Logical Inference

Some material adopted from notes by Andreas Geyer-Schulz,, Chuk Dyer, and Mary Getoor

Overview

- Inference in first-order logic
  - Inference rules and generalized modes ponens
  - Forward chaining
  - Backward chaining
  - Resolution
    - Clausal form
    - Unification
    - Resolution as search

Inference rules for FOL

- Inference rules for propositional logic apply to FOL as well
  - Modus Ponens, And-Introduction, And-Elimination, …
- New (sound) inference rules for use with quantifiers:
  - Universal elimination
  - Existential introduction
  - Existential elimination
  - Generalized Modus Ponens (GMP)

Automated inference for FOL

- Automated inference using FOL is harder than PL
  - Variables can potentially take on an *infinite* number of possible values from their domains
  - Hence there are potentially an *infinite* number of ways to apply the Universal Elimination rule of inference
- *Godel's Completeness Theorem* says that FOL entailment is only *semidecidable*
  - If a sentence is **true** given a set of axioms, there is a procedure that will determine this
  - If the sentence is **false**, then there is no guarantee that a procedure will ever determine this — i.e., it **may never halt**

Generalized Modus Ponens

- Modus Ponens
  - P, P=>Q |= Q
- Generalized Modus Ponens (GMP) extends this to rules in FOL
- Combines And-Introduction, Universal-Elimination, and Modus Ponens, e.g.
  - *from P(c) and Q(c) and* $\forall x\, P(x) \wedge Q(x) \rightarrow R(x)$ *derive R(c)*
- Need to deal with
  - more than one condition on left side of rule
  - variables

Generalized Modus Ponens

- General case: **Given**
  - **atomic sentences** $P_1, P_2, ..., P_N$
  - **implication sentence** $(Q_1 \wedge Q_2 \wedge ... \wedge Q_N) \rightarrow R$
    - $Q_1, ..., Q_N$ and R are atomic sentences
  - **substitution** $subst(\theta, P_i) = subst(\theta, Q_i)$ for i=1,...,N
  - **Derive new sentence: subst(θ, R)**
- Substitutions
  - subst(θ, α) denotes the result of applying a set of substitutions defined by θ to the sentence α
  - A substitution list $\theta = \{v_1/t_1, v_2/t_2, ..., v_n/t_n\}$ means to replace all occurrences of variable symbol v_i by term t_i
  - Substitutions made in left-to-right order in the list
  - subst({x/Cheese, y/Mickey}, eats(y,x)) = eats(Mickey, Cheese)

Our rules are Horn clauses

- A Horn clause is a sentence of the form:

  $P_1(x) \wedge P_2(x) \wedge \ldots \wedge P_n(x) \rightarrow Q(x)$

where

  - ≥ 0 P_is and 0 or 1 Q
  - the P_is and Q are positive (i.e., non-negated) literals

- Equivalently: $P_1(x) \vee P_2(x) \ldots \vee P_n(x)$ where the P_i are all atomic and *at most one* is positive
- Prolog is based on Horn clauses
- Horn clauses represent a *subset* of the set of sentences representable in FOL

Horn clauses

- Special cases
  - *Typical rule:* $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow Q$
  - *Constraint:* $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow$ false
  - *A fact:* true $\rightarrow Q$
- These are not Horn clauses:
  - $p(a) \vee q(a)$
  - $(P \wedge Q) \rightarrow (R \vee S)$
- Note: can't assert or conclude disjunctions, no negation
- No wonder reasoning over Horn clauses is easier

Horn clauses

- Where are the quantifiers?
  - Variables appearing in conclusion are universally quantified
  - Variables appearing only in premises are existentially quantified
- Example: grandparent relation
  - $parent(P1, X) \wedge parent(X, P2) \rightarrow grandParent(P1, P2)$
  - $\forall P1,P2\ \exists PX\ parent(P1,X) \wedge parent(X, P2) \rightarrow grandParent(P1, P2)$
  - Prolog: grandParent(P1,P2) :- parent(P1,X), parent(X,P2)

Forward & Backward Reasoning

- We usually talk about two reasoning strategies: Forward and backward ‘chaining’
- Both are equally powerful
- You can also have a mixed strategy

Forward chaining

- Proofs start with the given axioms/premises in KB, deriving new sentences using GMP until the goal/query sentence is derived
- This defines a **forward-chaining** inference procedure because it moves “forward” from the KB to the goal [eventually]
- Inference using GMP is **sound** and **complete** for KBs containing **only Horn clauses**

Forward chaining algorithm

procedure FORWARD-CHAIN(KB, p)

  if there is a sentence in KB that is a renaming of p **then return**
  Add p to KB
  for each $(p_1 \wedge \ldots \wedge p_n \Rightarrow q)$ **in** KB such that for some i, UNIFY$(p_i, p) = \theta$ succeeds **do**
    FIND-AND-INFER(KB, $[p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n], q, \theta$)
  end

---

procedure FIND-AND-INFER(KB, $premises$, $conclusion$, θ)

  if $premises = [\,]$ **then**
    FORWARD-CHAIN(KB, SUBST(θ, $conclusion$))
  else for each p' **in** KB such that UNIFY(p', SUBST(θ, FIRST($premises$))) $= \theta_2$ **do**
    FIND-AND-INFER(KB, REST($premises$), $conclusion$, COMPOSE(θ, θ_2))
  end

