Data Analysis: Assignment 04

1. The following data relate x, the moisture of a wet mix of a certain product, to Y, the density of the finished product.

x_i	Y_i
5	7.4
6	9.3
7	10.6
10	15.4
12	18.1
15	22.2
18	24.1
20	24.8

   a) Draw a scatter diagram.
   b) Fit a linear curve to the data (i.e., estimate α and β for a simple linear regression $y = \alpha + \beta x$).

2. The following data relate the number of units of a good that were ordered as a function of the price of the good at six different locations.

Number ordered	88	112	123	136	158	172
Price	50	40	35	30	20	15

   How many units do you think would be ordered if the price were 25?

3. An individual claims that the fuel consumption of his automobile does not depend on how fast the car is driven. To test the plausibility of this hypothesis, the car was tested at various speeds between 45 and 70 miles per hour. The miles per gallon attained at each of these speeds was determined, with the following data resulting:

Speed	Miles per Gallon
45	24.2
50	25.0
55	23.3
60	22.0
65	21.5
70	20.6
75	19.8

This problem is about refuting the claim that the mileage per gallon of gas is unaffected by the speed at which the car is being driven, using the above data.

a) Fit a linear relation (i.e., estimate α and β for a simple linear regression $y = \alpha + \beta x$).
b) When using the least square estimator of α and β, we observed the followings:
The t-statistic (or v-value) for testing that $\beta = 0$ is -8.138 and its p-value is 0.000455 while the t-statistic (or v-value) and p-value for testing that $\alpha = 0$ are 25.612 and 1.69×10^{-6}, respectively. How can we conclude?

4. **[Textbook Exercise 3.7: Problem 6]** Using textbook equation (3.4), which is also given below, argue that in the case of simple linear regression, the least squares line always passes through the point $(\bar{x}, \bar{y})$.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$