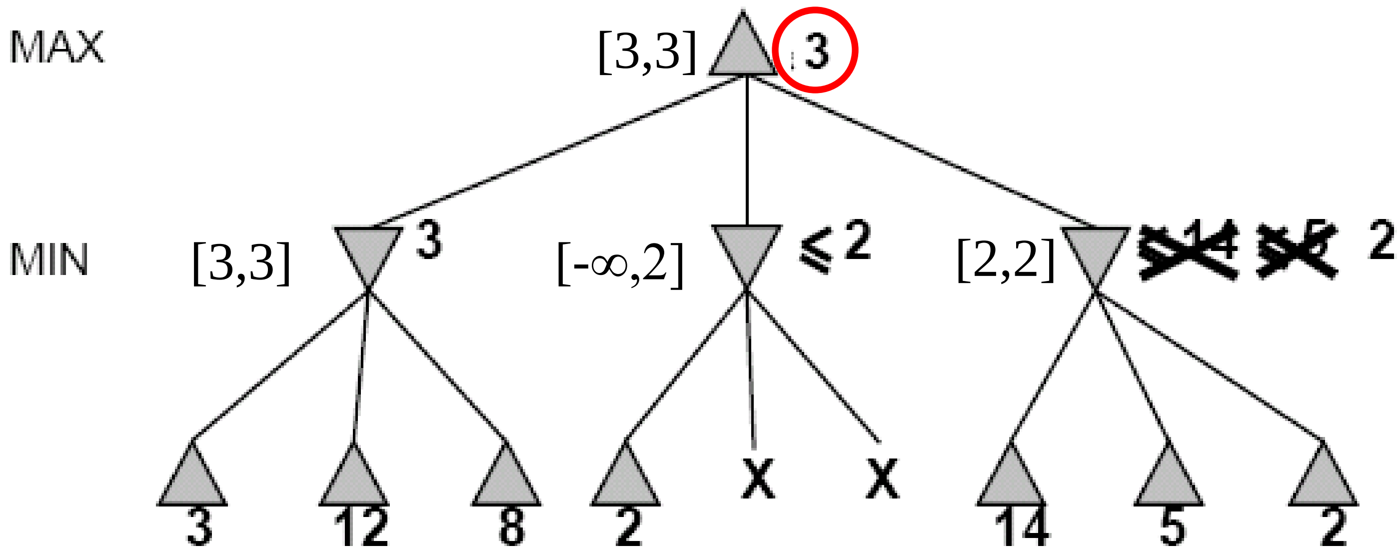
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-beta Algorithm

- Depth first search – only considers nodes along a single path at any time

α = highest-value choice that we can guarantee for MAX so far in the current subtree.

β = lowest-value choice that we can guarantee for MIN so far in the current subtree.

- update values of α and β during search and prunes remaining branches as soon as the value is known to be worse than the current α or β value for MAX or MIN.
- Alpha-beta Demo.

Effectiveness of Alpha-Beta Search

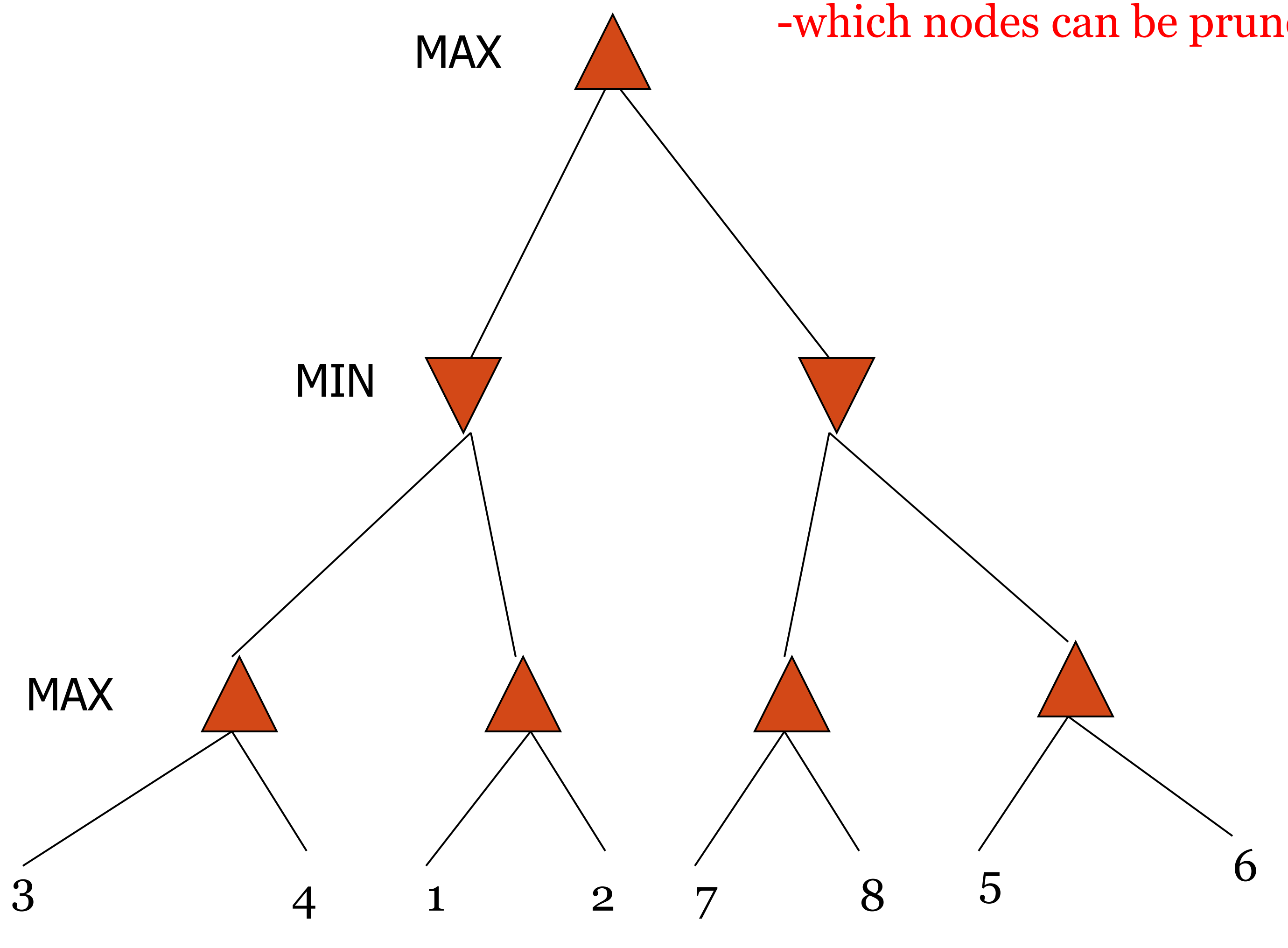
- Worst-Case
  - branches are ordered so that no pruning takes place. In this case alpha-beta gives no improvement over exhaustive search

- Best-Case
  - each player's best move is the left-most alternative (i.e., evaluated first)
  - in practice, performance is closer to best rather than worst-case

- In practice often get $O(b^{(d/2)})$ rather than $O(b^d)$
  - this is the same as having a branching factor of sqrt(b),
    - since $(\mathrm{sqrt}(b))^d = b^{(d/2)}$
    - i.e., we have effectively gone from b to square root of b
  - e.g., in chess go from b ~ 35 to b ~ 6
    - this permits much deeper search in the same amount of time
    - Typically twice as deep.

Example

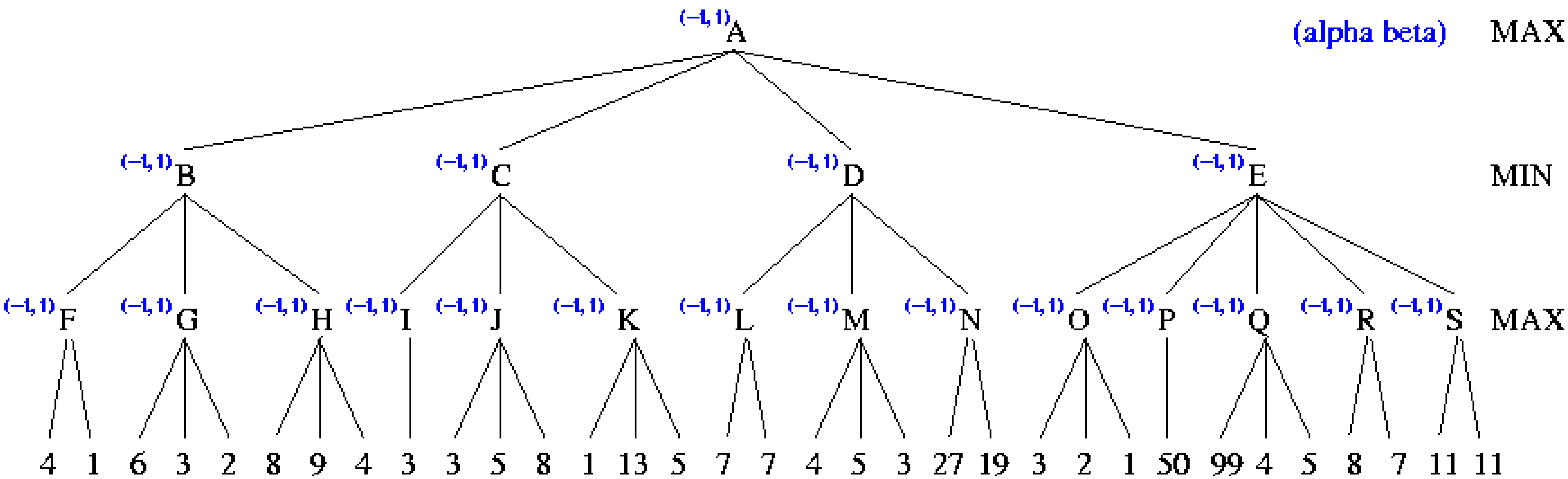
-which nodes can be pruned?



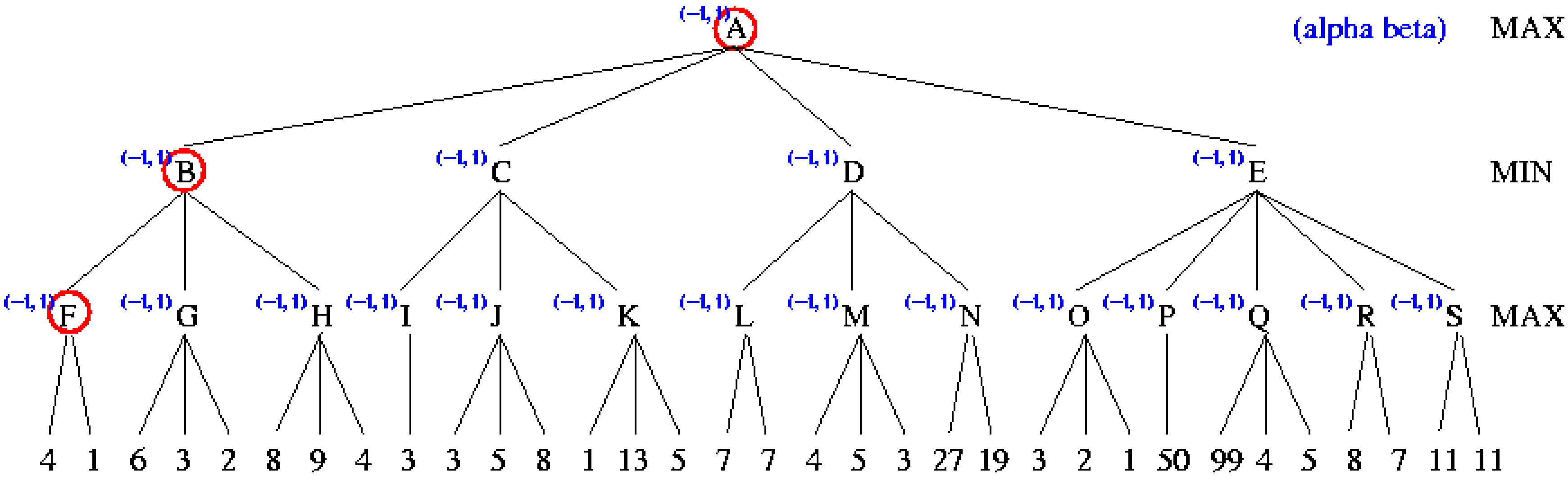
Final Comments about Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned.
- Good move *ordering* improves effectiveness of pruning
- Repeated states are again possible.
  - Store them in memory = transposition table

Example


(alpha beta)
MAX
MIN
MAX
(−I, I) A
(−I, I) B
(−I, I) C
(−I, I) D
(−I, I) E
(−I, I) F
(−I, I) G
(−I, I) H
(−I, I) I
(−I, I) J
(−I, I) K
(−I, I) L
(−I, I) M
(−I, I) N
(−I, I) O
(−I, I) P
(−I, I) Q
(−I, I) R
(−I, I) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
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MAX
(−I, I) A
(−I, I) B
(−I, I) C
(−I, I) D
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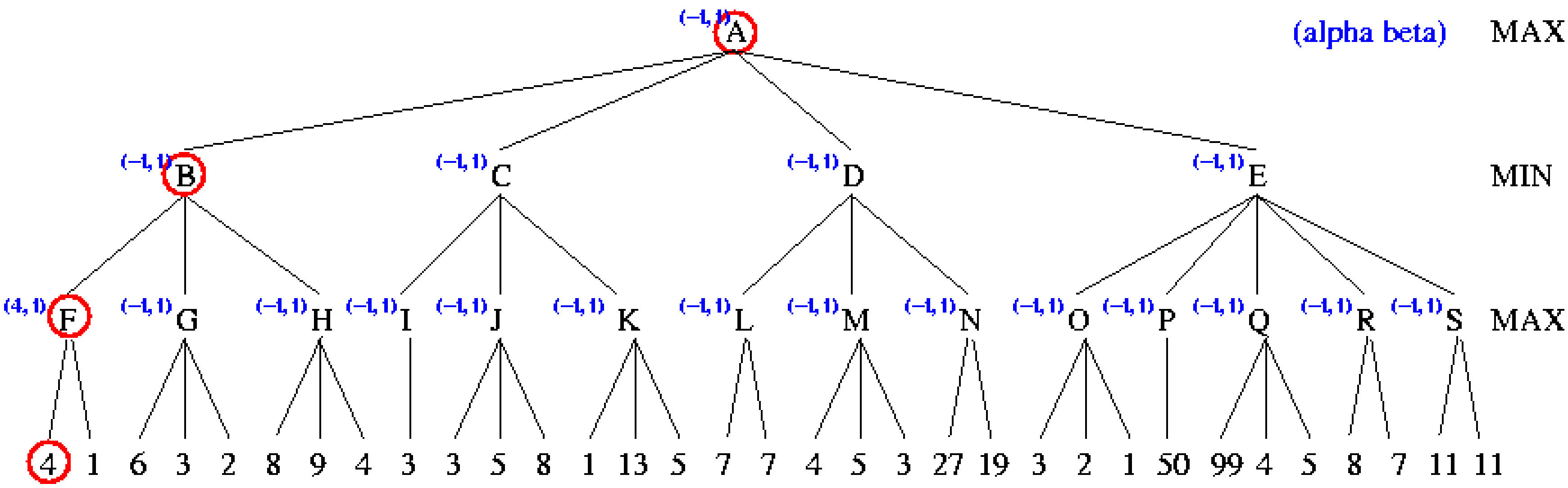
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(−I, I)
A
(alpha beta)
MAX
(−I, I) B
(−I, I) C
(−I, I) D
(−I, I) E
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(−I, I) H
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(−I, I) M
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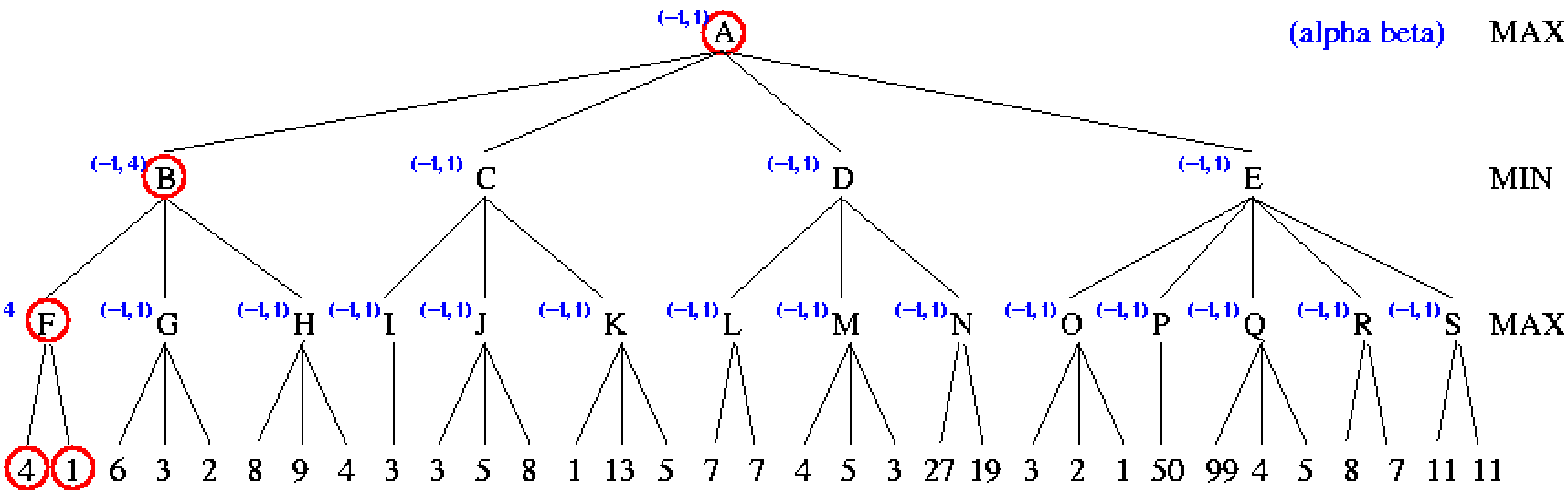
(−I, I) P
(−I, I) Q
(−I, I) R
(−I, I) S
MAX
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
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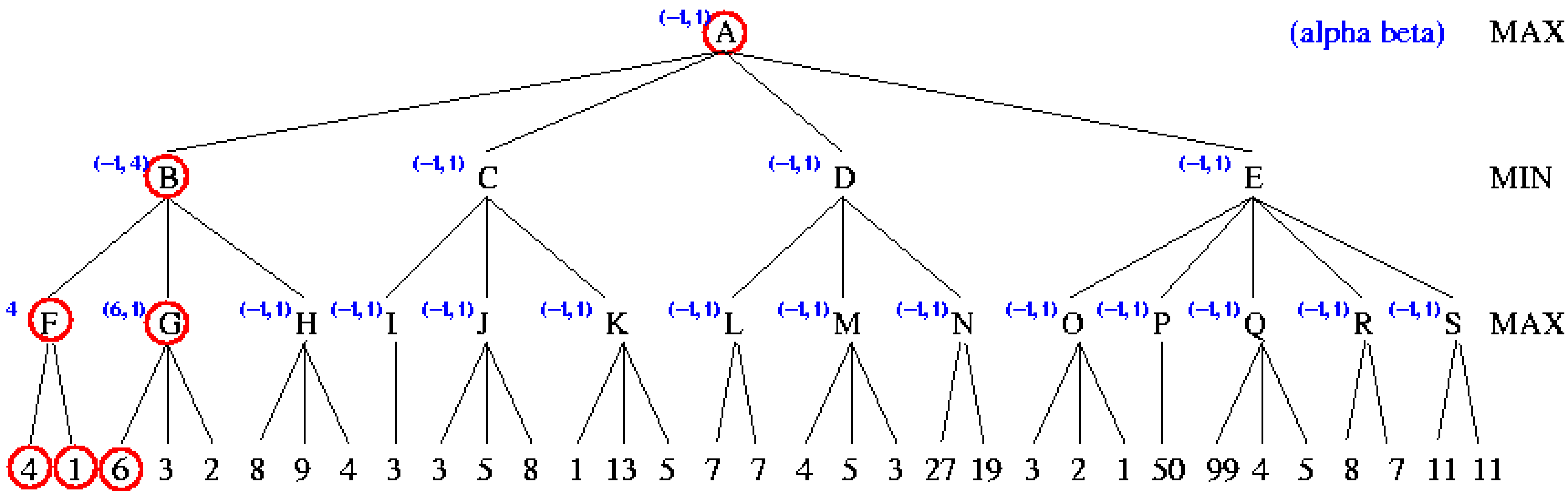
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
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(−1, 1) A
(−1, 4) B
(−1, 1) C
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(−1, 1) E
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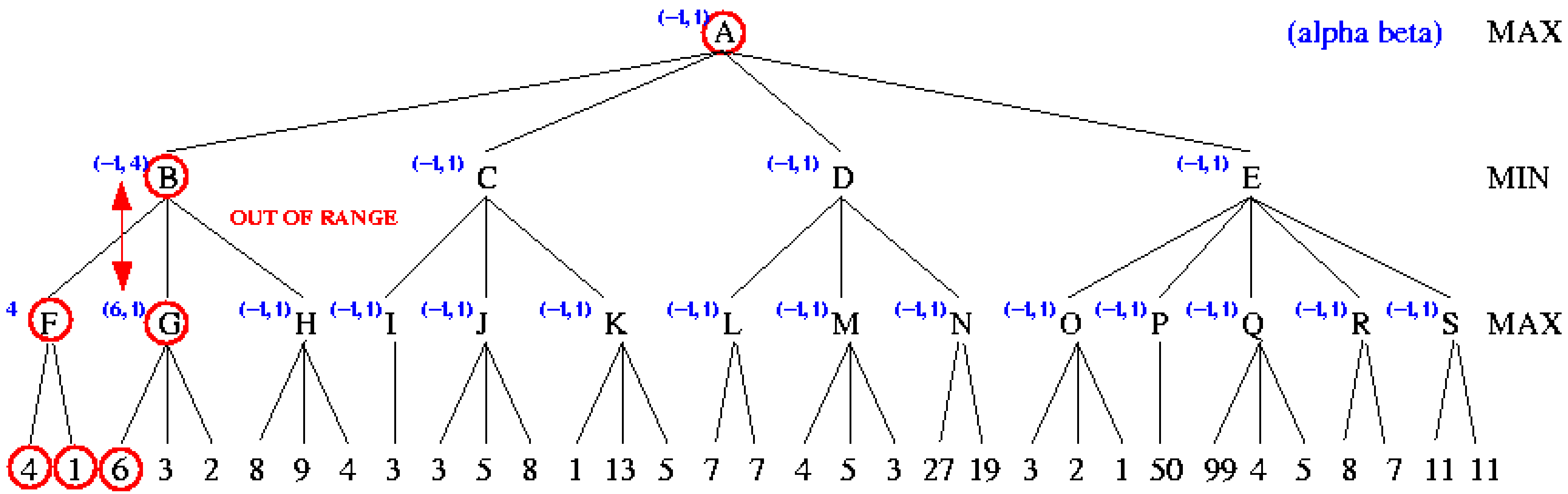
Example