Forward chaining example

- KB:
  - $allergies(X) \rightarrow sneeze(X)$
  - $cat(Y) \wedge allergicToCats(X) \rightarrow allergies(X)$
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Backward chaining

- **Backward-chaining** deduction using GMP is also **complete** for KBs containing **only Horn clauses**
- Proofs start with the goal query, find rules with that conclusion, and then prove each of the antecedents in the implication
- Keep going until you reach premises
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  - Has already been proved true
  - Has already failed

Backward chaining algorithm

function BACK-CHAIN(KB, q) **returns** a set of substitutions

BACK-CHAIN-LIST(KB, $[q]$, $\{\}$)

---

function BACK-CHAIN-LIST(KB, *qlist*, θ) **returns** a set of substitutions
inputs: KB, a knowledge base
qlist, a list of conjuncts forming a query (θ already applied)
θ, the current substitution
static: *answers*, a set of substitutions, initially empty

if *qlist* is empty **then return** $\{\theta\}$
$q \leftarrow$ FIRST(*qlist*)
for each q'_i **in** KB such that $\theta_i \leftarrow$ UNIFY(q, q'_i) succeeds **do**
Add COMPOSE(θ, θ_i) to *answers*
end
for each sentence ($p_1 \wedge \ldots \wedge p_n \Rightarrow q'_i$) **in** KB such that $\theta_i \leftarrow$ UNIFY(q, q'_i) succeeds **do**
answers $\leftarrow$ BACK-CHAIN-LIST(KB, SUBST(θ_i, $[p_1 \ldots p_n]$), COMPOSE(θ, θ_i)) $\cup$ *answers*
end
return the union of BACK-CHAIN-LIST(KB, REST(*qlist*), θ) for each $\theta \in$ *answers*

Backward chaining example

- KB:
  - allergies(X) → sneeze(X)
  - cat(Y) ^ allergicToCats(X) → allergies(X)
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Forward vs. backward chaining

- FC is *data-driven*
  - Automatic, unconscious processing
  - E.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
  - Efficient when you want to compute all conclusions
- BC is goal-driven, better for problem-solving
  - Where are my keys? How do I get to my next class?
  - Complexity of BC can be much less than linear in the size of the KB
  - Efficient when you want one or a few decisions

Mixed strategy

- Many practical reasoning systems do both forward and backward chaining
- The way you encode a rule determines how it is used, as in

  % this is a forward chaining rule

  spouse(X,Y) => spouse(Y,X).

  % this is a backward chaining rule

  wife(X,Y) <= spouse(X,Y), female(X).
- Given a model of the rules you have and the kind of reason you need to do, it’s possible to decide which to encode as FC and which as BC rules.

Completeness of GMP

- GMP (using forward or backward chaining) is complete for KBs that contain only Horn clauses
- ***not* complete** for simple KBs with **non-Horn clauses**
- The following entail that S(A) is true:
  1. $(\forall x)\ P(x) \rightarrow Q(x)$
  2. $(\forall x)\ \neg P(x) \rightarrow R(x)$
  3. $(\forall x)\ Q(x) \rightarrow S(x)$
  4. $(\forall x)\ R(x) \rightarrow S(x)$
- If we want to conclude S(A), with GMP we cannot, since the second one is not a Horn clause
- It is equivalent to $P(x) \vee R(x)$

Automating FOL Inference with Resolution

Resolution

- Resolution is a **sound** and **complete** inference procedure for unrestricted FOL
- Reminder: Resolution rule for propositional logic:
  - $P_1 \lor P_2 \lor ... \lor P_n$

    $\land \neg P_1 \lor Q_2 \lor ... \lor Q_m$
  - Resolvent: $P_2 \lor ... \lor P_n \lor Q_2 \lor ... \lor Q_m$
- We'll need to extend this to handle quantifiers and variables

