## Appendix: Proofs

### Proof of Proposition 1

*Proof.* Our greedy algorithm is as follows. For the case when $F(\boldsymbol{x}) = 1$, start with $t = t_{\boldsymbol{x}}$, and iterate over the literals $\ell$ of $t$ by checking whether $t$ deprived of $\ell$ is a UP-implicant given $Th$ of at least $\lfloor \frac{m}{2} \rfloor + 1$ decision trees of $F$. If so, remove $\ell$ from $t$ and proceed to the next literal. Once all literals in $t_{\boldsymbol{x}}$ have been examined, the final term $t$ is by construction a UP-implicant given $Th$ of a majority of decision trees in $F$, such that removing any literal from it would lead to a term that is no longer a UP-implicant given $Th$ of this majority. So, $t$ is by construction a UP-majoritary reason. The case when $F(\boldsymbol{x}) = 0$ is similar, by simply replacing each $T_i$ by its negation (which can be obtained in linear time by replacing every 0-leaf in $T_i$ by a 1-leaf and vice-versa). This greedy algorithm runs in time polynomial in the size of the input $t_{\boldsymbol{x}}$, $F$ and $Th$ since on each iteration, checking whether $t$ is a UP-implicant given $Th$ of $T_i$ (for each $i \in [m]$) can be done in time polynomial in the size of $t$, $T_i$, and $Th$. Indeed, in order to decide whether $t$ is a UP-implicant given $Th$ of a decision tree $T_i$ of $F$, it is enough to test that for every clause $\delta \in \texttt{CNF}(T_i)$, $\delta$ contains a literal derivable by unit propagation from $t \wedge Th$. The fact that this set can be derived in time linear in the size of $t \wedge Th$ completes the proof.