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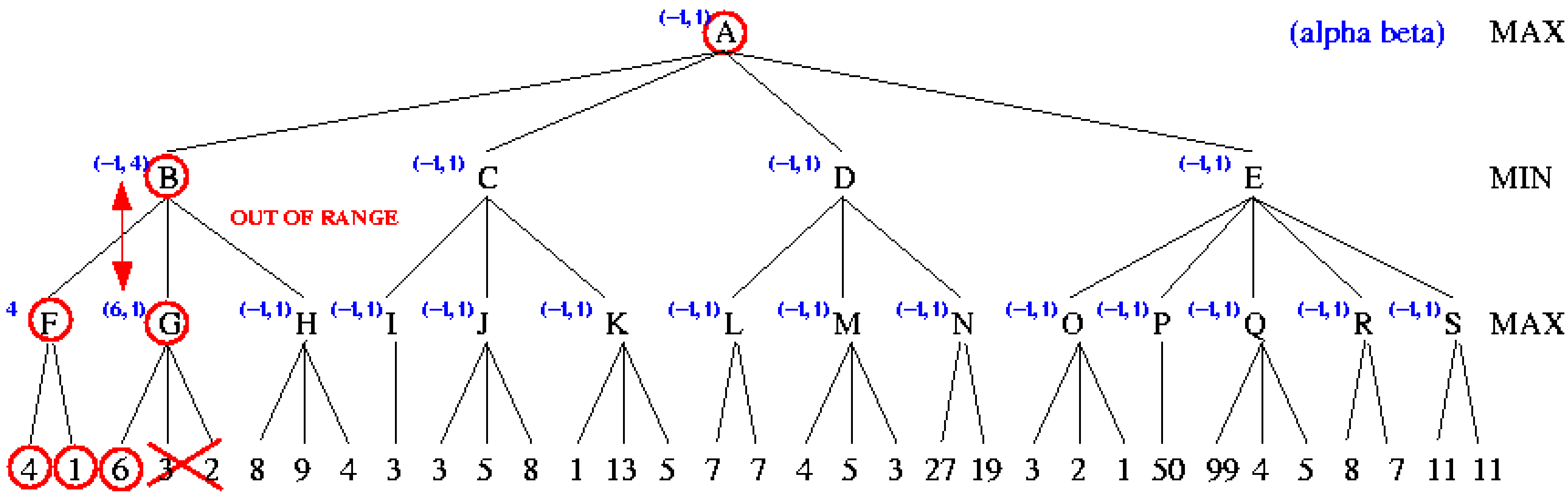
Example



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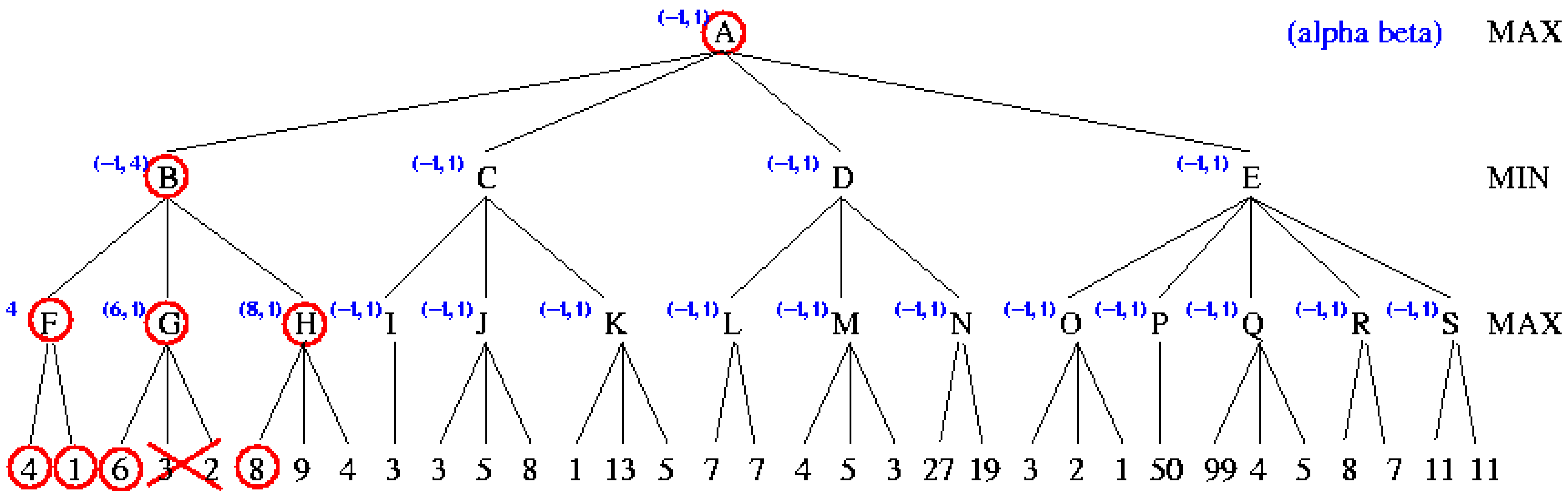
Example



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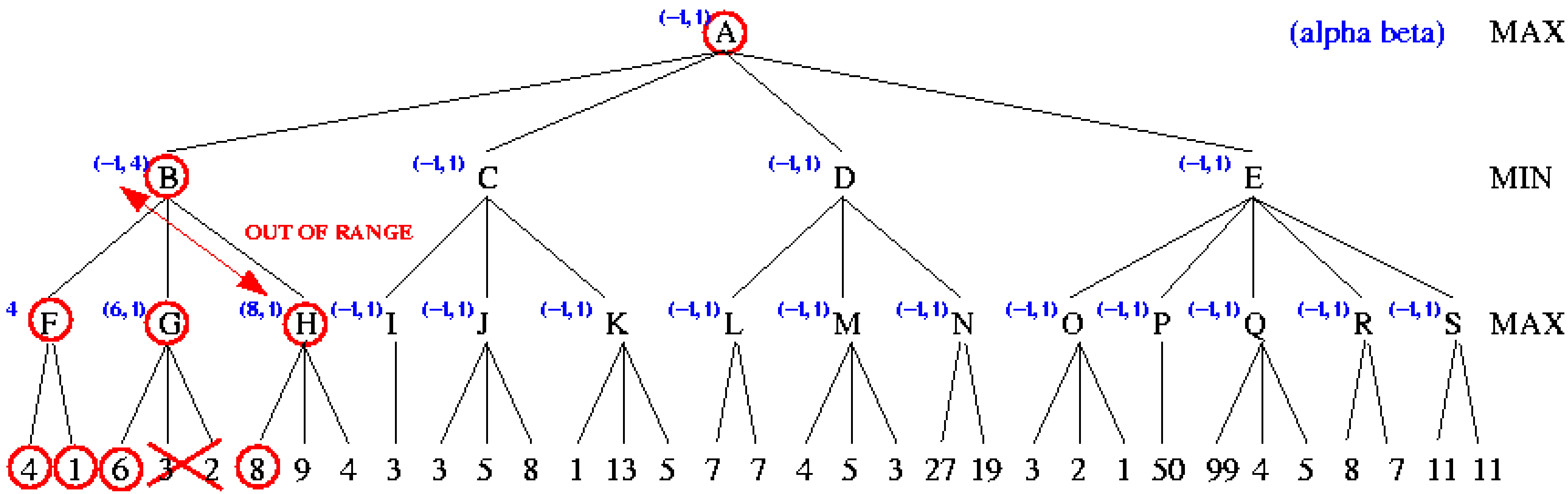
Example



(alpha beta)
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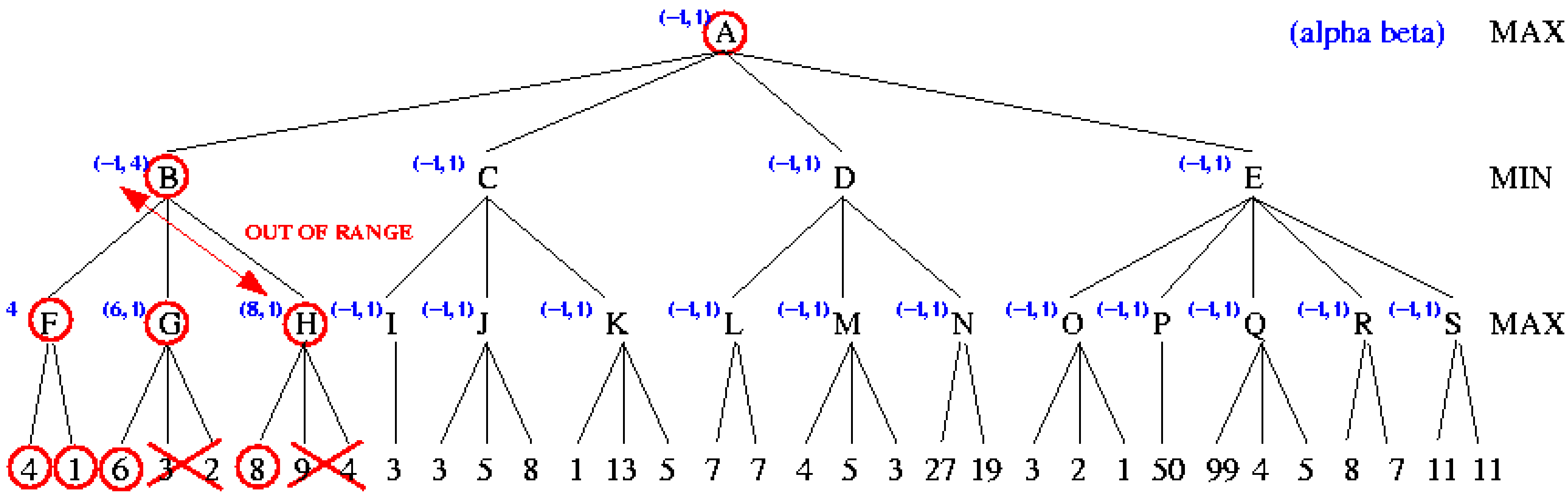
Example



(alpha beta)
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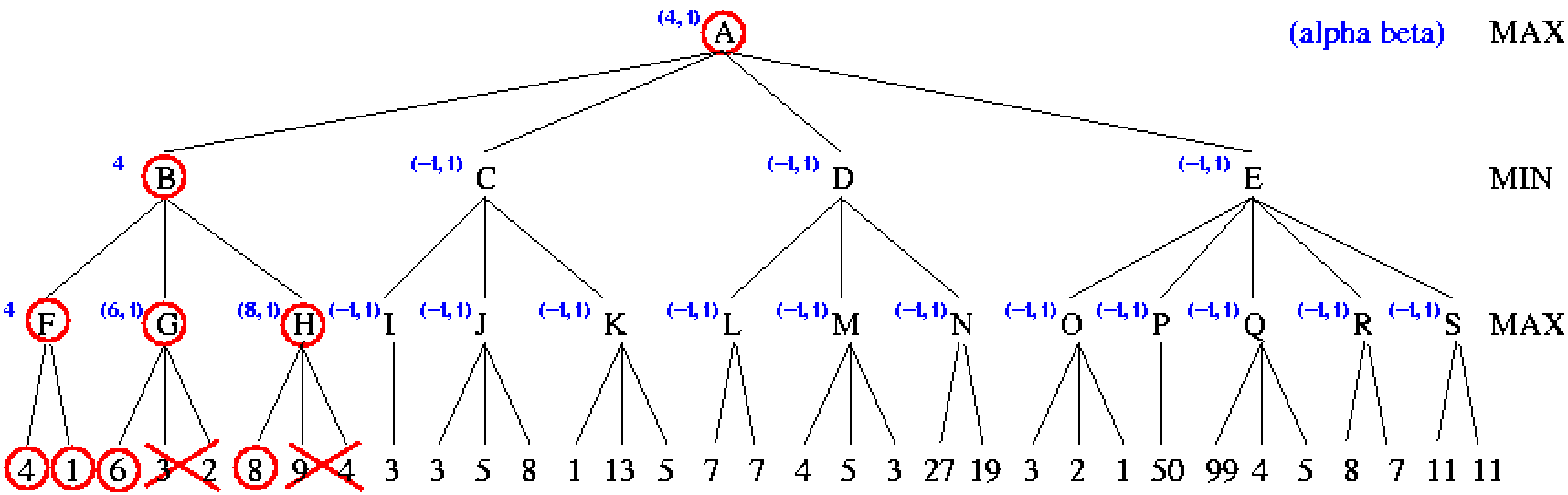
Example