Resolution covers many cases

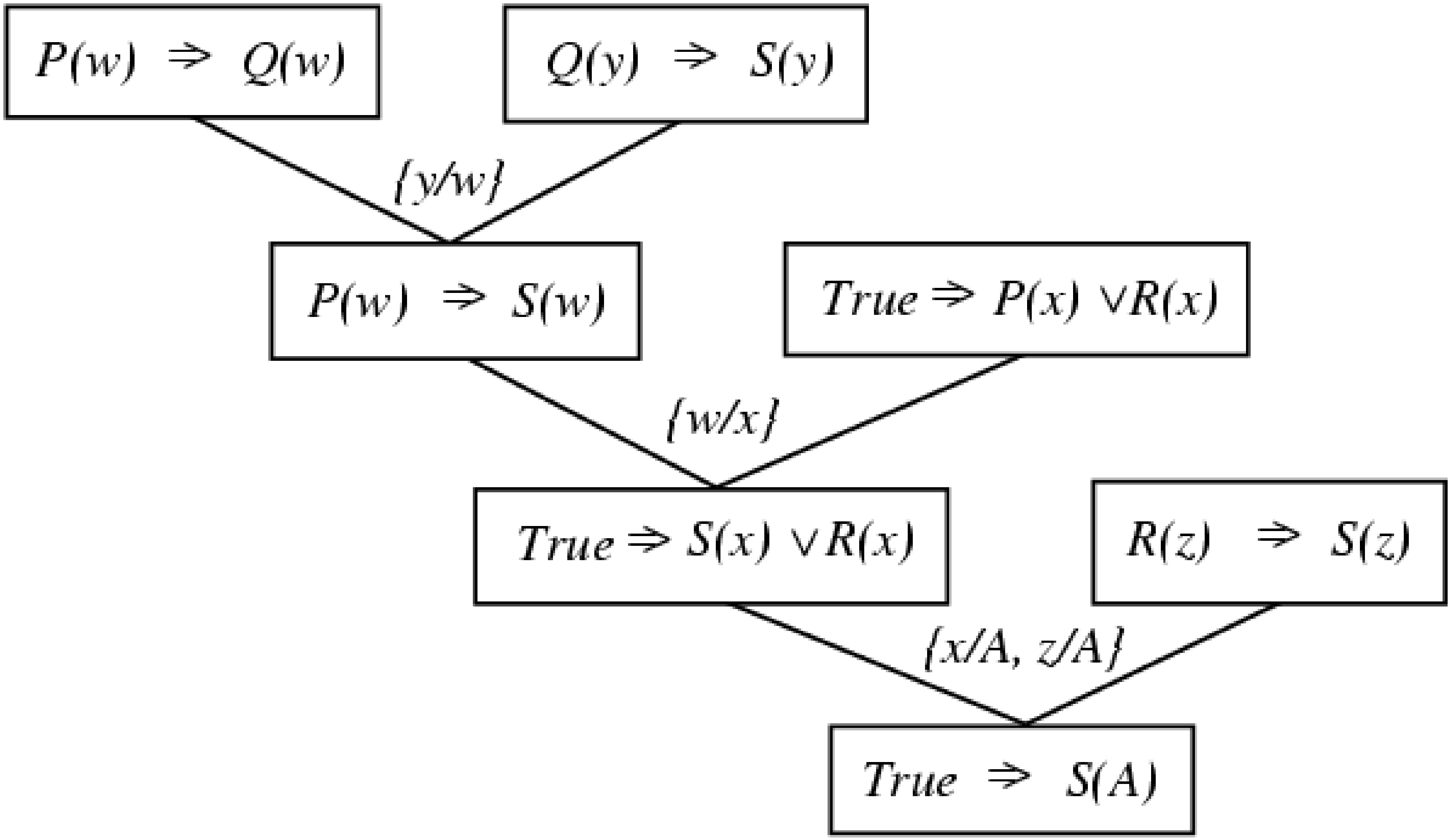
- Modes Ponens
  - from P and $P \rightarrow Q$ derive Q
  - from P and $\neg P \vee Q$ derive Q
- Chaining
  - from $P \rightarrow Q$ and $Q \rightarrow R$ derive $P \rightarrow R$
  - from $(\neg P \vee Q)$ and $(\neg Q \vee R)$ derive $\neg P \vee R$
- Contradiction detection
  - from P and $\neg P$ derive false
  - from P and $\neg P$ derive the empty clause (=false)

