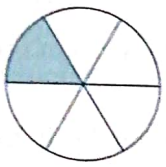
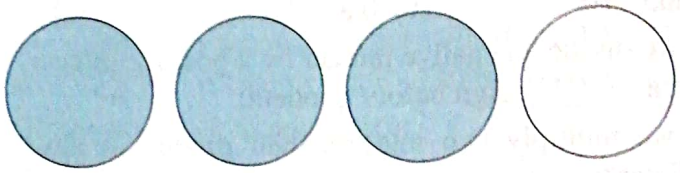
2 Fractions

Fractions are a part of a whole. This whole can be either one unit or a combination of many units. Fraction can also be considered as the ratio of two entities, like ratio of the number of boys to the number of girls in the class.



$\frac{1}{6}$



$\frac{3}{4}$

In fractions we have studied about proper, improper and mixed fractions, their addition, subtraction, comparison of fractions, equivalent fractions and their representation on the number line, etc. In this chapter, we will learn multiplication and division of fractions. Before we proceed further, let's recall what we have learnt earlier.

Fraction

A **fraction** is a number which can be written in the form $\frac{a}{b}$, where a and b are natural numbers. The number a is known as the numerator and b is called the denominator.

Proper fractions

If the numerator of a fraction is less than the denominator, then it is called a **proper fraction**. $\frac{1}{2}, \frac{2}{5}, \frac{6}{7}, \frac{3}{13}$, etc. are examples of proper fractions.

Improper fractions

If the numerator of a fraction is greater than or equal to the denominator, then it is called an **improper fraction**. In other words, a fraction which is not proper is called an improper fraction. $\frac{7}{4}, \frac{11}{6}, \frac{5}{5}, \frac{15}{8}$, etc. are examples of improper fractions.

Mixed fractions

Mixed fractions are a combination of a whole number and a proper fraction. For example, $7\frac{1}{2}, 5\frac{2}{3}, 3\frac{3}{4}$, etc. In fact $7\frac{1}{2}$ is equal to the sum of 7 and $\frac{1}{2}$.

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Like fractions

The fractions having same denominators are called like fractions. $\frac{1}{7}, \frac{2}{7}, \frac{5}{7}$, etc. are examples of **like fractions.**

Unlike fractions

The fractions having different denominators are called unlike fractions. $\frac{3}{7}, \frac{3}{5}, \frac{3}{11}$, etc. are examples of unlike fractions.

Unit fractions

The fractions having 1 as numerator are called unit fractions. $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}$, etc. are examples of unit fractions.

Example 1: Solve:

(a) $3 + \frac{7}{8}$ (b) $\frac{5}{13} - \frac{3}{8}$ (c) $\frac{9}{10} + \frac{2}{5} + \frac{5}{2}$ (d) $7\frac{1}{2} - 3\frac{5}{8}$

Solution: (a) $3 + \frac{7}{8} = \frac{24}{8} + \frac{7}{8} = \frac{24+7}{8} = \frac{31}{8} = 3\frac{7}{8}$

(b) $\frac{5}{13} - \frac{3}{8} = \frac{40}{104} - \frac{39}{104} = \frac{40-39}{104} = \frac{1}{104}$

(c) $\frac{9}{10} + \frac{2}{5} + \frac{5}{2} = \frac{9}{10} + \frac{4}{10} + \frac{25}{10} = \frac{9+4+25}{10} = \frac{38}{10} = 3\frac{8}{10} = 3\frac{4}{5}$

(d) $7\frac{1}{2} - 3\frac{5}{8} = \frac{15}{2} - \frac{29}{8} = \frac{60}{8} - \frac{29}{8} = \frac{60-29}{8} = \frac{31}{8} = 3\frac{7}{8}$

Example 2: Arrange $\frac{2}{9}, \frac{2}{3}, \frac{9}{21}$ in descending order.

Solution: $\frac{2}{9}, \frac{2}{3}, \frac{9}{21}$

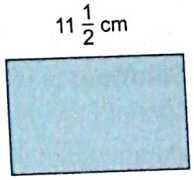
First convert the given fractions into like fractions. For this we need to find out the LCM of the denominators. LCM of 9, 3 and 21 = 3 × 3 × 7 = 63

3	9, 3, 21
	3, 1, 7

$\therefore \frac{2}{9} = \frac{14}{63}, \frac{2}{3} = \frac{42}{63}$ and $\frac{9}{21} = \frac{27}{63}$

$\because 42 > 27 > 14$

$\therefore \frac{42}{63} > \frac{27}{63} > \frac{14}{63} \Rightarrow \frac{2}{3} > \frac{9}{21} > \frac{2}{9}$



Example 3: A rectangular sheet of paper is $11\frac{1}{2}$ cm long and $10\frac{2}{3}$ cm wide. Find its perimeter.

Solution: Perimeter of rectangular sheet $= 2(l + b)$

$$= 2\left(11\frac{1}{2} + 10\frac{2}{3}\right) \text{cm} = 2\left(\frac{23}{2} + \frac{32}{3}\right) \text{cm}$$

$$= 2\left(\frac{69+64}{6}\right) \text{cm} = \frac{133}{3} \text{cm} = 44\frac{1}{3} \text{cm}$$

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Example 4: Vani wants to put a picture in a frame. The picture is $5\frac{3}{4}$ cm wide. To fit in the frame the picture cannot be more than $5\frac{1}{2}$ cm wide. How much should the picture be trimmed?

Solution: Width of the picture $= 5\frac{3}{4}$ cm $= \frac{23}{4}$ cm

Width of the frame $= 5\frac{1}{2}$ cm $= \frac{11}{2}$ cm

Width of the picture to be trimmed $= \left(\frac{23}{4} - \frac{11}{2}\right)$ cm

$= \left(\frac{23}{4} - \frac{22}{4}\right)$ cm $= \frac{1}{4}$ cm

Example 5: Ritu ate $\frac{3}{5}$ part of an apple and the remaining apple was eaten by her brother Somu. How much part of the apple did Somu eat? Who had the larger share? By how much? *(NCERT)*

Solution: Let the apple be considered as 1.

Part of the apple Ritu ate $= \frac{3}{5}$

$\therefore$ Somu ate $= 1 - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$ apple

$\because 3 > 2 \qquad \therefore \frac{3}{5} > \frac{2}{5}$

Hence, Ritu had a larger share.

By how much:

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

i.e., Ritu ate $\frac{1}{5}$ part more than Somu.

Extension

Decimal fractions are those in which the denominator is some power of 10, e.g., $\frac{3}{10}$ or $\frac{15}{100}$.