(alpha beta)
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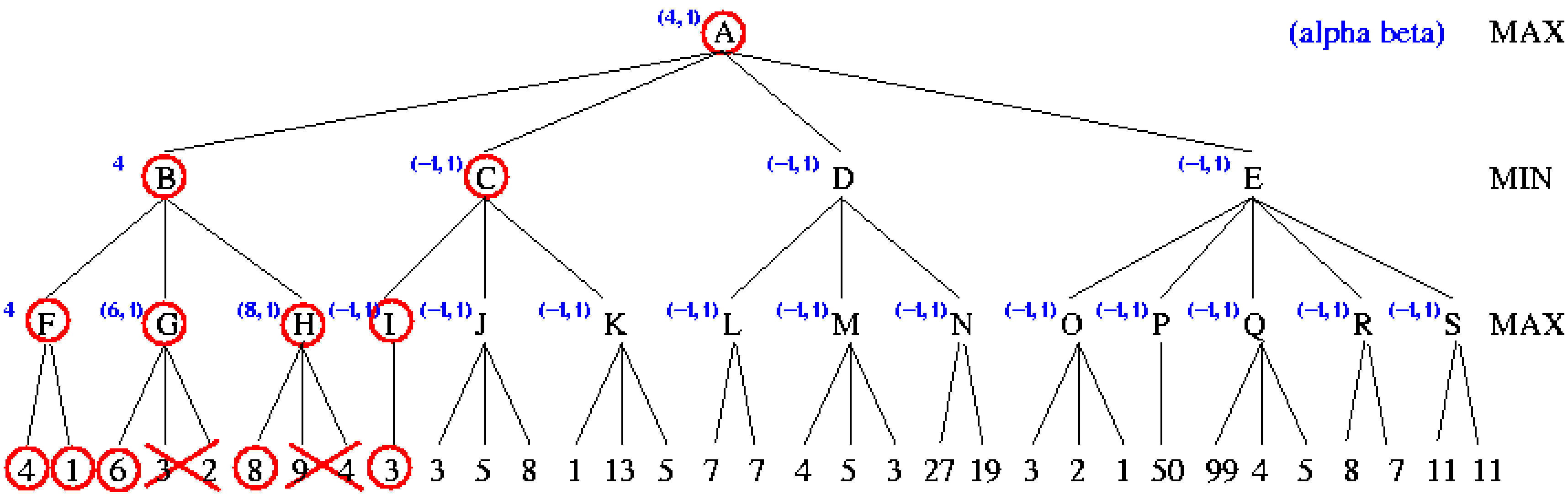
Example



(alpha beta)
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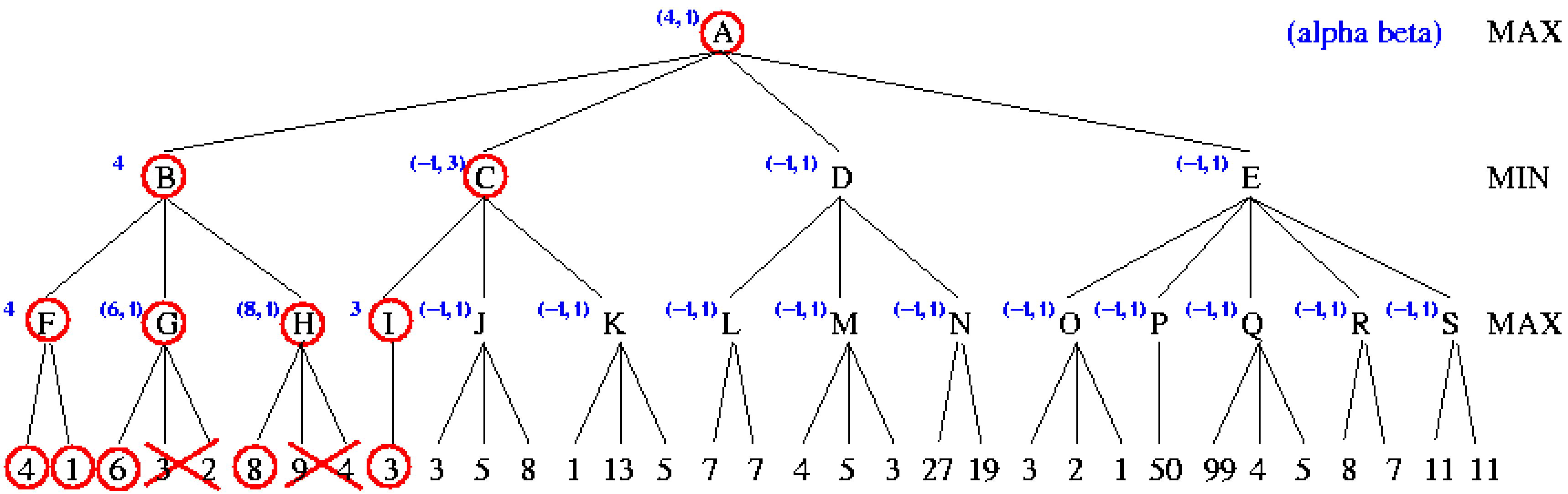
Example


(alpha beta)
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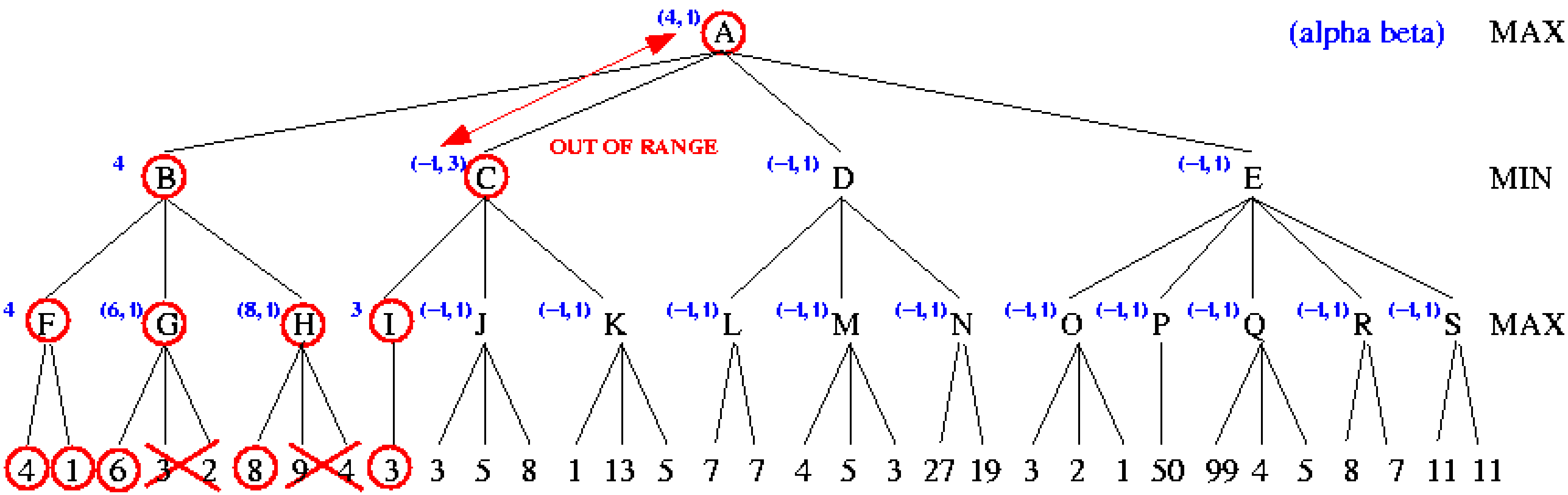
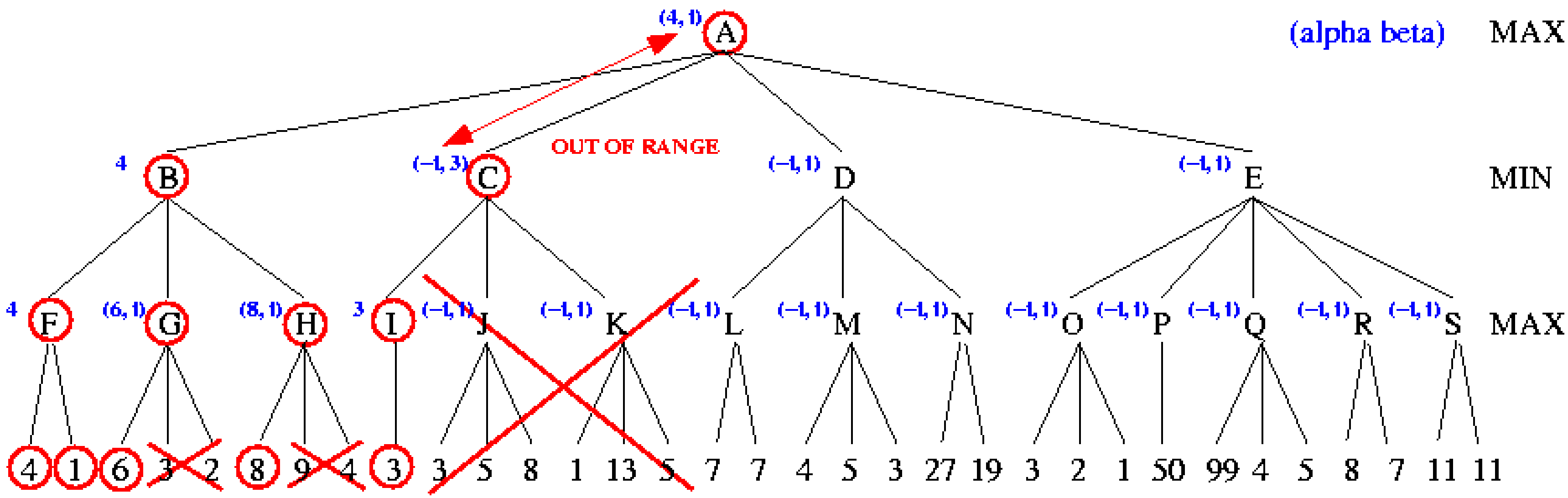
Example



(4, 1) A
(alpha beta)
MAX
OUT OF RANGE
4 B
(−1, 3) C
(−1, 1) D
(−1, 1) E
MIN
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
(−1, 1) L
(−1, 1) M
(−1, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
MAX
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(4, 1)
A
(alpha beta)
MAX
OUT OF RANGE
4
B
(–1, 3)
C
(–1, 1)
D
(–1, 1)
E
MIN
4
F
(6, 1)
G
(8, 1)
H
3
I
(–1, 1)
J
(–1, 1)
K
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L
(–1, 1)
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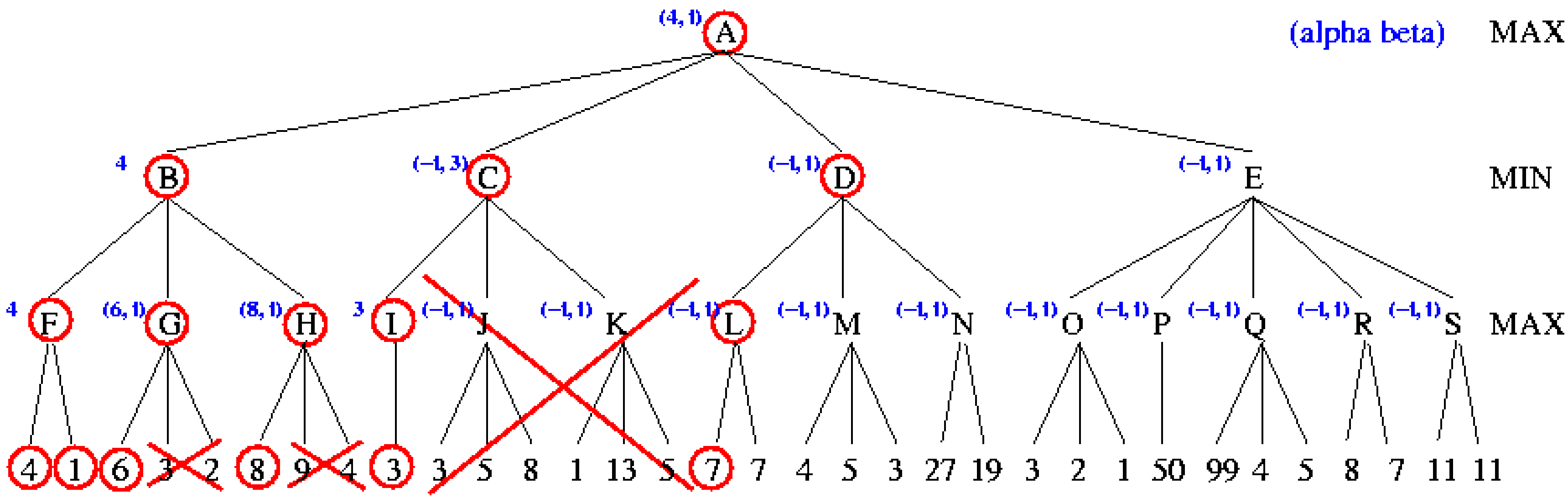
N
(–1, 1)
O
(–1, 1)
P
(–1, 1)
Q
(–1, 1)
R
(–1, 1)
S
MAX
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example


(alpha beta)
MAX
MIN
MAX
(4, 1) A
4 B
(−1, 3) C
(−1, 1) D
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4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
(−1, 1) L
(−1, 1) M
(−1, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
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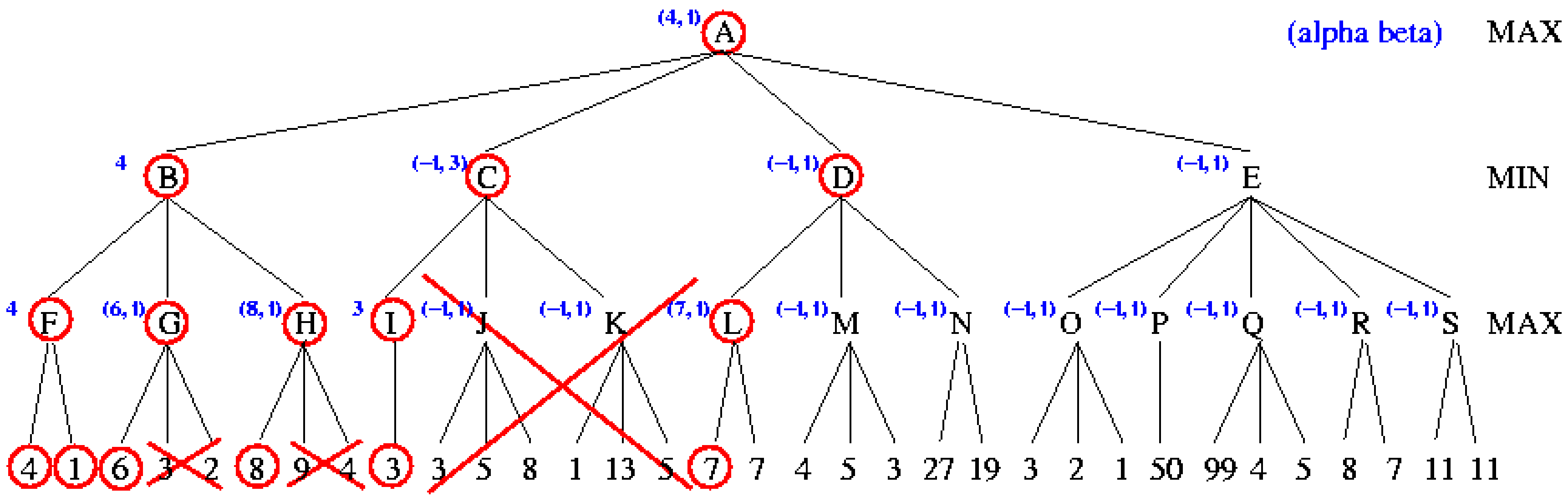
(−1, 1) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
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(4, 1)
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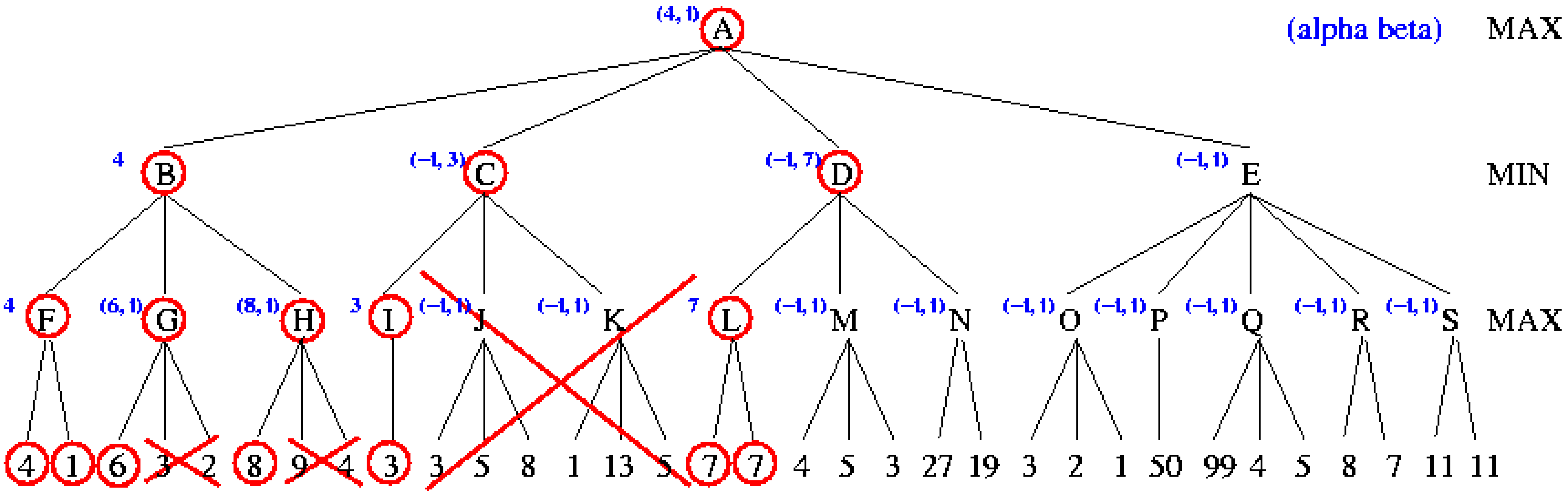
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example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MAX
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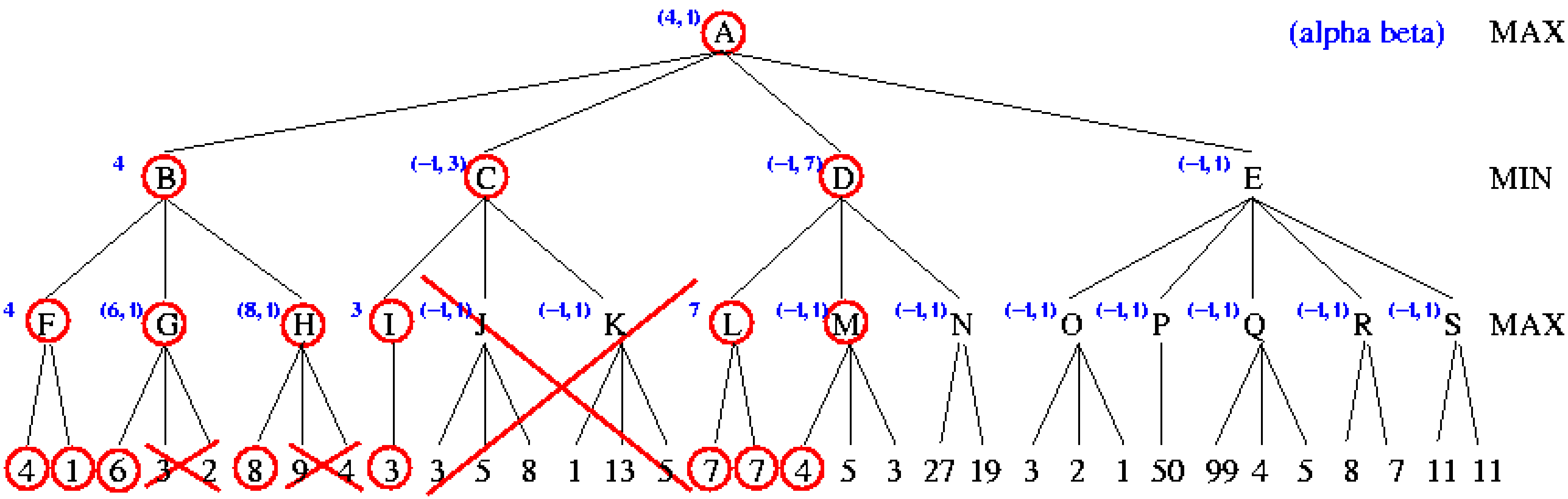
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example


