COMP409: COMPILER DESIGN

1. LEXICAL ANALYSIS

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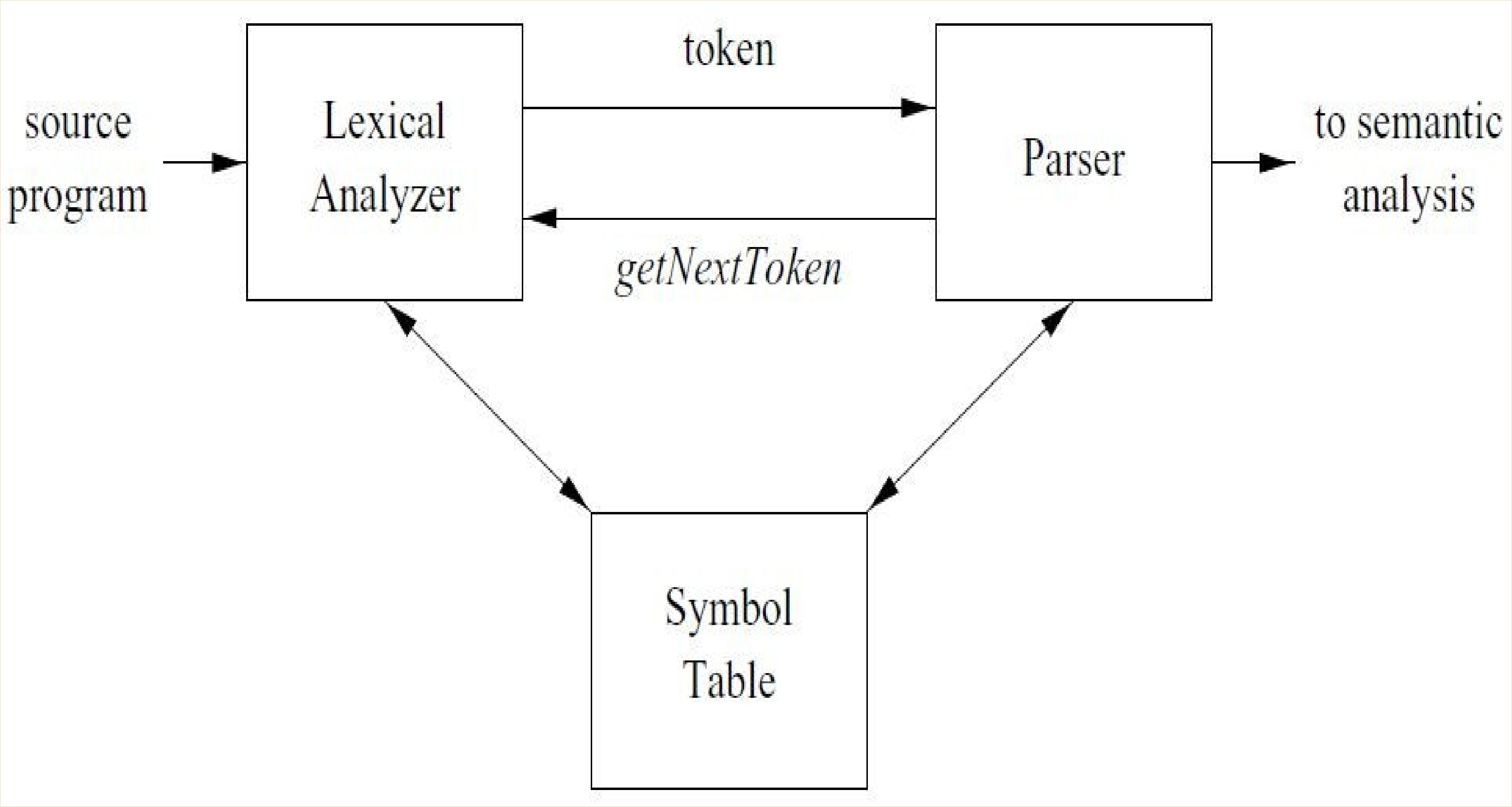
Lexical Analysis

- It is the initial part of reading and analyzing the program text
- The text is read and divided into tokens, each of which corresponds to a symbol in programming language, e.g. a variable name, keyword or number, etc.
- Lexical analyzer (also called lexer) will as its input take a string of individual letters and divide this string into tokens

The Role of Lexical Analyzer



The Role of Lexical Analyzer

- A lexical analyzer doesn't return a list of tokens at once, it returns a token when the parser asks a token from it
- The parser requests the lexical analyzer for the next token whenever it requires one using **getnexttoken( )**
- On the receipt of the command, lexer scans the input and processes until a token is matched

The Role of Lexical Analyzer

- The function of a lexical analyzer is to read the input stream representing the source program, one character at a time and to translate it into valid tokens
- Lexical analyzer may also perform other operations like
  - *removing redundant white spaces (i.e. blanks, tabs and newlines)*
  - *removing token separators (like semicolon)*
  - *removal of comments*
  - *providing line number to the parser for error reporting*

Reasons for Separation of Lexical Analysis and Parsing

- Simplicity of design is the most important consideration. The separation of lexical and syntactic analysis often allows us to simplify at least one of these tasks.
- Compiler efficiency is improved. A separate lexical analyzer allows us to apply specialized techniques that serve only the lexical task, not the job of parsing.
- Compiler portability is enhanced. Input-device-specific peculiarities can be restricted to the lexical analyzer.

Tokens, Patterns and Lexemes

- Tokens
  - *A token is a single word of source code input*
  - *A token is a pair consisting of a token name and an optional attribute value*
  - *Tokens are the separately identifiable blocks with collective meaning*

Tokens, Patterns and Lexemes

- *When a string representing a program is broken into sequence of substrings, such that each substring represents a constant, identifier, operator, keyword, etc of the language, these substrings are called the tokens of the language*
- *They are the building block of the programming language*
- *Eg: if, else, identifiers*

Tokens, Patterns and Lexemes

- <u>Lexemes</u>
  - *Lexemes are the actual string matched as token*
  - *They are the specific characters that make up of a token*
  - *For example,* ***abc*** *and* ***123***
  - *A token can represent more than one lexeme. i.e. token* ***intnum*** *can represent lexemes* ***123, 244, 4545,*** *etc*

Tokens, Patterns and Lexemes

- Patterns
  - *Patterns are rules describing the set of lexemes belonging to a token*
  - *For example: “letter followed by letters and digits” and “non-empty sequence of digits”*
  - *Regular expressions are usually used to specify patterns*
  - *Eg: **intnum** token can be defined as **[0-9][0-9]****

Attributes for Tokens

- When a token represents more than one lexeme, lexical analyzer must provide additional information about the particular lexeme
- This additional information is called as the **attribute** of the token
- Eg: If token ***id*** matched ***var1*** and ***var2*** both, then lexical analyzer must be able to represent ***var1*** and ***var2*** as different identifiers
- For obtaining actual value, each token is associated with attribute, generally pointer to the symbol table

Attributes for Tokens

- Some attributes:
  - *<id, attr> where attr is pointer to the symbol table*
  - *<assignop,_> no attribute is needed (if there is only one assignment operator)*
  - *<num, val> where val is the actual value of the number*
- Eg: dest = source + 5
  - *Tokens: <id, pt for dest>, <assignop>, <id, pt for source>, <num, 5>*
- Token type and its attribute uniquely identifies a lexeme

Attributes for Tokens

- Example: take statement,

  area = 3.1416 * r * r

1.getnexttoken( ) returns **(id, attr)** where **attr** is pointer to **area** in symbol table

2.getnexttoken( ) returns **(assignop)** where no attribute is needed, if there is only one assignment operator

3. getnexttoken( ) returns **(floatnum, 3.1416)** where **3.1416** is the actual value of floatnum etc

Lexical Errors

- Though error at lexical analysis is normally not common, there is possibility of errors
- When error occurs, the lexical analyzer must not halt the process
- It can print the error message and continue
- Error in this phase is found when there are no matching strings found as given by the pattern

Lexical Errors

- Some error recovery techniques
  - *Deletion of extraneous character*
  - *Inserting missing character*
  - *Replacing incorrect character by correct one*
  - *Transposition of adjacent characters*
- Lexical error recovery is normally an expensive process
- Recovery eg: finding the number of transformations that would make the correct tokens

Approaches to Implementing Lexical Analyzer

1. Use lexical analyzer generator like **Flex** that produces lexical analyzer from the given specification as regular expression. The generator provides routine for reading and buffering the input.

2. Write a lexical analyzer in general programming language like C. We need to use the I/O facility of the language for reading and buffering the input.

3. Use the assembly language to write the lexical analyzer. Explicitly manage the reading of input.

Approaches to Implementing Lexical Analyzer

- These strategies are in increasing order of difficulty and efficiency
- Since we deal with characters in lexical analysis, it is better to take some time during implementation to get efficient result

Input Buffering

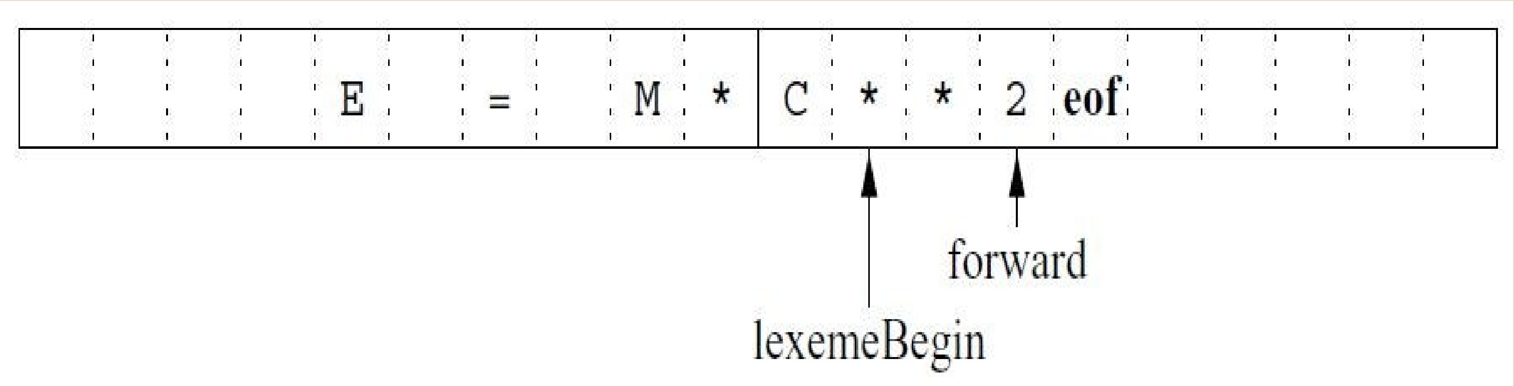
- Technique used to speed up reading the source program
- There are many situations where we need to look at least one (if not more) additional character ahead to recognize lexemes in the input
- For eg, **int** is a keyword in C but **intnum** is an identifier so when the scanner reads **i, n, t**, it has to look for other characters to see whether it is just **int** or some other word

Input Buffering

- In this case, when the token is read the next time, the scanner needs to move back to rescan the input again for the characters that are not used for the lexeme and this is time consuming
- In C, single-character operators like **-, =,** or **<** could also be the beginning of a two-character operator like **->, ==,** or **<=**
- To reduce the overhead and efficiently move back and forth, input buffering technique is used

Buffer Pairs (2N Buffering)

- Specialized buffering techniques have been developed to reduce the amount of overhead required to process a single input character



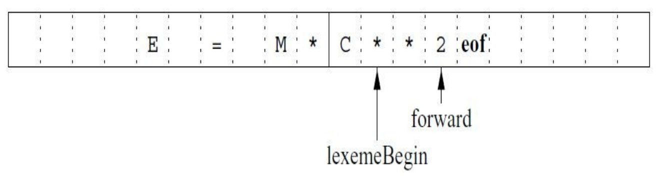
Buffer Pairs

- Each buffer is of the same size N, and N is usually the size of a disk block, e.g., 4096 bytes
- Using one system read command we can read N characters into a buffer, rather than using one system call per character
- If fewer than N characters remain in the input file, then a special character, represented by **eof** marks the end of the source file

Buffer Pairs

- Two pointers to the input are maintained:
  - *Pointer* ***lexemeBegin****, marks the beginning of the current lexeme, whose extent we are attempting to determine*
  - *Pointer* ***forward*** *scans ahead until a pattern match is found*
- Once the next lexeme is determined, ***forward*** is set to the character at its right end
- After the lexeme is recorded, ***lexemeBegin*** is set to the character immediately after the

Buffer Pairs



- In the above figure, ***forward*** has passed the end of the next lexeme, and must be retracted one position to its left
- Advancing ***forward*** requires that we first test whether we have reached the end of one of the buffers, and if so, we must reload the other buffer from the input, and move forward to the beginning of the newly loaded buffer

Code to advance forward pointer:

```
if forward at end of first half then begin
        reload second half;
        forward := forward + 1
end
else if forward at end of second half then begin
        reload first half;
        move forward to beginning of first half
end
else forward := forward + 1;
```

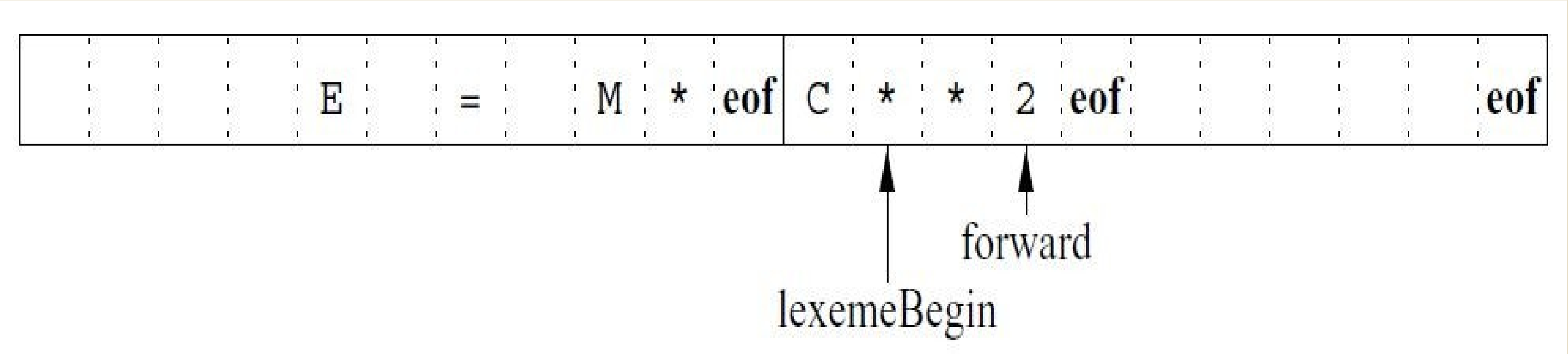
Sentinels

- If we use the 2N buffering scheme, we must check, each time we advance forward, that we have not moved off one of the buffers; if we do, then we must also reload the other buffer
- Thus, for each character read, we make two tests: one for the end of the buffer, and one to determine what character is read