Example 6: Mr John, a school principal, drives from his residence to school. It takes him about 40 minutes to travel a distance of 30 km everyday. One day after travelling 20 km, his car encountered a snag (fault) which took 10 minutes to repair.

(a) What fraction of the total distance had Mr John travelled before his car encountered a fault?

(b) What fraction of total time was spent in repairing the car?

Solution: (a) Fraction of distance travelled before the fault is detected $= \frac{20}{30} = \frac{2}{3}$

(b) Fraction of the total time spent to repair the car $= \frac{10}{50} = \frac{1}{5}$

EXERCISE 2.1

1. Compare the following:

(a) $\frac{3}{4}$ and $\frac{4}{5}$ (b) $\frac{2}{9}$ and $\frac{2}{3}$ (c) $\frac{9}{10}$ and $\frac{11}{15}$ (d) $\frac{3}{5}$ and $\frac{5}{7}$

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2. Arrange in ascending order: (a) $\frac{1}{3}, \frac{2}{3}, \frac{5}{6}, \frac{7}{9}$ (b) $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{3}{4}$

3. Arrange in descending order: (a) $\frac{7}{9}, \frac{5}{6}, \frac{13}{18}, \frac{11}{15}$ (b) $\frac{3}{5}, \frac{9}{10}, \frac{7}{15}, \frac{17}{20}$

4. Find the sum of the following:

   (a) $3 + \frac{4}{5}$ (b) $\frac{3}{5} + \frac{2}{15} + \frac{7}{10}$ (c) $\frac{5}{6} + \frac{3}{4} + 4$ (d) $1\frac{3}{5} + 2\frac{7}{10} + 3\frac{4}{15}$

5. Find the difference of the following:

   (a) $3 - \frac{6}{7}$ (b) $5\frac{3}{10} - 2\frac{2}{15}$ (c) $3\frac{1}{5} - \frac{7}{15}$ (d) $\frac{8}{3} - \frac{15}{7}$

6. What should be added to $6\frac{3}{4}$ to get 9?

7. What should be subtracted from 10 to get $3\frac{3}{4}$?

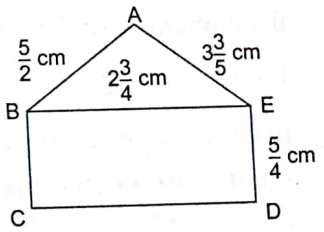
8. Jack and Jill took $10\frac{5}{11}$ minutes and $13\frac{3}{7}$ minutes respectively to complete an assignment. Who completed the assignment faster? By how much time?

9. Amit covered $7\frac{5}{12}$ km in 2 hours on bicycle and $4\frac{3}{8}$ km in another 1 hour on foot. Find the total distance covered by Amit in 3 hours.

10. The length and breadth of a rectangular room is $7\frac{1}{2}$ m and $5\frac{1}{6}$ m. Find the perimeter of the room.

11. On a TV channel, a popular film lasted for $3\frac{3}{4}$ hours. Out of this time the commercial advertisements were for $1\frac{1}{4}$ hour. What was the actual duration of the film?

12. Find the perimeter of (a) Δ ABE, (b) rectangle BCDE in the given figure. Whose perimeter is greater?



Multiplication of Fractions

We have learnt how to add or subtract fractions, but in many cases we need to multiply fractions. Consider the case of Sharda, a fruit vendor who bought 16 boxes each containing 20 apples from the wholesale fruit market. After reaching home, she found that the fruit merchant has cheated her, as in each box $\frac{1}{4}$ of the apples were rotten. How many apples were rotten? How many apples were in good condition?

Mr Sharma bought $\frac{4}{5}$ kg of sweets for the guests expected next morning. By the same evening he found that his son and daughter had $\frac{1}{2}$ of the sweets. How much sweet was left for the guests?

To get answers of all such problems we need to learn multiplication of fractions.

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Case I: Fraction multiplied by a whole number

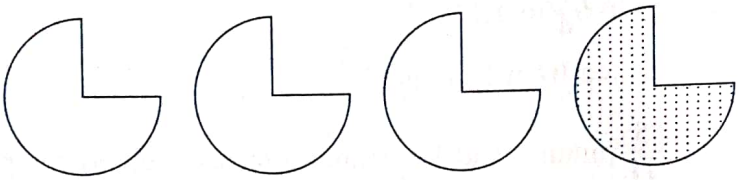
To understand this, let's perform the following activity:

Maths Lab Activity

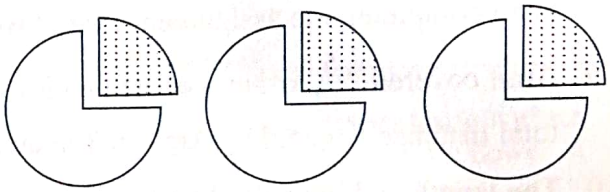
Multiplication of a Fraction by a Whole Number

Let's multiply $\frac{3}{4}$ by 4.

1. Represent $\frac{3}{4}$ as $\frac{3}{4}$th of a circle.
2. Now represent $\frac{3}{4} \times 4 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$



3. Re-arranging the above figures, we get



Hence, $\frac{3}{4} \times 4 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 3$

Thus, $\frac{3}{4} \times 4 = 3$

Multiplication Rules

- There is a simple way to multiply a fraction by a whole number. You just need to multiply the numerator of the fraction by the whole number and the denominator remains unchanged. For example, $\frac{2}{5} \times 3 = \frac{2 \times 3}{5} = \frac{6}{5}$
- If we have to multiply a mixed fraction by a whole number, we first convert the mixed fraction to an improper fraction and then proceed as above. For example, $3\frac{1}{4} \times 2 = \frac{13}{4} \times 2 = \frac{13}{2}$
- Likewise two mixed fractions can be multiplied by converting them into improper fractions. For example, $1\frac{1}{2} \times 2\frac{3}{4} = \frac{3}{2} \times \frac{11}{4} = \frac{33}{8}$
- To find the fraction of a fraction, we proceed as follows:

  $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, i.e., replace 'of' by '×' sign and proceed as above.
- The multiplicative inverse or reciprocal of a non-zero fraction may be obtained by interchanging its numerator and denominator. For example, multiplicative inverse of $\frac{5}{4}$ is $\frac{4}{5}$.