(alpha beta)
MAX
MIN
MAX
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C
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E
4
F
(6, 1)
G
(8, 1)
H
3
I
(−1, 1)
J
(−1, 1)
K
7
L
(−1, 1)
M
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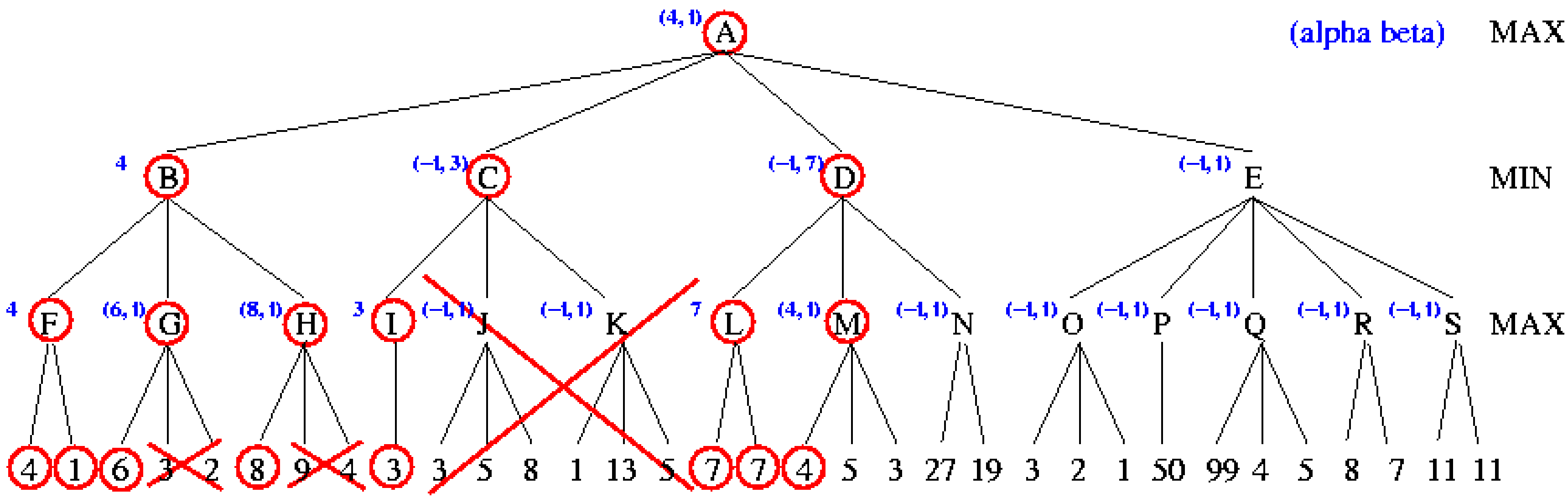
N
(−1, 1)
O
(−1, 1)
P
(−1, 1)
Q
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example


(alpha beta)
MAX
MIN
MAX
A
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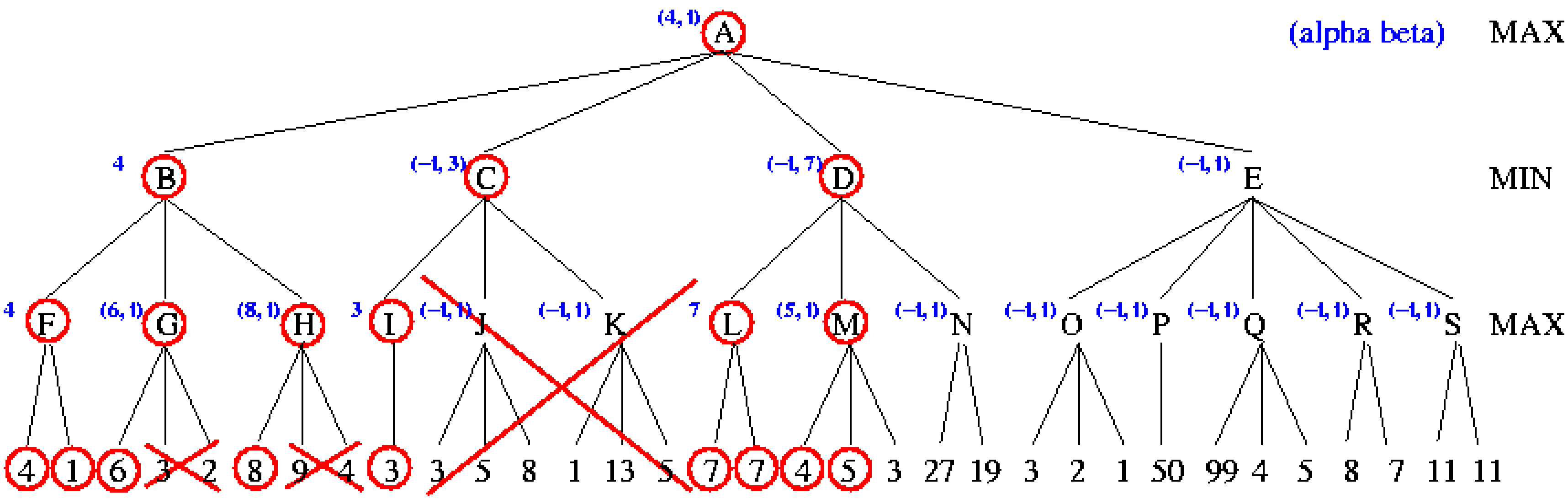
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
(4, 1)
A
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B
(−1, 3)
C
(−1, 7)
D
(−1, 1)
E
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F
(6, 1)
G
(8, 1)
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N
(−1, 1)
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(−1, 1)
P
(−1, 1)
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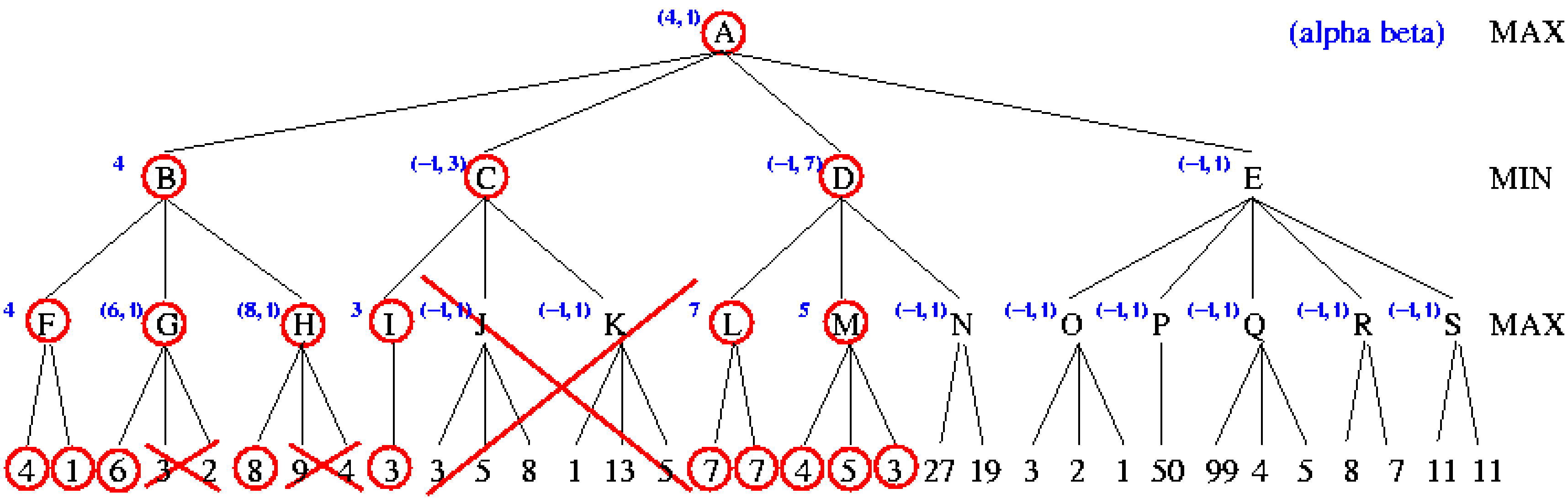
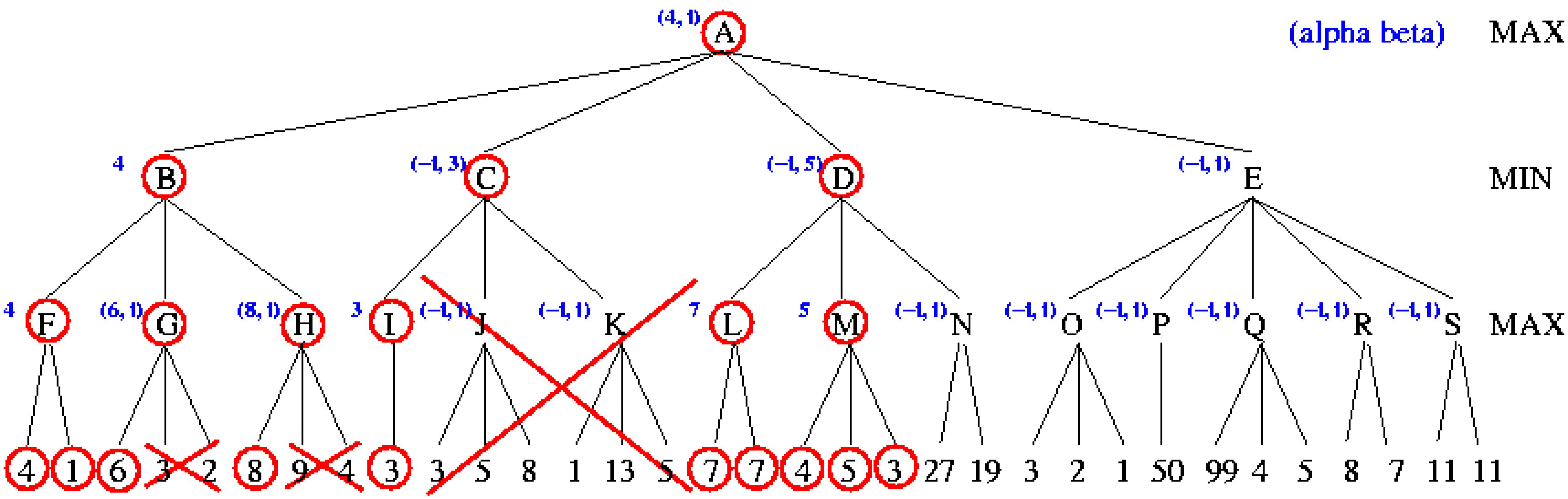
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



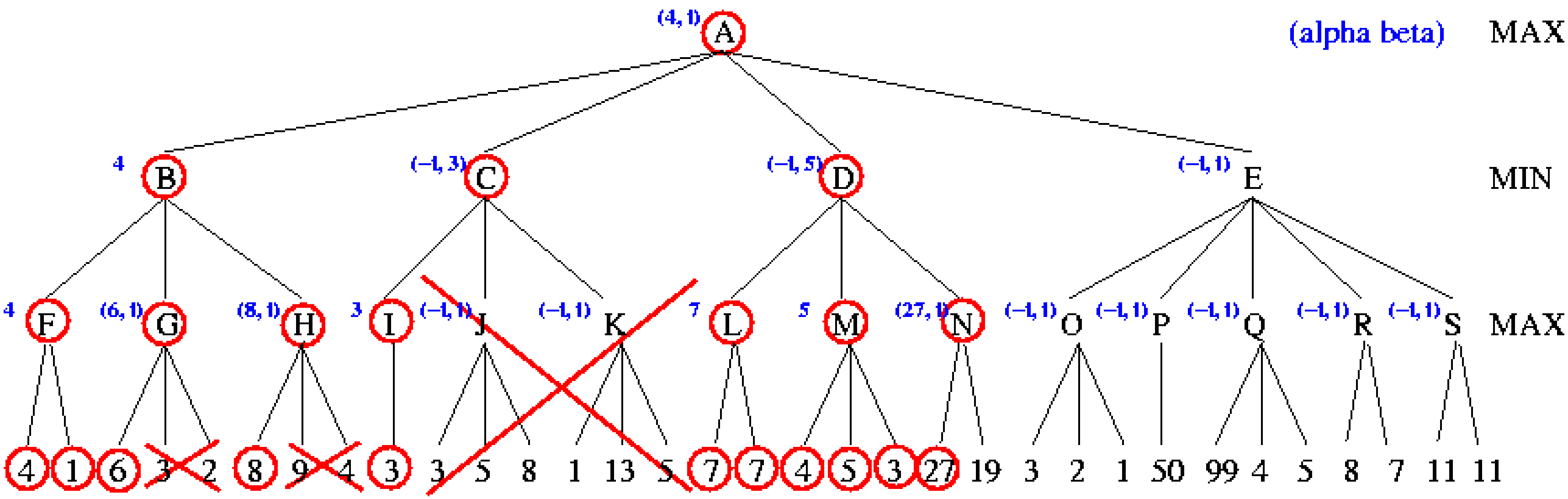
(4, 1)
A
(alpha beta)
MAX
4
B
(−1, 3)
C
(−1, 7)
D
(−1, 1)
E
MIN
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(−1, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
MAX
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



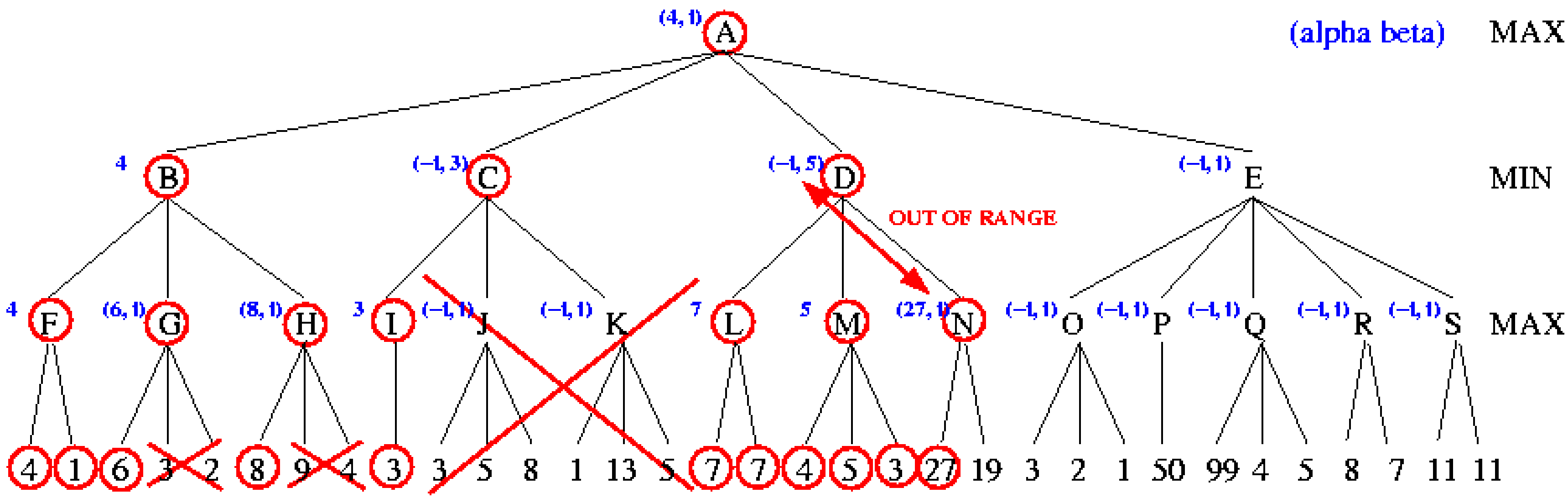
(alpha beta)
MAX
MIN
MAX
(4, 1)
A
4
B
(−1, 3)
C
(−1, 5)
D
(−1, 1)
E
4
F
(6, 1)
G
(8, 1)
H
3
I
(−1, 1)
J
(−1, 1)
K
7
L
5
M
(−1, 1)
N
(−1, 1)
O
(−1, 1)
P
(−1, 1)
Q
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example

