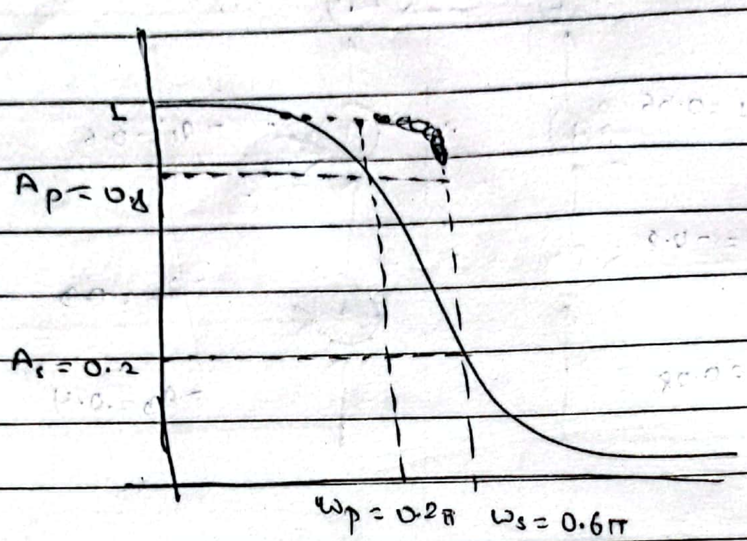
Assignment 5

1. Using bilinear transformation, design a butterworth filter which satisfies the following conditions.

$$0.8 \le |H_e(j\omega)| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H_e(j\omega)| \le 0.2 \qquad 0.6\pi \le \omega \le \pi$$

⇒



Finding analog filter specs using BLT Method:

$$\Omega_p = \frac{2}{T_s} \tan \frac{\omega_p}{2}$$

Assume $T_s = 1$

$$\Omega_p = 2 \tan \frac{\omega_p}{2} = 0.65$$

Similarly, $\Omega_s = \frac{2}{T_s} \tan \frac{\omega_s}{2} = 2\tan\left(\frac{0.6\pi}{2}\right) = 2.75$

Finding order of filter:

$$N = \frac{\frac{1}{2} \times \log\left[\left(\frac{1}{A_s^2} - 1\right) \Big/ \left(\frac{1}{A_p^2} - 1\right)\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

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$$= \frac{\frac{1}{2}\log\left[\left(\frac{1}{0.2^2}-1\right)\Big/\left(\frac{1}{0.8^2}-1\right)\right]}{\log\left(\frac{0.75}{0.65}\right)}$$

$\therefore N = 1.3 \simeq 2$

Finding cutoff frequency Ω_c,

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{1/2N}} = \frac{0.65}{\left(\frac{1}{0.8^2}-1\right)^{1/4}} = 0.75$$

Finding transfer function $H(s)$, poles are given by :

$$P_k = \pm \Omega_c e^{j(N+2k+1)\pi/2N}, \quad k = 0, 1, 2, \dots N-1$$

$$P_k = \pm 0.75 e^{j(3+2k)\pi/4}$$

For $k = 0$, $P_0 = \pm 0.75 e^{j(3\pi/4)} = \pm 0.75\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$

$$= \pm[-0.53 + j\,0.53]$$

For $k = 1$;

$$P_1 = \pm 0.75 e^{j(3+2)\pi/4} = \pm 0.75\left[\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right]$$

$$= \pm[-0.53 - j\,0.53]$$

Now, set $s_1 = -0.53 + j\,0.53$ and $s_1^* = 0.53 - j\,0.53$

$$\therefore H(s) = \frac{(\Omega_c)^N}{(s-s_1)(s-s_1^*)} = \frac{(0.75)^2}{(s+0.53-j0.53)(s+0.53+j0.53)}$$

$$= \frac{(0.75)^2}{s^2 + 0.53s + j0.53s + 0.53s + 0.28 + 0.28j - j0.53s - 0.28j + 0.28}$$

$$= \frac{(0.75)^2}{s^2 + 1.06s + 0.56}$$

$$= \frac{0.56}{s^2 + 1.06s + 0.56}$$

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Finding digital filter using BLT, where putting :

$$s = \frac{2}{15}\left[\frac{z-1}{z+1}\right] \text{ in } H(s)$$

$$H(z) = \frac{0.56}{4\left(\frac{z-1}{z+1}\right)^2 + (2\times 1.08)\left(\frac{z-1}{z+1}\right) + 0.56}$$

$$= \frac{0.56(z+1)^2}{4(z^2-2z+1) + (2.12z - 2.12z)(z+1) + 0.56(z+1)^2}$$

$$= \frac{0.56(z+1)^2}{4z^2 - 8z + 4 + 2.12z^2 + 2.12z - 2.12z - 2.1z + 0.56z^2 + 1.12z + 0.56}$$

$$= \frac{0.56(z+1)^2}{6.68z^2 + -6.88z + 2.44}$$

required digital filter.