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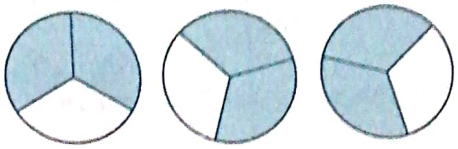
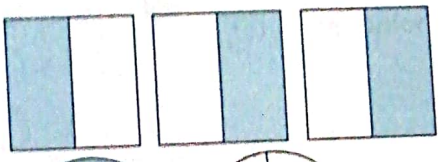
Example 7: Match the following product with the given figures:

(a) $2 \times \frac{3}{5}$ (b) $3 \times \frac{1}{2}$ (c) $3 \times \frac{2}{3}$ (d) $2 \times \frac{1}{4}$

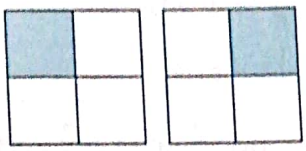
(i)



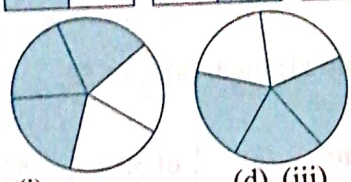
(ii)



(iii)



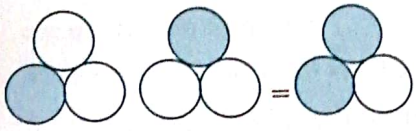
(iv)



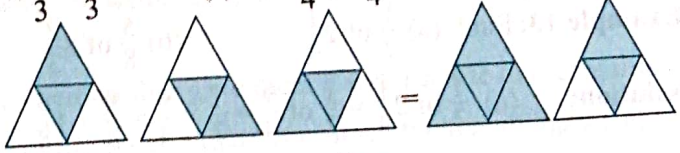
Solution: (a) (iv) (b) (ii) (c) (i) (d) (iii)

Example 8: Match the given figures with the following equalities involving product of a fraction and a whole number.

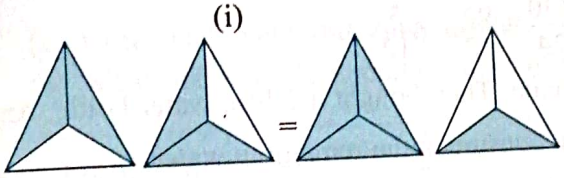
(a) $3 \times \frac{1}{5} = \frac{3}{5}$ (b) $2 \times \frac{1}{3} = \frac{2}{3}$ (c) $3 \times \frac{2}{4} = 1\frac{2}{4}$ (d) $2 \times \frac{2}{3} = 1\frac{1}{3}$



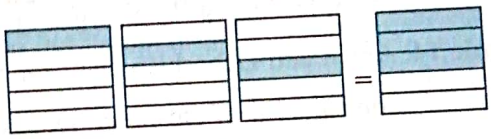
(i)



(ii)



(iii)



(iv)

Solution: (a) (iv) (b) (i) (c) (ii) (d) (iii)

Example 9: Multiply and reduce to lowest form:

(a) $4 \times \frac{2}{3}$ (b) $\frac{2}{3} \times 5$ (c) $\frac{5}{2} \times 6$ (d) $30 \times \frac{2}{5}$

Solution:

(a) $4 \times \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$

(b) $\frac{2}{3} \times 5 = \frac{10}{3} = 3\frac{1}{3}$

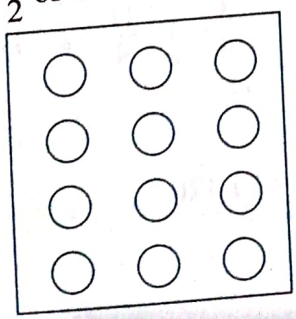
(c) $\frac{5}{2} \times 6 = \frac{30}{2} = 15$

(d) $30 \times \frac{2}{5} = \frac{60}{5} = 12$

Example 10: Shade:

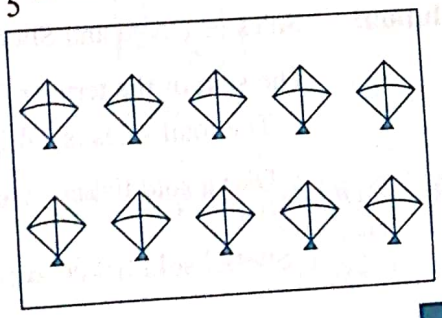
(a) $\frac{1}{2}$ of the circles in (i)

(i)



(b) $\frac{4}{5}$ of the kites in (ii)

(ii)



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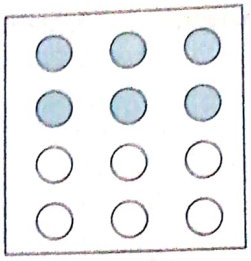
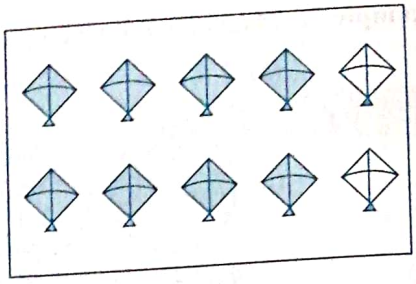
Solution: (a)



(b)



Example 11: Find: (a) $\frac{1}{2}$ of 52 (b) $\frac{2}{3}$ of 27

Solution: (a) $\frac{1}{2}$ of $52 = \frac{1}{2} \times 52 = \frac{52}{2} = 26$ (b) $\frac{2}{3}$ of $27 = \frac{2}{3} \times 27 = \frac{54}{3} = 18$

Example 12: Multiply and express as a mixed fraction: (a) $4 \times 5\frac{1}{5}$ (b) $5 \times 6\frac{1}{4}$

Solution: (a) $4 \times 5\frac{1}{5} = 4 \times \frac{26}{5} = \frac{104}{5} = 20\frac{4}{5}$ (b) $5 \times 6\frac{1}{4} = 5 \times \frac{25}{4} = \frac{125}{4} = 31\frac{1}{4}$

Example 13: Find: (a) $\frac{1}{2}$ of $2\frac{1}{4}$ (b) $\frac{5}{8}$ of $8\frac{2}{3}$

Solution: (a) $\frac{1}{2}$ of $2\frac{1}{4} = \frac{1}{2}$ of $\frac{9}{4} = \frac{1}{2} \times \frac{9}{4} = \frac{9}{8} = 1\frac{1}{8}$

(b) $\frac{5}{8}$ of $8\frac{2}{3} = \frac{5}{8}$ of $\frac{26}{3} = \frac{5}{8} \times \frac{26}{3} = \frac{130}{24} = \frac{65}{12} = 5\frac{5}{12}$

Example 14: Anjan and Debashish went for a picnic. They bought a 2 litre water bottle. Anjan consumed $\frac{3}{5}$ of the water. Debashish consumed the remaining water.

(a) How much water did Anjan drink?