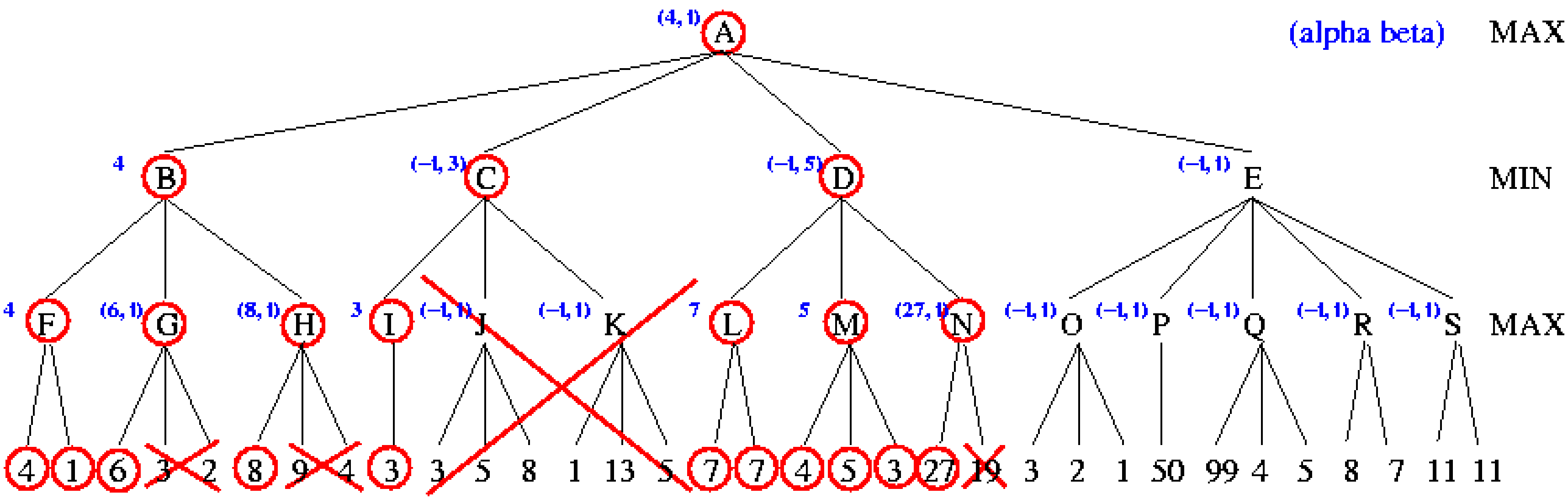


(4, 1) A
(alpha beta)
MAX
4 B
(−1, 3) C
(−1, 5) D
(−1, 1) E
MIN
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(27, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
MAX
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example

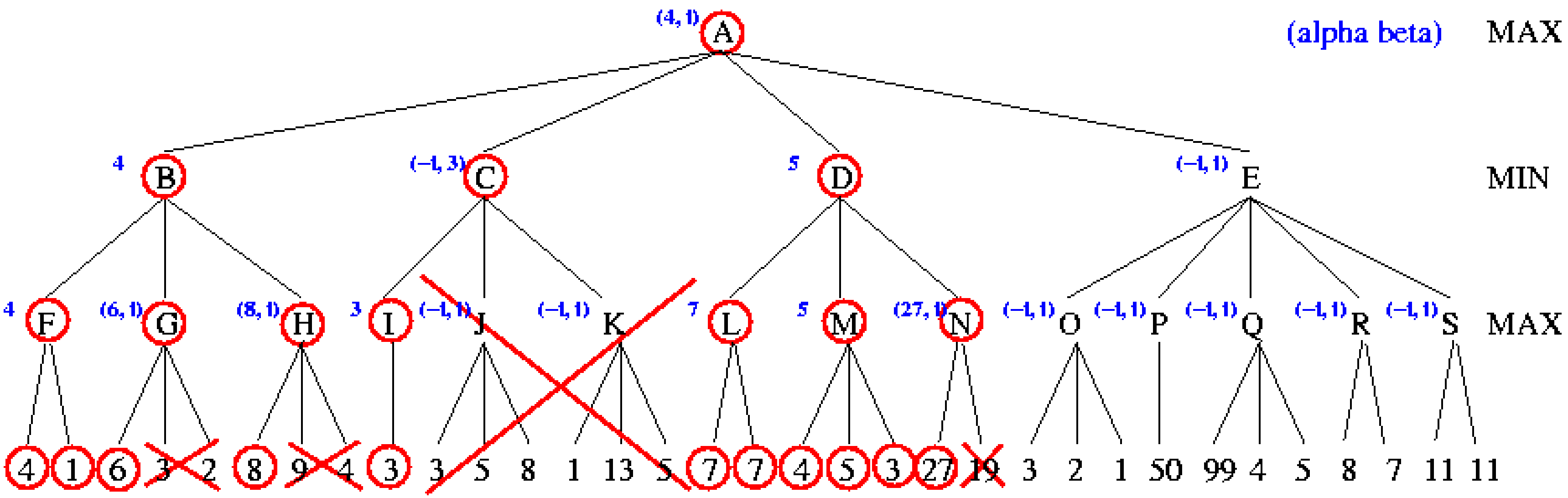

(alpha beta)
MAX
MIN
MAX
(4, 1)
A
4
B
(−1, 3)
C
(−1, 5)
D
(−1, 1)
E
OUT OF RANGE
4
F
(6, 1)
G
(8, 1)
H
3
I
(−1, 1)
J
(−1, 1)
K
7
L
5
M
(27, 1)
N
(−1, 1)
O
(−1, 1)
P
(−1, 1)
Q
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example

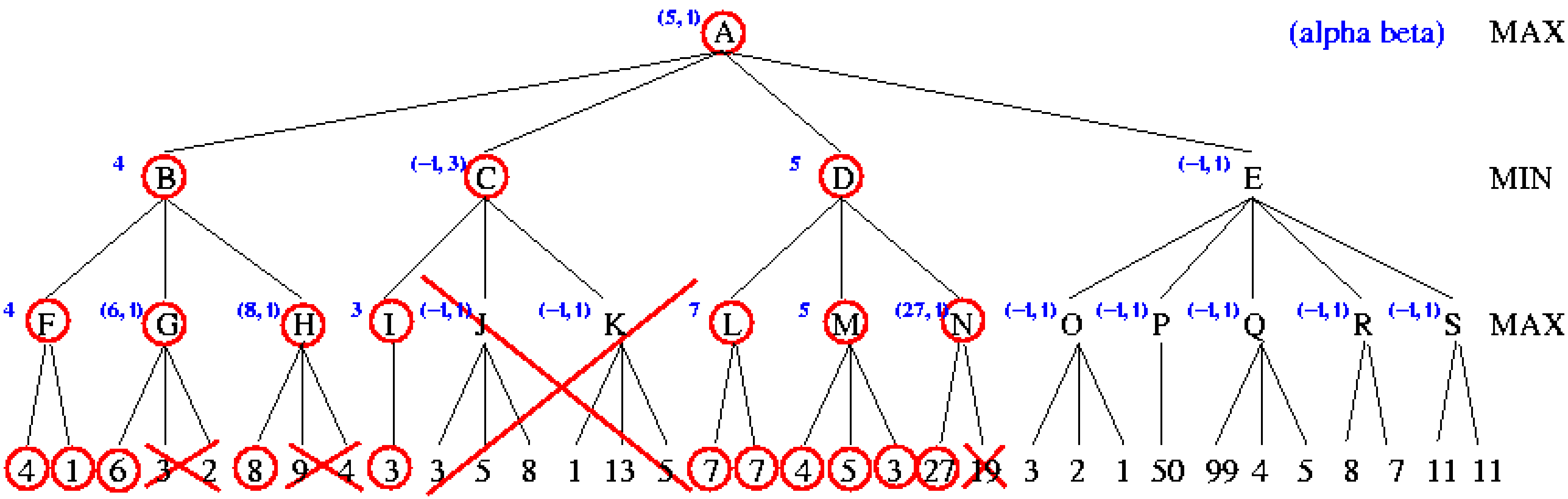


(alpha beta)
MAX
MIN
MAX
(4, 1) A
4 B
(−1, 3) C
(−1, 5) D
(−1, 1) E
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(27, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example


(alpha beta)
MAX
MIN
MAX
(4, 1) A
4 B
(−1, 3) C
5 D
(−1, 1) E
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(27, 1) N
(−1, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
(5, 1)
A
4
B
(−1, 3)
C
5
D
(−1, 1)
E
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F
(6, 1)
G
(8, 1)
H
3
I
(−1, 1)
J
(−1, 1)
K
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L
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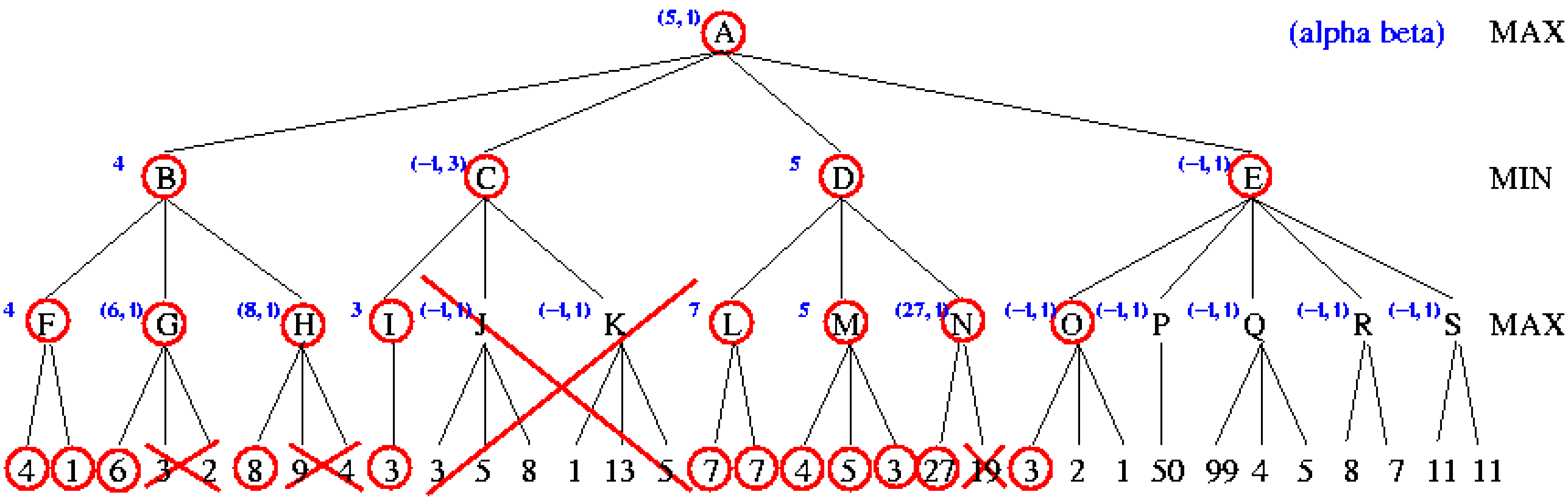
M
(27, 1)
N
(−1, 1)
O
(−1, 1)
P
(−1, 1)
Q
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
(5, 1)
A
4
B
(−1, 3)
C
5
D
(−1, 1)
E
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F
(6, 1)
G
(8, 1)
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(−1, 1)
J
(−1, 1)
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(27, 1)
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(−1, 1)
P
(−1, 1)
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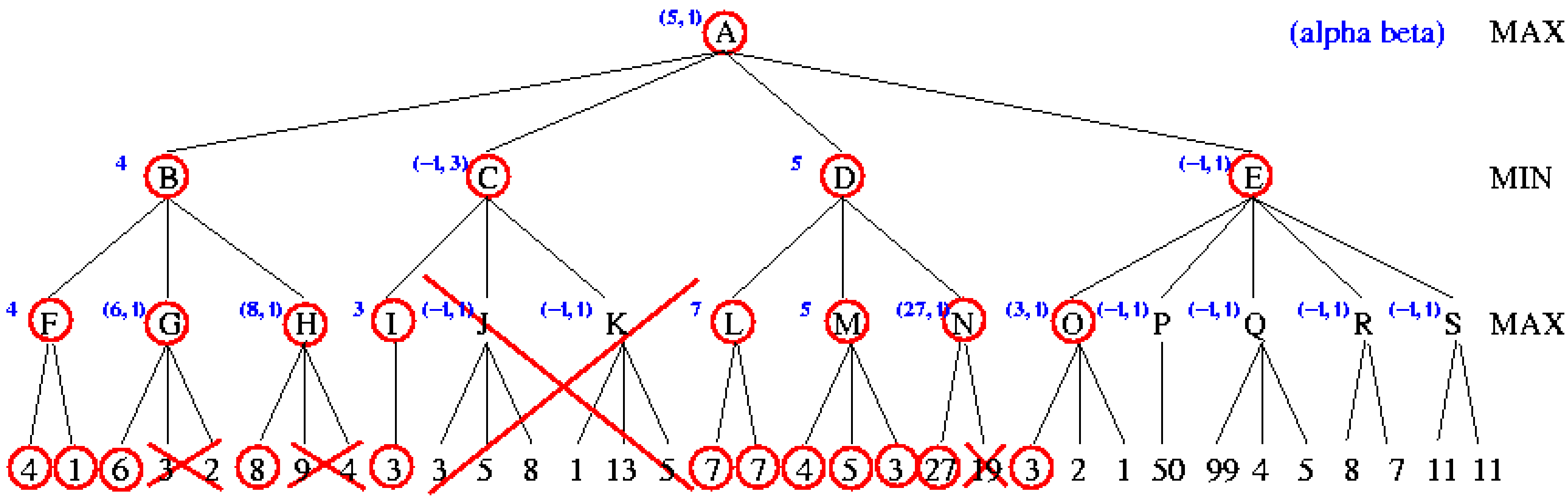
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
(5, 1)
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(−1, 3)
C
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D
(−1, 1)
E
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F
(6, 1)
G
(8, 1)
H
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I
(−1, 1)
J
(−1, 1)
K
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L
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M
(27, 1)
N
(3, 1)
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P
(−1, 1)
Q
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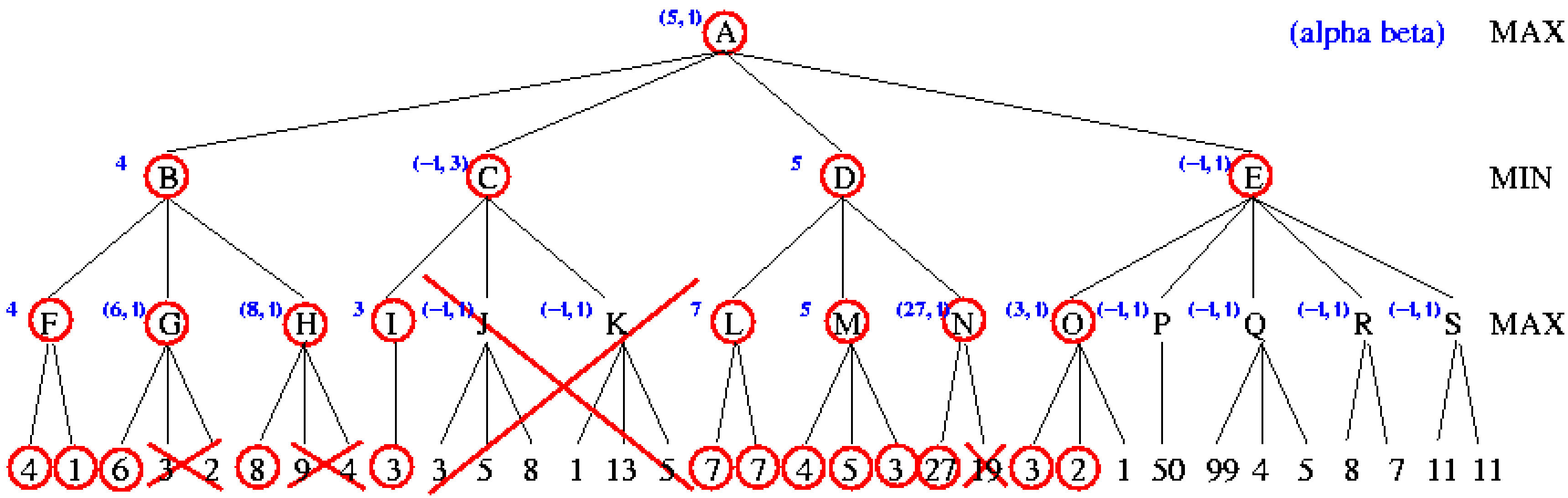
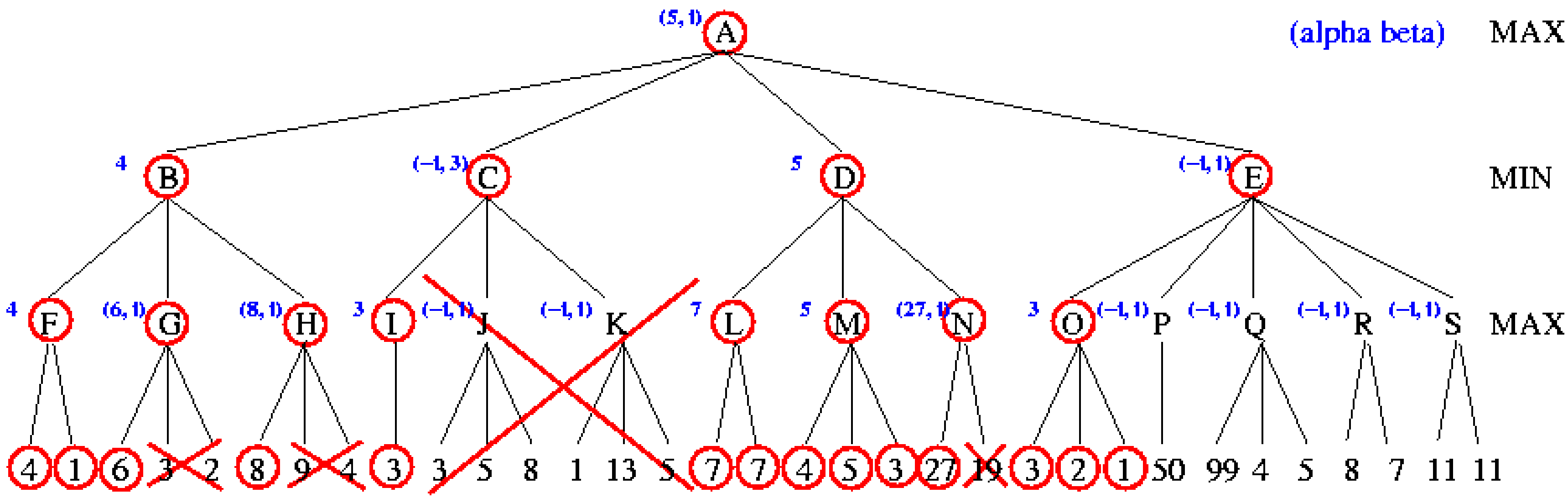
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
(5, 1) A
4 B
(−1, 3) C
5 D
(−1, 1) E
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(27, 1) N
(3, 1) O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
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(5, 1)
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E
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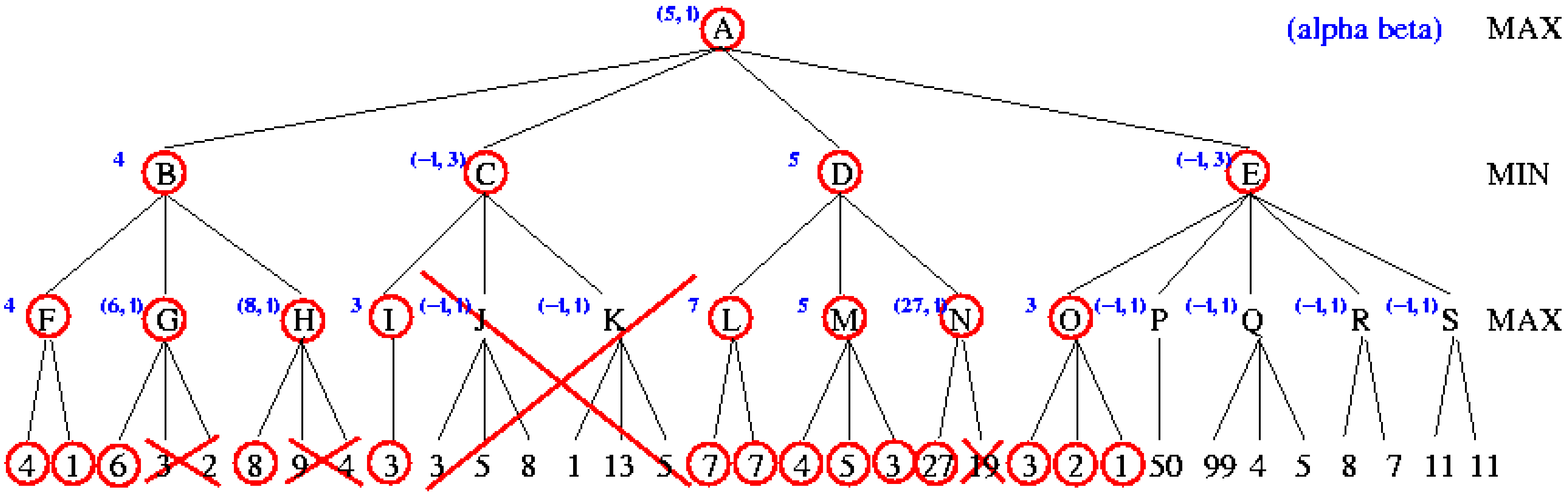
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example


