

## 1) (2 Pts): State Amdahl's Law in words

Answer: Amdahl's Law states that the overall speedup of a system is limited by the portion of the system that cannot be improved. Specifically, the maximum speedup achievable is determined by the fraction of the execution time that is affected by the enhancement. The law shows that focusing on improving the most time-consuming parts of a system yields the greatest overall performance gains.

Mathematical form:  $\text{Speedup} = 1 / [(1 - f) + f/s]$  where  $f$  = fraction of execution time affected by enhancement,  $s$  = speedup of the enhanced portion

## 2) (4 Pts): I/O System Speedup

Given:

- 65% of time spent on I/O
- Disk upgrade provides 50% greater throughput (1.5x speedup)

Solution: Using Amdahl's Law:

- $f = 0.65$  (fraction using I/O)
- $s = 1.5$  (I/O speedup factor)

$\text{Speedup} = 1 / [(1 - 0.65) + 0.65/1.5]$   $\text{Speedup} = 1 / [0.35 + 0.433]$   $\text{Speedup} = 1 / 0.783$   $\text{Speedup} = 1.28x$  (28% improvement)

## 3) (8 Pts): 30% Faster Servers

Given:

- Need 30% overall speedup (1.3x)
- 70% CPU time, 30% I/O time

### Part a: CPU Speedup Required

Using Amdahl's Law for CPU enhancement:  $1.3 = 1 / [(1 - 0.7) + 0.7/s_{\text{cpu}}]$   $1.3 = 1 / [0.3 + 0.7/s_{\text{cpu}}]$   $0.3 + 0.7/s_{\text{cpu}} = 1/1.3 = 0.769$   $0.7/s_{\text{cpu}} = 0.769 - 0.3 = 0.469$   $s_{\text{cpu}} = 0.7/0.469 = 1.49$

CPU needs to be 1.49x faster (49% improvement)

### Part b: Disk Speedup Required

Using Amdahl's Law for I/O enhancement:  $1.3 = 1 / [(1 - 0.3) + 0.3/s_{\text{disk}}]$   $1.3 = 1 / [0.7 + 0.3/s_{\text{disk}}]$   $0.7 + 0.3/s_{\text{disk}} = 0.769$   $0.3/s_{\text{disk}} = 0.069$   $s_{\text{disk}} = 0.3/0.069 = 4.35$

Disk needs to be 4.35x faster (335% improvement)

### 4) (4 Pts): Musical Instrument Processor

Given:

- Need 12% performance boost (1.12x speedup)
- Processor accounts for 25% of workload

Solution: Using Amdahl's Law:  $1.12 = 1 / [(1 - 0.25) + 0.25/s_{\text{proc}}]$   $1.12 = 1 / [0.75 + 0.25/s_{\text{proc}}]$   $0.75 + 0.25/s_{\text{proc}} = 1/1.12 = 0.893$   $0.25/s_{\text{proc}} = 0.893 - 0.75 = 0.143$   $s_{\text{proc}} = 0.25/0.143 = 1.75$

New processor needs to be 1.75x faster (75% improvement)

### 5) (16 Pts): Cost-Performance Analysis

Given:

- 60% CPU, 40% disk activity
- Disk upgrade: 2.5x speedup for \$8,000
- CPU upgrade: 1.4x speedup for \$5,000

#### Part a: Best Performance per Dollar

Disk upgrade speedup:  $\text{Speedup} = 1 / [0.6 + 0.4/2.5] = 1 / [0.6 + 0.16] = 1.32x$  Cost per 1% improvement =  $\$8,000 / 32\% = \$250$  per 1%

CPU upgrade speedup:  $\text{Speedup} = 1 / [0.4 + 0.6/1.4] = 1 / [0.4 + 0.429] = 1.21x$  Cost per 1% improvement =  $\$5,000 / 21\% = \$238$  per 1%

Answer: CPU upgrade provides better performance per dollar

#### Part b: Best Performance (Money No Object)

Disk upgrade provides 1.32x speedup vs CPU's 1.21x speedup. Answer: Choose disk upgrade for maximum performance

### Part c: Break-even Point

For equal cost per 1% improvement: Disk:  $\$8,000 / 32\% = \$250$  per 1% To match this, CPU price should be:  $21\% \times \$250 = \$5,250$

Answer: CPU should cost \$5,250 for break-even

### Part d: Changed Activity (55% CPU, 45% disk)

Disk upgrade speedup:  $\text{Speedup} = 1 / [0.55 + 0.45/2.5] = 1 / [0.55 + 0.18] = 1.37x$

CPU upgrade speedup:  $\text{Speedup} = 1 / [0.45 + 0.55/1.4] = 1 / [0.45 + 0.393] = 1.19x$

Answer: Disk upgrade still provides better absolute performance, and CPU still provides better cost-performance ratio

## 6) (8 Pts): Disk Drive Analysis #1

Given:

- 6 surfaces, 16,383 tracks/surface, 63 sectors/track, 512 bytes/sector
- 8.5ms seek time, 7,200 rpm

Part a: Capacity

Capacity =  $6 \times 16,383 \times 63 \times 512$  bytes Capacity = 3,145,521,216 bytes Capacity = 3.15 GB

Part b: Access Time

Rotational latency =  $(60 \text{ sec} / 7,200 \text{ rpm}) / 2 = 4.17 \text{ ms}$  Access time = Seek time + Rotational latency Access time =  $8.5 + 4.17 = 12.67 \text{ ms}$