(b) What fraction of the total quantity of water did Debashish drink?

Solution: (a) Anjan consumed $\frac{3}{5}$ of 2 L $= \frac{3}{5} \times 2$ L $= \frac{6}{5}$ L or $1\frac{1}{5}$ L

(b) Fraction of water consumed by Debashish $= 1 - \frac{3}{5}$

$= \frac{5-3}{5} = \frac{2}{5}$

Example 15: Divya and Shefali sold raffle tickets for Diwali mela worth ₹ 4,200. If Shefali sold $2\frac{3}{4}$ times of what Divya sold, how much did each sell?

Solution: Sales of Divya and Shefali are in the ratio $1 : 2\frac{3}{4} = 1 : \frac{11}{4} = 4 : 11$

The sum of the terms of the ratios = 4 + 11 = 15

∵ The total sales is ₹ 4,200

∴ Divya sold tickets worth $= \frac{4}{15} \times$ ₹ 4,200 = ₹ 1,120

Shefali sold tickets worth $= \frac{11}{15} \times$ ₹ 4,200 = ₹ 3,080

EXERCISE 2.2

1. Represent d
   (a) $2 \times \frac{2}{3} =$
2. Multiply ar
   (a) $2 \times \frac{3}{4}$
   (e) $12 \times \frac{2}{2}$
3. Find:
   (a) $\frac{1}{4}$ of
   (b) $\frac{5}{6}$ of
4. Vineeta
   complete
5. Vidya ar
   5 litres
   (a) How
   (b) Wh

Case II: Fract

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EXERCISE 2.2

1. Represent diagrammatically:

   (a) $2 \times \frac{2}{3} = 1\frac{1}{3}$ (b) $3 \times \frac{1}{6} = \frac{3}{6}$ (c) $4 \times \frac{1}{8} = \frac{4}{8}$ (d) $4 \times \frac{1}{3} = 1\frac{1}{3}$

2. Multiply and write the following in the simplest form:

   (a) $2 \times \frac{3}{4}$ (b) $12 \times \frac{3}{5}$ (c) $\frac{3}{4} \times 8$ (d) $6 \times \frac{1}{3}$

   (e) $12 \times \frac{2}{4}$ (f) $4 \times 5\frac{1}{2}$ (g) $14 \times 3\frac{1}{7}$ (h) $2\frac{3}{14} \times 21$

3. Find:

   (a) $\frac{1}{4}$ of (i) 24 (ii) 28 (iii) $18\frac{3}{4}$ (iv) $9\frac{1}{2}$

   (b) $\frac{5}{6}$ of (i) 20 (ii) 24 (iii) $36\frac{1}{6}$ (iv) $42\frac{5}{6}$

4. Vineeta completed $\frac{5}{6}$ of her homework on Saturday. How much homework is left to be completed on Sunday?

5. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

   (a) How much water did Vidya drink?

   (b) What fraction of the total quantity of water did Pratap drink?

Case II: Fraction multiplied by a fraction

To understand multiplication of two fractions, we perform the following activity:

Maths Lab Activity

Fraction Multiplied by a Fraction

Let's find the product of two fractions, i.e., $\frac{1}{2} \times \frac{4}{9}$.

We first represent $\frac{4}{9}$ by shaded region as shown in Fig. 1.

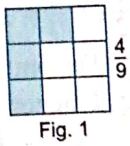

Fig. 1

Now $\frac{1}{2}$ of $\frac{4}{9}$ may be represented by half of the shaded region as shown in Fig. 2.

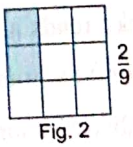

Fig. 2

Therefore, $\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$.

Evaluate: $\frac{1}{3} \times \frac{9}{16}$

The product of two fractions is obtained by simply multiplying the numerator with numerator and the denominator with denominator, i.e., $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

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Value of the Product

- The product of two proper fractions is always smaller than each of the fractions.
  For example, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$, which is smaller than each of the proper fractions, $\frac{3}{4}$ and $\frac{1}{2}$.
- The product of two improper fractions is always greater than each of the fractions.
  For example, $\frac{3}{2} \times \frac{4}{3} = 2$, which is greater than each of the improper fractions, $\frac{3}{2}$ and $\frac{4}{3}$.

Example 16: Find: (a) $\frac{1}{3}$ of $\frac{3}{5}$ (b) $\frac{1}{7}$ of $\frac{4}{9}$

Solution: (a) $\frac{1}{3}$ of $\frac{3}{5} = \frac{1}{3} \times \frac{3}{5} = \frac{3}{15} = \frac{1}{5}$ (b) $\frac{1}{7}$ of $\frac{4}{9} = \frac{1}{7} \times \frac{4}{9} = \frac{4}{63}$

Example 17: Multiply and reduce each of the following to the lowest terms. In case the result obtained is improper, convert it into mixed fractions.

(a) $\frac{2}{5} \times \frac{5}{9}$ (b) $\frac{11}{2} \times \frac{4}{10}$ (c) $\frac{3}{5} \times 5\frac{1}{4}$

Solution: (a) $\frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$

(b) $\frac{11}{2} \times \frac{4}{10} = \frac{44}{20} = 2\frac{1}{5}$

(c) $\frac{3}{5} \times 5\frac{1}{4} = \frac{3}{5} \times \frac{21}{4} = \frac{63}{20} = 3\frac{3}{20}$

Example 18: Which is greater: (a) $\frac{2}{7}$ of $\frac{2}{3}$ or $\frac{4}{5}$ of $\frac{2}{6}$ (b) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

Solution: (a) $\frac{2}{7}$ of $\frac{2}{3} = \frac{2}{7} \times \frac{2}{3} = \frac{4}{21}$; $\frac{4}{5}$ of $\frac{2}{6} = \frac{4}{5} \times \frac{2}{6} = \frac{8}{30} = \frac{4}{15}$

$\because$ Numerators of both the fractions are same.

$\therefore$ Fraction with smaller denominator would be greater.

i.e., $\frac{4}{15} > \frac{4}{21} \Rightarrow \frac{4}{5}$ of $\frac{2}{6} > \frac{2}{7}$ of $\frac{2}{3}$

(b) $\frac{1}{2}$ of $\frac{6}{7} = \frac{1}{2} \times \frac{6}{7} = \frac{6}{14} = \frac{3}{7}$; $\frac{2}{3}$ of $\frac{3}{7} = \frac{2}{5} \times \frac{3}{7} = \frac{6}{21} = \frac{2}{7}$

$\because$ Denominators of both the fractions are same.