(alpha beta)
MAX
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(5, 1)
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(−1, 3)
C
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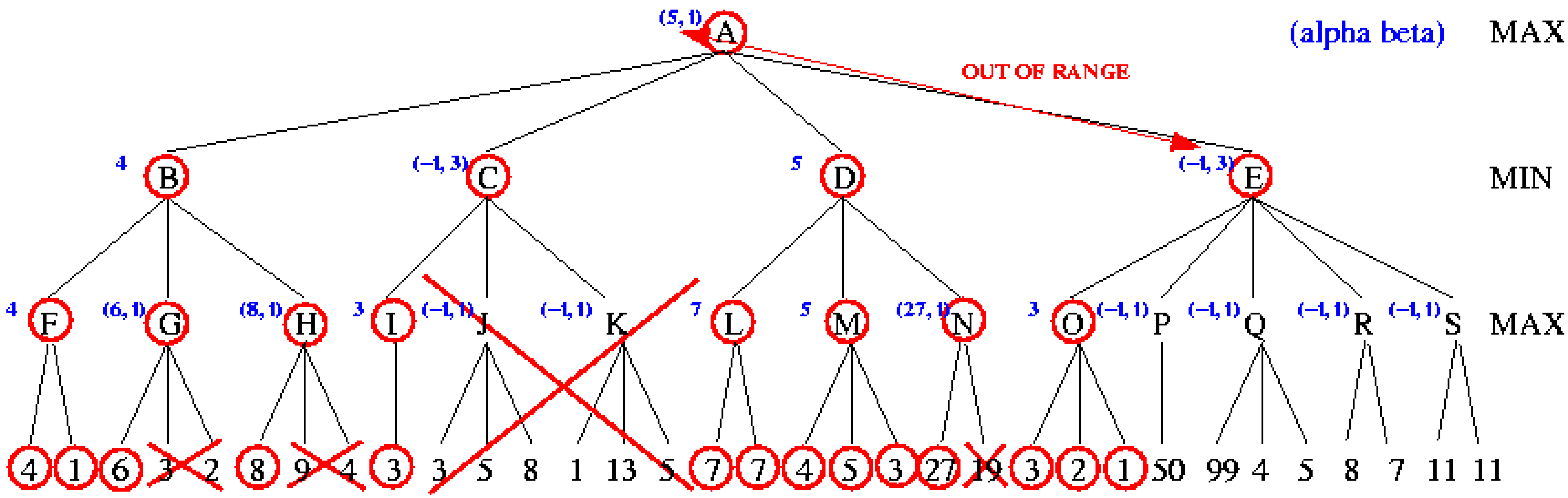
D
(−1, 3)
E
4 F
(6, 1) G
(8, 1) H
3 I
(−1, 1) J
(−1, 1) K
7 L
5 M
(27, 1) N
3 O
(−1, 1) P
(−1, 1) Q
(−1, 1) R
(−1, 1) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
OUT OF RANGE
(5, 1)
A
4
B
(−1, 3)
C
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D
(−1, 3)
E
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F
(6, 1)
G
(8, 1)
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(27, 1)
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(−1, 1)
P
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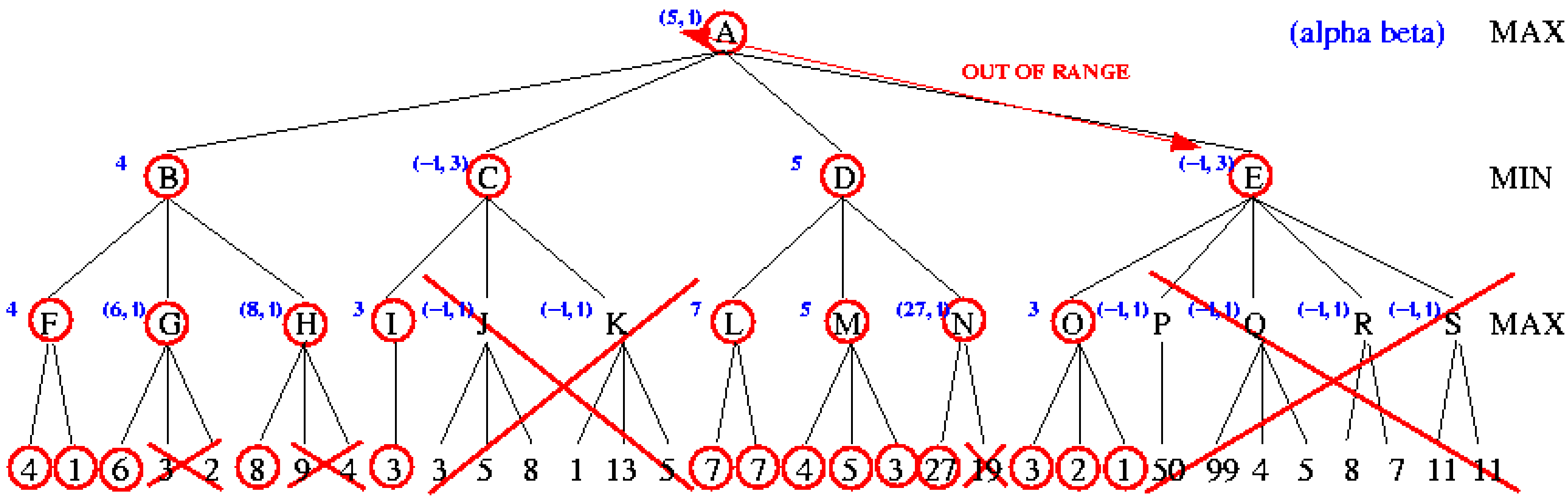
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



(alpha beta)
MAX
MIN
MAX
OUT OF RANGE
(5, 1)
A
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B
(−1, 3)
C
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D
(−1, 3)
E
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F
(6, 1)
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(8, 1)
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(−1, 1)
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(−1, 1)
K
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5
M
(27, 1)
N
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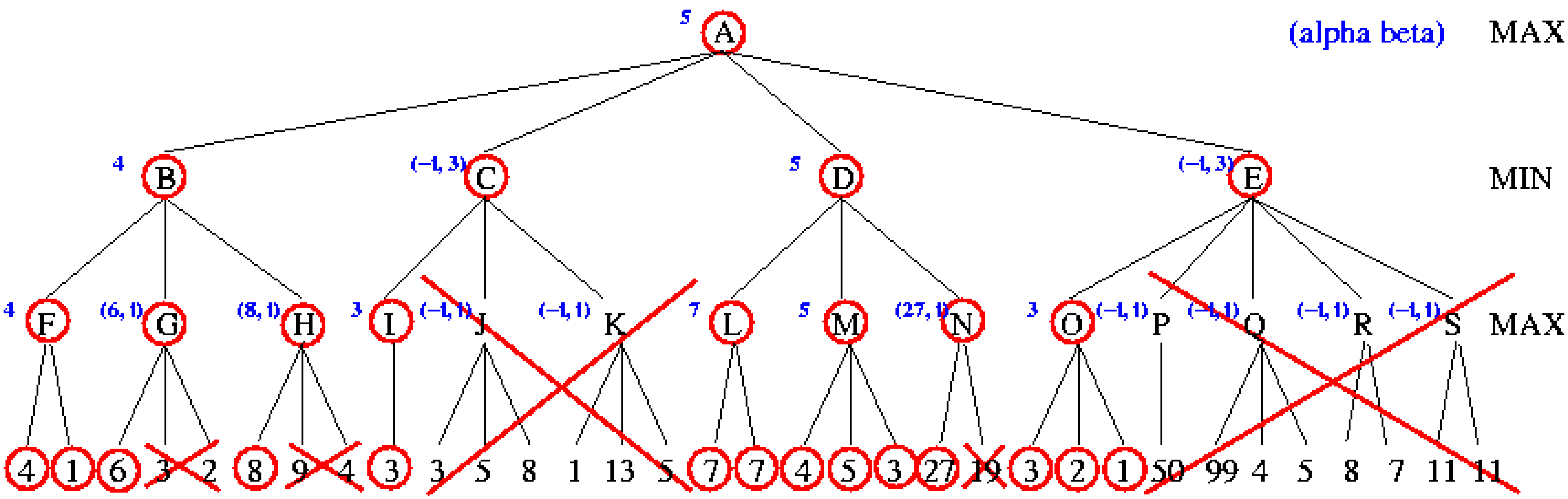
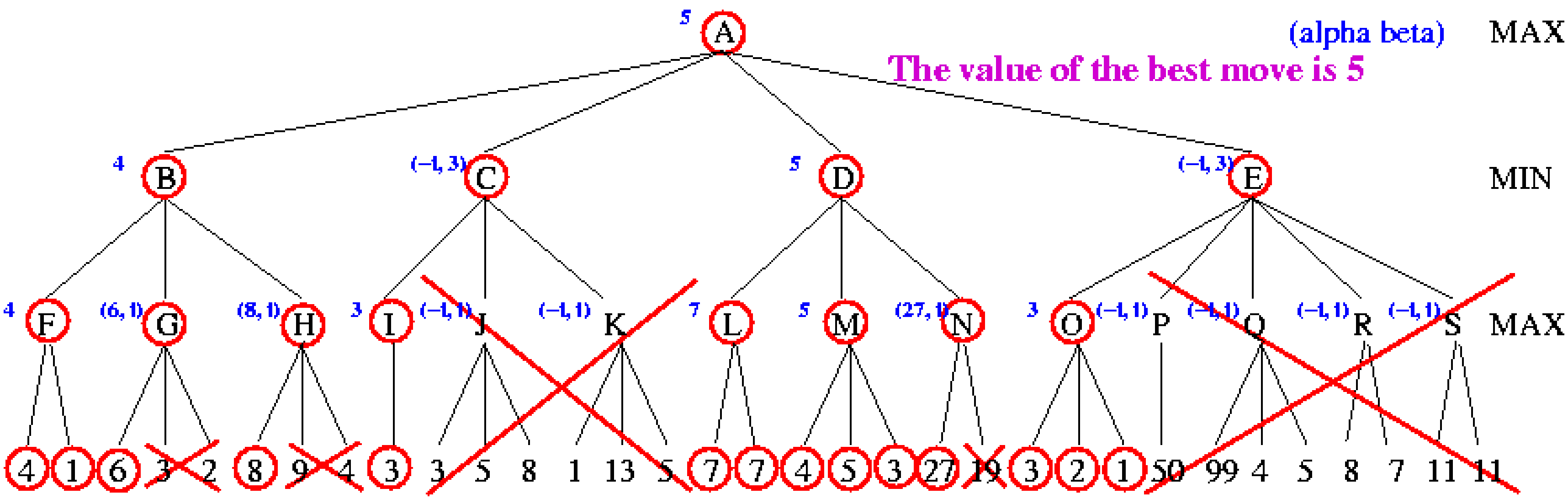
(−1, 1)
P
(−1, 1)
Q
(−1, 1)
R
(−1, 1)
S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Example



Example


(alpha beta)
The value of the best move is 5
MAX
MIN
MAX
5 A
4 B
(−I, 3) C
5 D
(−I, 3) E
4 F
(6, I) G
(8, I) H
3 I
(−I, I) J
(−I, I) K
7 L
5 M
(27, I) N
3 O
(−I, I) P
(−I, I) Q
(−I, I) R
(−I, I) S
4 1 6 3 2 8 9 4 3 3 5 8 1 13 5 7 7 4 5 3 27 19 3 2 1 50 99 4 5 8 7 11 11

Practical Implementation

How do we make these ideas practical in real game trees?

Standard approach:

- cutoff test: (where do we stop descending the tree)
  - depth limit
  - better: iterative deepening
  - cutoff only when no big changes are expected to occur next (quiescence search).

- evaluation function
  - When the search is cut off, we evaluate the current state
    by estimating its utility using **an evaluation function**.

Static (Heuristic) Evaluation Functions

- An Evaluation Function:
  - estimates how good the current board configuration is for a player.
  - Typically, one figures how good it is for the player, and how good it is for the opponent, and subtracts the opponents score from the players
  - Othello: Number of white pieces - Number of black pieces
  - Chess: Value of all white pieces - Value of all black pieces

- Typical values from -infinity (loss) to +infinity (win) or [-1, +1].
- If the board evaluation is X for a player, it's -X for the opponent.
- Many clever ideas about how to use the evaluation function.
  - e.g. null move heuristic: let opponent move twice.
- Example:
  - Evaluating chess boards,
  - Checkers
  - Tic-tac-toe

Summary

- Game playing can be effectively modeled as a search problem
- Game trees represent alternate computer/opponent moves
- Evaluation functions estimate the quality of a given board configuration for the Max player.
- Minimax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- For many well-known games, computer algorithms based on heuristic search match or out-perform human world experts.

