1.

$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3$

$f_1(x) = a_1 + b_1 x + c_1 x^2 + d_1 x^3$

, $f'(\xi) = f_1'(\xi)$, $f''(\xi) = f''(\xi)$

1) $f(\xi) = f_1(\xi)$

$a_1 + b_1\xi + c_1\xi^2 + d_1\xi^3 = \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3$

2) $f'(\xi) = f_1'(\xi)$

⋮

$a_1 = \beta_0 + \beta_1\xi - \beta_2\xi^2 - \beta_3\xi^3 + \beta_4\xi^3$

$b_1 = \beta_1 + 3\beta_3\xi^2 - \beta_4\xi^3$

$c_1 = (3\beta_3 - 2\beta_4\xi)/\xi^2$

$d_1 = (\beta_4\xi - 2\beta_3)/\xi^3$

b) Same as 1)

$a_2 = \beta_0 - \beta_1\xi - \beta_2\xi^2 - [(3\beta_3 - 3\beta_4\xi^2)/\xi^2]\xi^2 - [(\beta_0 - \beta_1\xi - \beta_2\xi^2)/\xi^3 - \beta_3]\xi^3$

$b_2 = \beta_1 - 2\beta_2\xi - 3\beta_3\xi^2 + [(6\beta_3 - 6\beta_4\xi^2)/\xi]$

$c_2 = (3\beta_3 - 3\beta_4\xi^2)/\xi^2$

$d_2 = (\beta_0 - a_2 - b_2\xi - c_2\xi^2)/\xi^3 + \beta_3$

c)

$f_1(\xi) = f_2(\xi)$

$\Rightarrow \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 + \beta_4(\xi-\xi)^3 = a_2 + b_2\xi + c_2\xi^2 + d_2\xi^3$

$\beta_3 = 6d_2$ $\beta_1 = b_2$ $a_2 = \beta_0$

$\beta_2 = 2c_2$ $\beta = 6d_2$.

d) Same as C

e)

4. $\hat{\beta}_0 = 1$, $\hat{\beta}_1 = 1$, $\hat{\beta}_2 = 3$. $-2 < x < 6$