$\therefore$ Fraction with greater numerator would be greater.

i.e., $\frac{3}{7} > \frac{2}{7} \Rightarrow \frac{1}{2}$ of $\frac{6}{7} > \frac{2}{5}$ of $\frac{3}{7}$

Example 19: Dipika reads a book for $1\frac{1}{4}$ hours every day. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

Solution: Number of hours Dipika spent every day = $1\frac{1}{4}$

$\because$ She reads the entire book in 6 days,

$\therefore$ Total number of hours required to read the book = $6 \times 1\frac{1}{4}$

$= 6 \times \frac{5}{4} = \frac{30}{4} = \frac{15}{2} = 7\frac{1}{2}$ hours

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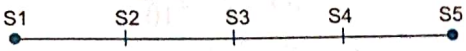
Example 20: Anny plants 5 saplings in a row in her garden. The distance between two adjacent saplings is $\frac{3}{5}$ m. Find the distance between the first and the last sapling.

Solution:



Let the saplings be planted at S1, S2, S3, S4 and S5.

Distance between two adjacent saplings $= \frac{3}{5}$ m

$\therefore$ Distance between S1 and S5 $= 4 \times \frac{3}{5} = \frac{12}{5} = 2\frac{2}{5}$ m

Example 21: A bike runs 64 km using 1 litre of petrol. How much distance will it cover using $3\frac{1}{4}$ litres of petrol?

Solution: Distance covered in 1 litre of petrol = 64 km

$\therefore$ Distance covered in $3\frac{1}{4}$ litres of petrol $= 3\frac{1}{4} \times 64 = \frac{13}{4} \times 64$

$= 13 \times 16 = 208$ km

Example 22: Write the fraction in the box, such that $\frac{8}{25} \times \boxed{} = \frac{16}{100}$. Reduce it in simplest form.

Solution: We know $8 \times 2 = 16$

$\therefore$ The numerator is 2.

Also, $25 \times 4 = 100$

$\therefore$ The denominator is 4.

Hence, the fraction in the box is $\boxed{\frac{2}{4}}$.

The simplest form of $\frac{2}{4}$ is $\frac{1}{2}$.

Example 23: Out of a group of 160 students, $\frac{2}{5}$ opted for Science stream. $\frac{3}{4}$ of the Science students chose to study Mathematics. How many Science students want to study Mathematics?

Solution: Total number of students = 160

Students opted for Science stream $= \frac{2}{5} \times 160 = 64$

Science students who want to study Mathematics $= \frac{3}{4} \times 64 = 48$

Thus, 48 Science students are interested in studying Mathematics.

EXERCISE 2.3

1. Multiply and reduce to the lowest form:

   (a) $\frac{4}{5}$ and $\frac{2}{7}$ (b) $2\frac{1}{4}$ and $1\frac{3}{4}$ (c) $4\frac{3}{4}$ and $3\frac{1}{2}$ (d) $\frac{11}{3}$ and $\frac{5}{6}$

2. Insert appropriate inequality sign, i.e., > or < in the given boxes.

   (a) $\frac{3}{4}$ of $\frac{6}{7}$ $\boxed{}$ $\frac{1}{2}$ of $\frac{15}{16}$ (b) $\frac{1}{4}$ of $\frac{20}{21}$ $\boxed{}$ $\frac{1}{2}$ of $\frac{3}{4}$

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3. Multiply and write the following in their simplest form:

(a) $\frac{8}{11} \times \frac{33}{64}$ (b) $\frac{11}{7} \times \frac{49}{55}$ (c) $7\frac{1}{4} \times 2\frac{3}{10}$ (d) $3\frac{4}{5} \times 4\frac{3}{4}$

4. Find the cost of $6\frac{3}{4}$ litres of milk if the cost of milk is ₹ $21\frac{4}{5}$ per litre.

5. Find the area of a rectangular plot with length $16\frac{1}{2}$ m and breadth $10\frac{1}{3}$ m.

6. In a class of 36 students, the number of boys is $\frac{2}{3}$ of the total strength. How many girls are there in the class?

7. In a basket having 144 oranges, $\frac{1}{6}$ of the total oranges are rotten. How many of them are in good condition?

8. To stitch a dress, $4\frac{1}{2}$ m cloth is required. If the cost of cloth is ₹ $77\frac{5}{6}$ per metre, find the cost of the cloth required for the dress.

Division of Fractions

We are well versed with the mathematical operation of division. Division of fractions also has the same meaning.

For example, a rod of $\frac{5}{2}$ m is cut into $\frac{1}{2}$ m long pieces. The number of pieces obtained would be

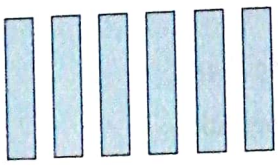
$$\frac{5}{2} \div \frac{1}{2} = \left(\frac{5}{2}\right) \times \left(\text{Reciprocal of } \frac{1}{2}\right) = \frac{5}{2} \times \frac{2}{1} = 5$$

Therefore, the number of such pieces would be 5.

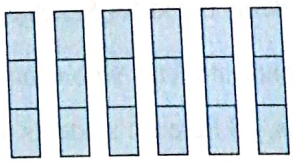
Case I: Division of a whole number by a fraction

Let's evaluate $6 \div \frac{1}{3}$, i.e., we need to divide 6 wholes into one-thirds.

We represent 6 wholes as 6 strips.



Now all the strips are divided into one-thirds.



Therefore, $6 \div \frac{1}{3} = 6 \times \left(\text{Reciprocal of } \frac{1}{3}\right) = 6 \times \frac{3}{1} = 18.$

Case II: Division of a fraction by a whole number

Let's divide $\frac{4}{5}$ by 2.

$$\frac{4}{5} \div 2 = \frac{4}{5} \times (\text{Reciprocal of } 2) = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10} = \frac{2}{5}$$

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Case III: Division of a fraction by another fraction

Let's divide $\frac{3}{8}$ by $\frac{1}{4}$.

$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \left(\text{Reciprocal of } \frac{1}{4}\right) = \frac{3}{8} \times \frac{4}{1} = \frac{12}{8} = \frac{3}{2}$

Division Rules

- To divide a whole number by a fraction, multiply the whole number by the reciprocal of the fraction.
- To divide a fraction by a non-zero whole number, multiply the fraction by the reciprocal of the whole number.
- To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the fraction by which it is to be divided.

Note: In division if mixed fractions are involved, we first convert the mixed fraction into improper fraction and then proceed as explained above.