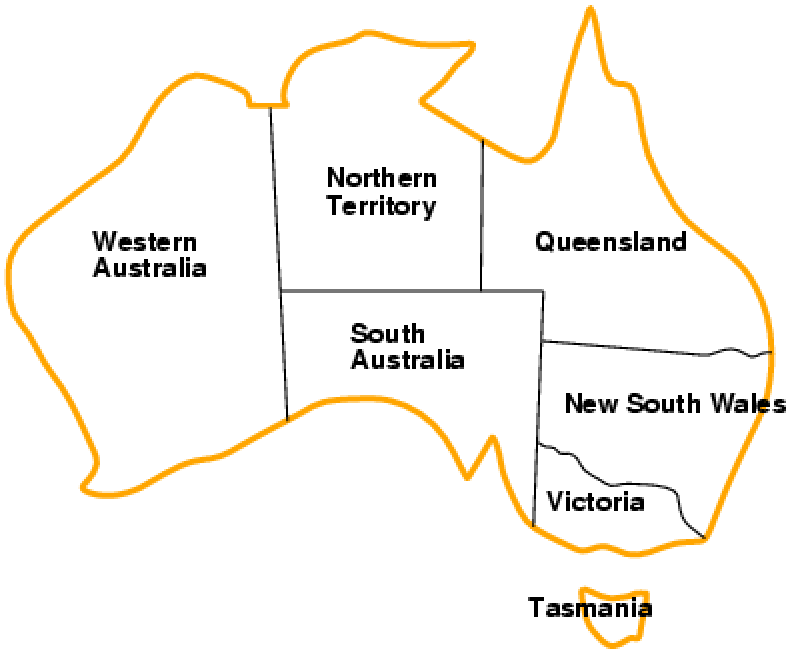
Constraint Satisfaction Problems

Constraint satisfaction problems (CSPs)

- CSP:
  - state is defined by variables X_i with values from domain D_i
  - goal test is a set of constraints specifying allowable combinations of values for subsets of variables

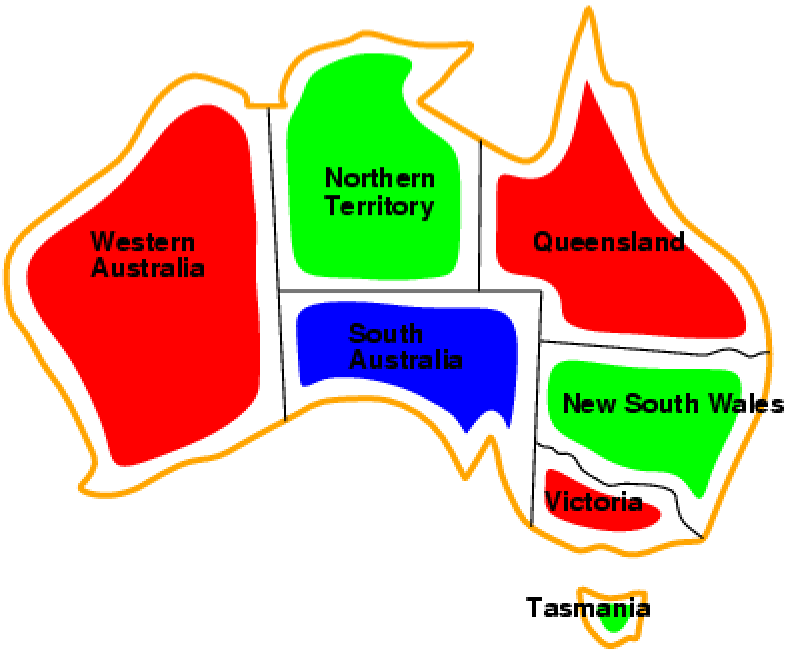
- Allows useful general-purpose algorithms with more power than standard search algorithms

Example: Map-Coloring



- Variables WA, NT, Q, NSW, V, SA, T
- Domains $D_i = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors
- e.g., WA ≠ NT

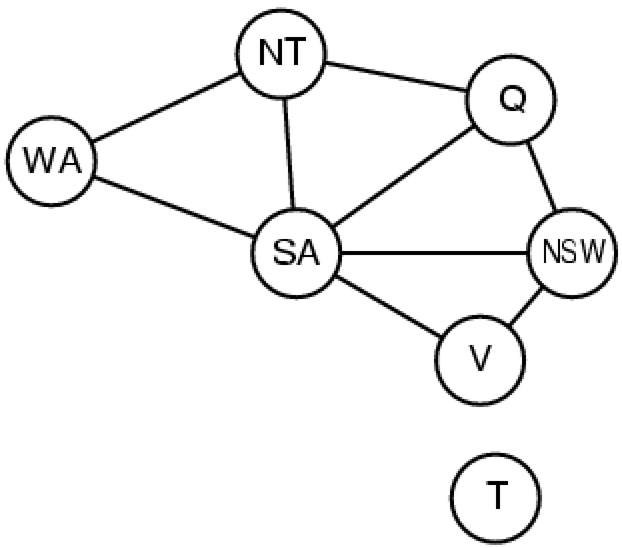
Example: Map-Coloring



- Solutions are complete and consistent assignments, e.g., WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green

Constraint graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints



Varieties of CSPs

- Discrete variables
  - finite domains:
    - n variables, domain size d → $O(d^n)$ complete assignments
    - e.g., 3-SAT (NP-complete)
  - infinite domains:
    - integers, strings, etc.
    - e.g., job scheduling, variables are start/end days for each job
    - need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$

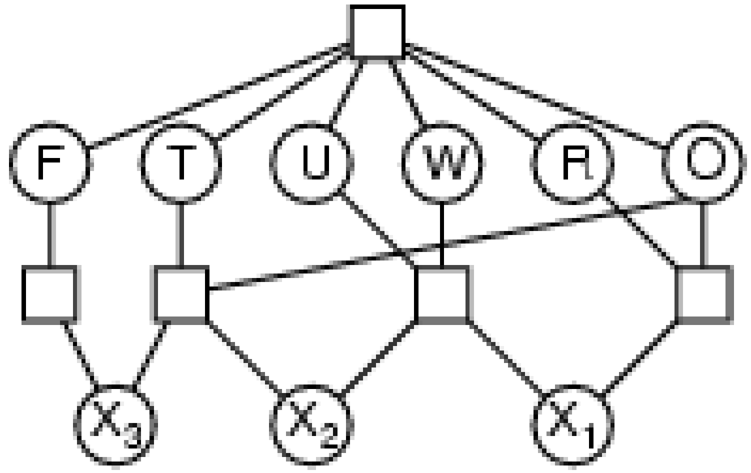
- Continuous variables
  - e.g., start/end times for Hubble Space Telescope observations
  - linear constraints solvable in polynomial time by linear programming

Varieties of constraints

- Unary constraints involve a single variable,
  - e.g., SA $\neq$ green
- Binary constraints involve pairs of variables,
  - e.g., SA $\neq$ WA
- Higher-order constraints involve 3 or more variables,
  - e.g., SA $\neq$ WA $\neq$ NT

Example: Cryptarithmetic

$$
\begin{array}{cccc}
 & T & W & O \\
+ & T & W & O \\
\hline
F & O & U & R
\end{array}
$$



- Variables: $F\ T\ U\ W\ R\ O$ $\quad X_1\ X_2\ X_3$
- Domains: $\{0,1,2,3,4,5,6,7,8,9\}$ $\quad \{0,1\}$
- Constraints: $Alldiff\,(F, T, U, W, R, O)$
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$

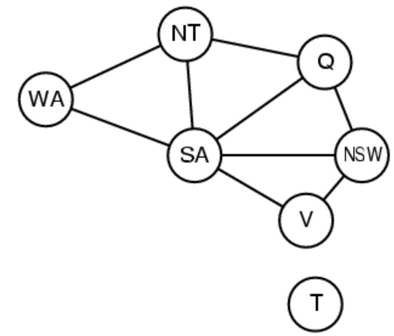
- SEND+MORE=MONEY
- BASE+BALL=GAMES
- LOGIC+LOGIC=PROLOG

Real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling

- Notice that many real-world problems involve real-valued variables

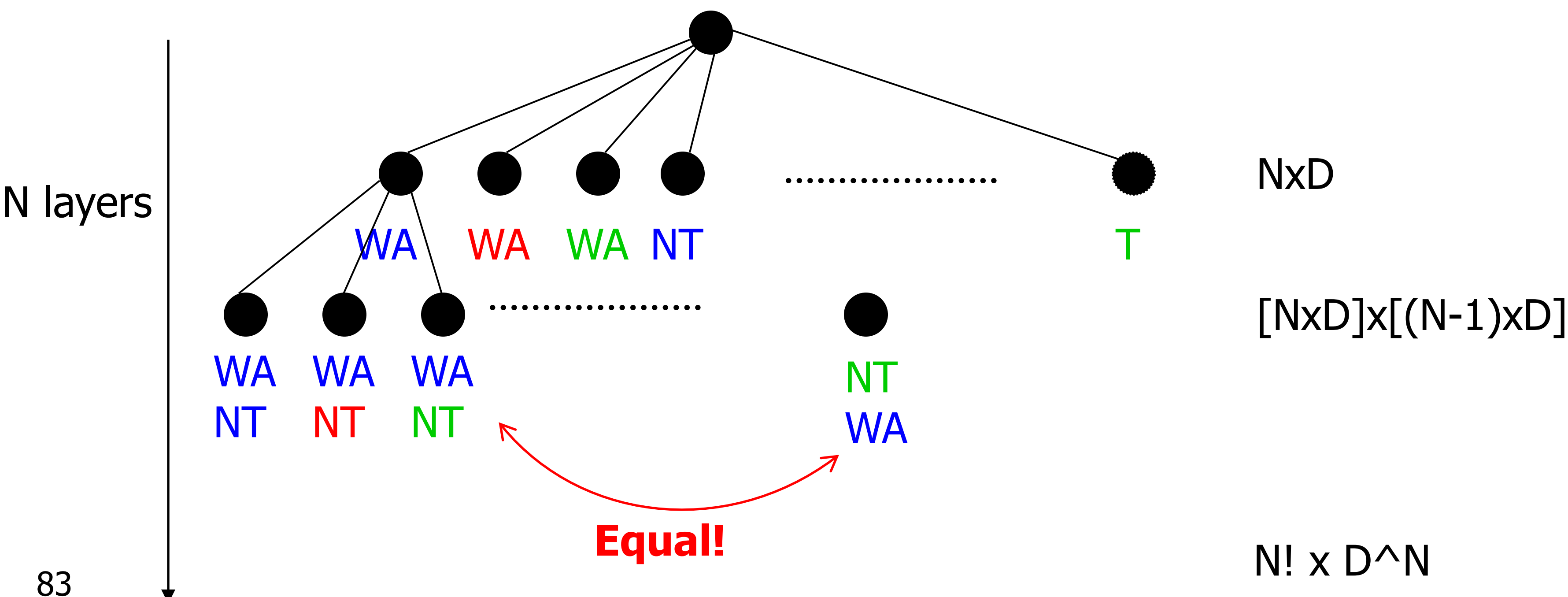
Standard search formulation



Let's try the standard search formulation.

We need:

- Initial state: none of the variables has a value (color)
- Successor state: one of the variables without a value will get some value.
- Goal: all variables have a value and none of the constraints is violated.

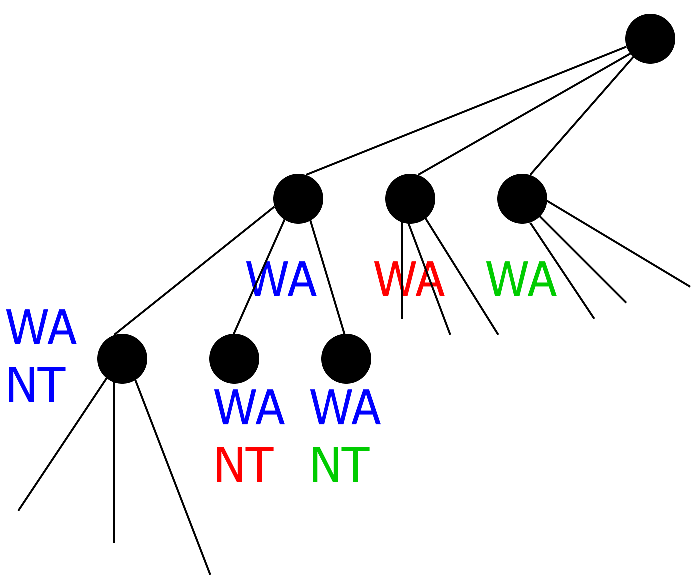


There are N! x D^N nodes in the tree but only D^N distinct states??

Backtracking (Depth-First) search

- Special property of CSPs: They are commutative:
  This means: the order in which we assign variables
  does not matter.
- Better search tree: First order variables, then assign them values one-by-one.

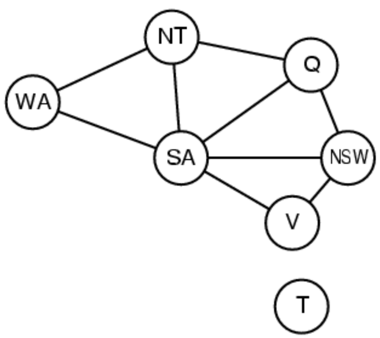
NT	=	WA
WA		NT



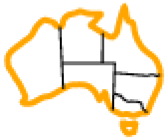
D

D^2

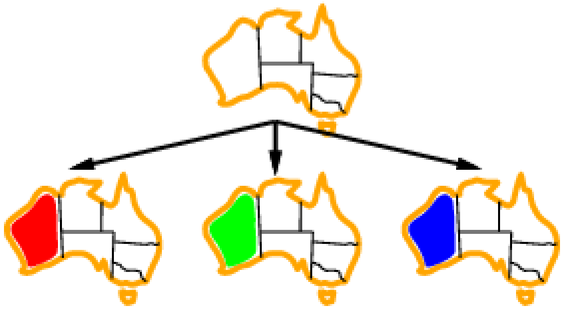
D^N



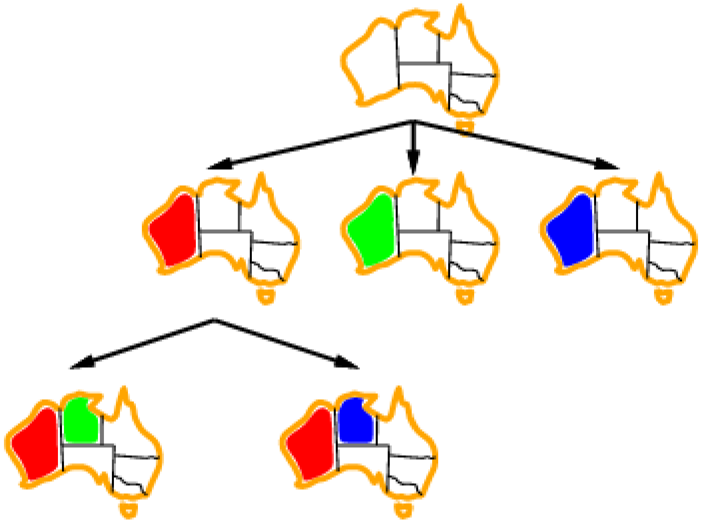
Backtracking example



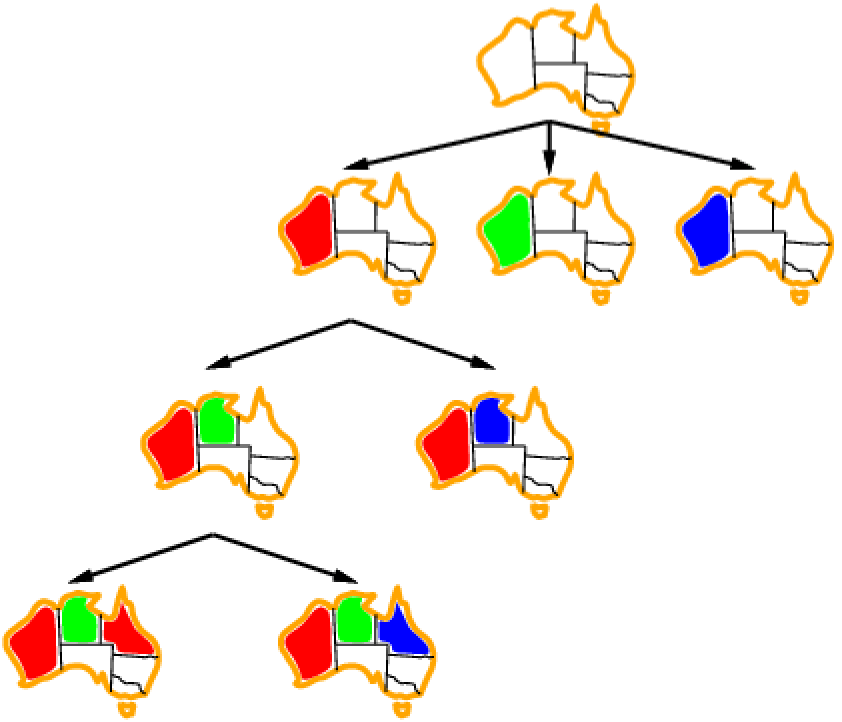
Backtracking example



Backtracking example



Backtracking example



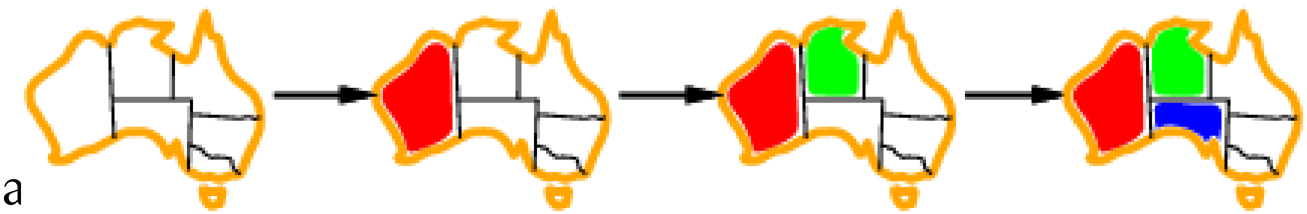
Improving backtracking efficiency

- General-purpose methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Most constrained variable

- Most constrained variable:

  choose the variable with the fewest legal values

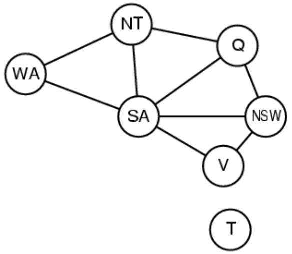


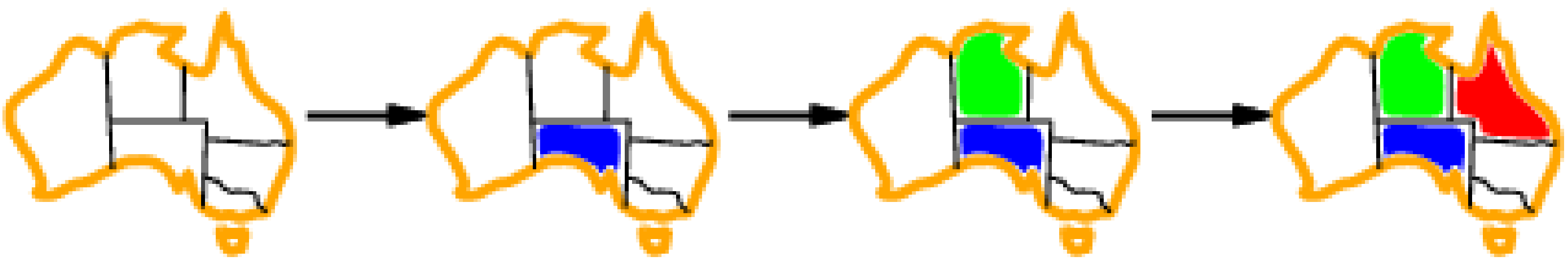
- a
- Picks a variable which will cause failure as soon as possible, allowing the tree to be pruned.