Sentinels

- We can combine the buffer-end test with the test for the current character if we extend each buffer to hold a sentinel character at the end
- The sentinel is a special character that cannot be part of the source program, and a natural choice is the character **eof**
- **eof** retains its use as a marker for the end of the entire input
- Any **eof** that appears other than at the end of a buffer means that the input is at an end

Sentinels



```
forward : = forward + 1;
if forward ↑ = eof then begin
        if forward at end of first half then begin
                reload second half;
                forward := forward + 1
        end
        else if forward at end of second half then begin
                reload first half;
                move forward to beginning of first half end
else /* eof within a buffer signifying end of input */
        terminate lexical analysis
end
```

Specification of Tokens

- Regular expression is the common way of specifying the patterns for tokens
- Some Definitions:
- **Alphabet:**
  - *Set of symbols that generate language. For e.g. {0-9} is an alphabet that is used to produce all the non-negative integer numbers*
  - *{0-1} is an alphabet that is used to produce all the binary strings*

Specification of Tokens

- **String:**
  - *Finite sequence of characters from the alphabet*
  - *Given the alphabet A, $A^2 = A.A$ is set of strings of length 2, similarly A^n is set of strings of length n*
  - *The **length** of the string **w** is denoted by |**w**| i.e. number of characters (symbols) in **w***
  - *We also have $A^0 = \{\varepsilon\}$, where ε is called empty string*

Specification of Tokens

- **Kleene Closure:**
  - *Kleene closure of an alphabet A denoted by* ***A**** *is set of all strings of any length (0 also) possible from A*
  - *Mathematically* $A^* = A^0 \cup A^1 \cup A^2 \cup \ldots\ldots$
  - *For any string, w over alphabet A,* $w \in A^*$

Specification of Tokens

OPERATION	DEFINITION AND NOTATION
Union of L and M	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
Concatenation of L and M	$LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L	$L^* = \cup_{i=0}^{\infty} L^i$
Positive closure of L	$L^+ = \cup_{i=1}^{\infty} L^i$

Specification of Tokens

- A **language** L over alphabet A is the set such that L ⊆ A*
- The string s is called **prefix** of w, if the string s is obtained by removing zero or more trailing characters from w. If s is a proper prefix, then s ≠ w.
- The string s is called **suffix** of w, if the string s is obtained by deleting zero or more leading characters from w. We say s is a proper suffix if s ≠ w.
- The string s is called **substring** of w if we can obtain s by deleting zero or more leading or trailing characters from w. We say s is a proper substring if s ≠ w.

Specification of Tokens

- **Regular Operators:**
- The following operators are called regular operators and the language formed called regular language.
  - $. \rightarrow$ *Concatenation operator,* $R.S = \{rs \mid r \in R \text{ and } s \in S\}$
  - $* \rightarrow$ *Kleene star operator,* $A^* = \bigcup_{i \geq 0} A^i$
  - $+ / \cup / \mid \rightarrow$ *Choice/union operator,* $R \cup S = \{t \mid t \in R \text{ or } t \in S\}$

Specification of Tokens

- **<u>Regular Expression (RE):</u>**
- We use regular expression to describe the tokens of a programming language.
- **Basic Symbol**
  - *ε is a regular expression denoting language { ε }*
  - *a ∈ A is a regular expression denoting {a}*

Specification of Tokens

- If r and s are regular expressions denoting languages L1(r) and L2(s) respectively, then
  - *r + s is a regular expression denoting L1(r) ∪ L2(s)*
  - *rs is a regular expression denoting L1(r) . L2(s)*
  - *r* is a regular expression denoting (L1(r))**
  - *(r) is a regular expression denoting L1(r)*

Specification of Tokens

- ***Practice Questions***

(a) RE in which every pair of adjacent zero's appear before any pair of adjacent ones.

(b) RE that gives binary strings having at most two 1s

(c) RE that denotes the language of all strings that ends with 00 (binary number multiple of 4)

(d) RE that denotes the set of all strings that describes alternating 1s and 0s

Specification of Tokens

- ***Answers***

(a) (01)*(0011)(01)*

(b) 0*10*10* + 0*10* + 0*

(c) (1+0)*00

(d) (01)* + (10)*

Specification of Tokens

- **Properties of RE**
  - *r+s = s+r (+ is commutative)*
  - *r+(s+t) = (r+s)+t ; r(st) = (rs)t (+ and . are associative)*
  - *r(s+t) = (rs)+(rt); (r+s)t =(rt)+(st) (. distributes over +)*
  - *εr = rε (ε is identity element)*
  - *r* = (r+ε)* (relation between * and ε)*
  - *r** = r* (* is idempotent)*

Specification of Tokens

- **Regular Definitions:**
- To write regular expression for some languages can be difficult, because their regular expressions can be quite complex
- In those cases, we may use ***regular definitions***
- We can give names to regular expressions, and we can use these names as symbols to define other regular expressions

Specification of Tokens

A ***regular definition*** is a sequence of the definitions of the form:

$d_1 \rightarrow r_1$ where d_i is a distinct name and

$d_2 \rightarrow r_2$ r_i is a regular expression over symbols in

. $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

$d_n \rightarrow r_n$

basic symbols previously defined names

Specification of Tokens

- Identifiers in Pascal are defined as a string of letters and digits beginning with a letter

  letter → A | B | … | Z | a |b| … |

  z digit → 0 | 1 | … | 9

  id → letter (letter | digit) *

- If we try to write the regular expression representing identifiers without using regular definitions, that regular expression will be complex:

  (A|...|Z|a|...|z) ((A|...|Z|a|...|z) | (0|...|9)) *

Recognition of Tokens

- A recognizer for a language is a program that takes a string **w**, and answers "YES" if **w** is a sentence of that language, otherwise "NO"
- The tokens that are specified using RE are recognized by using transition diagram or finite automata (FA)
- Starting from the start state we follow the transition defined

Recognition of Tokens

- If the transition leads to the accepting state, then the token is matched and hence the lexeme is returned, otherwise other transition diagrams are tried out until we process all transition diagrams or failure is detected
- Recognizer of tokens takes the language **L** and the string **s** as input and tries to verify whether **s** ∈ **L** or not

Recognition of Tokens

- We concentrate on a class of recognizer called Finite Automata (FA)
- There are two types of Finite Automaton:
  - *Deterministic Finite Automaton (DFA)*
  - *Non Deterministic Finite Automaton (NFA)*

Recognition of Tokens

Design of a Lexical Analyzer

First, we define regular expressions for tokens; Then we convert them into a DFA to get a lexical analyzer for our tokens.

Algorithm1:

Regular Expression ➔ NFA ➔ DFA (two steps: first to NFA, then to DFA)

Algorithm2:

Regular Expression ➔ DFA (directly convert a regular expression into a DFA)

Recognition of Tokens

- Why convert from NFA to DFA?
  - Computer programs generally need to know all possible transitions and states for a given state machine.
  - A non-deterministic finite automaton can have a transition that goes to any number of states for a given input and state.
  - This is a problem for a computer program because it needs precisely one transition for a given input from a given state.
  - The process of converting NFA to DFA eliminates this ambiguity and allows a program to be made.

Deterministic Finite Automata (DFA)

- FA is deterministic, if there is exactly one transition for each (state, input) pair
- It is a fast recognizer but takes large space
- DFA is a five tuple (S, $\sum$, q_0, δ, F) where,
  - $S \rightarrow$ *finite set of states*
  - $\sum \rightarrow$ *finite set of input alphabets*
  - $q_0 \rightarrow$ *starting state*
  - $\delta \rightarrow$ *transition function i.e.* $\delta : S \times \sum \rightarrow S$

Deterministic Finite Automata

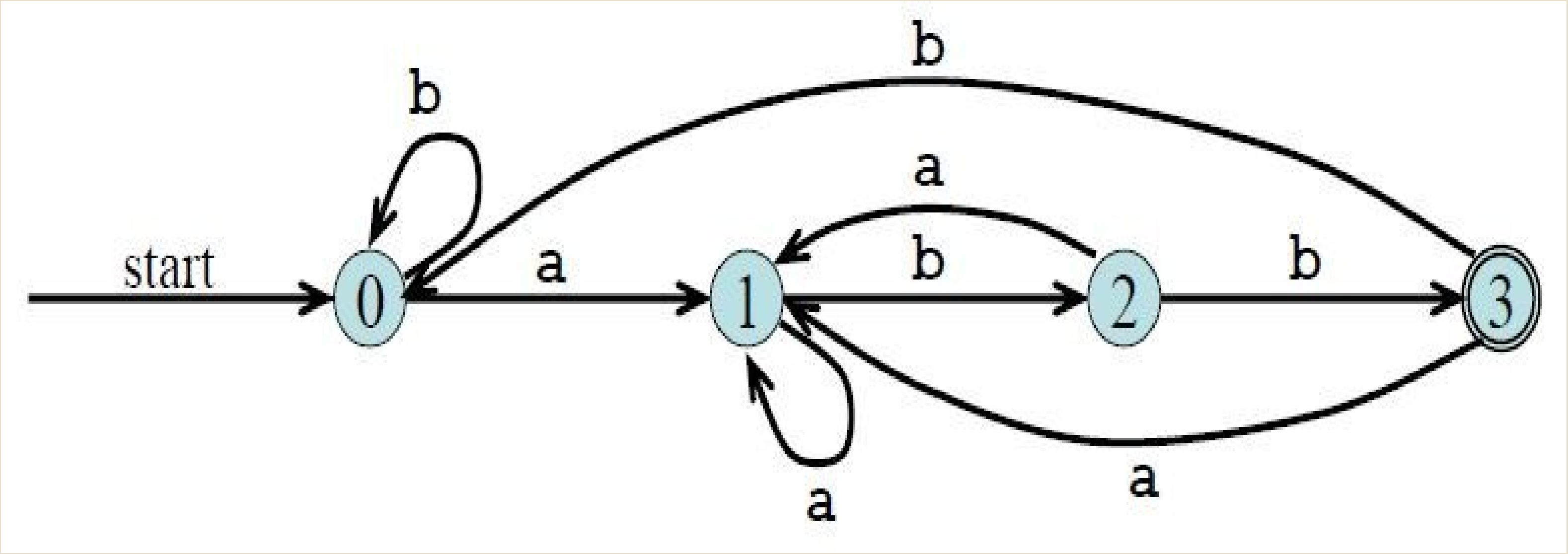
- The following is the algorithm for simulating DFA for recognizing given string
- For a given string **w**, in DFA **D**, with start state $\mathbf{q_0}$, with set of final states **F**, the output is “**YES**”, if **D** accepts **w**, otherwise “**NO**”

DFASim(D, q_0) {

```
q = q0
c = getchar( )
while (c != eof) {
    q = move(q, c)        // this is δ function.
    c = getchar( )
}
if (q is in F)            // if D accepts w
    return "yes"
else
    return "no"
}
```

Deterministic Finite Automata

- The figure shows DFA for the regular expression: (**a**+**b**)*__abb__



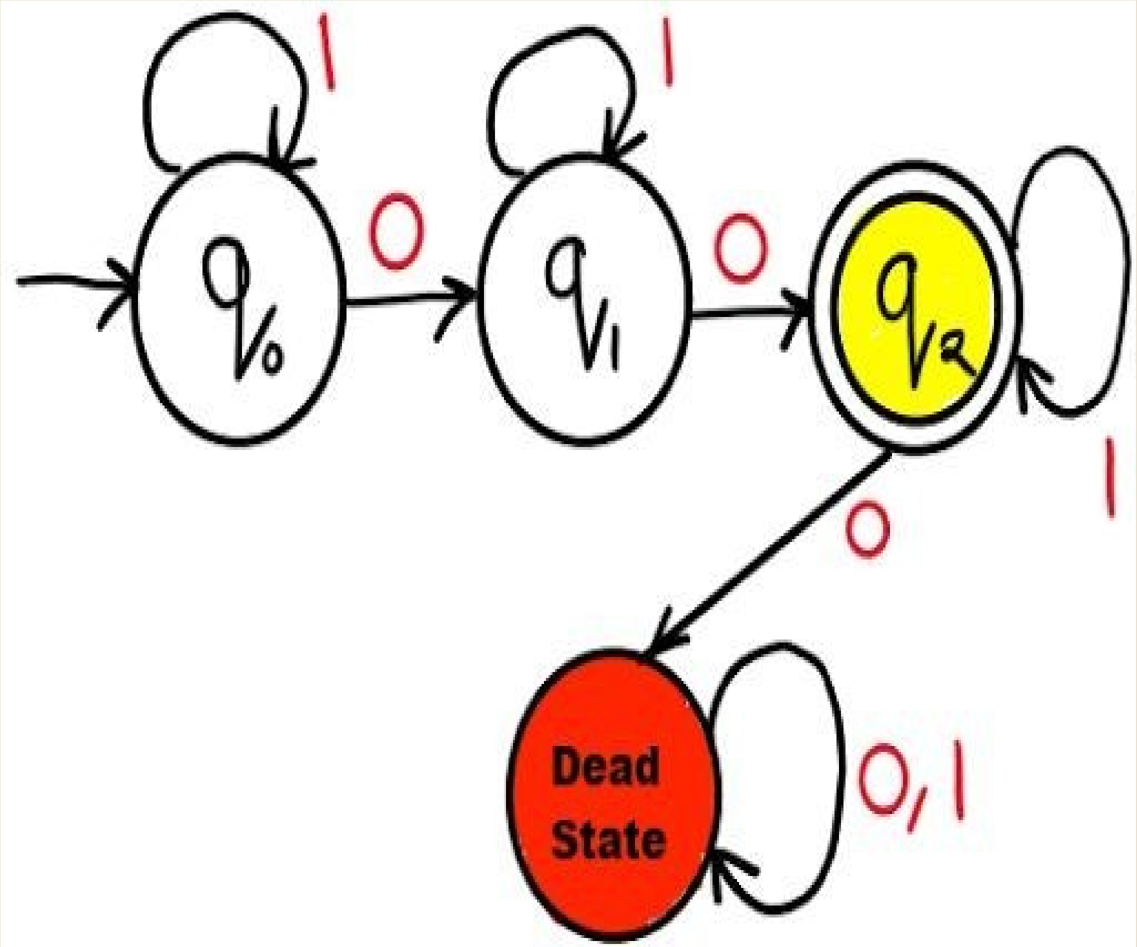
Practice Questions

Q) Write the regular expression and draw its corresponding DFA for the following:

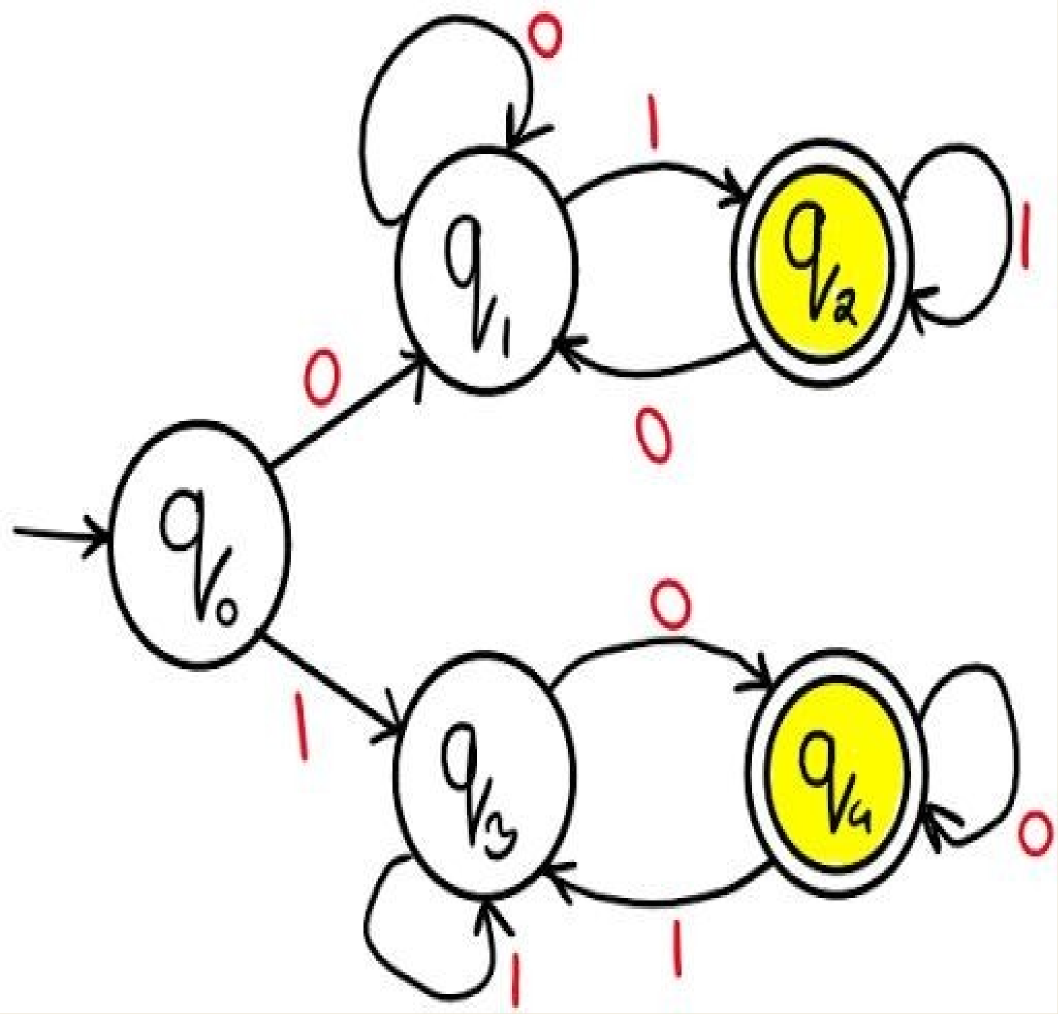
(1)Language accepting strings containing exactly two 0s over input alphabets $\sum = \{0, 1\}$.

(2)Language accepting strings starting and ending with different characters over input alphabets $\sum = \{0, 1\}$.

Ans 1:   1*01*01*



Ans 2: (0(0+1)*1)+(1(1+0)*0)

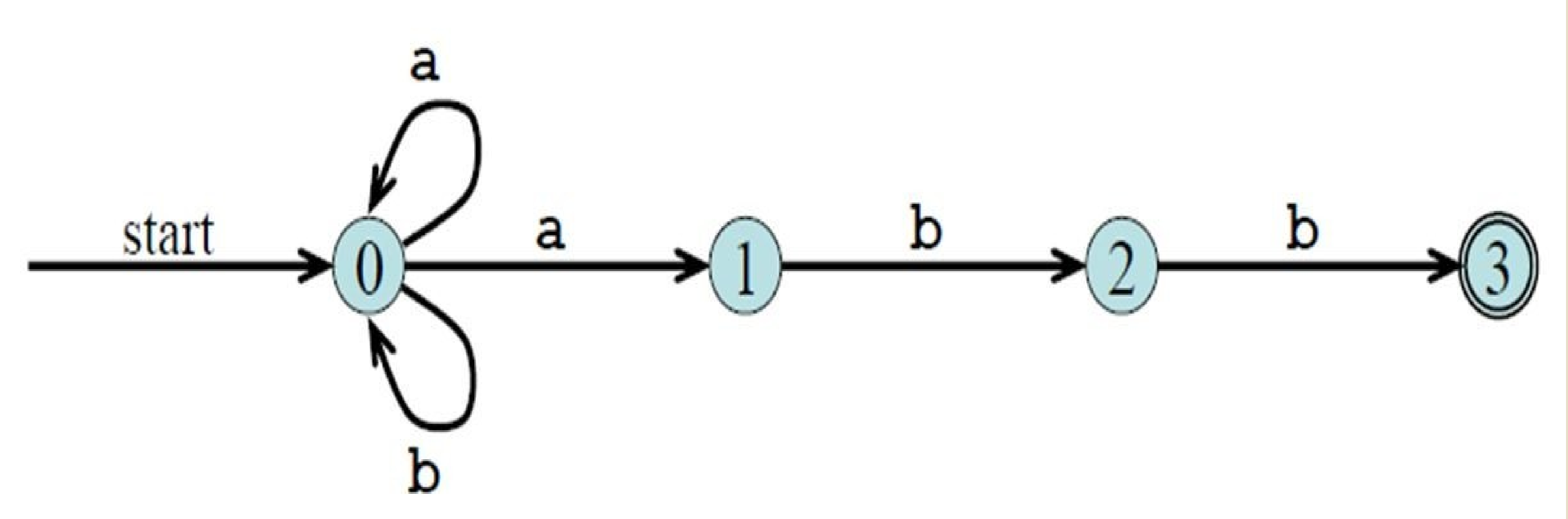


Non-Deterministic Finite Automata (NFA)

- FA is non-deterministic if there can be more than one transition (or none) for each (state, input) pair
- It is a slow recognizer but takes less space
- An NFA is a five tuple (S, ∑, q_0, δ, F) where,
  - *S → finite set of states*
  - *∑ → finite set of input alphabets*
  - *q_0 → starting state*
  - *δ → transition function i.e.* $\delta : S \times \Sigma \rightarrow 2^{|S|}$
  - *F → set of final states* $F \subseteq S$

Non-Deterministic Finite Automata

- The figure shows NFA for the regular expression: (**a**+**b**)***abb**

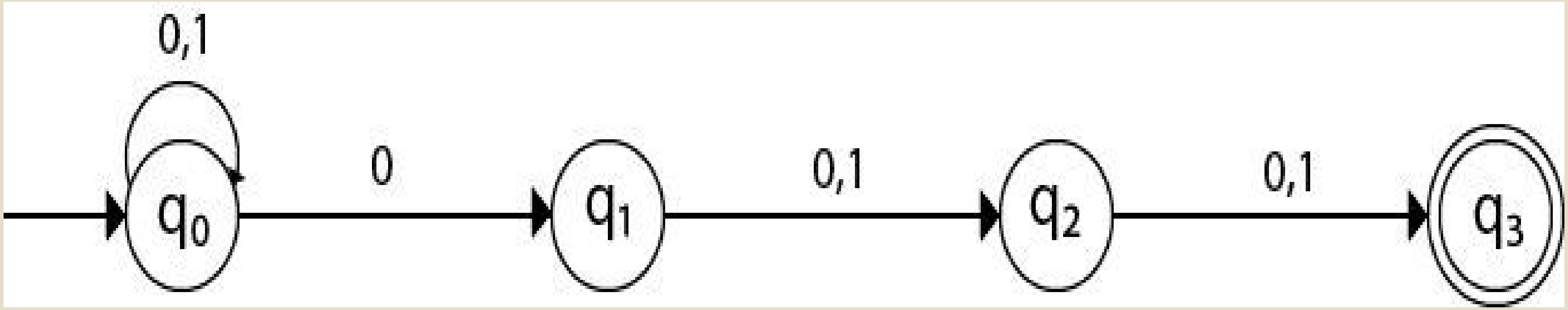


Practice Questions

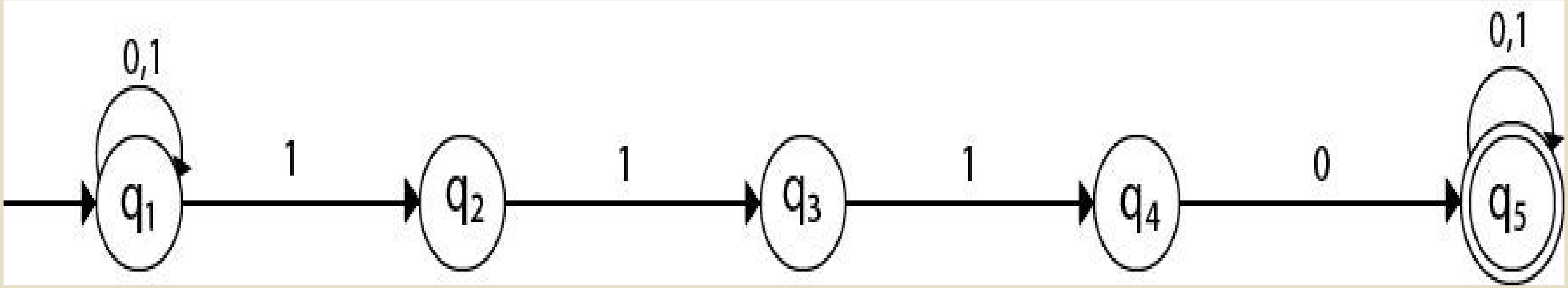
(1)Design an NFA with $\sum = \{0, 1\}$ that accepts all strings in which the third symbol from the right end is always 0.

(2)Design an NFA with $\sum = \{0, 1\}$ which accepts all strings containing the substring 1110.

Ans 1:



Ans 2:

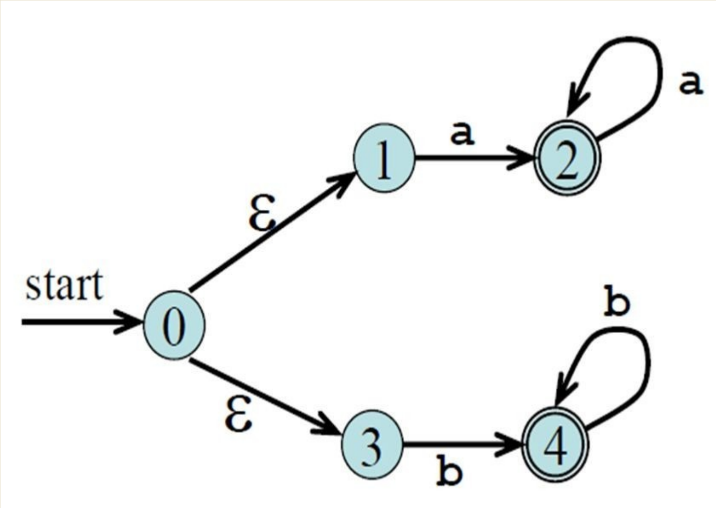


ε-NFA

- ε-transitions are allowed in ε-NFAs
- In other words, we can move from one state to another without consuming any symbol
- In case of ε-NFA the only difference is $\sum = \sum \cup \{\varepsilon\}$ and hence $\delta: S \times \sum \cup \{\varepsilon\} \rightarrow 2^{|S|}$

ε-NFA

- The figure shows a state machine with ε moves that is equivalent to the regular expression: **aa* +bb***



//SIMULATING NFA

S = ε-closure({S_0}) *// set of all states that can be accessed from S_0 by ε- transitions*

c = getchar( )

while(c != eof) {

S = ε-closure(move(S, c)) *//set of all states that can be accessible from a state in S by a transition on c*

c = getchar( )

}

if (S ∩ F ≠ Φ) then

*return "**YES**"*

else

*return "**NO**"*

RE to NFA (Thompson's Construction)

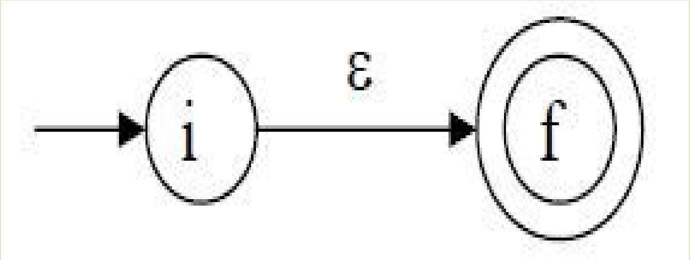
- Thompson's construction is a simple and systematic method
- It guarantees that the resulting NFA will have exactly one final state and one start state
- Construction starts from simplest parts(alphabet symbols)
- To create an NFA for a complex regular expression, NFAs of its subexpressions are combined

RE to NFA

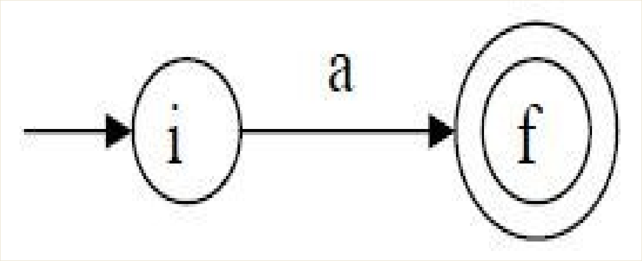
- **Input** → RE, r, over alphabet ∑
- **Output** → ε-NFA accepting L(r)
- **Procedure** → Process in bottom-up manner by creating ε-NFA for each symbol in ∑ including ε. Then recursively create for other operations as shown below.