Resolution in first-order logic

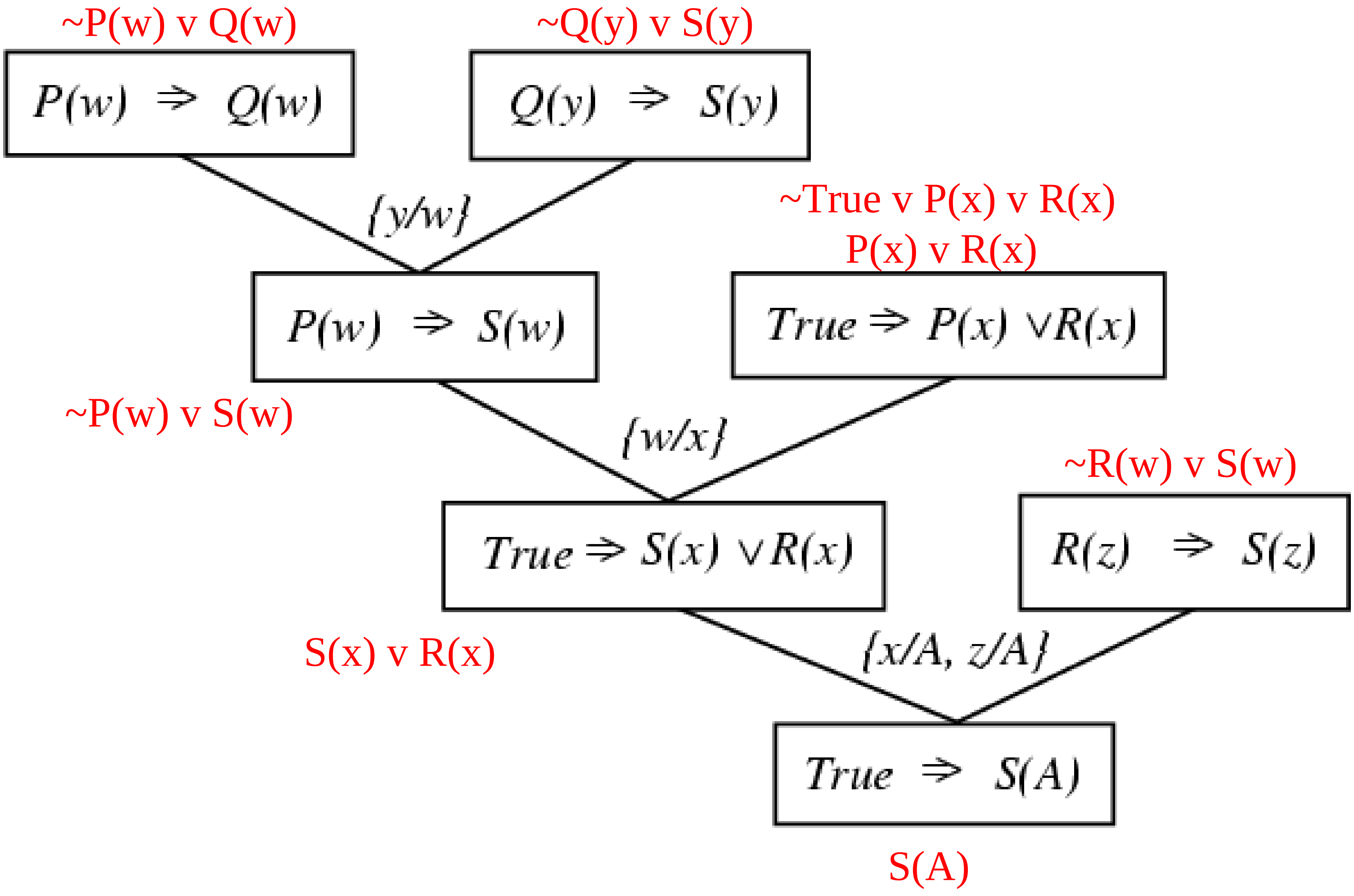
- Given sentences in *conjunctive normal form:*
  - $P_1 \vee \ldots \vee P_n$ and $Q_1 \vee \ldots \vee Q_m$
  - P_i and Q_i are literals, i.e., positive or negated predicate symbol with its terms
- if P_j and $\neg Q_k$ **unify** with substitution list θ, then derive the resolvent sentence:

  $\text{subst}(\theta, P_1 \vee \ldots \vee P_{j-1} \vee P_{j+1} \ldots P_n \vee Q_1 \vee \ldots Q_{k-1} \vee Q_{k+1} \vee \ldots \vee Q_m)$
- Example
  - from clause $\mathbf{P(x, f(a)) \vee P(x, f(y)) \vee Q(y)}$
  - and clause $\mathbf{\neg P(z, f(a)) \vee \neg Q(z)}$
  - derive resolvent $\mathbf{P(z, f(y)) \vee Q(y) \vee \neg Q(z)}$
  - Using $\boldsymbol{\theta = \{x/z\}}$

A resolution proof tree



A resolution proof tree



Resolution refutation

- Given a consistent set of axioms KB and goal sentence Q, show that KB |= Q
- **Proof by contradiction:** Add ¬Q to KB and try to prove false, i.e.:

  (KB |- Q) ↔ (KB ^ ¬Q |- False)
- Resolution is **refutation complete:** it can establish that a given sentence Q is entailed by KB, but can't (in general) generate all logical consequences of a set of sentences
- Also, it cannot be used to prove that Q is **not entailed** by KB
- Resolution **won't always give an answer** since entailment is only semi-decidable
  - And you can't just run two proofs in parallel, one trying to prove Q and the other trying to prove ¬Q, since KB might not entail either one

Resolution example

- KB:
  - allergies(X) → sneeze(X)
  - cat(Y) ∧ allergicToCats(X) → allergies(X)
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Refutation resolution proof tree

¬allergies(w) v sneeze(w)   ¬cat(y) v ¬allergicToCats(z) ˅ allergies(z)

w/z

¬cat(y) v sneeze(z) ˅ ¬allergicToCats(z)   cat(felix)

y/felix

sneeze(z) v ¬allergicToCats(z)   allergicToCats(mary)

z/mary

sneeze(mary)   ¬sneeze(mary)

false

negated query

Notation

old/new

questions to be answered

- How to convert FOL sentences to conjunctive normal form (a.k.a. CNF, clause form): **normalization and skolemization**
- How to unify two argument lists, i.e., how to find their most general unifier (**mgu**) q: **unification**
- How to determine which two clauses in KB should be resolved next (among all resolvable pairs of clauses) : **resolution (search) strategy**

Converting to CNF

Converting sentences to CNF

1. Eliminate all $\leftrightarrow$ connectives

   $(P \leftrightarrow Q) \Rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P))$