## 7) (12 Pts): Disk Drive Analysis #2

Given:

- 6 surfaces, 953 tracks/surface, 256 sectors/track, 512 bytes/sector
- 6.5ms seek time, 5,400 rpm

Part a: Capacity

Capacity =  $6 \times 953 \times 256 \times 512$  bytes = 754,974,720 bytes = 755 MB

### Part b: Access Time

Rotational latency =  $(60 \text{ sec} / 5,400 \text{ rpm}) / 2 = 5.56 \text{ ms}$  Access time =  $6.5 + 5.56 = 12.06 \text{ ms}$

### Part c: Speed Comparison

6 disk: 12.67 ms access time 7 disk: 12.06 ms access time 7 disk is slightly faster (by 0.61 ms) but has much smaller capacity

## 8) (4 Pts): Instruction Encoding

Given:

- 32-bit instructions, 12-bit addresses
- 250 two-address instructions

Solution: Two-address instruction format: [opcode][addr1][addr2] Bits needed: opcode + 12 + 12 = opcode + 24 Available for opcode:  $32 - 24 = 8$  bits Maximum opcodes with 8 bits:  $2^8 = 256$  Used for two-address: 250 Remaining:  $256 - 250 = 6$

One-address instruction format: [opcode][addr1] For remaining opcodes, we can use longer opcode fields. Since we have 6 remaining opcode values, and each can represent multiple one-address instructions:

Maximum one-address instructions =  $6 \times 2^{(32-12-8)} = 6 \times 2^{12} = 24,576$

## 9) (4 Pts): Addressing Modes

Given: Load 100, R1 = 0x300, Memory values as shown

| Mode      | Value loaded into AC                       |
|-----------|--|
| Mode      | Value loaded into AC                       |
| Immediate | 100 (0x64)                                 |
| Direct    | 0x600 (value at address 100)               |
| Indirect  | 0x300 (value at address pointed to by 100) |
| Indexed   | 0x500 (value at address $100 + R1 = 400$ ) |

## 10) (6 Pts): Expanding Opcodes

Given: 16-bit instructions, 32 registers (5 bits each)

- 60 instructions with two register operands
- 30 instructions with one register operand
- 3 instructions with one 10-bit address
- 26 instructions with zero operands

Analysis: Total instructions needed:  $60 + 30 + 3 + 26 = 119$

Format 1 (2 registers): [6-bit opcode][5-bit reg][5-bit reg] = 16 bits  
Format 2 (1 register): [6-bit opcode][5-bit reg] = 11 bits

Format 3 (10-bit addr): [6-bit opcode][10-bit addr] = 16 bits  
Format 4 (0 operands): [6-bit opcode] = 6 bits

Encoding scheme:

- Opcodes 000000-111011 (60 codes): Two register format
- Opcodes 111100 xxxxx (32 codes): One register format (uses 30)
- Opcodes 111101 (1 code): 10-bit address format (uses 3 sub-codes)
- Opcodes 111110-111111: Zero operand format (can encode 26)

Answer: Yes, this encoding is possible

## 11) (8 Pts): Pipeline Analysis #1

Given:

- Non-pipelined: 200ns per task
- Pipeline: 5 segments, 40ns clock cycle

Part a: Speedup for 200 tasks

Non-pipelined time:  $200 \times 200\text{ns} = 40,000\text{ns}$   
Pipeline time:  $(5-1) \times 40\text{ns} + 200 \times 40\text{ns} = 160\text{ns} + 8,000\text{ns} = 8,160\text{ns}$   
Speedup =  $40,000 / 8,160 = 4.9x$

Part b: Theoretical Maximum Speedup

Maximum speedup =  $200\text{ns} / 40\text{ns} = 5x$

## 12) (8 Pts): Pipeline Analysis #2

Given:

- Non-pipelined: 100ns per task
- Pipeline: 5 stages, 20ns clock cycle

### **Part a: Speedup for 100 tasks**

Non-pipelined time:  $100 \times 100\text{ns} = 10,000\text{ns}$  Pipeline time:  $(5-1) \times 20\text{ns} + 100 \times 20\text{ns} = 80\text{ns} + 2,000\text{ns} = 2,080\text{ns}$  Speedup =  $10,000 / 2,080 = 4.8x$

### **Part b: Theoretical Maximum Speedup**

Maximum speedup =  $100\text{ns} / 20\text{ns} = 5x$

## **13) (8 Pts): Memory and Instruction Format**

Given:

- 24 bits per word
- 150 different operations
- One address per instruction

### **Part a: Opcode Bits**

Bits needed =  $\lceil \log_2(150) \rceil = 8$  bits

### **Part b: Address Bits**

Address bits =  $24 - 8 = 16$  bits

### **Part c: Maximum Memory Size**

Maximum memory =  $2^{16} = 65,536$  words

### **Part d: Largest Unsigned Binary Number**

Largest number =  $2^{24} - 1 = 16,777,215$

## **14) (8 Pts): Computer Instruction Format**

Given:

- 256K words of 32 bits each
- 7 addressing modes, 60 registers
- 32-bit instructions

**Part a: Mode Field Size**

Mode field =  $\lceil \log_2(7) \rceil = 3$  bits

**Part b: Register Field Size**

Register field =  $\lceil \log_2(60) \rceil = 6$  bits

**Part c: Address Field Size**

Address field =  $\lceil \log_2(256K) \rceil = 18$  bits

**Part d: Opcode Field Size**

Opcode field =  $32 - 3 - 6 - 18 = 5$  bits