RE to NFA

(1) To recognize an empty string **ε**



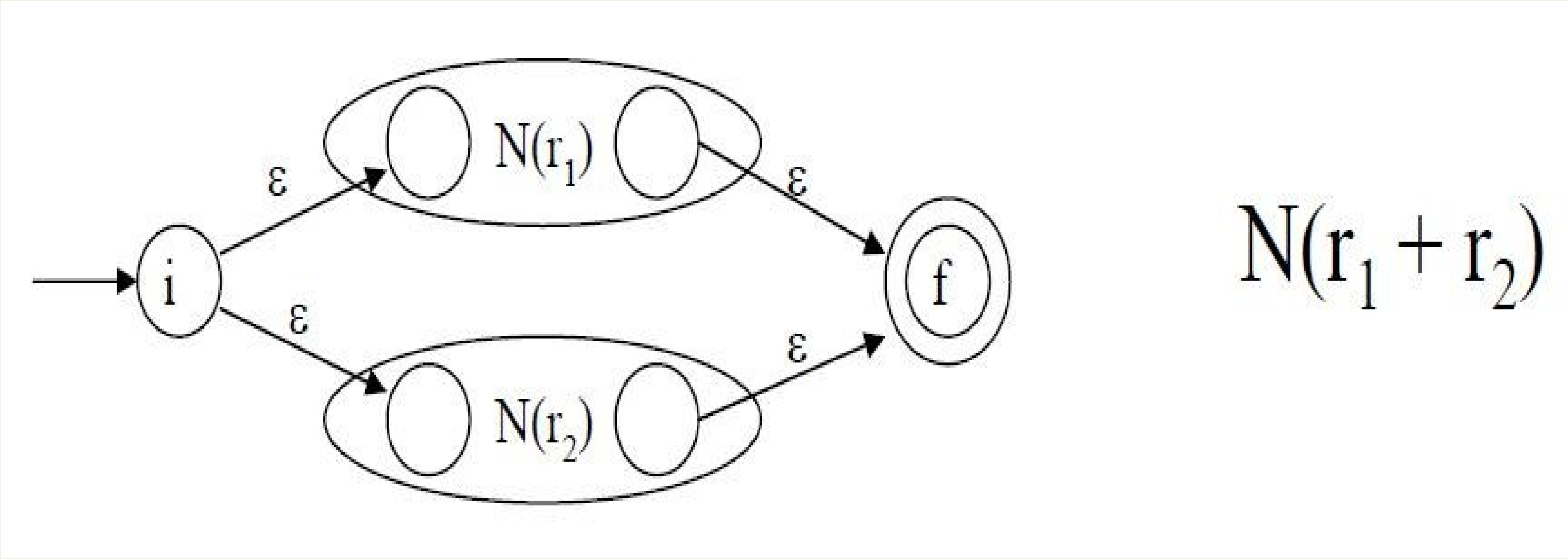
(2) To recognize a symbol **a** in the alphabet ∑



RE to NFA

(3) If $N(r_1)$ and $N(r_2)$ are NFAs for regular expressions r_1 and r_2

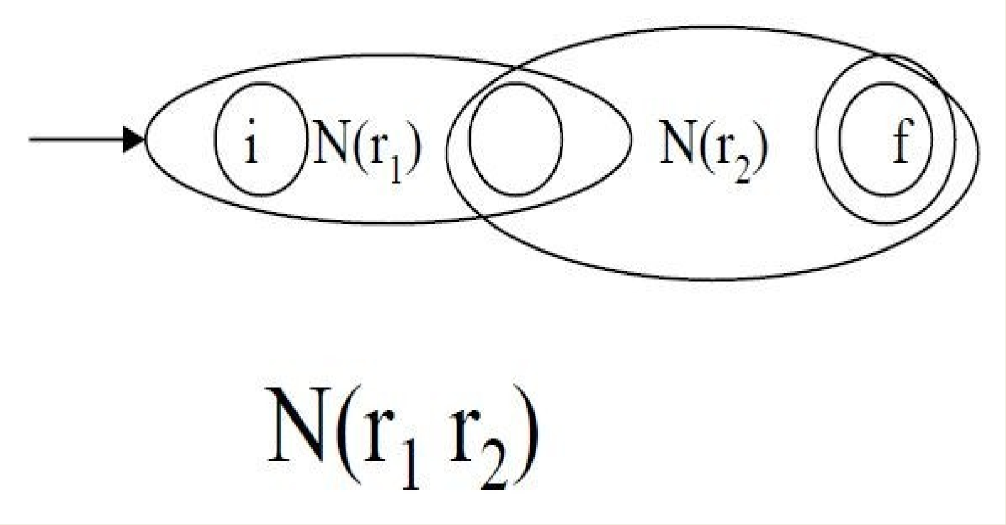
(A) For regular expression $r_1 + r_2$



RE to NFA

(B) For regular expression $r_1 . r_2$

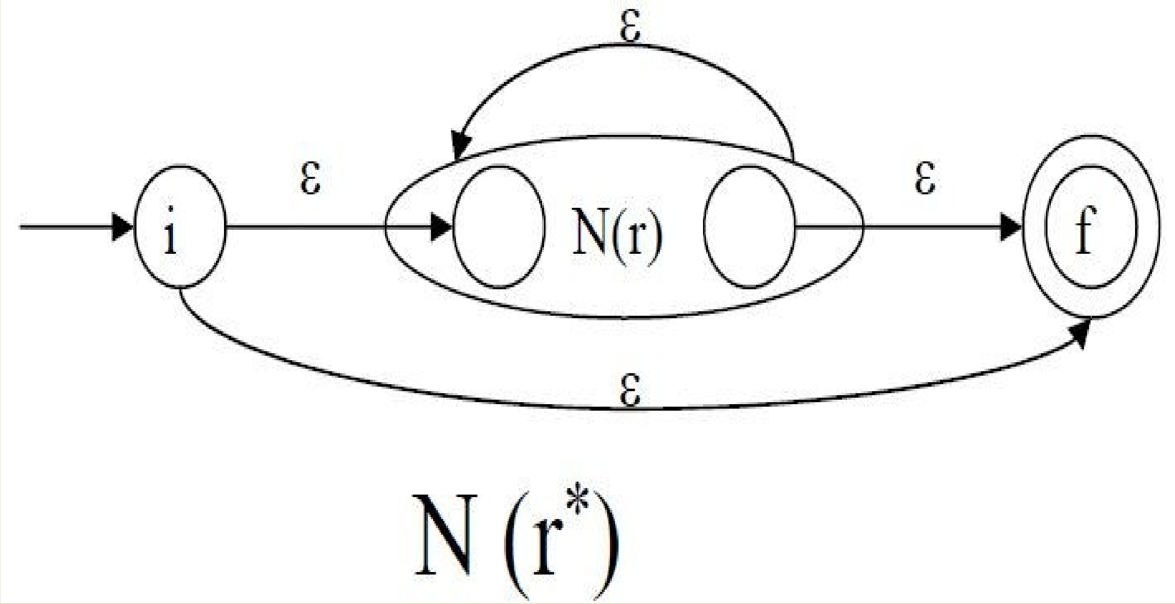
The start state of $N(r_1)$ becomes the start state of $N(r_1r_2)$ and final state of $N(r_2)$ become final state of $N(r_1r_2)$



RE to NFA

(C) For regular expression

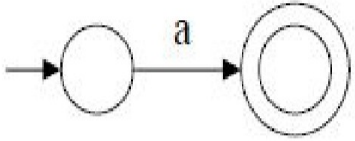
r*



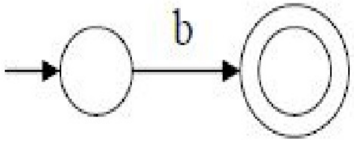
Using rules 1 and 2, we construct NFAs for each basic symbol in the expression; we combine these basic NFAs using rule 3 to obtain an NFA for the entire expression

Example:- NFA construction of RE **(a+b) * a**

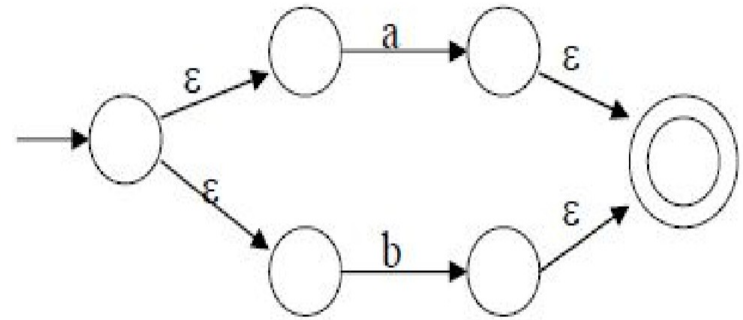
a:



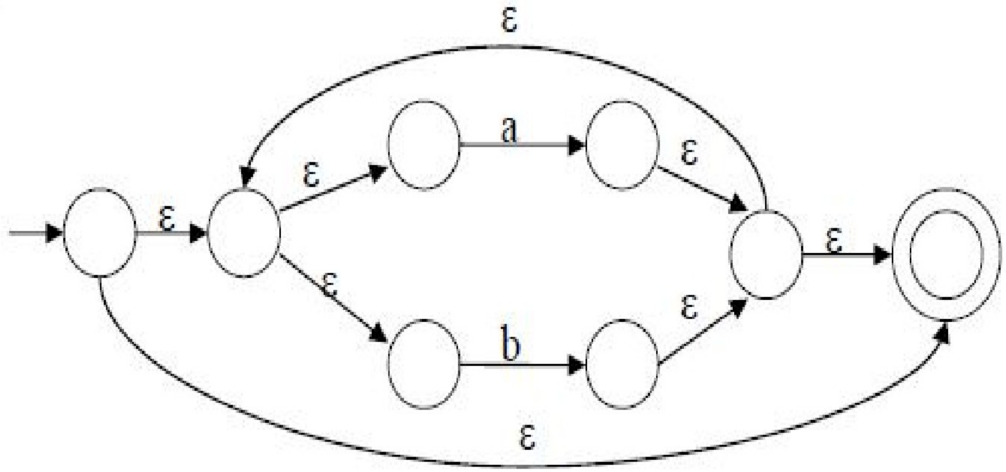
b:



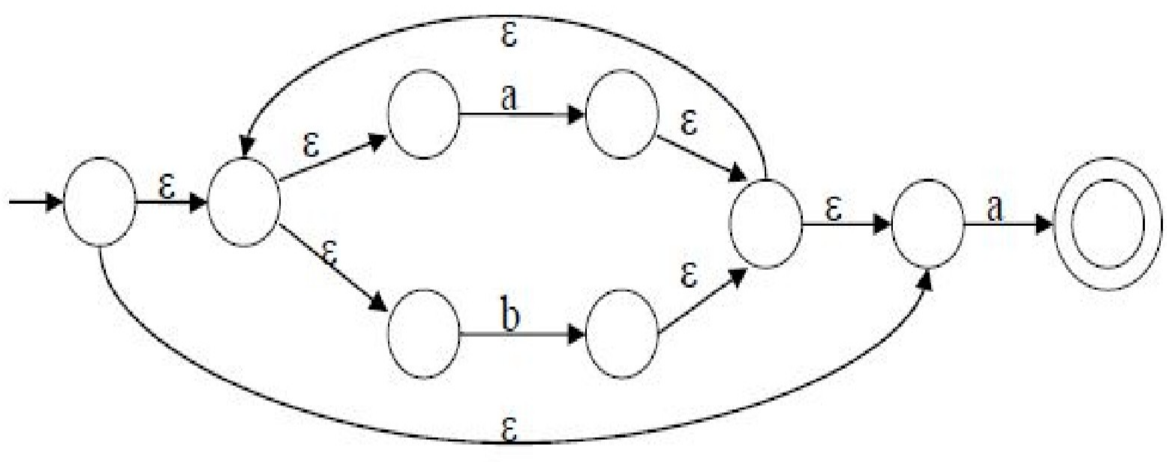
(a + b)



(a + b) *



(a+b) * a



NFA to DFA (Subset Construction)

- The subset construction algorithm converts an NFA into a DFA
- We use the following operations to keep the track of sets of NFA
  - *ε-closure(s) → the set of NFA states reachable from state s on ε- transition*
  - *ε-closure(T) → the set of NFA states reachable from state s in T on ε- transition*
  - *move(T, a) → the set of NFA states to which there is transition on input **a** from state s in T*

NFA to DFA

- The algorithm produces:
  - ***Dstates*** *is the set of states of the new DFA consisting of sets of states of the NFA*
  - ***Dtran*** *is the transition table of the new DFA*
- The start state of DFA is ε-closure(q_0)
- A set of **Dstates** is an accepting state of DFA if it is a set of NFA states containing at least one accepting state of NFA

Algorithm

Put ε-closure(q_0) as an unmarked state in *Dstates*

While there is an unmarked state T in *Dstates* **do**

Mark T

For each input symbol a ∈ ∑ **do**

U = ε-closure(move(T, a))

If U is not in *Dstates* **then**

Add U as an unmarked state to *Dstates*

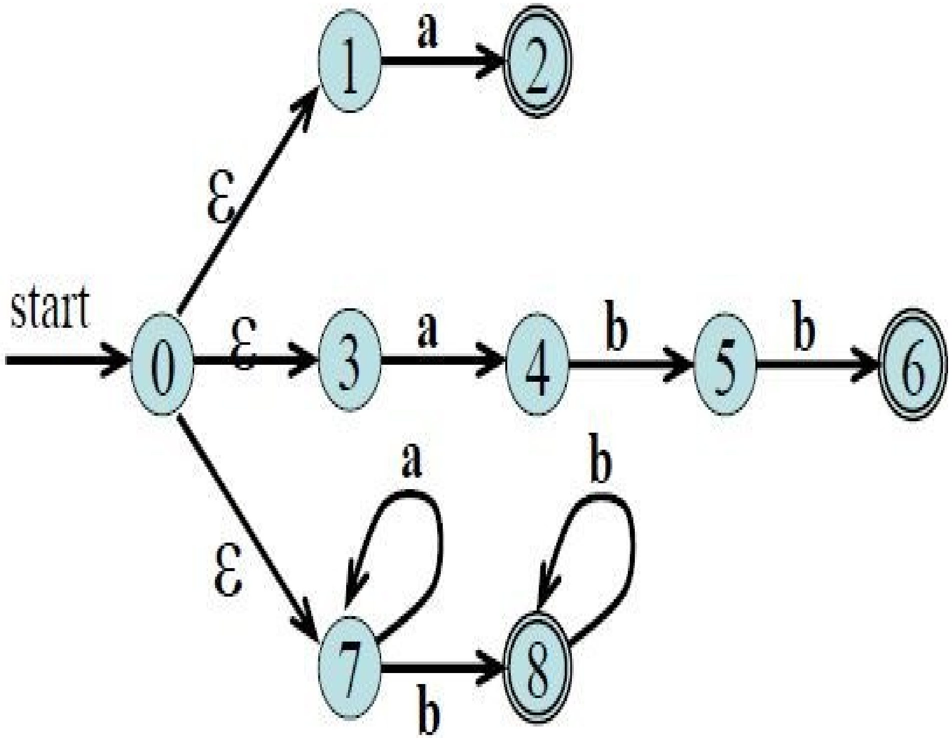
End if

Dtran[T, a] = U

End do

End do

ε-*closure* and *move* Examples



ε-*closure*({0}) = {0,1,3,7}

move({0,1,3,7},**a**) = {2,4,7}

ε-*closure*({2,4,7}) = {2,4,7}

move({2,4,7},**a**) = {7}

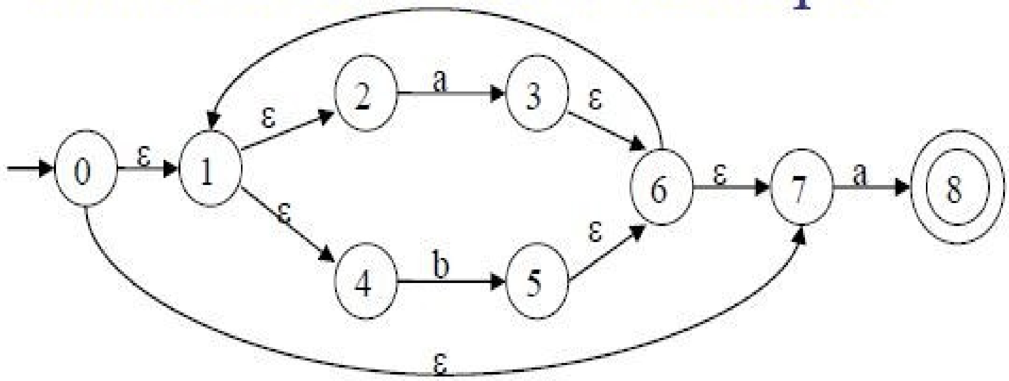
ε-*closure*({7}) = {7}

move({7},**b**) = {8}

ε-*closure*({8}) = {8}

move({8},**a**) = ∅

Subset Construction Example



S_0 = ε-closure({0}) = {0,1,2,4,7} S_0 into *Dstates* as an unmarked state

$\Downarrow$ mark S_0

ε-closure(move(S_0,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1 S_1 into *Dstates*