Most constraining variable

- Tie-breaker among most constrained variables

- Most constraining variable:
  - choose the variable with the most constraints on remaining variables (most edges in graph)

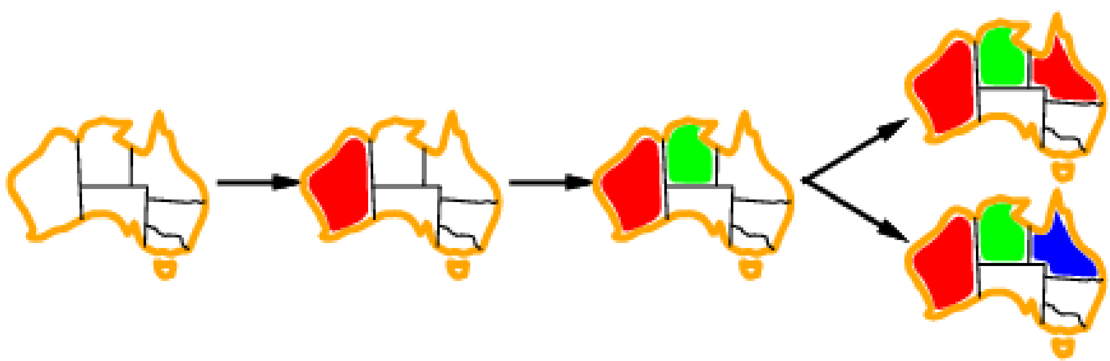




Least constraining value

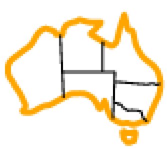
- *Given a variable*, choose the least constraining val
  - the one that rules out the fewest values in the ren variables

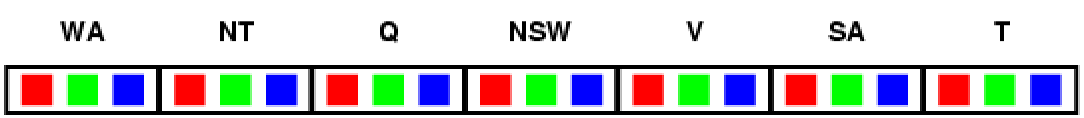
-
-

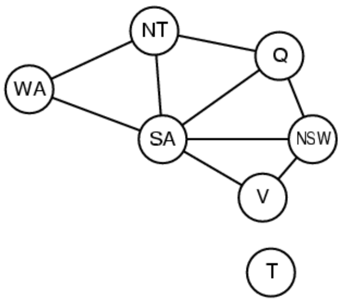


Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values







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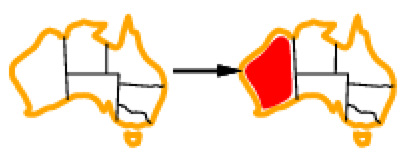

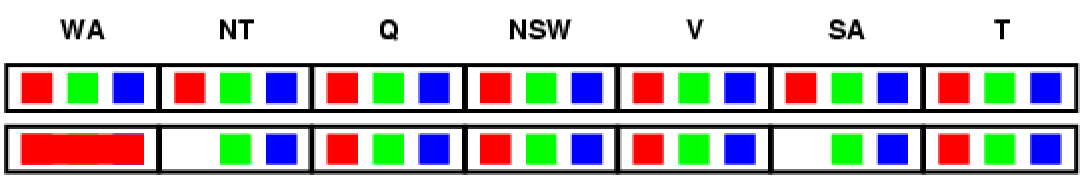

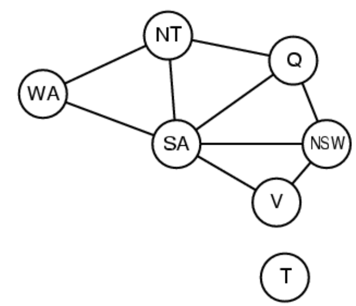

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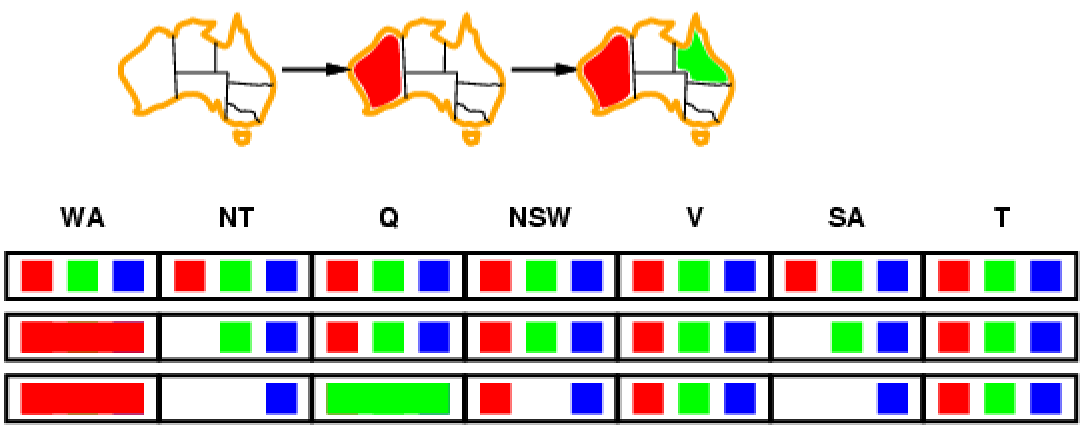

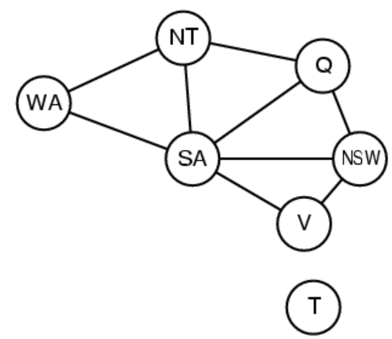

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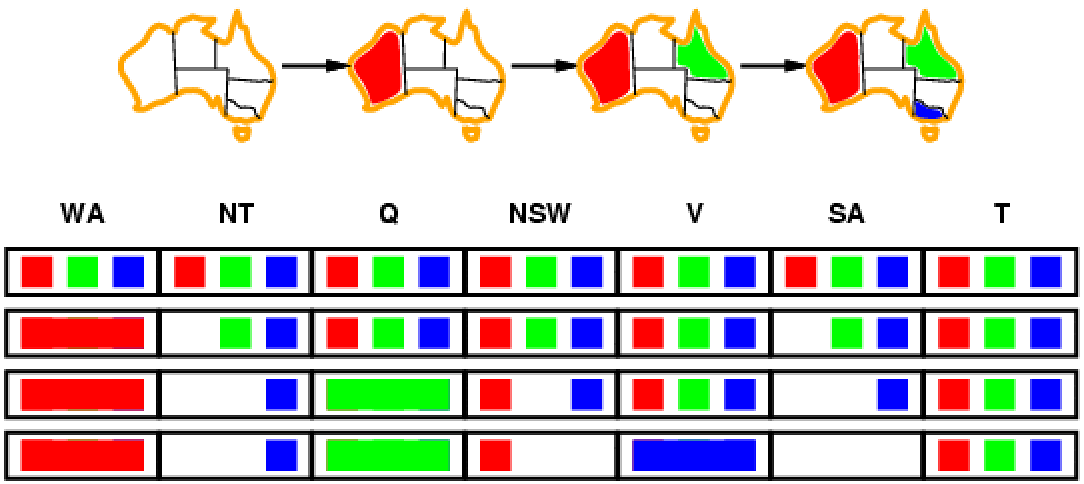
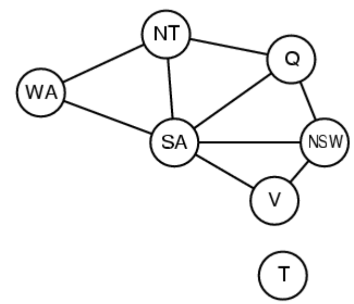
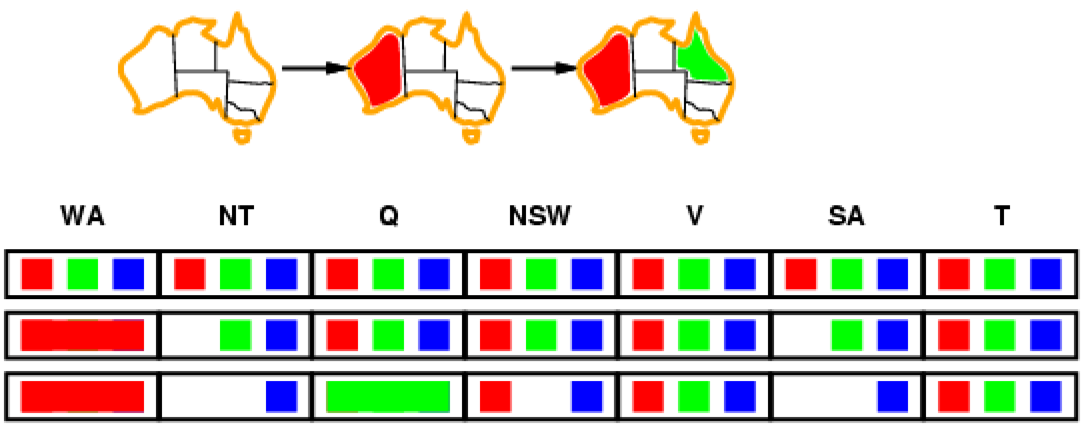
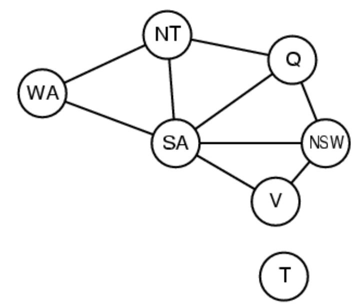




Constraint propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:



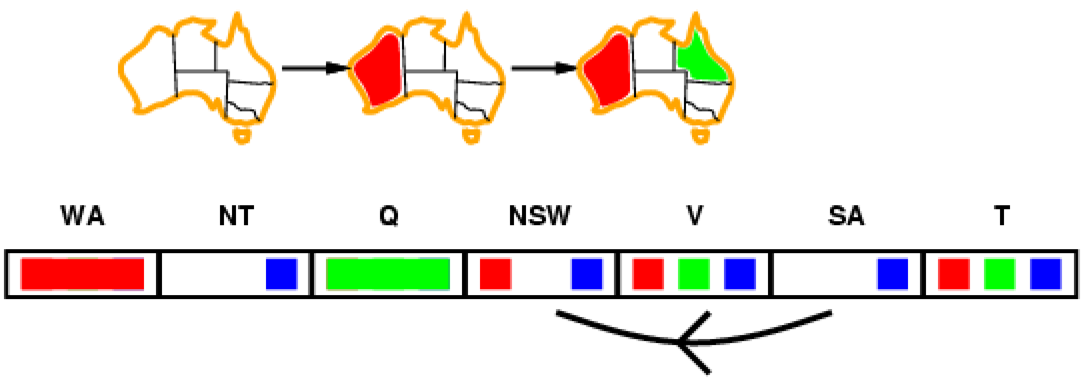


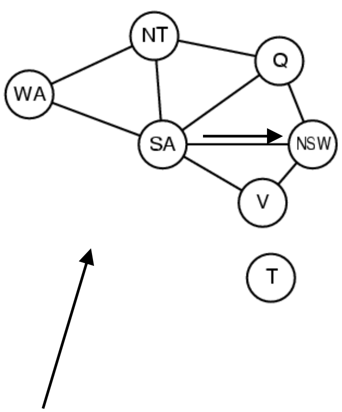
- NT and SA cannot both be blue!
- Constraint propagation repeatedly enforces constraints locally

Arc consistency

- Simplest form of propagation makes each arc consistent
- $X \rightarrow Y$ is consistent iff

  for every value x of X there is some allowed y





constraint propagation propagates arc consistency on the graph.

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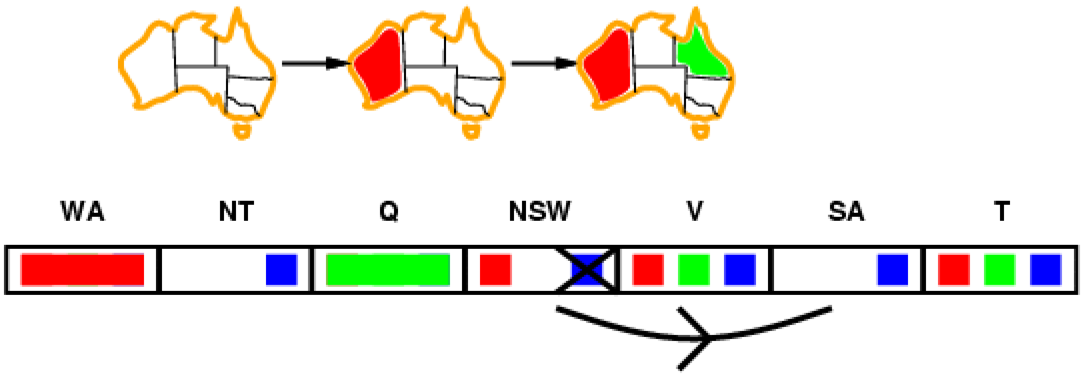

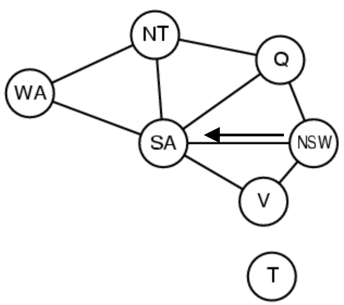

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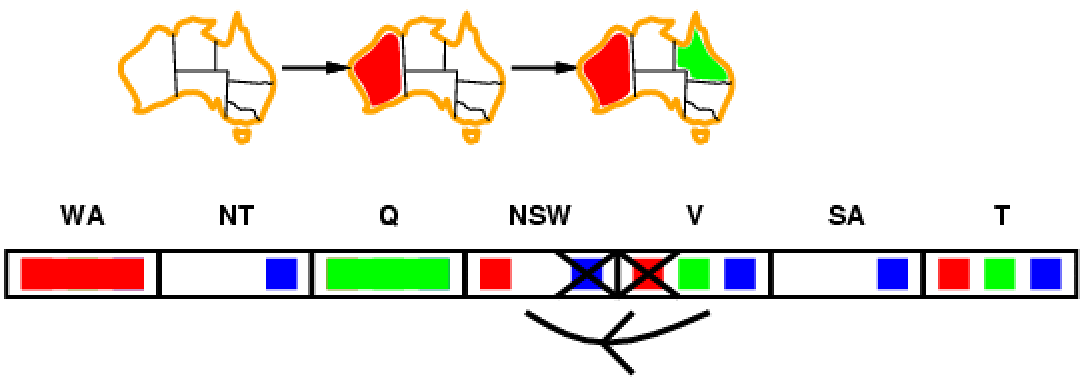

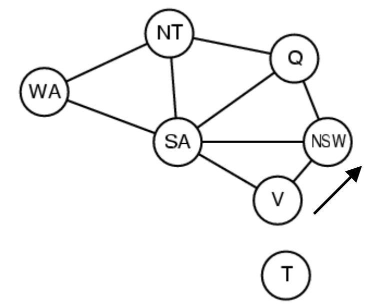

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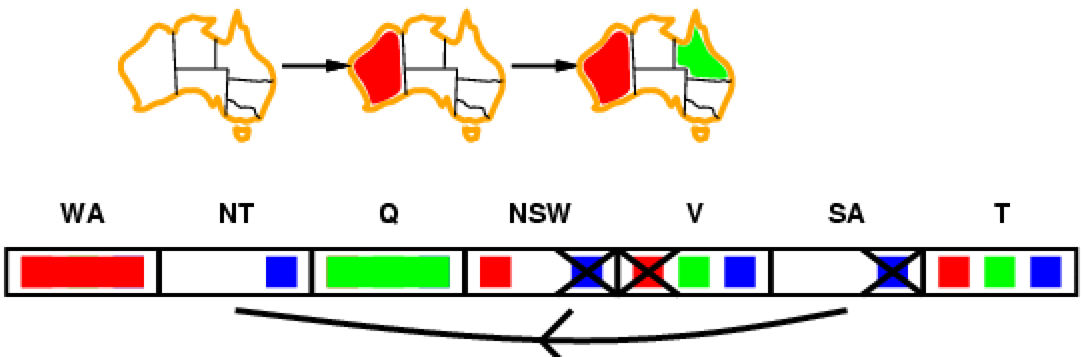

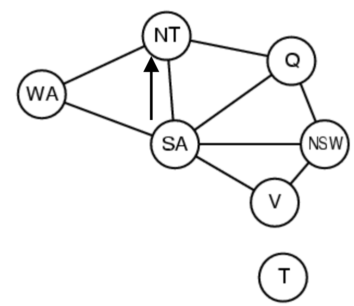

- If X loses a value, neighbors of X need to be rechecked
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment

Time complexity: $O(n^2d^3)$