Example 24: Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

(a) $\frac{3}{5}$ (b) $\frac{5}{9}$ (c) $\frac{13}{11}$ (d) $\frac{1}{6}$

Solution: (a) $\frac{5}{3}$ (improper) (b) $\frac{9}{5}$ (improper) (c) $\frac{11}{13}$ (proper) (d) 6 (whole number)

Example 25: Find:

(a) $9 \div \frac{3}{5}$ (b) $10 \div 3\frac{4}{7}$ (c) $\frac{6}{11} \div 7$ (d) $5\frac{1}{2} \div 4$

Solution: (a) $9 \div \frac{3}{5} = 9 \times \frac{5}{3} = 15$

(b) $10 \div 3\frac{4}{7} = 10 \div \frac{25}{7} = 10 \times \frac{7}{25} = \frac{14}{5}$

(c) $\frac{6}{11} \div 7 = \frac{6}{11} \times \frac{1}{7} = \frac{6}{77}$

(d) $5\frac{1}{2} \div 4 = \frac{11}{2} \div 4 = \frac{11}{2} \times \frac{1}{4} = \frac{11}{8}$

Example 26: Find:

(a) $\frac{3}{4} \div \frac{1}{2}$ (b) $3\frac{1}{5} \div 1\frac{1}{5}$

Solution: (a) $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$

(b) $3\frac{1}{5} \div 1\frac{1}{5} = \frac{16}{5} \div \frac{6}{5} = \frac{16}{5} \times \frac{5}{6} = \frac{8}{3}$

Order of Operations

The rules that we follow to decide the order of operation with integers are also true in calculations involving fractions.

- We first solve the expressions inside the brackets. If there are brackets inside other brackets, do calculations in the innermost bracket first and then proceed.
- If the calculation contains any combination of addition and/or subtraction together with division and/or multiplication, then we first do all the divisions and/or multiplications before the additions and/or subtractions OR one can simply follow the order given in BODMAS.

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EXERCISE 2.4

1. Find the reciprocals of : (a) $\frac{7}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{7}$ (d) $\frac{7}{3}$

2. Find: (a) $18 \div \frac{3}{4}$ (b) $14 \div \frac{7}{5}$ (c) $6 \div 2\frac{1}{3}$ (d) $25 \div 2\frac{1}{7}$

3. Divide: (a) $3\frac{1}{4}$ by $\frac{2}{3}$ (b) $\frac{7}{8}$ by $4\frac{1}{2}$ (c) $3\frac{7}{15}$ by $2\frac{3}{5}$ (d) 100 by $\frac{10}{3}$

4. A wire of length $6\frac{1}{2}$ m is cut into 13 pieces of equal length. Find the length of each piece.

5. By what number should $4\frac{3}{4}$ be multiplied to get $6\frac{1}{4}$?

6. The cost of $4\frac{3}{4}$ kg apples is ₹ 300. What is the rate of apples per kg?

7. A bucket contains 18 litres of water. How many $\frac{3}{5}$ litres jugs can be filled from the bucket?

8. In a hostel's mess, each student gets $\frac{1}{3}$ litre of milk everyday. If the total consumption of milk is $87\frac{2}{3}$ litres per day, how many students are there in the hostel?

9. Use the correct order of operations to calculate:

   (a) $2 + \frac{1}{2} \div \frac{1}{3} - \frac{1}{5}$ (b) $\frac{1}{3} \div \frac{1}{2} + \frac{1}{4}$

   (c) $\left(3 + \frac{1}{2}\right) \div \left(2 - \frac{1}{2}\right)$ (d) $5\frac{1}{2} \times \left(4\frac{1}{2} - \frac{7}{2}\right) \div 2\frac{1}{3}$

Reason It Out!

Write down the next three numbers after recognizing the pattern in each series.

(a) $2\frac{3}{8}, 2, 1\frac{5}{8}, 1\frac{1}{4}$, _____, _____, _____

(b) $9, 4\frac{1}{2}, 2\frac{1}{4}, 1\frac{1}{8}$, _____, _____, _____

(c) $\frac{7}{64}, \frac{7}{32}, \frac{7}{16}, \frac{7}{8}$, _____, _____, _____

(d) $100, 20, 4, \frac{4}{5}$, _____, _____, _____

(e) $\frac{11}{13}, \frac{5}{6}, \frac{9}{11}, \frac{4}{5}, \frac{7}{9}$, _____, _____, _____

[**Hint:** (a) *Subtract* $\frac{3}{8}$ *from each.* (b) *Multiply each by* $\frac{1}{2}$.
(c) *Multiply each by* 2. (d) *Divide each by* 5.
(e) *Subtract* 1 *from the numerator as well as the denominator.*]

Grid Fun

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Grid Fun

Find ten words and categorize them as a term connected to Addition, Multiplication, Division and Measurement of Distance or Time.

S	M	U	L	T	I	P	L	Y
D	I	N	C	H	O	F	T	U
I	N	S	F	D	I	A	O	R
Q	U	O	T	I	E	N	T	T
O	T	X	L	V	Y	E	A	R
M	E	Z	V	I	O	N	L	X
E	A	P	Q	S	U	M	A	N
T	A	G	R	I	R	C	E	P
R	T	P	R	O	D	U	C	T
E	P	R	Z	N	L	O	N	R

At a Glance

1. A **fraction** is a number which can be written in the form $\frac{a}{b}$, where both a and b are natural numbers. The number a is called the numerator and b the denominator.
2. A **proper fraction** is one in which the numerator is smaller than the denominator.
3. An **improper fraction** is one in which the numerator is greater than or equal to the denominator.
4. **Mixed fraction** is a combination of a whole number and a proper fraction.
5. The numerator of the **product of fractions** is the product of their numerators, and the denominator of the product of fractions is the product of their denominators.
6. The product of two proper fractions is less than each of the fractions. The product of a proper and an improper fraction is always less than the improper fraction.
7. A **reciprocal or multiplicative inverse** of a non-zero fraction is obtained by interchanging its numerator and the denominator.
8. When a number or fraction is **divided** by another number or fraction, we multiply the first number or fraction by the multiplicative inverse or reciprocal of the second number or fraction.