ε-closure(move(S_0,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2 S_2 into *Dstates*

Dtran[S_0,a] $\leftarrow$ S_1 *Dtran*[S_0,b] $\leftarrow$ S_2

$\Downarrow$ mark S_1

ε-closure(move(S_1,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1

ε-closure(move(S_1,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2

Dtran [S_1,a] $\leftarrow$ S_1 *Dtran* [S_1,b] $\leftarrow$ S_2

$\Downarrow$ mark S_2

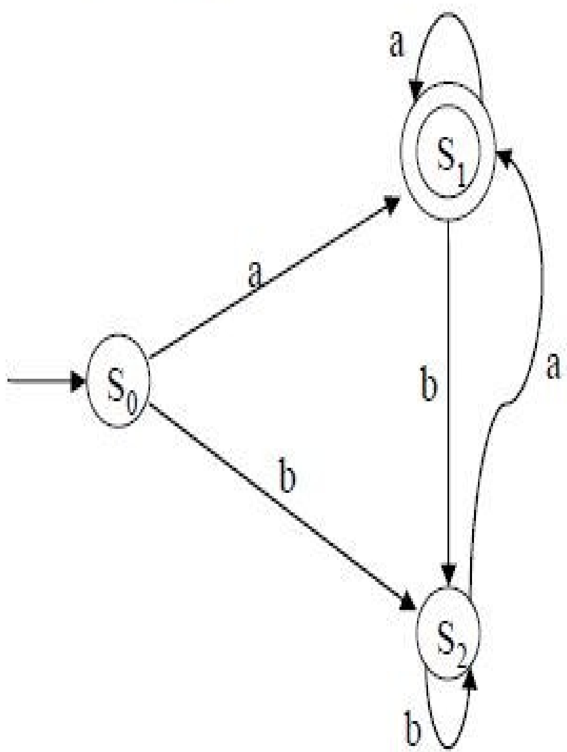
ε-closure(move(S_2,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1

ε-closure(move(S_2,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2

Dtran[S_2,a] 🡐 S_1 *Dtran*[S_2,b] 🡐 S_2

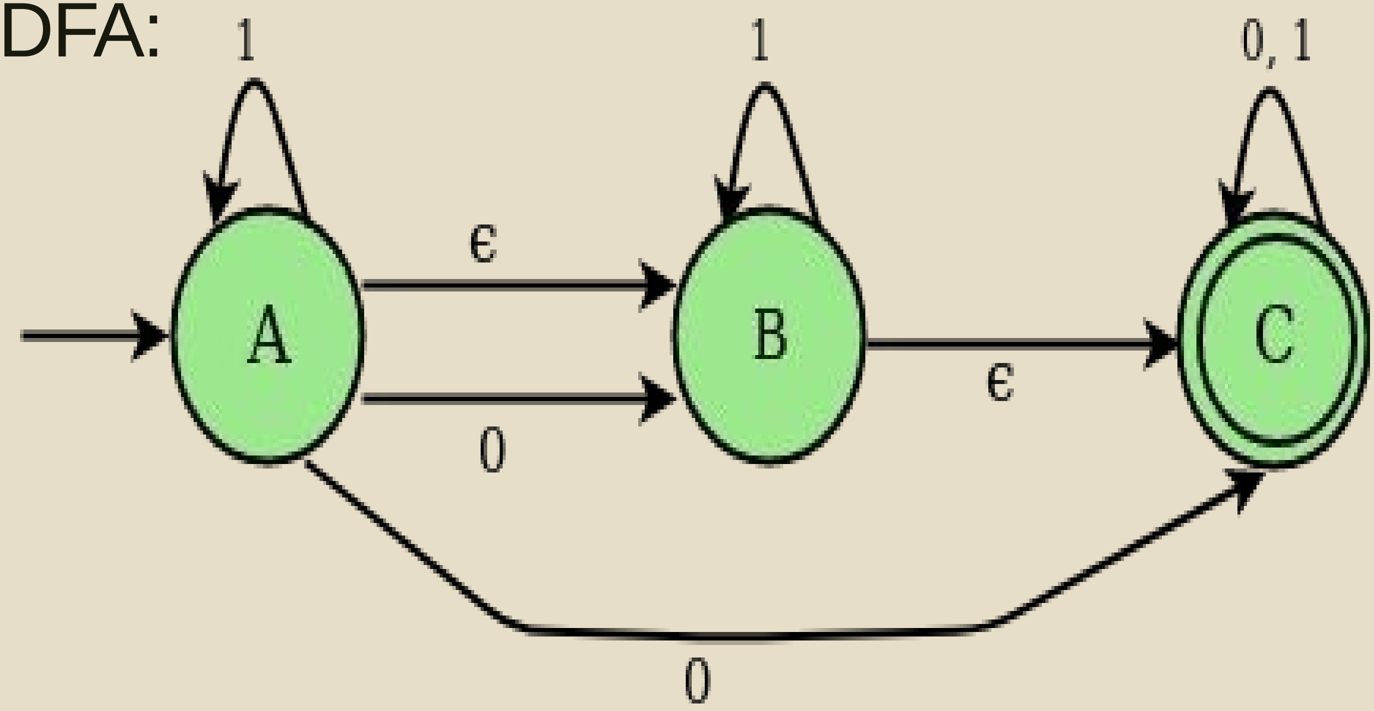
S_0 is the start state of DFA since 0 is a member of S_0={0,1,2,4,7}

S_1 is an accepting state of DFA since 8 is a member of S_1 = {1,2,3,4,6,7,8}



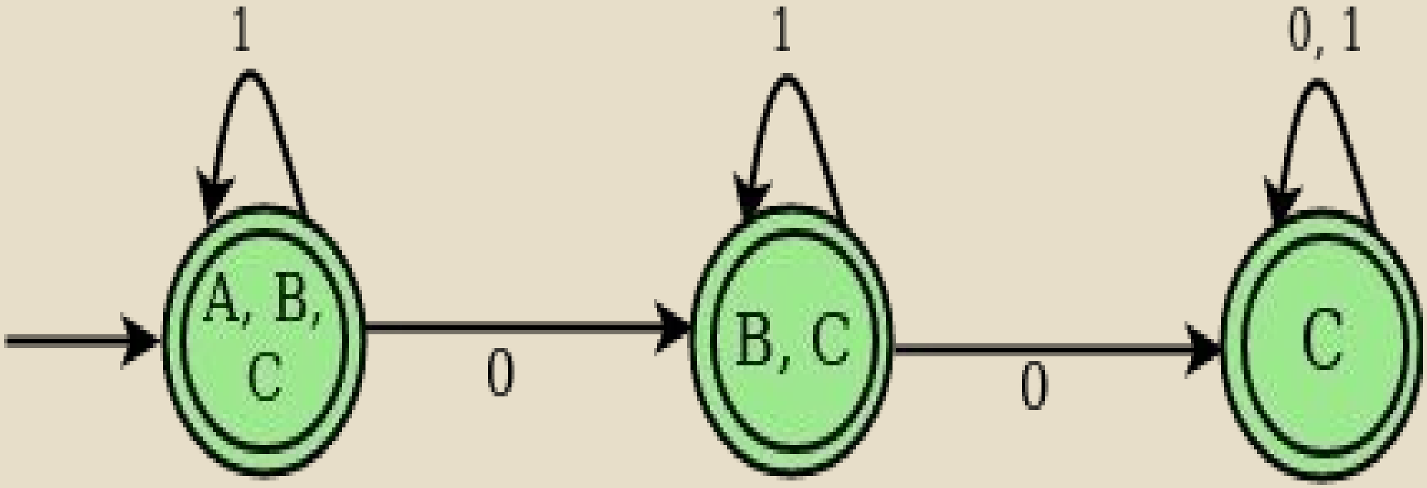
Practice Question

Q) Convert the following NFA to DFA:



Practice Question

Ans:



RE to DFA (directly)

<u>Important States:</u>

- *The "important states" of an NFA are those without a null transition; that is, if **move({s},a)** ≠ ø for some **a**, then **s** is an important state*
- *In an optimal state machine, all states are important states*
- *The subset construction algorithm uses only the important states when it determines **ε-closure(move(T,a))***

RE to DFA

Augmented Regular Expression:

- The ε-NFA created from RE has exactly one accepting state and it does not have any transition i.e. it is not important state
- We introduce an “augmented character” **#** from the accepting state to make it an important state
- The regular expression **(r)#** is called the augmented regular expression of the original expression **r**

Procedure:

1. Augment the given regular expression by concatenating it with special symbol # i.e r $\rightarrow$ (r)#

2. Create the syntax tree for this augmented regular expression
   - *In this syntax tree, all alphabet symbols (plus # and the empty string) in the augmented regular expression will be on the leaves, and all inner nodes will be the operators in that augmented regular expression.*

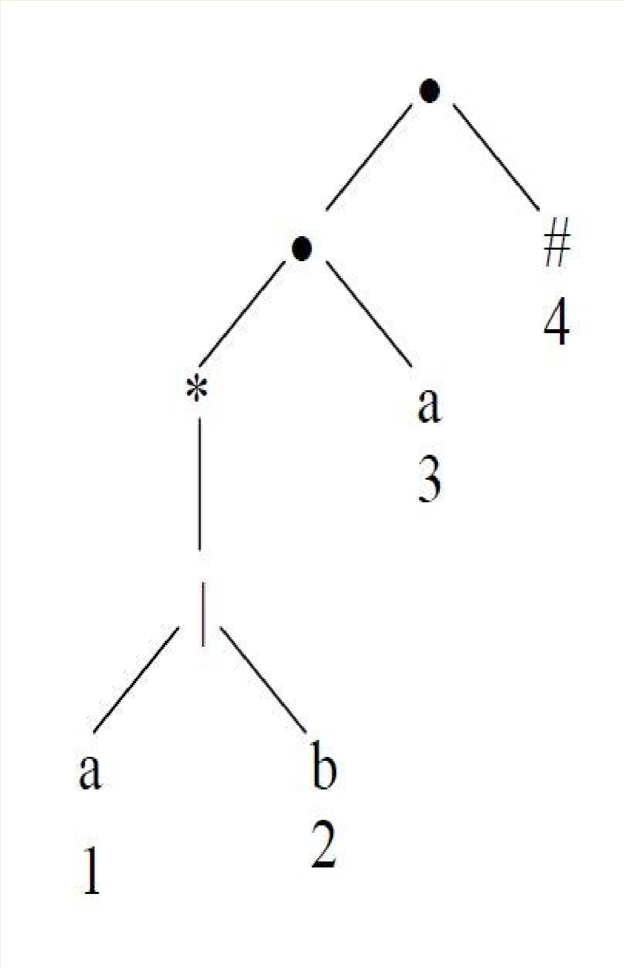
3. Then each alphabet symbol (plus #) will be numbered (position numbers)

4. Traverse the tree to construct functions ***nullable***, ***firstpos***, ***lastpos***, and ***followpos***

5. Finally construct the DFA from the ***followpos***

Syntax Tree Construction:

- (a|b)*a → (a|b)*a#   [augmented regular expression]



Syntax tree of (a|b)*a#

- each symbol is numbered (positions)
- each symbol is at a leaf
- inner nodes are operators

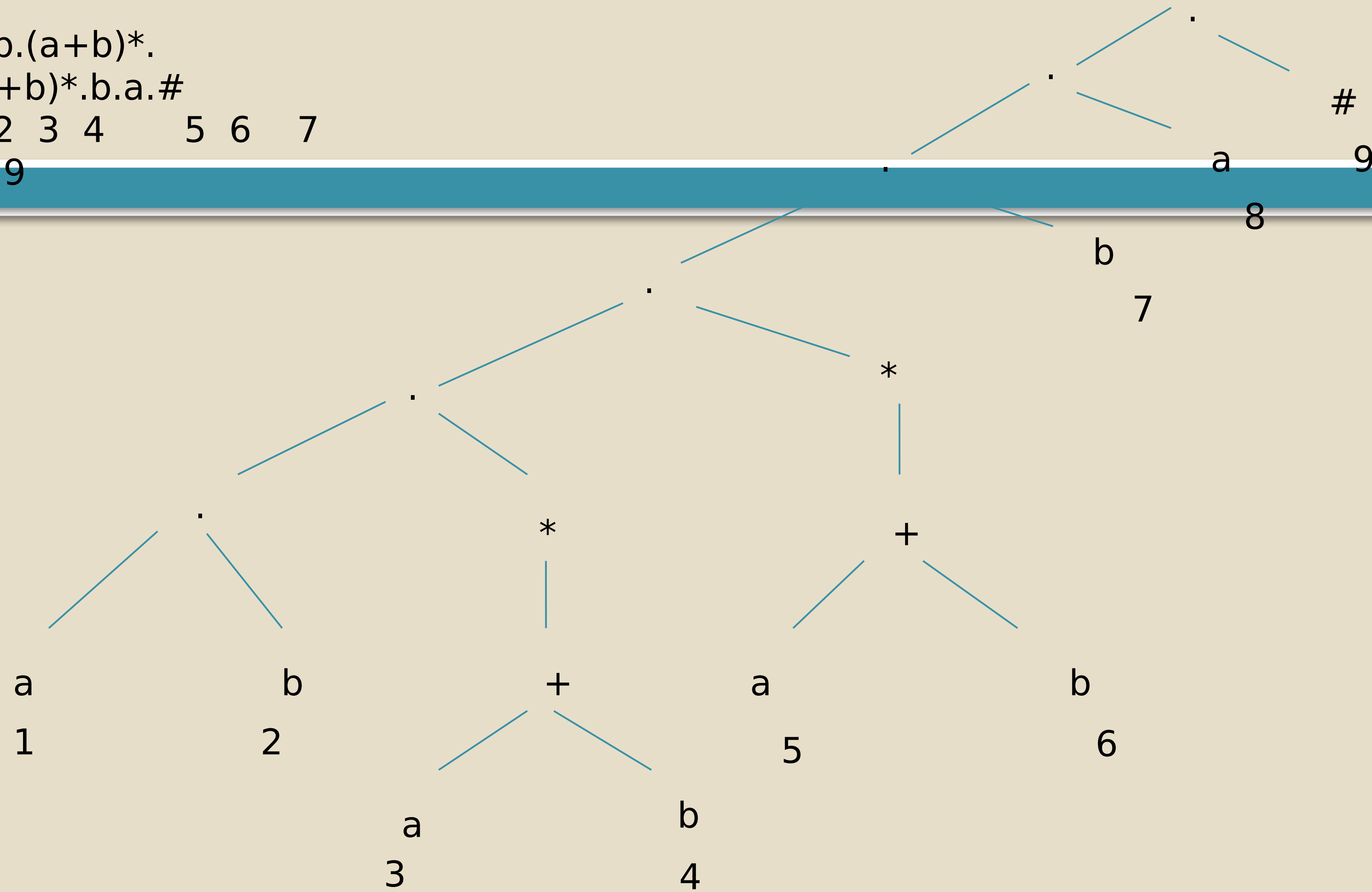
firstpos, lastpos, nullable

- To evaluate ***followpos***, we need three functions to define the nodes (not just for leaves) of the syntax tree:
- **firstpos(n)** → the set of the positions of the **first** symbols of strings generated by the sub-expression rooted by n
- **lastpos(n)** → the set of the positions of the **last** symbols of strings generated by the sub-expression rooted by n
- **nullable(n)** → ***true*** if the empty string is a member of strings generated by the sub-expression rooted by n, ***false*** otherwise