2. Eliminate all $\rightarrow$ connectives

   $(P \rightarrow Q) \Rightarrow (\neg P \vee Q)$

3. Reduce the scope of each negation symbol to a single predicate

   $\neg\neg P \Rightarrow P$

   $\neg(P \vee Q) \Rightarrow \neg P \wedge \neg Q$

   $\neg(P \wedge Q) \Rightarrow \neg P \vee \neg Q$

   $\neg(\forall x)P \Rightarrow (\exists x)\neg P$

   $\neg(\exists x)P \Rightarrow (\forall x)\neg P$

4. Standardize variables: rename all variables so that each quantifier has its own unique variable name

Converting sentences to clausal form

Skolem constants and functions

5. Eliminate existential quantification by introducing Skolem constants/functions

$(\exists x)P(x) \Rightarrow P(C)$

C is a Skolem constant (a brand-new constant symbol that is not used in any other sentence)

$(\forall x)(\exists y)P(x,y) \Rightarrow (\forall x)P(x, f(x))$

since $\exists$ is within scope of a universally quantified variable, use a **Skolem function f** to construct a new value that **depends on** the universally quantified variable

f must be a brand-new function name not occurring in any other sentence in the KB

E.g., $(\forall x)(\exists y)loves(x,y) \Rightarrow (\forall x)loves(x,f(x))$

In this case, f(x) specifies the person that x loves

a better name might be **oneWhoIsLovedBy**(x)

Converting sentences to clausal form

6. Remove universal quantifiers by (1) moving them all to the left end; (2) making the scope of each the entire sentence; and (3) dropping the "prefix" part

   Ex: $(\forall x)P(x) \Rightarrow P(x)$

7. Put into conjunctive normal form (conjunction of disjunctions) using distributive and associative laws

   $(P \wedge Q) \vee R \Rightarrow (P \vee R) \wedge (Q \vee R)$

   $(P \vee Q) \vee R \Rightarrow (P \vee Q \vee R)$

8. Split conjuncts into separate clauses

9. Standardize variables so each clause contains only variable names that do not occur in any other clause

An example

$(\forall x)(P(x) \rightarrow ((\forall y)(P(y) \rightarrow P(f(x,y))) \wedge \neg(\forall y)(Q(x,y) \rightarrow P(y))))$

2. Eliminate $\rightarrow$

$(\forall x)(\neg P(x) \vee ((\forall y)(\neg P(y) \vee P(f(x,y))) \wedge \neg(\forall y)(\neg Q(x,y) \vee P(y))))$

3. Reduce scope of negation

$(\forall x)(\neg P(x) \vee ((\forall y)(\neg P(y) \vee P(f(x,y))) \wedge (\exists y)(Q(x,y) \wedge \neg P(y))))$

4. Standardize variables

$(\forall x)(\neg P(x) \vee ((\forall y)(\neg P(y) \vee P(f(x,y))) \wedge (\exists z)(Q(x,z) \wedge \neg P(z))))$

5. Eliminate existential quantification

$(\forall x)(\neg P(x) \vee ((\forall y)(\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \wedge \neg P(g(x)))))$

6. Drop universal quantification symbols

$(\neg P(x) \vee ((\neg P(y) \vee P(f(x,y))) \wedge (Q(x,g(x)) \wedge \neg P(g(x)))))$

Example

7. Convert to conjunction of disjunctions

   $(\neg P(x) \vee \neg P(y) \vee P(f(x,y))) \wedge (\neg P(x) \vee Q(x,g(x))) \wedge$

   $(\neg P(x) \vee \neg P(g(x)))$

8. Create separate clauses

   $\neg P(x) \vee \neg P(y) \vee P(f(x,y))$

   $\neg P(x) \vee Q(x,g(x))$

   $\neg P(x) \vee \neg P(g(x))$

9. Standardize variables

   $\neg P(x) \vee \neg P(y) \vee P(f(x,y))$

   $\neg P(z) \vee Q(z,g(z))$

   $\neg P(w) \vee \neg P(g(w))$

Unification

Unification

- Unification is a **“pattern-matching”** procedure
  - Takes two atomic sentences, called literals, as input
  - Returns “Failure” if they do not match and a substitution list, θ, if they do
- That is, $unify(p,q) = \theta$ means $subst(\theta, p) = subst(\theta, q)$ for two atomic sentences, p and q
- **θ** is called the **most general unifier** (mgu)
- All variables in the given two literals are implicitly universally quantified
- To make literals match, replace (universally quantified) variables by terms

Unification algorithm

```
procedure unify(p, q, θ)
    Scan p and q left-to-right and find the first corresponding
      terms where p and q “disagree” (i.e., p and q not equal)
    If there is no disagreement, return θ  (success!)
    Let r and s be the terms in p and q, respectively,
      where disagreement first occurs
    If variable(r) then {
      Let θ = union(θ, {r/s})
      Return unify(subst(θ, p), subst(θ, q), θ)
    } else if variable(s) then {
      Let θ = union(θ, {s/r})
      Return unify(subst(θ, p), subst(θ, q), θ)
    } else return “Failure”
  end
```

Unification: Remarks

- *Unify* is a linear-time algorithm that returns the most general unifier (mgu), i.e., the shortest-length substitution list that makes the two literals match
- In general, there isn't a **unique** minimum-length substitution list, but unify returns one of minimum length
- Common constraint: A variable can never be replaced by a term containing that variable

  Example: x/f(x) is illegal.