Review Exercises

Multiple Choice Questions

1. If a mixed fraction is converted into fraction, then its reciprocal is:

   (a) proper (b) improper (c) equal to itself (d) none of these

2. The number of months in $\frac{3}{5}$th of a century is:

   (a) 60 (b) 600 (c) 720 (d) 7.3

3. 474 surgeons were invited for a conference on the latest surgical techniques and $\frac{5}{6}$ of those invited actually participated in it. The number of participants was:

   (a) 359 (b) 395 (c) 474 (d) 568

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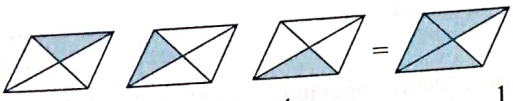
4. The figure  represents:

(a) $3 \times \frac{2}{3} = 2$ (b) $4 \times \frac{1}{3} = \frac{4}{3}$ (c) $3 \times \frac{1}{4} = \frac{3}{4}$ (d) $3 \times \frac{5}{12} = 1\frac{1}{4}$

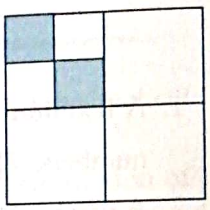
5. John can read a novel in 17 hours. If he devotes $2\frac{3}{7}$ hours to reading every day, he would finish the novel in:

(a) 5 days (b) 7 days (c) 9 days (d) 10 days

6. Smith bought a laptop from abroad in a special promotional drive for £ 45 which is $\frac{5}{8}$ of the original price. The original marked price of the laptop is:

(a) £ 72 (b) £ 28 (c) £ 90 (d) £ 112

7. The fraction representing the shaded part of the given square is:



(a) $\frac{2}{7}$ (b) $\frac{2}{4}$ (c) $\frac{2}{8}$ (d) $\frac{1}{8}$

Solve Mentally

True or False

1. The product of two proper fractions is greater than each of the fractions.
2. Product of $\frac{a}{b}$ and $\frac{c}{d}$ is $\frac{ad}{bc}$.
3. To divide one fraction by another, we first multiply the fractions and then take its reciprocal.
4. No proper fraction is equal to a mixed fraction.
5. The word '*of*' written between two fractions refers to their product.
6. The multiplicative inverse of a fraction is the same as its reciprocal.

Fill in the Blanks

1. The multiplicative inverse of $3\frac{1}{2}$ is _____.
2. The speed of a car is 30 km per hour. The distance covered by the car in $5\frac{1}{2}$ hours is _____.
3. The product of a non-zero fraction and its reciprocal is always _____.
4. If the price of 12 books is ₹ 189, then each book costs _____.
5. The number of pieces each measuring $3\frac{1}{4}$ metres that can be cut from a rope measuring $45\frac{1}{2}$ metres is _____.
6. There are _____ minutes in $\frac{3}{5}$th of 2 hours.
7. The value of the expression $2\frac{1}{2}\left(\frac{5}{4} - \frac{4}{5}\right)$ is _____.
8. A fraction equivalent to $\frac{1}{20}$ having denominator 60 is _____.

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Answer in One Word or a Line

1. What are like fractions?
2. If the product of two fractions is 1, what can be said about the fractions?
3. What is the reciprocal of 1?
4. What is the difference between improper and mixed fractions? Can they be converted into each other? Give an example.
5. What can be said about the value of the product of a proper and an improper fraction?
6. What is multiplicative inverse of a fraction? For which number it does not exist?
7. How many millimetres are there in $\frac{1}{5}$ of a metre?

LET'S EVALUATE

1. Simplify: (a) $2\frac{4}{15} + 4\frac{7}{10} + 2\frac{3}{5}$ (b) $7\frac{5}{6} - 4\frac{3}{8} + 2\frac{7}{12}$
2. Arrange $\frac{6}{12}, \frac{5}{12}, \frac{2}{3}, \frac{3}{8}$ in ascending order.
3. Arrange $\frac{3}{7}, \frac{9}{35}, \frac{11}{14}, \frac{13}{28}$ in descending order.
4. Simplify: (a) $2\frac{2}{17} \times 7\frac{2}{9} \times 1\frac{33}{52}$ (b) $\left(4\frac{3}{7} \div 3\frac{1}{2}\right) \div 1\frac{1}{2}$
5. Mr Srinivasan spends $\frac{4}{7}$ of his income on household expenses, $\frac{1}{7}$ of his income on conveyance and ₹ 2,000 per month on insurance. If his monthly income is ₹ 42,000, find his annual savings.
6. If the cost of a notebook is ₹ $12\frac{3}{4}$, how many notebooks can be purchased for ₹ $191\frac{1}{4}$?
7. By what number should $4\frac{5}{6}$ be multiplied to get $9\frac{1}{6}$?
8. By what fraction should $5\frac{3}{7}$ be divided to get $\frac{4}{7}$?

THINKING SKILLS

1. A shelf is $24\frac{6}{7}$ cm long. How many books can be arranged on the shelf if each book is $4\frac{1}{7}$ cm wide? How many encyclopedias can fit on the shelf if each occupies $5\frac{2}{7}$ cm of width?
2. A cat drinks $1\frac{3}{4}$ litres of milk contained in a utensil having $2\frac{1}{4}$ litres of milk. What fraction of milk did the cat drink?
3. A soccer team is to be selected from a group of probables numbering more than 16 but less than 30 attending the training-cum-fitness camp. Amongst these probables $\frac{1}{3}$ are below the age of 16 years. $\frac{1}{4}$ are in the age group 16 to 18 years and the remaining are above 18 years.
   (a) How many probables attended the camp?
   (b) What fraction of probables are above 18 years?

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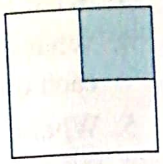
4. Gajender, a tailor has $37\frac{1}{2}$ m of material available to make baby suits. Each baby suit requires $\frac{5}{7}$ m of material.
   (a) How many baby suits can be made? (b) How much material will be left?
5. A farmer cultivated sugarcane in one fourth of his square field as shown. He wants to grow three different vegetables in equal areas in the remaining part of the field such that 2 parts are rectangular and one is square. Can you help him?



VALUES AND LIFE SKILLS

1. An NGO wanted to plant 6,000 plants as a part of their social service. All the members of the NGO participated and planted $\frac{7}{12}$ plants on a Sunday and the remaining on next Sunday. How many plants were planted on each Sunday by the NGO?
   Is planting trees and plants important for environment? Have you planted plants on the terrace or at the backyard of your house?
2. Rohan devotes 7 hours to his studies everyday. He devotes $2\frac{4}{5}$ hours to Mathematics, $1\frac{1}{4}$ hours to English and the remaining time to study other subjects. How much time does he spend on other subjects?
   What is the importance of time management for students?
3. Mr Rawat earns ₹ 24,500 per month. He spends $\frac{9}{10}$ of his income and deposits the remaining money in his bank. How much money does he save in a year?
   What is the importance of earning in life?
4. A sugar bag contains 30 kg of sugar. If the consumption of sugar in Mr Sharma's house is $\frac{2}{3}$ kg per day, for how many days will the sugar bag last?
   Is high consumption of sugar good for health? What are the consequences of consuming high sugar?