- <u>Rules for calculating **nullable**, **firstpos** & **lastpos**</u>

node <u>n</u>	<u>nullable(n)</u>	<u>firstpos(n)</u>	<u>lastpos(n)</u>
is leaf labeled ε	true	Φ	Φ
is leaf labeled with position i	false	{i} (position of leaf node)	{i}
n $\mid$ c_1 c_2	nullable(c_1) or nullable(c_2)	firstpos(c_1) $\cup$ firstpos(c_2)	lastpos(c_1) $\cup$ lastpos(c_2)
n $\bullet$ c_1 c_2	nullable(c_1) and nullable(c_2)	**if** (nullable(c_1)) **then** firstpos(c_1) $\cup$ firstpos(c_2) **else** firstpos(c_1)	**if** (nullable(c_2)) **then** lastpos(c_1) $\cup$ lastpos(c_2) **else** lastpos(c_2)
n $*$ c_1	true	firstpos(c_1)	lastpos(c_1)

a.b.(a+b)*.
(a+b)*.b.a.#
1 2 3 4 5 6 7
8 9

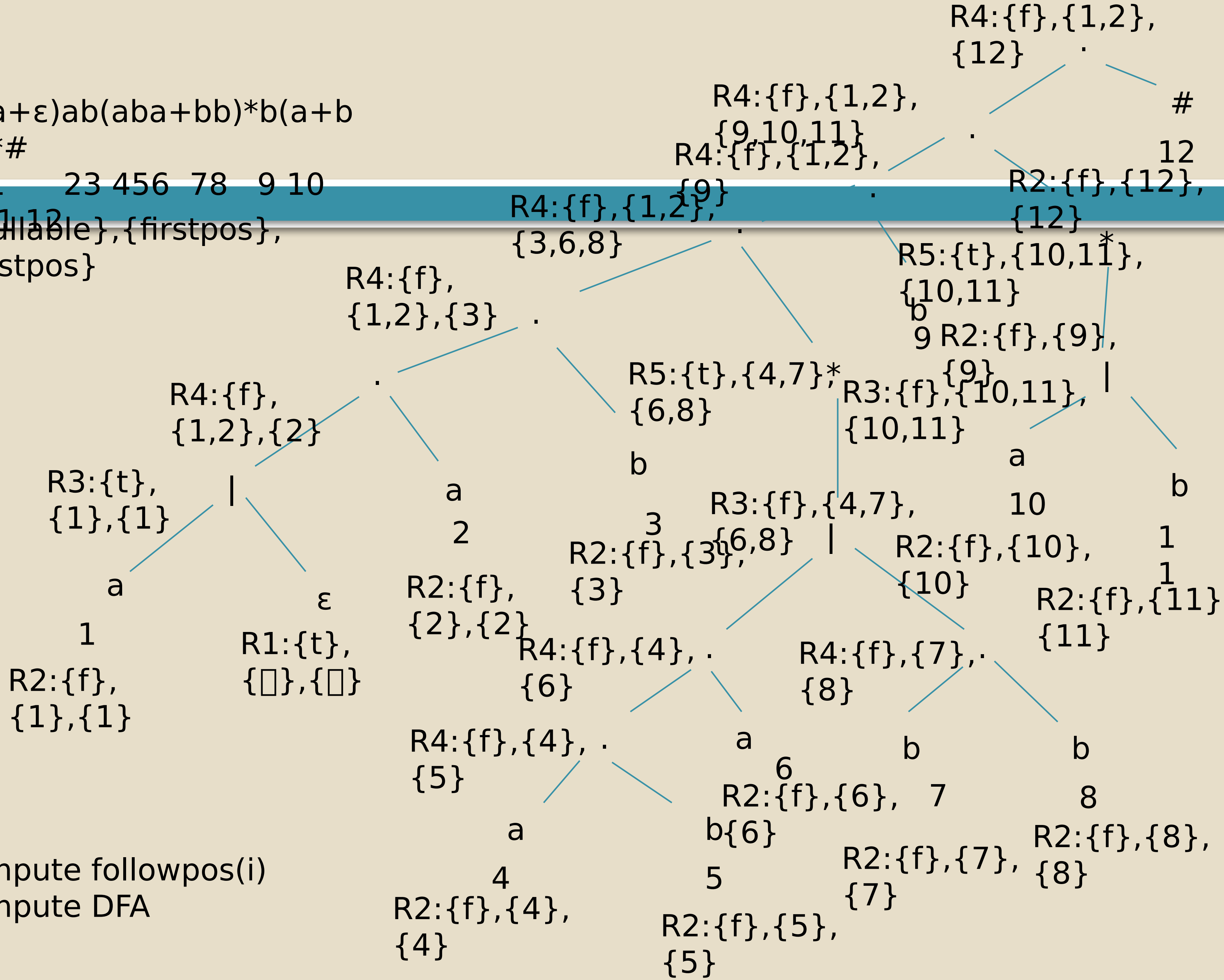


Compute nullable,
firstpos, lastpos
Compute followpos
Construct DFA

(a+ε)ab(aba+bb)*b(a+b)*#

1 23 456 78 9 10 11 12

{nullable},{firstpos},{lastpos}



Compute followpos(i)

Compute DFA

Evaluating **followpos**

for each node *n* in the tree **do**

if *n* is a cat-node with left child *c*1 and right child *c*2

then **for** each *i* in *lastpos*(*c*1) **do**

followpos(*i*) := *followpos*(*i*) ∪ *firstpos*(*c*2)

end do

else if *n* is a star-node

for each *i* in *lastpos*(*n*) **do**

followpos(*i*) := *followpos*(*i*) ∪ *firstpos*(*n*)

end do

end if

end do

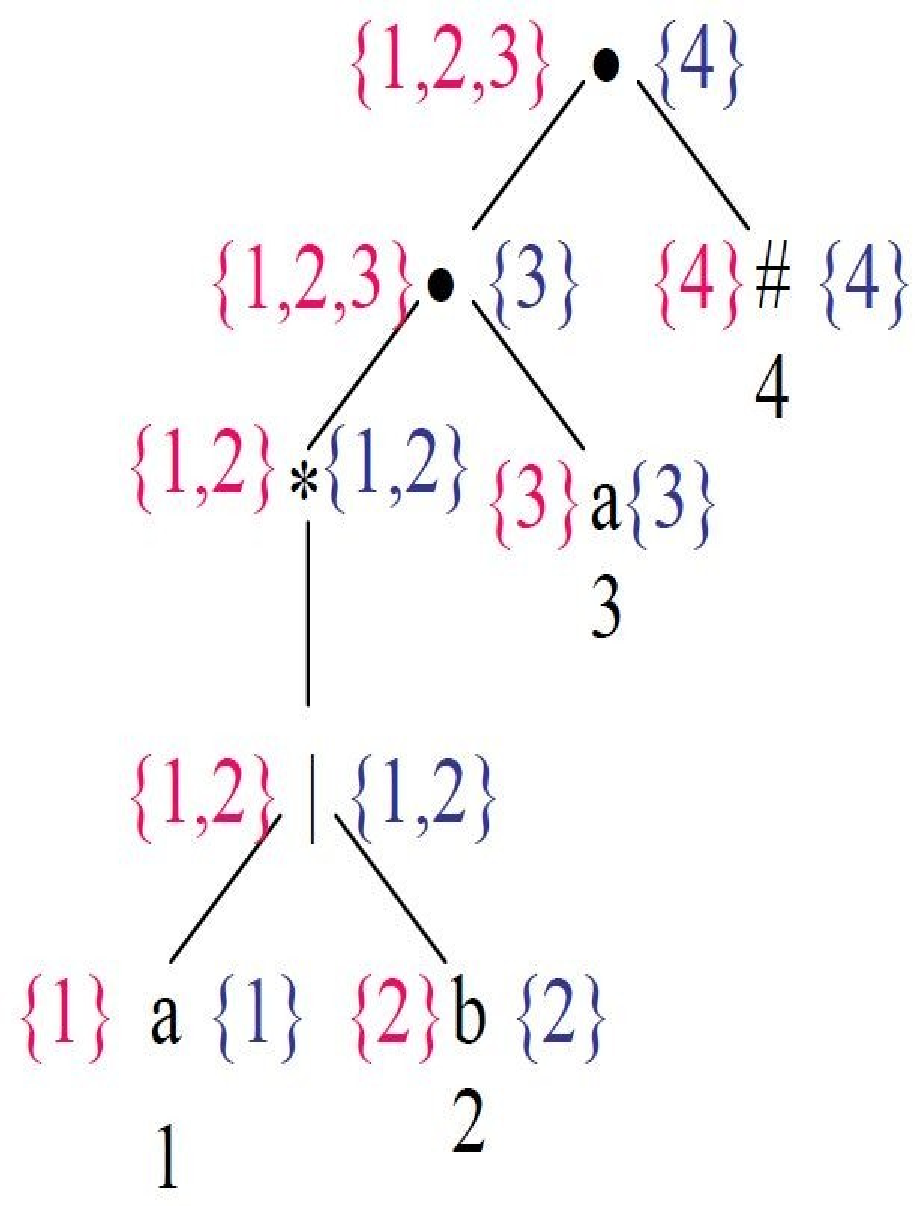
*Compute the **followpos** bottom-up for each node of* **Syntax tree**

followpos

- We define the function **followpos** for the positions (positions assigned to leaves)
- **followpos(i)** → is the set of positions which can follow the position **i** in the strings generated by the augmented regular expression
- For example, (a | b)* a #

  1 2 3 4
- followpos(1) = {1, 2, 3}
- followpos(2) = {1, 2, 3}
- followpos(3) = {4}
- followpos(4) = { }

Evaluating **followpos** example:



red – firstpos

blue – lastpos

Then we can calculate **followpos**

followpos(1) = {1,2,3}

followpos(2) = {1,2,3}

followpos(3) = {4}

followpos(4) = { }

After we calculate **followpos**, we

Algorithm for converting RE to DFA

1. Create the syntax tree of (r)#
2. Calculate the functions: **nullable**, **firstpos**, **lastpos** & **followpos**
3. Put **firstpos(root)** into the states of DFA as an unmarked state

4. *while* (there is an unmarked state S in the states of DFA) *do*

- mark **S**
- *for each* input symbol ***a*** *do*
  - let s1,...,sn are positions in **S** and symbols in those positions is ***a***
  - **S'** ← followpos(s1) ∪ ... ∪ followpos(sn)
  - **move(S, a)** ← **S'**
  - if (**S'** is not empty and not in the states of DFA)
    - put **S'** into the states of DFA as an unmarked state

The start state of DFA is firstpos(root)

The accepting states of DFA are all states containing

Example1

For the RE --- $(\underset{1}{a} \mid \underset{2}{b})^{*} \underset{3}{a} \underset{4}{\#}$

followpos(1)={1,2,3},followpos(2)={1,2,3},followpos(3)={4},followpos(4)={}

S_1=*firstpos(root)*={1,2,3}

mark S_1

for a: followpos(1) $\cup$ *followpos(3)*={1,2,3,4}=S_2 *move*(S_1,*a*)=S_2

for b: followpos(2)={1,2,3}=S_1 *move*(S_1,*b*)=S_1

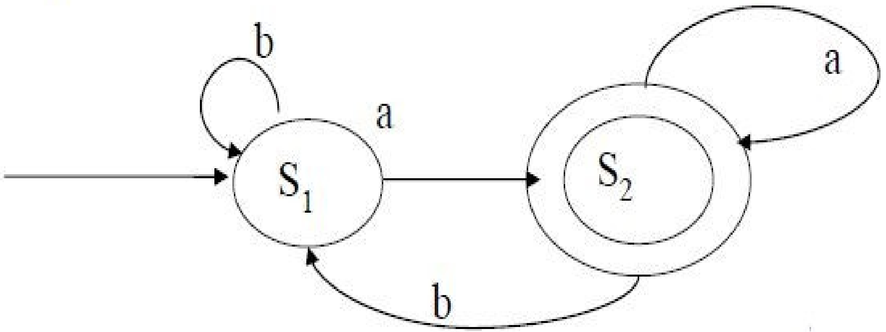
mark S_2

for a: followpos(1) $\cup$ *followpos(3)*={1,2,3,4}=S_2 *move*(S_2,*a*)=S_2

for b: followpos(2)={1,2,3}=S_1 *move*(S_2,*b*)=S_1

start state: S_1

accepting states: {S_2}



Example2

For RE---- $(\underset{1}{a} \mid \varepsilon)\ \underset{2}{b}\ \underset{3}{c}^{*}\ \underset{4}{\#}$

followpos(1)={2}   followpos(2)={3,4}   followpos(3)={3,4}   followpos(4)={}

S_1=firstpos(root)={1,2}

mark S_1

for a: followpos(1)={2}=S_2 move(S_1,a)=S_2

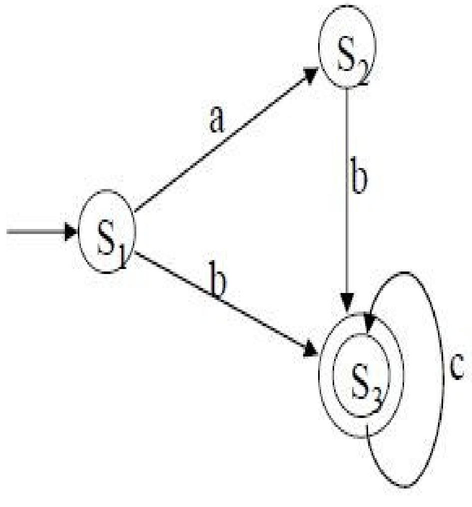
for b: followpos(2)={3,4}=S_3 move(S_1,b)=S_3

mark S_2

for b: followpos(2)={3,4}=S_3 move(S_2,b)=S_3

mark S_3

for c: followpos(3)={3,4}=S_3 move(S_3,c)=S_3



start state: S_1

accepting states: {S_3}

State Minimization in DFA

- DFA minimization refers to the task of transforming a given DFA into an equivalent DFA which has minimum number of states
- Two states **p** and **q** are called **equivalent** if for all input strings **w**, **δ(p, w)** is an accepting state iff **δ(q,w)** is an accepting state
- Otherwise they are called **distinguishable** states

State Minimization in DFA

- String **w** distinguishes state **s** from state **t** if, by starting with DFA **M** in state **s** and feeding it input **w**, we end up in an accepting state, but starting in state **t** and feeding it with same input **w**, we end up in a non accepting state, or vice-versa
- The procedure finds the states that can be distinguished by some input string
- Each group of states that cannot be distinguished is then merged into a single state

State Minimization in DFA

Suppose there is a DFA **D < Q, Σ, q_0, δ, F >** which

recognizes a language **L**. Then the minimized DFA **D <Q', Σ, q_0, δ', F' >** can be constructed for language **L** as:

Step 1: Divide **Q** (set of states) into two sets. One set will contain all final states and the other set will contain all non-final states. This partition is called P_0.