  – This "occurs check" should be done in the above pseudo-code before making the recursive calls

Unification examples

- Example:
  - parents(x, father(x), mother(Bill))
  - parents(Bill, father(Bill), y)
  - {x/Bill,y/mother(Bill)} yields parents(Bill,father(Bill), mother(Bill))
- Example:
  - parents(x, father(x), mother(Bill))
  - parents(Bill, father(y), z)
  - {x/Bill,y/Bill,z/mother(Bill)} yields parents(Bill,father(Bill), mother(Bill))
- Example:
  - parents(x, father(x), mother(Jane))
  - parents(Bill, father(y), mother(y))
  - Failure

Resolution example

Practice example
Did Curiosity kill the cat

- Jack owns a dog
- Every dog owner is an animal lover
- No animal lover kills an animal
- Either Jack or Curiosity killed the cat, who is named Tuna.
- Did Curiosity kill the cat?

Practice example

Did Curiosity kill the cat

- Jack owns a dog. Every dog owner is an animal lover. No animal lover kills an animal. Either Jack or Curiosity killed the cat, who is named Tuna. Did Curiosity kill the cat?
- These can be represented as follows:

A. $(\exists x)\ Dog(x) \wedge Owns(Jack,x)$

B. $(\forall x)\ ((\exists y)\ Dog(y) \wedge Owns(x, y)) \rightarrow AnimalLover(x)$

C. $(\forall x)\ AnimalLover(x) \rightarrow ((\forall y)\ Animal(y) \rightarrow \neg Kills(x,y))$

D. $Kills(Jack,Tuna) \vee Kills(Curiosity,Tuna)$

E. $Cat(Tuna)$

F. $(\forall x)\ Cat(x) \rightarrow Animal(x)$

G. $Kills(Curiosity, Tuna)$

GOAL

$\exists x$ Dog(x) $\wedge$ Owns(Jack,x)
$\forall x$ ($\exists y$) Dog(y) $\wedge$ Owns(x, y) $\rightarrow$ AnimalLover(x)
$\forall x$ AnimalLover(x) $\rightarrow$ ($\forall y$ Animal(y) $\rightarrow$ $\neg$Kills(x,y))
Kills(Jack,Tuna) $\vee$ Kills(Curiosity,Tuna)
Cat(Tuna)
$\forall x$ Cat(x) $\rightarrow$ Animal(x)
Kills(Curiosity, Tuna)

- **Convert to clause form**

  A1. (Dog(D))

  A2. (Owns(Jack,D))