SOME NCERT TEXTBOOK QUESTIONS

1. In a "magic square", the sum of the numbers in each row, in each column and along the diagonals is the same. Is this a magic square?

$\frac{4}{11}$	$\frac{9}{11}$	$\frac{2}{11}$
$\frac{3}{11}$	$\frac{5}{11}$	$\frac{7}{11}$
$\frac{8}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

(Along the first row $\frac{4}{11} + \frac{9}{11} + \frac{2}{11} = \frac{15}{11}$).

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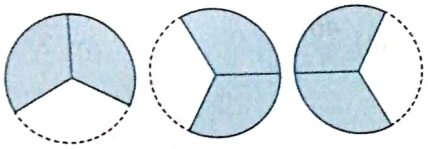
2. Salil wants to put a picture in a frame. The picture is $7\frac{3}{5}$ cm wide.

To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much should the picture be trimmed?

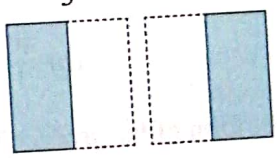
3. Which of the drawings (a) to (d) show:

(i) $2 \times \frac{1}{5}$ (ii) $2 \times \frac{1}{2}$ (iii) $3 \times \frac{2}{3}$ (iv) $3 \times \frac{1}{4}$

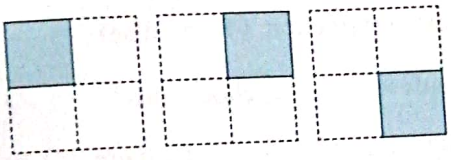
(a)



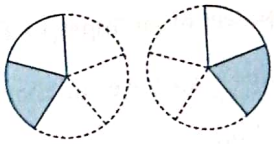
(b)



(c)

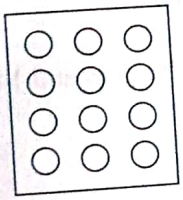


(d)

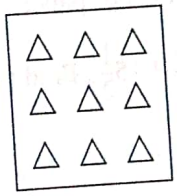


4. Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

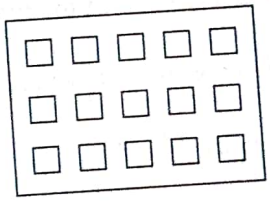
(iii) $\frac{3}{5}$ of the squares in box (c).



(a)



(b)



(c)

5. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

(i) How much water did Vidya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

6. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

8. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

(i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$

(v) $\frac{12}{7}$ (vi) $\frac{1}{8}$ (vii) $\frac{1}{11}$

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EXEMPLAR PROBLEMS

1. $\frac{3}{7}$ of $\frac{2}{5}$ is equal to:

   (a) $\frac{5}{15}$ (b) $\frac{5}{35}$ (c) $\frac{1}{35}$ (d) $\frac{6}{35}$

2. $2\frac{2}{3} \div 5$ is equal to:

   (a) $\frac{8}{15}$ (b) $\frac{40}{3}$ (c) $\frac{40}{5}$ (d) $\frac{8}{3}$

3. The lowest form of the product $2\frac{3}{7} \times \frac{7}{9}$ is _____ .

4. The reciprocal of an improper fraction is an improper fraction. (True/False)

5. How many $\frac{1}{16}$ kg boxes of chocolates can be made with $1\frac{1}{2}$ kg chocolates?

6. It takes 17 full specific type of trees to make one tonne of paper. If there are 221 such trees in a forest, then

   (i) What fraction of forest will be used to make:

   (a) 5 tonnes of paper (b) 10 tonnes of paper

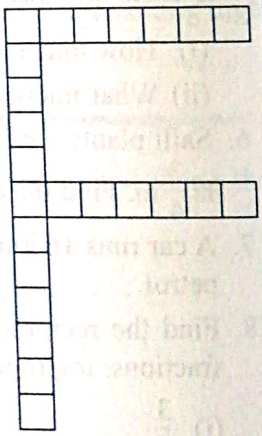
   (ii) To save $\frac{7}{13}$ part of the forest, how much of paper we have to save?

7. Cost of burger is ₹ $20\frac{3}{4}$ and of macpuff is ₹ $15\frac{1}{2}$. Find the cost of 4 burgers and 14 macpuffs.

8. Divide $\frac{3}{10}$ by $\left(\frac{1}{4} \text{ of } \frac{3}{5}\right)$.

Group Activity

Fun with Fractions (A Game based on Comparison of Fractions)



This activity-based game can be played either between two partners or in a small group consisting of upto 5 students.

- *Requirement:* Each participant should trace or draw F similar to the one given here.
- Cut thirteen identical squares on a thick paper or cardboard. Write one of these fractions on each square:

  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{2}{12}, \frac{4}{12}, \frac{5}{12}, \frac{1}{24}, \frac{3}{24}, \frac{6}{24}$

  Each of these squares with a fraction written on them is referred to as fraction cards.

Rules:

1. Keep all the fraction cards in a pile (one above the other) facing downwards.
2. Each person should pick a fraction card from the top of the pile.

3. Turn over
   many squa
   For exam
   getting th
   players g
4. Put all th
   and play
5. In case o
   fraction
   players
   top.
6. Continu
   game. T

[**Hint:** *For*
above wit
$\frac{6}{24} = \frac{6}{24}$.]

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3. Turn over the fraction cards and the player with the largest fraction must colour or cross as many squares in his/her traced copy of F that is equivalent to the fractional part of the whole F. For example, let's say amongst all players the highest value of the fraction is $\frac{3}{8}$. The player getting the fraction will colour or cross $\frac{3}{8}$ of the total squares in F, i.e., $\frac{3}{8} \times 24 = 9$. Other players getting lesser fraction will not cross or colour any squares.
4. Put all the fraction cards at the bottom of the pile and keep choosing new ones from the top and play as before.
5. In case of two or more players getting the same largest fraction or a player with the largest fraction not able to colour or cross the fractional part due to non-availability of squares, all players should put their cards back at the bottom of the pile and pick new cards from the top.
6. Continue playing till one of the players colours or crosses the whole F first and wins the game. The player to finish next comes second and so on.

[**Hint:** *For fast track game, prepare a table giving equivalents of all the fractions mentioned above with denominator* 24 *(like fractions), i.e.,* $\frac{1}{2} = \frac{12}{24}, \frac{1}{3} = \frac{8}{24}, \frac{1}{4} = \frac{6}{24}, \ldots, \frac{5}{12} = \frac{10}{24}, \frac{6}{24} = \frac{6}{24}$.]

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