Step 2: Initialize k = 1

State Minimization in DFA

Step 3: Find $\mathbf{P_k}$ by partitioning the different sets of $\mathbf{P_{k-1}}$. In each set of $\mathbf{P_{k-1}}$, take all possible pair of states. If two states of a set are distinguishable, split the states into different sets in $\mathbf{P_k}$.

Step 4: Stop when $\mathbf{P_k} = \mathbf{P_{k-1}}$ (No change in partition)

Step 5: All states of one set are merged into one. No. of states in minimized DFA will be equal to no. of sets in $\mathbf{P_k}$.

State Minimization in DFA

- In addition to the procedure, we also remove the following states from the DFA:
  - *Unreachable State:* *A state that cannot be reached through any transition from any other state in the DFA*
  - *Dead State:* *A* ***non-final*** *state, that when an automata reaches, cannot transit into any other state*

State Minimization in DFA

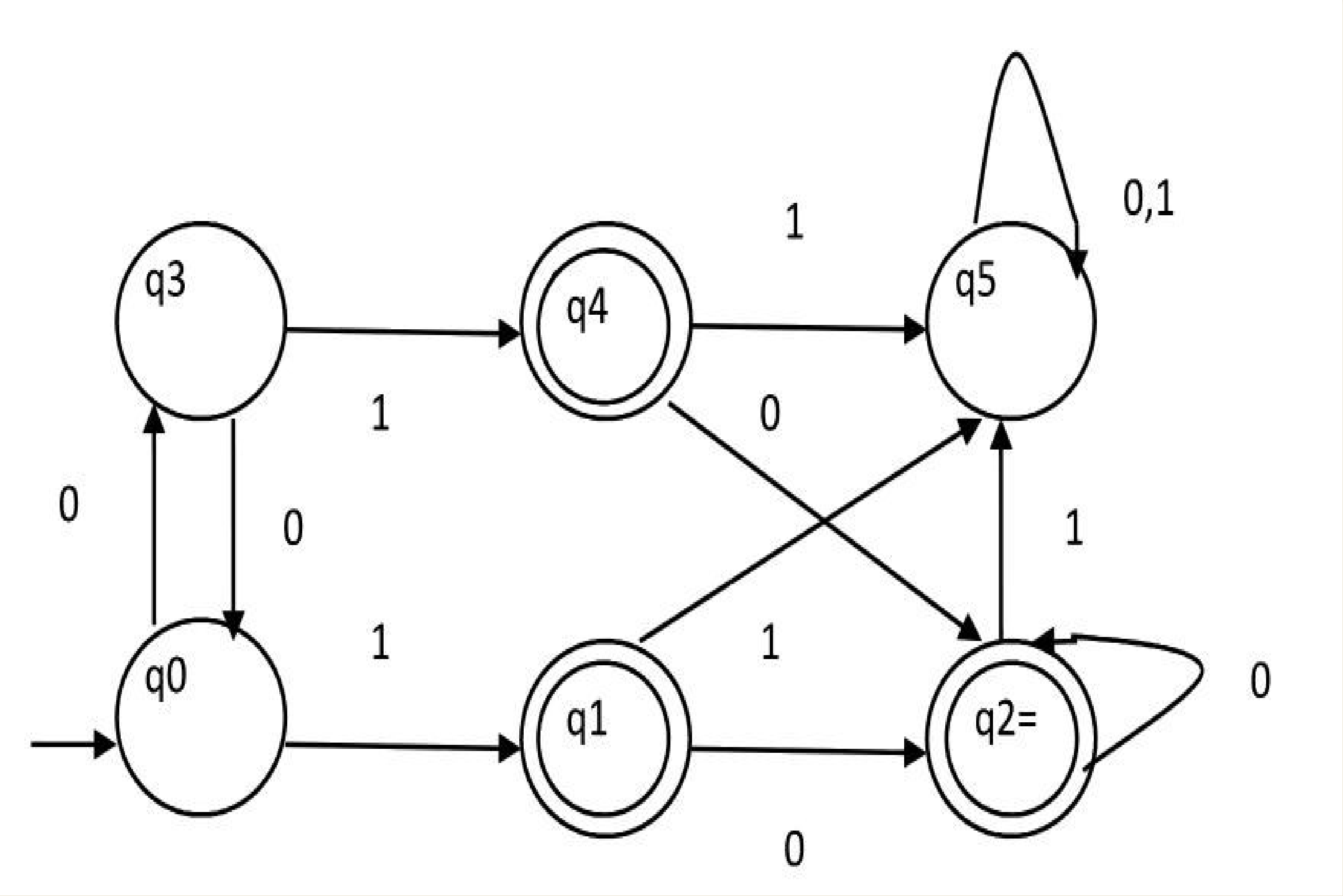
- How to find whether two states in partition $\mathbf{P_k}$ are distinguishable?
  - *Two states **(qi, qj)** are distinguishable in partition* $\boldsymbol{P_k}$ *if for any input symbol **a**, **δ(qi, a)** and **δ(qj, a)** are in different sets in partition* $\boldsymbol{P_{k-1}}$

State Minimization in DFA

- Start state of the minimized DFA is the group containing the start state of the original DFA
- Accepting states of the minimized DFA are the groups containing the accepting states of the original DFA

Example

Consider the following DFA shown in figure.



1. P_0 will have two sets of states. One set will contain q1, q2, q4 which are final states of DFA and another set will contain remaining states. So P_0 = { { q1, q2, q4 }, { q0, q3, q5 } }.

2. To calculate P_1, we will check whether sets of partition P_0 can be partitioned or not:

For set { q1, q2, q4 } :

δ (q1, 0) = δ (q2, 0) = q2 and δ (q1, 1) = δ (q2, 1) = q5, So q1 and q2 are not distinguishable.

Similarly, δ (q1, 0) = δ (q4, 0) = q2 and δ (q1, 1) = δ (q4, 1) = q5, So q1 and q4 are not distinguishable.

Since, q1 and q2 are not distinguishable and q1 and q4 are also not distinguishable, So q2 and q4 are not distinguishable. So, { q1, q2, q4 } set will not be partitioned in P_1.

ii) For set { q0, q3, q5 } :

δ (q0, 0) = q3 and δ (q3, 0) = q0   δ (q0, 1) = q1 and δ(q3, 1) = q4

Moves of q0 and q3 on input symbol 0 are q3 and q0 respectively which are in same set in partition P_0. Similarly, Moves of q0 and q3 on input symbol 1 are q1 and q4 which are in same set in partition P_0. So, q0 and q3 are not distinguishable.

δ (q0, 0) = q3 and δ (q5, 0) = q5 and δ (q0, 1) = q1 and δ (q5, 1) = q5 Moves of q0 and q5 on input symbol 1 are q1 and q5 respectively which are in different sets in partition P_0. So, q0 and q5 are distinguishable. So, set { q0, q3, q5 } will be partitioned into { q0, q3 } and { q5 }. So,

P_1 = { { q1, q2, q4 }, { q0, q3}, { q5 } }

To calculate P_2, we will check whether sets of partition P_1 can be partitioned or not:

iii)For set { q1, q2, q4 } :

δ (q1, 0) = δ (q2, 0) = q2 and δ (q1, 1) = δ (q2, 1) = q5, So q1 and q2 are not distinguishable.

Similarly, δ (q1, 0) = δ (q4, 0) = q2 and δ (q1, 1) = δ (q4, 1) = q5, So q1 and q4 are not distinguishable.

Since, q1 and q2 are not distinguishable and q1 and q4 are also not distinguishable, So q2 and q4 are not distinguishable. So, { q1, q2, q4 } set will not be partitioned in P_2.

iv)For set { q0, q3 } :

δ (q0, 0) = q3 and δ (q3, 0) = q0

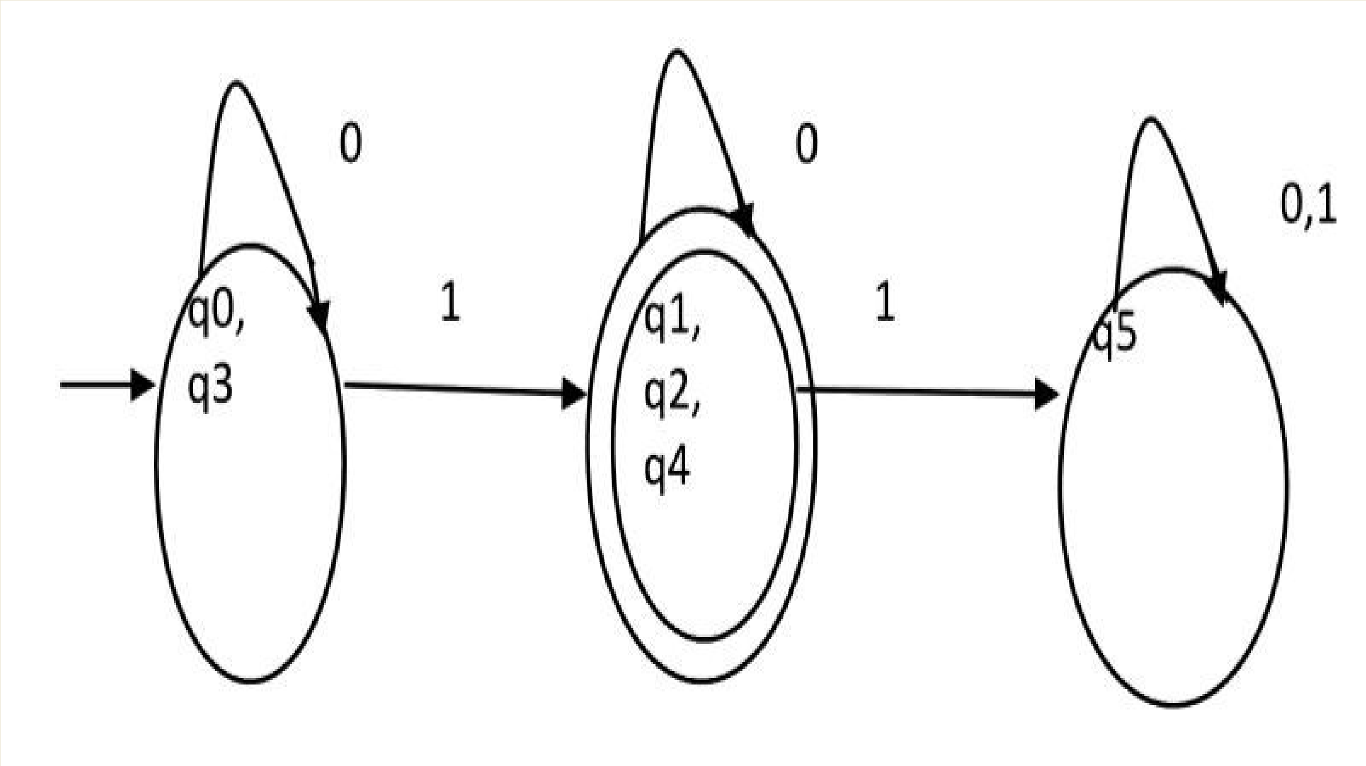
δ (q0, 1) = q1 and δ (q3, 1) = q4

Moves of q0 and q3 on input symbol 0 are q3 and q0 respectively which are in same set in partition P_1. Similarly, Moves of q0 and q3 on input symbol 1 are q1 and q4 which are in same set in partition P_1. So, q0 and q3 are not distinguishable.

v) For set { q5 }:

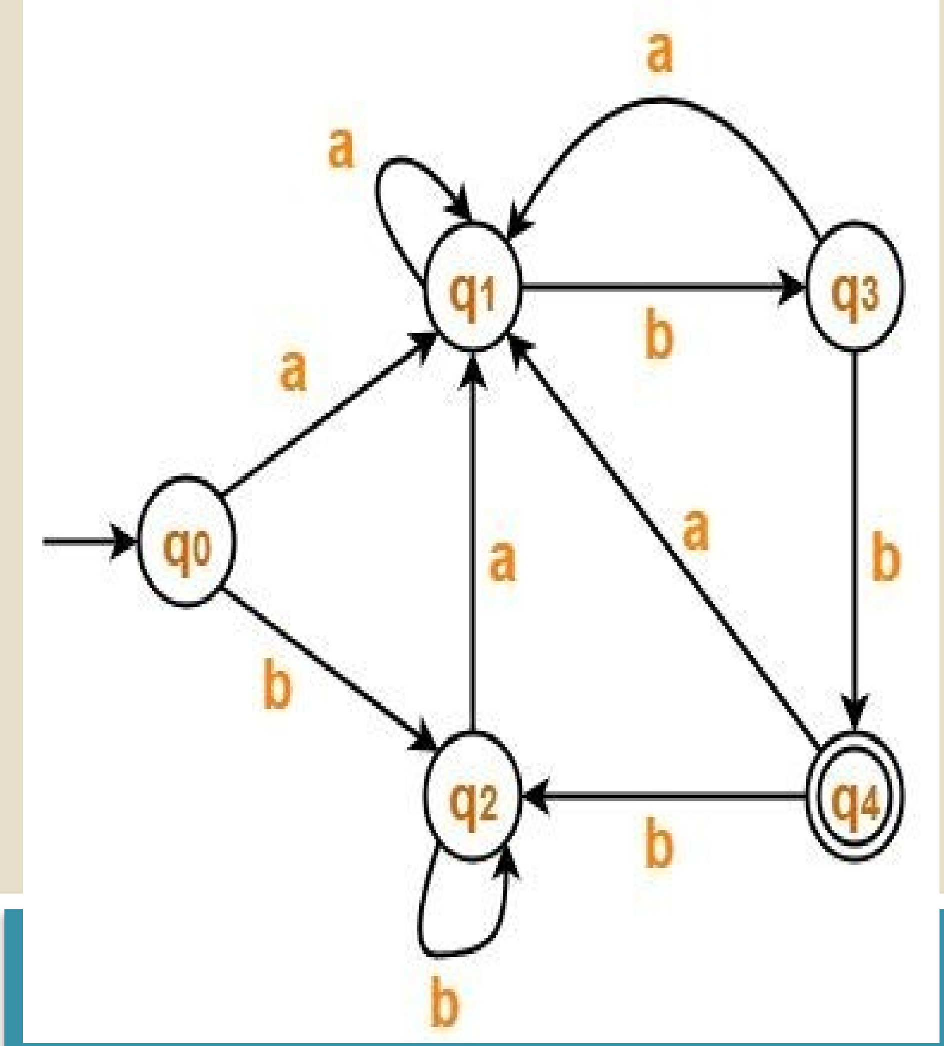
Since we have only one state in this set, it can’t be further partitioned. So, P_2 = { { q1, q2, q4 }, { q0, q3 }, { q5 } }