  B. ($\neg$Dog(y), $\neg$Owns(x, y), AnimalLover(x))

  C. ($\neg$AnimalLover(a), $\neg$Animal(b), $\neg$Kills(a,b))

  D. (Kills(Jack,Tuna), Kills(Curiosity,Tuna))

  E. Cat(Tuna)

  F. ($\neg$Cat(z), Animal(z))

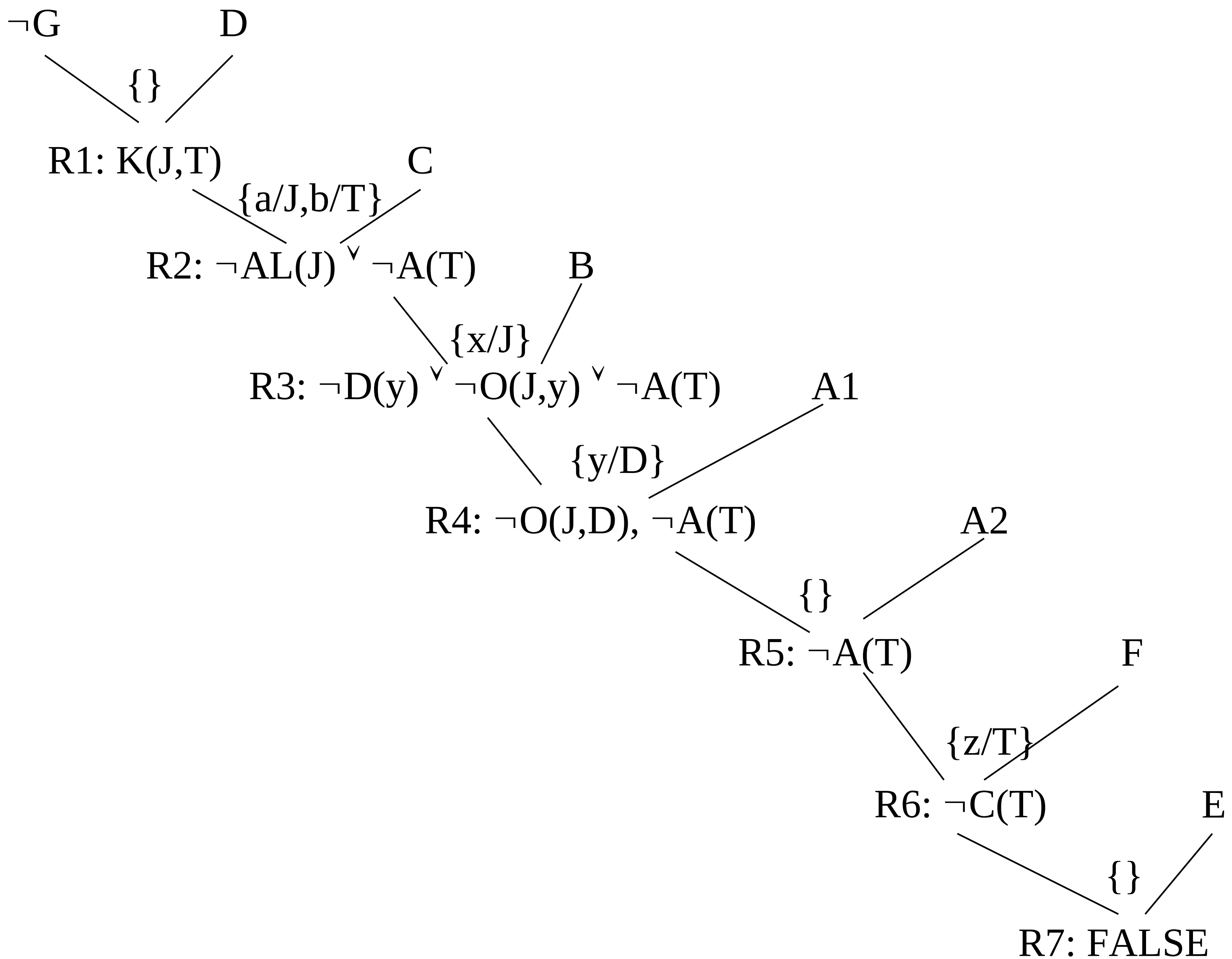
- **Add the negation of query:**

  $\neg$G: $\neg$Kills(Curiosity, Tuna)

The resolution refutation proof

R1: ¬G, D, {}	(Kills(Jack, Tuna))
R2: R1, C, {a/Jack, b/Tuna}	(~AnimalLover(Jack), ~Animal(Tuna))
R3: R2, B, {x/Jack}	(~Dog(y), ~Owns(Jack, y), ~Animal(Tuna))
R4: R3, A1, {y/D}	(~Owns(Jack, D), ~Animal(Tuna))
R5: R4, A2, {}	(~Animal(Tuna))
R6: R5, F, {z/Tuna}	(~Cat(Tuna))
R7: R6, E, {}	FALSE

The proof tree



Resolution search strategies

Resolution TP as search

- Resolution can be thought of as the **bottom-up construction of a search tree**, where the leaves are the clauses produced by KB and the negation of the goal
- When a pair of clauses generates a new resolvent clause, add a new node to the tree with arcs directed from the resolvent to the two parent clauses
- **Resolution succeeds** when a node containing the **False** clause is produced, becoming the **root node** of the tree
- A strategy is **complete** if its use guarantees that the empty clause (i.e., false) can be derived whenever it is entailed

Strategies

- There are a number of general (domain-independent) strategies that are useful in controlling a resolution theorem prover
- We'll briefly look at the following:
  - Breadth-first
  - Length heuristics
  - Set of support
  - Input resolution
  - Subsumption
  - Ordered resolution

Example

1. Battery-OK $\wedge$ Bulbs-OK $\rightarrow$ Headlights-Work
2. Battery-OK $\wedge$ Starter-OK $\rightarrow$ Empty-Gas-Tank $\vee$ Engine-Starts
3. Engine-Starts $\rightarrow$ Flat-Tire $\vee$ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. $\neg$Empty-Gas-Tank
8. $\neg$Car-OK
9. Goal: Flat-Tire ?

Example

1. ¬Battery-OK ˅ ¬Bulbs-OK ˅ Headlights-Work
2. ¬Battery-OK ˅ ¬Starter-OK ˅ Empty-Gas-Tank ˅ Engine-Starts
3. ¬Engine-Starts ˅ Flat-Tire ˅ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire ← **negated goal**

Breadth-first search

- Level 0 clauses are the original axioms and the negation of the goal
- Level k clauses are the resolvents computed from two clauses, one of which must be from level k-1 and the other from any earlier level
- Compute all possible level 1 clauses, then all possible level 2 clauses, etc.
- Complete, but very inefficient

BFS example

1. ¬Battery-OK ∨ ¬Bulbs-OK ∨ Headlights-Work
2. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts
3. ¬Engine-Starts ∨ Flat-Tire ∨ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire

1,4 10. ¬Battery-OK ∨ ¬Bulbs-OK

1,5 11. ¬Bulbs-OK ∨ Headlights-Work

2,3 12. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank ∨ Flat-Tire ∨ Car-OK

2,5 13. ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts

2,6 14. ¬Battery-OK ∨ Empty-Gas-Tank ∨ Engine-Starts

2,7 15. ¬Battery-OK ¬ Starter-OK ∨ Engine-Starts

16. … [and we’re still only at Level 1!]

