12 Days of Christmas Countdown Counter Design

Problem Analysis

We need to design a 4-bit counter that:

- Counts down from 12 (1100_2) to 1 (0001_2)
- Stays at 1 until Start pulse resets to 12
- Uses negative-edge triggered J-K flip-flops
- Uses asynchronous Preset/Clear for restart functionality

Part A: State Transition Table (Without Asynchronous Restart)

Let's define our states using 4 bits: $Q_3Q_2Q_1Q_0$

Current State	Q_3	Q_2	Q_1	Q_0	Decimal	Next State	Q_3^+	Q_2^+	Q_1^+	Q_0^+	Decimal$^+$
12	1	1	0	0	12	11	1	0	1	1	11
11	1	0	1	1	11	10	1	0	1	0	10
10	1	0	1	0	10	9	1	0	0	1	9
9	1	0	0	1	9	8	1	0	0	0	8
8	1	0	0	0	8	7	0	1	1	1	7
7	0	1	1	1	7	6	0	1	1	0	6
6	0	1	1	0	6	5	0	1	0	1	5
5	0	1	0	1	5	4	0	1	0	0	4
4	0	1	0	0	4	3	0	0	1	1	3
3	0	0	1	1	3	2	0	0	1	0	2
2	0	0	1	0	2	1	0	0	0	1	1
1	0	0	0	1	1	1 (stay)	0	0	0	1	1

Unused States (Don't Care Conditions):

- $0000_2 = 0$
- $1101_2 = 13$
- $1110_2 = 14$
- $1111_2 = 15$

These will be treated as don't care (X) conditions in our Karnaugh maps.

Part B: J-K Flip-Flop Input Determination

For J-K flip-flops, the characteristic equation is:

- $J = 1, K = 0 \rightarrow Q^+ = 1$ (Set)
- $J = 0, K = 1 \rightarrow Q^+ = 0$ (Reset)
- $J = 1, K = 1 \rightarrow Q^+ = \bar{Q}$ (Toggle)
- $J = 0, K = 0 \rightarrow Q^+ = Q$ (Hold)

J-K Input Truth Table for Each Flip-Flop

Q	**Q^+**	**J**	**K**
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

For Flip-Flop Q_3 (MSB):

$Q_3Q_2Q_1Q_0$	**Q_3**	**Q_3^+**	**J_3**	**K_3**
1100	1	1	X	0
1011	1	1	X	0
1010	1	1	X	0
1001	1	1	X	0
1000	1	0	X	1
0111	0	0	0	X
0110	0	0	0	X
0101	0	0	0	X
0100	0	0	0	X
0011	0	0	0	X
0010	0	0	0	X
0001	0	0	0	X

Karnaugh Map for J_3:

```
    Q1Q0
Q3Q2  00  01  11  10
00    X   0   0   0
01    0   0   0   0
11    X   X   X   X
10    X   X   X   X
```

Karnaugh Map for K_3:

```
    Q1Q0
Q3Q2  00  01  11  10
00    X   X   X   X
01    X   X   X   X
11    X   X   X   X
10    1   X   X   X
```

J_3 = 0 (always 0) **$K_3 = \bar{Q}_2\bar{Q}_1\bar{Q}_0$** (only high when $Q_2Q_1Q_0$ = 000, i.e., state 1000_2 = 8)

For Flip-Flop Q_2:

$Q_3Q_2Q_1Q_0$	Q_2	Q_2^+	J_2	K_2
1100	1	0	X	1
1011	0	0	0	X
1010	0	0	0	X
1001	0	0	0	X
1000	0	1	1	X
0111	1	1	X	0
0110	1	1	X	0
0101	1	0	X	1
0100	1	1	X	0
0011	0	0	0	X
0010	0	0	0	X
0001	0	0	0	X

Karnaugh Map for J_2:

```
    Q1Q0
Q3Q2  00  01  11  10
00    X   0   0   0
01    X   0   0   0
11    X   X   X   X
10    1   X   X   X
```

Karnaugh Map for K_2:

```
    Q1Q0
Q3Q2  00  01  11  10
00    X   X   X   X
01    0   1   X   0
11    1   X   X   X
10    X   X   X   X
```

$\mathbf{J_2 = Q_3\bar{Q}_1\bar{Q}_0}$ (high when in state $1000_2 = 8$) $\mathbf{K_2 = Q_3\bar{Q}_1\bar{Q}_0 + \bar{Q}_3Q_1\bar{Q}_0}$ (high when in states $1100_2 = 12$ or $0101_2 = 5$)

For Flip-Flop Q_1:

$Q_3Q_2Q_1Q_0$	Q_1	Q_1^+	J_1	K_1
1100	0	1	1	X
1011	1	1	X	0
1010	1	0	X	1
1001	0	0	0	X
1000	0	1	1	X
0111	1	1	X	0
0110	1	0	X	1
0101	0	0	0	X
0100	0	1	1	X
0011	1	1	X	0
0010	1	0	X	1
0001	0	0	0	X

Karnaugh Map for J_1:

```
    Q1Q0
Q3Q2 00 01 11 10
00   X  0  X  1
01   1  0  X  X
11   1  X  X  X
10   1  X  X  X
```

Karnaugh Map for K_1:

```
    Q1Q0
Q3Q2 00 01 11 10
00   X  X  0  X
01   X  X  0  1
11   X  X  X  1
10   X  X  0  1
```

$J_1 = \bar{Q}_2\bar{Q}_0 + Q_3\bar{Q}_0$ $K_1 = Q_0$

For Flip-Flop Q_0 (LSB):

$Q_3Q_2Q_1Q_0$	Q_0	Q_0^+	J_0	K_0
1100	0	1	1	X
1011	1	0	X	1
1010	0	1	1	X
1001	1	0	X	1
1000	0	1	1	X
0111	1	0	X	1
0110	0	1	1	X
0101	1	0	X	1
0100	0	1	1	X
0011	1	0	X	1
0010	0	1	1	X
0001	1	1	X	0

Karnaugh Map for J_0:

Q_1Q_0

Q_3Q_2	00	01	11	10
00	X	X	X	1
01	1	X	X	1
11	1	X	X	1
10	1	X	X	1

Karnaugh Map for K_0:

Q_1Q_0

Q_3Q_2	00	01	11	10
00	X	X	1	X
01	X	1	1	X
11	X	1	X	X
10	X	1	X	X

$J_0 = \bar{Q}_0$ (always complement when $Q_0 = 0$) **$K_0 = \bar{Q}_3 + Q_2 + Q_1$** (complement except when in state $0001_2 = 1$)

Final Simplified Expressions:

$J_3 = 0$ $K_3 = \bar{Q}_2\bar{Q}_1\bar{Q}_0$

$J_2 = Q_3\bar{Q}_1\bar{Q}_0$
$K_2 = Q_3\bar{Q}_1\bar{Q}_0 + \bar{Q}_3Q_1\bar{Q}_0$

$J_1 = \bar{Q}_2\bar{Q}_0 + Q_3\bar{Q}_0$ $K_1 = Q_0$

$J_0 = \bar{Q}_0$ $K_0 = \bar{Q}_3 + Q_2 + Q_1$

Asynchronous Restart Implementation:

To implement the Start pulse functionality:

- Connect Start signal to asynchronous PRESET inputs of Q_3 and Q_2
- Connect Start signal to asynchronous CLEAR inputs of Q_1 and Q_0
- This will immediately set the counter to $1100_2 = 12$ when Start is pulsed