Here, P_1=P_2. So, this is the final partition. Partition P_2 means that q1, q2 and q4 states are merged into one. Similarly, q0 and q3 are merged into one. Minimized DFA corresponding to the given input DFA is shown below:

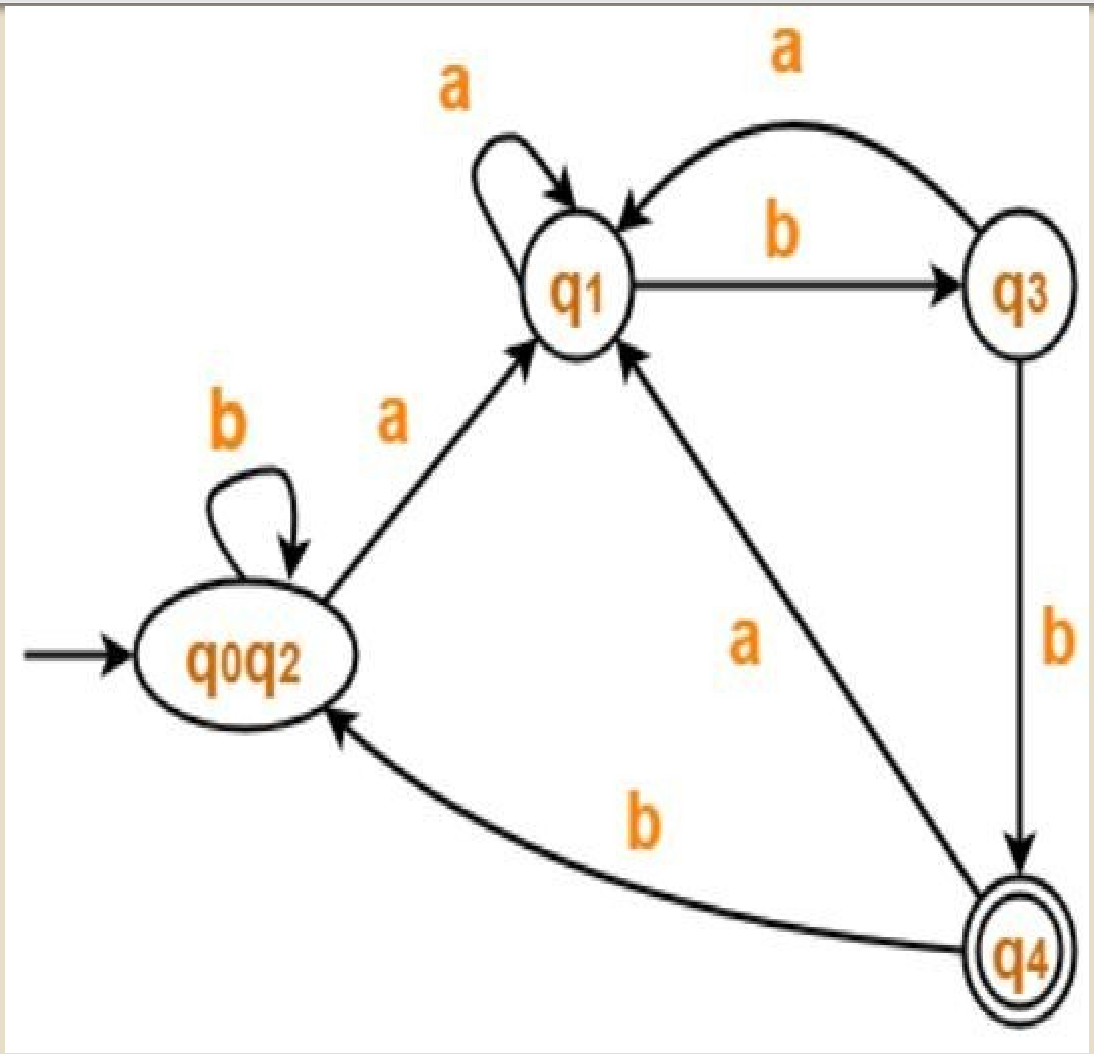


Practice Question

Q) Minimize the following DFA:



An
S:



Summary: Specification and Recognization

Consider the following grammar:

$$
\begin{array}{lcl}
\mathit{stmt} & \rightarrow & \textbf{if } \mathit{expr} \textbf{ then } \mathit{stmt} \\
 & | & \textbf{if } \mathit{expr} \textbf{ then } \mathit{stmt} \textbf{ else } \mathit{stmt} \\
 & | & \epsilon \\
\mathit{expr} & \rightarrow & \mathit{term} \textbf{ relop } \mathit{term} \\
 & | & \mathit{term} \\
\mathit{term} & \rightarrow & \textbf{id} \\
 & | & \textbf{number}
\end{array}
$$

A grammar for branching statements

- Tokens in the above grammar are:

$$
\begin{array}{rcl}
\textit{digit} & \rightarrow & \texttt{[0-9]} \\
\textit{digits} & \rightarrow & \textit{digit}^{+} \\
\textit{number} & \rightarrow & \textit{digits}\ (.\ \textit{digits})?\ (\ \texttt{E}\ \texttt{[+-]}?\ \textit{digits}\ )? \\
\textit{letter} & \rightarrow & \texttt{[A-Za-z]} \\
\textit{id} & \rightarrow & \textit{letter}\ (\ \textit{letter}\ |\ \textit{digit}\ )^{*} \\
\textit{if} & \rightarrow & \texttt{if} \\
\textit{then} & \rightarrow & \texttt{then} \\
\textit{else} & \rightarrow & \texttt{else} \\
\textit{relop} & \rightarrow & \texttt{<}\ |\ \texttt{>}\ |\ \texttt{<=}\ |\ \texttt{>=}\ |\ \texttt{=}\ |\ \texttt{<>}
\end{array}
$$

Patterns for tokens of branching statements

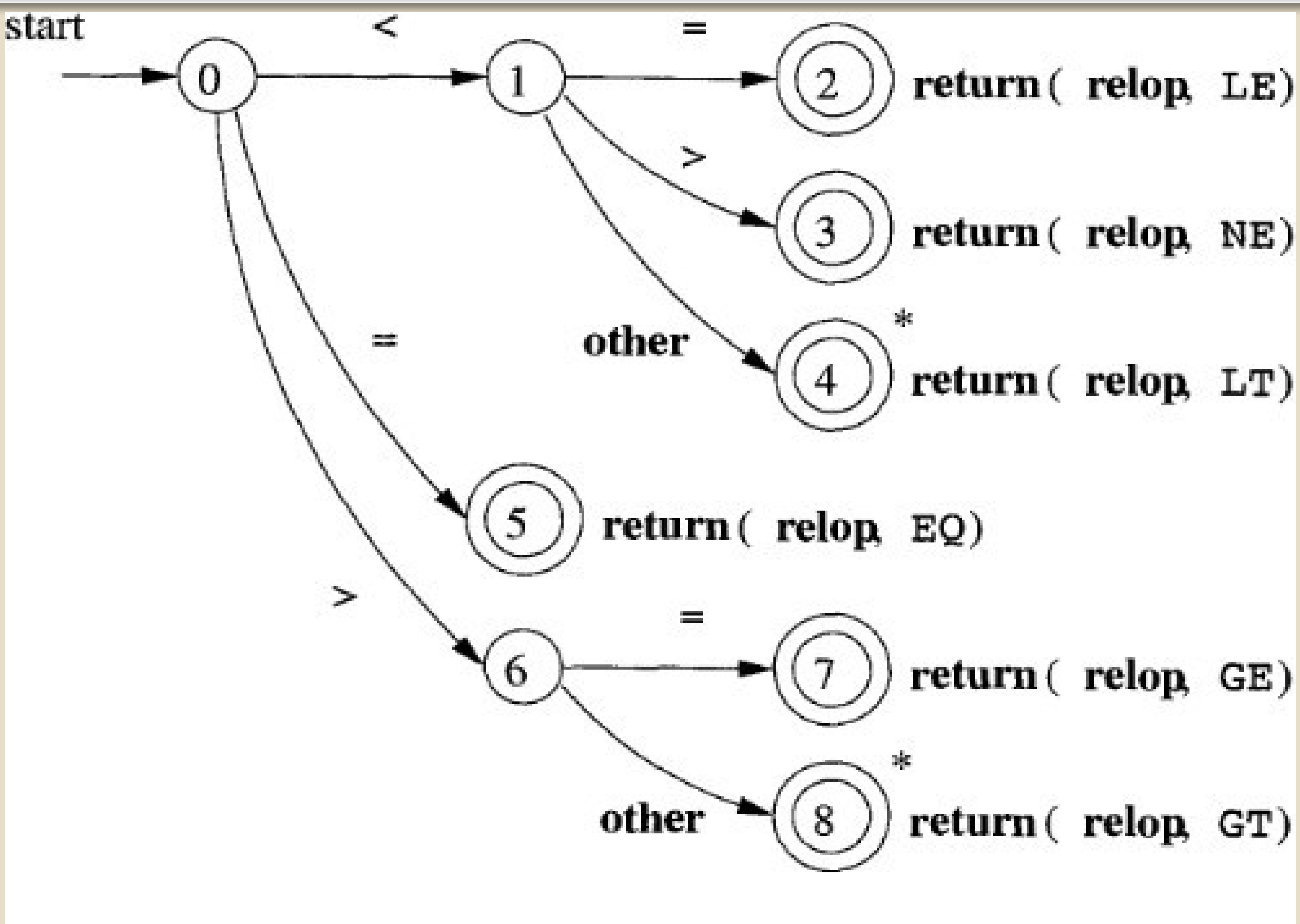
- Whitespace is identified as shown below:

$$
ws \rightarrow (\ \textbf{blank}\ |\ \textbf{tab}\ |\ \textbf{newline}\ )^{+}
$$

REGULAR-EXPRESSION PATTERNS FOR TOKENS

Regular Expression	Token	Attribute-value
ws	--	--
if	**if**	--
then	**then**	--
id	**id**	pointer to table entry
num	**num**	pointer to table entry
<	**relop**	LT
<=	**relop**	LE
=	**relop**	EQ
<>	**relop**	NE
>	**relop**	GT
>=	**relop**	GE

In the construction of lexical analyzer we first convert the patterns into flowcharts called “transition diagrams”.

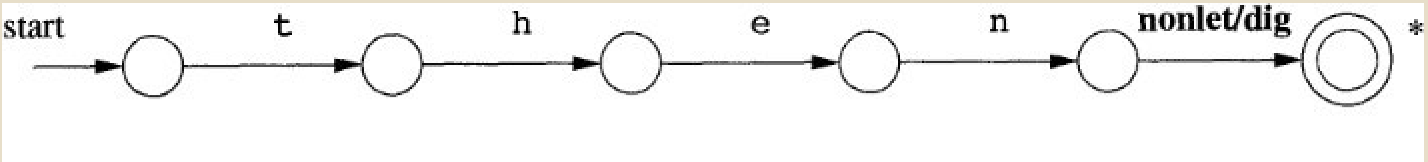


Transition diagram for **relop**

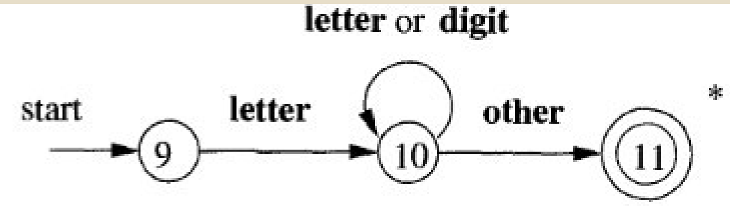
Implementation of Transition diagram


```
TOKEN getRelop()
{
    TOKEN retToken = new(RELOP);
    while(1) { /* repeat character processing until a return
                  or failure occurs */
        switch(state) {
            case 0: c = nextChar();
                    if ( c == '<' ) state = 1;
                    else if ( c == '=' ) state = 5;
                    else if ( c == '>' ) state = 6;
                    else fail(); /* lexeme is not a relop */
                    break;
            case 1: ...
            ...
            case 8:
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```

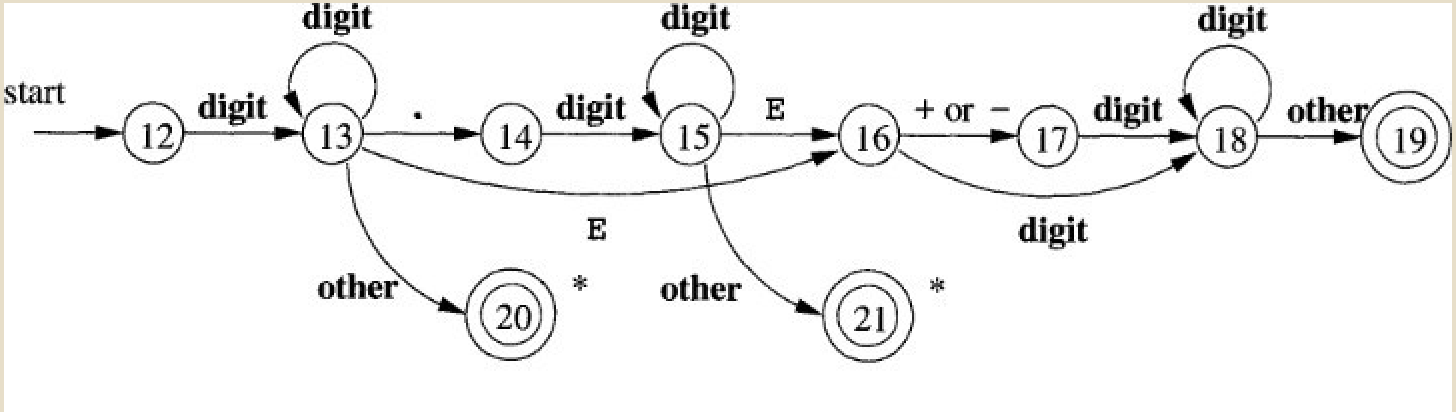
Sketch of implementation of **relop** transition diagram



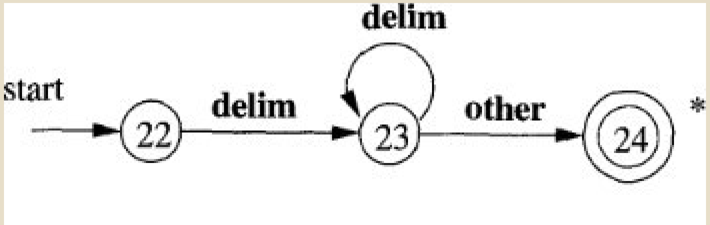
Hypothetical transition diagram for the keyword **then**



A transition diagram for **id**'s and keywords



A transition diagram for unsigned numbers



A transition diagram for whitespace