Length heuristics

- **Shortest-clause heuristic**:
  Generate a clause with the fewest literals first
- **Unit resolution**:
  Prefer resolution steps in which at least one parent clause is a "unit clause," i.e., a clause containing a single literal
  - Not complete in general, but complete for Horn clause KBs

Unit resolution example

1. ¬Battery-OK ∨ ¬Bulbs-OK ∨ Headlights-Work
2. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts
3. ¬Engine-Starts ∨ Flat-Tire ∨ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire

1,5 10. ¬Bulbs-OK ∨ Headlights-Work

2,5 11. ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts

2,6 12. ¬Battery-OK ∨ Empty-Gas-Tank ∨ Engine-Starts

2,7 13. ¬Battery-OK ¬ Starter-OK ∨ Engine-Starts

3,8 14. ¬Engine-Starts ∨ Flat-Tire

3,9 15. ¬Engine-Starts ¬ Car-OK

16. ... [this doesn’t seem to be headed anywhere either!]

Set of support

- At least one parent clause must be the negation of the goal *or* a “descendant” of such a goal clause (i.e., derived from a goal clause)
- *When there’s a choice, take the most recent descendant*
- Complete, assuming all possible set-of-support clauses are derived
- Gives a goal-directed character to the search (e.g., like backward chaining)

Set of support example

1. ¬Battery-OK ˅ ¬Bulbs-OK ˅ Headlights-Work
2. ¬Battery-OK ˅ ¬Starter-OK ˅ Empty-Gas-Tank ˅ Engine-Starts
3. ¬Engine-Starts ˅ Flat-Tire ˅ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire

9,3 10. ¬Engine-Starts ˅ Car-OK
10,2 11. ¬Battery-OK ˅ ¬Starter-OK ˅ Empty-Gas-Tank ˅ Car-OK
10,8 12. ¬Engine-Starts
11,5 13. ¬Starter-OK ˅ Empty-Gas-Tank ˅ Car-OK
11,6 14. ¬Battery-OK ˅ Empty-Gas-Tank ˅ Car-OK
11,7 15. ¬Battery-OK ˅ ¬Starter-OK ˅ Car-OK
16. … [a bit more focused, but we still seem to be wandering]

Unit resolution + set of support example

1. ¬Battery-OK ∨ ¬Bulbs-OK ∨ Headlights-Work
2. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts
3. ¬Engine-Starts ∨ Flat-Tire ∨ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire

9,3 10. ¬Engine-Starts ∨ Car-OK
10,8 11. ¬Engine-Starts
11,2 12. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank
12,5 13. ¬Starter-OK ∨ Empty-Gas-Tank
13,6 14. Empty-Gas-Tank
14,7 15. FALSE

[Hooray! Now that’s more like it!]

Simplification heuristics

- **Subsumption:**
  Eliminate sentences that are subsumed by (more specific than) an existing sentence to keep KB small
  - If P(x) is already in the KB, adding P(A) makes no sense – P(x) is a superset of P(A)
  - Likewise adding P(A) $\vee$ Q(B) would add nothing to the KB
- **Tautology:**
  Remove any clause containing two complementary literals (tautology)
- **Pure symbol:**
  If a symbol always appears with the same "sign," remove all the clauses that contain it

Example (Pure Symbol)

1. ¬~~Battery-OK ∨ ¬Bulbs-OK ∨ Headlights-Work~~
2. ¬Battery-OK ∨ ¬Starter-OK ∨ Empty-Gas-Tank ∨ Engine-Starts
3. ¬Engine-Starts ∨ Flat-Tire ∨ Car-OK
4. H~~eadlights-Work~~
5. Battery-OK
6. Starter-OK
7. ¬Empty-Gas-Tank
8. ¬Car-OK
9. ¬Flat-Tire

Input resolution

- At least one parent must be one of the input sentences (i.e., either a sentence in the original KB or the negation of the goal)
- Not complete in general, but complete for Horn clause KBs
- Linear resolution
  – Extension of input resolution
  – One of the parent sentences must be an input sentence *or* an ancestor of the other sentence
  – Complete

Ordered resolution

- Search for resolvable sentences in order (left to right)
- This is how Prolog operates
- Resolve the first element in the sentence first
- This forces the user to define what is important in generating the “code”
- The way the sentences are written controls the resolution

Prolog: logic programming language based on Horn clauses

- Resolution refutation
- Control strategy: goal-directed and depth-first
  - always start from the goal clause
  - always use new resolvent as one of parent clauses for resolution
  - backtracking when the current thread fails
  - complete for Horn clause KB
- Supports answer extraction (can request single or all answers)
- Orders clauses & literals within a clause to resolve non-determinism
  - Q(a) may match both Q(x) <= P(x) and Q(y) <= R(y)
  - A (sub)goal clause may contain >1 literals, i.e., <= P1(a), P2(a)
- Use "closed world" assumption (negation as failure)
  - If it fails to derive P(a), then assume ~P(a)

Summary

- Logical agents apply inference to a KB to derive new information and make decisions
- Basic concepts of logic:
  - Syntax: formal structure of sentences
  - Semantics: truth of sentences wrt models
  - Entailment: necessary truth of one sentence given another
  - Inference: deriving sentences from other sentences
  - Soundness: derivations produce only entailed sentences
  - Completeness: derivations can produce all entailed sentences
- FC and BC are linear time, complete for Horn clauses
- Resolution is a sound and complete inference method for propositional and first-order logic